Trade Finance and the Durability of the Dollar*

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January 4, 2021

Abstract

We propose a model in which the emergence of a single dominant currency is driven by the need to finance international trade. The model generates multiple stable steady states, each characterized by a different dominant asset, consistent with the historical durability of real-world currency regimes. The key force driving persistence of regimes is a positive interaction between the returns to saving in an asset and the use of that asset for financing trade. A calibrated version of the model implies that the welfare gains of dominance are substantial, but accrue primarily during the transition to dominance. We consider several counterfactual experiments that highlight the importance of path dependence and economic policy.

JEL Codes: E44, F02, F33, F41, G15

Keywords: trade finance; dollar dominance; exorbitant privilege; liquidity premium; global imbalances

*This paper originally circulated as Boston College Working Paper 934. We are grateful to our discussants, Ozge Akinci, Matteo Maggiori, Adrien Verdelhan and Martin Wolf, and to seminar participants at the AEA annual meetings, Banque de France, Barcelona Summer Forum, Boston Macro Juniors Workshop, Boston University, CEF, Chicago Booth IFM, EACBN-Mannheim, the Federal Reserve Banks of Boston, Dallas, Chicago, Richmond, and Philadelphia, Fordham Conference on Macroeconomics and International Finance, Harvard University, Indiana University, the Konstanz Seminar on Monetary Theory and Monetary Policy, the Spring 2018 NBER IFM, SED, SEM, SFS, UQAM, University of Notre Dame, and the University of Wisconsin-Madison. We thank Sherty Huang for excellent research assistance. Contact: ryan.chahrour@bc.edu, valchev@bc.edu.

1 Introduction

Throughout most of its history, the international financial system has featured a "dominant" safe asset that facilitates a large majority of international trade and financial flows. Since the 1940s, this role has been played by the US dollar and safe US assets, but this was not always the case. Throughout the nineteenth and early twentieth centuries, the British pound was the dominant international asset, while the Dutch guilder played a similar role in the eighteenth century. This pattern suggests that the international financial system favors the emergence of long-lasting periods of currency dominance. Moreover, historical accounts often emphasize the role that trade finance plays in establishing this dominance (Eichengreen, 2011; Eichengreen and Flandreau, 2012).

The purpose of this paper is to provide a quantitatively realistic theory of the durability of international currency regimes, based on their links with financing needs for international trade. The key friction in our mechanism is imperfect enforcement of contracts across borders, which necessitates the use of collateral guarantees in international transactions. Firms seek to borrow collateral via frictional trade finance markets, and this financing friction generates a feedback between the use of an asset as collateral and the households' incentives to save in that asset: high use of an asset for trade financing encourages savers to hold more of that asset, while high saver holdings of the asset encourage firms to seek it for financing their trade. This interaction gives rise to multiple steady states, each characterized by a different dominant asset, with unique and stable equilibrium dynamics around those steady states.

We isolate these forces in a tractable analytical model and then explore their quantitative implications in a quantitative general equilibrium theory. The structure of our model is motivated by the many frictions international trade faces in practice, including imperfect contracting but also transportation lags and others, that generate the need for financing to facilitate trade (Antràs, 2013). Indeed, empirical studies show that the majority of international trade transactions require some form of external financing (Auboin, 2016), and that this financing is scarce – e.g. Di Capria et al. (2016) estimate an unmet trade finance need of \$1.6 trillion for 2016. Meanwhile, BIS (2014) document that outside of the US and the Eurozone, the majority of trade finance is locally sourced, using local banks, and that trade finance contracts are heavily dollarized, showing that international trade is not only denominated in dollars (Gopinath, 2015), but also financed via dollar debt.[1]

[1] A related literature documents that disruptions to trade finance availability, for example when local banks lack dollar funding, cause significant reductions in international trade (e.g. Amiti and Weinstein, 2011; Ahn, 2014; Antras and Foley, 2015; Niepmann and Schmidt-Eisenlohr, 2017; Bruno and Shin, 2019)

Our model is composed of three regions: the United State (US), the Eurozone (EZ), and a continuum of rest-of-world (RW) small open economies. In each country, there is a continuum of trading firms seeking to engage in profitable transactions with traders from other countries. Imperfect contract enforcement across borders requires traders to collateralize transactions with safe assets that serve as performance guarantees. To obtain collateral, traders seek an intra-period loan of either US or EZ bonds in local bond-specific search and matching markets. On the other side of these credit markets are households, who form optimal portfolios and offer intra-period loans of their assets for a fee.

Search frictions in the collateral credit markets mean that trading firms prefer to try to borrow an asset that forms a substantial portion of local household portfolios, and thus is relatively plentiful in local funding markets. Conversely, households are aware that an asset that is actively used by traders is more likely to be loaned in the collateral markets and earn the associated fee. Thus, the incentives of households and traders are mutually reinforcing: wide holdings of an asset drive its adoption in trade finance activities globally, while higher adoption encourages households across the world to indeed concentrate savings in that asset.

Using our analytical model, we highlight several important implications of this strategic interaction between trade financing and household portfolio decisions. First, we show that the feedback between households and trading firms leads to steady-state multiplicity, including a dollar-dominant steady state in which US safe assets are both widely held across the world and the dominant means of trade finance. The other steady states are a mirror-image euro-dominant steady state, and a "multi-polar" steady-state in which portfolios and trade finance use are split equally across two assets.

While firm-household interaction leads to dominant-currency steady states, these outcomes may not be stable. Intuitively, when one asset is dominant, the market for loans of that asset is relatively congested. Hence, an off-equilibrium shift in the portfolio composition of households can drive firms to shift their finance use away from the dominant asset. To ensure stability, we introduce a currency mismatch cost that is incurred by trading pairs who use different types of collateral. This cost can be micro-founded as the expected cost of default by one of the transaction counterparties, since mismatched collateral means the two promises are not guaranteed to be equivalent in all states of the world.[2]

Mismatch costs add an additional strategic element to the model, namely a complementarity between the types of financing sought by trading firms in different countries. On its own, this complementarity would give rise to sunspot equilibria, since different trade finance choices could be supported as long as all firms expect that their respective foreign counter-

[2] Amiti et al. (2018) find evidence of such coordination incentives in firm-level trade data from Belgium.

parties to do the same. The friction in funding markets limits this kind of indeterminacy, however, because firms' funding choices become coordinated by their desire to finance using the asset most widely held by their respective households. Intuitively, firms anchor their expectations of what potential counterparties will do on the observable credit conditions in foreign markets, which themselves are driven by the composition of foreign portfolios.

We conclude our analysis of the analytical model by showing that the interaction between the two complementarity mechanisms can ensure that dominant-asset steady states are locally stable, in the sense that off-equilibrium shifts in the actions of one type of agent (e.g. households) will not precipitate a shift away by the other type (e.g. firms). Essentially, firms' actions are less affected by a small shift in portfolios, since such a change still leaves trade finance markets heavily favoring the dominant asset, and the coordination incentive across firms ensures they will not change their actions by much. In particular, we prove that for intermediate levels of currency mismatch costs, dominant-asset steady states are both locally stable and not subject to self-fulfilling sunspot shocks to traders' beliefs. Neither mechanism can achieve this on its own – without the interaction we outline above, equilibrium is either locally unstable or subject to sunspot indeterminacy.

Having established the intuition for our core mechanism, we turn to a generalized model to quantify its implications, both in and out of steady state. The main difference in this model is that it provides a full microfoundation for profitable international transactions via trade subject to search and matching frictions.[3] We calibrate the model by selecting parameters to match target moments on the size of government debt, trade, currency denomination of trade finance activities, and import markups. The model exactly matches the target moments and a number of additional, non-targeted moments are also closely aligned with the data.

Using the calibrated model, we first show that dominant-currency steady states are *dynamically* stable, and lie within large regions of the state space that uniquely converge to their respective dominant-asset steady state. Within those regions, the equilibrium paths of the economy are determinate (i.e. not subject to sunspot shocks) and the currency regime is uniquely determined by initial conditions. Essentially, the model tethers the equilibrium mix of assets used for trade finance to a slow moving endogenous state variable, bond holdings, so that the model is able to generate the co-existence of separate dollar- and euro-dominant steady states, each of which has its own unique region of attraction. The emerging dominant-currency regimes are thus durable, consistent with the historical record.

We next compute the welfare implications of issuing the dominant asset. At the steady-state, the welfare gain of the dominant country, relative to the other large country, is small:

[3] This is akin to Antras and Costinot (2011) with search frictions in both trade and financing markets.

only 0.03% of permanent consumption. At first sight this result seems surprising, since the dominant asset endogenously pays an interest rate that is 1.07% lower than that of the other safe asset, earning the dominant country a so-called "exorbitant privilege". As our theory shows, however, that dominance is supported by wide holdings of the central country's asset by foreigners. This high external demand generates a significant *negative* steady-state NFA position in the dominant country, which, in the long run, nearly offsets the benefits of the lower interest rate. This happens even though the mechanism can generate a situation where both the NFA and the trade balance are negative at steady state.

Though dominance has a small effect on long-run consumption, factoring in the transition to dominance dramatically changes welfare conclusions. Incorporating the effects of transition from the (unstable) symmetric steady state, the eventual dominant country gains an equivalent of 0.75% of permanent consumption, as in the process of converging to dominant status the country increases its external debt at a favorable price (i.e. a low interest rate) which helps fund a consumption that is temporarily higher than its eventual steady state level. Hence, our model shows that evaluating the benefits of currency dominance requires a nuanced analysis that properly capture the effects of the transition periods as well.

We conclude the paper with two counter-factual experiments that highlight the role of history and economic policy in determining the dominant asset. First, we use our model to analyze one of the most important recent developments of the international financial system, the formation of the Eurozone and ask, "Could this change have resulted in a change in the dominant international currency?" Consistent with historical experience, the model implies that the mere introduction of the Euro is not enough to precipitate a switch away from the dollar-dominant steady state. We find that a switch *could* have occurred, however, if the Eurozone's capacity of issuing safe assets had exceeded (at least eventually) that of the US by an additional 30%. On the one hand, this shows that switches in the currency regime require quite large developments. On the other hand, a Eurozone with 30% larger fiscal capacity could easily have occurred if the EZ had continued its anticipated expansion with the UK and/or additional Eastern European countries. Looking forward, the model predicts that another potential challenger, the Chinese renminbi, cannot play a significant role until the Chinese capital account is sufficiently liberalized. According to our theory, broad international holdings of an asset are essential for it to gain dominant status.

In our second counterfactual experiment, we consider the consequences of the trade policy of the central country. We find that a trade war in which the US raises tariffs on all imports by 15%, and its trading partners retaliate in kind, disproportionately hurts the US and could be enough to threaten the dominance of the dollar, but does not eliminate it for certain. We estimate that a potential switch in the dominant-asset in this case would lower US welfare by 1.5% of permanent consumption. Our analysis indicates that US policies favoring free trade have been quite helpful in establishing its preeminent role in the world financial system.

Relation to existing literature

In this paper, we develop a model where a dominant international asset emerges endogenously due to financial frictions in international trade, and dominance is a stable, persistent outcome. We contrast the co-existence of multiple potential long-run outcomes (e.g. dollar and euro steady states), with the fact that each of these long-run outcomes has its own unique region of attraction. As a result, the emerging dominant-asset regimes are stable (i.e. likely to actually occur) and persistent once they occur, consistent with the historical record.

The paper is novel, on the one hand, because it focuses explicitly on the durability of dominant-asset regimes, as previous studies either do not consider dynamic models or analyze models that exogenously assume some form of asymmetry in favor of one country, making dominance essentially hard-wired. In our model all assets are ex-ante identical, and the identity of the dominant-asset is an equilibrium outcome. On the other hand, our model is also unique in that it links the emergence of a dominant asset to financial frictions in international trade, whereas other studies have proposed other potential explanations.[4]

Concurrent work by Gopinath and Stein (2018) is perhaps the most closely related to our own. It also connects currency dominance to long-run differences in asset returns, however, their mechanism operates through a specific bonds-in-the-utility structure that favors saving in assets denominated in the currency of imports. Their model is also static, while our model focuses on the determination of dynamic stability in dominant currency regimes.

A broader literature focuses on linking dominant-asset status with the differential capacity of countries to generate store-of-value assets. Caballero et al. (2008) argue that the United States' superior ability to produce safe assets can explain the US experience of persistent trade deficits, falling interest rates, and the rising portfolio share of US assets in developing countries. Mendoza et al. (2009) and Maggiori (2017) focus on differences in financial development as a driver of global imbalances and the emergence of a dominant currency, while Liu et al. (2019) argue that financial market depth is also associated with trade currency choices. Brunnermeier and Huang (2018) explore the impact of this mechanism in emerging market crises. Gourinchas et al. (2017) propose a framework in which US households have lower risk-aversion than foreigners, and end up holding most of the world's risky assets. Bocola

[4] Gourinchas et al. (2019) provide an excellent overview of the literature.

and Lorenzoni (2017) provide a framework where dollarization of world financial markets occurs because of the dollar's unique risk profile. Meanwhile, Farhi and Maggiori (2016) consider the positive and normative implications of having a single dominant reserve asset versus a multipolar system. He et al. (2016) use a global-game in a world with two ex-ante identical assets to model how a single safe asset emerges in equilibrium.

Several features differentiate our work from the literature cited above. First, compared to models with ex-ante asymmetries, a single asset emerges as dominant in our theory in an otherwise symmetric world. Further, in our model, dominance is a determinate equilibrium outcome, allowing us to explore the notion of dynamic stability and explicitly consider transitions between currency regimes. We view the last two features as especially desirable, because (i) historically currency regimes have changed only infrequently and (ii) we show that accounting for transitions changes welfare implications substantially. Previous work has included some of these features, but not all together.

Another literature has considered the related question of why there is a dominant currency in trade invoicing. This literature emphasize the strategic complementarities in pricing decisions of firm in the presence of price stickiness (e.g. Engel (2006), Gopinath et al. (2010), Goldberg and Tille (2016), Mukhin et al. (2018)). This literature has generally also focused on static (or steady-state models) and characterized the potentially emergent equilibrium multiplicity, but does not speak to the question of the stability of the invoicing currency regime. Also, this literature does not generally trace the implications of currency dominance to equilibrium interest rates and asset positions, as we do in our paper.

Finally, there is a long literature on search-based theories of money in international settings, which examines the micro-foundations of different trading and payment structures, focusing on equilibrium multiplicity and the co-existence of multiple currencies (e.g. Matsuyama et al., 1993; Zhou, 1997; Wright and Trejos, 2001; Rey, 2001; Kannan, 2009; Devereux and Shi, 2013; Zhang, 2014; Doepke and Schneider, 2017; Liu et al., 2019). A separate strand of literature, including Vayanos and Weill (2008) and Weill (2008), uses search frictions to explain variation in liquidity premia for over-the-counter asset markets. Our mechanism does not operate through the endogenous emergence of "money", but rather through financial frictions, and also focuses on the dynamic stability of the dominant-asset equilibria.

2 Simplified Model

In this section we present a simplified version of our model that includes only the two essential ingredients of our mechanism – the savings decisions of households and the financing decisions of firms. The goal is to provide the reader with the core intuition of how our mechanism works. In Section 3, we present a fully-specified benchmark model that we use to examine the quantitative implications of the mechanism.

The world consists of two symmetric big countries, the United States (US) and the Eurozone (EZ), which are of equal size $\mu_{us} = \mu_{ez}$, and a continuum of small open economies with total mass μ_{rw} making up the *rest of the world* (RW). There are two assets in the economy, and both are universally recognized as safe and ex-ante identical – a bond issued by the US government and one issued by the EZ government. Both assets are available in exogenously fixed supply equal to $\bar{B}$, and serve as saving vehicles and, potentially, as collateral guaranteeing international transactions.

Countries are indexed by $j \in \{us, ez, [0, \mu_{rw}]\}$, and within each country j, there is representative consumer and a continuum of risk-neutral international trading firms. In our simplified model, the only decision of the household is how to allocate its savings between the two available assets and the only decision of a trading firm is which type of asset to seek out to finance international trade. We describe the decisions of the two agents types in turn.

Households

Households solve a standard consumption-savings problem, allocating their endowment income between consumption and the two safe assets. For now, we assume there is a single consumption good and both assets promise one unit of that good, hence the bonds only differ ex-ante because of their issuer. Nevertheless, they may serve different trade financing roles in equilibrium and, therefore, earn different interest rates. In Section 3 we generalize our argument to a setting with differentiated goods, where the bonds also differ in their payoffs due to the exchange rate – those extensions have reinforcing quantitative effects, but are not needed for the qualitative argument we pursue in this section.

