Scrambling for Dollars: International Liquidity, Banks and Exchange Rates*

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Abstract

We develop a theory of exchange rate fluctuations arising from financial institutions' demand for liquid dollar assets. Financial flows are unpredictable and may leave banks "scrambling for dollars". As a result of settlement frictions in interbank markets, a precautionary demand for dollar reserves emerges and gives rise to an endogenous convenience yield. In our framework, an increase in the volatility of idiosyncratic liquidity shocks leads to a rise in the convenience yield and an appreciation of the dollar---as banks scramble for dollars---while foreign exchange interventions matter because they alter the relative supply of liquidity in different currencies. We present empirical evidence on the relationship between exchange rate fluctuations for the G10 currencies and the quantity of dollar liquidity consistent with the theory.

Keywords: Exchange rates, liquidity premia, monetary policy
JEL Classification: E44, F31, F41, G20

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1 Introduction

The well-known "disconnect" puzzle in international finance holds that foreign exchange rates show little empirical relationship to the supposed economic drivers of currency values: monetary policy instruments, output, etc.[1] The literature has turned toward models of excess returns in international financial markets (for example, based on foreign-exchange risk or default risk) as the potential "missing link" in the exchange-rate puzzle.[2] Moreover, when measured in historical data on major currency pairs, a consistent pattern emerges: most currencies earn a significant and persistent premium over the US dollar. That is, saving in dollars has typically earned a low return.[3] But the source or sources of these expected excess returns remains a mystery that has not been fully resolved.

In this paper, we develop a theory of exchange rate fluctuations arising from the demand by financial institutions for liquid dollar assets. We build on the observation that banks that participate in the international payments system (global banks) incur dollar-denominated liabilities. In the international banking system, U.S. dollars are the dominant foreign-currency source of funding. According to the BIS locational banking statistics, in June 2020, the global banking and non-bank financial sector had cross-border dollar liabilities of over $14 trillion.

Banks domiciled in the U.S. and foreign banks (and other financial institutions, which we will call "banks") face uncertainty about funding. This uncertainty arises in part from the possibility of withdrawals by depositors, but also from the vicissitudes of very short-term lenders to these banks. While banks might typically turn to other financial institutions as a source of short-term liquidity, in times of uncertainty, the banks must rely on their own stocks of liquid assets to meet the demands of depositors and short-term lenders. U.S. banks might hold reserves at the Federal Reserve to maintain liquidity, but they also generally hold U.S. Treasury and agency obligations that are highly liquid. International banks have no central bank dollar reserves but hold other liquid dollar assets. We build a model of "global" banks, which could be domiciled either in the U.S. or abroad, but which have dollar and non-dollar (euro) liabilities. The global demand of the financial system for liquid dollar assets plays a pivotal role in our story.

Of course, the risk for an individual bank is that if it experiences a net outflow of liabilities, it will end up short of liquid assets to settle those flows. In the case of a shortfall, these banks might seek funding in the interbank market, but that market operates with frictions. In periods where transactions move in one direction, it may be costly to find a counterparty, and there may

[1] The term "exchange-rate disconnect" was coined by Obstfeld and Rogoff (2000). Devereux and Engel (2002), Duarte and Stockman (2005), and Itskhoki and Mukhin (2019) provide models of disconnect.

[2] McCallum and Nelson (1999), Kollmann (2002), Bergin (2006) and Itskhoki and Mukhin (2019) are examples of DSGE models that include random ex-ante excess returns in order to help explain exchange-rate behavior.

[3] See for example, Lustig and Verdelhan (2007) and Hassan (2013) which emphasize the relatively low rate of return on nominally risk-free dollar assets, while Gourinchas and Rey (2007) refer more broadly to the "exorbitant privilege" the U.S. enjoys by paying a lower return on its external liabilities than it earns on its foreign assets.

be times when banks may lose confidence in one another.

We model how frictions in the settlement of international deposit transactions emerge as a liquidity premium earned by the dollar. In our framework, ex ante excess returns arise as a function of monetary policy variables such as the quantity of outside money and policy rates in two currencies, as well as technology parameters such as the a matching efficiency among banks (which captures interbank confidence), the volatility of payments, and relative settlement demand in different currencies. Through the channel of UIP deviations, we link the determination of nominal dollar exchange rates (and a dollar return premium) to the reserve position of banks in different currencies, and to settlement risk and payments technology.

Many recent theories of exchange rate movements have focused primordially on a risk-premium or an external financing premium earned by the US dollar. Risk premium models chiefly explain the expected excess returns for foreign relative to dollar bonds as stemming from a greater exposure of currencies other than the dollar to global pricing factors.[4] Theories of the external financing premia can explain the dollar premium as a funding advantage in dollar liabilities. The interpretation in this paper is an alternative, a liquidity premium. On the surface, the model resembles the early monetary exchange rate model of Lucas (1982).[5] In that model, two currencies earn a liquidity premium over bonds because certain goods must be bought with corresponding currencies. A money demand equation determines prices in both currencies, and relative prices determine the exchange rate. Our model shares the segmentation of transactions and the exchange-rate determination of Lucas. However, in our model, the demand for reserves in either currency stems from the settlement demand by banks. This distinction is important, because our model leads to predictions about the direction of exchange rates as functions of the ratio of reserves to deposits in different currencies, the size and volatility of flows in different currencies and the dispersion of interbank rates in different currencies.

Literature Review

The expected excess return on foreign interest earning assets, or the deviation from "uncovered interest parity (UIP)" is important not just for understanding international pricing of interest-bearing assets, or the expected depreciation (or appreciation) of the currency, but also for the level of the exchange rate. This point is brought out clearly by Obstfeld and Rogoff (2003), which shows how the expected present value of current and future foreign exchange risk premiums affect the current exchange rate in a simple DSGE model. They refer to this present value as the "level risk premium."[6]

[4]See, for example, Lustig, Roussanov, and Verdelhan (2011).

[5]See also Svensson (1985) and Engel (1992a,b).

[6]This present value plays a key role in the analysis of Engel and West (2005), Froot and Ramadorai (2005), and Engel (2016). See Engel (2014)'s survey of exchange rates for an overview of the effect of the risk premium on exchange rates.

Potentially, a better understanding of the role of ex ante excess returns can help account for the empirical failure of exchange-rate models (Meese and Rogoff, 1983; Obstfeld and Rogoff, 2000), and the excess volatility of exchange rates (Frankel and Meese, 1987; Backus and Smith, 1993; Rogoff, 1996). Much of the literature has been directed toward explaining the expected excess return as arising from foreign exchange risk. Another branch of the literature has explored limits to capital mobility and frictions in asset markets. A third branch has looked at deviations from rational expectations. A line of research closely related to this paper has been the role of the "convenience yield" in driving exchange rates.

Foreign exchange risk premium. The modeling of failures of uncovered interest parity as arising from foreign exchange risk has a long history. Early contributions include Solnik (1974), Roll and Solnik (1977), Kouri (1976), Stulz (1981), and Dumas and Solnik (1995). Much theoretical work has been devoted toward building models of the risk premium that are consistent with the Fama (1984) puzzle, which finds a positive correlation between the expected excess return and the interest rate differential.[7] Bansal and Shaliastovich (2013), Colacito (2009), Colacito and Croce (2011, 2013), Colacito, Croce, Gavazzoni, and Ready (2018b); Colacito, Croce, Ho, and Howard (2018a), and Lustig and Verdelhan (2007) examine models with recursive preferences. Verdelhan (2010) presents a model with habit formation to account for the Fama puzzle. Ilut (2012) proposes ambiguity aversion as a solution to the puzzle. Some recent studies, such as Burnside et al. (2011), Farhi and Gabaix (2016), and Farhi, Fraiberger, Gabaix, Ranciere, and Verdelhan (2015), model the risk premium as arising from risks associated with rare events.

Limited Capital Mobility. Other models attribute these uncovered interest parity differentials to financial premia earned by foreign currency because of limited market participation as in the segmented markets models of (Alvarez et al., 2002, 2009; Itskhoki and Mukhin, 2019) or limited capital flows (Gabaix and Maggiori, 2015; Amador et al., 2019). Models in which order flow matters for exchange rate determination also require some frictions in the foreign exchange market. See, for example, Evans and Lyons (2002, 2008). Relatedly, Bacchetta and Van Wincoop (2010) posit that slow adjustment of portfolios can account for the expected excess returns on foreign bonds.

Deviations from Rational Expectations. A simple alternative story for the UIP deviations is that agents expectations are not fully rational. Empirical studies, such as Frankel and Froot (1987), Froot and Frankel (1989), and Chinn and Frankel (2019) have used survey measures of expectations to uncover possible deviations from rational expectations. Models that incorporate systematically skewed expectations include Gourinchas and Tornell (2004) and Bacchetta and Van Wincoop (2006).

[7] See Tryon (1979) and Bilson (1981) for earlier empirical studies that find this relationship. Engel (1996, 2014) surveys empirical and theoretical models.

Convenience Yield. Our model is closely connected to the recent examination of the "convenience yield" - the low return on riskless government liabilities - and exchange rates. We posit that our model provides one possible channel for the emergence of the convenience yield on U.S. government bonds. See Engel (2016), Valchev (2020), Jiang, Krishnamurthy, and Lustig (2018; 2020), Engel and Wu (2018), and Kekre and Lenel (2020).

The paper is organized as follows. Section 2 presents the empirical analysis. Section 3 presents the model. Section 4 presents the calibration of the model and the quantitative results. Section 5 concludes. All proofs are in the appendix.

2 Motivating Facts

We begin with a look at the data relating the banking sector's balance sheet data to the foreign currency price of U.S. dollars. Our thesis, at its simplest, is that the financial sector increases its demand for dollar liquid assets—US government obligations, including reserves held at the Federal Reserve for banks in the Federal Reserve system—when funding becomes more uncertain. The global banking system relies heavily on U.S. dollars for funding, much of which is raised through money market funding for banks located outside of the US.

We examine the behavior of the dollar against the other nine of the so-called G10 currencies, with special attention given to the euro. The euro area is especially important in our analysis because it encompasses a large economy with a financial system that relies heavily on short-term dollar funding. The other currencies are the Australian dollar, Canadian dollar, Japanese yen, New Zealand dollar, Norwegian krone, Swedish krona, Swiss franc, and the U.K. pound.

We look at two sources of data for the US banking system. Detailed data on short-term dollar funding and on liquid dollar assets is not readily available for the global financial system, so we use the US data as a proxy for the dollar-denominated elements of the global banking balance sheets. That is, we presume that foreign banks' demand for liquid dollar assets responds in a similar way to banks located in the U.S. (including U.S.-based subsidiaries of foreign banks) when faced with uncertainty about dollar funding. This approach is also followed by Adrian et al. (2010), a study that aims to show how the price of risk is related to banks' balance sheets and the expected change in the exchange rate (rather than the level of the exchange rate, which is our concern here), and presents a simple partial-equilibrium model of the banking sector. More precisely, Adrian et al. (2010) focuses on the state of the balance sheet at time t in forecasting $e_{t+1} - e_t$, as they are concerned with understanding the expected excess return on foreign bonds between t and $t+1$. Our interest is centered on how changes in the balance sheet between $t-1$ and t contribute to changes in the exchange rate between $t-1$ and t, that is $e_t - e_{t-1}$.

We consider two measures of short-term funding to financial intermediaries. The first is used by

Adrian et al. (2010), U.S. dollar financial commercial paper (series DTBSPCKFM from FRED, the Federal Reserve Economic Data website maintained by the Federal Reserve Bank of St. Louis.) Another major source of short-term funding to U.S. banks is demand deposits, measured by DEMDEPSL from FRED. We construct a variable that measures the level of funding and the response of financial intermediaries to uncertainty about that funding. We look at the ratio of the sum of reserves held at Federal Reserve banks and government (Treasury and agency) securities held by commercial banks (the sum of RESBALNS and USGSEC from FRED) to short term funding (DTBSPCKFM + DEMDEPSL from FRED). This variable is endogenous in our model, but its movements are a key indicator of how the demand for dollars is affected by the financial sector's demand for liquid assets when uncertainty increases. As dollar funding becomes more volatile for banks, they will increase their ratio of safe dollar assets to liabilities. That in turn will lead to a global increase in dollar demand, leading to a dollar appreciation.

Figure 1 plots this ratio of liquid government assets holdings to short-term funding of the financial sector.[8] During this period, bank reserve balances rose from around 10 billion dollars in August, 2008 to nearly 800 billion dollars one year later, and then continued to climb to a peak of around 2.3 trillion dollars by late 2017 before gradually declining to 1.4 trillion dollars by the end of 2019. However, the liquidity ratio does not show movement anywhere near that magnitude. It is true that it rose during the onset of the global financial crisis, but this movement is not largely driven by the increase in reserves, because demand deposits rose almost proportionately. Mechanically, a large part of the rise in the overall liquidity ratio is driven by a fall in financial commercial paper funding, which works to lower the denominator of the ratio.

In Table 1, we present the parameter estimates of the regression:

$$\Delta e_t = \alpha + \beta_1 \Delta(LiqRat_t) + \beta_2(\pi_t - \pi_t^*) + \beta_3 LiqDepRat_{t-1} + \epsilon_t \tag{1}$$

In this regression, $\Delta(x_t)$ means the "change from $t-1$ to t" in the variable x_t; e_t is the log of the exchange rate expressed as the G10 currency price of a U.S. dollar; $LiqRat_t$ is the variable described above; $\pi_t - \pi_t^*$ is the difference between year-on-year inflation rates in each of the 9 countries and the U.S. All data is monthly.[9] The inflation variable is meant to capture the effects of monetary policy on exchange rates. As much of the empirical literature has found, there is a negative relationship between the change in a country's inflation rate and its exchange rate. When inflation is rising in a country, markets anticipate future monetary tightening, and that leads to a currency appreciation.

If uncertainty is driving the $LiqRat_t$, then we should also expect a positive relationship between this variable and e_t, i.e., β_1 positive. During times of high uncertainty, banks hold greater amounts

[8] The figure also plots an alternative measure described below.

[9] An exception is that inflation for Australia and New Zealand are reported only quarterly. We linearly interpolate the data to get monthly series.

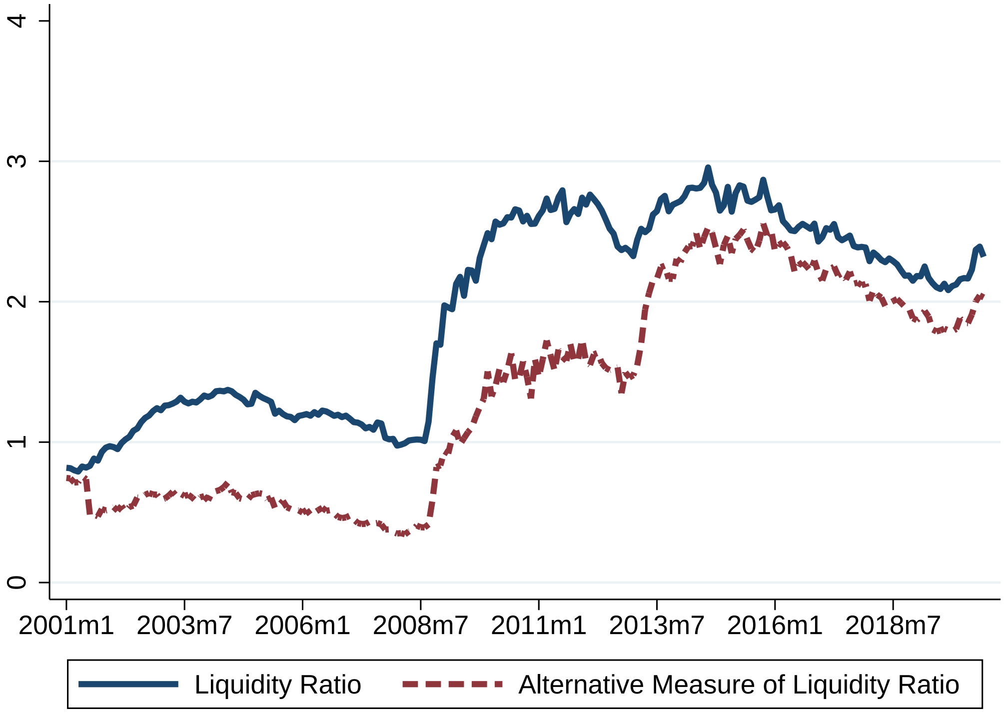

Figure 1: The ratio of liquid assets to short-term liabilities

of liquid dollar assets (reserves and Treasury securities) relative to demand deposits, so $LiqRat_t$ is higher. That increased demand for safe dollar assets leads to a stronger dollar (an increase in e_t.)

