Persistent Crises and Levered Asset Prices*

Lars-Alexander Kuehn
Tepper School of Business
Carnegie Mellon University

David Schreindorfer
W.P. Carey School of Business
Arizona State University

Florian Schulz
Michael G. Foster School of Business
University of Washington

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Abstract

We rationalize the joint behavior of aggregate consumption, asset prices, and financial leverage by incorporating persistent macroeconomic crises into a structural credit risk model. As in the data, longer-lasting crises are associated with more severe macroeconomic contractions and larger increases in leverage ratios, credit risk, and return volatility. Leverage provides a strong propagation mechanism for fundamental shocks because it continues to rise while crises endure. The model replicates the firm-level implied volatility curve and its cross-sectional relation with observable proxies of default risk. Lastly, a structural estimation reveals that common idiosyncratic risk is an important driver of credit spreads.

JEL Classification: G12, G13, G33

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Macroeconomic crises typically last for multiple years and are characterized by large drops in aggregate economic activity and asset values, and increases in financial leverage and risk. At the same time, measures of equity and default risk tend to comove strongly with leverage, as depicted in Figure 1 for the Great Depression and Great Recession.[1] These observations are important because the nature of potential crises is a crucial determinant of asset valuations even in normal times. Existing asset pricing models, however, cannot match the aforementioned facts jointly because they model crises as instantaneous events or abstract from financial leverage.

In this paper, we rationalize the joint behavior of macroeconomic aggregates, asset prices, and financial leverage by incorporating persistent macroeconomic crises into a structural credit risk model. Our model features a representative agent with recursive preferences and Markov-switching processes for aggregate consumption and firms' cash flows. Firms issue debt trading off tax benefits and bankruptcy costs, refinance when their interest coverage ratio is too high, and optimally default when their interest coverage ratio is too low.[2] The Markov chain is key for modeling crises as long-lasting: When the economy switches into the crisis state, consumption and earnings growth are persistently depressed and more volatile. Consequently, equity risk premia, option-implied volatilities and credit spreads rise. As the crisis continues to unfold, leverage continues to rise, further increasing risk premia and measures of equity and default risk. Persistent crises and optimal financial leverage thus provide a strong propagation mechanism for fundamental shocks, in line with the dynamics in Figure 1.

Building on this economic mechanism, we structurally estimate the model to quantify the impact of time-varying cash flow risk and bankruptcy losses on credit spreads. To identify these channels, the estimation relies on information in firm-level CDS rates and option prices. While credit spreads are sensitive to both default probabilities and loss rates to bond holders, equity option prices are only sensitive to default risk because bond holders' losses do not affect equity values ex post. At the same time, equity options are very informative about the level and cyclicality of conditional equity risk. For the estimation, we focus on firms in the S&P 100 index, excluding financials and utilities, for which we observe liquid option and CDS contracts for the period from 2004 to 2019. The estimation targets the first and second moments of

[1] The link between economic crises and financial fragility has long been recognized; see, e.g. Gertler (1988), Bernanke (1983), Bernanke, Gertler, and Gilchrist (1999), Bhamra, Fisher, and Kuehn (2011), Schularick and Taylor (2012), and Gilchrist and Zakrajsek (2012).

[2] Our structural credit risk model builds closely on Bhamra, Kuehn, and Strebulaev (2010), and Chen (2010). In contrast to them, we model lasting crises and structurally estimate the model. The literature review addresses additional details.

leverage, firm-level and aggregate excess returns, CDS rates, at-the-money implied volatility and the implied volatility skew, and matches all of these moments well.

Long-lasting crises enable the model to explain risk premia across equity, credit, and option markets. Specifically, the model resolves the credit spread puzzle. It generates low leverage and low physical default probabilities, but credit spreads are large because defaults cluster in crises where marginal utility is very high. When we instead calibrate consumption as in the long-run risks literature, the resulting model is inconsistent with observed credit spreads. The model also matches well the shape and time variation of the implied volatility curve. This fact is surprising because Backus, Chernov, and Martin (2011) show that the disaster model of Barro (2006) generates an unrealistically steep implied volatility curve. While Barro (2006) assumes that consumption drops on impact when a disaster occurs, our consumption process captures the positive correlation between disaster size and duration: Larger cumulative consumption drops tend to occur over longer periods. Even though consumption losses in our model are similar to those in Barro (2006), the fact that they tend to accumulate over many periods makes them considerably less painful to investors. As a result, our model is consistent with the implied volatility curve.

While crises are disastrous on average, some crises last shorter than expected and result in only small consumption losses. The model therefore allows us to ask whether the Great Recession, which led to a peak-to-trough drop in aggregate consumption of only 2.1%, can be attributed to a brief crisis. Such a characterization is clearly not within the scope of traditional disaster models, since Barro (2006) assumes that consumption drops by at least 10% during crises. Overall, asset prices during a brief crisis in the model look strikingly similar to those observed during the Great Recession. The onset of the crisis is characterized by a large increase in equity risk premia, leverage, implied volatility, and credit spreads. Importantly, due to the persistent nature of financial leverage, all these financial variables remain elevated for a prolonged period of time. Our model therefore provides a novel characterization of deep recessions as short-lived crises.[3] The model is also consistent with long-lasting crises such as the Great Depression. Compared to the short-lived crises, the additional increase in financial leverage during longer-lasting crises amplifies conditional asset pricing moments. As such, equity risk premia, credit spreads, and option-implied volatility continue to rise.

It is precisely the model's ability to fit derivative data that allows us to recover firms' deep

[3] Bianchi (2020) provides additional evidence in favor of this view. He identifies a Great Depression regime by using a Markov-switching VAR. The probability of this regime has remained close to zero for many decades, but spiked for a short period during the Great Recession.

economic parameters via a structural estimation. Specifically, the model allows us to identify five interesting economic parameters based on information in options and CDS data: the cyclicality of idiosyncratic and aggregate cash flow risk, the level and cyclicality of bankruptcy costs, as well as risk aversion of the representative investor.

In line with existing literature, we find that bankruptcy costs are sizable on average at 26%. While bankruptcy costs capture a variety of factors such as reputation costs, asset fire sales, loss of customer or supplier relationships, legal and accounting fees, and costs of changing management, loss rates on bonds additionally reflect reduced asset values that result from a history of adverse shocks. The corporate finance literature, including Jankowitsch, Nagler, and Subrahmanyam (2014), has provided evidence in favor of countercyclical loss rates on bonds, but we find that bankruptcy costs are strongly countercyclical as well. This finding is new to the literature. Gauging the time-varying nature of bankruptcy costs requires a structural model because empirical loss rates can be time varying, even when bankruptcy costs are constant. Because the representative investor is risk averse, a wedge arises between the expected bankruptcy costs under the physical and risk-neural measure. In particular, average risk-neutral bankruptcy costs amount to 36%, compared to 26% under the physical measure. Had we assumed a risk-neutral investor instead, as is common in corporate finance models, the model would have required bankruptcy costs of around 36% to replicate the low leverage ratios in the data. In contrast, our framework implies that observable bankruptcy costs are much smaller, while the model is still consistent with low leverage. Our exercise therefore highlights the importance of risk adjustments in structural corporate finance models.

How important are time-varying loss rates and bankruptcy costs for the valuation of CDS contracts? A structural decomposition reveals that time-variation in bankruptcy costs accounts for 2% of the level of credit spreads and 2% of the changes in spreads. Surprisingly, this effect is small even though the wedge between average physical and risk-neutral bankruptcy costs is large. The reason is that credit spreads are primarily driven by time variation in cash flow risks. Even when bankruptcy costs are constant, the model features state-dependent asset values and default thresholds, generating time-variation in loss rates for bond investors. When we further eliminate any time-variation in loss rates, we find that it accounts for 14% of the level of credit spreads and 13% of the changes in spreads.

Our estimation allows for time variation in both aggregate and common idiosyncratic risk. While the former is common in the asset pricing literature, the latter has only recently been identified as a robust feature of firms' cash flows by Herskovic, Kelly, Lustig, and Nieuwerburgh (2016). The structural credit risk literature has thus far ignored this channel. A structural decomposition reveals that the time variation in common idiosyncratic risk is more important than time variation in aggregate risk. Quantitatively, eliminating time-varying common idiosyncratic risk from the valuation of CDS contracts leads to a drop of 57% in average CDS rates, while the elimination of time-varying aggregate risk causes a reduction of only 33%.

We estimate a value of 9.1 for the representative agent's coefficient of relative risk aversion. This value falls between the ones in Barro (2006), who assumes a value of 3 to 4, and Bansal and Yaron (2004), whose main specification assumes a value of 10. In line with this finding, consumption shocks in our model are smaller than in the rare disaster model but larger than in the long-run risks model.

Lastly, we explore the link between option prices and default risk. Cross-sectional sorts on firms' distance-to-default show that increased default risk is associated with significantly higher levels of option-implied volatilities and steeper implied volatility curves. The first finding is intuitive because default risk reflects a combination of asset volatility and financial leverage, both of which make stock returns more volatile. The second finding arises because default risk leads to more conditional left skewness, as equity holders' return equals -100% in default. Our model replicates both patterns quantitatively very well, even though these moments are not targeted in the estimation. In contrast, popular reduced-form option pricing models in the spirit of Black and Scholes (1973) and Heston (1993) are not able to capture these effects because they abstract from capital structure decisions. Instead, this literature models stock returns as an exogenous process.

Related Literature

Our consumption-based pricing kernel is motivated by the long-run risk literature initiated by Bansal and Yaron (2004) and the disaster risk literature following Rietz (1988), Barro (2006), Gabaix (2012), and Wachter (2013).

Our work differs from canonical disaster models in three important ways. First, we model crises as persistent. When disasters are modeled as single-period i.i.d. events, as in Barro (2006), Gabaix (2012), or Wachter (2013), the occurrence of a disaster leaves equity volatility, expected returns, and other conditional asset pricing moments unchanged. While it is common to explain fluctuations in these moments with time variation in the probability of disasters, this channel cannot account for the co-movement between (conditional) asset prices and the real economy during crises, because shocks to the disaster probability and consumption growth are typically assumed to be independent. Second, we explicitly model financial leverage. Because leverage rises during crises, it acts as an important propagation mechanism for fundamental shocks. For example, leverage increases expected returns and credit spreads during crises because it induces higher equity volatility and default risk.

Lastly, we follow a different calibration strategy. In the international dataset of Barro and Ursua (2012), the average crisis lasts 3.7 years and results in an average cumulative drop in total consumption expenditures of 21.6% – numbers that are very similar to the U.S. experience during the Great Depression (3.7 years; 20.8% drop). While the Great Depression represents only about 4.1% of the U.S. sample, however, the international dataset features a much higher crisis frequency of about 13.3%. This large wedge suggests that the international disaster sample is not representative for the U.S. economy.[4]

Rather than relying on international data, we calibrate crises based on U.S. data only. Besides implying a lower crisis frequency, this approach has the important advantage of allowing us to calibrate consumption dynamics based on nondurables and services consumption – the common measure of consumption in macro-finance models. In contrast, international data is only available for total consumption expenditures. An important predecessor to our work and the first study to model persistent disasters is Nakamura, Steinsson, Barro, and Ursua (2013). Their model is calibrated based on international disaster data (including their frequency) and used to explain the equity premium. Unlike us, they do not consider financial leverage or the pricing of derivatives.

Our model builds closely on structural credit risk models such as Hackbarth, Miao, and Morellec (2006), Chen, Collin-Dufresne, and Goldstein (2009), Bhamra, Kuehn, and Strebulaev (2010), and Chen (2010). We differ from these papers along three dimensions. First, we find that long-lasting consumption crises are an important source of aggregate risk for the valuation of option and CDS contracts. Second, we model equity issuance costs and common idiosyncratic volatility, two important features of the data. Third and most importantly, we structurally estimate our model.

In contrast to Bhamra, Kuehn, and Strebulaev (2010) and Chen (2010), we find that a long-run risks pricing kernel based on the framework of Bansal and Yaron (2004) cannot explain the credit spread puzzle. The difference arises because we structurally estimate our

[4] In line with this argument, Barro (2006) does not calibrate the disaster frequency in his model based on the frequency in international data, but rather based on the frequency of entering a disaster. This probability equals 3.6%.

model, while Bhamra, Kuehn, and Strebulaev (2010) and Chen (2010) follow a calibration exercise. In particular, Chen (2010) measures the average 10-year CDS rate at times when firms issue debt. Bhamra, Kuehn, and Strebulaev (2010) study a dynamic panel of firms. They select firms from a simulated panel, such that at the beginning of the simulation the leverage distribution matches BBB-rated firms from Compustat. Thus, both papers compare credit spreads in the data with those of a subset of firms in the model. In contrast, our structural estimation targets unconditional moments from the full panel of firms. Moreover, while both papers calibrate leverage and credit spreads based on data for different sets of firms, our structural estimation relies on a common dataset. In our data, the credit spread puzzle is particularly pronounced, with leverage ratios below 20% and CDS rates around 70 basis points. In contrast, both Bhamra, Kuehn, and Strebulaev (2010) and Chen (2010) calibrate to leverage ratios of nearly 40%.

While many consumption-based models have considered the pricing of index options, this literature has thus far ignored the pricing of single-name equity options and the implications of financial leverage.[5] We are not the first, however, to explore the link between credit risk and option pricing. Prior literature has recognized that option prices contain information about the dynamics and pricing of risk that is useful for the valuation of credit instruments.[6] Two closely related papers are Collin-Dufresne, Goldstein, and Yang (2012) and Seo and Wachter (2018). They provide structural credit risk models with disaster risk and use them to price index options and CDX contracts, i.e., market-level debt and equity derivatives. A limitation of their frameworks is that they model asset risk instead of modeling cash flow risk to shareholders and bondholders separately. In their models, equity value therefore does not approach zero when firms default. In contrast, we model optimal capital structure and default decisions, which allows us to price derivatives written on individual firms' debt and equity.

Similar to Hennessy and Whited (2007), we provide a structural estimate of bankruptcy costs. Their model features endogenous capital accumulation and it is based on risk neutral agents, while we model earnings exogenously but allow for endogenous risk premia. Our work is also related to that of Glover (2016), who argues that the sample of observed defaults significantly understates the average firm's true expected cost of default due to a sample

[5]The consumption-based option pricing literature, which has focus on index options, includes Liu, Pan, and Wang (2005), Drechsler and Yaron (2011), Backus, Chernov, and Martin (2011), Drechsler (2013), Bekaert and Engstrom (2017), Seo and Wachter (2019), and Schreindorfer (2020).

[6]Notable contributions include Cremers, Driessen, and Maenhout (2008), Coval, Jurek, and Stafford (2009), Carr and Wu (2011), Culp, Nozawa, and Veronesi (2018).

selection bias. Firms with higher cost of default endogenously opt for a lower level of leverage, and thus default at lower frequency all else equal. Davydenko, Strebulaev, and Zhao (2012) extract the cost of default from the change in the market value of a firm's assets upon default.

1 Model

The goal of the model is to capture the joint dynamics of firms' debt and equity values, and of derivatives that are written on them. To this end, we assume a representative investor with recursive preferences and consumption dynamics with Markov-switching drift and volatility. Firms choose capital structures by issuing perpetual debt, trading off tax shields and bankruptcy costs. They optimally issue more debt after a sequence of positive shocks and default after a sequence of negative shocks. The basic structure of our model is similar to Bhamra, Kuehn, and Strebulaev (2010) and Chen (2010), but we allow for equity issuance costs and time-variation in common idiosyncratic risk. Both features turn out to be important for the pricing of CDS contracts. A more central innovation of our model lies in the specification of macroeconomic risk, which features long-lasting crises, as detailed in Section 2.

1.1 Pricing Kernel

To allow risk premia to fluctuate with macroeconomic conditions, we follow the consumption-based asset pricing literature and model the pricing kernel as the marginal utility of consumption of a representative investor. Specifically, log aggregate consumption growth $g_{c,t+1}$ follows a Markov-switching modulated random walk

$$g_{c,t+1} = \mu_{c,t} + \sigma_{c,t}\varepsilon_{c,t+1}, \tag{1}$$

where the conditional mean $\mu_{c,t}$ and volatility $\sigma_{c,t}$ of consumption growth depend on the aggregate Markov state ξ_t and $\varepsilon_{c,t+1}$ are standard normal innovations. The aggregate state ξ_t follows a Markov chain with transition matrix $\mathcal{P}$.