Households in each country $j \in \{us, ez, [0, \mu_{rw}]\}$ solve

$$\max_{C_{jt}, B^{\$}_{jt}, B^{€}_{jt}} E_0 \sum_{t=0}^{\infty} \beta^t \frac{C_{jt}^{1-\sigma}}{1-\sigma}$$

subject to

$$C_{jt} + Q^{\$}_t B^{\$}_{jt} + Q^{€}_t B^{€}_{jt} = B^{\$}_{jt-1} + B^{€}_{jt-1} + \Delta^{\$}_{jt} B^{\$}_{jt} + \Delta^{€}_{jt} B^{€}_{jt} + Y_{jt},$$

and a non-negativity constraint on bond-holdings. In the above, Q^c_t is the price of a c-

denominated bond for $c \in \{\$, €\}$, B^c_{jt} is the amount of that bond held by the household, and Y_{jt} is an exogenous endowment of consumption goods. Besides the interest rate, the two assets earn an additional endogenous return due to the collateral-use fees that firms pay to the households whose bonds they borrow, a process we describe in detail below. We refer to these extra payments as the bonds' "liquidity premia" and denote them by Δ^c_j.

The optimal choice of asset holdings $B^{\$}_{jt}$ and $B^{€}_{jt}$ implies the Euler equations

$$1 = \beta E_t \left[\left(\frac{C_{jt+1}}{C_{jt}} \right)^{-\sigma} \frac{1}{Q^{\$}_t \left(1 - \Delta^{\$}_{jt}\right)} \right] \tag{1}$$

$$1 = \beta E_t \left[\left(\frac{C_{jt+1}}{C_{jt}} \right)^{-\sigma} \frac{1}{Q^{€}_t \left(1 - \Delta^{€}_{jt}\right)} \right]. \tag{2}$$

In steady-state, these Euler equations imply that returns are equalized across assets and across countries, so that

$$\frac{1}{\beta} = \frac{1}{Q^{\$} - \Delta^{\$}_j} = \frac{1}{Q^{€} - \Delta^{€}_j}. \tag{3}$$

Since our analytical results regard steady-states, we now time subscripts until Section 3.

Trading Firms

Trading firms in each country have the opportunity to make a profitable transaction with a randomly-matched foreign trading partner. If executed, this transaction generates a joint surplus of 2π that is split evenly between the two counterparties. The potential profit π is treated as an exogenous parameter in this simplified version of the model, and is endogenized as the equilibrium profit in import/export operations in our fully-specified model in Section 3. In either case, due to imperfect contract enforcement across borders (as documented for example by Antras and Foley (2015) and Hoefele et al. (2016)), to execute the deal and realize this profit, each firm must post collateral to guarantee their side of the transaction.[5]

Both the US and the EZ safe assets can serve the role of collateral and the firms' optimal choice regarding which collateral type to seek out is central to our mechanism. To obtain this collateral, firms seek an intra-period loan of one of the assets in local bond-specific search and matching credit markets. On the other side of these credit markets are domestic households, who make their holdings of safe assets available for loan. Lastly, we assume that

[5] The two-sided friction is convenient because it makes the problems of an exporter and an importer symmetric. It is also in line with the evidence of e.g. Schmidt-Eisenlohr (2013).

traders look for a fixed amount of funding, which we normalize to one, and that they make the binary choice of either seeking dollar or euro collateral (i.e. a US or an EZ safe asset).

The probability that a country-j firm seeking to borrow a US asset is successful is given by $p_j^{\$}$, while the probability of borrowing an EZ asset is $p_j^{€}$. If a firm successfully borrows a unit of collateral, it pays a fee, $r^{\$}$ or $r^{€}$ respectively, to the household for the use of the asset, and proceeds to trade in the international market and earn profits. If the firm is not successful in these credit markets, it continues on to trade using a "backup" funding plan that still provides its chosen collateral, but absorbs all surplus from the transaction.[6]

The only equilibrium requirement for the funding fees $r^{\$}$ and $r^{€}$ is that they leave firms with a positive ex-post surplus. In parallel with the labor search literature, these prices can be treated as fixed parameters themselves so long as they are fixed to a value within the surplus range of the trading firms, or they could be determined via some bargaining paradigm, which can then be parameterized itself. For simplicity, we follow the first of these paths and fix the funding prices to a common value, $r^{\$} = r^{€} = r < \pi$.[7]

Once in the trading market, the firm is randomly matched with a counterparty from another country $j' \neq j$. Upon matching, the pair transacts using their collateral to clear any payments needed and splits the gross transaction surplus of 2π. In the event that the two counterparties' collateral is mismatched — i.e. that one side of the match uses US assets and the other side EZ assets as collateral — the transaction's surplus is reduced by a collateral mismatch cost of 2κ. Throughout we assume that $\kappa < \pi - r$, so that the transaction is profitable even in the event of such a mismatch.

This "collateral mismatch" cost can be micro-founded as the expected cost in case of default, since with collateral denominated in different currencies the two promises are not guaranteed to be equivalent in all states of the world. However, given that our model abstracts from aggregate shocks, we introduce this cost via the parameter κ. We think this is innocuous, since a number of the analytical results we discuss below hold for $\kappa = 0$ and, in our quantitative analysis in Section 3.2, the calibrated value of κ is very small.

Putting everything together, a country-j firm chooses to apply for dollar trade financing if the expected payoffs of seeking dollar vs euro financing is positive. The differential payoff

[6] This assumption simplifies our analytical derivations; in the quantitative model, unfunded firms exit without trading and so that trade finance constitutes a real constraint on equilibrium trade flows.

[7] This approach helps achieve analytical tractability for the results of this section, but more generally it does not have an appreciable impact on our main results either way – for example assuming Nash bargaining over r, instead, has no effect on the quantitative analysis in Section 3.

of seeking a US asset over seeking a EZ asset, denoted by $V_j^{\$}$, is given by

$$V_j^{\$} = p_j^{\$}\left[\pi - r_j^{\$} - \kappa(1-\bar{X})\right] - p_j^{€}\left[\pi - r_j^{€} - \kappa\bar{X}\right] > 0, \tag{4}$$

where $\bar{X}$ is the fraction of all trading firms in the world that use dollar funding and, hence, the probability an individual firm gets matched with another firm using US assets as collateral.

Since our primary aim is to explain third-party use of a dominant currency, we focus on characterizing the optimal choice of collateral type for trading firms operating in the RW, while assuming that the traders in the US and EZ only seek funding supported by their own domestic assets. Nevertheless, the US and EZ firms face similar financial frictions and need to obtain funding before participating in international trade, it is just that the type of trade financing they seek is set exogenously. Thus,

$$\bar{X} \equiv \mu_{us}X_{us} + \mu_{ez}X_{ez} + \int_{j\in\mu_{rw}} X_j dj = \mu_{us} + \int_{j\in\mu_{rw}} X_j dj,$$

where X_j is the fraction of firms in country $j \in [0, \mu_{rw}]$ that choose to apply for dollar financing.

We assume that the number of matches in a given country-asset credit market is governed by the constant returns to scale den Haan et al. (2000) matching function

$$M^F(B,X) = \frac{BX}{B+X},$$

where B represents units of the asset on offer in that market and X, the number of trading firms demanding that asset. Thus, a country-j trading firm searching for e.g. dollar funding succeeds with probability

$$p_j^{\$} = \frac{M^F(B_j^{\$}, X_j)}{X_j} = \frac{B_j^{\$}}{B_j^{\$} + X_j}.$$

Substituting expressions for the funding probabilities into equation (4) yields

$$V_j^{\$} = \frac{B_j^{\$}}{B_j^{\$} + X_j}\left[\pi - r - \kappa(1-\bar{X})\right] - \frac{B_j^{€}}{B_j^{€} + 1 - X_j}\left[\pi - r - \kappa\bar{X}\right]. \tag{5}$$

Equation (5) reveals the individual firm's strategic three incentives, two with respect to other trading firms and one with respect to the domestic household. First, with respect to other domestic firms, collateral choices are strategic substitutes: when a larger share of the other country-j traders apply for dollar funding (higher X_j), the local dollar funding market becomes more congested, lowering the probability of any given trader's loan being approved. Naturally, a lower probability of successfully borrowing a US safe asset lowers the relative payoff of seeking this type of funding.

Second, with respect to the choices of foreign trading firms ($\bar{X}$), the collateral mismatch costs imply that funding choices are strategic complements. This complementarity can lead to equilibrium multiplicity, in which both $\bar{X} = 0$ and $\bar{X} = 1$ can be sustained as equilibrium outcomes of self-fulfilling sunspot shocks to expectations of other firms' actions. The existence of the financial friction, however, serves to limit the scope of such indeterminacy.

To see how financial frictions work against indeterminacy, notice that equation (5) also captures a third strategic interaction, this one between the collateral choice of trading firms and household savings choices, $B_j^{\$}$ and $B_j^{€}$. For example, a trading firm's expected payoff of choosing to seek dollar financing increases with the household's holdings of US bonds ($B_j^{\$}$), as larger household US asset holdings make the dollar credit market less tight from the perspective of firms and increase the firm's probability of successfully obtaining dollar funding.

The interaction described above helps anchor the equilibrium choice of a trade finance currency: intuitively, the funding search friction makes the households' bond position a *coordination device* for international traders, and disciplines their equilibrium funding choice. To analyze this effect formally, we define the notion of quasi-equilibrium in trade finance use, in analogy to Mas-Colell et al. (1995, p. 551). We focus on symmetric equilibria where all small open economies are the same, thus $X_j = X_{rw}$, $B_j^{\$} = B_{rw}^{\$}$, $B_j^{€} = B_{rw}^{€}$ for all $j \in [0, \mu_{rw}]$.

Definition 1 (Quasi-equilibrium). *Given household asset holdings in rest-of-world countries $\{B_{rw}^{\$}, B_{rw}^{€}\}$, a symmetric quasi-equilibrium in funding choice is a rest-of-world traders' funding choice X_{rw} such that no trader has an incentive to change its trade finance choice.*

Quasi-equilibrium describes the equilibrium funding choice among traders, as *a function of the asset holdings of the households.* Hence, we denote the set of quasi-equilibria by the correspondence $X(B_{rw}^{\$}, B_{rw}^{€})$. Using equation (5), the set of quasi-equilibria can be characterized by the condition

$$V^{\$} X_{rw}(1 - X_{rw}) = 0, \tag{6}$$

with $V^{\$} > 0$ only if $X_{rw} = 1$ and $V^{\$} < 0$ only if $X_{rw} = 0$. If there is a unique value of X_{rw} satisfying (6), then $X(B_{rw}^{\$}, B_{rw}^{€})$ becomes a function.

Lemma 1 characterizes some key properties of the quasi-equilibria in our economy:

Lemma 1. *Given household portfolio holdings, the currency quasi-equilibrium is unique for any feasible bond allocation if and only if*

$$\kappa < \kappa^{sunspot} \equiv \frac{\pi - r}{\bar{B} + \frac{\mu_{rw}}{2} + \frac{1}{2}}.$$

In that case, if $B_{rw}^{\$} > B_{rw}^{€}$, then

$$X(B_{rw}^{\$}, B_{rw}^{€}) \in \left(\frac{B_{rw}^{\$}}{B_{rw}^{\$} + B_{rw}^{€}}, 1 \right].$$

Proof. Proved in Appendix A. ■

Lemma 1 shows that the quasi-equilibrium in trade finance currency can indeed be unique even when trade finance choices are strategic complements across firms in different countries (i.e. $\kappa > 0$). This happens because credit market conditions make different types of funding more or less accessible to the different counterparties. Limited funding opportunities, in turn, mean that bond holdings acts as a coordination device synchronizing international trade firms' financing choices: differences in relative market tightness make the plentiful asset a more attractive source of financing. When financing incentives are strong enough, quasi-equilibrium is unique *and* biased towards the plentiful asset.

Naturally, if we reduce the effective financial friction, by for example by raising both $B_{rw}^{\$}$ and $B_{rw}^{€}$, we increase the scope for indeterminacy and the threshold $\kappa^{sunspot}$ falls. This happens because increasing both of the households' asset holdings makes both types of funding easier to obtain, which reduces the anchoring effects of the friction in credit markets. If asset holdings of both types become arbitrarily large, potential counterparties' choices become the only payoff-relevant factor in the funding choice and the quasi-equilibrium is not unique for any level of κ:

Corollary 1. *In the limit of $B_{rw}^{\$} \to \infty$ and $B_{rw}^{€} \to \infty$, for any $\kappa > 0$*

$$X(B_{rw}^{\$}, B_{rw}^{€}) \to \{0, 1/2, 1\}.$$

We refer to indeterminacy in the funding choice quasi-equilibrium as "sunspot" multiplicity, because in that case the funding choices are solely determined by the beliefs of what other trading firms would do, i.e. a sunspot shock in the expectations of others' actions.

Lemma 1 describes how firms' funding choices are influenced by household bond holdings; we now turn to effects that firm funding decisions have on household savings allocations. To start, observe that the equilibrium holding premium earned by each bond type is equal to the expected intra-period loan fees that the bond earns – which are equal to the probability that the country j household successfully lends this kind of bond multiplied by the funding fee r that it receives when it does so. Hence, the premia can be expressed as

$$\Delta_j^{\$} = \frac{M^F(B_j^{\$}, X_j)}{B_j^{\$}} \times r = \frac{X_j}{B_j^{\$} + X_j} r, \tag{7}$$

$$\Delta_j^{€} = \frac{M^F(B_j^{€}, 1 - X_j)}{B_j^{€}} \times r = \frac{(1 - X_j)}{B_j^{€} + (1 - X_j)} r. \tag{8}$$

Since the Euler equation (3) holds for all households j, the liquidity premia earned by each asset are equalized across countries: $\Delta_j^c = \Delta^c$ for all $j \in \{us, ez, [0, \mu_{rw}]\}$. Using this observation, the Euler equations, and the market clearing conditions in bond markets ($\mu_{us} B_{us}^c + \mu_{ez} B_{ez}^c + \int B_j^c dj = \bar{B}$), we can derive Lemma 2.

Lemma 2. *Equilibrium household portfolios, as a function of traders' currency choices, are:*

$$B_j^{\$} = \bar{B} \frac{X_j}{\int_{\mu_{rw}} X_j dj + \mu_{us}} \tag{9}$$

$$B_j^{€} = \bar{B} \frac{1 - X_j}{\int_{\mu_{rw}} (1 - X_j) dj + \mu_{ez}}. \tag{10}$$

Proof. Proved in Appendix A. ■

Lemma 2 describes the determination of household portfolios, *as a function of the mix of assets used for trade financing.* The Lemma shows this relationship is upward sloping: higher X_j implies higher $B_j^{\$}$. Intuitively, an asset that is heavily used for funding international transactions will deliver a higher liquidity premium ceteris paribus, increasing households' incentive to hold that bond. Since bond premia are equalized everywhere, countries with a higher dollar usage (higher X_j) must also have higher portfolio holdings of US bonds ($B_j^{\$}$).

Together, Lemmas 1 and 2 capture a strategic interaction between trading firms and households that lies at the heart of our mechanism. Specifically, the model implies positive feedback between the asset allocations of RW households and the RW firms' trade finance currency choice: higher bond holdings of a given type tilt the quasi-equilibrium funding choice towards the same asset, which reinforces the households' decision to favor saving in that asset. This additional source of complementarity is distinct from the cross-country complementarity among trading counterparties and does not rely on $\kappa > 0$.

Steady-state equilibria

Having characterized both the optimal choice of trade finance made by firms and the optimal savings choices of households, we are ready to analyze the resulting steady state equilibria. We consider symmetric steady states in which the strategies of the ex-ante identical RW agents are the same.

Definition 2. *A steady-state equilibrium is a RW currency usage X_{rw}, a set of asset holdings $\{B^{\$}_{rw}, B^{€}_{rw}, B^{\$}_{us}, B^{€}_{us}, B^{\$}_{ez}, B^{€}_{ez}\}$, bond prices $\{Q^{\$}, Q^{€}\}$ and premia $\{\Delta^{\$}, \Delta^{€}\}$ such that*

1. *There is a quasi-equilibrium in currency choice*
2. *The optimality conditions of household bond holdings are satisfied.*
3. *Bond markets clear:*

$$\bar{B} = \mu_{rw} B^c_{rw} + \mu_{us} B^c_{us} + \mu_{ez} B^c_{ez}, \ \text{for } c \in \{\$, €\}.$$

4. *The bond liquidity premia equal fees paid by firms as per equations (7) and (8)*

Proposition 1 summarize the characteristics of the emerging steady state equilibria.

Proposition 1. *For any $\kappa \geq 0$, the economy has the three steady-state equilibria:*

(i) a dollar-dominant steady state with $X_{rw} = 1$ and $B^{€}_{rw} = 0$;

(ii) a euro-dominant steady state with $X_{rw} = 0$ and $B^{\$}_{rw} = 0$;

(iii) a multipolar steady state with $X_{rw} = 1/2$ and $B^{\$}_{rw} = B^{€}_{rw}$.

Outside of the knife-edge case where $\kappa = \frac{\pi - r}{\bar{B}+1}$, these are also the only steady states.

Proof. Proved in Appendix A. ■

The steady-state multiplicity that emerges even when $\kappa = 0$ highlights the feedback loop between household and trader choices generated by the financing frictions in trade. For example, in that case the dollar-dominant steady state exists not because the firms in the rest-of-world SOEs choose dollar funding in an attempt to match behavior with their trading counterparties, but because the RW households happen to hold a lot of US assets, making dollar financing the most convenient for RW firms. The rest-of-world households, in turn, are happy to concentrate their holdings in US assets because the traders' demand for dollar funding gives rise to a liquidity premium on dollar assets.

