Coherent Preferences and Asset Prices in Stock Market

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ABSTRACT

We study individual coherent preferences underlying asset prices and propose a set of explicit models for nonlinear V-shaped price pressure utility using a new mathematical method. Coherent preferences are consistent interactive choices between momentum trading and reversal trading in stock market where market dynamic equilibrium exists. We find that: 1) coherent preferences generate nonlinear V-shaped price pressure and market dynamic equilibrium whereas beliefs contribute to discrepancy between market equilibrium prices and fundamental prices; and 2) subject individuals can display either disposition effect or inverse disposition effect. Our study suggests a better asset price model with trading volume distribution in finance.

KEYWORDS. Nonlinear V-shaped utility, coherent preference, asset price, dynamic equilibrium, volume distribution over price, disposition effect

JEL CLASSIFICATIONS. C60, D53, G40, G10.
“From the time of Adam Smith’s Wealth of Nations in 1776, one recurrent theme of economic analysis has been the remarkable degree of coherence among the vast numbers of individual and seemingly separate decisions about the buying and selling of commodities”—Arrow (1992)

1. Introduction

A central theme in financial economics is asset prices which include two parts: the market equilibrium price and the fundamental value that is sometimes called rational asset price. It is widely recognized that the former is formulated by price pressure from supply-demand quantity and the latter is determined with reference to macroeconomic variables pertaining to the trading asset. While the stock fundamental value may be figured out in terms of its future dividends in accounting (Shiller 1981), market prices are much more complex and hard to predict. Naturally, we ask what kind of individual trading behavior generate the price pressure and the market equilibrium point, how we measure and determine these variables, and why it is important to understand the mechanism of market dynamics in equilibrium.

We study individual coherent preferences adaptive to a price equilibrium point in market dynamic equilibrium (Shi et al. 2021) when they face price fluctuations in stock markets. Coherent preferences specify individual consistent choices by those traders who adapt themselves to buy and sell, keep constant interactive trading between momentum trading and reversal trading across all prices, and generate
market dynamic equilibrium. Recently, Ben-David and Hirshleifer (2012) analyze the probability schedule of selling a stock with returns since purchasing it and propose a V-shaped value function in stock market based on empirical results (see Figure 1). They point out that disposition effect is not necessarily evidence in support of preference-based explanations for individual trading behaviors; instead, belief-based interpretations may come into play. Disposition effect specifies that investors are more likely to sell winners than losers (Shefrin and Statman 1985). They identified higher price pressure over gains than over losses, which accounts for disposition effect (An 2016).

![V-shaped selling propensity in response to profits](image)

Figure 1: Asymmetry V-shaped value function (Ben-David and Hirshleifer 2012)

The empirical result (Ben-David and Hirshleifer 2012) comes into our notice for several reasons. First, it provides a simple model of how dynamic equilibrium works in stock market. Second, some key information in the model loses such as trading volume (effectual supply-demand quantity) and the individual interaction reflected among investors themselves in actually nonlinear V-shaped price pressure utility (see
Figure 2). However, the main reason is that our explicit models of coherent preferences predict the nonlinear V-shaped price pressure utility. We interpret individual trading behaviors as coherent preferences in a nonlinear market dynamic equilibrium. This will help understand asset prices that include both market equilibrium prices and fundamental prices with reference to macroeconomic variables.

Guided by Shi’s probability wave equation (Shi 2006), we measure individual trading preferences by intraday cumulative trading volume distribution over a price range and determine a market equilibrium point in beliefs by the maximum trading volume utility price. We come up with an individual coherent preferences hypothesis and examine it in the framework of the price-volume probability wave differential equation in a new mathematical method. In order to better understand the mechanism of market dynamic equilibrium, we propose a set of explicit models for nonlinear

\(^{(1)}\) Individual interaction is called social interaction (Hong et al. 2004, Hirshleifer 2020) or causal interaction (Stavrogloou et al. 2019) in some financial papers.
V-shaped price pressure utility function (see Figure 2).

The contributions of this paper are as follows. First, we define measurable coherent preferences or choices over a price range in equilibrium, distinguish between market equilibrium prices and rational asset prices, and explain the mechanism of market dynamics in behavioral finance. Second, we offer a set of explicit models for actually nonlinear V-shaped price pressure utility and explain the price pressure utility by coherent preferences in a new paradigm. We perfect the model Ben-David and Hirshleifer (2012) have proposed. There is a V-shaped value function unless individual traders are independent and homogeneous. Finally, we find that subject individuals as a whole can display either disposition effect or inverse disposition effect.