The representative agent has recursive Epstein and Zin (1989) preferences over consumption such that the pricing kernel is given by

$$M_{t,t+1} = \beta^{\theta} \left(\frac{\lambda_{t+1}^{c} + 1}{\lambda_{t}^{c}} \right)^{-(1-\theta)} e^{-\gamma g_{c,t+1}}, \tag{2}$$

where β is the time discount rate, γ denotes relative risk aversion, ψ the elasticity of intertemporal substitution (EIS), $\theta = \frac{1-\gamma}{1-1/\psi}$, and the wealth-consumption ratio λ_t^c solves

$$(\lambda_t^c)^\theta = \beta^\theta e^{-\gamma\mu_{c,t}+\frac{1}{2}(1-\gamma)^2\sigma_{c,t}^2}\mathbb{E}_t\left[\left(\lambda_{t+1}^c+1\right)^\theta\right]. \tag{3}$$

The h-period risk-free rate is given by $R_{t,t+h}^f = 1/\mathbb{E}_t[M_{t,t+h}]$, where $M_{t,t+h} = \prod_{i=1}^h M_{t+i-1,t+i}$.

1.2 Unlevered Firm Value

Firm i's earnings before interest and taxes (EBIT) $E_{i,t}$ are exogenous and follow a Markov-switching modulated random walk, whose conditional growth rate and volatility depend on the aggregate Markov state ξ_t. Specifically, log earnings growth $g_{i,t+1}$ follows

$$g_{i,t+1} = \mu_t + \sigma_t\varepsilon_{t+1} + \zeta_t\nu_{i,t+1}, \tag{4}$$

where $\nu_{i,t+1}$ is an idiosyncratic standard normal shock, which is independent across time and firms, and ε_{t+1} is an aggregate standard normal shock, which is independent across time and has correlation ρ with the consumption shock $\varepsilon_{c,t+1}$. We assume that the drift μ_t, systemic risk σ_t, and idiosyncratic risk ζ_t of earnings growth depend on the aggregate state ξ_t. The dynamics of idiosyncratic risk captures the idea of common idiosyncratic volatility in the spirit of Herskovic, Kelly, Lustig, and Nieuwerburgh (2016).

The unlevered firm value is the present value of after-tax earnings. The tax environment consists of three tax rates: constant tax rate τ_i on personal interest income, τ_e on equity income, and τ_c on corporate earnings. The after-tax cum-earnings asset value $A_{i,t}$ is given by

$$A_{i,t} = (1-\tau_c)(1-\tau_e)E_{i,t} + \mathbb{E}_t[M_{t,t+1}A_{i,t+1}]. \tag{5}$$

1.3 Debt Value

Firms issue perpetual debt to take advantage of the tax benefits of debt financing. Optimal leverage is determined by the trade-off between tax shields and bankruptcy costs. In addition, firms can issue more debt in the future or declare default.

The interest coverage ratio $\kappa_{i,t}$ is defined as the ratio of current earnings $E_{i,t}$ to coupon payments $c_{i,s}$ based on the most recent debt issuance date s

$$\kappa_{i,t} = \frac{E_{i,t}}{c_{i,s}}.$$

When firms are hit by a sequence of negative earning shocks, earnings might not be sufficient to cover interest expenses. Eventually, the interest coverage ratio falls below the optimal aggregate state-dependent threshold $\kappa_t^D = \kappa^D(\xi_t)$ and equity holders optimally declare default.

In the opposite case, firms receive a sequence of positive earnings news and the interest coverage ratio rises. A rising interest coverage ratio implies lower leverage and shrinking tax shields. Consequently, when the interest coverage ratio exceeds the optimal aggregate state-dependent threshold $\kappa_t^I = \kappa^I(\xi_t)$, firms issue more debt, such that the interest coverage ratio resets to the optimal state-dependent interest coverage ratio $\bar{\kappa}_t = \bar{\kappa}(\xi_t) = \frac{E_{i,t}}{c_{i,t}}$. Importantly, firms do not refinance every period because of proportional debt issuance costs. Both issuance and target interest coverage ratios are set to maximize total firm value.

Given these dynamics, the after-tax cum-coupon debt value $D_{i,t}$ is given by

$$\begin{aligned} D_{i,t} &= 1_{\{\kappa_{i,t} \leq \kappa_t^D\}}(1-\omega_t)A_{i,t} \\ &+ 1_{\{\kappa_t^D < \kappa_{i,t} < \kappa_t^I\}}\Big((1-\tau_i)c_{i,s} + \mathbb{E}_t[M_{t,t+1}D_{i,t+1}]\Big) \\ &+ 1_{\{\kappa_t^I \leq \kappa_{i,t}\}}\Big((1-\tau_i)c_{i,s} + \frac{c_{i,s}}{c_{i,t}}\mathbb{E}_t[M_{t,t+1}D_{i,t+1}]\Big), \end{aligned} \tag{6}$$

where ω_t are aggregate state-dependent bankruptcy costs. The first term captures the cash-flows in default, when $\kappa_{i,t} \leq \kappa_t^D$. When the interest coverage ratio varies between the default and issuance thresholds, $\kappa_t^D < \kappa_{i,t} < \kappa_t^I$, firms optimally do not adjust their capital structure and continue to pay coupon $c_{i,s}$ per period, as described by the second term. The last term captures debt value when firms issue additional debt in case the interest coverage ratio exceeds the issuance threshold, $\kappa_t^I \leq \kappa_{i,t}$. When new debt is issued at date t, coupon payments increase from $c_{i,s}$ to $c_{i,t}$. While current bond holders are entitled to coupon payments $c_{i,s}$ in perpetuity, they also receive the fraction $\frac{c_{i,s}}{c_{i,t}}$ of the new bond's value, $\mathbb{E}_t[M_{t+1}D_{i,t+1}]$. Hence, we assume pari-passu – equal seniority for old and new bond holders.

To model time-variation in bankruptcy costs, we posit an affine function in the ratio of consumption growth to consumption volatility, denoted by $R_t = \mu_{c,t}/\sigma_{c,t}$, and given by

$$\omega_t = \omega_0 - \omega_1(R_t - \bar{R}), \tag{7}$$

where ω_0 controls the average and ω_1 the cyclicality of bankruptcy costs. Intuitively, bankruptcy costs should be higher when the conditional mean of consumption growth $\mu_{c,t}$ is low or when the conditional volatility of consumption growth $\sigma_{c,t}$ is high, requiring that the coefficient ω_1 is positive.

1.4 Equity Value

Equity holders decide about the optimal timing of default by maximizing the after-tax cum-dividend equity value $S_{i,t}$

$$
\begin{aligned}
S_{i,t} &= \max\Big\{0, 1_{\{\kappa_{i,t}<\kappa_t^I\}}\Big((1-\tau_c)(1-\tau_e)(E_{i,t}-c_{i,s})1_{\{E_{i,t}\geq c_{i,s}\}} \\
&+ (1-\tau_c+\phi_e)(E_{i,t}-c_{i,s})1_{\{E_{i,t}<c_{i,s}\}} + \mathbb{E}_t[M_{t,t+1}S_{i,t+1}]\Big) \\
&+ 1_{\{\kappa_t^I\leq\kappa_{i,t}\}}\Big((1-\tau_c)(1-\tau_e)(E_{i,t}-c_{i,s}) + \Delta_{i,t} + \mathbb{E}_t[M_{t,t+1}S_{i,t+1}]\Big)\Big\},
\end{aligned} \tag{8}
$$

where

$$\Delta_{i,t} = (1-\phi_d)D^x_{i,t}(c_{i,t}) - D^x_{i,t}(c_{i,s})$$

are after-tax debt issuance proceeds and $D^x_{i,t}$ denotes the ex-coupon bond value.[7]

Intuitively, when the interest coverage ratio falls below the issuance thresholds, $\kappa_{i,t} < \kappa_t^I$, firms pay after-tax dividends of $(1-\tau_c)(1-\tau_e)(E_{i,t}-c_{i,s})$. However, when earnings are not sufficient to cover interest expenses, $E_{i,t} < c_{i,s}$, the firm has to raise equity, which triggers proportional equity issuance costs ϕ_e. When the present value of after-tax dividends drops to zero, it is optimal for share holders to shut down the firm, which is captured by the first term in the max-operator. When the interest coverage ratio rises and exceeds the issuance threshold, $\kappa_t^I \leq \kappa_{i,t}$, firms lever up and equity holders receive a special dividend in the amount of the ex-coupon price of newly issued debt $D^x_{i,t}(c_{i,t})$ net of debt issuance costs ϕ_d and existing bond holder ex-coupon value $D^x_{i,t}(c_{i,s})$.

The prior structural credit risk literature has ignored equity issuance costs, i.e., it imposed $\phi_e = 0$. Without equity issuance costs, firms declare bankruptcy at very low interest coverage ratios, driving up losses to bond holders. In contrast, equity issuance costs reduce firms' incentive to raise equity, resulting in realistic average loss rates to bond holders.

Given the cum-dividend equity value, the optimal state-dependent default threshold satisfies

$$\kappa^D(\xi_t) = \max\{\kappa_{i,t} : S(\kappa_{i,t}, \xi_t) \leq 0\}.$$

Equity holders declare default when the interest coverage ratio falls below this default trigger.

1.5 Levered Firm Value

Levered firm value is the sum of the value of debt (6) and equity (8). Management chooses the optimal issuance threshold κ_t^I and the optimal coverage ratio $\bar{\kappa}_t$ to maximize the after-tax

[7] This is a special dividend coming from debt issuance proceeds and does not represent ordinary dividend income.

cum-cash flow firm value, $F_{i,t}$, which is given by

$$\begin{aligned} F_{i,t} &= 1_{\{\kappa_{i,t}\leq\kappa_t^D\}}(1-\omega_t)A_{i,t} + 1_{\{\kappa_t^D<\kappa_{i,t}<\kappa_t^I\}}\Big((1-\tau_c)(1-\tau_e)(E_{i,t}-c_{i,s})1_{\{E_{i,t}\geq c_{i,s}\}} \\ &+ (1-\tau_c+\phi_e)(E_{i,t}-c_{i,s})1_{\{E_{i,t}<c_{i,s}\}} + (1-\tau_i)c_{i,s} + \mathbb{E}_t\big[M_{t,t+1}F_{i,t+1}\big]\Big) \quad (9) \\ &+ 1_{\{\kappa_t^I\leq\kappa_{i,t}\}}\Big((1-\tau_c)(1-\tau_e)(E_{i,t}-c_{i,s}) + (1-\tau_i)c_{i,s} - \phi_d D_{i,t}^x + \mathbb{E}_t\big[M_{t,t+1}F_{i,t+1}\big]\Big). \end{aligned}$$

When the interest coverage ratio drops below the default threshold, $\kappa_{i,t} \leq \kappa_t^D$, the firm defaults and bond holders recover a fraction $1-\omega_t$ of asset value. When the interest coverage ratio varies between the default and issuance thresholds, $\kappa_t^D < \kappa_{i,t} < \kappa_t^I$, firms optimally do not adjust their capital structure and cash-flows equal after-tax earnings plus tax shields in the amount $(1-\tau_i)/\kappa_{i,t}$. When the interest coverage ratio exceeds the issuance threshold, $\kappa_t^I \leq \kappa_{i,t}$, firms issue additional debt and cash-flows equal after-tax earnings plus tax shields $(1-\tau_i)/\kappa_{i,t}$ net off issuance costs $\phi_d D_{i,t}^x$.

Maximizing firm value (9) implies that the state-dependent optimal interest coverage ratio is given by

$$\bar{\kappa}(\xi_t) = \arg\max_{\kappa}\Big\{-\phi_d D_{i,t}^x(\kappa,\xi_t) + \mathbb{E}_t\big[M_{t,t+1}F_i(\kappa e^{g_{i,t+1}},\xi_{t+1})\big]\Big\},$$

which captures the trade-off between marginal issuance costs and expected tax benefit. Similarly, the state-dependent optimal issuance threshold is given by the interest coverage ratio for which the firm continuation value of issuance dominates the one without issuance,

$$\kappa^I(\xi_t) = \min\Big\{\kappa_{i,t} : \mathbb{E}_t\big[M_{t,t+1}F(\bar{\kappa}_t e^{g_{i,t+1}},\xi_{t+1})\big] \geq \mathbb{E}_t\big[M_{t,t+1}F(\kappa_{i,t}e^{g_{i,t+1}},\xi_{t+1})\big]\Big\}.$$

1.6 Credit Default Swaps

Given the price dynamics of debt and equity, we are equipped to price derivative securities written on them. In this section, we tackle the pricing of credit default swaps and in the following section the pricing of put option contracts.

Firm i defaults at time t_i when its interest coverage ratio $\kappa_{i,t}$ drops below the default threshold κ_t^D, so that

$$t_i \equiv \inf\{t : \kappa_{i,t} \leq \kappa_t^D\}. \quad (10)$$

The CDS rate $z_{i,t,s}^T$ for horizon T equates the present value of the payments made by the insurance seller and the insurance buyer. The insurance seller receives the CDS rate until the expiration date T or firm default. In the case of default, bond holders take over the firm and recover the after-tax firm value net of bankruptcy costs, $(1-\omega_{t+h})A_{i,t+h}$, such that their losses relative to the initial investment, $D_{i,s}$, are given by

$$x_{i,t+h,s} = 1 - \frac{(1-\omega_{t+h})A_{i,t+h}}{D_{i,s}}. \tag{11}$$

By no-arbitrage, the periodic CDS rate for horizon T rate is given by[8]

$$z^T(\kappa_{i,t}, \xi_s, \xi_t) = \frac{\sum_{h=1}^{T} \mathbb{E}_t\left[M_{t+h} 1_{\{t_i = t+h\}} x_{i,t+h,s}\right]}{\sum_{h=1}^{T} \mathbb{E}_t\left[M_{t+h}(1 - 1_{\{t_i \le t+h\}})\right]}, \tag{12}$$

which depends on the firm's interest coverage ratio $\kappa_{i,t}$, the most recent debt issuance state ξ_s, the aggregate state ξ_t, and realized loss rates to bond holders $x_{i,t+h,s}$.

1.7 Equity Option Pricing

The value of a one period put option with strike price K is given by

$$P_{i,t} = \mathbb{E}_t\left[M_{t,t+1} \max\left\{K - 1_{\{\kappa^I_{t+1} \le \kappa_{i,t+1}\}} \Delta_{i,t+1} - S^x_{i,t+1}, 0\right\}\right], \tag{13}$$

where $S^x_{i,t}$ denotes the ex-dividend share price and the term $1_{\{\kappa^I_{t+1} \le \kappa_{i,t+1}\}} \Delta_{i,t+1}$ reflects an adjustment for special dividends. Because special dividends and other corporate actions (stock splits, mergers, etc.) lead to discrete jumps in the underlying's price, the contractual terms of equity options are altered when such events occur. In the case of a special dividend, the strike price is lowered by the amount of the dividend. Note that Equation (13) prices options that are written on the ex-dividend stock price, as in the data. We convert model-based option prices to Black-Scholes-Merton implied volatilities (IV), taking into account the state-dependent risk-free rate and dividend yield based on ordinary dividends, i.e., excluding special dividends.

2 Macroeconomic Crises

This section discusses how we calibrate the consumption process to capture persistent macroeconomic crises in the model. Using this calibration, we illustrate the model's ability to replicate the dynamics of macroeconomic aggregates, financial leverage, and asset prices during crises with different durations. We do so before detailing the parameter values that were used to solve the model to enable readers to get to the main insights more quickly. Section 3 discusses the SMM estimation of model.

[8] Appendix B.1 contains closed-form expressions for CDS rates.

2.1 Consumption Dynamics

We assume a three-state Markov chain for the aggregate state, where the drift and volatility of log consumption growth are given by

$$\mu_c = \begin{bmatrix} \mu_c^{(1)} \\ \mu_c^{(2)} \\ \mu_c^{(3)} \end{bmatrix} \qquad \sigma_c = \begin{bmatrix} \sigma_c^{(1)} \\ \sigma_c^{(2)} \\ \sigma_c^{(3)} \end{bmatrix}. \tag{14}$$

States 1 and 2 capture business cycle variation in consumption, whereas state 3 captures macroeconomic crises. The transition matrix is given by

$$\mathcal{P} = \begin{bmatrix} p & 1-p & \pi \\ 1-p & p & \pi \\ (1-p_3)/2 & (1-p_3)/2 & p_3 \end{bmatrix}, \tag{15}$$

where p denotes the persistence of states 1 and 2, π the probability of switching into the crisis state, and p_3 the persistence of crises. For parsimony, the transition matrix imposes that (i) states 1 and 2 are symmetric, i.e., equally likely and persistent, (ii) the crisis probability π does not vary across states 1 and 2, and (iii) crises are equally likely to end with a transition into either states 1 or 2.

We calibrate the consumption process at the monthly frequency, targeting moments of (time-aggregated) annual log consumption growth. Consumption is defined as the sum of real nondurable and services consumption per capita from the BEA. Table 1 summarizes the calibration and shows associated moments.