We also include the lagged level of $LiqRat_t$. This is included because the depreciation of the dollar might depend on lagged as well as current levels of this variable. The regressions we report would have the identical fit if we included current and lagged levels of this variable, instead of the change in the variable and the lagged level. We specify the regression as above for two reasons: First, specifying the regression so that the change in the liquidity variable influences the change in the exchange rate leads to a more natural interpretation. Second, while the current and lagged levels of the variable are highly correlated, which leads to multicollinearity and imprecise coefficient estimates, the change and the lagged level are much less highly correlated.

Table 1 reports the regression findings for the nine exchange rates. The sample period is February 2001 to July 2020.[10] (Data on financial commercial paper starts in January 2001.) With the exception of Japan, the liquidity ratio variable has the expected sign and is statistically significant at the 1 percent level for all exchange rates. The relative inflation variable also has the correct sign for all the currencies and is statistically significant for most countries

It is commonplace to look at short-term interest rate movements to account for the effects of monetary policy changes on exchange rates. During much of our sample period, interest rates were

[10] Australia and New Zealand's sample end in May 2020 because of availability of inflation data.

near the zero-lower bound, and do not appear to do a good job measuring the monetary policy stance. In Table 1i, we include $i_t - i_t^*$, the interest rate in each of the 9 countries relative to the U.S. as regressors. It is only statistically significant at the 5 percent level for Japan, and none of the major conclusions are altered by its inclusion.

We highlight that the key regressor, $\Delta LiqRat_t$, is not simply a market price. That is, these regressions "explain" exchange rate movements but are not relying on other market prices to do the job. It is the balance sheet variables that play the pivotal role.

We argue that uncertainty about funding drives the balance sheet variables, but what if we were to include a direct measure of uncertainty in the regressions? Many asset-pricing studies have used *VIX* to quantify market uncertainty, and *VIX* has power in explaining the movements of many asset prices. However, *VIX* does not directly measure uncertainty about dollar funding for banks. Indeed, *VIX* might measure some dimensions of uncertainty, but it might also be capturing global risk, and global risk might be driving the dollar, as in the model of Farhi and Gabaix (2016). In Table 2, we have included the change in *VIX* along with the other variables.

As expected, *VIX* has positive coefficients in all cases (except Japan) and is statistically significant. An increase in *VIX* is associated with an appreciation of the dollar. However, the introduction of this variable does not reduce the significance of the liquidity ratio variable, for any of the countries, and for most only has a small effect on the magnitude of the coefficient. This suggests that the uncertainty that is quantified by the *VIX* does not include all of the forces that drive the liquidity ratio and lead to its positive association with dollar appreciation. (Table 2i also includes the interest rate differential, and as in Table 1i, we see that its inclusion has little influence on the findings and its effect is mostly small and insignificant.)

It is important to note that the liquidity ratio is not an exogenous driver of exchange rates—either in our model or in the real world. We will show that a calibrated version of our model implies a positive association between the change in the liquidity ratio and the value of the dollar under multiple driving shocks, including uncertainty shocks, monetary policy shocks and liquidity demand shocks.

We can, however, use instrumental variables to isolate the effects of uncertainty on the liquidity ratio, and its transmission to exchange rates. To that end, Table 3A uses two measures of uncertainty as instruments for the liquidity ratio: the cross-section standard deviation at each time period of the inflation rates of the G10 countries, and the cross-section standard deviation of the rates of depreciation for these currencies. The findings are largely the same as in Table 2. For most of the countries, the magnitude of the effect of the liquidity ratio on the exchange rate is increased, and for some (such as Canada), the statistical significance greatly increases. The model still fits poorly for the Japanese yen, and we now find statistical significance of the liquidity ratio on the Swiss franc exchange rate.

In Table 3B, we take the alternative tack of including *VIX* as an instrument for the liquidity ratio. It is possible for *VIX* to be a valid instrument that is uncorrelated with the regression error, even though it is statistically significant when included in the regression separately (as in Tables 2 and 3A) if the other forces that drive the liquidity ratio are uncorrelated with *VIX*. That is, Table 3B reports the influence of the liquidity ratio on exchange rates when the liquidity ratio is driven by *VIX* and other measures of uncertainty, while any other forces that might influence the exchange rate are relegated to the regression error. *VIX* is a valid instrument if it is uncorrelated with those forces. If one takes such a stance, then the estimates reported in Table 3B reveal a strong channel of uncertainty on exchange rates working through the liquidity ratio.

We consider next an alternative measure of the liquidity ratio that includes "net financing" of broker-dealer banks. This is a measure of "funds primary dealers borrow through all fixed-income security financing transactions," as described in Adrian and Fleming (2005). We include this as another source of short-term liabilities, similar to how net repo financing is included as a measure of short-term liabilities in the liquidity ratio calculated by the IMF (Global Financial Stability Report, 2018). Figure 1 also plots this alternative measure of the liquidity ratio, which is smaller than our baseline measure because it includes another class of liabilities of the banking system in the U.S.

Tables 4, 5, 6A, and 6B are analogous to Tables 1, 2, 3A, and 3B, respectively. The conclusions using this alternative measure are virtually unchanged qualitatively, though of course the numerical values of the estimated coefficients are different. The liquidity ratio is still highly statistically significant for all currencies, except for Japan, and, in the case of the instrumental variable regressions, Switzerland.

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Table 1: Relationship of Exchange Rates and Banking Liquidity Ratio Feb. 2001 – July 2020

	Euro	Australia	Canada	Japan	New Zealand	Norway	Sweden	Switz	U.K.
$\Delta(\text{LiqRat}_t)$	0.225***	0.243***	0.131***	-0.152***	0.295***	0.189***	0.213***	0.145***	0.165***
	(4.525)	(3.597)	(2.643)	(-3.036)	(4.302)	(3.023)	(3.555)	(2.654)	(3.320)
$\pi_t - \pi_t^*$	-0.542***	-0.422**	-0.412*	0.008	-0.718***	-0.117	-0.492**	-0.666***	-0.390**
	(-3.718)	(-2.226)	(-1.928)	(0.055)	(-3.757)	(-0.803)	(-2.521)	(-2.803)	(-2.114)
LiqRat_{t-1}	0.011**	0.006	0.007	0.002	0.009	0.010*	0.006	0.005	0.009*
	(2.425)	(1.065)	(1.578)	(0.315)	(1.508)	(1.763)	(1.145)	(0.990)	(1.735)
Constant	-0.012***	-0.004	-0.006*	-0.001	-0.009**	-0.007*	-0.009**	-0.017***	-0.006
	(-3.452)	(-1.053)	(-1.832)	(-0.108)	(-2.095)	(-1.653)	(-2.069)	(-3.169)	(-1.597)
N	234	232	234	234	232	234	234	234	234
adj. R^2	0.11	0.05	0.03	0.03	0.10	0.03	0.05	0.04	0.04

t statistics in parentheses.

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Table 1i: Relationship of Exchange Rates and Banking Liquidity Ratio Feb. 2001 – July 2020

	Euro	Australia	Canada	Japan	New Zealand	Norway	Sweden	Switz	U.K.
$\Delta(\text{LiqRat}_t)$	0.220***	0.233***	0.125**	-0.141***	0.277***	0.184***	0.215***	0.140**	0.163***
	(4.422)	(3.359)	(2.519)	(-2.830)	(3.947)	(2.914)	(3.561)	(2.557)	(3.254)
$\Delta(i_t - i_t^*)$	-1.284	-0.491	-1.131	-1.793**	-1.114	-0.513	0.249	-1.541*	-0.269
	(-1.486)	(-0.599)	(-1.304)	(-2.336)	(-1.245)	(-0.638)	(0.311)	(-1.663)	(-0.351)
$\pi_t - \pi_t^*$	-0.546***	-0.408**	-0.382*	0.050	-0.729***	-0.118	-0.504**	-0.649***	-0.394**
	(-3.750)	(-2.136)	(-1.780)	(0.343)	(-3.815)	(-0.807)	(-2.528)	(-2.737)	(-2.129)
LiqRat_{t-1}	0.010**	0.006	0.006	-0.001	0.008	0.010	0.006	0.004	0.009*
	(2.141)	(0.940)	(1.405)	(-0.142)	(1.279)	(1.643)	(1.181)	(0.775)	(1.705)
Constant	-0.011***	-0.004	-0.006*	0.002	-0.008*	-0.006	-0.009**	-0.016***	-0.005
	(-3.239)	(-0.950)	(-1.679)	(0.349)	(-1.892)	(-1.560)	(-2.081)	(-2.980)	(-1.571)
N	234	232	234	234	232	234	234	234	234
adj. R^2	0.11	0.05	0.03	0.05	0.10	0.03	0.05	0.05	0.04

t statistics in parentheses.

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Table 2: Relationship of Exchange Rates and Banking Liquidity Ratio with VIX Feb. 2001 – July 2020

	Euro	Australia	Canada	Japan	New Zealand	Norway	Sweden	Switz	U.K.
$\Delta(\text{LiqRat}_t)$	0.195***	0.163***	0.086*	-0.137***	0.232***	0.139**	0.173***	0.129**	0.140***
	(3.999)	(2.798)	(1.909)	(-2.741)	(3.674)	(2.358)	(3.017)	(2.337)	(2.831)
$\pi_t - \pi_t^*$	-0.427***	-0.185	-0.277	-0.016	-0.519***	-0.032	-0.415**	-0.595**	-0.304*
	(-2.950)	(-1.130)	(-1.436)	(-0.112)	(-2.941)	(-0.235)	(-2.234)	(-2.491)	(-1.660)
ΔVIX_t	0.001***	0.004***	0.002***	-0.001**	0.003***	0.002***	0.002***	0.001*	0.001***
	(3.881)	(9.339)	(7.513)	(-2.250)	(6.909)	(5.934)	(5.153)	(1.928)	(3.116)
LiqRat_{t-1}	0.011**	0.007	0.007*	0.002	0.009*	0.010*	0.007	0.005	0.008
	(2.492)	(1.427)	(1.829)	(0.334)	(1.696)	(1.892)	(1.390)	(1.063)	(1.617)
Constant	-0.011***	-0.005	-0.006*	-0.001	-0.009**	-0.007*	-0.008**	-0.016***	-0.005
	(-3.277)	(-1.504)	(-1.943)	(-0.212)	(-2.210)	(-1.711)	(-2.128)	(-2.973)	(-1.459)
N	234	232	234	234	232	234	234	234	234
adj. R^2	0.16	0.31	0.22	0.05	0.25	0.16	0.15	0.05	0.07

t statistics in parentheses.

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Table 2i: Relationship of Exchange Rates and Banking Liquidity Ratio with VIX Feb. 2001 – July 2020

	Euro	Australia	Canada	Japan	New Zealand	Norway	Sweden	Switz	U.K.
$\Delta(\text{LiqRat}_t)$	0.192***	0.155**	0.084*	-0.123**	0.208***	0.137**	0.174***	0.126**	0.138***
	(3.922)	(2.592)	(1.856)	(-2.477)	(3.230)	(2.291)	(3.012)	(2.291)	(2.769)
$\Delta(i_t - i_t^*)$	-1.075	-0.404	-0.466	-1.994***	-1.404*	-0.289	0.096	-1.285	-0.266
	(-1.277)	(-0.578)	(-0.593)	(-2.615)	(-1.725)	(-0.385)	(0.126)	(-1.374)	(-0.353)
$\pi_t - \pi_t^*$	-0.432***	-0.174	-0.266	0.027	-0.530***	-0.033	-0.419**	-0.589**	-0.308*
	(-2.989)	(-1.054)	(-1.370)	(0.191)	(-3.016)	(-0.240)	(-2.210)	(-2.468)	(-1.676)
ΔVIX_t	0.001***	0.004***	0.002***	-0.001**	0.003***	0.002***	0.002***	0.001*	0.001***
	(3.794)	(9.317)	(7.385)	(-2.538)	(7.018)	(5.896)	(5.133)	(1.683)	(3.110)
LiqRat_{t-1}	0.010**	0.007	0.007*	-0.001	0.008	0.010*	0.007	0.004	0.008
	(2.240)	(1.299)	(1.736)	(-0.177)	(1.390)	(1.808)	(1.385)	(0.871)	(1.587)
Constant	-0.011***	-0.005	-0.006*	0.002	-0.008*	-0.006	-0.009**	-0.015***	-0.005
	(-3.095)	(-1.399)	(-1.863)	(0.291)	(-1.941)	(-1.648)	(-2.095)	(-2.837)	(-1.433)
N	234	232	234	234	232	234	234	234	234
adj. R^2	0.16	0.31	0.22	0.07	0.26	0.15	0.15	0.05	0.07

t statistics in parentheses.

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Table 3A: Relationship of Exchange Rates and Banking Liquidity Ratio Instrumental Variable Regression: StDev(Inf) and StDev(XRate) instrument for Δ(LiquidityRatio) Feb. 2001 – July 2020

	Euro	Australia	Canada	Japan	New Zealand	Norway	Sweden	Switz	U.K.
$\Delta(\text{LiqRat}_t)$	0.450***	0.593***	0.441***	-0.306**	0.507***	0.526***	0.473***	0.019	0.717***
	(3.150)	(3.182)	(3.274)	(-2.266)	(2.804)	(2.831)	(2.800)	(0.121)	(3.547)
$\pi_t - \pi_t^*$	-0.566***	-0.444**	-0.496**	0.051	-0.658***	-0.242	-0.595***	-0.476	-0.981***
	(-3.339)	(-2.111)	(-2.149)	(0.328)	(-3.250)	(-1.370)	(-2.728)	(-1.640)	(-3.034)
LiqRat_{t-1}	0.014***	0.013**	0.013**	-0.002	0.013**	0.017**	0.011*	0.004	0.024***
	(2.912)	(2.119)	(2.589)	(-0.255)	(2.123)	(2.596)	(1.897)	(0.733)	(2.922)
ΔVIX_t	0.001***	0.003***	0.002***	-0.001*	0.003***	0.002***	0.002***	0.001**	0.000
	(2.693)	(6.879)	(5.423)	(-1.680)	(5.659)	(4.278)	(3.971)	(2.049)	(0.856)
Constant	-0.015***	-0.010**	-0.011***	0.003	-0.012***	-0.013**	-0.013***	-0.013*	-0.017***
	(-3.641)	(-2.263)	(-2.859)	(0.433)	(-2.645)	(-2.528)	(-2.708)	(-1.834)	(-2.896)
N	234	232	234	234	232	234	234	234	234

t statistics in parentheses.

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Table 3B: Relationship of Exchange Rates and Banking Liquidity Ratio Instrumental Variable Regression: Δ(VIX), StDev(Inf) and StDev(XRate) instrument for Δ(LiquidityRatio) Feb. 2001 – July 2020

	Euro	Australia	Canada	Japan	New Zealand	Norway	Sweden	Switz	U.K.
$\Delta(\text{LiqRat}_t)$	0.604***	1.099***	0.688***	-0.380***	0.888***	0.832***	0.710***	0.153	0.799***
	(4.231)	(4.663)	(4.421)	(-2.904)	(4.357)	(4.087)	(3.961)	(1.058)	(4.249)
$\pi_t - \pi_t^*$	-0.717***	-0.892***	-0.732***	0.094	-0.985***	-0.450**	-0.778***	-0.673**	-1.094***
	(-4.120)	(-3.245)	(-2.627)	(0.596)	(-4.176)	(-2.234)	(-3.217)	(-2.465)	(-3.519)
LiqRat_{t-1}	0.016***	0.018**	0.016***	-0.003	0.017**	0.023***	0.012**	0.005	0.026***
	(3.063)	(2.177)	(2.618)	(-0.466)	(2.349)	(2.862)	(1.972)	(0.951)	(3.208)
Constant	-0.018***	-0.014**	-0.015***	0.005	-0.017***	-0.017***	-0.017***	-0.017***	-0.019***
	(-4.085)	(-2.325)	(-3.065)	(0.723)	(-3.049)	(-2.932)	(-3.068)	(-2.618)	(-3.219)
N	234	232	234	234	232	234	234	234	234

t statistics in parentheses.