The following describes the structure of the paper. Section 2 proposes the coherent preference hypothesis; section 3 presents two sets of explicit models and empiric tests; Section 4 is discussions; and finally we conclude.

1.1 Related Literature

Finance literature has traditionally focused on asset price and return patterns and to a much lesser extent on trading volume. Trading volume plays no role in neoclassical finance models, such as CAPM (Sharpe 1964), ICAPM (Merton 1973), option pricing model (Black and Scholes 1973), and arbitrage pricing theory (Ross 1976) etc. Moreover, these models assume that subject individuals behave in a rational and independent manner and that returns follow a Brownian motion that shifts around a fundamental level in random or no preferences (Samuelson 1965). However, a large
number of empiric tests reveal that they are not true in reality.

Over the past two decades, behavioral finance has tried to make sense of financial data using models that make psychologically accurate assumptions about people’s beliefs, preferences, and cognitive limits. For example, the trading volume linked to asset prices has become increasingly popular for individual mental representation. Individual mental representation is an external and measurable variable to represent individual internal and intangible mental behaviors such as preferences and beliefs. Lee and Swaminathan (2000) find that past trading volume predicts both the magnitude and persistence of price momentum. Specifically, price momentum effects reverse over the next five years, and high (low) volume winners (losers) experience faster reversals. Collectively, past volume helps to reconcile intermediate-horizon “underreaction” and long-horizon “overreaction” effects. Odean (1999) evidence that overconfidence increases expected trading volume and decreases the expected utility of overconfident traders. Overconfident investors tend to perceive themselves to be more competent, and thus are more willing to act on their beliefs, leading to higher trading frequency (Graham et al. 2009). In addition, Barber and Odean (2008) argued that attention is a major factor determining the stocks that individual investors buy. Hereafter, Huo et al. (2009) measure cross-sectional and time-series variations of investor attention by trading volume and market state, and find that the role of investor attention explains the profitability of price and earnings momentum strategies. Nevertheless, there are more other behavioral explanations on trading volume such as psychological biases (Barber et al. 2009), entertainment (Dorn and Sengmueller 2009),
speculative motive (Mei et al. 2009), sentiment (Han 2007), disagreement (Hong and Stein 2007), sensation seeking (Grinblatt and Keloharju 2001), and gender (Barber and Odean 2001) etc. They are belief-based accounts for trading volume linked to market prices at which every trading price is an equilibrium point. Today, it is recognized that beliefs with cognitive limits such as emotion, overreaction, overconfidence and so on contribute to discrepancy between a stock market equilibrium price and its fundamental price.

We classify the behavioral finance approaches to understanding asset prices with a trading volume dimension into three blocks: cognitive limits, limits of arbitrage with liquidity constraints, and the gain-loss utility inspired by prospect theory. Cognitive limits emphasize that more realistic assumptions about individual trading behaviors such as sentiment, overreaction, overconfidence, and attention etc are required to incorporate into a financial model. The behavioral factors in beliefs help explain a spread between a stock market equilibrium price and its fundamental price. Limits of arbitrage indicate that rational investors who are short constraint often fail to bring misprice back to its fundamental immediately in trading (Shleifer and Vishny 1997). Liquidity trading utility generates momentum trading and the price momentum force that departs price far away from an equilibrium point in a small trading volume probability (Shi et al. 2021). A gain-loss utility is a reference-dependent explanation for asset prices although Kahneman and Tversky (1979) tell little how a reference point is determined. The utility is further modified by introducing decision weights which are computed indirectly with the help of a weighting function in cumulative
prospect theory (Tversky and Kahneman 1992). However, there is no distinction about trading volume at different prices in these studies. Thus, a lot of information on trading volume loses.

In experimental settings, subjects are typically given a representation for any risk they are asked to consider. For example, the most classical laboratory empirical finding deals with a bet offering 85% chances of winning 100 yuan versus 15% chances of zero-win, compared with a scenario in which 80 yuan is the secure 100% outcome. On the other hand, what probability does a subject individual prefer to trade a specific stock at two slightly different prices, saying 5.66 and 5.70 yuan, in the stock market, respectively? Subject individuals are sensitive to the price fluctuation relative to a reference point. They prefer to buy low and sell high rather than behave no preference or random. Shi et al. (2021) select intraday cumulative trading volume distribution over a price range to represent individual trader’s preferences, determine a price equilibrium point by the maximum trading volume price, and study nonlinear market dynamic equilibrium. However, they have not yet identified what kind of individual trading behaviors creates intraday nonlinear market dynamic equilibrium in behavioral finance.