Following Barro (2006), disaster models are typically calibrated based on international data of observed macroeconomic crises. The dataset of Barro and Ursua (2012) features 125 consumption declines of at least 10% in 28 countries, dating back as far as 1870 and ending in 2009. In this dataset, disasters have an average duration of 3.7 years (44 months), are associated with an average drop in total consumption expenditures of 21.6%, and account for 13.3% of all years.[9] The Great Depression in the U.S. is representative of these events, with a drop in total consumption expenditures of 20.8% over 44 months (August 1929 to March 1933). Given this fact, it appears reasonable to assume that investors consider events of the same magnitude and duration as in international data when pricing U.S. assets. In our view, however, it is less reasonable to assume that investors expect the U.S. economy to spend

[9] The fraction of years spent in disasters can be computed by multiplying the probability of entering a disaster, which Barro and Ursua (2012) report as 3.6%, by the average disaster duration of 3.7 years.

13.3% of time in severe macroeconomic crises, because this number is far removed from the U.S. experience. In particular, only 4.1% of all years in the BEA sample period correspond to depression years.

We therefore calibrate state 3 of the Markov chain based on U.S. data only. p_3 is set so that the average crisis lasts 44 months, like the Great Depression. π is set to imply a crisis frequency of 4.1%, the value for U.S. data. While the international dataset features a much higher crises frequency of 13.3%, it is worth noting that Barro (2006) does not calibrate the disaster frequency based on this number either, but rather based on the empirical probability of entering a crisis, which equals 3.6%. This number is close to the crisis frequency we calibrate based on U.S. data.

In contrast to Barro and Ursua (2012), we measure consumption as the sum of real nondurable and services consumption per capita. $\mu_c^{(3)}$ is set to generate a consumption drop of 16.58% for the average crisis, which equals the drop in U.S. nondurables and services consumption during the Great Depression. Lastly, $\sigma_c^{(3)}$ is estimated based on the sample standard deviation of annual log consumption growth rates during the Great Depression.[10]

Figure 3 shows the distributions of crisis durations and cumulative consumption drops during crises for our calibration. Many crises are short-lived. Half of all crisis episodes last less than 33 months and are associated with only moderate declines in consumption. However, crises with durations above 4 years and cumulative consumption declines of 16% or more, as experienced in the U.S. during the Great Depression, have sizable probabilities. For comparison, the figure also shows the distributions of crises durations and magnitudes in the international data of Barro and Ursua (2012). Our calibration provides a good match for both distributions.

Our approach for modeling crises has important implications for the calibration of non-disaster-related parameters. Barro (2006) defines disasters as consumption drops of at least 10%, which implies that the U.S. did not experience any disasters over the post-war period. In the disaster literature, it is therefore common practice to calibrate non-disaster-related parameters based on stratified model data. For example, Wachter (2013) simulates her model

[10]Specifically, we set $\sigma_c^{(3)} = 2.44\%/\sqrt{12}$, where 2.44% is the volatility of annual log growth rates. This is an approximation because it ignores time-aggregation. The average duration of the Markov chain in state 3 is $\sum_{i=1}^{\infty} p_3^{i-1}(1-p_3)i = 44$ months and thus pins down p_3. Conditional on remaining in state 3 for i periods, the consumption growth rate is log-normally distributed with mean of $i \times \mu_c^{(3)}$ and variance of $i \times (\sigma_c^{(3)})^2$ for the associated normal distribution. Denoting this conditional density by f_i, the unconditional distribution of log consumption growth, while the Markov chain remains in state 3, is $f(x) = \sum_{i=1}^{\infty} p_3^{i-1}(1-p_3)f_i(x)$. Given p_3 and $\sigma_c^{(3)}$, the mean of this distribution pins down $\mu_c^{(3)}$ and equals -0.1658.

unconditionally, and then filters out years with disaster occurrences. For our calibration, which allows for short-lived crises with small consumption drops, it is not obvious whether a disaster occurred post-war. We therefore calibrate the parameters of states 1 and 2 based on simulated model data that mimics the U.S. post-war experience, but does not rule out crises. Specifically, we simulate 10,000 years of monthly consumption data and ensure that the worst peak-to-trough consumption drop of every 73 year block equals $2.14\% \pm 0.5\%$, as in the period from the second quarter of 2008 to the fourth quarter of 2009.[11] Given the values for crisis parameters $(p_3, \pi, \mu_c^{(3)}, \sigma_c^{(3)})$, the non-crisis parameters $(\mu_c^{(1)}, \mu_c^{(2)}, \sigma_c^{(1)}, \sigma_c^{(2)}, p)$ are chosen such that the simulated model data perfectly matches the mean, standard deviation, skewness, kurtosis, and first-order autocorrelation of annual (time-aggregated) log consumption growth for the period 1947–2019, as shown in Table 1.

Our Markov-switching consumption process has several features that compare favorably with existing models. First, similar to Rietz (1988) and Barro (2006), it implies that finite samples with and without severe macroeconomic crises both have substantial probabilities. As such, the process overcomes an important criticism of the long-run risks model, which is very unlikely to generate finite samples that mimic the post-war U.S. sample for typical calibrations, as shown by Beeler and Campbell (2012). Second, unlike typical disaster models, but as in the data, our model implies that crises are long-lasting and more severe if they extend over longer periods. Third, unlike both typical disaster and long-run risk models, it results in a pricing kernel that is consistent with option prices, as we will show in Section 4.3.

2.2 Empirical Crises

We now turn to the behavior of firm-level moments during crises, first in the data (this section) and then in the model (next section). The Great Recession serves as an example of a short-lived crisis and the Great Depression as an example of a long-lived crisis. We collect data on firms' leverage ratios, CDS rates, and IVs. For the Great Recession period, we focus on firms in the S&P 100 index, excluding utilities and financial services firms and firms without valid CDS quotes. The data collection is straightforward and details are discussed in Section 3.1.

Crating a comparable sample for the Great Depression is more challenging. We take monthly data on the market value of firms' equity from CRSP, and hand-collect annual data on firms' book value of debt over the years from 1927 to 1935 from Moody's Industrial Manuals. We include all sources of long-term debt, common examples are funded debt, bonded debt,

[11] The simulated data implies a crisis frequency of 0.081%.

notes and loans payable, as well as bonds of subsidiaries.[12] Starting with the 100 largest non-financial, non-utility companies with available book debt and market equity data, we eliminate firms with zero leverage (mimicking the elimination of firms without CDS quotes). Because option and CDS contracts were not traded at the time of the Great Depression, we compute the corresponding series via a projection. Specifically, using data from January 2004 to June 2019, we regress the time series of average at-the-money IVs and CDS rates on (1) average leverage among the set of firms in our sample, (2) the realized volatility of daily returns on the CRSP value-weighted stock market index over the past calendar month, and (3) the Moody's Baa-Aaa corporate bond yield spread from FRED. The regressions result in R^2's of 89.5% for implied volatility and 82.9% for CDS rates. Based on these estimated regressions, we infer implied average IVs and CDS rates for the Great Depression using data over the period from August 1928 to August 1936.

Figure 1 shows the resulting time series in event time. Time 0 refers to the start date of each recession according to the NBER, which is August 1929 for the Great Depression and December 2007 for the Great Recession. The onset of both crises is characterized by nearly identical changes in firms' leverage ratios, which increased by about 50% over the initial 14 recession months. After that point, the trajectory of both crises looks strikingly different. In the case of the Great Recession, the contraction ended and leverage ratios returned to their pre-crisis level over the next year and a half. In contrast, the Great Depression witnessed a continued increases in leverage, which peaked at more than 250% of its original level three years into the recession.

Because higher leverage results in increased stock return volatility and default risk, financial leverage is important for understanding the behavior of asset prices during crises. In particular, the middle and bottom panels of Figure 1 show that firms' IVs and credit spreads followed a similar trajectory as leverage in both crises episodes. In the Great Recession, both quantities peaked after about one year and then quickly returned to their pre-crises levels. In the Great Depression, IVs and credit spreads peaked after about 3 years and, like leverage, stayed elevated for many years thereafter. This evidence suggests that both the persistence of shocks and financial leverage are important for understanding the joint behavior of macroeconomic quantities and asset prices during crises. We return to the relation between default risk and option prices in Section 4.4, where we examine it cross-sectionally.

[12] We thank Sunil Wahal for sharing PDF documents of the Moody's manuals with us. See Wahal (2019) for additional information on the data.

2.3 Simulated Crises

To evaluate whether the model is consistent with the behavior of leverage ratios and asset prices during crises, we rely on simulations to mimic specific crises in the model. In particular, we simulate one million paths for the aggregate Markov chain and impose that it enters the crisis state at time 0. We keep paths with consumption drops of $-2.14\% \pm 0.5\%$ to mimic the Great Recession, and paths with consumption drops of $-16.58\% \pm 0.5\%$ to mimic the Great Depression. For each crisis path, we then evaluate leverage policies and asset prices for 100 firms and average them in each period. Figure 2 shows the mean of these firm-level averages across all paths for the simulated Great Recession and Great Depression.

We focus on results for the Great Recession first. The onset of the crisis is characterized by a sharp increase in risk premia. Expected excess returns increase by 4.1 percentage points to 12.7% (top-right panel), and the associated drop in stock prices increases leverage ratios by nearly four percentage points to 22.5% (top-left panel). In turn, higher leverage ratios cause equity volatility to increase more than asset volatility, with option-implied volatilities reaching 67% on an annualized basis (bottom-left panel). Higher leverage and equity volatility increase default rates from near zero to 26 basis points (middle-right panel). Bankruptcy costs increase and raise the loss rates of defaulting firms from 77% to 88% (bottom-right panel). The increase in default risk and loss rates causes CDS rates to spike to 211 basis points. These effects are quantitatively similar to their data counterparts in Figure 1. In the data, the financial crisis saw average leverage ratios increase to 29%, implied volatilities to 69%, and CDS rates to 333 basis points.

Turning to results for the Great Depression, the figure shows that asset prices behave identically in the initial months. As the crisis continues to unfold, however, the model generates interesting and realistic propagation effects. The ongoing drop in earnings causes leverage to continue to rise, as a larger fraction of firms' cash flows is required for debt servicing. Leverage rises by an additional 14 percentage points after the onset of the crises, reaching 37% at its peak (top-left panel). Increasing leverage causes implied volatility to increase by another 8 percentage points to 75% p.a. (middle-left panel), despite no further rise in asset volatility. The joint increases in leverage and equity volatility cause default rates to reach 2.1% p.a. – eight times their level in the simulated Great Recession (middle-right panel). Higher volatility and default risk lead to a further increase in expected stock returns (top-right panel). Loss rates do not rise further because our model ascribes variation in bankruptcy costs to the aggregate economic state only (bottom-right panel). Nevertheless, the rise in default risk causes CDS rates to double from their initial spike as the crisis continues to evolve, reaching nearly 477 basis points at its peak (bottom-left panel).

Once again, these effect are quantitatively similar to the data shown in Figure 1. The Great Depression saw average leverage ratios increase to 48%, implied volatilities to 79%, and CDS rates to 523 basis points. Additionally, in line with the predictions of our model, Giesecke, Longstaff, Schaefer, and Strebulaev (2011) and Ou, Irfan, Liu, and Kanthan (2018) show that default rates for both investment grade and speculative debt were substantially higher during the Great Depression than the financial crisis.

It is important to point out that, while we calibrate the consumption process to data from the Great Depression, the dynamic behavior of leverage and asset prices during prolonged crises does not serve as an input to the model estimation. The model's ability to match these dynamics can therefore be viewed as an external validation of its mechanism, and it contrasts sharply with the implications of standard disaster models. When disasters are modeled as i.i.d. jump events, as in Barro (2006), Gabaix (2012), and Wachter (2013), the occurrence of a disaster results in a single and proportional drop in cash flows and equity prices. Valuation ratios, expected returns, implied volatilities, and other conditional asset prices remain unchanged. Instead, the literature has explained time-variation in these quantities via variation in the probability of disasters, which evolves independently from disaster events themselves. As such, existing mechanisms cannot account for the joint dynamics of economic aggregates and asset prices during crises. We overcome this shortcoming by modeling crises as persistent events. In addition, the presence of financial leverage in our model implies that equity volatility and default risk increase during crises, which amplifies the effect of cash flow shocks on conditional asset prices.

3 Estimation

In this section, we describe data sources and explain how we obtain parameter values for the quantitative evaluation of the model. Firm level parameters are estimated with the simulated method of moments, which minimizes a distance criterion between key moments from actual data and simulated model data. Because the model has to be solved numerically, an estimation of all model parameters is not computationally feasible. We therefore focus the estimation on those parameters for which the existing literature provides only weak priors – the parameters associated with firms' cash flow risk, the level and cyclicality of bankruptcy costs, and risk aversion. The other parameters are based on values in the literature.

3.1 Firm Level Data

The estimation relies on data from a number a sources. Daily data on credit default swaps (CDS) for the period from January 2004 to June 2019 are obtained from ICE Data Services (formerly known as Credit Market Analysis Ltd. (CMA)). The dataset contains information on pricing (bid and ask quotes) and contract terms of the underlying debt and credit default swaps (e.g., currency, debt seniority, credit event of restructuring, and tenor of the CDS contract). Mayordomo, Pena, and Schwartz (2014) find that the CMA database leads the price discovery process in comparison with other CDS databases, including Markit. We focus on five-year credit spreads, which represent the most actively traded CDS tenor.

Because we are interested in the joint behavior of option and CDS contracts, the empirical analysis focuses on constituents of the S&P 100 index, an index of blue chip companies across multiple industry groups that requires constituents to have listed stock options. As is standard, we exclude regulated utilities (SIC codes between 4,900 and 4,999) and financial firms (SIC codes between 6,000 and 6,999) because they have a non-standard capital structure.

Equity option data are obtained from OptionMetrics, which provides end-of-day bid and ask quotes on traded put and call options, as well as implied volatilities on synthetic option contracts that are standardized across maturity and moneyness (the so-called implied volatility surface). Individual equity options are American style, and OptionMetrics uses binomial trees to compute implied volatilities which account for early exercise. We use the implied volatility surface data, as it provides a relatively stable number of contracts and allows a more homogenous analysis across firms than would be the case with option quotes.

We retain observations on CDS contracts with no restructuring (XR) or modified restructuring clause (MR), on senior debt, denominated in US dollars. After applying these filters and merging with OptionMetrics, we are left with a sample of 104 unique firms with at least 10 monthly observations, covering a total of 11,694 firm-month observations over the period from 2004 to 2019.

Daily mid-quotes of the CDS contracts are averaged within a calendar month to obtain a time series of monthly average credit default swap rates for each firm. Over the entire sample period, the average (median) firm's CDS rates equals 69.1 basis points (40.1 bp), with considerable variation both across firms and within firms over time.

We summarize the cross section of option prices for each firm with the level and slope of its IV curve at the 1-month maturity. The level is measured by the at-the-money IV, which we compute by linearly interpolating the IV of out-of-the money call and put options with strike prices closest to the current stock price. The slope, commonly referred to as IV skew, is estimated by the difference between the IV of a put option that is one standard deviation out of the money (using the at-the-money IV to measure standard deviations) and the at-the-money IV. We then average the daily estimates for each firm to obtain a monthly time series of firm-level IV level and skew. The average firm has an IV level of 24.9% and an IV skew of 4.0%.

Equity market data are taken from CRSP and data on corporate policies from the quarterly Compustat files. Debt is measured as debt in current liabilities (DLCQ) plus long-term debt (DLTTQ). Using CRSP data, market equity is the product of share price (PRC) and number of shares outstanding (SHROUT). Leverage is defined as debt divided by debt plus market equity. The average firm in our sample has a leverage ratio of 19.5%.

3.2 Cash Flow Risks

Before solving the model, we need to define functional forms for firms' cash flow risks. We assume that the conditional moments of earnings growth (4) are governed by the aggregate state, ξ_t, so that earnings and consumption are subject to the same aggregate trends. In particular, the earnings drift is parameterized as

$$\mu_t = \mathbb{E}[\mu_{c,t}] + \phi_\mu(\mu_{c,t} - \mathbb{E}[\mu_{c,t}]), \tag{16}$$

which implies that earnings and consumption grow at the same long-run rate $\mathbb{E}[\mu_{c,t}]$ and that earnings growth is low when the macroeconomy is doing poorly, if $\phi_\mu > 0$. For $\phi_\mu > 1$, the conditional growth rate of earnings varies more than that of consumption, similar to how Bansal and Yaron (2004) scale the long run risk factor in the dividend process of their model.

We also assume that the conditional volatility of systematic cash flow risk varies with the aggregate state according to

$$\sigma_t = \phi_{\sigma,0} + \phi_{\sigma,1}(\sigma_{c,t} - \mathbb{E}[\sigma_{c,t}]), \tag{17}$$

where $\phi_{\sigma,0}$ measures the level and $\phi_{\sigma,1}$ the cyclicality of aggregate earnings risk. For $\phi_{\sigma,0} > \mathbb{E}[\sigma_{c,t}]$, aggregate earnings are more volatile than aggregate consumption, which captures sources of operating leverage such as sticky wages. For $\phi_{\sigma,1} > 1$, the volatility of aggregate earnings fluctuates more than the volatility of aggregate consumption, similar to how Bansal and Yaron (2004) scale long-run volatility risk in the dividend process of their model.