This logic also explains why steady-state multiplicity is robust to generalizing a number of the details of our model of international trade. For example, any strategy that traders may take to avoid paying currency mismatch costs — such as directing their search to counterparties holding a particular type of collateral or renegotiating the settlement currency ex-post — will not eliminate the key interaction between households and firms. Similarly, the result is robust to allowing households to lend their assets as collateral in foreign markets, since the optimal allocation of assets across credit markets in different countries j will still satisfy (9) and (10), as those optimality conditions simply rely on return equalization.

Overall, the complementarity between trading firms and households implies that dominant-asset equilibria are characterized by large holdings of the dominant asset by RW households. This showcases how in our mechanism wide holdings of a given asset beget and support that asset's dominant position in the broader international financial system.[8]

Stability of steady states

Our chief objective in this paper is to model why dominant-asset equilibria emerge endogenously and to understand why such outcomes might be *stable*. When $\kappa < \kappa^{sunspot}$, the equilibria are not subject to sunspot shocks that can change the equilibrium in trade finance markets without a concurrent shift in household actions. Thus, next we examine under what conditions the interactions between households and firms results in locally stable dominant-asset steady states, in the sense that the best response functions of each jointly define a local contraction map. Intuitively, this ensures that if one type of agent deviates from their optimal steady-state action, this would not give an incentive to the other type to change behavior, and hence ultimately the economy would converge back to that steady state.

We find that for intermediate values of κ the model can generate the desired outcome of dominant-asset steady states that are both locally stable and not subject to sunspot shocks. They key for generating this result is a calibration in which the two types of strategic complementarities in the model — (i) between households and firms and (ii) between firms in different countries — interact in a way such that neither dominates.

Proposition 2. *For $\kappa > \bar{\kappa}$ the dominant-currency steady states are locally stable, where*

$$\bar{\kappa} \equiv \frac{\pi - r}{\bar{B} + 1} < \kappa^{sunspot}.$$

Proof. Proved in Appendix A. ■

[8]Appendix B provides empirical evidence supporting the link between asset holdings and currency use.

The result can be understood by considering the two kinds of incentives traders face in their funding choice – the differential availability of financing, due to the feedback between household and trader choices, and the cross-country coordination incentive across traders.

Consider a situation in which the economy begins at the dollar-dominant steady state, with RW household portfolios concentrated in US assets and RW firms using only US safe assets for financing their trade. Suppose now that households shift their savings towards EZ assets. The increased availability of euro trade financing gives firms an incentive to shift towards funding via EZ assets. However, as long as $\kappa > \bar{\kappa}$, this change in firms' incentives is weak, because a small shift in portfolios still leaves a high overall usage of dollar trade financing internationally, and firms have an incentive to match behavior with their potential counterparties. Thus, given a local shift in household portfolios, the cross-country coordination of firms prevents the currency quasi-equilibrium (X) from changing significantly, which in turn means that the conjectured shift in household portfolios is suboptimal.

Crucially, there is an interaction between the two complementarity mechanisms which ensures that the dominant-asset steady states become stable at a relatively low level of κ, such that the cross-trader coordination incentives are too weak to generate sunspot multiplicity. Intuitively, when household bond holdings are asymmetric, firms tends to prefer to fund themselves in the more broadly available asset not only because of its more favorable credit conditions, but also because the firms understand the credit conditions in other countries will similarly favor this kind of funding for their potential counterparties. As a result, even a relatively "low" κ, once amplified with the coordination-device effect of asymmetric bond holdings, is enough to ensure the local stability of dominant-asset steady states.

Hence, there exists an interval of intermediate $\kappa \in (\bar{\kappa}, \kappa^{sunspot})$, such that the dominant-asset steady states are locally stable and the trade financing choice remains uniquely determined *given* bond holdings. This is at the heart of our model's ability to deliver persistent dominant-asset regimes, and the interaction between the two complementarity mechanisms is what makes this possible. Neither the asset availability mechanism nor the cross-country complementarity can do so on their own: If $\kappa = 0$, the dominant-asset steady state equilibria are unstable. While if $\kappa > 0$ and bond supplies are infinite (so credit frictions are non-existent), the firms' funding choices are subject to sunspot shocks.

In the numerical analysis of our full model (to which we turn next), we exploit this feature, and calibrate κ to an intermediate value such that the interaction between the two mechanisms allows the model to obtain both dynamically stable dominant-asset steady states and determinate transition paths (i.e. no sunspot indeterminacy). These features are crucial for the empirical realism of the model, since historically the international financial system has experienced prolonged dominant asset regimes.

3 Dynamic General Equilibrium Model

Having illustrated the key intuition behind our mechanism, we now embed it within a rich dynamic general equilibrium model. We calibrate the model, and show it can match both targeted and untargeted moments, quantify the welfare effects of dominance, and perform counterfactuals to better understand under what conditions dominant currencies can fall.

3.1 Setup

Like our stylized model, our general environment consists of households and firms in the US, EZ, and a continuum of rest-of-world small open economies, but each of those agents now have several margins of choice. Households choose a basket of consumption, as well as their optimal savings patterns. Trading firms make optimal choices about whether to operate and how/where to trade, as well as the choice of how to finance their activities. Prices and quantities are all are determined in general equilibrium.

3.1.1 Households

The household sector in any country $j \in \{us, ez, [0, \mu_{rw}]\}$ consists of a representative consumer who seeks to maximize the present discounted value of utility,

$$E_0 \sum_{t=0}^{\infty} \beta^t \frac{C_{jt}^{1-\sigma}}{1-\sigma}.$$

In contrast to the model in Section 2, each household is endowed with *a country-specific good*, and the consumption basket C_{jt} is a Cobb-Douglas aggregate of the all domestic and foreign goods. The consumption share of the domestic good is $a_h \in (0, 1)$, and the consumption shares for foreign goods are proportional to the respective size of the origin country. For example, the US consumption basked is

$$C_{us,t} = (C_{us,t}^{us})^{a_h} (C_{us,t}^{ez})^{\frac{(1-a_h)\mu_{ez}}{\mu_{ez}+\mu_{rw}}} (C_{us,t}^{rw})^{\frac{(1-a_h)\mu_{rw}}{\mu_{ez}+\mu_{rw}}}. \tag{11}$$

In the above, C_{jt}^{i} denotes consumption of good i in country j and $C_{jt}^{rw} \equiv (\int_0^{\mu_{rw}} (C_{jt}^{i})^{\frac{\eta-1}{\eta}} di)^{\frac{\eta}{\eta-1}}$

aggregates goods from the small open economies. The price index corresponding to (11) is

$$P_{us,t} = K^{-1}(P^{us}_{us,t})^{a_h}(P^{ez}_{us,t})^{\frac{(1-a_h)\mu_{ez}}{\mu_{ez}+\mu_{rw}}}(P^{rw}_{us,t})^{\frac{(1-a_h)\mu_{rw}}{\mu_{ez}+\mu_{rw}}},$$

where K is a proportionality constant and $P^{j}_{us,t}$ is the price of country j's differentiated good in the US. The consumption baskets and price indexes of the EZ and the small open economies are analogous.

Because of the frictions in international trade, the law of one price does not hold in our economy and goods have different equilibrium prices in different locations, with generally $P^i_{j,t} > P^j_{j,t}$. This effective markup on imports relative to the origin-country price is endogenous and depends on the equilibrium patterns of trade, as we describe below. Nevertheless, since our economy is real, all prices can be expressed in terms of a numeraire good, which we take as the domestic price of small open economy goods, i.e. $P^{rw}_{rw,t} \equiv 1$.

In addition to consumption, households choose how much to save and how to allocate savings among US and EZ bonds, each of which yields a risk-free unit of their respective domestic good. The household in country j faces the budget constraint:

$$\begin{aligned} P_{jt}C_{jt} + (1-\Delta^{\$}_{jt})P^{us}_{us,t}Q^{\$}_t B^{\$}_{jt} + (1-\Delta^{€}_{jt})P^{ez}_{ez,t}Q^{€}_t B^{€}_{jt} + \text{adj. costs}_t \\ = P^{us}_{us,t}B^{\$}_{jt-1} + P^{ez}_{ez,t}B^{€}_{jt-1} + P^{j}_{jt}Y_{jt} + \Pi^T_{jt} + T_{jt}, \end{aligned} \tag{12}$$

where $Q^{\$}_t$ and $Q^{€}_t$ are the prices of the US and the EZ bonds, Y_{jt} is the household's endowment of its domestic good, Π^T_{jt} is the total profit of country j's import/export firms (described below), and T_{jt} are lump-sum taxes. As in our simple model, a household's bond holdings earn an endogenous liquidity premium, given by the intra-period cash flows $\Delta^{\$}_{jt}$ and $\Delta^{€}_{jt}$. We focus on a perfect foresight, symmetric model hence we assume that all endowments are constant through time and equal, $Y_{jt} = \bar{Y}$ for all j.

Households are also subject to external portfolio adjustment costs, given by

$$\text{adj. costs}_t \equiv P^{us}_{us,t}Q^{\$}_t\tau(B^{\$}_{jt}, \underline{B}^{\$}_{j,t-1}) + P^{ez}_{ez,t}Q^{€}_t\tau(B^{€}_{jt}, \underline{B}^{€}_{j,t-1}).$$

These costs are parameterized by the function $\tau(B, \underline{B}) \equiv \frac{\tau}{2}\left(\frac{B-\underline{B}}{\underline{B}}\right)^2 \underline{B}$, which is quadratic in terms of percent deviations from the country-wide bond holdings entering the period, $\underline{B}^{\$}_{j,t-1}$ and $\underline{B}^{€}_{j,t-1}$. These adjustment costs are zero at (any) steady state, and thus have no effect on steady states, but serve to limit the volatility of capital flows outside of steady state.

Intertemporal optimality implies the following household Euler equations:

$$1 = \beta E_t \left[\left(\frac{C_{jt+1}}{C_{jt}} \right)^{-\sigma} \frac{P_{jt}}{P_{jt+1}} \frac{P^{us}_{us,t+1}}{P^{us}_{us,t}} \frac{1}{Q^{\$}_t \left(1 - \Delta^{\$}_{jt} + \tau'(B^{\$}_{jt}, \underline{B}^{\$}_{j,t-1}) \right)} \right] \tag{13}$$

$$1 = \beta E_t \left[\left(\frac{C_{jt+1}}{C_{jt}} \right)^{-\sigma} \frac{P_{jt}}{P_{jt+1}} \frac{P^{ez}_{ez,t+1}}{P^{ez}_{ez,t}} \frac{1}{Q^{€}_t \left(1 - \Delta^{€}_{jt} + \tau'(B^{€}_{jt}, \underline{B}^{€}_{j,t-1}) \right)} \right]. \tag{14}$$

The equations are analogous to (1)-(2), with exception that they now reflect the consequences of relative price differences among goods and across time, as well as influence of adjustment costs on effective bond returns.

3.1.2 The Import-Export Sector

International goods trade is subject to search and matching frictions as emphasized by the recent trade literature (e.g. Antras and Costinot, 2011). Specifically, we now assume that trade flowing through import/export firms directly affects the consumption possibilities in each country: trading firms organize to sell country-j's differentiated good in country i, via a match between a country-j export firm with a country-i import firm.

Once matched, the exporting firm buys goods at the prevailing domestic market price and sells them to the matched foreign importer, who then resells the good to the country-i household at the prevailing market price in that location. Firms optimally choose the intensity with which they search for different types of trade partners (e.g. import from US vs import from EZ), and the resulting matching patterns determine the size and direction of equilibrium trade flows. The import/export firms operate within the period, return profits to households, and disband.[9]

As before, international trade is subject to a financial friction, which implies a need for trade finance. Firms look for a fixed amount of funding, which we normalize to one unit of the numeraire, and firms again make the binary choice of either seeking US or EZ safe assets. Both of these assumptions can be relaxed. While stylized, this framework closely resembles the actual letter of credit mode of international trade financing, and serves as tractable abstraction for a wider range of trade finance arrangements.[10] This environment

[9] This market structure aligns well with the empirical evidence of significant churn in the firm-to-firm trade relationships (Eaton et al., 2016).

[10] For example, it can also be interpreted as a financial friction in raising the working capital needed to produce the export good or maintain the distribution network in the destination country. The importance of such financing frictions to international trade is documented by Manova (2013) and Manova et al. (2015).

also captures the first two key empirical features of trade finance outlined in the Introduction: financing is essential for trade and it is provided locally.

A trading firm's choices occur in two stages. In the first, the firm chooses whether or not to pay a fixed cost and become operational in a given period and the likelihood that, if operational, it will pursue an import or export opportunity and in what partner country. Second, the firm chooses the type of trade financing to apply for. We consider each stage in detail before characterizing equilibrium in the model.

Entry and trading pattern choice

Trading firms must pay a fixed cost ϕ in units of the domestic good before entering the international trade market. Potential trading firms enter only if the ex-ante expected profits from trading net of this cost are positive. Firms that do enter make a probabilistic choice regarding the direction in which they will trade. Specifically, an active country-j firm chooses the probability that it will become an importer from any country i or an exporter to any country i, which we denote p^{im}_{jit} and p^{ex}_{jit} respectively.

Let $\Pi^{\$}_{jt}$ and $\Pi^{€}_{jt}$ be the profits a firm expects to earns if it chooses to operate with US or EZ funding, respectively, and let X_{jt} be the fraction of country-j firms that optimally choose to use dollar funding. Since X_{jt} also corresponds to the probability that the firm itself will choose dollar funding funding in Stage 2, the ex-ante value of operating a firm is given by

$$W_{jt} = \max_{\{p^{im}_{jit}, p^{ex}_{jit}\}} X_{jt}\Pi^{\$}_{jt} + (1 - X_{jt})\Pi^{€}_{jt} - \phi P_{jt} \quad \text{s.t.} \quad \sum_{i\neq j} p^{im}_{jit} + \sum_{i\neq j} p^{ex}_{jit} = 1. \tag{15}$$

Firms enter as long as $W_{jt} > 0$. Thus, the equilibrium mass of active firms in country j, which we label m_{jt}, is determined by the condition $W_{jt} = 0$, and the optimal trade pattern is such that firms are indifferent between operating as an importer or exporter in any direction.

Funding Choice

In order to complete an international transaction, firms must arrive to the international trade markets with either US or EZ safe assets as collateral. With the mass of country-j trading firms, m_{jt}, determined by the zero-profit entry condition, the total mass of country-j trading firms applying for dollar funding is $m_{jt}X_{jt}$.

In each country, the domestic household supplies collateral by offering its bond holdings for intra-period loan in asset-specific funding markets. Relative to the analytical model in Section 2, we enrich our model of asset supply in a two ways. First, we tie the potential liquidity service of an asset to the total *market value* of its supply, rather than simply the available units of the asset. Second, because we will calibrate our model to annual data while the typical trade finance arrangement is much shorter, we introduce a parameter ν that corresponds to the number of times a given bond could be used to intermediate trade within one model period. Thus, the total value of trade flows that the country j holdings of US and EZ safe assets can intermediate is given by $\nu P^{us}_{us,t} B^{\$}_{jt} Q^{\$}_t$ and $\nu P^{ez}_{ez,t} B^{€}_{jt} Q^{€}_t$ respectively.

With these assumptions, the probability of success faced by a country-j trading firm seeking US financing is

$$p^{\$}_{jt} = \frac{M^F\left(m_{jt}X_{jt}, \nu P^{us}_{us,t} B^{\$}_{jt} Q^{\$}_t\right)}{m_{jt}X_{jt}} = M^F\left(1, \frac{\nu P^{us}_{us,t} B^{\$}_{jt} Q^{\$}_t}{m_{jt}X_{jt}}\right), \tag{16}$$

where we use the general form of the den Haan et al. (2000) matching function $M^F(u,v) = \frac{uv}{(u^{\frac{1}{\xi_F}} + v^{\frac{1}{\xi_F}})^{\xi_F}}$, which allows for an elasticity parameter ε_F that we calibrate. Similarly, a country-j trading firm seeking EZ bonds finds a credit match with a probability

$$p^{€}_{jt} = \frac{M^F\left(m_{jt}(1-X_{jt}), \nu P^{ez}_{ez,t} B^{€}_{jt} Q^{€}_t\right)}{m_{jt}(1-X_{jt})} = M^F\left(1, \frac{\nu P^{ez}_{ez,t} B^{€}_{jt} Q^{€}_t}{m_{jt}(1-X_{jt})}\right). \tag{17}$$

In sum, the market tightness of the financing markets is given by ratio of supply to demand in each: $\frac{\nu P^{us}_{us,t} B^{\$}_{jt} Q^{\$}_t}{m_{jt}X_{jt}}$ and $\frac{\nu P^{ez}_{ez,t} B^{€}_{jt} Q^{€}_t}{m_{jt}(1-X_{jt})}$. We note here that tying the effective trade finance supply to the market value of assets introduces a new channel that generally reinforces the emergence of a dominant asset: Since a dominant asset carries a high equilibrium price, an asset's ability to facilitate trade increases as it becomes dominant.