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Table 4: Relationship of Exchange Rates and Alternative Measure of Banking Liquidity Ratio

	Euro	Australia	Canada	Japan	New Zealand	Norway	Sweden	Switz	U.K.
$\Delta(\text{LiqRat2}_t)$	0.098***	0.109***	0.069***	-0.012	0.123***	0.101***	0.088***	0.079***	0.103***
	(3.718)	(3.073)	(2.675)	(-0.450)	(3.388)	(3.100)	(2.789)	(2.768)	(4.049)
$\pi_t - \pi_t^*$	-0.511***	-0.406**	-0.421*	-0.082	-0.673***	-0.126	-0.440**	-0.640***	-0.346**
	(-3.480)	(-2.135)	(-1.944)	(-0.540)	(-3.519)	(-0.849)	(-2.253)	(-2.731)	(-2.065)
LiqRat2_{t-1}	0.006**	0.005	0.005*	0.004	0.005	0.006*	0.004	0.003	0.004
	(2.118)	(1.415)	(1.833)	(1.217)	(1.388)	(1.796)	(1.377)	(1.129)	(1.280)
Constant	-0.006***	-0.001	-0.002	-0.002	-0.003	-0.001	-0.005*	-0.014***	-0.001
	(-2.699)	(-0.257)	(-1.249)	(-0.659)	(-1.414)	(-0.496)	(-1.765)	(-3.175)	(-0.311)
N	234	232	234	234	232	234	234	234	234
adj. R^2	0.09	0.04	0.04	-0.00	0.07	0.04	0.04	0.04	0.06

t statistics in parentheses.

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Table 5: Relationship of Exchange Rates and Alternative Measure of Banking Liquidity Ratio with VIX

	Euro	Australia	Canada	Japan	New Zealand	Norway	Sweden	Switz	U.K.
$\Delta(\text{LiqRat2}_t)$	0.090***	0.088***	0.059**	-0.008	0.106***	0.088***	0.078***	0.074***	0.096***
	(3.501)	(2.933)	(2.544)	(-0.283)	(3.233)	(2.911)	(2.625)	(2.616)	(3.833)
$\pi_t - \pi_t^*$	-0.393***	-0.188	-0.299	-0.112	-0.480***	-0.052	-0.379**	-0.575**	-0.274*
	(-2.721)	(-1.163)	(-1.543)	(-0.742)	(-2.748)	(-0.373)	(-2.052)	(-2.451)	(-1.655)
ΔVIX_t	0.001***	0.004***	0.002***	-0.001**	0.003***	0.002***	0.002***	0.001**	0.001***
	(4.215)	(9.677)	(7.735)	(-2.589)	(7.238)	(6.145)	(5.446)	(2.126)	(3.330)
LiqRat2_{t-1}	0.006**	0.006*	0.005**	0.004	0.006*	0.007*	0.005*	0.003	0.003
	(2.224)	(1.860)	(2.137)	(1.259)	(1.656)	(1.951)	(1.663)	(1.220)	(1.268)
Constant	-0.005**	-0.001	-0.002	-0.003	-0.003	-0.001	-0.004*	-0.013***	-0.000
	(-2.372)	(-0.660)	(-1.235)	(-0.829)	(-1.466)	(-0.502)	(-1.698)	(-2.931)	(-0.277)
N	234	232	234	234	232	234	234	234	234
adj. R^2	0.15	0.32	0.23	0.02	0.24	0.17	0.15	0.06	0.10

t statistics in parentheses.

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Table 6A: Relationship of Exchange Rates and Alternative Measure of Banking Liquidity Ratio Instrumental Variable Regression: StDev(Inf) and StDev(XRate) instrument for Δ(LiquidityRatio)

	Euro	Australia	Canada	Japan	New Zealand	Norway	Sweden	Switz	U.K.
$\Delta(\text{LiqRat2}_t)$	0.362***	0.491***	0.352***	-0.239*	0.411**	0.450***	0.373**	0.018	0.604***
	(2.758)	(2.791)	(3.033)	(-1.872)	(2.579)	(2.625)	(2.593)	(0.146)	(2.674)
$\pi_t - \pi_t^*$	-0.607***	-0.548**	-0.403	0.100	-0.698***	-0.324	-0.552**	-0.490	-1.149**
	(-2.991)	(-2.065)	(-1.573)	(0.483)	(-2.996)	(-1.496)	(-2.347)	(-1.630)	(-2.436)
LiqRat2_{t-1}	0.008**	0.008*	0.007**	0.000	0.007*	0.011**	0.007*	0.003	0.011**
	(2.318)	(1.966)	(2.124)	(0.091)	(1.832)	(2.310)	(1.795)	(1.087)	(1.989)
ΔVIX_t	0.001***	0.004***	0.002***	-0.001*	0.003***	0.002***	0.002***	0.001**	0.001
	(2.617)	(6.194)	(5.240)	(-1.693)	(5.402)	(4.018)	(3.955)	(2.146)	(0.859)
Constant	-0.007***	-0.002	-0.004	0.002	-0.005*	-0.003	-0.007**	-0.011*	-0.004
	(-2.717)	(-0.832)	(-1.636)	(0.484)	(-1.839)	(-1.137)	(-2.146)	(-1.945)	(-1.228)
N	234	232	234	234	232	234	234	234	234

t statistics in parentheses.

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Table 6B: Relationship of Exchange Rates and Alternative Measure of Banking Liquidity Ratio Instrumental Variable Regression: Δ(VIX), StDev(Inf) and StDev(XRate) instrument for Δ(LiquidityRatio)

	Euro	Australia	Canada	Japan	New Zealand	Norway	Sweden	Switz	U.K.
$\Delta(\text{LiqRat2}_t)$	0.471***	0.822***	0.484***	-0.296**	0.651***	0.649***	0.507***	0.105	0.686***
	(3.276)	(3.327)	(3.420)	(-2.251)	(3.250)	(3.135)	(3.076)	(0.882)	(3.036)
$\pi_t - \pi_t^*$	-0.774***	-1.007***	-0.552*	0.171	-1.023***	-0.527**	-0.678**	-0.677**	-1.315***
	(-3.466)	(-2.679)	(-1.734)	(0.788)	(-3.468)	(-1.978)	(-2.466)	(-2.353)	(-2.760)
LiqRat2_{t-1}	0.008**	0.010	0.008*	-0.001	0.008	0.013**	0.007	0.003	0.012**
	(2.177)	(1.597)	(1.831)	(-0.120)	(1.598)	(2.219)	(1.559)	(1.149)	(2.074)
Constant	-0.009***	-0.002	-0.005	0.004	-0.007*	-0.004	-0.008**	-0.015***	-0.004
	(-2.938)	(-0.601)	(-1.641)	(0.780)	(-1.841)	(-1.240)	(-2.235)	(-2.662)	(-1.300)
N	234	232	234	234	232	234	234	234	234

t statistics in parentheses.

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

3 A Model of Banking Liquidity and Exchange Rates

We present a dynamic equilibrium model of global banks that intermediate international financial flows and are subject to idiosyncratic liquidity shocks. The model has two countries, the EU and the US, and two currencies. To fix ideas, we think about the euro as the domestic currency and the dollar as the foreign currency. In each country, there is a continuum of households and a central bank that sets monetary policy. Production of the single tradable consumption good is carried out globally by multinationals. We assume that the law of one price holds.

3.1 Banks

Timing. Time is discrete and there is an infinite horizon. Every period is divided in two sub-stages: a lending stage and a balancing stage. In the lending stage, banks make their equity payout, Div_t, and portfolio decisions. In the balancing stage, banks face liquidity shocks and re-balance their portfolio.

Notation. We use "asterisk" to denote the foreign currency (i.e., the "dollar") variable and "tilde" to denote a real variable. The vector of aggregate shocks is indexed by X. The exchange rate is defined as the amount of euros necessary to purchase one dollar—hence, a higher e indicates an appreciation of the dollar.

Preferences and budget constraint. Payouts are distributed to households that own bank shares and have linear utility with discount factor β. Banks' objective is to maximize shareholders' value and therefore they maximize the net present value of dividends:

$$\sum_{t=0}^{\infty} \beta^t Div_t \tag{2}$$

Banks enter the lending stage with a portfolio of assets/liabilities and collect/make associated interest payments. The portfolio of initial assets is given by corporate loans, l_t, and liquid assets m_t, both in euros and dollars, and corporate loans, b_t, denominated in consumption goods. Note that we refer to liquid assets as "reserves", for simplicity, but they should be understood as capturing also government bonds—the important property, as we will see, is that these are assets that can be used as settlement instruments. On the liability side, banks issue demand deposits, d_t, discount window loans, w_t, and net interbank loans, f_t (if the bank has borrowed funds, f_t is positive, and vice versa), again in both currencies. Deposits and interbank market loans have market returns given by i^d and $\bar{i}^f$ while central banks set the corridor rates for reserves and discount window, respectively i^m and i^w. A yield i_t paid in period t is pre-determined in period $t-1$. Meanwhile, R^b is the real return on loans.

Their budget constraint is given by

$$P_t^* Div_t + \frac{m_{t+1} - d_{t+1}}{e_t} + b_{t+1}P_t^* + m_{t+1}^* - d_{t+1}^* \leq \; P_t^* b_t R_t^b + m_t^*(1 + i_t^{m,*}) - d_t^*(1 + i_t^{d,*})$$
$$+ f_t^*(1 + i_t^{f,*}) + w_t^*(1 + i_t^{w,*}) - \frac{m_t(1 + i_t^m) - d_t(1 + i_t^d) + f_t(1 + i_t^f) + w_t(1 + i_t^w)}{e_t} \quad (3)$$

Withdrawal shocks. In the balancing stage, banks are subject to random withdrawal of deposits in either currencies. As in Bianchi and Bigio (2020), withdrawals have zero mean—hence deposits are reshuffled but preserved within the banking system. In addition, we assume that the distribution of these shocks is time-varying. As a way to capture the prevalence of the dollar for international settlements, we will be focus on an environment where the volatility of dollar deposits is larger than the euro.

The inflow/outflow of deposits across banks generates, in effect, a transfer of liabilities. We assume that these liabilities are settled using reserves in the same currency of the deposit. Importantly, reserves must remain positive at the end of the period. We denote by s^j the euro surplus of a bank facing a withdrawal shock ω_t^j. This surplus is given by the amount of euro reserves a bank brings from the lending stage minus the withdrawals of deposits:

$$s_t^j = m_{t+1} + \omega_t^j d_{t+1}, \quad (4)$$

Notice that we omit the subscript of bank choices in deposits and reserves, because it is without loss of generality that all banks make the same choices in the lending stage. If a bank faces a negative withdrawal shock, lower than $\tilde{\omega} \equiv -m/d$, the bank has a deficit of reserves. Conversely, if the withdrawal shock is larger than $\tilde{\omega}$, the bank has a surplus. Notice that if $m = 0$, the sign of the surplus has the same sign as the withdrawal shock. A higher liquidity ratio makes more likely that the bank will be in surplus.

Similarly, we have the following surplus in dollars

$$s_t^{j,*} = m_{t+1}^* + \omega_t^{j,*} d_{t+1}^* \quad (5)$$

Interbank market. After the withdrawal shocks are realized, we have a distribution of banks in surplus and deficits in both currencies. We assume that there is an interbank for each currency in which banks that have a deficit in one currency borrow from banks that have a surplus in the same currency. These two interbank market behave symmetrically, so it suffices to show how one of the market works.[11]

[11] We are assuming in the background an extreme form of segmented interbank markets: penalties in dollars and euros are independent because dollar surpluses cannot be used to patch Euro deficits and vice versa. This assumption can be relaxed to some extent but some form of segmentation of asset markets is necessary to obtain liquidity premia. See the discussion below.

We model the interbank market an over-the-counter (OTC) market, which is in line with institutional features of this market (see Ashcraft and Duffie, 2007; Afonso and Lagos, 2015). Modeling the interbank market using search and matching is also natural considering that the interbank market is a credit market in which banks on different sides of the market (surplus and deficit) must find a counterpart they trust.

As a result of the search frictions, only a fraction of the surplus (deficit) will be lent (borrowed) in the interbank market. We assume, in particular, that each bank gives an order to a continuum of traders to either lend or borrow, as in Atkeson, Eisfeldt, and Weill (2015). A bank with surplus s is able to lend a fraction Ψ^+ to other banks. The remainder fraction is kept in reserves. Conversely, a bank that has a deficit is able to borrow a fraction Ψ^- from other banks, and the remainder deficit is borrowed at a penalty rate i^w. The penalty rate can be thought of as the discount window rate or as an overdraft-rate charged by correspondent banks that have access to the Fed's discount window.

The fractions $\Psi^+$and Ψ^- depend on the abundance of reserve deficits relative to surpluses. Assuming a constant returns to scale matching function, the probabilities depend entirely on market tightness, defined as

$$\theta_t \equiv S_t^- / S_t^+ \tag{6}$$

where $S_t^+ \equiv \int_0^1 \max\left\{s_t^j, 0\right\} dj$ and $S_t^- \equiv -\int_0^1 \min\left\{s_t^j, 0\right\} dj$ denote the aggregate surplus and deficit, respectively. Notice that because $m \geq 0$ and $\mathbb{E}(\omega) = 0,$ we have that in equilibrium $\theta \leq 1$. That is, there is a relatively larger mass of banks in surplus than deficit.

The interbank market rate is the outcome of a bargaining problem between banks in deficit and surplus, as in Bianchi and Bigio (2020). There are N trading rounds, in which banks trade with each other. If banks are not able to match by the N trading rounds, they deposit the surplus of reserves at the central bank or borrow from the discount window. Throughout the trading, the terms of trade at which banks borrow/lend, i.e., the interbank market rate, depend on the probabilities of finding a match in a future period. We denote by $\bar{i}^f$ the average interbank market rate at which banks trade on average. Ultimately, we can define a real penalty function χ that captures the benefit of having a real surplus or deficit $\tilde{s}$ upon facing the withdrawal shock as follows:

$$\chi(\theta, \tilde{s}; X, X') = \begin{cases} \chi^+(\theta; X, X')\tilde{s} & \text{if } \tilde{s} \geq 0, \\ \chi^-(\theta; X, X')\tilde{s} & \text{if } \tilde{s} < 0 \end{cases} \tag{7}$$

where χ^+ and $\chi^-$are given by

$$\chi^-(\theta; X, X') = \Psi^-(\theta)[R^f(X, X') - R^m(X, X')] + (1 - \Psi^-(\theta))[R^w(X, X') - R^m(X, X')] \tag{8}$$

$$\chi^+(\theta; X, X') = \Psi^+(\theta)[R^f(X, X') - R^m(X, X')] \tag{9}$$

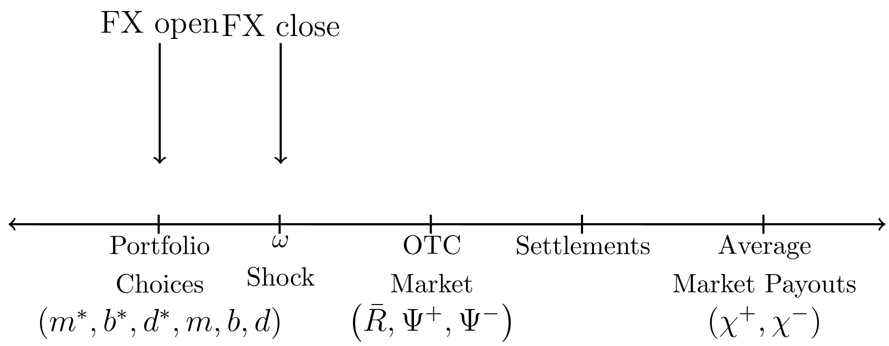

Figure 2: Timeline

In these expressions, $R^y(X, X') \equiv \frac{1+i^y(X)}{1+\pi(X,X')}$, denote the realized gross real rate of an asset/liability y and $\pi(X, X') \equiv \frac{P(X')}{P(X)} - 1$ denotes the inflation rate when the initial state is X and the next period state is X'. (When it does not lead to confusion, we streamline the argument (X, X') in these expressions.) We will also denote by $\bar{R}^y \equiv E_X \frac{1+i^y(X)}{1+\pi(X,X')}$ as the expected real rate—recall that the nominal rate is pre-determined but the ex-post real return depends on the inflation rate.