Lastly, idiosyncratic cash flow risk is parameterized as an affine function of consumption volatility,

$$\zeta_t = \phi_{\zeta,0} + \phi_{\zeta,1}(\sigma_{c,t} - \mathbb{E}[\sigma_{c,t}]), \tag{18}$$

where $\phi_{\zeta,0}$ measures the level and $\phi_{\zeta,1}$ the cyclicality of idiosyncratic earnings risk. While idiosyncratic shocks are firm-specific, our parameterization of ζ_t implies that their volatility is time-varying and common across firms. Such common idiosyncratic volatility in earnings growth is a robust feature of the data, as recently documented by Herskovic, Kelly, Lustig, and Nieuwerburgh (2016). In what follows, we structurally estimate the six parameters governing' firms cash flow risks, $(\phi_\mu, \phi_{\sigma,0}, \phi_{\sigma,1}, \phi_{\zeta,0}, \phi_{\zeta,1})$, and show that common idiosyncratic risk has important implications for the pricing of CDS contracts.

3.3 Predefined Parameters

Table 2 summarizes predefined parameters. We follow Bansal and Yaron (2004) in assuming a value of $\psi = 1.5$ for the representative agent's EIS, which results in a smooth risk-free rate. The time discount rate $\beta = 0.9977$ is chosen so that the average 1-year risk-free rate equals 1%. Different from Bansal and Yaron (2004), our model features a crisis state, which necessitates the estimation of risk aversion. The correlation between consumption and earnings is set at 20%, as estimated by Bhamra, Kuehn, and Strebulaev (2010).

While in reality, equity investors receive payoffs in the form of both cash dividends and capital gains, our model abstracts from retained earnings. It is therefore appropriate to calibrate the tax rate on cash dividends in the model based on the tax rate for total equity income in the data. We follow Graham (2000) in computing the tax rate on equity income as $\tau_e = (d + (1-d)\delta)\tau_d$, where d equals the payout ratio, τ_d the tax rate on dividend income, and δ measures the benefit of deferring capital gains. For our sample period, the average tax rate on dividend income equals 17.2%, while the average payout ratio of firms in our sample equals 45.6%. Using the same estimate of $\delta = 0.25$ as in Graham (2000), we find $\tau_e = 11.2\%$.[13] We also follow Graham (2000) in assuming a tax rate on interest income of

[13]The tax rate on dividend income equalled 15% over 2004-2013, and 23.8% over 2014-2019. The latter includes the 3.8% Net Investment Income Tax that applies to dividends received by married taxpayers with modified adjusted gross income over $250,000, and single taxpayers with modified adjusted gross income over $200,000. These numbers are substantially lower than those in Graham's sample period due to the tax cuts under president Bush in 2003.

$\tau_i = 29.6\%$. The corporate tax rate is based on Graham (1996a,b). Using Graham's data for tax rates of individual firms and years, we compute the pooled average marginal tax rate (before interest expenses) for our sample of firms and sample period as $\tau_c = 32.9\%$.

Finally, we have to calibrate debt and equity issuance costs. We set the debt issuance costs at 1% and equity issuance costs at 4% based on the empirical evidence in Altinkilic and Hansen (2000) for large firms.

3.4 Simulated Method of Moments

The remaining model parameters are estimated with the simulated method of moments (SMM). Given the predefined parameters and the vector $\theta = (\phi_\mu, \phi_{\sigma,0}, \phi_{\sigma,1}, \phi_{\zeta,0}, \phi_{\zeta,1}, \omega_0, \omega_1, \gamma)$, we solve the model numerically and simulate a panel of firms. Numerical details of the model solution are discussed in Appendix C. Based on the simulated panel, we calculate the model moments $\Psi^M(\theta)$. The SMM objective function equals a weighted distance metric between moments from actual data Ψ^D and model moments $\Psi^M(\theta)$, $J(\theta) = [\Psi^D - \Psi^M(\theta)]' W [\Psi^D - \Psi^M(\theta)]$. The optimal weighting matrix W is based on the spectral density, which is computed via influence functions as in Hennessy and Whited (2007). The parameter estimate $\hat{\theta}$ is found by searching globally over the parameter space, which we implement via a particle swarm algorithm. Computing standard errors for the parameter estimate requires the Jacobian of the moment vector, which we find numerically via a finite difference method.

The estimation builds on a simulated monthly panel of 100 firms over 10,000 years, which is roughly 650 times the size of the actual data sample. We use the same path for the aggregate Markov chain and aggregate Gaussian shocks in the simulation, which was used to calibrate the consumption process. As such, every 73-year block of simulated data contains one consumption drop of $2.14\% \pm 0.5\%$, just as the observed post-war sample. The first 100 periods are discarded to eliminate the effect of initial conditions. Defaulting firms are replaced with new ones, initialized at optimal (state-dependent) leverage. Since the market value of debt is not observable for public firms, we compute leverage based on book value of debt, which is typically referred to as quasi-market leverage. We mimic quasi-market leverage in the model by relying on the value of debt from the most recent issuance date.

The 7 estimated parameters $(\phi_\mu, \phi_{\sigma,0}, \phi_{\sigma,1}, \phi_{\zeta,0}, \phi_{\zeta,1}, \omega_0, \omega_1, \gamma)$ are identified based on 11 moments. At the firm-level, we include the mean and variance of quasi-market leverage, excess returns, the 5-year CDS rate, at-the-money IV, and the IV skew. At the aggregate level, we include the variance of the market return. While each parameter affects multiple moments, it is useful to discuss the main sources of identification.

As in the long-run risk framework, the representative investor's recursive utility function implies aversion against persistent variation in conditional growth rates. An increase in the drift scaling parameter ϕ_μ therefore increases risk premia, and it makes returns more volatile by increasing the variability of firms' valuation ratios. Similarly, risk aversion γ increases all risk premia, and is therefore identified by average excess returns, CDS rates, and IVs.

The level of idiosyncratic $\phi_{\zeta,0}$ and aggregate risk $\phi_{\sigma,0}$ are positively related to the volatility of firm-level returns and average IV and negatively with average leverage. Because idiosyncratic shocks average out in the aggregate, however, only the aggregate volatility scaling parameter $\phi_{\sigma,0}$ increases the volatility of the market return. The cyclicality of idiosyncratic $\phi_{\zeta,1}$ and aggregate risk $\phi_{\sigma,1}$ have a large impact on the time variation of the IV curve and credit risk. As the level of bankruptcy costs ω_0 rises, it is optimal for firms to lower leverage, which depresses CDS rates. The cyclicality of bankruptcy costs, ω_1, decreases average leverage and CDS rates, while increasing their volatility.

3.5 Results

Tables 3 and 4 summarize the SMM estimation. Table 3 reports parameter estimates, whereas Table 4 reports moments in the model and the data. The estimation targets variances, but we also report implied standard deviations for easier interpretation. The data column is based on the sample of S&P100 firms, excluding financials and utilities, as explained in the previous section. Using the same moments, we also estimate the model with a traditional long-run risks pricing kernel, denoted by LRR. Standard errors are reported in parenthesis.

The benchmark estimation matches the target moments well. In the model, quasi-market leverage averages 19.4% (data: 19.5%) with a standard deviation of 14.8% (data: 15.0%). To match these moments, the estimation sets the level of bankruptcy costs to $\omega_0 = 0.259$ and their cyclicality to $\omega_1 = 0.096$. While true market leverage is not observable in the data, our model implies that it has an average of 17.6% and a standard deviation of 11.5%. Interestingly, average quasi-market leverage is therefore an upward-biased proxy of average true leverage. The reason is that leverage is a concave function of the bond price, so that more bond price variability translates to higher average leverage due to a Jensen effect. Similarly, because stock and bond returns are positively correlated, leverage ratios based on market values are less volatile. To our knowledge, this finding is new, and it appears useful for interpreting the findings of empirical work on firms' capital structure.

In the model, average firm-level excess returns equal 0.72% (data: 0.78%) per month. Risk premia in our model are generated by persistent drift and volatility differences across states, as in the long-run risks literature. In fact, the estimated magnitude of the drift scaling parameter $\phi_\mu = 3.5$ is very close to the calibrated value of 3 in Bansal and Yaron (2004). The volatility of excess returns equals 7.1% (data: 7.3%) at the individual firm level and 3.9% (data: 4.0%) for the market – both close to their data counterparts. To replicate the ratio of idiosyncratic to aggregate risk, the estimation selects average idiosyncratic risk of $\phi_{\zeta,0} = 0.045$ and a smaller average aggregate risk of $\phi_{\sigma,0} = 0.032$.

Related to stock return volatility are option moments. The model closely matches the average firm-level at-the-money IV of 24.3% (data: 24.9%) with a standard deviation of 9.3% (data: 11.3%). The IV skew, which equals the difference between the one standard deviation out-of-the money IV and at-the-money IV, averages 3.9% (data: 4.0%) with a standard deviation of 1.7% (data: 2.1%). The volatility scaling parameters for idiosyncratic $\phi_{\zeta,1} = 22.4$ and aggregate risk $\phi_{\sigma,1} = 17.1$ are crucial for the variability of the IV curve. Because only aggregate risk commands a risk premium, however, variation of the two sources of risk affect the level and slope of the IV curve differently. Lastly, the composition of cash flow risk is also important for credit risk. The model generates an average 5-year CDS rate at 63 basis points (data: 69 basis points) and somewhat understates its standard deviation at 1.3% (data: 2.0%).

We estimate a value of 9.1 for the representative agent's coefficient of relative risk aversion γ. This value falls between the ones in Barro (2006), who assumes a value of 3 to 4, and Bansal and Yaron (2004), whose main specification assumes a value of 10. In line with this finding, consumption shocks in our model are smaller than in the rare disaster model but larger than in the long-run risks model.

In Table 4, the column Population refers to a simulation based on the unconditional distribution of the aggregate state, i.e., including severe consumption disasters. With more severe crises, risk premia increase. Average excess stock returns increase to 0.74%, the 5-year CDS rate to 107 basis points, at-the-money IV to 27.6%, and the IV skew to 4.0%.

The column LRR in Table 4 refers to an estimation where we calibrate consumption as in the long-run risks literature but we leave assumptions about preferences and firms unchanged. The resulting model resembles those of Bhamra, Kuehn, and Strebulaev (2010) and Chen (2010). The Markov-switching process for consumption approximates the long-run risks process of Bansal and Yaron (2004), and is calibrated as in Bonomo, Garcia, Meddahi, and Tedongap (2010). The purpose of this exercise is to illustrate the effect of the crisis state on corporate policies and asset prices.

The long-run risks estimation fails to explain risk premia. In particular, the average excess return, 5-year CDS rates, at-the-money IV, and IV skew are all much lower than in the data. In contrast, Bhamra, Kuehn, and Strebulaev (2010) and Chen (2010) find that the long-run risks model can explain the credit spread puzzle. The difference arises because we structurally estimate our model, while Bhamra, Kuehn, and Strebulaev (2010) and Chen (2010) follow a calibration exercise. In particular, Chen (2010) measures the average 10-year CDS rate at times when firms issue debt. Bhamra, Kuehn, and Strebulaev (2010) study a dynamic panel of firms. They select firms from a simulated panel, such that at the beginning of the simulation the leverage distribution matches BBB-rated firms from Compustat. Thus, both papers compare credit spreads in the data with those of a subset of firms in the model. In contrast, our structural estimation targets unconditional moments from the full panel of firms. Moreover, while both papers calibrate leverage and credit spreads based on data for different sets of firms, our structural estimation relies on a common dataset. In our data, the credit spread puzzle is particularly pronounced, with leverage ratios below 20% and CDS rates around 70 basis points. In contrast, both Bhamra, Kuehn, and Strebulaev (2010) and Chen (2010) calibrate to leverage ratios of nearly 40%.

4 Determinants of Credit Spreads and Option Prices

This section explores the economic mechanism behind the model's ability to match CDS and option prices. We show how default risk and loss rates vary with financial leverage and the aggregate state, and use counterfactuals to illustrate the quantitative importance of cash flow risk and bankruptcy costs for credit spreads. Next, we discuss the importance of leverage for option prices, and illustrate the relation between default risk and option prices via cross-sectional sorts.

4.1 Decomposing Credit Risk

Figure 4 depicts the 5-year CDS rate as a function of leverage for the benchmark estimation. The red solid line represents booms (high growth rate and low volatility regime), the dashed blue line recessions (low growth rate and high volatility regime), and the black dash-dotted line crisis. Not surprisingly, credit spreads are much higher in crises than recessions. Credit spreads are also strongly convex in leverage, raising average credit spreads through a Jensen effect. For very low leverage ratios, credit spreads are actually falling in leverage. This effect arises because as leverage falls, the probability of firms issuing more debt rises, thereby increasing credit risk.

In Table 5, we report credit market moments that were not targeted by the estimation. In line with existing literature, we find that average bankruptcy costs are sizable on average at 25.9% with a standard deviation of 3.6%. For instance, Andrade and Kaplan (1998) estimate distress costs of 10-23% of firm value for a sample of 31 highly leveraged transactions, Hennessy and Whited (2007) find that bankruptcy costs equal to 8.4% of capital (not asset value) for large and 15.1% for small firms, Davydenko, Strebulaev, and Zhao (2012) estimate an average default cost of 21.7% of the market value of assets from a sample of 175 defaulted firms, and Glover (2016) structurally estimates average cost of 25% among defaulted firms.

While bankruptcy costs capture a variety of factors such as reputation costs, asset fire sales, loss of customer or supplier relationships, legal and accounting fees, and costs of changing management, loss rates on bonds additionally reflect reduced asset values that result from a history of adverse shocks. The corporate finance literature, including Jankowitsch, Nagler, and Subrahmanyam (2014) and Nozawa (2017), has provided evidence in favor of countercyclical loss rates on bonds, we find that bankruptcy costs are strongly countercyclical as well. This finding is new to the literature. Gauging the time-varying nature of bankruptcy costs requires a structural model, because empirical loss rates can be time varying even when bankruptcy costs are constant. Because the representative investor is risk averse, cyclicality drives a wedge between expected bankruptcy costs under the physical and risk-neural measure. In particular, average risk-neutral bankruptcy costs amount to 36.4%, compared to 25.9% under the physical measure. Had we assumed a risk-neutral investor, as is common in corporate finance models, the model would have required bankruptcy costs of around 36.4% to replicate firms' low observed leverage ratios. In contrast, our framework implies that observable bankruptcy costs are much smaller, while the model is still consistent with low leverage. Our exercise therefore highlights the importance of risk adjustments in structural corporate finance models.

A solution to the credit spread puzzle requires that firms have low leverage and low physical default probabilities. As shown in Table 4, the model generates low leverage of around 19%, as in the data. Similarly, the model implies a low default probability of only 4 basis points per year (untabulated). Empirically, we observe a single default event in our sample (General Motors in 2009), which equates to an annual default probability of 10 basis points.[14] Obviously, this is a very noisy estimate because of the small sample size. Table 5 also reports a low average 5-year cumulative default probability of 0.25%, compared with 0.9% for investment grade firms for the period from 1970 to 2017 as reported by Ou, Irfan, Liu, and Kanthan (2018, Exhibit 33). Their sample provides an imperfect comparison with our model, however, because it differs in both the sample period and set of firms.

While the majority of papers studying the credit spread puzzle have focused on the Moody's sample between 1970 and 2017 to measure average default rates, Bhamra, Kuehn, and Strebulaev (2010) and Feldhütter and Schaefer (2018) stress the importance of using default data going back to 1920. Since default events are rare, using only 47 years of data can introduce large sampling error in the observed average rate. As a result, both papers suggest to use the average 5-year default probability for investment grade firms for the years 1920 to 2017, which Ou, Irfan, Liu, and Kanthan (2018, Exhibit 32) report as 1.5%.

In Table 5, the column Population shows credit market moments from the unconditional distribution of the aggregate Markov chain, which includes severe macroeconomic crises such as the Great Depression. In the long sample, severe crises lead to waves in default. As a result, the average 5-year default probability rises fourfold to 0.95%. Overall, the consumption and earnings processes are equipped to generate both post-war and long-sample dynamics, which are close to the data. This is a contribution relative to the existing credit risk literature, which has focused either on post-war or long sample dynamics.

Even though actual default probabilities are low, CDS rates are sizable on average due to a risk compensation for cyclical default risk. Specifically, the conditional risk-neutral h-period ahead default probability $q_{i,t}^h$ can be decomposed into the actual default probability $p_{i,t}^h$ and a risk adjustment, measured by a covariance with the pricing kernel, given by

$$q_{i,t}^h = \mathbb{E}_t^{\mathbb{Q}}\left[1_{\{t_i=t+h\}}\right] = p_{i,t}^h + \mathrm{Cov}_t\left(\frac{M_{t,t+h}}{\mathbb{E}[M_{t,t+h}]}, 1_{\{t_i=t+h\}}\right). \tag{19}$$

Since defaults tend to cluster in crises when marginal utility is high, the covariance is positive and significant. In the model, the average risk-neutral 5-year cumulative default probability is more than 10 times larger than the average physical one, at about 3.3%. At the same time, risk-neutral default probabilities are more volatile than physical ones.