As in Section 2, we fix the collateral use in the big countries exogenously, though now we calibrate the probabilities $X_{us} = 1 - X_{ez}$ to the domestic-currency trade finance usage in the US and Eurozone data, which is high but not exactly 100%. We continue to assume that the US and EZ firms face the same financing frictions as small open economies, so equations (16) and (17) apply without modification for these countries as well.

In making their funding choice, the rest-of-world firms compare the respective expected profits of seeking dollar and euro financing. Upon a successful funding match, the expected profit of a country-j trading firm using US safe assets as a collateral guarantee is given by

$$\tilde{\Pi}^{\$}_{jt} = \sum_{i \neq j} p^{im}_{jit} \pi^{\$,im}_{jit} + \sum_{i \neq j} p^{ex}_{jit} \pi^{\$,ex}_{jit},$$

where $\pi_{jit}^{\$,im}$ is the expected profit of a firm importing from i to j that is financed via US bonds, and $\pi_{jit}^{\$,ex}$ is the analogous values for a country-j exporter looking to match with a country-i importer. The corresponding expected profits of a country-j firm funded with EZ assets instead is

$$\tilde{\Pi}_{jt}^{€} = \sum_{i \neq j} p_{jit}^{im} \pi_{jit}^{€,im} + \sum_{i \neq j} p_{jit}^{ex} \pi_{jit}^{€,ex}.$$

We describe the determinants of these trading profits in detail below.

In return for the intra-period use of the household's bonds, the firm pays a fee r. Thus, the expected net payoff to a country-j firm seeking dollar funding is given by

$$\Pi_{jt}^{\$} = p_{jt}^{\$}(\tilde{\Pi}_{jt}^{\$} - r), \tag{18}$$

which is simply the probability of obtaining dollar funding, $p_{jt}^{\$}$, times the expected profit net of the dollar funding costs. The expected payoff to seeking Euro funding is $\Pi_{jt}^{€} = p_{jt}^{€}(\tilde{\Pi}_{jt}^{€} - r^{€})$.

Lastly, to match the empirical fact that, despite its dominance, the dollar is not the *only* currency used to finance global trade, we introduce an i.i.d. additive idiosyncratic preference for the type of trade financing $\theta_{jt}^{(l)} \sim N(0, \sigma_\theta^2)$. This generates some idiosyncratic heterogeneity across firms and thus results in an interior equilibrium value for the currency mix X_{jt}, which we can then calibrate to the data.

Combing the the probabilities of obtaining each type of funding, (16) and (17), the expressions for profits, $\Pi_{jt}^{\$}$ and $\Pi_{jt}^{€}$, and the disturbance $\theta_{it}^{(l)}$, we can compute an individual firm's net benefit of seeking financing via US assets:

$$V_{jt}^{\$,(l)} = \frac{1}{\left[1 + \left(\frac{m_{jt} X_{jt}}{\nu P_{us,t}^{us} B_{jt}^{\$} Q_t^{\$}}\right)^{\frac{1}{\xi_F}}\right]^{\xi_F}} (\tilde{\Pi}_{jt}^{\$} - r) - \frac{1}{\left[1 + \left(\frac{m_{jt}(1 - X_{jt})}{\nu P_{ez,t}^{ez} B_{jt}^{€} Q_t^{€}}\right)^{\frac{1}{\xi_F}}\right]^{\xi_F}} (\tilde{\Pi}_{jt}^{€} - r) + \theta_{it}^{(l)}.$$

Firm l in country j will then choose to seek dollar funding if and only if $V_{jt}^{\$,(l)} > 0$. Given that the expected payoff of seeking dollar funding is increasing in $\theta_{jt}^{(l)}$, we can express the optimal choice in terms of a threshold strategy, where the firm seeks dollar funding if and only if their idiosyncratic shock exceeds a country-specific threshold $\bar{\theta}_{jt}$. Thus, the fraction of country-j trading firms using US safe assets is

$$X_{jt} = \int_0^1 \mathbb{1}(\theta_{jt}^{(l)} >= \bar{\theta}_{jt}) dl = 1 - \Phi\left(\frac{\bar{\theta}_{jt}}{\sigma_\theta}\right),$$

where $\Phi(\cdot)$ denotes the standard normal CDF.

In equilibrium, the cutoff $\bar{\theta}_{jt}$ is defined by $V_{jt}^{\$,(l)} = 0$, the value of the idiosyncratic preference shock that leaves a country-j trader indifferent between choosing one asset or the other. We focus on symmetric equilibria where all ex-ante identical RW countries have the same equilibrium allocations, hence $\bar{\theta}_{jt} = \bar{\theta}_t$ for all $j \in [0, \mu_{rw}]$.

Exchange of Goods

The remaining steps in our model of trade unfold without further decisions on the part of trading firms. Firms that are successful in obtaining financing search for a foreign trading counterpart. Country-j exporters match with country-i importers according to the technology $M^T(u, v) = \frac{uv}{(u^{\frac{1}{\varepsilon_T}} + v^{\frac{1}{\varepsilon_T}})^{\varepsilon_T}}$, which is of the same functional form as the matching function in credit markets, but allows for a different elasticity parameter ε_T.

The probability of a country-j exporter matching with a country-i importer is

$$p_{jit}^{ei} = \frac{M^T\left(\tilde{m}_{jit}^{ex}, \tilde{m}_{ijt}^{im}\right)}{\tilde{m}_{jit}^{ex}} = \left(1 + \left(\tilde{m}_{jit}^{ex}/\tilde{m}_{ijt}^{im}\right)^{1/\varepsilon_T}\right)^{-\varepsilon_T}.$$

where $\tilde{m}_{ijt}^{im} = p_{ijt}^{im} m_{it}(p_{it}^{\$} X_{it} + p_{it}^{€}(1 - X_{it}))$ is the mass of *funded* importing firms in country i (given by the term $m_{it}(p_{it}^{\$} X_{it} + p_{it}^{€}(1 - X_{it}))$) seeking trade with country-j firms that are looking to export to i, which are themselves of mass $\tilde{m}_{jit}^{ex} = p_{jit}^{ex} m_{it}(p_{it}^{\$} X_{it} + p_{it}^{€}(1 - X_{it}))$. Using analogous derivations, the probability of a country-j importer matching with a country-i exporter is $p_{jit}^{ie} = \left(1 + \left(\tilde{m}_{jit}^{im}/\tilde{m}_{ijt}^{ex}\right)^{1/\varepsilon_T}\right)^{-\varepsilon_T}$.

In a successful match between a country-j exporter and a country-i importer, the exporter buys the j good at its domestic market price P_{jt}^j and the importer then sells it to the country-i household at the prevailing market price in that location P_{it}^j. The transaction thus generates a gross surplus of $P_{it}^j - P_{jt}^j$. In addition, the transaction is subject to a collateral mismatch cost κ as discussed in Section 2.

The importer and exporter in a trading match split the surplus of their transaction via Nash bargaining, with the exporter having a Nash bargaining share of α. The effective "wholesale" price at which a country-j exporter sells to a country-i importer is thus

$$P_{jit}^{whol} = P_{jt}^j + \alpha(P_{it}^j - P_{jt}^j).$$

Then, the profit that a firm looking to export from country j to i *expects* to make is

$$\pi_{jit}^{\$,ex} = p_{jit}^{ei} \frac{\alpha}{P_{jit}^{whol}} \left[P_{it}^j - P_{jt}^j - \kappa P_{jit}^{whol}(1 - \tilde{X}_{it})\right]. \tag{19}$$

The term in square brackets is the net expected surplus per unit of goods traded, which is given by the gross markup on the imported good, net of the expected currency mismatch cost $\kappa P^{whol}_{ji,t}(1 - \tilde{X}_{it})$. In this expression,

$$\tilde{X}_{it} \equiv \frac{p^{\$}_{it} X_{it}}{p^{\$}_{it} X_{it} + p^{€}_{it}(1 - X_{it})}$$

is the average use of dollar trade financing among the funded country-i firms (which are thus actively searching for trade counterparts), hence $1 - \tilde{X}_{i,t}$ is the probability of matching with a EUR-funded country-i importer, and thus having to incur the expected default cost κ.[11]

Lastly, the financing friction limits the overall value of the transaction to the value of the attached safe collateral. Since we assume each firm borrows one unit of safe assets, to obtain the net expected profit from the view point of a country-j exporter (who earns α fraction of the total surplus), the expected per-unit profit is then scaled by $\frac{\alpha}{P^{whol}_{jit}}$.

Government

We assume that government purchases are zero, and thus governments play a role only in the large countries $j \in \{us, ez\}$, where they issue bonds in fixed supply $\bar{B} = B^{\$} = B^{€}$ and set the level of lump-sum taxes so as to keep their stock of debt constant at $\bar{B}$:

$$\bar{B} = T_{jt} + Q^{j}_{t} \bar{B}.$$

The small rest-of-world countries $j \in [0, \mu_{rw}]$ do not issue debt and set $T_{jt} = 0$.

Equilibrium

In equilibrium, the liquidity premia a country-j household can earn on lending US and EZ bonds respectively are equal to the frequency with which the household successfully lends the asset in its respective credit market multiplied by the funding fee r:

$$\Delta^{\$}_{jt} = \frac{\nu m_{jt} X_{jt}}{\left[(m_{jt} X_{jt})^{1/\xi_F} + (\nu P^{us}_{us,t} B^{\$}_{jt} Q^{\$}_{t})^{1/\xi_F}\right]^{\xi_F}} r \tag{20}$$

[11] Note that since a potential trading partner's funding is uncertain, it would be costly to first match with a counterparty, agree on a particular type of collateral, and only then seek that financing. If the financing falls through, the firm looses the opportunity to earn π, which is simply not worth the risk for potentially saving the small mismatch cost κ.

$$\Delta_{jt}^{€} = \frac{\nu m_{jt}(1 - X_{jt})}{\left[(m_{jt}(1 - X_{jt}))^{1/\xi_F} + (\nu P_{ez,t}^{ez} B_{jt}^{€} Q_t^{€})^{1/\xi_F}\right]^{\xi_F}} r \tag{21}$$

Given those expressions, the rest of the equilibrium is determined by the household and firms' optimal decisions, and market clearing in real goods and bond markets. We focus on the class of symmetric equilibria where the strategies of the ex-ante identical rest-of-world traders and households are the same, e.g. $X_{jt} = X_{rw,t}$ for all $j \in [0, \mu_{rw}]$.

Definition 3 (**Equilibrium**). *A symmetric equilibrium is a pair of bond prices* $\{Q_t^{\$}, Q_t^{€}\}$, *and a set of country specific allocations* $\{C_{jt}^{us}, C_{jt}^{ez}, C_{jt}^{rw}, B_{jt}^{\$}, B_j^{€}, X_{jt}, m_{jt}, p_{jit}^{im}, p_{jit}^{ex}\}$, *prices* $\{P_{jt}^{us}, P_{jt}^{ez}, P_{jt}^{rw}\}$, *and liquidity premia* $\{\Delta_{jt}^{\$}, \Delta_{jt}^{€}\}$ *for* $j \in \{us, ez, rw\}$ *such that*

1. *The household optimality conditions are satisfied.*
2. *The trading firms optimality conditions are satisfied.*
3. *Liquidity premia earned by households are given by* (20)-(21).
4. *The mass of successful cross-border trading matches is consistent with consumption of foreign goods* $C_{jt}^{j'}$ *for all* $j \neq j'$.

3.2 Calibration

We fix a set of parameters to standard values, then use the remaining parameters to target several moments, which the model is able to exactly replicate, as described below.

The exogenously-fixed parameters are listed in Table 1. Specifically, we set $\mu_{us} = \mu_{ez} = 0.2$, consistent with the sizes of the US and the EZ in world GDP. One model period represents a year, hence we set $\beta = 0.96$; we also assume log preferences ($\sigma = 1$). Next, we minimize the role of the search friction between exporters and importers by using a low value for the elasticity of the trade matching function, $\varepsilon_T = 0.01$, which ensures that firms on the less crowded side of the market are virtually guaranteed a match. We fix $\alpha = 0.5$, implying that importers and exporters have equal bargaining power and set the currency use in the big countries (X_{us} and X_{ez}) so that 90% of their firms use the domestic asset, to match the evidence on domestic currency usage in trade finance for the US and the EZ (BIS, 2014).

To match the observed maturity and cost of a typical letter of credit contract in the data (45 days on average), we set $\nu = 8$ so that every safe asset can be used up to 8 times per year for trade finance operations. To match the typical cost of letters of credit – which include

Parameter	Concept	Value
β	Time preference	0.960
$\mu_{us} = \mu_{ez}$	Big country measure	0.200
κ	Mismatch cost	0.010
r	Funding fee	0.005
υ	Exog. velocity	8.000
X_{us}	US dollar share	0.900
X_{ez}	EZ dollar share	0.100
α	Exporters bargaining parameter	0.500
σ	Risk aversion	1.000
ε_T	Elasticity of trade matching function	0.010
σ_θ^2	Variance of idio. shock	1e-06
τ	Portfolio adj. costs	0.040

Table 1: Exogenously Fixed Parameters

a substantial fixed component, on average 40 basis points of the principal, plus a spread on top of the LIBOR – we set $r = 0.005$.[12]

We calibrate additional parameters by targeting specific model implications and moments. First, we set the collateral mismatch cost κ to the minimum value such that the dominant-currency steady states of the model are locally stable, in the sense that the linearized solution around each of those steady states has stable eigenvalues, which leads us to $\kappa = 0.01$. This value is both small in absolute terms – mismatch costs are just 1% of the value of a transaction – and also turns out to be small enough to prevent sunspot multiplicity in the collateral choice quasi-equilibrium, given bond holdings.

Next we set $\tau = 0.04$ to be as small as possible, while still preventing large instantaneous jumps in the composition of bond portfolios (e.g. a shift from a portfolio concentrated in US asset, to one concentrate in EZ assets within a period) that could lead to equilibrium multiplicity. This value implies that a 10% change in bond positions incurs a cost of just 2 basis points on the portfolio. We also make the currency preference shocks as small as possible (σ_θ^2 =1e-06), while still ensuring numerically reliable interior solutions for X_{rw}.

We calibrate the remaining parameters of our model to match a set of target steady-state moments. As foreshadowed by the analytical analysis, the model has three steady states – dollar and euro dominant ones, and a symmetric one. Since in our data sample (1984-2017) the dollar has been the dominant currency, we match the empirical moments to those at the dollar-dominant steady state of the model. Panel (a) of Table 2 summarizes the targeted

[12] See guidance by the US Commerce Dept.: acetool.commerce.gov/cost-risk-topic/trade-financing-costs.

Concept	Target Value
Gross debt/GDP	0.60
ROW trade/GDP	0.55
ROW USD invoice shr.	0.80
Import markup	1.10

(a) Calibration Targets

Parameter	Concept	Value
$\bar{B}$	US/EZ asset supply	0.613
a_j^h	Home bias	0.718
ε^f	Funding match. elas.	0.294
ϕ	Fixed cost of entry	0.038

(b) Implied Parameter Values

Table 2: Calibration Strategy

moments: (1) government debt of 60% of GDP, consistent with the US average; (2) rest-of-world trade share ($\frac{\text{Imports + Exports}}{\text{GDP}}$) of 55%, consistent with trade data for non-US and non-EZ countries from the World Bank; (3) dollar share in trade financing used by RW trading firms of 80%, consistent with the evidence of on the fraction of letters of credit and trade finance loans denominated in dollars BIS (2014); and (4) import markups of 10%, consistent with micro-level estimates on import markups in Coşar et al. (2018).

We target these four moments with the four remaining free parameters $\{\bar{B}, a^h, \varepsilon^F, \phi\}$. These parameters are: (1) the supply of government debt $\bar{B}$; (2) the home bias parameter in consumption preferences a_h which determines the trade share; (3) the elasticity of the funding matching function ε^F which helps determine the equilibrium level of currency coordination; and (4) the fixed cost of entry in the trading sector ϕ which helps determine import markups. We find the model can exactly match the targeted moments, with the implied parameter values given in Panel (b) of Table 2.

3.3 Main Results

We now consider the model's quantitative implications for non-targeted moments in steady-state and then proceed to use global techniques to solve for the model's (perfect-foresight) transition dynamics. We then explore the model's implications for two counterfactual scenarios. In the first, we examine what could have happened to dollar dominance had EZ continued it expansion beyond its current size. In the second, we explore the consequences — including welfare implications — of trade wars initiated by the United States.