We will propose a coherent preference hypothesis for individual trading choices in market dynamic equilibrium and find its application.

2. COHERENT PREFERENCES HYPOTHESIS
Individual traders are intelligent and adaptive (Staddon 2016). They compete for limited resources one another in intraday trading. They prefer to buy more and sell lesser at a low price, buy lesser and sell more at a high price, and thus generate market equilibrium. It can be traced back to Adam Smith’s Wealth of Nations 200 years ago (Smith 2011). Recently, Shi et al (2021) model nonlinear market dynamic equilibrium in an intraday interval in stock market, where individual traders display momentum trading and reversal trading in social interaction.

2.1 Assumptions

We separate individual decision makings into two phases. First, they evaluate a stock value in beliefs with expectation on return and decide whether it is worthy to be traded in risk, based on their own information and resources. Here, everyone has a different reference point in judgment and decision making.

Once they get ready to buy or sell with expectation on return in a time series, in the second phase they adapt themselves to sell more or less in terms of a gain or loss relative to a price equilibrium point in narrow framing (Tversky and Kahnema 1981). Narrow framing occurs when an agent who is offered a new gamble evaluates that gamble to some extent in isolation, separately from other risks (Barberis et al. 2006). As a result, individual traders generate cumulative trading volume distribution over a price range (effectual supply and demand quantity) and a nonlinear market dynamic equilibrium. It leads us to have assumption one.

Assumption One: Intraday cumulative trading volume distribution over a price range represents individual trading preferences and determines a price reference point
to which individual traders adapt sell or buy in stock market.

Price behaves momentum and reversal instead of random walks or no preferences (Lo and Mackinlay 1988). Intraday cumulative trading volume gradually distributes in a certain pattern over a price range, a consequence of coherence in interaction (Shi 2006). If we choose the maximum volume price or the maximum marginal utility price as a price equilibrium point at which price pressure can be defined zero (Ben-David and Hirshleifer 2012), then we have the other two assumptions.

**Assumption Two**: Subject individuals tend to engage in a price reversal process from price pressure in narrow framing which brings a stock price back to a price equilibrium point in beliefs.

![Figure 3](image_url)

Figure 3: The maximum volume price reference point P3 is ¥2.432 yuan.

Huaxia 50ETF (510050) on January 25, 2019

**Assumption Three**: Subject individuals prefer to trade most at a price equilibrium point concerning to individual beliefs at which trading volume utility is the maximum but price pressure is zero, and adapt to it by assigning trading volume weights over a
price range in the allocation of final trading wealth (see Figure 3).

2.2 Coherent preferences hypothesis

Individuals are heterogeneous in beliefs. They make judgment and decision separately according to their own information. As long as they behave a significant degree of coherent preferences, a large number of apparently separate and independent participants have the tendency to bring a stock price back to a market equilibrium point at which individuals maximize trading volume utility over the price. We can define that the price pressure is zero at the price equilibrium point. Now, we are ready to have a hypothesis as follow:

A Coherent Preferences Hypothesis: Subject individuals behave coherent preferences and generate a nonlinear market dynamic equilibrium as long as intraday cumulative trading volume distribution exhibits a significant degree of coherence in the framework of a price-volume probability wave differential equation. That is, it follows a set of the absolute of zero-order Bessel eigenfunctions (see Figure 4).

Figure 4: The absolute of zero-order Bessel eigenfunctions

In Figure 4, price is the horizontal coordinate and cumulative trading volume probability in a time interval is the vertical coordinate, respectively. The origin is a reference point.
3. MODELS AND EMPIRICAL RESULTS

We introduce two sets of explicit models for subject individuals in trading from a price-volume probability wave differential equation and do empirical test using tick by tick high frequency trading data in Chinese stock market in 2019.