Figure 5 illustrates the gap between physical and risk-neutral default probabilities. The top figure shows physical and the bottom one risk-neutral default probabilities as a function of firms' leverage ratio, where the red solid line represents booms, the dashed blue line recessions,

[14] This number is computed as $12 \times \frac{1}{11{,}694}$, where 11,694 equals the number of firm-month observations.

and the black dash-dotted line crises. In our model, risk-neutral default probabilities dominate physical ones because preferences make the disaster state much more likely under the risk-neutral measure. Since risk-neutral dynamics are key for pricing, credit spreads are large.

The pricing of multi-period CDS contracts also requires the term structure of conditional expected risk-neutral loss rates given default (LGD). The h-period ahead expected risk-neutral LGD can be decomposed into the expected loss rate under the physically measure $L^{\mathbb{P},h}_{i,t,s}$, the ratio of risk-neutral to physical h-period ahead conditional default probabilities, and a risk compensation, measured by a covariance between the pricing kernel and futures losses, given by

$$L^{\mathbb{Q},h}_{i,t,s} = \mathbb{E}^{\mathbb{Q}}_t\left[x_{i,t+h,s}|t_i = t+h\right] = L^{\mathbb{P},h}_{i,s,t}\frac{q^h_{i,t}}{p^h_{i,t}} + \mathrm{Cov}_t\left(\frac{M_{t,t+h}}{\mathbb{E}[M_{t,t+h}]}, x_{i,t+h,s}\middle| 1_{\{t_i=t+h\}}\right). \qquad (20)$$

Since marginal utility and loss rates are countercyclical in our model, the covariance is positive. It is common practice in reduced-form credit risk models to assume a constant LGD. The advantage of a structural credit risk model is that we can recover the term structure of expected physical and risk-neutral LGD. Importantly, long-term CDS rates depend on the entire term structure of expected risk-neutral LGD.

Given our bankruptcy costs estimates, the model generates average one-month LGD of 75% under the physical measure, which is slightly higher than in the data.[15] Ou, Irfan, Liu, and Kanthan (2018, Exhibit 8) report an average senior unsecured bond loss rates of 52.1% for the years 1987 to 2017. Because the model abstracts from indirect equity issuance costs, such as costs associated with informational frictions, it is relatively cheap for equity holders to keep the firm solvent, even in bad times. As a result, share holders declare default at very low asset values, inflating average LGD.

At the short end of the term structure, average loss rates given default do not differ much under the physical and risk-neutral measure. At the 5-year horizon, average expected LGD under the risk-neutral measure rise to 87%, while under the physical measure the term structure of LGD is fairly flat. This effect arises because preferences make the crisis state more likely under the risk-neutral measure than it is under physical one. As a result, the majority of defaults occur in the crisis state, where losses and risk premia are large. Figure 6 illustrates the term structure of LGD under the physical and risk-neutral measure.

[15] We compute realized LGD as expected loss rates (20), evaluated at the optimal default boundary.

4.2 Credit Risk Counterfactuals

In Table 6, we decompose the level and volatility of CDS rates into time-varying bankruptcy costs, time-varying loss rates given default, time-varying risk, and disaster risk. In all these counterfactual experiments, the optimal default, issuance, and target thresholds are taken from the benchmark model. Consequently, these counterfactuals only affect the pricing of derivative claims, but not the firms' capital structure.

In the first experiment, we assume that bankruptcy costs are not time-varying, $\omega_1 = 0$. As a result, the term structure of expected LGD flattens. This shift in the term structure of LGD causes a reduction in the average CDS rate by 2% and in CDS volatility by 2%. Surprisingly, this effect is small even though the wedge between average physical and risk-neutral bankruptcy costs is large. The reason is that time-varying cash flow risks are more important for credit risk, as we will show below.

In the second experiment, we assume a flat time structure of LGD. Specifically, we fix the term structure of expected LGD at the one-month average under the physical measure. This experiment mimics common practice in the reduced-form credit risk literature. This change is more restrictive than the first one, so that the average CDS rate drops by 14% and CDS volatility by 13%. Consequently, reduced-form credit risk models, which typically assume a constant LGD, have to compensate by inflating the degree of time variation in default probabilities.

Our estimation allows for time variation in both aggregate and idiosyncratic risk. While the former is common in the asset pricing literature, the latter has only recently been identified as a robust feature of firms' cash flows by Herskovic, Kelly, Lustig, and Nieuwerburgh (2016). The structural credit risk literature has thus far ignored this channel. A structural decomposition reveals that the time variation in common idiosyncratic risk (third experiment) is more important than time variation in aggregate risk (fourth experiment).

In the third experiment, we assume no time variation in idiosyncratic risk, $\phi_{\zeta,1} = 0$. This experiment has very little effect on the term structure of LGD but it dramatically affects the term structure of default risk under the risk-neutral measure. In particular, the 5-year risk-neutral default probability drops from 3.3% to 1.4% and it also becomes less volatile. Consequently, the average CDS rate drops by 57% and CDS volatility by 23%.

In the fourth experiment, we assume constant aggregate earnings risk, $\phi_{\sigma,1} = 0$. Compared to eliminating time-varying common idiosyncratic risk, constant aggregate earnings risk only leads to a reduction in CDS rates of 33%. These two experiments illustrate that the right composition of risk is crucial to match firm level prices and price dispersions. While the long-run risks literature has stressed the importance of time-varying aggregate risk to generate time-varying risk premia, our results suggest that models with cross-sectional firm heterogeneity require variation in idiosyncratic risk to explain the level and dispersion of credit and option markets.

In the fifth experiment, we assume that there are no persistent crises, $\pi = 0$. Without crisis risk, average credit spreads collapse to 3 basis points compared to 63 basis points in the benchmark model. Crisis risk is the main driver for risk premia. Without crisis risk, there is no wedge between the 5-year physical and risk-neutral default probability and between the 5-year physical and risk-neutral LGD. Consequently, post war consumption dynamics without the fear of dropping into persistent crises fail to generate realistic credit risk premia.

4.3 Option Prices

This section examines how the model generates realistic option prices. We follow the common approach of expressing option prices in Black-Scholes-Merton implied volatility units. Figure 7 shows IVs for the 1-month maturity as a function of standardized moneyness, defined as $\frac{K/S}{\sigma^{ATM}/\sqrt{12}}$. The dashed black line corresponds to the data and the solid red line to the model. The remaining lines are discussed below. We average IVs across firms and months, relying on the same simulation as for the SMM estimation.

The model replicates the shape of the empirical IV curve well. Because options are contingent claims on firms' equity and priced by imposing the absence of arbitrage, a model's ability to match option prices is closely linked to its ability to provide a realistic account of stock market risk premia. Nevertheless, many popular asset pricing theories remain inconsistent with properties of options, as shown by Martin (2017) and Beason and Schreindorfer (2020). The remainder of this section discusses the mechanism underlying our model's success along this important data dimension. We illustrate the role of leverage and crisis risk, and provide a comparison with popular disaster models.

4.3.1 At-the-money IV

The level of the IV curve in Figure 7 is determined by the risk-neutral volatility of stock returns, which reflects cash flow volatility, volatility due to financial leverage, and a variance risk premium.

The variance premium for the aggregate stock market is commonly measured by the difference between the squared VIX index and an estimate of the conditional return variance under the physical measure. The squared VIX measures the risk-neutral entropy of market returns, $VIX^2 = \log \mathbb{E}^{\mathbb{Q}}[R] - \mathbb{E}^{\mathbb{Q}}[\log(R)]$.[16] Being defined in terms of the log function, an equivalent to the VIX at the firm level does not exist, because the possibility of default implies a non-zero probability for returns of -100%. We therefore quantify the variance premium by the difference between the average at-the-money IV and the annualized volatility of monthly returns instead. For the S&P 100 index, this difference is large and positive over our sample period, with an average at-the-money IV of 17.9% and a return volatility of 13.7%. Hence, investors are willing to pay a substantial premium to insure against high aggregate stock market volatility. In contrast, Table 4 shows that the firm level variance premium is negligible: the average at-the-money IV equals 24.3% and return volatility $\sqrt{12} \times 7.22\% = 25.0\%$. Our model replicates this fact, as it matches both the volatility of firms' returns and the average at-the-money IV well.

To quantify the amount of volatility due to financial leverage, we also compute an average IV curve for options that are written on firms' unlevered assets. A comparison between the solid red and dotted blue lines in Figure 7 shows that the at-the-money asset volatility is on average 5.3 percentage points lower than the at-the-money equity volatility. Despite its low average, financial leverage therefore has a sizable effect on the volatility of equity returns. We view this gap as a realistic estimate because our model matches the level and variability of firms' leverage ratios. The decomposition implies that the remaining 19.1% (in absolute terms) of firms at-the-money IV reflect asset volatility.

4.3.2 The IV Skew

The slope of the IV curve, i.e., the IV skew, reflects negative conditional risk-neutral skewness in returns. While cash flow shocks in our model are conditionally Gaussian, the possibility of macroeconomic crises and firm defaults implies that stock returns are not. In particular, crises are associated with low cash flow growth rates and high volatility, so that firms' valuation ratios drop when the economy enters a crises. In normal times, the potential of a crisis therefore induces negative skewness. Similarly, defaults induce negative skewness because they are associated with returns of -100%. The fact that our model replicates the shape of the IV curve, as shown by the solid red line in Figure 7, suggests that it produces a realistic

[16] See Result 3 in Martin (2017).

amount of risk-neutral skewness in returns.

To gauge the relative importance of these two sources of skewness, we again turn to the IV curve for options on firms' unlevered assets.[17] Obviously, default risk plays no role for the pricing of asset options. The dotted blue line in Figure 7 shows that the asset option skew equals 3.57, which is only 9% lower than the corresponding equity option skew. Macroeconomic crises, rather than default risk due to financial leverage, are therefore the main source of skewness in the model. The realism of this implication will become clear in the next section, where we show that the model is consistent with the relationship between default risk and the IV skew in the data.

The IV skew in our model differs substantially from that in prior disaster models. In particular, Backus, Chernov, and Martin (2011) show that the premium for disaster insurance in the model of Barro (2006) is considerably larger than in the data, which is reflected in an implied volatility curve that is much too steep (see their Figure 5). To understand why our model produces a more realistic IV curve, note that there are three differences between the IVs plotted in Figure 7, and those computed by Backus et al.

First, the standard disaster model abstracts from financial leverage and default. This difference has a minor effect on the slope of the IV curve, as we saw above. Second, we price options that are written on individual firms' equity, whereas Backus, Chernov, and Martin (2011) price equity index options. To determine the effect of this difference, we set idiosyncratic risk to zero, $\zeta_t = 0$, and compute the price of options written on a claim to unlevered aggregate earnings. The dash-dotted blue line in Figure 7 shows that the slope of the IV curve for aggregate asset options increases to 5.26. Intuitively, the (unlevered) aggregate IV curve is steeper than the (unlevered) firm level IV curve because systematic tail risk carries a risk premium, while idiosyncratic tail risk does not. Nevertheless, the aggregate IV curve in our model remains substantially flatter than its equivalent in the Barro (2006) model. The third difference to standard disaster models – the fact that we model crises as long-lasting – is therefore the main reason why our model produces a more realistic IV curve. In particular, crises are considerably less painful to investors (in utility terms) if they unfold slowly, rather than materialize in an instant. This implication is reflected in the fact that our estimation requires a risk aversion coefficient of 9.1 to match risk premia, while Barro (2006) matches the equity premium based on a value of 3.

[17] A third source of skewness is the leverage effect of Black (1976), which induces negative skewness by increasing volatility after negative returns. Because this channel only plays a role for multi-period returns, however, it does not affect the pricing of 1-month options in our model.

These results show that the disaster risk implied by option prices are not necessarily small and frequent, as suggested by Backus, Chernov, and Martin (2011), but instead can also be characterized as large and rare when they are modeled as being persistent. An important benefit of our data generating process is that it is capable of generating samples that include severe disasters (such as the long U.S. sample going back to the 1930s) as well as samples without large consumption drops (such as the U.S. post-war sample). In contrast, the preferred calibration of Backus et al. makes it nearly impossible to observe episodes like the Great Depression.

4.4 Default Risk and Option Prices

To motivate our estimation approach, we have argued that option prices contain valuable information about cash flow risks and default probabilities. A natural question is therefore whether firms' IV curves vary systematically with default risk in the data. To test this implication, we sort firms into deciles based on Merton's distance-to-default (DD) measure, a proxy for default risk that can be easily computed in the model and data. DD is defined as $\frac{\ln[(E+D)/D]+(r_i-0.5\sigma_V^2)T}{\sigma_V\sqrt{T}}$, where E is the value of equity, D the value of debt, r_i the firm's stock return over the past year, T the horizon in years, and σ_V the firm's asset volatility. We follow Bharath and Shumway (2008) and implement their so-called naïve DD measure.[18] They find that the naïve DD performs better than the original Merton's DD measure in predicting defaults out-of-sample, while being based on the same inputs and easier to compute. Because we are interested in whether short-term option contracts are informative about the longer-horizon default probabilities reflected in CDS, we set the DD horizon to $T = 5$ years.

Within each DD decile, we compute the average at-the-money IV, IV skew, and CDS rate across firms. Table 7 shows that, in the model, at-the-money IV decreases monotonically from 32.1% (data: 34.7%) for firms with the highest default risk to 22.3% (data: 20.0%) for firms with the lowest default risk. This pattern is intuitive, because higher default risk reflects a combination of higher asset volatility and higher leverage, both of which are associated with higher stock return volatility. We also find that the IV skew decreases from 4.6% (data: 4.4%) to 3.7% (data: 3.7%) across deciles. The reason is that default events result in equity returns of -100%, so that higher default probabilities are associated with more negative conditional

[18]For the naïve implementation of the DD measure, Bharath and Shumway approximate asset volatility by $\sigma_V = \frac{E}{E+D}\sigma_E + \frac{D}{E+D}\sigma_D$, and debt volatility by $\sigma_D = 0.05 + 0.25\sigma_E$. Equity volatility σ_E is measured by the annualized standard deviation of monthly returns over the past year. In the model and data, we use the market value of equity but the book value of debt.

return skewness. Clearly, the model does a nice job of capturing the relation between default risk and option prices, despite the fact that these moments were not targeted by the estimation. While perhaps qualitatively unsurprising, the model provides a good match for the quantitative relationship between DD and CDS rates, which drop from 2.6% (data: 2.3%) for firms with the highest default risk to 0.1% (data: 0.3%) for firms with the lowest default risk.

These findings suggest that the model provides a realistic mapping between the volatility of firms' cash flows, their endogenous capital structure decisions, and the conditional distribution of their stock returns. In contrast, popular reduced-form option pricing models in the spirit of Black and Scholes (1973) and Heston (1993) are not able to capture these effects because they abstract from capital structure decisions. Instead, this literature models stock returns as an exogenous process.

5 Conclusion

Macroeconomic crises provide a unique window into connections between asset markets and the real economy. In this paper, we propose an asset pricing model that matches the joint dynamics of aggregate consumption, firms' leverage ratios, and asset prices during macroeconomic crises. In particular, the model replicates the fact that longer-lasting crises are associated with bigger macroeconomic contractions, larger increases in financial leverage, credit spreads, and equity volatility, and significantly more firm defaults. Capturing these dynamics is important because the nature of potential crises is a central driver of asset prices even during normal times.

Long-lasting crises enable the model to explain risk premia across equity, credit, and option markets. Specifically, we show that leverage explains a significant part of the time-series and cross-sectional variation in equity option prices. Because options reflect equity risks, default risk should also not be ignored when modeling stock prices. Yet, common reduced-form approaches for capturing leverage in representative agent asset pricing models, such as the one in Abel (1999), ignore time-variation in leverage. As a result, many models abstract from a crucial source of time-variation in equity values.

Lastly, a structural estimation of our model reveals a large gap between historical and risk-neutral bankruptcy costs. This finding highlights the importance of risk-adjustments in structural corporate models, where investors have traditionally been modeled as risk-neutral. We also show that time variation in idiosyncratic risk plays a more important role than time variation in aggregate risk in rationalizing the credit spread puzzle. Overall, we believe that our work provides a step towards a joint framework for macroeconomics, asset pricing, and corporate finance.

Appendix

The model solution relies on a risk-neutral probability measure, which is derived via a change of measure in Appendix A. Appendix B expresses the model in stationary units and solves for asset prices. Details of our numerical solution approach are discussed in Appendix C.