Equation (8) reflects that a bank that borrows from the the interbank market or from the discount window, obtains the interest on reserves—hence the cost of being in deficit is given by $R^f - R^m$ in the former and $R^w - R^m$ in the latter. By the same token, (9) reflects that the benefit from lending in the interbank market in case of surplus is $R^f - R^m$.

Figure 2 present a schematic version of the complete timeline within each period. We next turn to describe the bank optimization problem.

Banks' Problem. The objective of the bank is to choose dividends and portfolios to maximize 2 subject to the budget constraint. Critically, when choosing the portfolio, banks anticipate how withdrawal shocks may lead to a surplus or deficit of reserves and associated costs and benefits. We express the bank's optimization problem in terms of real portfolio holdings $\{\tilde{b}, \tilde{m}^*, \tilde{d}^*, \tilde{d}, \tilde{m}\}$and real returns. For example, we define for example $\tilde{m}_t \equiv m_t/P_{t-1}$. We can show that the problem can be expressed recursively as follows:

Problem 1. The recursive problem of a bank is

$$v(n, X) = \max_{\{Div, \tilde{b}, \tilde{m}^*, \tilde{d}^*, \tilde{d}, \tilde{m}\}} Div + \beta \mathbb{E}\left[v(n', X') \mid X\right] \tag{10}$$

subject to

the budget constraint:

$$Div + \tilde{b} + \tilde{m}^* + \tilde{m} = n + \tilde{d} + \tilde{d}^* \tag{11}$$

and the evolution of bank networth:

$$n' = \underbrace{R^b(X,X')\tilde{b} + R^m(X,X')\tilde{m} + R^{m,*}(X,X')\tilde{m}^* - R^d(X,X')\tilde{d} - R^{*,d}(X,X')\tilde{d}^*}_{\text{Portfolio Returns}}$$
$$+ \underbrace{\mathbb{E}_{\omega^*}\chi^*(\theta^*(X), \tilde{m}^* + \omega^*\tilde{d}^*; X, X') + \mathbb{E}_{\omega}\chi(\theta(X), \tilde{m} + \omega\ \tilde{d}; X, X')}_{\text{Settlement Costs}} \quad (12)$$

The variable n represents the banks' net worth at the beginning of the period.[12] Because of the linearity of banks' payoffs, we can show that the value function is linear in net worth and that dividends and that the level of the portfolio is indeterminate at the individual level. On the other hand, portfolio weights are determined. Lemma (1) summarizes these results.

Lemma 1. *The solution to* (10) *is* $v(n, X) = n$ *and the law of motion of bank net-worth satisfies:*

$$n' = \frac{1}{\beta}(n - Div) + \Pi^*(X)$$

where $\Pi^\star(X)$ *are the expected intermediation profits: given expected real returns and market tightness* $\{\theta, \theta^*\}$, $\Pi^\star(X)$ *solves*

$$\Pi^\star(X) = \max_{\{\tilde{m},\tilde{d}^*,\tilde{d},\tilde{m}\}} \left(R^b(X,X') - R^{*,d}(X,X')\right)\tilde{d}^* - \left(R^b(X) - R^{*,m}(X)\right)\tilde{m}^*$$
$$+ \left(R^b(X,X') - R^d(X,X')\right)\tilde{d} - \left(R^b(X,X') - R^m(X,X')\right)\tilde{m}$$
$$+ \mathbb{E}_{\omega^*}\chi^*(\theta^*(X,X'), \tilde{m}^* + \omega^*\tilde{d}^*; X, X') + \mathbb{E}_{\omega}\chi(\theta(X,X'), \tilde{m} + \omega\ \tilde{d}; X, X'). \quad (13)$$

In equilibrium $\Pi^*(X) = 0$ *and dividends are indeterminate at the individual bank level. Furthermore,* $R^b(X) = 1/\beta$.

Proof. In the appendix □

Central to this optimization problem are the liquidity costs, as captured by χ and χ^*. Deposits in either currency have direct interest costs given by the real returns on deposits, but also affect indirectly the banks' settlement needs. Reserves in each currency yield direct real returns, correspondingly, but have the additional indirect benefit of leading to higher average positions in the interbank market.

It is important to note that the bank problem is homogeneous of degree one: As a result, the scale of dollar and euro deposits of each bank is indeterminate for an individual bank. The liquidity ratio and the leverage ratio, however, are not. In effect, the kink in the liquidity cost

[12] To obtain (12), we use the definition of χ as expressed in (7)-(9) and the real returns. Implicit in the law of motion for net worth is the convention that a bank that faces a withdrawal covers the associated interest payments net of the interest on reserves.

function creates concavity in the bank objective, generating strictly interior solutions for the ratios.[13] Thus, liquidity risk generates an endogenous bank risk-averse behavior, which will be critical for the determination of the exchange rate, as will become clear below.

3.2 Non-Financial Sector

This section presents the description of the non-financial block: This block is composed of a representative household, one in each country. Households supply labor and save in deposits in both currencies. Firms are multinationals which use labor for production and are subject to working capital constraints, giving rise to a demand for loans. This block delivers an endogenous demand schedule for loans, and deposit demands in both currencies.

To keep the model as simple as possible, we purposely make assumptions so that the decisions for loan demand and deposit supplies are static, in the sense that they do not depend explicitly on future variables. In particular, we will be able to treat loan demand and deposit supply as exogenous schedules with only two parameters: an intercept that controls the scale, and an elasticity that controls how much they respond to changes in interest rates. As we show in the appendix, we obtain the following schedules

$$\bar{R}^{b}_{t+1} = \Theta^{b}\left(B_t\right)^{\epsilon},\ \epsilon > 0,\ \Theta^{b}_t > 0, \tag{14}$$

$$\bar{R}^{*,d}_{t+1} = \Theta^{*,d}\left(D^{*,s}_t\right)^{-\varsigma^*},\ \varsigma > 0,\ \Theta^{*,d} > 0, \tag{15}$$

$$\bar{R}^{d}_{t+1} = \Theta^{d}\left(D^{s}_t\right)^{-\varsigma},\ \varsigma^* > 0,\ \Theta^{d} > 0\cdot \tag{16}$$

where ϵ is the semi-elasticity of credit demand and ς, ς^* are the semi-elasticity of the deposit supply with respect to the real return in either currency. These parameters are linked to the production structure and preference parameters in the micro-foundations developed in the appendix.

3.3 Government/Central Bank

Both central banks choose the rates for reserves i^m_t and discount window i^w_t. Central banks in each country also set the supply of reserves $\left\{M_{t+1}, M^*_{t+1}\right\}$, which for now, we assume are constant for simplicity. Absent any aggregate shocks, this would imply that the price level would be constant over time in each country.

To balance the payments on reserves and the revenues from discount window loans, we assume that central banks use lump sum taxes/transfers rebated to households from the same country. Because households have linear utility in the tradable consumption good, these lump sum taxes

[13] This behavior is analogue to the behavior of productive firms with Cobb-Douglas technologies: firms earn zero-profits, their production scale is indeterminate, but the ratio of production inputs is determined in equilibrium as a function of relative factor prices.

only affect the level of consumption, but have no other implications. Using W_t to denote the discount window loans, we have the following budget constraint.

$$M_t + T_t + W_{t+1} = M_{t-1}(1 + i_t^m) + W_t(1 + i_t^w).$$

3.4 Competitive equilibrium

We study recursive competitive equilibria where all variables are indexed by the vector of aggregate shocks, X. We consider shocks to the nominal interest rates on reserves, deposit supply, matching efficiency and the volatility of withdrawals. Without loss of generality, we restrict to a symmetric equilibrium, in which all banks choose the same portfolios.

Definition 1. Given central bank policies for both countries $\{M(X), i^m(X), i^w(X), W(X)\}$, $\{M^*(X), i^{m,*}(X), i^{w,*}(X), W^*(X)\}$ a recursive competitive equilibrium is a set of price levels $\{P(X), P^*(X)\}$, exchange rates $e(X)$, real returns for loans, $R^b(X)$, nominal returns for deposits $\{i^d(X), i^{d,*}(X)\}$, an interbank market rate $\bar{i}^f(X)$, market tightness $\theta(X)$, bank portfolios $\{d(X), d^*(X), m(X), m^*(X), \tilde{b}(X)\}$, interbank and discount window loans $\{f(X), f^*(X), w(X), w^*(X)\}$ and aggregate quantities of loans $\{\tilde{B}(X)\}$ and deposits $\{D(X), D^*(X)\}$ such that:

(i) Households are on their deposit supply and firms are on their loan demand. That is, equations (14)-(16) are satisfied given real returns and quantities $\{\tilde{B}(X), D(X), D^*(X)\}$.

(ii) Banks choose portfolios $\{\tilde{d}(X), \tilde{d}^*(X), \tilde{m}(X), \tilde{m}^*(X), \tilde{b}(X)\}$ to maximize expected profits, as stated in (13)

(iii) The law of one price holds

$$P(X) = P^*(X)e(X). \tag{17}$$

(iii) All market clear. For deposits, we have

$$\tilde{d}(X) \quad = D^{s,}(X), \tag{18}$$

$$\tilde{d}^*(X) \quad = D^{s,*}(X). \tag{19}$$

For reserves:

$$\tilde{m}\ (X)\ P(X)\ = M(X), \tag{20}$$

$$\tilde{m}^*(X)P^*(X) = M^*(X). \tag{21}$$

For loans:

$$\tilde{b}(X) = B(X). \tag{22}$$

For interbank loans

$$\Psi^+(X)S^+ = \Psi^-(X)S^-. \tag{23}$$

(vi) Market tightness $\theta(X)$ is consistent with the portfolios and the distribution of withdrawals while the matching probabilities $\{\Psi^+(X), \Psi^-(X)\}$ and the fed funds rate $\bar{i}^f(X)$ are consistent with market tightness θ.dddf

Combining (20) and (21) and using the law of of one price (17), we arrive to a condition for the determination of the nominal exchange rate:

$$e(X) = \frac{P(X)}{P^*(X)} = \frac{\frac{M(X)}{\tilde{m}\ (X)}}{\frac{M^*(X)}{\tilde{m}^*(X)}} \tag{24}$$

Condition (24) is a Lucas-style exchange rate determination equation. Given a real demand for reserves in euro and dollars that emerge from the bank portfolio problem (13), the dollar will be stronger (i.e., higher e) the larger is the nominal supply of euro reserves relative to dollar reserves. Similarly, for given nominal supplies of euro and dollar reserves, the dollar will be stronger the larger is the demand for real dollar reserves.

The novelty here relative to the canonical model is that liquidity factors play a role in the real demand for currencies, and hence affect the value of the exchange rate. We turn next to analyze this mechanism.

3.5 Liquidity Premia and Exchange Rates

To understand how liquidity affects exchange rates, it is useful to inspect the bank portfolio problem (13). Using (7), we can write the expected profits of a bank with portfolio (m, m^*, d, d^*) as follows:

$$\begin{aligned}
\Pi^*(X) = {} & \left(\bar{R}^b - \bar{R}^{*,d}\right)\tilde{d}^* - \left(\bar{R}^b - R^{*,m}\right)\tilde{m}^* + \\
& \left(\bar{R}^b - \bar{R}^d\right)\tilde{d} - \left(\bar{R}^b - \bar{R}^m\right)\tilde{m} + \\
& \chi^{*,-}(\theta)\int_{-1}^{-m^*/d^*}(\tilde{m}^* + \omega^*\tilde{d}^*)d\Phi^*(\omega^*) + \chi^{*,+}(\theta)\int_{-m^*/d^*}^{\infty}(\tilde{m}^* + \omega^*\tilde{d}^*)d\Phi^*(\omega^*) + \\
& \chi^-(\theta)\int_{-1}^{-m/d}(\tilde{m} + \omega\tilde{d})d\Phi(\omega) + \chi^+(\theta)\int_{-m/d}^{\infty}(\tilde{m} + \omega\tilde{d})d\Phi(\omega).
\end{aligned} \tag{25}$$

The first two lines in (25) are the excess returns on loans relative to deposits and reserves. The last two lines represent the expected liquidity costs in the two currencies.

The first-order condition with respect to m^* is

$$\bar{R}^b - \bar{R}^{m,*} = (1 - \Phi^*(-m^*/d^*))\chi^{+,*}(\theta^*) + \Phi(-m^*/d^*)\chi^{-,*}(\theta^*). \tag{26}$$

At the optimum, banks equate the expected marginal return of investing in loans, which is equal to $\bar{R}^b$, with the marginal return on investing in reserves. The latter is given by the interest on reserves R^m plus a stochastic liquidity value. If the bank ends up in surplus, which occurs with probability $1-\Phi(-m^*/d^*)$, the marginal value is given by $\chi^{+,*}$ and if the bank ends up in deficit which occurs with probability $\Phi(-m^*/d^*)$, the marginal value is given by $\chi^{-,*}$. A useful observation is that for a given $\chi^{+,*}, \chi^{-,*}$ a higher ratio of reserves to deposits is associated with a smaller $R^b - R^m$ premium. We label the difference the excess bond premium, $\mathcal{EBP} \equiv \bar{R}^b - \bar{R}^m$.

We have an analogous condition for m:

$$\bar{R}^b = \bar{R}^m + \ [(1 - \Phi(-m/d))\chi^{+}(\theta) + \Phi(-m/d)\chi^{-}(\theta)]. \tag{27}$$

Combining (26) and (27), we obtain a condition that relates the difference in the real return of reserves to the difference in excess bond premium:

$$\begin{aligned} \bar{R}^m - \bar{R}^{m*} \ &= \ [(1 - \Phi^*(-m^*/d^*))\chi^{+,*}(\theta^*) + \Phi(-m^*/d^*)\chi^{-,*}(\theta^*)] \\ &\quad -[(1 - \Phi(-m/d))\chi^{+}(\theta) + \Phi(-m/d)\chi^{-}(\theta)]. \end{aligned} \tag{28}$$

We label this difference, the dollar liquidity premium (DLP), $\mathcal{DLP} \equiv \bar{R}^m - \bar{R}^{*,m}$.

Using that $1 + \pi = (1 + \pi^*)e_2/e_1$ by the law of one price, we can express (28) as:

$$\mathbb{E}_t \left\{ \frac{1}{1+\pi_{t+1}} \left[1 + i_t^m - (1 + i_t^{m,*}) \cdot \frac{e_{t+1}}{e_t} \right] \right\} = \underbrace{\mathbb{E}_{\omega^*} \left[\chi_{m^*} \left(s^*; \theta^* \right) \right] - \mathbb{E}_{\omega} \left[\chi_{m} \left(s; \theta \right) \right]}_{\text{dollar liquidity premium } (\mathcal{DLP})}. \tag{29}$$

We have then arrived to a *liquidity premium adjusted interest parity condition*. Absent any liquidity premia, (29) would reduce to a canonical uncovered interest parity (UIP) condition that simple equates, to a first order, the difference in nominal returns to the expected exchange rate depreciation. However, whenever the marginal liquidity value of a dollar reserve is larger than the marginal liquidity value of a euro reserves (i.e.,. when the dollar liquidity premium (DLP) is positive), and the nominal rates are equal, this implies that the dollar must be expected to depreciate over time. In effect, the dollar reserve delivers a lower expected real return compensating for the higher liquidity value. Notice also that because banks are owned by risk neutral shareholders, there is no risk premium in the model. The premia operates entirely through liquidity.[14]

[14] Another consequence of the absence of risk premia is that the UIP deviation coincides with the CIP deviations, assuming a Walrasian market for the forward market in the lending stage. In the data, these objects are, of course, different. Our focus is on understanding the former.