3.1 Two sets of explicit models for subject individuals

Shi (2006) assumed an individual trading utility function in stock market as follows

\[ -U + p \frac{v^2}{V} + W(p - p_0) = 0, \]  

(1)

with

\[ U = \frac{M}{t^2} = p \frac{v}{t^2} = pv_{tt} \]  

(2)

and

\[ W(p - p_0) = A(p - p_0), \]  

(3)

where \( U \) is the liquidity utility expressed in terms of trading amount, the second item \( \frac{v}{V} U \) is interactive or trading volume weight utility, and the third item \( W(p-p_0) \) is price pressure or gain-loss utility in equation (1), respectively; \( p \) is a trading price and \( p_0 \) is a price equilibrium point; \( v \) is intraday cumulative trading volume at the price \( p \) and \( V \) is total intraday cumulative trading volume across all trading prices; \( v_t = \frac{v}{t} \) is momentum trading; \( v_{tt} = \frac{v}{t^2} \) is momentum trading force rather than the second order partial differential \( \frac{\partial^2 v}{\partial t^2} \); \(-A\) is reversal trading force and the minus sign means that the direction of reversal trading force is always to a price equilibrium point; and \( t \) is a time interval which we can choose one for convenience.

From the utility function, equation (1), a price-volume probability wave differential...
equation is derived. It is written by

$$\frac{B^2}{V} \left( p \frac{d^2 \psi}{dp^2} + \frac{d\psi}{dp} \right) + [U - W(p - p_0)]\psi = 0. \quad (4)$$

Then, we have two sets of explicit models for intraday cumulative trading volume distribution over a price range from equation (4). They represent individual trading behaviors in stock market. One is as follows

$$\psi_m(p) = C_m J_\alpha(\omega_m(p - p_0)). \quad (m = 0,1,2\ldots) \quad (5)$$

subject to

$$\omega^2_m = v_{tt,m} - A_m \quad (\omega_m > 0) \quad (m = 0,1,2\ldots) \quad (6)$$

and

$$\omega^2_m = \frac{v}{v_{tt,m}} v_{tt,m} = \pi \cdot v_{tt,m} = \text{constant}, \quad (m = 0,1,2\ldots) \quad (7)$$

where $J_\alpha(\omega_m(p - p_0))$ is a zero-order Bessel eigenfunction, $\omega_m$ is an eigenvalue, $v_{tt,m}$ is momentum trading force, $-A_m$ is reversal trading force, $\omega^2_m = \frac{v}{v_{tt,m}} v_{tt,m}$ is interactive trading force between momentum trading and reversal trading, $\pi$ is a trading volume probability or weight which is equal to cumulative trading volume at a price over total cumulative trading volume across all trading prices, $C_m$ is a normalized constant, and $|\psi_m(p)|$ is the cumulative trading volume distribution function over a price range.

According to coherent preference hypothesis, the absolute of equation (5) is a set of models of coherent preferences in individual interactive trading. It is expressed by

$$|\psi_m(p)| = C_m |J_\alpha(\omega_m(p - p_0))|. \quad (m = 0,1,2\ldots) \quad (8)$$

Equation (8) is plotted in Figure 4. Equations (6) and (7) are necessary and sufficient conditions for individual coherent preferences in interactive trading. We can measure coherent preferences by an eigenvalue $\omega_m$.  

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The other is as follows

\[ \psi_{n,m}(p) = C_{n,m}e^{-\sqrt{A_{n,m}|p-p_0|}} \cdot F\left(-n,1,2,\sqrt{A_{n,m}|p-p_0|}\right), \]

where \( U_{n,m} \) is a constant at any a trading price \( p \), \( A_{n,m} \) is an eigenvalue and the magnitude of reversal trading force, \( C_{n,m} \) is a normalized constant, \( n = 0,1,2,3 \ldots \) is the order in this multiple-order functions, \( m = 0,1,2,3 \ldots \) represents states with eigenvalue \( \sqrt{A_{n,m}} \), \( F\left(-n,1,2,\sqrt{A_{n,m}|p-p_0|}\right) \) is a set of \( n \)-order confluent hypergeometric eigenfunctions (the first Kummer’s eigenfunctions) \( F(\alpha,\gamma,\xi) \). It is defined as

\[ F(\alpha,\gamma,\xi) = 1 + \frac{\alpha}{\gamma} \xi + \frac{\alpha(\alpha + 1)}{2! \gamma(\gamma + 1)} \xi^2 + \frac{\alpha(\alpha + 1)(\alpha + 2)}{3! \gamma(\gamma + 1)(\gamma + 2)} \xi^3 + \cdots \]

\[ = \sum_{k=0}^{\infty} \frac{(\alpha)_k}{(\gamma)_k} \xi^k, \]

where \( (\alpha)_k = \alpha(\alpha + 1) \cdots (\alpha + k - 1) \), \( (\gamma)_k = \gamma(\gamma + 1) \cdots (\gamma + k - 1) \).

\[ F(\alpha,\gamma,\xi) \] has meaning if and only if \( \gamma \neq 0 \) and \( \alpha = -n \) \( (n = 0,1,2,\ldots) \) are satisfied.