A Change of Measure

We value all financial claims under the risk-neural measure $\mathbb{Q}$. Given the pricing kernel (2), the change of measure is

$$\frac{d\mathbb{Q}}{d\mathbb{P}} = \frac{M_{t,t+1}}{\mathbb{E}_t[M_{t,t+1}]} = e^{(s_{t+1}-\bar{s}_t)-\gamma\sigma_{c,t}\varepsilon_{c,t+1}-\frac{1}{2}\gamma^2\sigma_{c,t}^2},$$

where

$$S_{t+1} = \left(\frac{\lambda_{t+1}^c + 1}{\lambda_t^c}\right)^{-(1-\theta)} \qquad s_{t+1} = \ln S_{t+1} \quad \bar{S}_t = \mathbb{E}_t[S_{t+1}] \quad \bar{s}_t = \ln \bar{S}_t$$

The distribution of consumption growth under $\mathbb{Q}$ is

$$\begin{aligned} f^{\mathbb{Q}}(\varepsilon_{c,t+1}) &= f^{\mathbb{P}}(\varepsilon_{c,t+1})\frac{d\mathbb{Q}}{d\mathbb{P}} \\ &= \frac{1}{\sqrt{2\pi}}\exp\left\{-\frac{\varepsilon_{c,t+1}^2}{2}\right\}\exp\{-\gamma\sigma_{c,t}\varepsilon_{c,t+1} - \frac{1}{2}\gamma^2\sigma_{c,t}^2\} \\ &= \frac{1}{\sqrt{2\pi}}\exp\left\{-\frac{(\varepsilon_{c,t+1}+\gamma\sigma_{c,t})^2}{2}\right\} \end{aligned}$$

Thus, $\varepsilon_{c,t+1}^{\mathbb{Q}} = \varepsilon_{c,t+1}^{\mathbb{P}} + \gamma\sigma_{c,t}$ and the dynamics of consumption growth is

$$g_{c,t+1} = (\mu_{c,t} - \gamma\sigma_{c,t}^2) + \sigma_{c,t}\varepsilon_{c,t+1}^{\mathbb{Q}}.$$

Similarly, the change of measure for the Markov chain is given by

$$\mathcal{Q} = \mathcal{P}\frac{d\mathbb{Q}}{d\mathbb{P}} = \mathcal{P}e^{(s_{t+1}-\bar{s}_t)},$$

where $\mathcal{Q}$ is the transition matrix under the risk-neutral measure.

The distribution of log earnings growth under $\mathbb{Q}$ is

$$g_{i,t+1} = \mu_t^{\mathbb{Q}} + \bar{\sigma}_t\varepsilon_{i,t+1}^{\mathbb{Q}}$$

where $\mu_t^{\mathbb{Q}} = \mu_t - \gamma\rho\sigma_t\sigma_{c,t}$ and $\bar{\sigma}_t = \sqrt{\sigma_t^2 + \zeta_t^2}$

B Stationary Model

Since earnings grow exponentially, we detrend all valuation equations by earnings and solve for stationary valuations ratios under the risk-neutral measure.

The stationary cum-earnings asset-to-earnings ratio, $\lambda_t^a = A_{i,t}/E_{i,t}$, solves

$$\lambda^a(\xi_t) = (1-\tau_c)(1-\tau_d) + e^{-r_t^f + \mu_t^{\mathbb{Q}} + \frac{1}{2}\bar{\sigma}_t^2} \mathbb{E}_t^{\mathbb{Q}}[e^{g_{i,t+1}} \lambda^a(\xi_{t+1})], \tag{B.1}$$

which depends on the aggregate state ξ_t and $r_t^f = \ln(R_{t,t+1}^f)$ is the one-period log risk-free rate. For an n-state Markov chain, define an $n \times 1$ vector X whose i-th element equals $e^{-r_t^f + \mu_t^{\mathbb{Q}} + \frac{1}{2}\bar{\sigma}_t^2}$ when ξ_t is in state i. The cum-earnings asset-earnings ratio vector can be solved exactly as

$$\lambda^a = (1-\tau_c)(1-\tau_d)(I_n - ((X \cdot \iota_n^\top) \odot Q))^{-1} \iota_n,$$

where I_n is the identity matrix of dimension n and ι_n is a vector of ones with length n.

The cum-coupon debt-to-earnings ratio, $\lambda_t^d = D_{i,t}/E_{i,t}$, depends on the firm's interest coverage ratio and aggregate state and solves

$$\begin{aligned} \lambda^d(\kappa_{i,t}, \xi_t) &= 1_{\{\kappa_{i,t} \leq \kappa_t^D\}} (1-\omega_t) \lambda^a(\xi_t) \\ &+ 1_{\{\kappa_t^D < \kappa_{i,t} < \kappa_t^I\}} \left(\frac{(1-\tau_i)}{\kappa_{i,t}} + e^{-r_t^f} \mathbb{E}_t^{\mathbb{Q}}[e^{g_{i,t+1}} \lambda^d(\kappa_{i,t} e^{g_{i,t+1}}, \xi_{t+1})] \right) \\ &+ 1_{\{\kappa_t^I \leq \kappa_{i,t}\}} \left(\frac{(1-\tau_i)}{\kappa_{i,t}} + \frac{\bar{\kappa}_t}{\kappa_{i,t}} e^{-r_t^f} \mathbb{E}_t^{\mathbb{Q}}[e^{g_{i,t+1}} \lambda^d(\bar{\kappa}_t e^{g_{i,t+1}}, \xi_{t+1})] \right). \end{aligned}$$

The stationary cum-dividend equity-to-earnings ratio, $\lambda_t^s = S_{i,t}/E_{i,t}$, depends on the firm's interest coverage ratio and aggregate state and solves

$$\begin{aligned} \lambda^s(\kappa_{i,t}, \xi_t) &= \max \Bigg\{ 0, 1_{\{\kappa_{i,t} < \kappa_t^I\}} \Bigg((1-\tau_c)(1-\tau_d) \left(1 - \frac{1}{\kappa_{i,t}}\right) 1_{\{\kappa_{i,t} \geq 1\}} \\ &+ (1-\tau_c+\phi_e) \left(1 - \frac{1}{\kappa_{i,t}}\right) 1_{\{\kappa_{i,t} < 1\}} + e^{-r_t^f} \mathbb{E}_t^{\mathbb{Q}} \left[e^{g_{i,t+1}} \lambda^s(\kappa_{i,t} e^{g_{i,t+1}}, \xi_{t+1}) \right] \Bigg) \\ &+ 1_{\{\kappa_t^I \leq \kappa_{i,t}\}} \left((1-\tau_c)(1-\tau_d) \left(1 - \frac{1}{\kappa_{i,t}}\right) + \tilde{\Delta}_{i,t} + e^{-r_t^f} \mathbb{E}_t^{\mathbb{Q}}[e^{g_{i,t+1}} \lambda^s(\bar{\kappa}_{i,t} e^{g_{i,t+1}}, \xi_{t+1})] \right) \Bigg\}, \end{aligned}$$

where the debt issuance proceeds are

$$\tilde{\Delta}_{i,t} = \left(1 - \phi_d - \frac{\bar{\kappa}_t}{\kappa_{i,t}}\right) \lambda^{d,x}(\bar{\kappa}_t, \xi_t)$$

and $\lambda^{d,x} = D_{i,s}^x(c_{i,t})/E_{i,t}$ is the ex-coupon debt-to-earnings ratio.

In stationary terms, the levered firm value-to-earnings ratio, λ_t^f, is given by

$$\begin{aligned}
\lambda^f(\kappa_{i,t},\xi_t) &= 1_{\{\kappa_{i,t}\le\kappa_t^D\}}(1-\omega_t)\lambda^a(\xi_t)\\
&+ 1_{\{\kappa_t^D<\kappa_{i,t}<\kappa_t^I\}}\Bigg((1-\tau_c)(1-\tau_d)\left(1-\frac{1}{\kappa_{i,t}}\right)1_{\{\kappa_{i,t}\ge 1\}}+\frac{(1-\tau_i)}{\kappa_{i,t}}\\
&+ (1-\tau_c+\phi_e)\left(1-\frac{1}{\kappa_{i,t}}\right)1_{\{\kappa_{i,t}<1\}}+e^{-r_t^f}\mathbb{E}_t^{\mathbb{Q}}\big[e^{g_{i,t+1}}\lambda^f(\kappa_{i,t}e^{g_{i,t+1}},\xi_{t+1})\big]\Bigg)\\
&+ 1_{\{\kappa_t^I\le\kappa_{i,t}\}}\Bigg((1-\tau_c)(1-\tau_d)\left(1-\frac{1}{\kappa_{i,t}}\right)+\frac{(1-\tau_i)}{\kappa_{i,t}}-\phi_d\lambda^{d,x}(\bar{\kappa}_{i,t},\xi_t)\\
&+ e^{-r_t^f}\mathbb{E}_t^{\mathbb{Q}}\big[e^{g_{i,t+1}}\lambda^f(\bar{\kappa}_t e^{g_{i,t+1}},\xi_{t+1})\big]\Bigg).
\end{aligned}$$

Maximizing firm value implies that the state-dependent optimal interest coverage ratio is given by

$$\bar{\kappa}(\xi_t)=\arg\max_{\kappa}\Big\{-\phi_d\lambda^{d,x}(\kappa,\xi_t)+e^{-r_t^f}\mathbb{E}_t^{\mathbb{Q}}\big[e^{g_{i,t+1}}\lambda^f(\kappa e^{g_{i,t+1}},\xi_{t+1})\big]\Big\}$$

which captures the trade-off between marginal issuance costs and future tax benefit. Similarly, the state dependent optimal issuance threshold is given by the interest coverage ratio when the firm continuation value of issuance dominantes the one without issuance,

$$\kappa^I(\xi_t)=\min\Big\{\kappa_{i,t}:\mathbb{E}_t^{\mathbb{Q}}\big[e^{g_{i,t+1}}\lambda^f(\bar{\kappa}_t e^{g_{i,t+1}},\xi_{t+1})\big]\ge\mathbb{E}_t^{\mathbb{Q}}\big[e^{g_{i,t+1}}\lambda^f(\kappa_{i,t}e^{g_{i,t+1}},\xi_{t+1})\big]\Big\}.$$

B.1 CDS Valuation

By no-arbitrage, the one-period CDS rate $z_{i,t,s}^1$ is given by

$$z^1(\kappa_{i,t},\xi_s,\xi_t)=\frac{\mathbb{E}_t^{\mathbb{Q}}\left[1_{\{t_i=t+1\}}x_{i,t+1,s}\right]}{1-\mathbb{E}_t^{\mathbb{Q}}\left[1_{\{t_i=t+1\}}\right]}=\frac{H_{i,t,s}^1}{1-q_{i,t}^1},$$

where $H_{i,t,s}^1$ is the risk-neutral one-period expected loss rate and $q_{i,t}^1$ the risk-neutral one-period conditional default probability.

Given the Gaussian-Markov switching environment, we can derive a closed-form expression for both the risk-neutral default probability as well as the risk-neutral loss rate. This step facilitates the structural estimation of the model because we do not have to rely on Monte-Carlo pricing for CDS rates. More specifically, the risk-neutral one-period default probability is given by

$$\begin{aligned}
q_{i,t}^1 &= \mathbb{E}_t^{\mathbb{Q}}\left[1\{\kappa_{i,t+1}\le\kappa_{t+1}^D\}\right]\\
&= \mathbb{E}_t^{\mathbb{Q}}\left[1\{\mu_t^{\mathbb{Q}}+\bar{\sigma}_t\varepsilon_{i,t+1}^{\mathbb{Q}}\le\log(\kappa_{t+1}^D/\kappa_{i,t})\}\right]\\
&= \mathbb{E}_t^{\mathbb{Q}}[\Phi(a_{i,t+1})] \qquad a_{i,t+1}=\frac{\log\left(\frac{\kappa_{t+1}^D}{\kappa_{i,t}}\right)-\mu_t^{\mathbb{Q}}}{\bar{\sigma}_t}
\end{aligned}$$

where Φ denotes the standard normal CDF, $a_{i,t+1}$ the negative of the distance-to-default, and $\bar{\sigma}_t$ is total asset volatility.

The risk-neutral one-period expected loss rate $H^1_{i,t,s}$ is given by

$$\begin{aligned}
H^1_{i,t,s} &= \mathbb{E}^{\mathbb{Q}}_t \left[1_{\{t_i=t+1\}} x_{i,t+1,s}\right] \\
&= \mathbb{E}^{\mathbb{Q}}_t \left[1\{\kappa_{i,t+1} \leq \kappa^D_{t+1}\} \left(1 - \frac{(1-\omega_{t+1})\lambda^a_{i,t+1}}{\lambda^d_i(\bar{\kappa}_{i,s}, \xi_s)} \frac{\kappa_{i,t+1}}{\bar{\kappa}_s}\right)\right] \\
&= \mathbb{E}^{\mathbb{Q}}_t \left[1\{\kappa_{i,t+1} \leq \kappa^D_{t+1}\}\right] - \mathbb{E}^{\mathbb{Q}}_t \left[1\{\kappa_{i,t+1} \leq \kappa^D_{t+1}\} \frac{(1-\omega_{t+1})\lambda^a_{i,t+1}}{\lambda^d_i(\bar{\kappa}_s, \xi_s)} \frac{\kappa_{i,t+1}}{\bar{\kappa}_s}\right] \\
&= \mathbb{E}^{\mathbb{Q}}_t[\Phi(a_{i,t+1})] - \mathbb{E}^{\mathbb{Q}}_t \left[\frac{(1-\omega_{t+1})\lambda^a_{i,t+1}}{\lambda^d_i(\bar{\kappa}_s, \xi_s)\bar{\kappa}_s} \kappa_{i,t} e^{\mu^{\mathbb{Q}}_t + 1/2\bar{\sigma}^2_t} \Phi(a_{i,t+1} - \bar{\sigma}_t)\right]
\end{aligned}$$

The last term follows from $\mathbb{E}\{1\{\epsilon \leq a\} e^{b\epsilon}] = \mathbb{E}[e^{1/2b^2} \Phi(a-b)]$.

The risk-neutral one-period expected loss rate given default can be computed from

$$L^{\mathbb{Q},1}_{i,t,s} = \mathbb{E}^{\mathbb{Q}}_t \left[x_{i,t+1,s} | t_i = t+1\right] = \frac{H^1_{i,t,s}}{q^1_{i,t}}.$$

By no-arbitrage, the T-period CDS rate is given by

$$z^T(\kappa_{i,t}, \xi_s, \xi_t) = \frac{\sum_{h=1}^T \mathbb{E}^{\mathbb{Q}}_t \left[1_{\{t_i=t+h\}} x_{i,t+h,s}\right] / R^f_{t,t+h}}{\sum_{h=1}^T (1 - \mathbb{E}^{\mathbb{Q}}_t \left[1_{\{t_i \leq t+h\}}\right]) / R^f_{t,t+h}},$$

where the realized loss rate is

$$x_{i,t+h,s} = 1 - \frac{(1-\omega_{t+h})\lambda^a_{i,t+h}}{\lambda^d_{i,s}(\bar{\kappa}_{i,s}, \xi_s)} \frac{\kappa_{i,t+h}}{\bar{\kappa}_{i,s}}.$$

By the law of iterated expectations, the h-period ahead conditional risk-neutral default probability $q^h_{i,t}$ and expected loss rate $H^h_{i,t,s}$ can be computed recursively as

$$\begin{aligned}
q^h_{i,t} &= \mathbb{E}^{\mathbb{Q}}_t \left[q^{h-1}_{i,t+1}\right] \\
H^h_{i,t,s} &= \mathbb{E}^{\mathbb{Q}}_t \left[H^{h-1}_{i,t+1,s}\right],
\end{aligned}$$

starting with the one-period probability and expected loss rate.

B.2 Option Valuation

The put price-earnings ratio $\lambda^p_{i,t} = P_{i,t}/E_{i,t}$ is given by

$$\lambda^p_{i,t} = e^{-r^f_t} \mathbb{E}^{\mathbb{Q}}_t \left[\max\left\{\lambda^{s,x}_{i,t} X - 1_{\{\kappa^I_{t+1} \leq \kappa_{i,t+1}\}} e^{g_{i,t+1}} \tilde{\Delta}_{i,t+1} - e^{g_{i,t+1}} \lambda^{s,x}_{i,t+1}, 0\right\}\right],$$

where $X = K/S^x_{i,t}$ is the option's moneyness and $\lambda^{s,x}_{i,t}$ is the ex-dividend price-earnings ratio. Because the Black-Scholes-Merton formula is homogeneous of degree zero in the spot price of the underlying asset and the strike price, implied volatilities can then be found based on $\lambda^p_{i,t}$ and X from

$$\lambda^{BS}_{i,t} = N(-d_2)e^{-r}X(\lambda^{s,x}_{i,t} - \lambda^{div}_{i,t}) - N(-d_1)(\lambda^{s,x}_{i,t} - \lambda^{div}_{i,t}),$$

where $\lambda^{div}_{i,t}$ is the ratio of ordinary dividends-to-earnings and

$$d_1 = \frac{1}{\sigma}\left(\ln\frac{1}{X} + (r + \sigma^2/2)\right) \qquad d_2 = d_1 - \sigma.$$

C Numerical Solution

We use a standard non-linear equation solver to find the state-dependent wealth-consumption ratio $\lambda^c(\xi_t)$ from (3). Based on the pricing kernel (2), we can then compute the 1-period risk-free rate as $R^f_t = 1/\mathbb{E}_t[M_{t+1}]$ analytically. The asset-to-earnings ratio $\lambda^a(\xi_t)$ of an unlevered firm can be computed based on (B.1). We use Gaussian quadrature with 51 nodes to evaluate the expectation over $\varepsilon^{\mathbb{Q}}_{i,t+1}$ and $\mathcal{Q}$ to evaluate expectations over the future aggregate state.