Theoretical properties. We now provide a theoretical characterization of how the exchange rate and the liquidity premia vary with various shocks in the model. We make two simplifying assumptions to simplify the characterization. First, we assume that the supply of deposits are perfectly inelastic. That is, the supply of dollar and euro deposits are fixed in real terms and the interest rates on dollar and euro deposits adjust to clear the two markets. Second, we assume that withdrawal shocks satisfy a bi-nominal distribution: :

$$\omega^* = \begin{cases} \delta^* & \text{with probability} 1/2 \\ -\delta^* & \text{with probability} 1/2, \end{cases}$$

With this distribution, we can think about banks having a surplus when they face a positive withdrawal shock and a deficit when they face a negative shock. How large is the surplus/deficit will depend on the portfolios chosen in the lending stage.

We focus on shocks that affect the dollar interbank market. Note that the same shocks to the euro interbank market will carry the opposite effect on the exchange rate and the $\mathcal{DLP}$. Throughout, a useful object for the characterization is the liquidity ratio, which defined in terms of aggregates is given by $\mu \equiv \frac{M/P}{D}$.

We first consider the a positive supply to the amount of dollar deposits.

Proposition 1 (Deposit scale effects)**.** *Consider a shock that increases the real supply for dollar deposits, D^*. We have the following*

1) If the shock is temporary (i.i.d.), then, there is an appreciation of the dollar a a reduction in the dollar liquidity ratio and increase in $\mathcal{DLP}$. In particular,

$$\frac{d\log e}{d\log D^*} = -\frac{d\log P^*}{d\log D^*} = \frac{-\frac{1}{2}\left(\chi^+_{\theta^*} + \chi^-_{\theta^*}\right)\theta_{\mu^*}\mu^*}{R^b - \frac{1}{2}\left(\chi^+_{\theta^*} + \chi^-_{\theta^*}\right)\theta_{\mu^*}\mu^*} \in [0,1),$$

$$\frac{d\log \mu^*}{d\log D^*} = -\left(\frac{R^b}{R^b - \frac{1}{2}\left(\chi^+_{\theta^*} + \chi^-_{\theta^*}\right)\theta_{\mu^*}\mu^*}\right) \in [-1,0)$$

and

$$d\left(\mathcal{DLP}\right) = R^{*,m} d\log e.$$

2) If the shock is permanent (random walk). Then,

$$\frac{d\log e^*}{d\log D^*} = -\frac{d\log P^*}{d\log D^*} = 1,$$

and

$$\frac{d\log\mu^*}{d\log D^*} = d\left(\mathcal{DLP}\right) = 0.$$

Proof. In the appendix. □

Proposition 1 establishes that a higher supply of dollar deposits appreciate the dollar. The logic is simple: a higher amount of real dollar deposits increases the demand for real dollar reserves. Given a fixed nominal supply, we must have an appreciation of the dollar.

There is an important distinction, however, depending on whether the shock is temporary or permanent. When the shock is temporary, the exchange rate is expected to revert to the initial value in the following period. Given nominal rates, this reduces the expected real return of holding dollar reserves. As a result, the demand for dollar liquidity by banks fall. In equilibrium, the increase in the scarcity of dollar reserves leads to an increase in the $\mathcal{DLP}$. Overall, we then have that in response to an increase in the supply of dollar deposits by households, the dollar appreciates, the dollar liquidity ratio falls and the $\mathcal{DLP}$ increases.

When the shock is permanent, the effect on the exchange rate is expected to be permanent. Absent any expected depreciation effects, the dollar liquidity ratio remains constant. As a result, there are no permanent effects on the $\mathcal{DLP}$. Important for this result is the assumption of constant returns to scale in matching technology. As banks scale up proportionally dollar deposits and reserves, given the same real returns on dollar and euro reserves, the original liquidity ratio remains consistent with the new equilibrium.[15]

Next, we consider the effects of a rise in the volatility of dollar deposits.

Proposition 2 (Volatility effects)**.** *Consider a shock that increases the size of dollar payment shocks, δ^*. We have the following*

1) If the shock is temporary (i.i.d.), then, the shock appreciates the dollar, lowers the dollar liquidity ratio:

$$\frac{d\log e_t}{d\log\delta_t^*} = \frac{d\log\mu_t^*}{d\log\delta_t^*} = -\frac{d\log P_t^*}{d\log\delta_t^*} = \frac{\frac{1}{2}\left[\chi_{\theta^*}^{+} + \chi_{\theta^*}^{-}\right]\theta_{\delta^*}^{*}\delta^*}{R^b - \frac{1}{2}\left[\chi_{\theta^*}^{+} + \chi_{\theta^*}^{-}\right]\theta_{\mu^*}^{*}\mu^*} \in [0, \frac{(1-\theta)}{(1+\theta)}\frac{\delta}{\mu})$$

and raises the $\mathcal{DLP}$

$$d\left(\mathcal{DLP}\right) = R^{*,m} d\log e.$$

2) If the shock is permanent (random walk). Then,

$$\frac{d\log e_t}{d\log\delta_t^*} = \frac{d\log\mu_t^*}{d\log\delta_t^*} = -\frac{d\log P_t^*}{d\log\delta_t^*} = \frac{(1-\theta)}{(1+\theta)}\frac{\delta}{\mu_t}$$

[15] Given our results about fully temporary and permanent shocks, our conjecture is that in response to persistent shocks, we would have still have an appreciation of the dollar and an increase in the $\mathcal{DLP}$.

and $d(\mathcal{DLP}) = 0$.

Proof. In the appendix. □

Proposition 2 presents a central result in the paper. In response to a rise in the volatility of dollar deposits, the dollar appreciates, and there is an increase in the dollar liquidity ratio and the $\mathcal{DLP}$. Intuitively, when there is larger dollar volatility, banks demand larger real holdings of dollars. With the nominal supplies given, this must lead to an appreciation of the dollar. Again, there is a relevant distinction on whether the shock is temporary or permanent. When the shock is temporary, the expected depreciation of the dollar reduces the expected real return of holding dollar reserves. Given the nominal rates, this implies that the $\mathcal{DLP}$ must be larger in equilibrium for (29) to hold. When the shock is permanent, the volatility shock appreciates the dollar without any real effects on the liquidity ratio or the $\mathcal{DLP}$.

The result in Propositions 1 and 2 highlight that episodes of large supply of dollar deposits and high dollar volatility and go hand in hand with appreciations of the dollar. There is, however, an important difference in the prediction of these two shocks. While the supply of deposit shock leads to a reduction in the liquidity ratio, the volatility shock leads to an increase in the liquidity ratio. Recall from the empirical relationship documented in Section 2, there is a positive relationship between the dollar liquidity ratio and the appreciation of the dollar. These results suggest that to reconcile the empirical observations, volatility shocks ought to play an important role in driving the exchange rate. In section 4, we show that a calibrated version of the model with both types of shocks is indeed consistent with the empirical results.

3.6 Monetary Policy and Exchange Rates

We now study how monetary policy affects the exchange rate. We start by considering the effect of a change in the nominal policy rates.

Proposition 3 (Effects of Changes in Policy Rates)**.** *Consider a temporary change in the dollar interest rate on reserves, $i^{*,m}$ while holding fixed the spread $(i^{*,w} - i^{*,m})$. We have that the dollar appreciates, the liquidity ratio increases and the DLP decreases. In particular:*

If the shock is temporary (iid), the shock appreciates the dollar and raises the liquidity ratio:

$$\frac{d\log e}{d\log(1+i^{*,m})} = \frac{d\log\mu}{d\log(1+i^{*,m})} = -\frac{d\log P^{*}}{d\log(1+i^{*,m})} = \frac{R^{*,m}}{\left(R^{b} - \frac{1}{2}\left[\chi_{\theta}^{+} + \chi_{\theta}^{-}\right]\theta_{\mu}\mu\right)} \in (0,1].$$

Furthermore $d(\mathcal{DLP}) = -R^{m}\left(d\log(1+i^{m}) - d\log e\right) < 0$.

Part 2. If the shock is permanent (random walk), then

$$\frac{d\log e}{d\log(1+i^{*,m})}=\frac{d\log\mu^{*}}{d\log(1+i^{*,m})}=-\frac{d\log P^{*}}{d\log(1+i^{*,m})}=-\frac{1}{\left[\chi_{\theta^{*}}^{*,+}+\chi_{\theta^{*}}^{*,-}\right]\theta_{\mu^{*}}^{*}\mu^{*}}>0.$$

Furthermore $d\mathcal{DLP}=-R^{m}d\log(1+i^{m})$.

Proof. In the appendix. □

Proposition 3 establishes that in response to an increase in the US nominal rate, the dollar appreciates, the liquidity ratio increases and the liquidity premium falls. The appreciation of the dollar follows a standard effect: a higher nominal rate leads to a larger demand for dollars, which in equilibrium requires a dollar appreciation. Absent liquidity premia, the difference in nominal returns across currencies would be equally offset by the expected depreciation of the dollar, following today's appreciation. Given the larger abundance of dollar reserves, however, there is a decrease in the marginal value of holding dollars and a reduction in the dollar liquidity premium takes place. Overall, the appreciation of the dollar is lower than the one that would prevail absent liquidity premia. This result is important because it breaks the tight connection between interest rate differentials and expected depreciation, at the heart of the forward premium puzzle, also called the Fama puzzle.

The next policy we study are foreign exchange interventions. We consider purchases of dollar reserves by the European Central Bank, which are financed by expanding the amount of dollar reserves. For simplicity, we assume that this operation is reverted in the following period. Any operating losses or profits are financed with transfers in the following period.

Proposition 4 (FX intervention.)**.** *Let M^{**} be the holdings of dollars reserves by the central bank of Europe. Let*

$$\mathcal{A}=\frac{M^{**}}{M^{*}+M^{**}},$$

be the ratio of the holdings of foreign reserves to total dollar reserves and let

$$\mathcal{M}_t\equiv e_t\frac{M_t^{**}}{M_t}$$

the ratio of foreign reserves to Euro reserves in Europe. Then, consider an increase in European holding of dollar reserves financed with euro reserves. We have the following:

$$\frac{d\log e}{d\log M^{**}}=\frac{d\log e}{d\log M^{**}}=(1-\mathcal{M}\cdot\mathcal{A}\cdot\Gamma^{*})\cdot\Gamma-\mathcal{A}\Gamma^{*}\geq 0,$$

where

$$\Gamma^* \equiv \frac{\frac{1}{2}\frac{d[\chi^{*,+}+\chi^{*,-}]}{d\theta^*}\frac{d\theta^*}{d\mu^*}\mu^*}{R^b - \frac{1}{2}\frac{\partial[\chi^{*,+}+\chi^{**,-}]}{\partial\theta^*}\frac{\partial\theta^*}{\partial\mu^*}\mu^*} < 0, \text{ and } \Gamma \equiv \frac{-\frac{1}{2}\frac{d[\chi^{+}+\chi^{-}]}{d\theta}\frac{d\theta}{d\mu}\mu}{R^b - (1-\mathcal{M})\,\frac{1}{2}\frac{\partial[\chi^{+}+\chi^{-}]}{\partial\theta}\frac{\partial\theta}{\partial\mu}\mu} > 0.$$

measure the sensitivity of dollar and euro prices respectively, to a change in the liquidity ratio. The change in the liquidity ratio is given by:

$$\frac{d\log\mu}{d\log M^{**}} = \frac{R^b\cdot(1-\mathcal{M}\cdot\mathcal{A}\cdot\Gamma^*)}{R^b - (1-\mathcal{M})\,\frac{1}{2}\frac{\partial[\chi^{+}+\chi^{-}]}{\partial\theta}\frac{\partial\theta}{\partial\mu}\mu} \geq 0.$$

And the change in the liquidity premia is given by:

$$\frac{d\mathcal{DLP}}{d\log M^{**}} = R^m\Gamma\left(1-\mathcal{M}\cdot\mathcal{A}\cdot\Gamma^*\right) - R^{*,m}\mathcal{A}\Gamma^* \geq 0 \text{ and } \frac{d\mathcal{EBP}}{d\log M^{**}} = -R^{*,m}\mathcal{A}\Gamma^* > 0.$$

Proof. In the appendix □

Proposition 4 establishes that a purchase of dollar reserves by the European Central Bank appreciates the dollar. Moreover, the purchases of dollar reserves increases the euro liquidity ratio and decreases the dollar liquidity ratio. As a result, we see an increase in the dollar liquidity premium.

Foreign exchange interventions matter in our framework, a result that contrasts with the classic irrelevance result of Backus and Kehoe 1989. An active literature has recently studied how in the presence of limits to international arbitrage, these interventions can have effects on real variables and nominal exchange rates (see e.g. Gabaix and Maggiori 2015; Amador et al. 2019; Fanelli and Straub 2020). However, our model provides a different channel by which FX intervention affects the exchange rate. In fact, banks do not face any leverage constraints and there is perfect financial arbitrage. The channel by which FX intervention affects allocations operates by altering the relative abundance of liquid assets in the market. When the domestic central bank purchases foreign reserves, it increases the relative scarcity of these assets. Given the original nominal and returns, an excess demand for dollars emerges. In equilibrium, the dollar appreciates and restores market clearing.

3.7 UIP vs. CIP.

Motivated by findings in Du et al. (2018), a large literature analyzes both theoretically and empirically the sources of deviations from covered-interest parity (CIP). A novel consideration in our model is that our model features assets with different liquidity properties, leading to potentially a different CIP deviations for different assets. Namely, given a forward exchange rates, the extent that nominal-rate differentials vary across assets imply unequivocally different CIP deviations per asset. So far, we have not allowed for a forward market. We now do so by considering a perfectly competitive market for forwards in the lending stage.

A forward traded at time t promises to exchange one dollar for a given amount of euros in the lending stage in the following period. The first-order condition with respect to the quantity of forwards purchased yields

$$0 = \beta \mathbb{E}_t \left[\frac{1}{P^*_{t+1}} (f_{t,t+1} - e_{t+1}) \right], \tag{30}$$

Given risk neutrality, a constant foreign price would imply that the forward equals the expected exchange rate.

The definition of CIP implies that

$$\mathbb{E}_t \left\{ \frac{1}{1+\pi_{t+1}} \left[1 + i^m_t - (1 + i^{m,*}_t) \cdot \frac{f_{t+1}}{e_t} \right] \right\} = CIP \tag{31}$$

Comparing (29) and (31) delivers that the $\mathcal{DLP}$ is captured precisely by the CIP deviation.

One interesting observation made by Du et al. (2018) is that low interest rate countries have had positive CIP gap since the global financial crisis, (i.e., low interest rate currencies deliver a risk-free excess return relative to the high interest rate currencies). This pattern contrasts sharply with the carry trade phenomenon by which low interest rate countries have negative UIP deviation (i.e. low interest rate countries deliver a low expected return relative to high interest rate currencies). As we showed earlier, an increase in the nominal interest rate reduces the liquidity premium. Our model is thus consistent with this observation.[16]

4 Simulation Results

Calibration. We set the model period to be one month. We consider a calibration for the two countries that is symmetric, except that the US has a time-varying volatility of dollar deposits and the policy rate in the US shifts; there is a larger scale of dollar deposits but this aspect of the model is immaterial. The volatility in Europe is fixed and so is the policy rate.