Steady State

Table 3 summarizes several key untargeted moments for each of the three steady states in the calibrated economy. The resulting dollar-dominant steady state matches a number of untargeted empirical regularities. First, since the RW countries primarily use dollars for trade finance (i.e. $X_{rw} = 0.8$), the US bond earns a higher equilibrium liquidity premium: $\Delta^{\$} > \Delta^{€}$. But since the Euler equations ensure that the total returns on the two bonds are equalized in equilibrium, this leads to an interest parity violation, where the interest rate on the EZ is higher than that of the US bond in order to offset its worse liquidity return:

$$\frac{1}{Q^{€}} - \frac{1}{Q^{\$}} = \frac{\Delta^{\$} - \Delta^{€}}{\beta} = 1.07\%$$

At our calibration, this interest differential is equal to 1.07%. Thus, the US earns a significant "exorbitant privilege" on its external net foreign assets position, as the interest rate it pays on its foreign liabilities (i.e. foreign holdings of US bonds) is significantly lower than the interest it earns on its foreign assets (i.e. US holdings of EZ bonds). This excess return magnitude is consistent with evidence on the US exorbitant privilege by Gourinchas and Rey (2007) and also the US Treasury convenience yield estimated by Jiang et al. (2020).[13]

The third line of the table (seignorage) provides a standard back-of-the-envelope estimate of the net benefit the US receives from its exorbitant privilege. It is computed as the counterfactual additional debt servicing payments the US would face, if it actually paid an interest rate equal to the inverse of the time discount, holding everything else constant. Essentially, this is the seignorage the US earns from the liquidity premium on the dominant international asset, and our model estimated this to be quite substantial – it is equal to 0.88% of GDP.

This is a straightforward measure and similar calculations have often been used to estimate the benefits of exorbitant privilege. However, our model implies that this is an *incomplete* and potentially misleading measure of privilege, because it takes asset positions as given. A key insight of our theory is that widespread foreign holdings of a country's assets are needed to support its special status. But such strong external demand leads to a negative steady-state NFA for the central country, and hence the seignorage benefits of being dominant are at least partially offset by the need to service the concomitant negative NFA position.

Indeed, the fourth line in the table shows that the dominant country has a significant negative net foreign asset position equal to -42% of GDP at steady state. The other big country, the EZ, also has a smaller negative NFA position of -26% of GDP. This is a manifestation of the fact that US assets play the dominant role in international trade financing,

[13]It is also in line with data on the average UIP deviation for the USD against other major currencies. Note that while our perfect foresight model does not explicitly differentiate between covered and uncovered interest parity, our interpretation of the mechanism is most consistent with a violation of UIP, not CIP: Since the financial friction in trade is about guaranteeing future delivery and payment, the resulting convenience yield is earned by an asset's ability to deliver a promised future value. Hence an EZ bond with its exchange rate exposure sold forward is equivalent to a US bond, implying CIP holds, but UIP does not.

Moments	USD Dominant			Multipolar			EUR Dominant		
	US	EZ	RW	US	EZ	RW	US	EZ	RW
					Panel A: Benchmark model				
USD share trade fin. (X_j)	0.90	0.10	0.80	0.90	0.10	0.50	0.90	0.10	0.20
$100\times(i^{\$}-i^{€})$	1.07	-	-	0.00	- 0	-	-	-1.07	-
100×Seignorage/GDP	0.88	0.23	-	0.56	0.56	-	0.23	0.88	-
NFA/GDP	-0.42	-0.26	0.18	-0.38	-0.38	0.19	-0.26	-0.42	0.18
Gross Foreign Assets/GDP	0.04	0.02	0.18	0.02	0.02	0.19	0.02	0.04	0.18
100×Trade bal./GDP	0.87	0.86	-0.45	1.01	1.01	-0.52	0.86	0.87	-0.45
					Panel B: Rest-of-World asset $(\bar{B}_{rw}/(P_{rw}\bar{Y})=0.40)$				
$100\times(i^{\$}-i^{€})$	1.03	-	-	0.00	-	-	-	-1.03	-
NFA/GDP	-0.14	0.01	0.03	-0.10	-0.10	0.05	0.01	-0.14	0.03
Gross Foreign Assets/GDP	0.31	0.29	0.18	0.30	0.30	0.20	0.29	0.31	0.18
100×Trade bal./GDP	-0.25	-0.28	0.14	-0.12	-0.12	0.07	-0.28	-0.25	0.14

Table 3: Steady-state values for baseline model.

hence RW households concentrate their savings in US assets. Overall, we find that two-thirds of RW portfolios are invested in US assets, amounting to a long position in US assets equal to 12% of RW GDP, consistent with the evidence in Caballero et al., 2008.

Though the excess return the US earns on its external position helps support its worse NFA position, its overall steady-state trade balance is similar to that of the EZ. Thus, while the US has the benefit of issuing low-yield assets and investing in high-yield assets, at the steady state its position as a significant net debtor eats away the benefit of exorbitant privilege. This suggests that the welfare implications of dominance are nuanced. As we discuss in detail below, they are still substantial, but are concentrated in the transition period to the dominant steady state, during which the would-be dominant country finances temporarily elevated consumption via increasing external debt.

Lastly, we want to emphasize that while the US trade balance is positive in the benchmark model, the general mechanism can indeed generate a negative NFA and a trade deficit at the same time. The trade surplus in the benchmark model arises due to the fact that the only foreign asset the US can invest in is the EZ bond, which remains in high demand by the EZ households for trade financing purposes. Consequently, the US households have tiny foreign asset holdings, just 4% of GDP, which limits the overall benefit of the exorbitant privilege, as it depends on the size of the gross asset position the excess return applies to.

Panel B of Table 3 demonstrates that this is not limitation of the core mechanism. The panel considers a version of the model with a richer asset structure, where in addition to the two big countries, each of the small RW economies also issues government debt equal to 40% of their GDP. Each of those assets is measure zero, and hence play no role in financing international trade, but households in all countries hold the basket of RW bonds for investment purposes, since absent a liquidity premium they offer high returns.[14]

In this version of the model, the US has a significantly higher gross asset position – 31% of GDP – as now there is a foreign asset which does not have a high liquidity value to foreigners. This changes the portfolio composition of the US, which ends up with a significant, leveraged position in the high-yielding RW assets financed with low-yielding US assets. This amplifies the effect of the exorbitant privilege, and results in a positive trade balance at steady state, despite the fact that NFA is still negative.[15] This is an important success of our framework, and something a large class of other models of the exorbitant privilege cannot generate (e.g. Caballero et al. (2008)). Still, we abstract from this third type of asset in the benchmark model because we want to minimize the number of state variables in order to facilitate the global, out-of-steady-state solution of the model. Importantly, the key result that much of the welfare benefit of dominance is concentrated in the transition to dominance, which we discuss next, is equally true in both versions of the model.

Dynamics

We now consider dynamics of the calibrated model outside of steady state, with a particular focus on determining the stability properties of the dominant-currency steady states.

To this end, Figure 1 plots the respective attraction regions of the models' three steady states. We compute these regions by defining a fine grid on the state space of the model, which is depicted in the axes of the figure.[16] For each grid point, we compute all perfect foresight equilibrium paths that start from there (i.e. the grid defines the initial conditions) and converge to one of the three steady states. Thus, for each grid point we run three separate attempts to compute an equilibrium path – one that ends at the dollar-dominant steady state, one that ends at the multipolar steady state, and one that ends at the euro-

[14] We provide more details on this version of the model in Appendix D.

[15] This compositional effect is also consistent with the data – see Gourinchas and Rey (2007).

[16] Note that there are four state variables: RW holding of US and EZ bonds, US holdings of US bonds, and EZ holdings of EZ bonds (with US and EZ foreign holdings determined by market clearing.) To display the Figure 1 in 2D, we initialize US and EZ portfolios shares at their symmetric steady-state level.

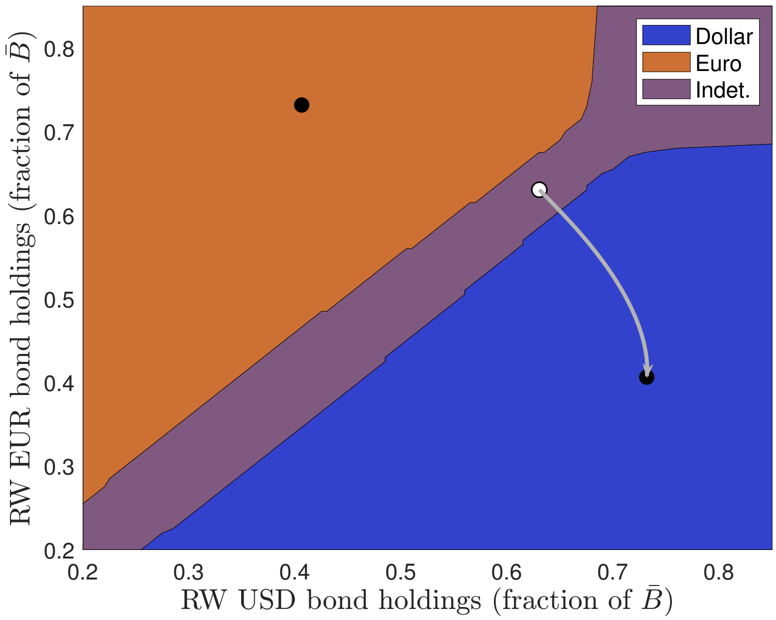


Figure 1: Steady state attraction regions.

dominant steady state. For each point in the blue region, we find that there is only a single possible equilibrium path, which converges to the dollar-dominant steady state. Conversely, for points in the orange region, the only feasible outcome is the euro-dominant steady state. Finally, the purple region corresponds to points where we found perfect foresight paths that arrive at both coordinated steady states – this is a region of dynamic indeterminacy.

A key result of the Figure is that only the dominant-currency steady states are dynamically stable, i.e. dynamic paths that are initialized away from the symmetric steady state never converge there.[17] Moreover, both dominant-asset steady states are contained within large regions from which the economy uniquely converges to them. For example, whenever the RW households' initial portfolio positions are sufficiently biased towards US assets (towards the bottom right corner of the graph), the unique equilibrium path converges to the dollar-dominant steady state. This is a manifestation of the interactions we explored in detail in Section 2: When RW household portfolios are sufficiently biased towards US bonds, firms tilt their actions towards financing international trade with US assets, perpetuating the households' decision to save primarily in US assets.

Hence, the figure shows that the dominant-asset regimes in the model are endogenously persistent and sustainable indefinitely, so long as no large shocks push the economy out of the respective regions of attraction. And even in that case, the model will still converge

[17]We have also confirmed via linearization that the dominant equilibria are locally-stable but the symmetric steady state is not.

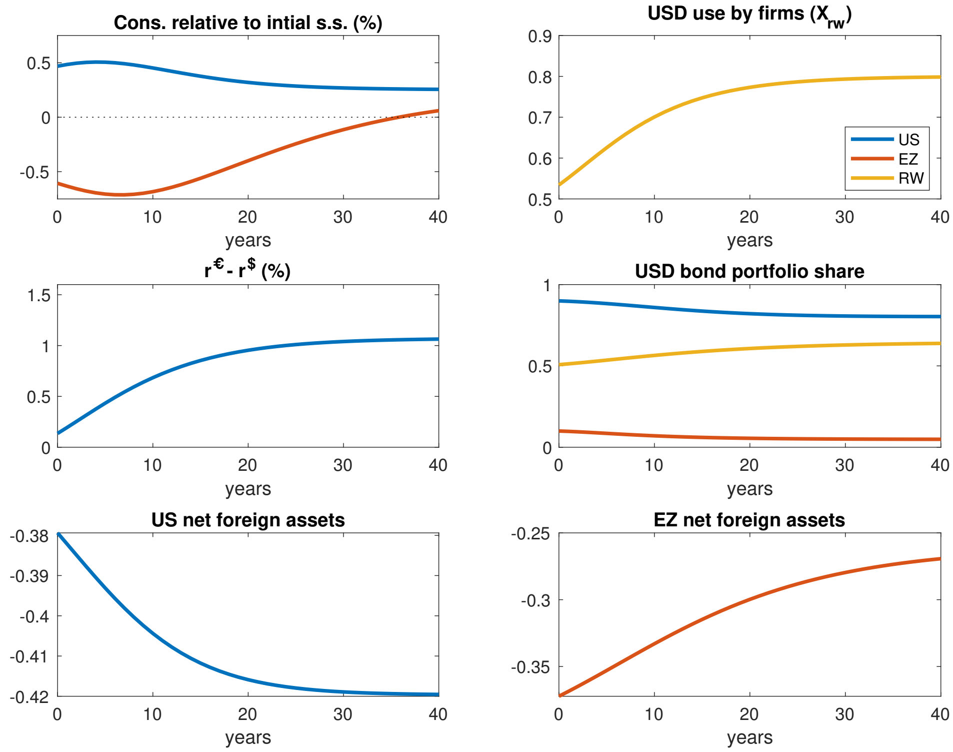


Figure 2: Transition from the symmetric steady state to dollar-dominant steady state.

to one or the other dominant-currency steady states, confirming that dominant-currency regimes as a whole are the likely outcome in any given simulation.

We next explore the transition paths underlying Figure 1 in more detail. As an example, Figure 2 plots the transition paths of several endogenous variables, when the economy starts at the (unstable) symmetric steady state and converges to the dollar-dominant steady state. The top right panel shows the evolutions of the equilibrium mix of collateral used (X_{rw}), which starts close to equally balanced and then gradually converges to dominant dollar usage over the subsequent 15-20 years. Along this transition path, the exorbitant privilege of the US gradually builds as RW's household portfolios (right column, middle plot and also the gray line in Panel (a) of Figure 1) shift towards US assets. The shift towards US assets, in turn, worsens the US net foreign asset position.

Perhaps most importantly, the top-left panel plots the paths of US and EZ consumption, showing that during this transition period US consumption is elevated for an extended period of time. This is a result of the *increasing* foreign demand for US assets, which allows the US to steadily increase its borrowing from the rest of the world at a low interest rate. Meanwhile, EZ consumption is significantly depressed, as the EZ increases savings in order to repatriate some of its assets, which were previously held abroad but are now no longer in high demand

	US	EZ	RW
steady-state only	0.25%	0.22%	-0.12%
incl. transition	0.37%	-0.37%	-0.00%

Table 4: Welfare gain/loss at dollar-dominant steady state, as percentage of symmetric steady state consumption.

externally as they lose their role in international trade.

Interestingly, we also find that consumptions in the US and the EZ eventually converge to roughly the same steady-state levels. Thus, even though the resulting steady state is one where the dollar is dominant, the US steady-state consumption is virtually identical to that of the EZ. This showcases that, at our benchmark calibration, the offsetting effects of the exorbitant privilege in interest rate differentials (which benefits the dominant country) and the necessarily worse NFA position of the dominant country roughly cancel each other out.

In Table 4 we summarize the welfare effects of dominance. The top line shows that as discussed above, at the dollar-dominant steady state itself the US and the EZ are similarly well off. Thus, a comparison of the eventual steady-state outcome alone suggests both US and EZ should be roughly indifferent about which country's asset is dominant.

While steady-state consumption levels suggest there is no harm of dollar dominance for the EZ, incorporating the transition dynamics to the dollar-dominant steady state completely reverses this conclusion. Taking into account the transition period, in which the US is able to maintain consumption significantly above the eventual steady-state level for an extended period of time, the US permanent consumption equivalent is 0.75% higher than that of the EZ. This is an order of magnitude larger than the 0.03% welfare gain implied by steady-state consumption difference alone. Thus, considering transitional dynamics is crucial for properly assessing the welfare implications of owning the dominant currency, highlighting the need for a model like this, where dominant currency regimes are determinate, endogenous outcomes.

3.4 Path Dependence: Historical context of dollar dominance

As is clear by now, the model can readily rationalize the historical experience of stable and prolonged dominant-asset regimes – e.g. the British pound was the dominant world safe asset from until the 1940s, and the US dollar has played that role since. Given that the model implies that only the dominant-asset steady states are stable, we should indeed expect that the world is typically in a long-lived dominant currency regime.

Still, why do we currently live in a USD-centric world as opposed to some other currency playing the dominant role? Our model points to the role of path dependence and an initial big shock. Naturally, in the case of the recent US dominance, the big shock was likely World War II. From the view point of our model, the post-WWII period is characterized by the ability of the US to credibly issue a much larger supply of safe assets ($\bar{B}^{\$} > \bar{B}^{£}$) than the UK, leading savers across the world to tilt portfolios towards the abundant US assets.

In addition, under the Bretton Woods agreement, the US was the *only* country with virtually no capital controls and thus free international access to its (abundant) supply of safe assets.[18] Second, the Marshall plan stimulated large quantities of international trade sourced from the US and financed by US dollars. The combination of the US capital markets being the only ones allowing unfettered access to foreign investors, the significantly larger ability of the US government to credibly issue safe assets and the Marshall plan directly providing USD trade finance to foreign countries all combined to shift the equilibrium and pushed the world inside the attraction region of the dollar-dominant steady state.

The historical perspective is interesting, but perhaps more currently relevant is the question of what would it take to cause a transition away from the US dollar today. One recent event that many academics and policy makers speculated may precipitate such a shift in the equilibrium was the introduction of the euro (e.g. Chinn and Frankel, 2007). Yet, our experience since shows that this was not sufficient to drive such a transition.