The absolute of equation (9) is

\[ |\psi_{n,m}(p)| = C_{n,m}e^{-\sqrt{A_{n,m}|p-p_0|}} \cdot |F\left(-n,1,2,\sqrt{A_{n,m}|p-p_0|}\right)|, \]

subject to

\[ \sqrt{A_{n,m}} = \frac{U_{n,m}}{1+2n} = \text{constant} > 0, \quad (n,m = 0,1,2,\ldots) \]

where \( U_{n,m} \) is a constant at any a trading price \( p \), \( A_{n,m} \) is an eigenvalue and the magnitude of reversal trading force, \( C_{n,m} \) is a normalized constant, \( n = 0,1,2,3 \ldots \) is the order in this multiple-order functions, \( m = 0,1,2,3 \ldots \) represents states with eigenvalue \( \sqrt{A_{n,m}} \), \( F\left(-n,1,2,\sqrt{A_{n,m}|p-p_0|}\right) \) is a set of \( n \)-order confluent hypergeometric eigenfunctions (the first Kummer’s eigenfunctions) \( F(\alpha,\gamma,\xi) \). It is defined as

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\[ = \sum_{k=0}^{\infty} \frac{(\alpha)_k}{(\gamma)_k} \xi^k, \]

where \( (\alpha)_k = \alpha(\alpha + 1) \cdots (\alpha + k - 1) \), \( (\gamma)_k = \gamma(\gamma + 1) \cdots (\gamma + k - 1) \).

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The absolute of equation (9) is

\[ |\psi_{n,m}(p)| = C_{n,m}e^{-\sqrt{A_{n,m}|p-p_0|}} \cdot |F\left(-n,1,2,\sqrt{A_{n,m}|p-p_0|}\right)|, \]

where

\[ (n,m = 0,1,2,\ldots) \]

Equation (14) is a set of multiple-order trading volume distribution eigenfunctions

(see Figure 5). It represents individual independent trading since there is no interaction in equation (10). It is a set of multiple-order models for those who are
homogeneous (see (a) in Figure 5) or heterogeneous (see (d) in Figure 5) in independent trading, respectively.

(a) zero-order  
(b) First order

(c) second order  
(d) tenth order

Figure 5: Multiple-order eigenfunctions

3.2 Empirical tests and results

We test the coherent preferences hypothesis by a set of explicit models of coherent preferences, equation (8), against a large number of intraday cumulative trading volume distributions over a price range using tick by tick high frequency data in Huaxia SSE 50ETF (510050) in Chinese stock market from January 3, 2019 to February 28, 2019.

We collected the ETF’s price list with intraday cumulative trading volume every

© Price is at a horizontal coordinate, cumulative trading volume probability is at a vertical coordinate, and a price reference point \( p_0 \) is origin in Figure 5.
trading day directly from a client’s trading system at Bank of China International (China) Co. Ltd., one of the largest state-owned securities companies in China (see Figure 6). We gathered a total number of 34 intraday cumulative trading volume distributions over a price range. Then, we converted each of the price-volume distribution data into an Excel table. Origin7.0 analysis software is used in fittings and R-square is applied in significant tests. The data is produced every trading day. It is reliable, repeatable, and reusable.

We examine them by a set of explicit models of coherence preferences, equation (8). 32 price-volume distributions or 94% of test results (32 over 34) shows significantly at 95% level. Subject individuals usually behave significant degree of coherent preferences in interactive trading, which results in nonlinear market dynamic equilibrium (Please see a fitting example in Figure 3).

However, there are two anomalies among 34 distributions in our tests. When we look at two abnormal distributions closely, we can find that there are two price
equilibrium points in the distribution dated on February 13, 2919. We model it by a linear superposition of two models. Then, the test shows significance. It indicates that individuals update a price reference point and make the price equilibrium point jump (with 2.9% probability). The other distribution is uniform on February 15, 2019. It exhibits significance when we test it by the first-order eigenfunction, equation (14). It shows that subject individuals are heterogeneous and display no preferences in independent trading. Price change behaves random walks. It happens with 2.9% probability, too (see METADATA for details). Generally speaking, our test results are consistent with previous studies (Shi, 2006). Individual traders behave coherent preferences widely in stock market.