The state space of an individual firm consists of the aggregate Markov state ξ_t and the continuous-valued interest coverage ratio $\kappa_{i,t}$. We discretize κ on an equally-spaced grid with 8,000 points between 0.1 and 100, and use value function iteration to compute the joint fixed point for $\lambda^s(\kappa_{i,t}, \xi_t)$ and $\lambda^d(\kappa_{i,t}, \xi_t)$ based on their equations in the appendix. In doing so, we solve for the optimal aggregate state-dependent thresholds $\kappa^D(\xi_t)$, $\kappa^I(\xi_t)$, and $\bar{\kappa}(\xi_t)$. Because the thresholds are associated with kinks in the value function, the optimization restricts them to grid values of κ, which makes it essential to rely on a large number of grid points. We use Gaussian quadrature with 51 nodes to evaluate expectations, and linear interpolation to evaluate λ^s_t and λ^d_t at off-grid values of κ that result from the quadrature. We iterate on λ^s_t and λ^d_t until the maximum absolute change across all values in the state space falls below 10^{-4} for both functions, and the thresholds κ^D_t, κ^I_t, and $\bar{\kappa}_t$ remain unchanged at their respective grid values.

Given solutions for the equity-to-earnings and debt-to-earnings rations, CDS and option prices can be evaluated based on the formulae in the appendix. We rely on the same quadrature routine as before to evaluate expectations.

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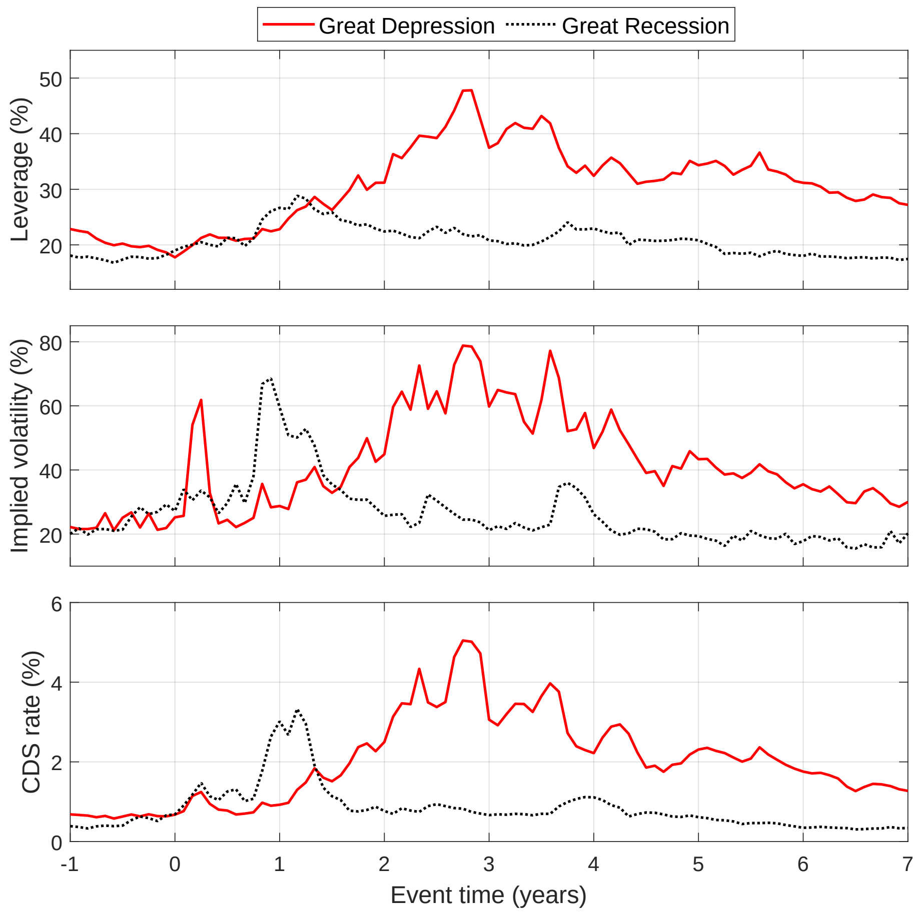

Figure 1: Crises Dynamics

This figures depicts average leverage, at-the-money implied volatilities, and 5-year CDS rates for a set of large public firms for the Great Depression and Great Recession. Time zero corresponds to the start date of the respective NBER recession. Implied volatilities and CDS rates for the Great Depression period are based on projections.

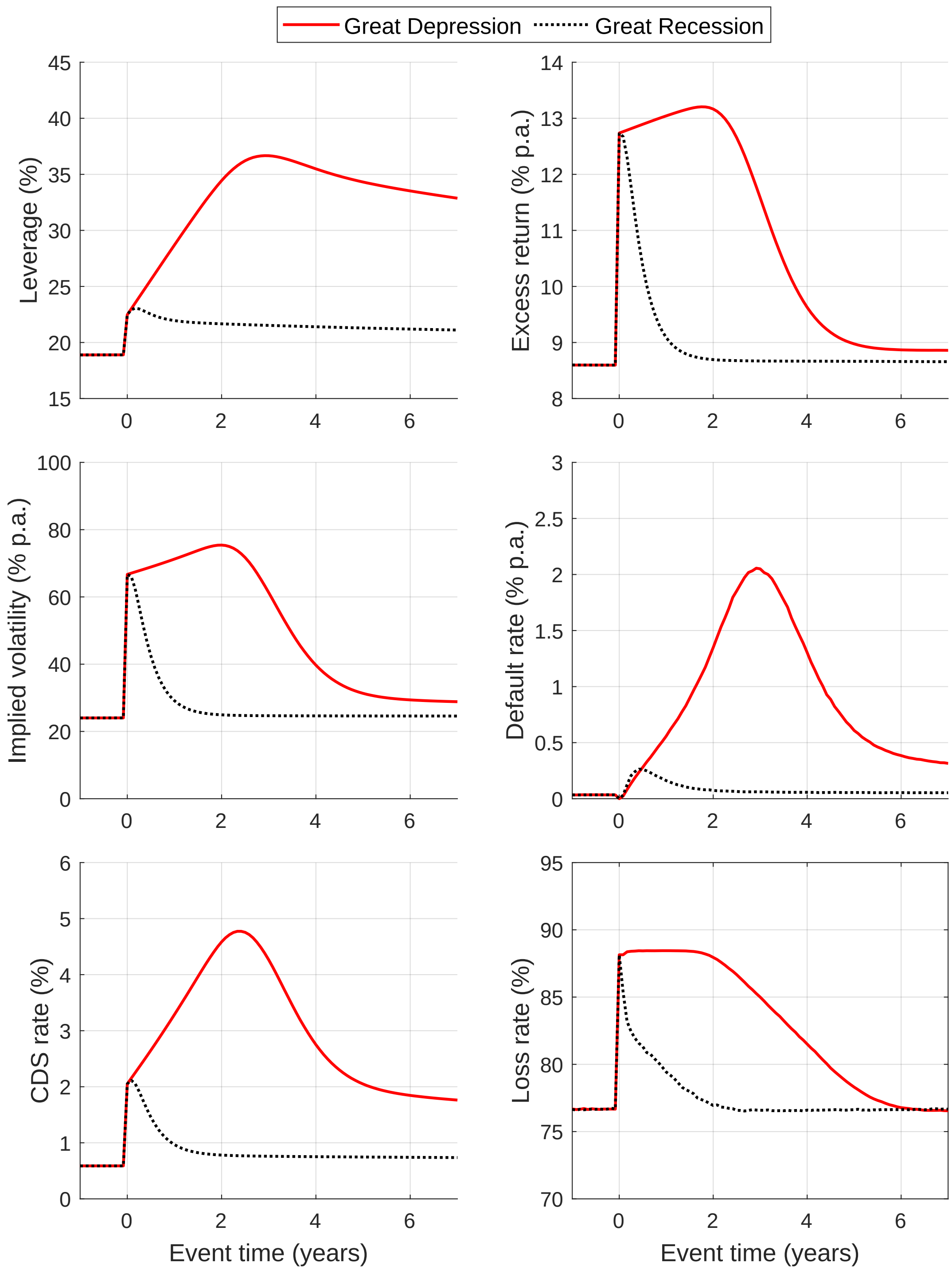

Figure 2: Simulated Crises in the Model

This figure shows firm-level asset prices during simulated crises in the model. We simulate the aggregate state, impose that the economy enters the crisis state at time zero, and keep 1,000,000 state paths for which the cumulative consumption drop during the crisis equals $-16.58\% \pm 0.5\%$ to mimic the Great Depression or $-2.14\% \pm 0.5\%$ to mimic the Great Recession. We then simulate a panel of 100 firms for each path, average their asset prices in each period, and plot the mean value across paths.

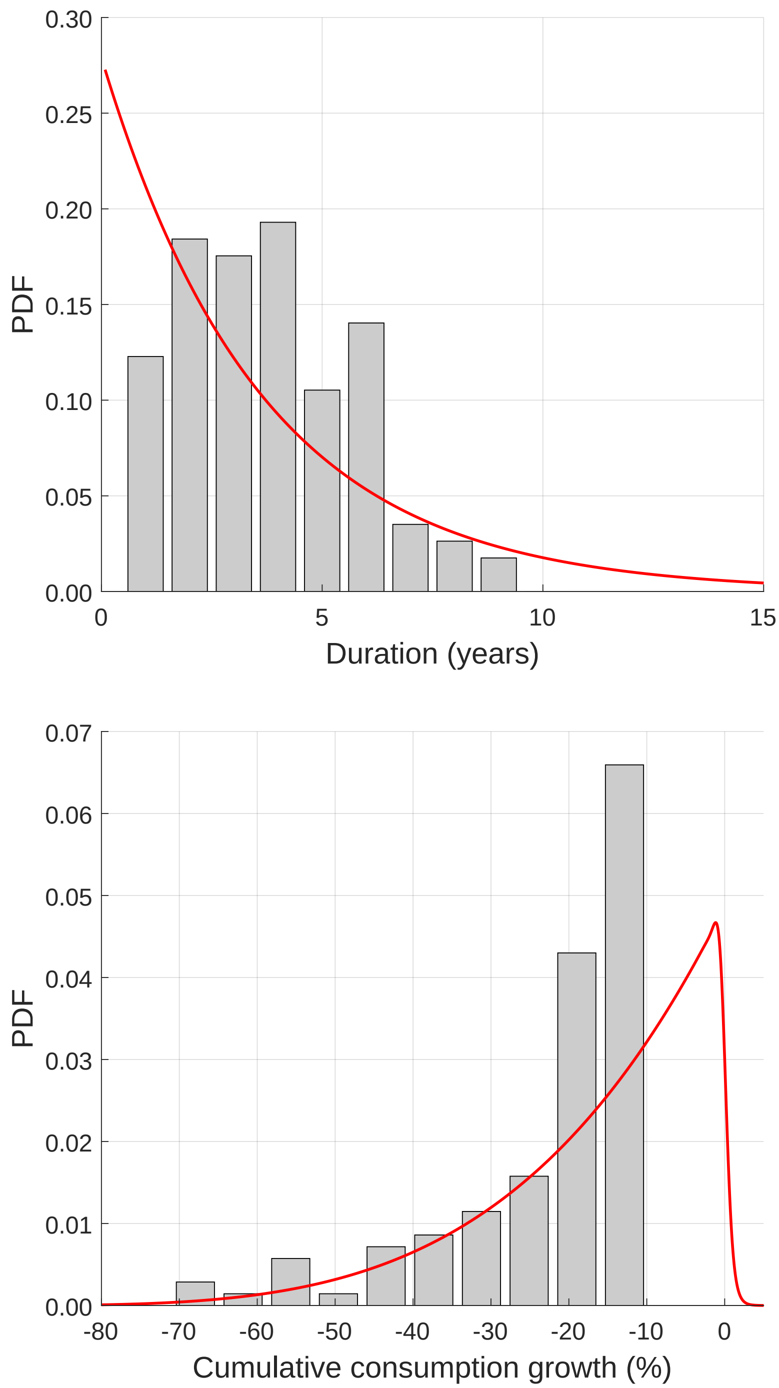

Figure 3: Characteristics of Macroeconomic Crises
The top panel shows the distribution of crises durations and the bottom panel the distribution of cumulative consumption drops during crises. Crises correspond to state 3 in the model. The gray bars show the distribution of peak-to-trough consumption declines of 10% or more in the international dataset of Barro and Ursua (2012).

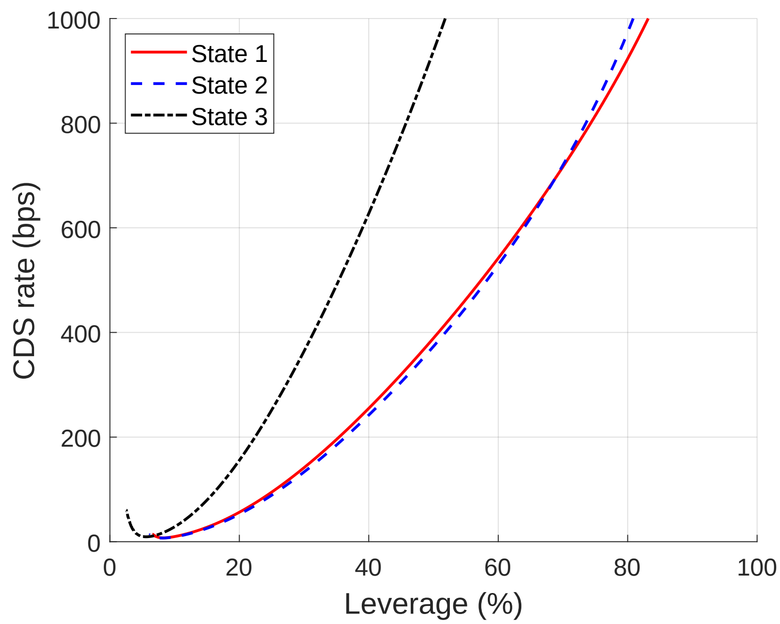

Figure 4: CDS Rates

This figure depicts the 5-year CDS rate in the model as a function of the aggregate state and the firm's leverage ratio.

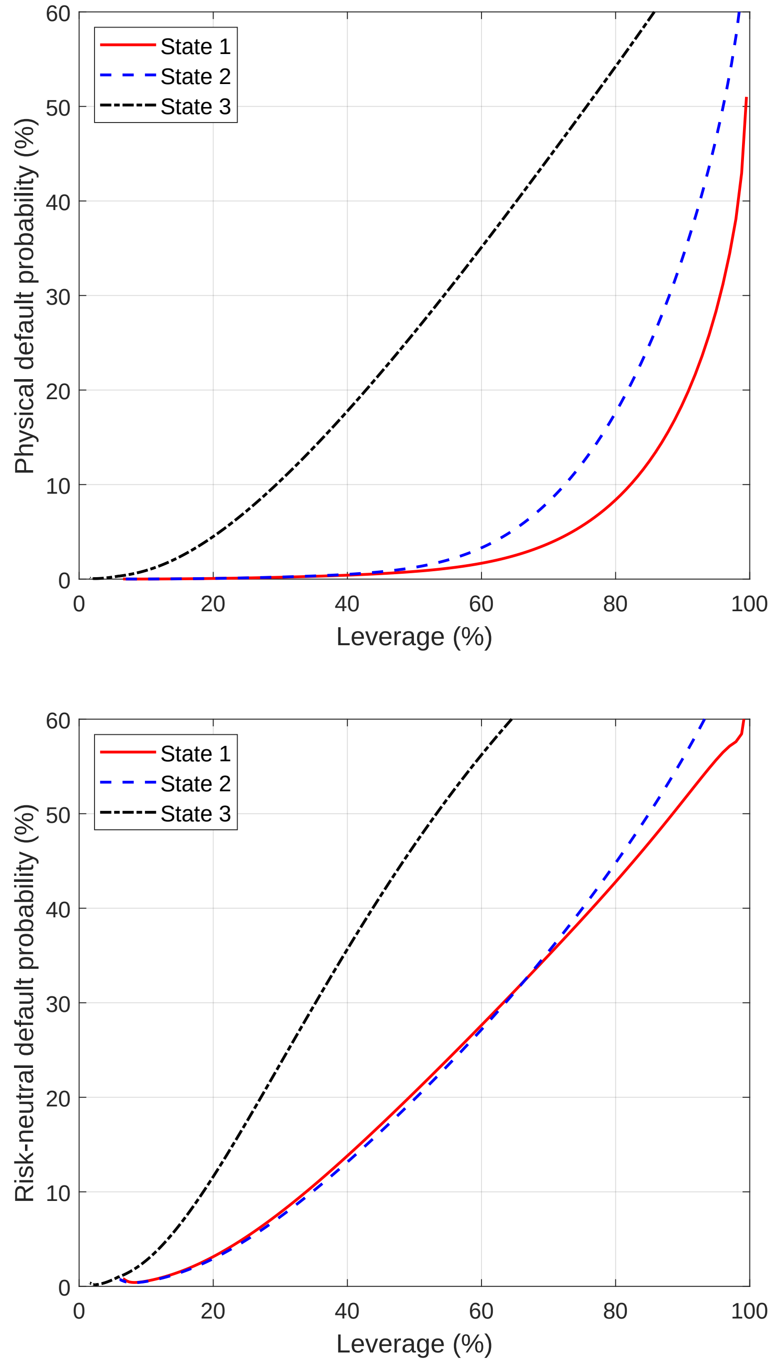

Figure 5: Default Probabilities
For the 5-year horizon, this figures depicts the probability of default under the physical (top panel) and risk-neutral (bottom panel) measure in the model as a function of the aggregate state and the firm's leverage ratio.