For the distrubutions of ω shocks, we assume that these are distributed as two sided exponentials (formally known as Laplace distributions) with shocks centered at zero. Each distribution is therefore indexed by a single time-varying dispersion parameter σ and σ^*_t. All shocks are assumed to follow a an AR(1) process which is approximated by a Markov process numerically. A first

[16] An alternative explanation provided by Amador et al. (2019) is related to limits to international arbitrage and central bank policies of resisting an appreciation at the zero lower bound. The policy of keeping an exchange rate temporarily depreciated together with a lower bound on the nominal rate, created excess returns on domestic currency assets. The excess returns, in turn, generated capital inflows, which had to be offset with central bank purchases of foreign reserves.

subset of parameters is calibrated externally. These parameters are the nominal rates, which we take directly from the data, and the elasticities, which we take from Bianchi and Bigio (2020). In addition, the relative supply of reserves and the intercept in loan demand that can be normalized.

A second set of parameters is calibrated to match second moments. The matching efficiency is calibrated to match an average excess bond premim on loans relative to dollar reserves of 100bps. Moreover, the standard deviation and the autcorrelation of the withdrawal shock is set to match the standard deviation and the autcorrelation of the for the exchange rate. The average dollar withdrawal risk is chosen to match a dollar liquidity premium of 20bps.

Table 6B: Parametrization

Parameter	**Description**	**Target**
Fixed Parameters		
$i_t^m = 2.14\%$	EU Safe Asset Rate	data
M^*/M	Relative Supplies of Reserves	normalized to match average e
$\Theta^b = 100$	Global loan demand scale	normalization
$\epsilon = -35$	Loan Elasticity	Bianchi and Bigio (2020)
$\Theta^{d,*} = 40$	US Deposit Demand Scale	Liquidity ratio of 20%
$\varsigma^* = 35$	US Deposit Demand Elasticity	Bianchi and Bigio (2020)
$\Theta^d = 40$	EU Deposit Demand Scale	symmetry
$\varsigma = 35$	US Deposit Demand Elasticity	symmetry
$\sigma = 4\%$	EU withdrawal risk	$R^b - R^d = 2\%$
$\lambda^* = 3.1$	US interbank market matching efficiency	$\mathcal{EBP} = R^b - R^{*,m} = 1\%$
$\lambda = 3.1$	EU interbank market matching efficiency	symmetric value of λ^*
Process for US withdrawal volatility (AR(1) process)		
$\mathbb{E}(\sigma_t^*) = 4\%$	average US withdrawal risk	empirical average $\mathcal{LP}$
$std(\sigma_t^*) = 0.12\%$	standard deviation	empirical std of $log(e)$
$\rho(\sigma_t^*) = 0.98$	mean reversion coefficient	empirical autocorrelation of $log(e)$
Process for US policy rate $i^{m,*}$ (AR(1) process)		
$\mathbb{E}(i_t^{*,m}) = 1.95\%$	average annual US policy rate	data
$std(i_t^{*,m}) = 2.1652\%$	std annual US policy rate	data
$\rho(i_t^{*,m})$=0.99	autocorrelation annual US policy rate	data

The model and data moments are reported in Table 6B. The model is successful at matching the targeted moments and, in addition, delivers untargeted moments that are close to the counterparts in the data.

Table 6B: Model and Data Moments

Statistic	Description	Data/Target	Model
Targets			
$std(\log e)$	Std. Dev. of log exchange rate	0.1538	0.154
$\rho\,(\log e)$	Autocorrelation of log exchange rate	0.9819	0.9922
$\mathbb{E}\,(\mathcal{LP})$	Average bond premium	20bps	19.8bps
$\mathbb{E}\,(\mathcal{EBP})$	Average bond premium	100bps	100.1bps
Non-Targeted			
$std(\log \mu^{*})$	Std. Dev. of dollar liquidity ratio	0.422	0.0656
$\rho\,(\log \mu)$	Autocorrelation of dollar liquidity ratio	0.9961	0.9924
$std(\pi_{eu} - \pi_{us})$	Std. Dev. of inflation differential	1.29	1.84
$\rho\,(\pi_{eu} - \pi_{us})$	Autocorrelation of inflation differential	0.925	0.98

4.1 Volatility, Liquidity Premia, and Exchange Rates

In this section, we present results of a version of the model with a Markov process for the volatility of dollar withdrawals. Figure 3 shows all endogenous variables as a function of the volatility of the withdrawal of dollar deposits. In line with the results of Proposition 2, we can see that a higher volatility appreciates the dollar and generates a positive liquidity premium. In addition, we can see a rise in the differential rate on deposits. That is, the rate on euro deposits increases relative to the dollar rate as the rise in volatility makes euro deposits more attractive. Furthermore, there is a rise in the loan rate because higher volatility increases, in effect, the liquidity frictions and reduces the demand for loans. Finally, we also see an increase in the dollar liquidity ratio concomitantly with a reduction in the euro liquidity ratio. The latter occurs because a higher lending rate makes euro reserves relatively less attractive (in the absence of any shocks to the euro market).

Figure 4 shows the simulations of the economy for a given path of volatility shocks. The red line denotes the realization of the volatility shock. The overall message, in line with the previous figure is that episodes of high volatility lead to appreciation of the dollar. Notice that there is mean reversion in the exchange rate and all other variables. Importantly, the simulations are consistent with the empirical analysis presented in Section 2. Indeed, we see a positive correlation between the strength of the dollar and the dollar liquidity ratio,

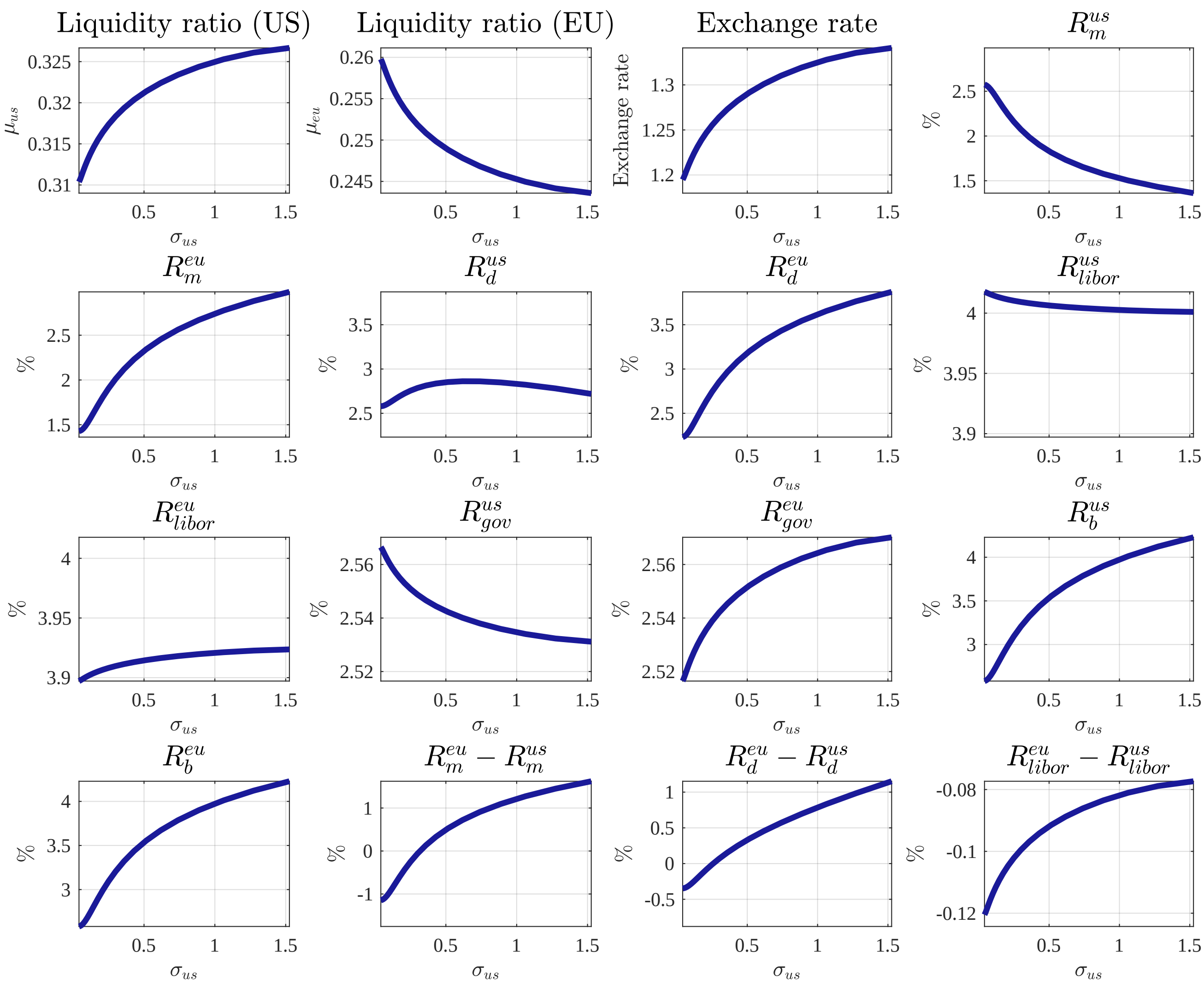

Figure 3: Equilibrium solution for a range of values of volatility

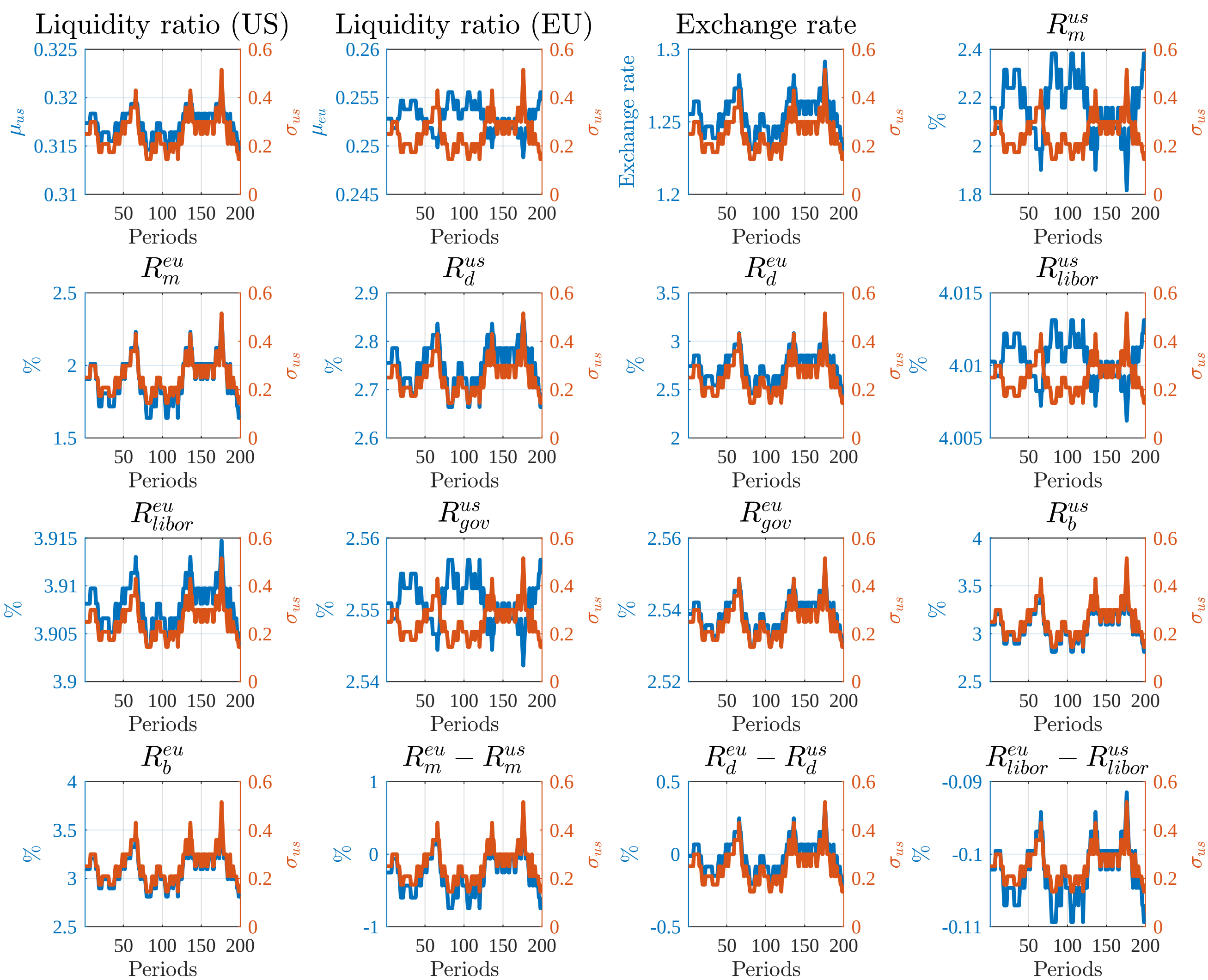

Figure 4: Simulation of the model. Red line is the volatility process

4.2 Regressions with Simulated Data

In this section, we simulate our model, study the second moments and run the same regressions as we did with simulated data. Table (6B) shows that the model simulations are consistent with the date time series. Namely, using a regression with the simulated data, we estimate a positive coefficient on the liquidity ratio, just like we found in the data. This result is in line with the comovement observed in the simulations in Figure 4.

Table 6B: Regression Coefficients with Simmulated Data

	σ^*-shocks only	$i^{*,m}-$shocks only	both shocks
$\Delta(\text{LiqRat}_t)$	2.2484***	1.0763***	1.9735***
	(0.0015)	(0.0440)	(0.0450)
(LiqRat_{t-1})	-0.0007	-0.0014	-0.0037
	(0.0004)	(0.0007)	(0.0015)
$\Delta(i_t^m - i_t^{*,m})$		-42.4640***	-14.5032***
		(1.5185)	(1.6027)
constant	-0.0	-0.015	-0.039
	0.01	0.008	0.0017
adj. R^2	0.999	0.9987	0.9953

t statistics in parentheses.

*** $p < 0.01$

5 Conclusion

We developed a theory of exchange rate determination as arising from the demand by financial institutions for liquid dollar assets. Periods of increased funding volatility generate an increase in the dollar liquidity premium and appreciates the dollar. The effect is empirically validated as we document that a higher liquidity ratio is associated with a stronger dollar.

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A Expressions for $\{\Psi^+, \Psi^-, \chi^+, \chi^-\}$

Here we reproduce formulas derived from Proposition 1 in Bianchi and Bigio (2017). In Bianchi and Bigio (2017), there is a market structure for interbank trades that delivers these functional forms. This proposition gives us the formulas for the liquidity yield function and the matching probabilities as functions of the tightness of the interbank market. The formulas are the following.