4. DISCUSSIONS

We discuss disposition effect and explain nonlinear V-shaped price pressure utility in trading by individual coherent preferences.

Ben-David and Hirshleifer (2012) examine how investor preferences and beliefs affect trading in relation to past gains and losses. They find that investors are more likely to sell a security when the magnitude of their gains or losses on it increases. They illustrate the finding by proposing an asymmetry V-shaped value function (see Figure 1) and conclude that investors has higher probability to sell over gains than over losses. An (2016) explains the asymmetry V-shaped value function by higher price pressure over gain in Grinblatt and Han’s (2005) analytical framework,
suggesting that investors’ trading behavior can aggregate to affect equilibrium price dynamics.

Followed the insights, we find that subject individuals can behave either disposition effect or inverse disposition effect in trading. Although there is higher price pressure over gains than over losses some days—disposition effect (see (a) in Figure 7), we also detect that there exist higher price pressure over losses than over gains on some other days. Individuals tend to sell fewer and generate lesser price pressure or reversal force over gains than over loses. It is inverse disposition effect (see (b) Figure 7).

(a) Higher price pressure over gains than losses on January 25, 2019
(b) Lower price pressure over gains than losses on January 3, 2019

Figure 7: Black solid lines represent asymmetrical price pressure in a V-shaped value function, and the lines of dashes are a supply line and a demand line, respectively.

According to equations (3), (10), and (14), moreover, cumulative trading volume distribution over a price range is log-normal if there is a V-shaped price pressure utility function in stock market (see (a) in Figure 5). In such scenarios, individual traders are independent since there is no interactive item in equation (10). A price reference point exists unless they are homogeneous and everyone has the same value in beliefs for the stock they trade. They behave no preferences over a price range in trading and price follows random walks in statistics. It is not true in reality. However, when we check carefully the empiric test results Ben-David and Hirshleifer (2012) did, we find that the price pressure utility is actually nonlinear V-shaped (see Figure 2). From equations (3), (1), and (8), we can rewrite equation (3). For convenience, we choose a natural unit $U=I$ and have
\[ W_m(p) = A_m(p - p_0) = U - p \frac{\omega_m^2}{2} - 1 - \| \omega_m(p - p_0) \| \quad , \]

\[ (m = 0, 1, 2 \ldots) , \quad (15) \]

where reversal force \( A_m \) is a variable instead of a constant; \( \omega_m \) is a set of eigenvalues or constants expressed by equations (6) and (7) and represents the magnitude of coherent preferences; and \( W_m(p) \) is price pressure utility function. It is plotted in Figure 8 if we choose \( \omega_m = 25 \) and \( p_0 = 0 \). The model of equation (15) predicts the nonlinear V-shaped price pressure caused by individual coherent preferences. On the other hand, Ben-David and Hirshleifer (2012) and An (2016) have well done empiric tests that demonstrate Shi’s probability wave equation validity to some extent. Thus, we can conclude that individual coherent preferences generate a nonlinear V-shaped price pressure utility and nonlinear market dynamic equilibrium in stock market.

Figure 8: An asymmetrical nonlinear V-shaped price pressure utility function

5. CONCLUSIONS

We study individual coherent preferences and asset prices in stock market where
nonlinear market dynamic equilibrium exists in a new paradigm. We distinguish a stock market equilibrium prices and its fundamental price in asset prices and mainly focus on the market equilibrium price and the mechanism of its formulation. We conclude that coherent preferences exist widely and generate nonlinear market dynamic equilibrium in stock market whereas beliefs contribute to discrepancy between two parts of price. Second, individual traders as a whole can behave either disposition effect or inverse disposition effect in trading. More importantly, we propose a set of explicit models for the nonlinear V-shaped price pressure utility detected in prior empiric tests (Ben-David and Hirshleifer 2012, An 2016). Otherwise, individual traders would be independent and homogeneous if there were a V-shaped value function. In another word, Ben-David and Hirshleifer (2012) demonstrate Shi’s probability wave equation validity to some extent whereas we in turn perfect the model of price pressure utility. Our study suggests better asset price models with a trading volume dimension in finance.

Next, we will study how market evolution with information, events, liquidity, and environments affects subject individuals in decision making. They adapt themselves to update a price reference point, make a price equilibrium point jump, and cause excessive price fluctuation in a positive feedback loop (Shiller 1981).
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