To understand why the creation of the Eurozone was not enough to shift the currency regime (and what it would have taken), we use our model to simulate two alternative, counter-factual scenarios of the EUR introduction. In both cases we consider the starting point to be a situation where the supply of the non-US safe asset is 60% of the size of the US safe asset supply: $\bar{B}^{€} = 0.6\bar{B}^{\$}$. Essentially, we assume that before the Eurozone was created the only European safe assets were generated by Germany. We then model the introduction of the Eurozone as a gradual increase in the supply, $\bar{B}^{€}$, of this rival asset over time.[19]

We find that initially, when the supply of EZ assets is only 60% of the supply of US assets, the model has a unique, dollar-dominant steady state. Thus, the fundamental asymmetry in the availability of the two types of safe assets is so strong that it is impossible to support a steady state where RW household holdings and international trade are all concentrated in

[18] See Ghosh and Qureshi (2016) for a detailed description of the evidence on capital controls. They also note that the US Treasury secretary at the time was well aware of the benefit of an open US capital account, having stated in his 1948 testimony to Congress that controlling capital inflows would require exchange controls that "would do maximum violence to our position as a world financial center."

[19] The gradual increase is motivated by the fact that interest rates on euro-area sovereigns did not collapse to the interest rate of the German Bund at the moment of the introduction of the euro, but took several years to converge. This suggests markets only gradually accepted euro bonds as a homogeneous safe asset.

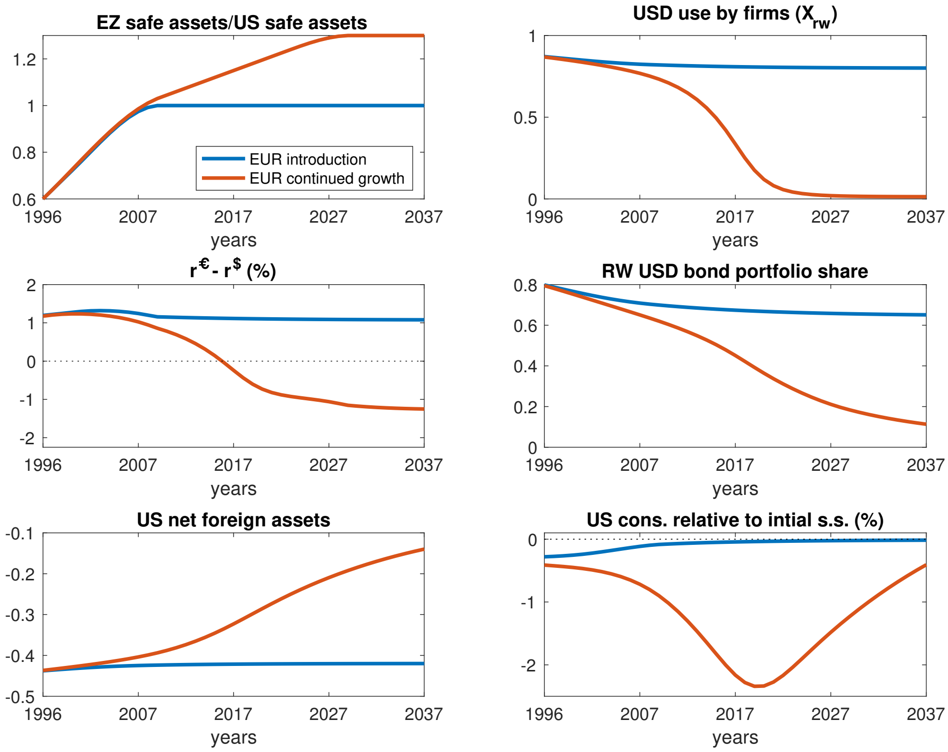


Figure 3: Introduction of euro

the smaller asset. We consider this to be the situation in the mid-1990s, and initialize the economy at the start of both counter-factuals at that steady state, and consider the resulting transition path as the supply of EZ assets grows with the introduction of the Eurozone.

The results of these exercises are displayed Figure 3. In the first scenario (blue line), we assume that the total supply of EZ safe assets converges to the same level as that of the US asset over a 10 year period (i.e. to our benchmark calibration). We find that there is a unique path that leads out of the pre-euro steady state, and that this path converges uniquely to the new dollar-dominant steady state. Along the transition, dollar use, the interest rate differential, and the US net foreign asset position are essentially unchanged, consistent with the continued dominance of the dollar in the data. Thus, our model agrees that path dependence is too strong for the introduction of the euro by itself to change currency regimes. The euro introduction creates a new euro-centric stable steady state, but without further shocks, the world economy will not converge to it.

In the second scenario, we consider the counter-factual possibility that the EZ grew larger than the US over time. Namely, we look for the threshold value for $\bar{B}^€$ such that the EZ asset base becomes so large that the unique stable steady state is the euro-dominant one. It turns out that for this to happen, the supply of EZ safe assets must exceed the supply of

US safe assets by at least 30% in the long-run: i.e. $\frac{\bar{B}^{€}}{\bar{B}^{\$}} > 1.3$. We plot this scenario with the red line in Figure 3, and note that this growth in EUR safe assets could have been quite plausible if the UK had indeed joined the EZ as initially anticipated to happen eventually.

Along the path to the new, uniquely-stable euro-dominant steady state, the US NFA position shrinks towards zero, and the US's exorbitant privilege benefits disappears as the US safe assets largely leave international markets and return to US portfolios. The figure shows that anticipation effects are important, as much of the shift in portfolios and usage occurs before the end of the rise in EZ asset supply. The welfare impact is also considerable: the welfare swing between the US and the EZ is equivalent to 1.4% of permanent consumption.

Thus, we conclude that our model can account for the evolution of the international financial system over the last century, and highlights in particular the important role played by inertia and the changing availability of safe assets internationally. In the current situation, merely introducing a rival safe asset that is equivalent in fundamentals to the dollar is not enough to shift the currency regime – for that to happen, the alternative safe asset needs to offer a substantially bigger base than the dollar. Yet the 30% threshold we found is not necessarily insurmountable, and could potentially be achieved by either the EZ or the Chinese renminbi in the future. However, the model does caution that the Chinese renminbi can not play a significant role until Chinese capital markets are sufficiently liberalized.

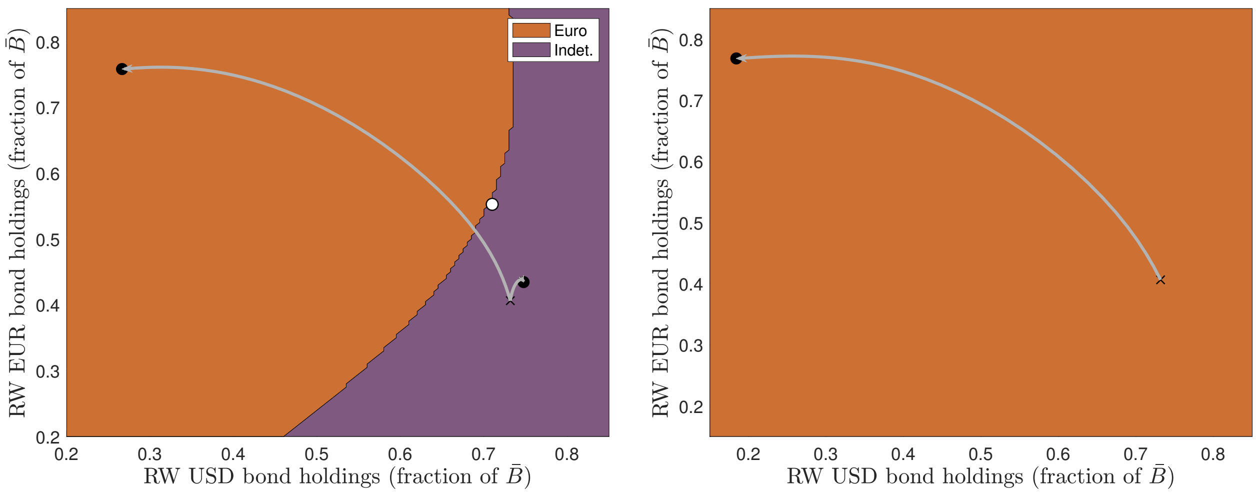
3.5 Trade Policy and Barriers

A natural question is whether trade barriers can affect the currency regime. To shed some light, in this section, we consider two counter-factual exercises that are particularly relevant, given the recent trade conflicts between the US and its trading partners.

We consider two scenarios of a "trade war" between the US and its trade partners, where the US introduces a proportional tariff γ on all imports and the EZ and RW countries respond in kind, and levy the same tariff on US imports. Tariffs are implemented as taxes paid by consumers, so while US importers receive the price of $P^j_{us,t}$ for each imported type j good, US consumers pay the effective price

$$\tilde{P}^j_{us,t} = (1+\gamma)P^j_{us,t}$$

Similarly, EZ and RW consumers pay import taxes on US imports, and hence face the prices $\tilde{P}^{us}_{jt} = (1+\gamma)P^{us}_{jt}$. In all countries, the import tax revenues (e.g. $\gamma P^{us}_{j,t}$) are then reimbursed lump-sum to households. Even though they are refunded, the tariffs lead to expenditure switching on the consumer side, and thus also shift the equilibrium patterns



(a) Attraction regions, moderate trade war

(b) Attraction regions, acute trade war

Figure 4: Attraction regions under trade ware scenarios.

of trade flows. We consider two scenarios, a "moderate" trade war where $\gamma = 0.15$ and an "acute" one where we set $\gamma = 0.3$.

Moderate Trade War

Figure 4, Panel (a) depicts the consequences of a permanent trade war, with tariffs set to $\gamma = 0.15$. The tariffs change the position of the steady states and also their respective attraction regions (old steady states are marked with a $\times$; the new steady states with a dot). As the figure shows, the region of unique attraction to the dollar-dominant steady state is eliminated, and both the old and new dollar-dominant steady states lie within the region of equilibrium indeterminacy. Moreover, the region of unique attraction ot the euro-dominant steady state is significantly increased. Hence, even a moderate trade war, as long as it is permanent, potentially endangers the position of the dollar, as it makes the dollar-dominant regime unstable and subject to sunspots.

The trade war weakens dollar dominance for two reasons. First, it diverts RW trade away from the US towards the EZ. Since EZ firms are far more likely to use EZ assets in their trade, RW firms become more likely to encounter euro-funded trading partners and, hence, to prefer euros. Second, the overall world trade level falls, decreasing trade quantities relative to total asset supply. As the relative scarcity of trade finance falls, the equilibrium anchoring effects of the financial friction become weaker, increasing the indeterminacy region.

The first row of Table 5 reports the welfare implications of the moderate trade war,

	US	EZ	RW
Dollar remains dominant	-1.34%	0.07%	0.09%
Euro becomes dominant	-2.13%	0.79%	0.10%

Table 5: Gain/loss as percentage of dollar-dominant steady state consumption.

assuming the dollar remains dominant. Even in that case the US is disproportionately hurt by the trade war for two reasons. First, there is the standard effect from the fact that the distortions created by the tariffs hit all of its exports, while the EZ and RW face tariffs only on their direct trade with the US. Second, as world trade levels fall, the US' seignorage revenue decreases as both fewer firms require liquidity and a smaller portion choose dollars.

Since the starting position of the economy (i.e. the dollar-dominant steady state before the tariffs) is now inside the indeterminate region, however, this is not the only possible outcome. In the second row of Table 5, we report the welfare implications of the same trade war scenario, but assuming the alternative long-run outcome that is now possible – that the dollar looses dominance and the economy transitions to the euro steady state.

In this second case, the US is significantly worse off and the EZ welfare is also substantially improved. On the one hand, at the new steady state the US loses all of its seignorage, as the world currency use and the resulting exorbitant privilege shift to the EZ. On the other hand, the transition to this new steady state itself, depicted by the long gray line in Panel (a) of Figure 4, is particularly painful because during the transition the US runs significant trade surpluses to reduce its NFA position as external demand for dollars dries up.

The welfare difference between the US and EZ when the Euro becomes dominant is 2.9% of permanent consumption. Comparing to the first line of the table, where the US-EZ welfare differential is 1.4%, we conclude that the loss of currency dominance is worth 1.5% of permanent consumption. However, we find that the transition dynamics generate two-thirds of the effect, hence a steady-state-only model would understate the loss by a factor of three.

Acute Trade War

Panel (b) of Figure 4 depicts the implications of a permanent 30% tariff between the US and EZ/RW, a scenario we call an "acute" trade war. In this case, the effects of the trade barriers are strong enough to eliminate both the symmetric and the dollar-dominant steady states, thereby guaranteeing a transition to the now unique euro-dominant steady state.

This is a strong implication, but a *permanent* 30% tariff on all imports is (we hope) implausible. Hence, we also consider the effects of a temporary trade war of the same

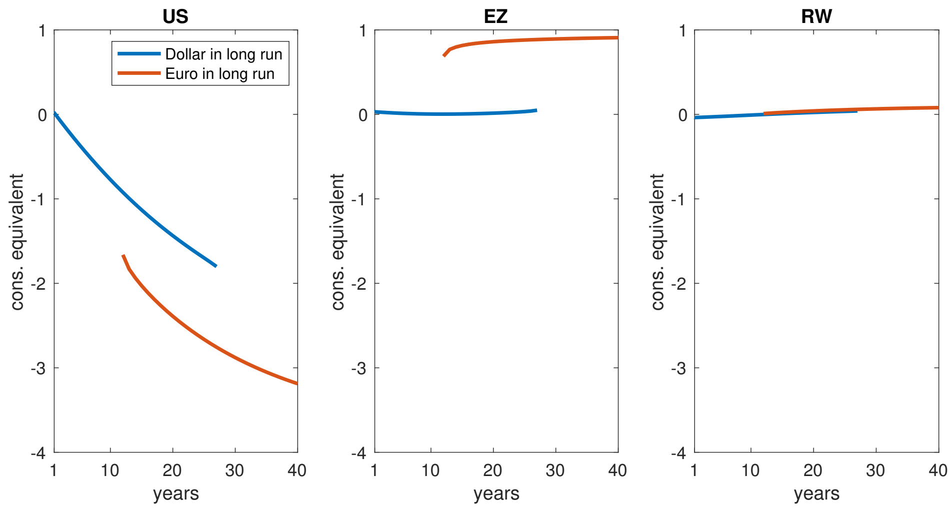


Figure 5: Welfare costs of an acute trade war as a function of duration.

magnitude. Figure 5 depicts the welfare cost for different possible durations, ranging from one to 40 years. The figure shows that for trade wars lasting under 10 years, the economy cannot transition to the euro steady state absent other shocks. Within this range, longer trade wars are worse, but not discretely so, and showcases the general stability of the model – even 10 years of a very acute trade war is not enough to shift the currency regime.

For trade wars lasting more than 10 years, however, transition to the euro-dominant steady state becomes possible as in this case we enter the indeterminacy region. If a transition occurs, the US is discretely worse off, and suffers an additional loss of 1% of permanent consumption. If the trade war lasts longer than 28 years, the *unique* outcome is a transition to the euro-dominant steady state, despite the fact that tariffs return to zero in the long-run. Thus, certain temporary shocks, could have *permanent* effects.

4 Conclusions

This paper presents a new theory describing the emergence of dominant international assets. Our model is quantitatively realistic and tractable enough to use for standard macroeconomic analysis. Throughout, we have abstracted from risk: both the potential for short run shocks that perturb the economy around a given steady state and possible longer-run stochastic transitions between currency regimes. Both of these extensions are straightforward: Business-cycle analysis can be conducted using policy functions approximated locally around a given steady state, or via global solution techniques, such as the ones suggested by Richter et al. (2013). Such extensions could help the model address the observation of

Gourinchas et al. (2017) that an "exorbitant duty" coincides with the privilege of being the dominant currency. We leave exploration of this issue to future work.

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Appendix

A Proofs

Proof of Lemma 1. First, we prove uniqueness. We aim to show that, under the condition in the Lemma, the quasi-equilibrium correspondence $X(B^{\$}_{rw}, B^{€}_{rw})$ is a scalar valued function for all possible asset holdings. Note that, due to the fact that the total supply of each asset is $\bar{B}$, then market clearing ($\bar{B} = \mu_{rw}B^c_{rw} + \mu_{us}B^c_{us} + \mu_{ez}B^c_{ez}$) puts an upper bound on the feasible RW asset holdings:

$$B^{\$}_{rw} < \frac{\bar{B}}{\mu_{rw}}$$
$$B^{€}_{rw} < \frac{\bar{B}}{\mu_{rw}}$$

If either $B^{\$}_{rw} = 0$ or $B^{€}_{rw} = 0$, it is trivially true that the quasi-equilibrium is unique. For example, if $B^{€}_{rw} = 0$ then

$$V^{\$}(X_{rw}) = \frac{B^{\$}_{rw}}{B^{\$}_{rw} + X_{rw}} \left(\pi - r - \kappa(1 - X_{rw}\mu_{rw} - \mu_{us})\right) > 0$$

since $\kappa < \pi - r$. Thus, the only quasi-equilibrium in funding is $X_{rw} = 1$. Similarly, if $B^{\$}_{rw} = 0$, the only quasi-equilibrium is $X_{rw} = 0$.