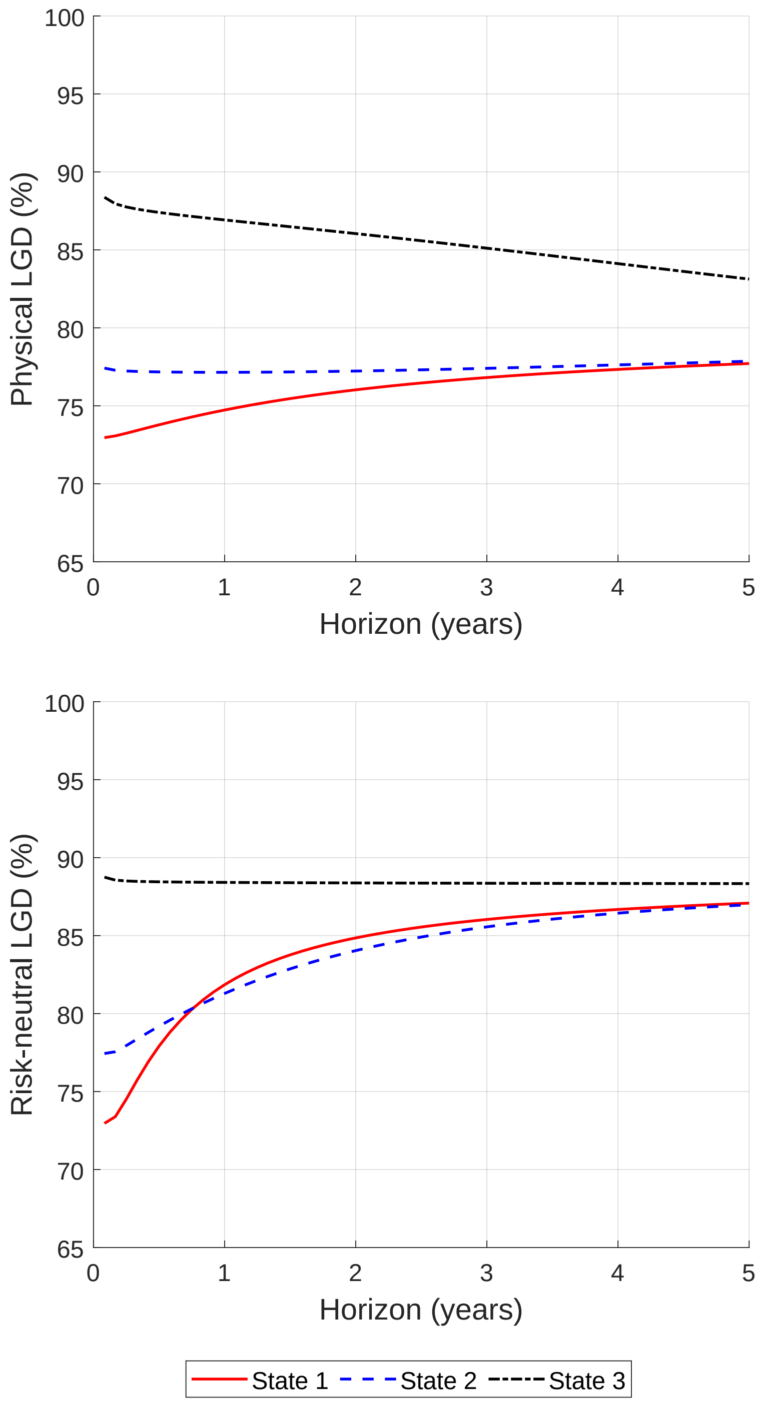

Figure 6: Loss Rates
This figures depicts the term structure of expected loss rates given default under the physical and risk-neutral measure, evaluated at the optimal default boundary.

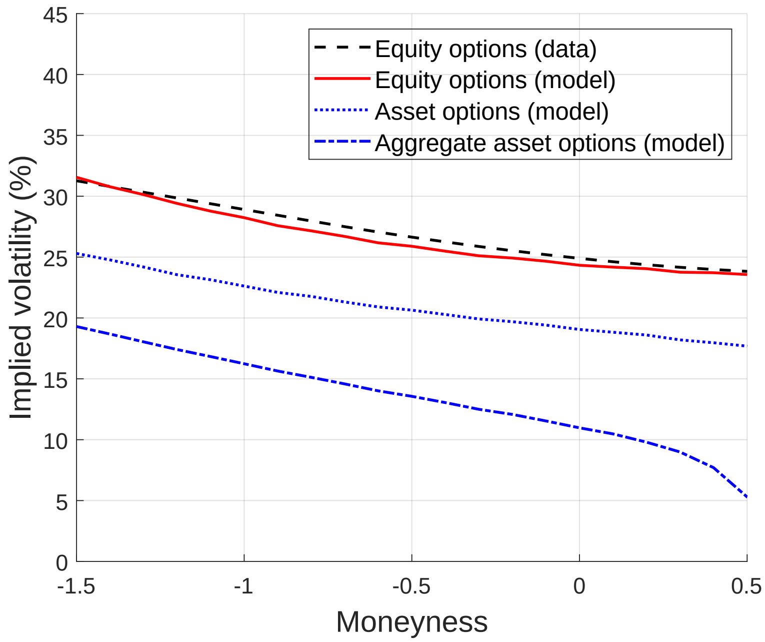

Figure 7: Implied Volatility Curve

This figure shows Black-Scholes-Merton IVs for for the 1-month maturity against standardized moneyness, defined as $\frac{K/S}{\sigma^{ATM}/\sqrt{12}}$. Both in the model and data, IVs are averaged across firms and months. Model data is generated based on the same simulation as the SMM estimation. Asset options are written on the firm's ex-earnings after-tax asset value. For aggregate asset options, we assume that earnings growth is subject to systematic risk only. The sample spans 2004 to 2019.

Table 1: Consumption Dynamics

This table shows the calibration of the Markov-switching process defined in equations (1), (14) and (15) and implied moments for annual (time-aggregated) log consumption growth. Consumption is defined as the sum of real non-durable and services consumption per capita. Crisis moments are based on the Great Depression from August 1929 to March 1933. Moments of post-war growth rates (expressed as percentages) are computed for the period 1947 to 2019.

Panel A: Parameters

State	1	2	3
$\mu_c \times 12$	0.0275	0.0108	-0.0544
$\sigma_c \times \sqrt{12}$	0.0094	0.0130	0.0244
$\mathcal{P}$	p	p_3	$\pi \times 12$
	0.9816	0.9773	0.0116

Panel B: Crises Moments

Average duration (months)	44
Average cumulative consumption drop (%)	16.58
Unconditional probability (%)	4.07
Conditional volatility of g_c (%/year)	2.44

Panel C: Post-War Moments

Moment	$\mathbb{E}[g_c]$	$\sigma(g_c)$	Skew(g_c)	Kurt(g_c)	AC$_1(g_c)$
Data	1.89	1.20	-0.35	2.85	0.46
Model	1.89	1.20	-0.35	2.85	0.46

Table 2: Predefined Parameters

This table lists calibrated model parameters. Details are discussed in Section 3.3.

Parameter		Value
Time discount rate	β	0.9977
Elasticity of intertemporal substitution	ψ	1.5
Consumption-earnings correlation	ρ	0.2
Corporate tax rate	τ_c	0.341
Equity payout tax rate	τ_e	0.135
Interest tax rate	τ_i	0.35
Debt issuance costs	ψ_d	0.01
Equity issuance costs	ψ_e	0.04

Table 3: SMM Parameter Estimates

This table shows the SMM parameter estimates for the model, along with standard errors in parenthesis. The estimation targets eleven moments: mean and variance of leverage, stock returns, 5-year CDS rate, at-the-money implied volatility, and implied volatility skew, as well as the variance of market returns. The benchmark model features long-lasting consumption crises. The column LRR replaces the consumption process with a Markov chain approximation of the long-run risks process in Bansal and Yaron (2004) and assumes risk aversion of 10.

Parameter		Model	LRR
Idiosyncratic volatility	$\phi_{\zeta,0}$	0.0457	0.0395
		(0.0003)	(0.0003)
Idiosyncratic volatility scaling	$\phi_{\zeta,1}$	22.4387	0.6180
		(0.6399)	(0.0723)
Aggregate volatility	$\phi_{\sigma,0}$	0.0319	0.0097
		(0.0004)	(0.0008)
Aggregate volatility scaling	$\phi_{\sigma,1}$	17.0692	0.205
		(0.4883)	(0.0194)
Drift scaling	ϕ_{μ}	3.4992	2.6263
		(0.0307)	(0.0136)
Bankruptcy cost level	ω_0	0.2589	0.2381
		(0.0153)	(0.0037)
Bankruptcy cost cyclicality	ω_1	0.0961	0.0747
		(0.0127)	(0.0070)
Risk aversion	γ	9.1274	
		(0.2641)	
J-statistic	J	113.5728	13398.6720

Table 4: Moments Targeted in the SMM Estimation

This table shows the moments targeted by the SMM estimation of the model. We report standard deviations (untargeted) in addition to variances (targeted) for ease of interpretability. The benchmark model features long-lasting consumption crises. The column LRR replaces the consumption process with a Markov chain approximation of the long-run risks process in Bansal and Yaron (2004) and assumes risk aversion of 10. The column Population is based on simulated model data from the unconditional distribution of the aggregate state, rather than the post-war simulation used to calibrate the consumption process.

Moment	Data	Model	Population	LRR
Average leverage	19.51	19.44	23.13	19.40
Average equity return	0.78	0.72	0.74	0.57
Average 5-year CDS rate	0.69	0.63	1.07	0.02
Average at-the-money IV	24.90	24.33	27.55	18.93
Average IV skew	4.01	3.91	3.97	3.17
Variance of leverage	2.26	2.18	3.59	2.86
Variance of returns	0.53	0.50	0.90	0.59
Variance of 5-year CDS rate	0.04	0.02	0.07	0.00
Variance of at-the-money IV	1.27	0.87	3.29	0.22
Variance of IV skew	0.05	0.03	0.12	0.02
Variance of market return	0.16	0.15	0.26	0.14
Std. Dev. of leverage	15.02	14.75	18.95	16.91
Std. Dev. of equity return	7.26	7.06	9.47	7.66
Std. Dev. of 5-year CDS rate	1.96	1.29	2.60	0.31
Std. Dev. of at-the-money IV	11.29	9.34	18.15	4.68
Std. Dev. of IV skew	2.14	1.70	3.48	1.56
Std. Dev. of market return	4.03	3.89	5.09	3.75

Table 5: Credit Market Moments

We tabulate moments that are important for the pricing of CDS contracts, but were not targeted in the model estimation. LGD denotes the loss rate given default and PD the probability of default. $\mathbb{P}$ refers to the physical probability measure and $\mathbb{Q}$ the risk-neutral one. The benchmark model features long-lasting consumption crises. The column LRR replaces the consumption process with a Markov chain approximation of the long-run risks process in Bansal and Yaron (2004) and assumes risk aversion of 10. The column Population is based on simulated model data from the unconditional distribution of the aggregate state, rather than the post-war simulation used to calibrate the consumption process.

Moment	Model	Population	LRR
Average bankruptcy costs under $\mathbb{P}$	25.89	25.89	23.81
Std. Dev. of bankruptcy costs under $\mathbb{P}$	3.63	3.63	2.00
Average bankruptcy costs under $\mathbb{Q}$	36.41	36.41	27.15
Std. Dev. of bankruptcy costs under $\mathbb{Q}$	2.15	2.15	1.60
Average 5-year PD under $\mathbb{P}$	0.25	0.95	0.03
Average 5-year PD under $\mathbb{Q}$	3.34	5.26	0.10
Std. Dev. of 5-year PD under $\mathbb{P}$	1.93	4.95	0.73
Std. Dev. of 5-year PD under $\mathbb{Q}$	6.00	9.59	1.40
Average LGD under $\mathbb{P}$	75.21	75.77	88.52
Average LGD under $\mathbb{Q}$	75.22	75.79	88.91
Average 5-year LGD under $\mathbb{P}$	77.76	78.05	89.12
Average 5-year LGD under $\mathbb{Q}$	87.01	87.12	89.75

Table 6: CDS Decomposition

This table shows the result of several counterfactuals, which aim to assess the importance of different factors for the pricing of CDS contracts. The benchmark model features long-lasting consumption crises. In experiment 1, bankruptcy costs are not time varying, $\omega_b = 0$. In experiment 2, we assume constant loss rates. In experiment 3, there is no time variation in idiosyncratic risk, $\phi_{\zeta,1} = 0$, and in experiment 4 no time variation in aggregate risk, $\phi_{\sigma,1} = 0$. In experiment 5, there is no crisis risk, $\pi = 0$. In all experiments, firm policies taken from the benchmark model but CDS contracts are revalued. LGD denotes the loss rate given default and PD the probability of default. $\mathbb{P}$ refers to the physical probability measure and $\mathbb{Q}$ the risk-neutral one.

Moment	Model	Exp. 1	Exp. 2	Exp. 3	Exp. 4	Exp. 5
Average bankruptcy costs	25.89	25.89	25.89	25.89	25.89	25.89
Std. Dev. of bankruptcy costs	3.63	0	0	3.63	3.63	3.63
Average risk	5.58	5.58	5.58	5.69	5.66	5.58
Std. Dev. of risk	2.56	2.56	2.56	1.09	1.79	2.56
Average 5-year CDS rate	0.63	0.62	0.54	0.27	0.42	0.03
Std. Dev. of 5-year CDS rate	1.29	1.26	1.12	0.98	1.12	0.48
Average 5-year PD under $\mathbb{P}$	0.25	0.25	0.25	0.18	0.21	0.15
Average 5-year PD under $\mathbb{Q}$	3.34	3.34	3.34	1.41	2.25	0.18
Average LGD under $\mathbb{P}$	75.21	75.42	75.21	75.28	75.25	75.17
Average LGD under $\mathbb{Q}$	75.22	75.43	75.21	75.30	75.27	75.19
Average 5-year LGD under $\mathbb{P}$	77.76	77.23	75.21	76.91	77.29	76.37
Average 5-year LGD under $\mathbb{Q}$	87.01	85.20	75.21	86.64	86.83	76.70

Table 7: Default Risk in Option and CDS Contracts

Every month, we sort firms into deciles based on the naïve distance to default (DD) measure of Bharath and Shumway (2008) and then average at-the-money IV, IV skew, and 5-year CDS rate across firms in each bin. The table reports time-series averages for each decile for the period 2004 to 2019.

Decile	IV		IV Skew		CDS	
	Data	Model	Data	Model	Data	Model
Low DD	0.347	0.321	0.044	0.046	2.296	2.603
2	0.290	0.260	0.042	0.041	0.883	1.156
3	0.265	0.246	0.041	0.040	0.683	0.742
4	0.254	0.239	0.041	0.039	0.590	0.520
5	0.244	0.234	0.040	0.038	0.551	0.380
6	0.232	0.231	0.038	0.038	0.482	0.286
7	0.228	0.229	0.039	0.038	0.439	0.218
8	0.221	0.226	0.038	0.038	0.404	0.166
9	0.211	0.225	0.038	0.037	0.364	0.128
High DD	0.200	0.223	0.037	0.037	0.322	0.101