Given θ, the market tightness after the interbank-market trading session is over:

$$\bar{\theta} = \begin{cases} 1 + (\theta - 1)\exp(\lambda) & \text{if } \theta > 1 \\ 1 & \text{if } \theta = 1 \\ \left(1 + (\theta^{-1} - 1)\exp(\lambda)\right)^{-1} & \text{if } \theta < 1 \end{cases}.$$

Trading probabilities are given by

$$\Psi^+ = \begin{cases} 1 - e^{-\lambda} & \text{if } \theta \geq 1 \\ \theta\left(1 - e^{-\lambda}\right) & \text{if } \theta < 1 \end{cases}, \qquad \Psi^- = \begin{cases} \left(1 - e^{-\lambda}\right)\theta^{-1} & \text{if } \theta > 1 \\ 1 - e^{-\lambda} & \text{if } \theta \leq 1 \end{cases}. \tag{32}$$

The parameter λ captures the matching efficiency of the interbank market. A reduced-form bargaining parameter is obtained as:

$$\phi \equiv \begin{cases} \frac{\theta}{\theta-1}\left(\left(\frac{\bar{\theta}}{\theta}\right)^{\eta} - 1\right)\left(\exp(\lambda) - 1\right)^{-1} & \text{if } \theta > 1 \\ \eta & \text{if } \theta = 1 \\ \frac{\theta(1-\bar{\theta})-\bar{\theta}}{\bar{\theta}(1-\theta)}\left(\left(\frac{\bar{\theta}}{\theta}\right)^{\eta} - 1\right)\left(\exp(\lambda) - 1\right)^{-1} & \text{if } \theta < 1 \end{cases}.$$

where η is a parameter associated with the bargaining power of banks with reserve deficits in each trade—a Nash bargaining coefficient. Hence, ϕ is an effective bargaining weight. The average interbank rate is:

$$R^f = (1-\phi)R^w + \phi R^m$$

The slopes of the liquidity yield function are given by

$$\chi^+ = (R^w - R^m)\left(\frac{\bar{\theta}}{\theta}\right)^{\eta}\left(\frac{\theta^{\eta}\bar{\theta}^{1-\eta} - \theta}{\bar{\theta} - 1}\right) \text{ and } \chi^- = (R^w - R^m)\left(\frac{\bar{\theta}}{\theta}\right)^{\eta}\left(\frac{\theta^{\eta}\bar{\theta}^{1-\eta} - 1}{\bar{\theta} - 1}\right). \tag{33}$$

These formulas are consistent with:

$$\chi^+ = \Psi^+\left(R^f - R^m\right) \text{ and } \chi^- = \Psi^-\left(R^f - R^m\right) + \left(1 - \Psi^-\right)\left(R^w - R^m\right).$$

B Proofs

B.1 Proof of Lemma 1

Proof. Substitute the budget constraint (11) into (12). We obtain:

$$\begin{aligned} n' &= R^b(X)n - R^b(X)Div - \left(R^b(X) - R^m(X)\right)\tilde{m} - \left(R^b(X) - R^{m,*}(X)\right)\tilde{m}^* \\ &+ \left(R^b(X) - R^d(X)\right)\tilde{d} + (R^b(X) - R^{*,d}(X))\tilde{d}^* \\ &+ \mathbb{E}_{\omega^*}\chi^*(\theta^*(X), \tilde{m}^* + \omega^*\tilde{d}^*) + \mathbb{E}_\omega\chi(\theta(X), \tilde{m} + \omega\tilde{d}). \end{aligned}$$

and thus:

$$n' = R^b(X)n - R^b(X)Div + \Pi^*(X).$$

Conjecture that $v(n, X) = n$. Then, substituting $v(n', X') = n'$, into (10) we obtain:

$$v(n, X) = \max_{\{Div, \tilde{b}, \tilde{m}^*, \tilde{d}^*, \tilde{d}, \tilde{m}\}} Div + \beta\mathbb{E}\left[R^b(X)n - R^b(X)Div + \Pi(X)|X\right].$$

Note that if $\beta R^b(X) \neq 1$,dividends are either ∞ or $-\infty$. Thus, in equilibrium, it must be the case that $R^b(X) = 1/\beta$. As a result,

$$\begin{aligned} v(b, X) &= \max_{\{Div, \tilde{b}, \tilde{m}^*, \tilde{d}^*, \tilde{d}, \tilde{m}\}} Div + \beta\mathbb{E}\left[1/\beta b - 1/\beta Div + \Pi^*(X)|X\right] \\ &= b + \beta \max_{\{\tilde{b}, \tilde{m}^*, \tilde{d}^*, \tilde{d}, \tilde{m}\}} \mathbb{E}\left[\Pi\left(\tilde{m}^*, \tilde{d}^*, \tilde{d}, \tilde{m}, X\right)\right]. \end{aligned} \tag{34}$$

where

$$\begin{aligned} \Pi\left(\tilde{m}^*, \tilde{d}^*, \tilde{d}, \tilde{m}, X\right) &= \left(R^b(X) - R^d(X)\right)\tilde{d} + (R^b(X) - R^{*,d}(X))\tilde{d}^* \\ &- \left(R^b(X) - R^m(X)\right)\tilde{m} - \left(R^b(X) - R^{m,*}(X)\right)\tilde{m}^* \\ &+ \mathbb{E}_{\omega^*}\chi^*(\theta^*(X), \tilde{m}^* + \omega^*\tilde{d}^*) + \mathbb{E}_\omega\chi(\theta(X), \tilde{m} + \omega\tilde{d}). \end{aligned}$$

Next, consider the first order conditions for $\{\tilde{m}^*, \tilde{d}^*, \tilde{d}, \tilde{m}\}$. We have:

$$d : \Pi_d\left(\tilde{m}^*, \tilde{d}^*, \tilde{d}, \tilde{m}, X\right) = R^b(X) - R^d(X) - \mathbb{E}_\omega\left[\chi_{\tilde{d}}\right] = 0.$$

$$d^* : \Pi_{d^*}\left(\tilde{m}^*, \tilde{d}^*, \tilde{d}, \tilde{m}, X\right) = R^b(X) - R^{*,d}(X) - \mathbb{E}_\omega\left[\chi^*_{\tilde{d}}\right] = 0.$$

$$m : \Pi_m\left(\tilde{m}^*, \tilde{d}^*, \tilde{d}, \tilde{m}, X\right) = R^b(X) - R^m(X) - \mathbb{E}_\omega\left[\chi^*_m\right] = 0.$$

□

$$m^*: \Pi_{m^*}\left(\tilde{m}^*, \tilde{d}^*, \tilde{d}, \tilde{m}, X\right) = R^b\left(X\right) - R^{*,m}(X) - \mathbb{E}_{\omega}\left[\chi^*_{\tilde{m}}\right] = 0.$$

Thus, in equilibrium, these conditions must hold. Next, observe that $\Pi\left(\tilde{m}^*, \tilde{d}^*, \tilde{d}, \tilde{m}, X\right)$ is homogeneous of degree 1 in $\{\tilde{m}^*, \tilde{d}^*, \tilde{d}, \tilde{m}\}$. Hence, by Euler's Theorem for Homogeneous Functions:

$$\Pi^*\left(X\right) = \max_{\{\tilde{b},\tilde{m}^*,\tilde{d}^*,\tilde{d},\tilde{m}\}} \Pi\left(\tilde{m}^*, \tilde{d}^*, \tilde{d}, \tilde{m}, X\right) = \begin{bmatrix} \Pi_d & \Pi_{d^*} & \Pi_m & \Pi_{m^*} \end{bmatrix} \cdot \begin{bmatrix} d \\ d^* \\ m \\ m^* \end{bmatrix} = 0.$$

Hence, we verify that, $\Pi^*\left(X\right) = 0$ and as a result, replacing this result in (34), we verify the conjecture that $v\left(e, X\right) = e$.

B.2 Preliminary Observations

Liquidity Costs. To produce theoretical results, we derive some observations. First, recall that the market tightness in both currencies is always lower than one:

$$\theta = \frac{\delta - \mu}{\delta + \mu} < 1.$$

For this reason, we have that tightness is increasing in the size of the dispersion shocks:

$$\theta_{\delta} \equiv \frac{\partial \theta}{\partial \delta} = \frac{1}{\mu + \delta} - \frac{\delta - \mu}{(\mu + \delta)} \frac{1}{(\mu + \delta)} = \frac{(1 - \theta)}{(\mu + \delta)} ¿0. \tag{35}$$

Also, note that:

$$\theta_{\mu} \equiv \frac{\partial \theta}{\partial \mu} = \frac{-1}{\delta + \mu} - \frac{\delta - \mu}{\delta + \mu} \frac{1}{\delta + \mu} = \frac{-1}{\delta + \mu} (1 + \theta) < 0. \tag{36}$$

Moreover, the penalties are increasing in tightness.

$$\frac{\partial \chi^{+}}{\partial \delta} = \frac{\partial \chi^{+}}{\partial \theta} \frac{\partial \theta}{\partial \delta} > 0,$$

and

$$\frac{\partial \chi^{-}}{\partial \delta} = \frac{\partial \chi^{-}}{\partial \theta} \frac{\partial \theta}{\partial \delta} > 0.$$

Likewise, we have that the tightness is decreasing in the liquidity ratio:

$$\frac{\partial \theta}{\partial \mu} = -\frac{1 + \theta}{\delta + \frac{M/P}{D}} < 0.$$

We also know that the penalty rates are increasing in tightness, and hence:

$$\frac{\partial \chi^{+}}{\partial \mu} = \frac{\partial \chi^{+}}{\partial \theta} \frac{\partial \theta}{\partial \mu} < 0, \text{ and } \frac{\partial \chi^{-}}{\partial \mu} = \frac{\partial \chi^{-}}{\partial \theta} \frac{\partial \theta}{\partial \mu} < 0.$$

Differential Form of Money-Market Equilibrium. Consider the equilibrium in the money demand for Euros,

$$\frac{M}{P} = \mu D,$$

The differential form of this equations is:

$$\frac{M}{P} \left(\frac{dM}{M} - \frac{dP}{P} \right) = d\mu D + \mu dD.$$

Then, manipulating the equation we obtain:

$$\frac{M}{P}\left(\frac{dM}{M} - \frac{dP}{P}\right) = \mu D \frac{d\mu}{\mu} + \mu D \frac{dD}{D}.$$

Using (xxx, first line above,) we obtain:

$$d\log M - d\log P = d\log \mu + d\log D. \tag{37}$$

Likewise, for dollars we obtain:

$$d\log M^* - d\log P^* = d\log \mu^* + d\log D^*. \tag{38}$$

Differential Form of Exchange Rate. We have that

$$e = \frac{P}{P^*}$$

Then,

$$de = \frac{dP}{P^*} - \frac{P}{P^*} \cdot \frac{dP^*}{P^*}.$$

From here, we have that:

$$d\log e = d\log P - d\log P^*. \tag{39}$$

B.3 Proof of Proposition 1

In this appendix, we proof the following proposition, together with the one for dollars in the body of the paper. [Scale Effects] Part 1. Consider a temporary (i.i.d.) shock that increases the demand for Euro deposits. Then, the shock appreciates the Euro and lowers the liquidity ratio:

$$\frac{d\log e}{d\log D}=\frac{\frac{1}{2}\left(\chi_{\theta}^{+}+\chi_{\theta}^{-}\right)\theta_{\mu}\mu}{R^{b}-\frac{1}{2}\left(\chi_{\theta}^{+}+\chi_{\theta}^{-}\right)\theta_{\mu}\mu}\in(-1,0]\text{ and }\frac{d\log\mu}{d\log D}=-\left(\frac{R^{b}}{R^{b}-\frac{1}{2}\left(\chi_{\theta}^{+}+\chi_{\theta}^{-}\right)\theta_{\mu}\mu}\right)\in(-1,0].$$

Hence, $\frac{d\log e}{d\log\mu}>0$. Furthermore $d\left(\mathcal{LP}\right)=-d\left(\mathcal{EUBP}\right)=R^{m}d\log e$.

Part 2. Consider a permanent (random walk) shock that increases the demand for Euro deposits. Then,

$$\frac{d\log e}{d\log D}=-1\text{ and }\frac{d\log\mu}{d\log D}=0,$$

Hence, $\frac{d\log e}{d\log\mu}=\infty$. Furthermore $d\left(\mathcal{LP}\right)=d\left(\mathcal{EBP}\right)=0$. Along the proofs, we employ the implicit function theorem. The proposition in the draft just follows by symmetry. First, some preliminary results.

Consider a small increase in D. Recall that

$$\theta\equiv\frac{\delta-\mu}{\mu+\delta}=\frac{\delta-\frac{M/P}{D}}{\frac{M/P}{D}+\delta}.$$

Hence,

$$\theta_{D}\equiv\theta_{\mu}\mu_{D}=-\theta_{\mu}\mu\frac{1}{D}>0.$$

When the loans supply is perfectly elastic, R^{b} is a constant. In this case, the equilibrium condition for Euro reserve holdings yields

$$R^{b}=\left(1+i^{m}\right)\frac{P}{\mathbb{E}\left[p\left(X'\right)\right]}+\frac{1}{2}[\chi^{+}+\chi^{-}].$$

The price level appears in the real rate on reserves, on the value of real balances, and the real penalties $\{\chi^{+},\chi^{-}\}$.

Temporary Shocks. From the liquidity premium, compute the total differential with respect to D. We obtain:

$$\left(1+i^{m}\right)\frac{P}{\mathbb{E}\left[p\left(X'\right)\right]}\frac{dP}{P}+\frac{1}{2}[\chi^{+}+\chi^{-}]\frac{dP}{P}+\left(\frac{1}{2}\frac{d[\chi^{+}+\chi^{-}]}{d\theta}\theta_{\mu}\right)\left(-\mu d\log P+\mu_{D}dD\right)=0. \quad (40)$$

Observe that:

$$\mu_D dD = -\mu \frac{dD}{D} = -\mu d \log D.$$

and that:

$$\frac{\partial \mu}{\partial P} dP = -\mu d \log P.$$

Hence

$$\left(R^b - \frac{1}{2} \left(\chi_\theta^+ + \chi_\theta^- \right) \theta_\mu \mu \right) d \log P = \frac{1}{2} \left(\chi_\theta^+ + \chi_\theta^- \right) \theta_\mu \mu \cdot d \log D.$$

Thus, we obtain that:

$$-1 \leq d \log P = \frac{\frac{1}{2} \left(\chi_\theta^+ + \chi_\theta^- \right) \theta_\mu \mu}{R^b - \frac{1}{2} \left(\chi_\theta^+ + \chi_\theta^- \right) \theta_\mu \mu} d \log D \leq 0$$

where the sign follows from $\left\{ \chi_\theta^+, \chi_\theta^- \right\} > 0$ and $\theta_\mu < 0$.

Next, using (39), we have that:

$$-1 \leq d \log e = \frac{\frac{1}{2} \left(\chi_\theta^+ + \chi_\theta^- \right) \theta_\mu \mu}{R^b - \frac{1}{2} \left(\chi_\theta^+ + \chi_\theta^- \right) \theta_\mu \mu} d \log D \leq 0.$$

Holding nominal reserve balances and real deposits fixed, we obtain that (37) becomes

$$d \log \mu = - \left(d \log D + d \log P \right)$$

hence:

$$d \log \mu = - \left(1 + \frac{\frac{1}{2} \left(\chi_\theta^+ + \chi_\theta^- \right) \theta_\mu \mu}{R^b - \frac{1}{2} \left(\chi_\theta^+ + \chi_\theta^- \right) \theta_\mu \mu} \right) d \log D < 0.$$

Combining:

$$\frac{d \log e}{d \log \mu} = \frac{-\frac{1}{2} \left(\chi_\theta^+ + \chi_\theta^- \right) \theta_\mu \mu}{R^b} > 0.$$

The differential of the liquidity premium is then:

$$\begin{aligned}
d \left(\mathcal{DLP} \right) &= \frac{1}{2} [\chi^+ + \chi^-] d \log P - \frac{1}{2} \left(\chi_\theta^+ + \chi_\theta^- \right) \theta_\mu \left(\mu d \log P + \mu d \log D \right) \\
&= \left(R^b - R^m \right) \frac{\frac{1}{2} \left(\chi_\theta^+ + \chi_\theta^- \right) \theta_\mu \mu}{R^b - \frac{1}{2} \left(\chi_\theta^+ + \chi_\theta^- \right) \theta_\mu \mu} d \log D - \frac{1}{2} \left(\chi_\theta^+ + \chi_\theta^- \right) \theta_\mu \mu \left(1 + \frac{\frac{1}{2} \left(\chi_\theta^+ + \chi_\theta^- \right) \theta_\mu \mu}{R^b - \frac{1}{2} \left(\chi_\theta^+ + \chi_\theta^- \right) \theta_\mu \mu} \right) d \log \\
&= \left(R^b - R^m \right) \frac{\frac{1}{2} \left(\chi_\theta^+ + \chi_\theta^- \right) \theta_\mu \mu}{R^b - \frac{1}{2} \left(\chi_\theta^+ + \chi_\theta^- \right) \theta_\mu \mu} - \frac{1}{2} \left(\chi_\theta^+ + \chi_\theta^- \right) \theta_\mu \mu \left(\frac{R^b}{R^b - \frac{1}{2} \left(\chi_\theta^+ + \chi_\theta^- \right) \theta_\mu \mu} \right) d \log D \\
&= -R^m \frac{\frac{1}{2} \left(\chi_\theta^+ + \chi_\theta^- \right) \theta_\mu \mu}{R^b - \frac{1}{2} \left(\chi_\theta^+ + \chi_\theta^- \right) \theta_\mu \mu} d \log D = R^m d \log e^*.
\end{aligned}$$