Next, we show that for any pair of bond holdings $\{B^{\$}_{rw}, B^{€}_{rw}\}$, such that $B^{\$} \in (0, \frac{\bar{B}}{\mu_{rw}})$ and $B^{€} \in (0, \frac{\bar{B}}{\mu_{rw}})$, the net payoff of using dollars crosses zero exactly once, thus the quasi-equilibrium is unique. Using $\mu_{us} = \mu_{eu} = \frac{1-\mu_{rw}}{2}$, and evaluating equation (5):

$$V^{\$}(1) = -\frac{\pi - r}{B^{\$}_{rw} + 1} + \kappa\frac{\mu_{rw}(B^{\$}_{rw} + \frac{1}{2}) + \frac{1}{2}}{B^{\$}_{rw} + 1}$$

This is strictly negative (and thus $X_{rw} = 1$ is not a quasi-equilibrium) if and only if

$$\kappa < \frac{\pi - r}{\mu_{rw}(B^{\$}_{rw} + \frac{1}{2}) + \frac{1}{2}}.$$

A similar argument implies that $V^{\$}(0) > 0$ if and only if

$$\kappa < \frac{\pi - r}{\mu_{rw}(B^{€}_{rw} + \frac{1}{2}) + \frac{1}{2}}.$$

But since $B^{\$}_{rw} < \frac{\bar{B}}{\mu_{rw}}$,

$$\frac{\pi - r}{\mu_{rw}(B^{\$}_{rw} + \frac{1}{2}) + \frac{1}{2}} > \frac{\pi - r}{\bar{B} + \frac{\mu_{rw}}{2} + \frac{1}{2}} > \kappa$$

Thus, $X_{rw} = 1$ is not a quasi-equilibrium for any feasible allocation of asset holdings when $B^{\$} > 0$ and $B^{€} > 0$. Similar argument, shows that $X_{rw} = 0$ is not a quasi-equilibrium either in this case. Hence, all existing quasi-equilibria must be interior and thus solve $V^{\$}(X_{rw}) = 0$.

Evaluating equation (5) gives:

$$V^{\$}(X_{rw}) = \frac{B^{\$}_{rw}}{B^{\$}_{rw} + X_{rw}} [\pi - r - \kappa(1 - X_{rw}\mu_{rw} - \mu_{us})] - \frac{B^{€}_{rw}}{B^{€}_{rw} + (1 - X_{rw})} [\pi - r - \kappa(X_{rw}\mu_{rw} + \mu_{ez})] .$$

Setting the above expression equal to zero and multiplying by $\frac{1}{\kappa}(B^{\$}_{rw} + X_{rw})(B^{€}_{rw} + (1 - X_{rw}))$ results in a quadratic equation in X_{rw}. Simplifying further, and dividing through by κ, allows us to express the resulting quadratic polynomial as $P(X_{rw})$:

$$\begin{aligned} P(X_{rw}) = {} & (B^{€}_{rw} - B^{\$}_{rw})\mu_{rw} X^2_{rw} \\ & + \left(B^{\$}_{rw} \left(\frac{1}{2} + \frac{3}{2}\mu_{rw} - 2\frac{\pi - r}{\kappa} \right) + B^{€}_{rw} \left(\frac{1}{2} - \frac{\mu_{rw}}{2} + 2B^{\$}_{rw}\mu_{rw} - \frac{\pi - r}{\kappa} \right) \right) X_{rw} \\ & + B^{\$}_{rw} \left(\frac{\pi - r}{\kappa} - \left(\frac{1}{2} + \mu_{rw}(B^{€}_{rw} + 1) \right) \right) \end{aligned}$$

Since the quadratic polynomial $P(X_{rw})$ has different signs at $P(0)$ and $P(1)$, it can only have a single crossing in the range $X \in (0, 1)$ and thus there is a unique quasi-equilibrium.

Let $X^* \in (0, 1)$ be the unique quasi-equilibrium value that satisfied $P(X^*) = 0$ for a given level of bond holdings $\{B^{\$}_{rw}, B^{€}_{rw}\}$. To see that $X^* > \frac{B^{\$}_{rw}}{B^{\$}_{rw} + B^{€}_{rw}}$ whenever $B^{\$}_{rw} > B^{€}_{rw}$, notice that if $B^{\$}_{rw} > B^{€}_{rw}$

$$P(X_{rw} = \frac{B^{\$}_{rw}}{B^{\$}_{rw} + B^{€}_{rw}}) = \frac{B^{\$}_{rw} B^{€}_{rw} \mu_{rw} \left(B^{\$}_{rw}(1 + B^{\$}_{rw}) - B^{€}_{rw}(1 + B^{€}_{rw}) \right)}{(B^{€}_{rw} + B^{\$}_{rw})^2} > 0.$$

Hence since $P(1) < 0$, the zero of the quadratic polynomial $P(X)$ must be at a value $X^* > \frac{B^{\$}_{rw}}{B^{\$}_{rw} + B^{€}_{rw}}$.

To prove that $\kappa < \frac{\pi - r}{\bar{B}^{\$} + \frac{\mu_{rw}}{2} + \frac{1}{2}}$ is a sufficient condition for uniqueness, let $\kappa > \frac{\pi - r}{\bar{B} + \frac{\mu_{rw}}{2} + \frac{1}{2}}$ and note that if $B^{\$}_{rw} = \frac{\bar{B}}{\mu_{rw}}$ and $B^{€}_{rw} = \frac{\bar{B}}{\mu_{rw}}$:

$$V^{\$}(1) = -\frac{\pi - r}{B^{\$}_{rw} + 1} + \kappa \frac{\mu_{rw}(B^{\$}_{rw} + \frac{1}{2}) + \frac{1}{2}}{B^{\$}_{rw} + 1} > 0$$

and

$$V^{\$}(0) = \frac{\pi - r}{B^{\$}_{rw} + 1} - \kappa \frac{\mu_{rw}(B^{\$}_{rw} + \frac{1}{2}) + \frac{1}{2}}{B^{\$}_{rw} + 1} < 0$$

Thus, in this case both 1 and 0 are quasi-equilibria, and hence $X(\frac{\bar{B}}{\mu_{rw}}, \frac{\bar{B}}{\mu_{rw}})$ is set valued, Hence, $X(B^{\$}_{rw}, B^{€}_{rw})$ is not a scalar-valued function across the whole state space, and we conclude the quasi-equilibrium is not always unique.

■

Proof of Lemma 2. Throughout, let $j \in [0, \mu_{rw}]$ be an arbitrary index of one of the small open economies. Accounting explicitly for the inequality constraint in bonds, the steady-state Euler equations (3) for dollars can be written

$$\frac{1}{\beta} = \frac{1}{Q^{\$} - \Delta^{\$}_{j}} + \lambda^{\$}_{j} = \frac{1}{Q^{\$} - \Delta^{\$}_{us}} + \lambda^{\$}_{us} = \frac{1}{Q^{\$} - \Delta^{\$}_{eu}} + \lambda^{\$}_{eu}, \tag{22}$$

where the weakly positive λ's are appropriately-scaled Lagrange multipliers and the complementarity slackness conditions

$$\lambda_j B^{\$}_{j} = \lambda_{us} B^{\$}_{us} = \lambda_{eu} B^{\$}_{eu} = 0 \tag{23}$$

must hold for all countries.

We being by proving the additional Lemma:

Lemma 3. *$B^{\$}_{j} = 0$ if and only if $X_j = 0$. Similarly, $B^{€}_{j} = 0$ if and only if $X_j = 1$.*

We prove the statement for dollar bond holdings, the statement for Euro holdings follows by a parallel argument.

If: If $X_j = X_{ez} = 0$ the premia $\Delta^{\$}_{j} = \Delta^{\$}_{eu} = 0$ by equation (7), while $\Delta^{\$}_{us} > 0$. Hence equation (22) reduces to

$$\frac{1}{Q^{\$}} + \lambda^{\$}_{j} = \frac{1}{Q^{\$}} + \lambda^{\$}_{ez} = \frac{1}{Q^{\$} - \Delta^{\$}_{us}} + \lambda^{\$}_{us}. \tag{24}$$

But this equation shows that $\lambda_j > \lambda_{us}$ and $\lambda_{ez} > \lambda_{us}$. Since $\lambda_{us} \geq 0$, we know that $\lambda_j > 0$ and $\lambda_{ez} > 0$ and, from complementary slackness, that $B^{\$}_{j} = B^{\$}_{eu} = 0$.

Only If: To find a contradiction, suppose that $X_j > 0$ but $B^{\$}_{j} = 0$. Since $B^{\$}_{j} = 0$, there exists some other country j' (it could be the US or it could be another small country) which holds positive bonds and pays premium $\Delta^{\$}_{j'} < r$. In this case, equation (22) implies

$$\frac{1}{Q^{\$} - \Delta^{\$}_{j'}} = \frac{1}{Q^{\$} - r} + \lambda^{\$}_{j}. \tag{25}$$

which cannot be true since $\lambda^{\$}_{j} \geq 0$. Hence, $B^{\$}_{j} > 0$. And the helper Lemma 3 is proved.

An implication of the proof above is that $B^{\$}_{ez} = B^{€}_{us} = 0$. We can now use the equality of the remaining premia along with market clearing conditions to compute the expressions in the text. For example, $\Delta^{\$}_{us} = \Delta^{\$}_{j}$ implies that

$$\frac{X_j}{B^{\$}_{j} + X_j} = \frac{X_{us}}{B^{\$}_{us} + X_{us}},$$

which simplifies to

$$B_{us}^{\$} = B_j^{\$}\frac{X_{us}}{X_j}.$$

Using this expression and the fact that $B_{ez} = 0$, $B_{us}^{\$}$ and $B_{ez}^{\$}$ can be eliminated in the market clearing condition for dollar bonds.

$$\bar{B} = \int_{\mu_{rw}} \frac{B_{us}^{\$}}{X_{us}} X_j dj + \mu_{us} B_{us}^{\$} \tag{26}$$

$$\bar{B} = \frac{B_j^{\$}}{X_j} \int_{\mu_{rw}} X_j dj + \mu_{us} \frac{B_j^{\$}}{X_j} X_{us}. \tag{27}$$

Solving expression (27) for $B_j^{\$}$ gives equation (9) in the Lemma 2. The same steps for Euro bonds imply equation (10).

■

Proof of Proposition 1. To show that the dollar-dominant steady state exists, conjecture that the RW traders all use dollars and thus $X_{rw} = 1$. Using Lemma 2, the optimal bond holdings of the RW households are then

$$B_{rw}^{\$} = \frac{\bar{B}}{\bar{B} + \mu_{rw} + \mu_{us}}$$
$$B_{rw}^{€} = 0.$$

Plugging those expression into the relative payoff of seeking dollar vs euro funding for a RW trader ($V^{\$}$), we have:

$$V^{\$} = \frac{\bar{B}}{\bar{B} + \mu_{rw} + \mu_{us}} \left(\pi - r - \kappa(1 - \mu_{rw} - \mu_{us})\right) > 0.$$

Thus, using dollars is strictly preferred by any given RW trader, and the dollar dominant steady state where $X_{rw} = 1$ is indeed sustained. Conjecturing $X_{rw} = 0$, instead, and following a similar argument shows that the euro-dominant steady state $X = 0$ also exists.

Lastly, we look for interior steady-state equilibria where $X_{rw} \in (0, 1)$. In that, case the optimal bond holdings for the RW households are given by:

$$B_{rw}^{\$} = \bar{B}\frac{X_{rw}}{\mu_{rw}X_{rw} + \mu_{us}}$$
$$B_{rw}^{€} = \bar{B}\frac{1 - X_{rw}}{\mu_{rw}(1 - X_{rw}) + \mu_{ez}}$$

Substituting in those expressions for bond holdings in the value of seeking dollar collateral relative to euro collateral for a RW trader, we have

$$V^{\$}(X_{rw}) = \frac{\bar{B}}{\bar{B} + \mu_{rw}X_{rw} + \mu_{us}}\left[\pi - r - \kappa(1 - X_{rw}\mu_{rw} - \mu_{us})\right] - \frac{\bar{B}}{\bar{B} + \mu_{rw}(1 - X_{rw}) + \mu_{ez}}\left[\pi - r - \kappa(X_{rw}\mu_{rw} + \mu_{us})\right]. \tag{28}$$

Any interior equilibrium must satisfy $V(X_{rw}) = 0$ – these are the points at which the traders are indifferent between seeking dollar and euro financing. To find the zeros of $V^{\$}(X_{rw})$, we set (28) equal to 0, and multiply through with $(\bar{B} + \mu_{rw}X_{rw} + \mu_{us})(\bar{B} + \mu_{rw}(1 - X_{rw}) + \mu_{ez})$. Then further dividing by $\bar{B}$, gives us the condition

$$(\bar{B}+\mu_{rw}(1-X_{rw})+\mu_{ez})\left[\pi - r - \kappa(1 - X_{rw}\mu_{rw} - \mu_{us})\right]-(\bar{B}+\mu_{rw}X_{rw}+\mu_{us})\left[\pi - r - \kappa(X_{rw}\mu_{rw} + \mu_{us})\right] = 0$$

Using the fact that $\mu_{us} = \mu_{ez}$ and $\mu_{us} + \mu_{eu} + \mu_{rw} = 1$, this equation simplifies to

$$\mu_{rw}(\kappa(\bar{B} + 1) - \pi - r)(2X_{rw} - 1) = 0.$$

This linear equation has the unique solution $X_{rw} = \frac{1}{2}$ when $\kappa \neq \frac{\pi - r}{\bar{B}+1}$, and admits any $X_{rw} \in [0, 1]$ as a solution in the knife edge case $\kappa = \frac{\pi - r}{\bar{B}+1}$. Thus, for any $\kappa \geq 0$ that is different from $\frac{\pi - r}{\bar{B}+1}$ there are three steady states, $X_{rw} \in \{0, \frac{1}{2}, 1\}$, and when $\kappa = \frac{\pi - r}{\bar{B}+1}$ there is a continuum of steady states $X_{rw} \in [0, 1]$. The associated steady-state bond holdings are then immediately implied by Lemma 2. ■

Proof of Proposition 2. To prove local stability of a given steady state, we need to show that the best-response functions define a contraction in the neighborhood of that steady state. Define the vector of best response functions of trading firms and households in country j, given the actions of all other firms, X_{rw}, and households in the rest of the world $B^{\$}_{rw}$ and $B^{€}_{rw}$:

$$\varphi_X(X, B^{\$}, B^{€}) = \frac{B^{\$}(\pi - \kappa(B^{€} + 1) + \kappa(\mu_{rw}X + \mu_{us})(2B^{€} + 1))}{(B^{\$} + B^{€})\pi + \kappa((\mu_{rw}X + \mu_{us})(B^{\$} - B^{€}) - B^{\$})}$$

$$\varphi_{B^{\$}}(X, B^{\$}, B^{€}) = \bar{B}\frac{X}{\mu_{rw}X + \mu_{us}}$$

$$\varphi_{B^{€}}(X, B^{\$}, B^{€}) = \bar{B}\frac{1 - X}{\mu_{rw}(1 - X) + \mu_{ez}}$$

Stacking these in the vector $\Phi \equiv [\varphi_X, \varphi_{B^{\$}}, \varphi_{B^{€}}]$, we want to show that Φ is a local contraction map, which is the case whenever the eigenvalues of the Jacobian $\nabla\Phi$ lie inside the unit circle.

The Jacobian has the form

$$\nabla\Phi = \begin{bmatrix} \frac{\partial\varphi_X}{\partial X} & \frac{\partial\varphi_X}{\partial B^{\$}} & \frac{\partial\varphi_X}{\partial B^{€}} \\ \frac{\partial\varphi_{B^{\$}}}{\partial X} & 0 & 0 \\ \frac{\partial\varphi_{B^{€}}}{\partial X} & 0 & 0 \end{bmatrix}$$

hence its eigenvalues are given by the roots of the characteristic polynomial

$$\lambda\left(\lambda^2 - \lambda\frac{\partial\varphi_X}{\partial X} - \frac{\partial\varphi_X}{\partial B^{\$}}\frac{\partial\varphi_{B^{\$}}}{\partial X} - \frac{\partial\varphi_X}{\partial B^{€}}\frac{\partial\varphi_{B^{€}}}{\partial X}\right) = 0.$$

Clearly, one of the solutions is $\lambda = 0$, so we just need to ensure that the roots of the quadratic expression in the parenthesis are inside the unit circle We proceed to check this condition for each steady state.

Case I: Symmetric Steady State

At the symmetric steady state we have that $\frac{\partial\varphi_X}{\partial B^{\$}} = -\frac{\partial\varphi_X}{\partial B^{€}}$ and $\frac{\partial\varphi_{B^{\$}}}{\partial X} = -\frac{\partial\varphi_{B^{€}}}{\partial X}$. Hence, the relevant condition for the eigenvalues reduces to

$$\lambda^2 - \lambda\frac{\partial\varphi_X}{\partial X} - 2\frac{\partial\varphi_X}{\partial B^{\$}}\frac{\partial\varphi_{B^{\$}}}{\partial X} = 0$$

with roots

$$\lambda^* = \frac{1}{2}\left(\frac{\partial\varphi_X}{\partial X} \pm \sqrt{\left(\frac{\partial\varphi_X}{\partial X}\right)^2 + 8\frac{\partial\varphi_X}{\partial B^{\$}}\frac{\partial\varphi_{B^{\$}}}{\partial X}}\right).$$

At the symmetric steady state.

$$\frac{\partial\varphi_X}{\partial X} = \frac{(1+2\bar{B})\kappa\mu_{rw}}{2\pi - \kappa} > 0$$

since $\kappa < \pi$. Hence, the bigger root (in absolute value) is

$$\lambda^* = \frac{1}{2}\left(\frac{\partial\varphi_X}{\partial X} + \sqrt{\left(\frac{\partial\varphi_X}{\partial X}\right)^2 + 8\frac{\partial\varphi_X}{\partial B^{\$}}\frac{\partial\varphi_{B^{\$}}}{\partial X}}\right).$$

Lastly, since we also have that

$$\frac{\partial\frac{\partial\varphi_{B^{\$}}}{\partial X}}{\partial\kappa} = \frac{\partial\frac{\partial\varphi_X}{\partial B^{\$}}}{\partial\kappa} = 0,$$

the root is growing in κ. The threshold $\bar{\kappa}$ that ensures the root is within the unit circle solves $\lambda^* = 1$, which after some re-arranging results in:

$$1 - \frac{\partial\varphi_X}{\partial X} - 2\frac{\partial\varphi_X}{\partial B^{\$}}\frac{\partial\varphi_{B^{\$}}}{\partial X} = 0.$$

Solving for the threshold κ, we obtain

$$\bar{\kappa} = \frac{\pi - r}{\bar{B} + 1}$$

Hence, in the neighborhood of the symmetric steady state, the roots of the characteristic polynomial are inside the unit circle so long as $\kappa < \bar{\kappa}$.

Case II: Dollar-dominant Steady State

At the dollar dominant steady state $X_{rw} = 1$ and

$$\frac{\partial \varphi_X}{\partial X} = \frac{\partial \varphi_X}{\partial B^{\$}} = 0.$$

Hence the roots λ are given by

$$\lambda^2 = \frac{\partial \varphi_{B^{€}}}{\partial X} \frac{\partial \varphi_X}{\partial B^{€}}$$

where

$$\frac{\partial \varphi_{B^{€}}}{\partial X} \frac{\partial \varphi_X}{\partial B^{€}} = \frac{(1 + \mu_{rw})(\pi - r) - \kappa(\frac{1}{2} + \mu_{rw}(1 + 2\bar{B} + \frac{\mu_{rw}}{2}))}{(1 - \mu_{rw})(\pi - r - \frac{\kappa}{2}(1 - \mu_{rw}))}.$$

If $\kappa < \frac{(\pi-r)(1+\mu_{rw})}{\frac{1}{2}+\mu_{rw}(1+2\bar{B}+\frac{\mu_{rw}}{2})}$ then the above expression is positive. In that case, if it is also true that

$$\kappa > \frac{\pi - r}{\bar{B} + 1} = \bar{\kappa}$$

(i.e. $\kappa \in (\bar{\kappa}, \frac{(\pi-r)(1+\mu_{rw})}{\frac{1}{2}+\mu_{rw}(1+2\bar{B}+\frac{\mu_{rw}}{2})})$ then $\lambda < 1$.

On the other hand, if $\kappa > \frac{(\pi-r)(1+\mu_{rw})}{\frac{1}{2}+\mu_{rw}(1+2\bar{B}+\frac{\mu_{rw}}{2})}$ the best response function φ_X hits its upper bound of 1. In particular, in that case $\frac{\partial \varphi_X}{\partial B^{€}} > 0$ in the neighborhood of the dollar-dominant steady state. This implies that starting at the dollar-dominant steady state, a small increase in $B^{€}$ will increase φ_X even further, going over 1. However, $X_{rw} = 1$ is the upper bound on X, and enforcing this, means that for $\kappa > \frac{(\pi-r)(1+\mu_{rw})}{\frac{1}{2}+\mu_{rw}(1+2\bar{B}+\frac{\mu_{rw}}{2})}$ effectively $\frac{\partial \varphi_X}{\partial B^{€}} = 0$ and thus $\lambda^2 = 0$. Thus, all eigenvalues of $\nabla \Phi$ are zero, and the system is very stable locally.

Thus, the dollar-dominant steady state is stable for any $\kappa > \bar{\kappa} = \frac{\pi - r}{\bar{B}+1}$.

Case III: Euro-dominant Steady State

Can be proven with identical steps to Case II.

■

B US bond holdings and trade invoicing in the data

A key implication of the model is that holdings of US assets is positively correlated with the intensity of dollar use in the international trade. To test this in the data, we regress

Table 6: Dollar invoice share and portfolio share of dollar bond holdings

	(1)	(2)	(3)	(4)	(5)
$\frac{B_i^{us}}{\sum_{c\in Countries} B_i^c}$	0.70***		0.62***		
	(0.16)		(0.2)		
$\frac{EX_i^{us}+IM_i^{us}}{\sum_c (EX_i^c+IM_i^c)}$		1.00***	0.26		0.44
		(0.34)	(0.39)		(0.32)
$\frac{B_i^{\$}}{\sum_{c\in currencies} B_i^c}$				0.72***	0.65***
				(0.10)	(0.13)
R^2	0.34		0.35	0.67	0.70
N	42	42	42	28	28

the share of dollar invoicing of a country's trade on the share of US bonds in that country's aggregate portfolio.[20] In particular, we estimate the regression

$$X_j = \alpha + \beta_{B_{usd}} \frac{\text{Holdings of US bonds}_j}{\text{Total Foreign Bond Holdings}_j} + \beta_{UStrade} \frac{\text{Trade with US}_j}{\text{Total Trade}_j} + \varepsilon_j$$

where X_j is the share of dollar invoicing in country j's trade (from Gopinath (2015)), while portfolio data is from the IMF's CPIS database. The estimates are presented in Table 6.

As predicted by the model, we find that the portfolio share of US bond holdings is highly positively correlated with the share of trade invoiced in dollars (column (1)). Moreover, US bond holdings do not simply proxy for the share of direct trade with the US – controlling for the share of direct trade with the US does not change the magnitude or strength of the relationship with bond holdings (column (3)). In fact, direct US trade is not significantly associated with invoicing once we control for bond holdings. Lastly, for a subset of the countries, we also have data on the currency composition of their foreign bonds holdings.[21] We find that the relationship between bond holdings and invoicing is even stronger once we use the share of dollar denominated bonds (see columns (4) and (5)).

This regression is related, but distinct, from the ones in Gopinath and Stein (2018) who use the share of dollars in the aggregate *bank liabilities* of country j as the regressor, not the US asset share in the country's gross foreign asset position. In their mechanism, dollar usage is driven by a shortage of dollar assets, which necessitates banks to create dollar deposits for local households to save in. It is a story of bank sector dollarization due to a shortage of US

[20] As documented by BIS (2014), the currency of invoicing is closely related to the currency in which trade is settled and financed, but we do not have direct data on the settlement currency and use invoicing instead.

[21] Maggiori et al. (2018) show that investors have an affinity for dollar denominated foreign assets, even when the issuer has a different local currency.

safe assets, while in our model, instead, dollar usage is motivated by the relative *abundance* of foreign-issues US assets in household portfolios. As a result, our regressions uses the composition of the aggregate foreign asset position of countries (not liabilities). And indeed, we find that when a country owns large holdings of US-issued safe assets, that is indeed associated with a high usage of dollars in the trade activities of that country.

C Additional Quantitative Model Details

C.1 Households

For $j \in \{us, ez\}$, foreign imports consist of the good of the other big country and an aggregate of rest-of-the world goods. Hence, the big country consumption aggregator is

$$C_{jt} = (C_{jt}^{j})^{a_h} \left((C_{jt}^{j'})^{\frac{\mu_{j'}}{\mu_{j'}+\mu_{rw}}} (C_{jt}^{rw})^{(\frac{\mu_{rw}}{\mu_{j'}+\mu_{rw}})} \right)^{1-a_h}, \tag{29}$$

where j' is the complement of j and $C_{jt}^{j'}$ is the consumption in country j of the good of country j', and a_h controls the degree of home bias in consumption. Rest-of-world consumption goods are aggregated according to $C_{jt}^{rw} = (\int (C_{jt}^{i})^{\frac{\eta-1}{\eta}} di)^{\frac{\eta}{\eta-1}}$. The corresponding aggregate consumption price index is

$$P_{jt} = \frac{1}{K}(P_{jt}^{j})^{a_h} \left((P_{jt}^{j'})^{\frac{\mu_{j'}}{\mu_{j'}+\mu_{rw}}} (P_{jt}^{rw})^{\frac{\mu_{rw}}{\mu_{j'}+\mu_{rw}}} \right)^{1-a_h}. \tag{30}$$

where $K \equiv a_h^{a_h}(1-a_h)^{1-a_h} \left(\frac{\mu_{j'}}{\mu_{j'}+\mu_{rw}}\right)^{(1-a_h)\frac{\mu_{j'}}{\mu_{j'}+\mu_{rw}}} \left(\frac{\mu_{rw}}{\mu_{j'}+\mu_{rw}}\right)^{(1-a_h)\frac{\mu_{rw}}{\mu_{j'}+\mu_{rw}}}$.

For small countries $j \in [0, \mu_{rw}]$, the consumption basket includes imports from both big countries and all other rest-of-world small countries:

$$C_{jt} = C_{jt}^{j\ a_h} \left((C_{jt}^{us})^{\frac{\mu_{us}}{\mu_{us}+\mu_{ez}+\mu_{rw}}} (C_{jt}^{ez})^{\frac{\mu_{ez}}{\mu_{us}+\mu_{ez}+\mu_{rw}}} (C_{jt}^{rw})^{\frac{\mu_{rw}}{\mu_{us}+\mu_{ez}+\mu_{rw}}} \right)^{1-a_h}. \tag{31}$$

The associated price index is

$$P_{jt} = \frac{1}{K_{rw}}(P_{jt}^{j})^{a_h} \left((P_{jt}^{us})^{\frac{\mu_{us}}{\mu_{us}+\mu_{ez}+\mu_{rw}}} (P_{jt}^{ez})^{\frac{\mu_{ez}}{\mu_{us}+\mu_{ez}+\mu_{rw}}} (P_{jt}^{rw})^{\frac{\mu_{rw}}{\mu_{us}+\mu_{ez}+\mu_{rw}}} \right)^{1-a_h}. \tag{32}$$

where $K_{rw}$is defined analogously to K above.

C.2 The Import-Export Sector

The following subsection provide additional details on the trading structure of the general equilibrium model.

Trading Round and Profits

Let $c = (j, j')$ be a double index, capturing an arbitrary country pair, and let $\tilde{m}^{im}_{ct}$ be the mass of *funded* importing firms in country j seeking trade with funded exporting firms in country j' at time t. Then the probability of a country j importer matching with a country j' exporter is

$$p^{ie}_{ct} = \frac{\tilde{m}^{ex}_{c',t}}{\left[(\tilde{m}^{ex}_{c',t})^{1/\varepsilon_T} + (\tilde{m}^{im}_{ct})^{1/\varepsilon_T}\right]^{\varepsilon_T}},$$

where $c' \equiv (j', j)$. Using analogous definitions, the probability of a country j exporter matching with a country j' importer is

$$p^{ei}_{ct} = \frac{\tilde{m}^{im}_{c',t}}{\left[(\tilde{m}^{im}_{c',t})^{1/\varepsilon_T} + (\tilde{m}^{ex}_{ct})^{1/\varepsilon_T}\right]^{\varepsilon_T}}.$$

Let $\tilde{X}_{jt}$ be the fraction of funded country j firms who hold dollar collateral. Then the expected profits of country-j importer importing from j' who hold dollars is given by

$$\pi^{\$,im}_{c,t} = p^{ie}_{c,t}\frac{(1-\alpha)}{P^{whol}_{c',t}}\left[P^{j'}_{j,t} - P^{j'}_{j',t} - \kappa P^{whol}_{c',t}(1-\tilde{X}_{jt})\right],$$

while if it hold euros, expected profits are

$$\pi^{€,im}_{c,t} = p^{ie}_{c,t}\frac{(1-\alpha)}{P^{whol}_{c',t}}\left[P^{j'}_{j,t} - P^{j'}_{j',t} - \kappa P^{whol}_{c',t}\tilde{X}_{jt}\right].$$

Similar expressions hold for exporters:

$$\pi^{\$,ex}_{c,t} = p^{ei}_{c,t}\frac{(1-\alpha)}{P^{whol}_{c,t}}\left[P^{j}_{j',t} - P^{j}_{j,t} - \kappa P^{whol}_{c,t}(1-\tilde{X}_{j',t})\right]$$
$$\pi^{€,ex}_{c,t} = p^{ei}_{c,t}\frac{(1-\alpha)}{P^{whol}_{c,t}}\left[P^{j}_{j',t} - P^{j}_{j,t} - \kappa P^{whol}_{c,t}\tilde{X}_{j',t}\right].$$

Firm Formation

Equilibrium with interior p^{im}_{ct} and p^{ex}_{ct} requires that, prior to learning their private currency choice, firms are ex post indifferent between importing and exporting to the various countries. Hence, for example in the US, we must have

$$X_{us}\pi^{\$,im}_{(us,j),t} + (1-X_{us})\pi^{€,im}_{(us,j),t} = X_{us}\pi^{\$,im}_{(us,j'),t} + (1-X_{us})\pi^{€,im}_{(us,j'),t}$$

for all US potential trading partners j and j'. Similarly,

$$X_{us}\pi^{\$,im}_{(us,j),t} + (1-X_{us})\pi^{€,im}_{(us,j),t} = X_{us}\pi^{\$,ex}_{(us,j),t} + (1-X_{us})\pi^{€,ex}_{(us,j),t}.$$

Moments	USD Coord. US	USD Coord. EZ	USD Coord. RW	Symmetric US	Symmetric EZ	Symmetric RW	EUR Coord. US	EUR Coord. EZ	EUR Coord. RW
Dollar Share	0.90	0.10	0.80	0.90	0.10	0.50	0.90	0.10	0.20
$100\times(i^{\$} - i^{€})$	1.03	-	-	0.00	0.00	-	-	-1.03	-
100×Seignorage/GDP	0.85	0.22	-	0.54	0.54	-	0.22	0.85	-
100×Trade bal./GDP	-0.25	-0.28	0.14	-0.12	-0.12	0.07	-0.28	-0.25	0.14
Gross debt/GDP	0.60	0.59	0.40	0.60	0.60	0.40	0.59	0.60	0.40
NFA/GDP	-0.14	0.01	0.03	-0.10	-0.10	0.05	0.01	-0.14	0.03
Home Bias	0.12	0.39	0.06	0.23	0.23	0.00	0.39	0.12	0.06
Consumption	11.492	11.492	12.264	11.466	11.466	12.279	11.492	11.492	12.264

Table 7: Steady-state values for baseline model.

The above equations are sufficient to pin down the equilibrium probabilities for importing and exporting to and from each country pair.

Given this and all of the above choices, prospective firms then decide whether or not to pay the fixed cost $\phi > 0$ in order to become operational this period. Firms enter the import-export sector until the zero-profit condition

$$W_{jt} = \max_{\{p^{im}_{jit}, p^{ex}_{jit}\}} X_{jt}\Pi^{\$}_{jt} + (1 - X_{jt})\Pi^{€}_{jt} - \phi P_{jt} = 0 \quad \text{s.t.} \quad \sum_{i\neq j} p^{im}_{jit} + \sum_{i\neq j} p^{ex}_{jit} = 1.$$

is satisfied.

D Rest-of-World Asset Supply

Our baseline economy abstracts from the presence of any savings vehicle issued by the rest of the world. This is in part because, absent a liquidity premium term, adding such an asset would create an indeterminacy in long-run wealth levels. The same sort of indeterminacy is pervasive in open economy models with incomplete asset markets.

As simple way to include a rest-of-world asset market is to assume there exists an exogenous liquidity demand z_j for the rest of world asset. Though we don't model this role explicitly, we assume it is proportional to the measure of firms in the economy, so that the liquidity wedge for the RW asset is given by

$$\Delta^{RW}_{jt} = \frac{M^f\left(m_{jt}z_j, \nu P^{rw}_{rw,t}B^{RW}_{jt}Q^{\$}_t\right)}{\nu P^{rw}_{rw,t}B^{RW}_{jt}Q^{\$}_t} r.$$

The household euler equation for the RW bond is

$$1 = \beta E_t \left[\left(\frac{C_{jt+1}}{C_{jt}} \right)^{-\sigma} \frac{P_{jt}}{P_{jt+1}} \frac{P^{rw}_{rw,t+1}}{P^{rw}_{rw,t}} \frac{1}{Q^{RW}_t (1 - \Delta^{RW}_{jt} + \tau'(B^{RW}_{j,t}, B^{RW}_{j,t-1})} \right].$$

A desirable feature of this approach to determining holding of the RW bond is that the steady state portfolio allocations are independent of a scale shift in the z_j.

Table 7 reproduces the moments for a calibration targeted to the moments in Panel (a) of Table 1, except that the share of RW assets is increased from zero to 40% of rest-of-world GDP and $z_j = 0.10$. Consumption for the US and EZ remain extremely close to each other, but two other differences stand out. First, the EZ NFA position is now roughly zero, while the US NFA position is reduced to -14%, more consistent with its average value over the past 40 years. Second, as a result of the more moderate NFA position, the US runs a permanent trade deficit, despite its negative NFA position.