Re-arranging, we obtain:

$$d\log\left(\mathcal{DLP}\right)=-\left[\frac{\mathcal{DLP}}{R^m}\right]^{-1}\frac{dP}{P}=-\left[\frac{\mathcal{LP}}{R^m}\right]^{-1}d\log e.$$

Likewise, the excess bond premium satisfies:

$$d\left(\mathcal{EUBP}\right)=-d\left(\mathcal{DLP}\right)=R^m d\log e,$$

and re-arranging:

$$d\log\left(\mathcal{EUBP}\right)=\left[\frac{\mathcal{EBP}}{R^m}\right]^{-1}\frac{dP}{P}=-\left[\frac{\mathcal{EBP}}{R^m}\right]^{-1}d\log e.$$

The statement in the body of the paper follows by symmetry. Namely, if we shock dollar loans:

$$-1\leq d\log P^*=\frac{\frac{1}{2}\left(\chi^+_{\theta^*}+\chi^-_{\theta^*}\right)\theta^*_{\mu^*}\mu^*}{R^b-\left(\chi^+_{\theta^*}+\chi^-_{\theta^*}\right)\theta^*_{\mu^*}\mu^*}d\log D^*\leq 0.$$

In this case, the effect on the exchange rate is:

$$0\leq d\log e=-\frac{\frac{1}{2}\left(\chi^+_{\theta^*}+\chi^-_{\theta^*}\right)\theta^*_{\mu^*}\mu^*}{R^b-\left(\chi^+_{\theta^*}+\chi^-_{\theta^*}\right)\theta^*_{\mu^*}\mu^*}d\log D\leq 1.$$

Thus, for the dollar

$$d\left(\mathcal{DLP}\right)=R^{*,m}d\log e\geq 0,$$

and likewise the

$$d\left(\mathcal{EBP}\right)=d\left(\mathcal{DLP}\right)=R^{*,m}d\log e\geq 0.$$

Permanent Shocks. From the liquidity premium, compute the total differential with respect to D. Because the shock is permanent, expected inflation is zero. Hence, optimality in reserves yields:

$$\frac{1}{2}\left(\chi^+_{\theta}+\chi^-_{\theta}\right)\theta_{\mu}\left(-\mu d\log P+\mu d\log D\right)=0. \tag{41}$$

Thus,

$$d\log P=-d\log D,$$

and thus,

$$d\log e=-d\log D.$$

We know then that the liquidity ratio must not change in this case,

$$d\log \mu=0,$$

and as a result:

$$d\left(\mathcal{EBP}\right)=d\left(\mathcal{DLP}\right)=0.$$

Thus, the same is true if the increase is in dollar funding.

From the liquidity premium, compute the total differential with respect to D. Because the shock is permanent, expected inflation is zero. Hence, optimality in reserves yields:

$$\frac{1}{2}\left(\chi_{\theta}^{+}+\chi_{\theta}^{-}\right)\theta_{\mu}\left(-\mu d\log P+\mu d\log D\right)=0. \tag{42}$$

Thus,

$$d\log P=-d\log D,$$

and thus,

$$d\log e=-d\log D.$$

Likewise, for dollars:

$$d\log P=-d\log D^{*},$$

but in this case:

$$d\log e=-d\log P^{*}=d\log D^{*}.$$

Hence, in both cases, $d\log\mu=d\log\mu^{*}=0$. Since the liquidity ratio is constant and inflation is not expected to move, the effects on premia are constant.

B.4 Proof of Proposition 2

[Scale Effects] Part 1. Consider a temporary (i.i.d.) shock that increases the demand for Euro payment volaititiy. Then, the shock appreciates the Euro and lowers the liquidity ratio:

$$\frac{d\log e}{d\log\delta}=-\frac{d\log\mu}{d\log\delta}=\frac{-\frac{1}{2}\left[\chi_{\theta}^{+}+\chi_{\theta}^{-}\right]\theta_{\delta}\delta}{R^{b}-\frac{1}{2}\left[\chi_{\theta}^{+}+\chi_{\theta}^{-}\right]\theta_{\mu}\mu}\leq 0.$$

Hence, $\frac{d\log e}{d\log\mu}=-1$. Furthermore $d\left(\mathcal{LP}\right)=-d\left(\mathcal{EUBP}\right)=R^{m}d\log e$.

Part 2. Consider a permanent (random walk) shock that increases the demand for Euro deposits. Then,

$$\frac{d\log e}{d\log\delta}=-\frac{d\log\mu}{d\log\delta}=-\frac{(1-\theta)}{(1+\theta)}\cdot\frac{\delta}{\mu}.$$

Hence, $\frac{d\log e}{d\log\mu}=-1$. Furthermore $d\left(\mathcal{LP}\right)=-d\left(\mathcal{EBP}\right)=R^{m}d\log e$. When the loans supply is perfectly elastic, R^{b} is a constant. In this case, we have that the liquidity premium is given by:

$$R^{b}=\left(1+i^{m}\right)\frac{P}{\mathbb{E}\left[p\left(X'\right)\right]}+\frac{1}{2}[\chi^{+}+\chi^{-}].$$

Recall that the price level level appears in the real rate on reserves, on the value of real balances, and the real penalties $\{\chi^+, \chi^-\}$. Note that we already established that $\theta_\delta > 0$.

Temporary Euro Funding Shocks. From the liquidity premium, we have that:

$$(1+i^m)\frac{P}{\mathbb{E}\left[p\left(X'\right)\right]}\frac{dP}{P}+\frac{1}{2}[\chi^++\chi^-]\frac{dP}{P}+\frac{1}{2}\frac{d[\chi^++\chi^-]}{d\theta}\theta_\delta d\delta+\frac{1}{2}\frac{\partial[\chi^++\chi^-]}{\partial\theta}\frac{\partial\theta}{\partial\mu}\frac{d\mu}{dP}dP=0. \quad (43)$$

Holding nominal reserve balances and real deposits fixed, we obtain that

$$d\mu=\frac{M}{P^2}\frac{dP}{D}=-\mu\frac{dP}{P}=-\mu d\log P \quad (44)$$

Thus, re-arranging (43) we obtain:

$$d\log\left(P\right)=\frac{-\frac{1}{2}\left[\chi_\theta^++\chi_\theta^-\right]\theta_\delta\delta}{R^b-\frac{1}{2}\left[\chi_\theta^++\chi_\theta^-\right]\theta_\mu\mu}d\log\delta<0.$$

where we used $R^b=R^m+\frac{1}{2}\left(\chi^++\chi^-\right)$ and the sign follows from $\theta_\delta>0$ and $\theta_\mu<0$. Next, using (39), we have that:

$$d\log e=\frac{-\frac{1}{2}\left[\chi_\theta^++\chi_\theta^-\right]\theta_\delta\delta}{R^b-\frac{1}{2}\left[\chi_\theta^++\chi_\theta^-\right]\theta_\mu\mu}d\log\delta\leq 0.$$

Note that:

$$\theta_\delta=\frac{(1-\theta)}{(\mu+\delta)}<\frac{(1+\theta)}{\delta+\mu}=-\theta_\mu,$$

but $\mu<\delta$, hence, we cannot bound the effect on the exchange rate.

Next, we obtain the effect on the liquidity ratio. We know that

$$d\log\left(\mu\right)=-d\log\left(P\right)=\frac{\frac{1}{2}\left[\chi_\theta^++\chi_\theta^-\right]\theta_\delta\delta}{R^b-\frac{1}{2}\left[\chi_\theta^++\chi_\theta^-\right]\theta_\mu\mu}d\log\delta>0.$$

Consider next the liquidity premium. We have that

$$\begin{aligned}
d\left(\mathcal{DLP}\right) &= \frac{1}{2}[\chi^++\chi^-]d\log P-\frac{1}{2}\left[\chi_\theta^++\chi_\theta^-\right]\theta_\mu\mu d\log\left(P\right)-\frac{1}{2}\left[\chi_\theta^++\chi_\theta^-\right]\theta_\delta\delta \\
&= \left[\frac{\frac{1}{2}\left[\chi_\theta^++\chi_\theta^-\right]\theta_\mu\mu}{R^b-\frac{1}{2}\left[\chi_\theta^++\chi_\theta^-\right]\theta_\mu\mu}+1\right]\frac{1}{2}\left[\chi_\theta^++\chi_\theta^-\right]\theta_\delta\delta \\
&= \frac{1}{2}[\chi^++\chi^-]d\log P-R^b d\log e. \\
&= -R^m d\log e.
\end{aligned}$$

Likewise the excess bond premium in euros:

$$d\,(\mathcal{EUBP}) = -d\,(\mathcal{DLP}) = R^m d\log e.$$

Temporary Dollar Funding Shocks. The statement in the body of the paper follows by symmetry. Namely, if we shock dollar loans:

$$d\log P^* = \frac{-\frac{1}{2}\left[\chi^+_{\theta^*} + \chi^-_{\theta^*}\right]\theta^*_{\delta^*}\delta^*}{R^b - \frac{1}{2}\left[\chi^+_{\theta^*} + \chi^-_{\theta^*}\right]\theta^*_{\mu^*}\mu^*} d\log\delta^* \le 0.$$

Since the money supply and deposits are fixed:

$$d\log\mu^* = -d\log P^* = \frac{\frac{1}{2}\left[\chi^+_{\theta^*} + \chi^-_{\theta^*}\right]\theta^*_{\delta^*}\delta^*}{R^b - \frac{1}{2}\left[\chi^+_{\theta^*} + \chi^-_{\theta^*}\right]\theta^*_{\mu^*}\mu^*} d\log\delta^* \ge 0.$$

In this case, the effect on the exchange rate is:

$$0 \le d\log e = \frac{-\frac{1}{2}\left[\chi^+_{\theta^*} + \chi^-_{\theta^*}\right]\theta^*_{\delta^*}\delta^*}{R^b - \frac{1}{2}\left[\chi^+_{\theta^*} + \chi^-_{\theta^*}\right]\theta^*_{\mu^*}\mu^*} d\log\delta^* \ge 0.$$

Thus, for the dollar

$$d\,(\mathcal{DLP}) = R^{*,m} d\log e \ge 0,$$

and likewise the

$$d\,(\mathcal{EBP}) = d\,(\mathcal{DLP}) = R^{*,m} d\log e \ge 0.$$

Permanent Shocks. From the liquidity premium, compute the total differential with respect to D. Because the shock is permanent, expected inflation is zero. Hence, optimality in reserves yields:

$$\frac{1}{2}\left(\chi^+_\theta + \chi^-_\theta\right)\left(-\theta_\mu \mu d\log P + \theta_\delta \delta d\log\delta\right) = 0. = 0. \tag{45}$$

Thus,

$$d\log P = d\log e = \frac{\theta_\delta \delta}{\theta_\mu \mu} d\log\delta = \frac{\frac{(1-\theta)}{(\mu+\delta)}}{-\frac{(1+\theta)}{\delta+\mu}}\frac{\delta}{\mu} = -\frac{(1-\theta)}{(1+\theta)}\cdot\frac{\delta}{\mu} d\log\delta,$$

and thus,

$$d\log\mu = \frac{(1-\theta)}{(1+\theta)}\frac{\delta}{\mu}\cdot d\log\delta.$$

We know then that the liquidity ratio must not change in this case,

$$d\log\theta = 0,$$

and as a result:

$$d(\mathcal{EBP}) = d(\mathcal{DLP}) = 0.$$

Thus, the same is true if the increase is in dollar funding.

B.5 Proof of Proposition 2 (Interest Rate Pass-Through)

[Scale Effects] Part 1. Consider a temporary (i.i.d.) shock that increases the demand for Euro payment volaititiy. Then, the shock appreciates the Euro and lowers the liquidity ratio:

$$-1 \leq \frac{d\log e}{d\log(1+i^m)} = -\frac{d\log\mu}{d\log\delta} = -\frac{R^m}{\left(R^b - \frac{1}{2}\left[\chi_\theta^+ + \chi_\theta^-\right]\theta_\mu\mu\right)} < 0.$$

Hence, $\frac{d\log e}{d\log\mu} = -1$. Furthermore $d(\mathcal{LP}) = -d(\mathcal{EBP}) = R^m\left(1 - \frac{d\log e}{d\log(1+i^m)}\right)d\log(1+i^m)$.

Part 2. Consider a permanent (random walk) shock that increases the demand for Euro deposits. Then,

$$\frac{d\log e}{d\log\delta} = -\frac{d\log\mu}{d\log\delta} = .$$

Hence, $\frac{d\log e}{d\log\mu} = -1$. Furthermore $d(\mathcal{LP}) = d(\mathcal{EBP}) = R^m d\log e$. The gist of the proof is similar to the ones in the previous Propositions.

Temporary shocks to Euro Policy Rate. From the excess bond premium in Euros, we obtain that:

$$\frac{\mathbb{E}[p(X')]}{P}d(1+i^m) + (1+i^m)\frac{\mathbb{E}[p(X')]}{P}\frac{dP}{P} + \frac{1}{2}[\chi^+ + \chi^-]\frac{dP}{P} + \left[\chi_\theta^+ + \chi_\theta^-\right]\theta_\mu\mu\frac{dP}{P} = 0. \quad (46)$$

Therefore, re-arranging terms we have:

$$d\log(P) = -\frac{\frac{P}{\mathbb{E}[p(X')]}d(1+i^m)}{\left(R^b - \frac{1}{2}\left[\chi_\theta^+ + \chi_\theta^-\right]\theta_\mu\mu\right)}.$$

Multiplying and dividing by $(1+i^m)$ we obtain:

$$d\log(P) = -\frac{R^m}{\left(R^b - \frac{1}{2}\left[\chi_\theta^+ + \chi_\theta^-\right]\theta_\mu\mu\right)}d\log(1+i^m).$$

We know that:

$$R^m \leq R^b \text{ and } \frac{1}{2}\left[\chi_\theta^+ + \chi_\theta^-\right]\theta_\mu\mu \leq 0$$

Thus,

$$-1 \leq d\log(P) = -\frac{R^m}{\left(R^b - \frac{1}{2}\left[\chi_\theta^+ + \chi_\theta^-\right]\theta_\mu\mu\right)}d\log(1+i^m) < 0.$$

with equality only under satiation. Next, using:

$$\frac{d\mu}{dP} = -\mu\frac{dP}{P},$$

we obtain:

$$\frac{d\log\left(\mu\right)}{d\log\left(1+i^{m}\right)} = -\frac{d\log\left(P\right)}{d\log\left(1+i^{m}\right)} \in (0,1].$$

Now consider the dollar liquidity premium

$$\mathcal{DLP} = R^{m} - R^{*,m}.$$

Then, we have that:

$$\begin{aligned} d\mathcal{DLP} &= R^{m} d\log\left(1+i^{m}\right) - R^{m} d\log\left(P\right) \\ &= R^{m}\left(\log\left(1+i^{m}\right) - d\log\left(P\right)\right) \\ &= R^{m}\left(\frac{\frac{1}{2}\left[\chi^{+}+\chi^{-}\right]}{\left(R^{b}-\frac{1}{2}\left[\chi_{\theta}^{+}+\chi_{\theta}^{-}\right]\theta_{\mu}\mu\right)}\right) d\log\left(1+i^{m}\right) \in [0, R^{m}). \end{aligned}$$

Hence, we have that:

$$\frac{d\mathcal{DLP}}{d\log\left(1+i^{*,m}\right)} = -\left[\frac{\mathcal{DLP}}{R^{*,m}}\right]^{-1}\left(1+\frac{d\log\left(P\right)}{\log\left(1+i^{*,m}\right)}\right) = -\left[\frac{\mathcal{DLP}}{R^{*,m}}\right]^{-1}\left(1-\frac{d\log\left(e\right)}{\log\left(1+i^{*,m}\right)}\right) \leq 0,$$

because $\frac{d\log(P)}{\log(1+i^{*,m})} \geq -1$ with strict inequality away from satiation. Thus, we also have that:

$$d\mathcal{EBP} = -d\mathcal{DLP} = -R^{m}\left(\frac{\frac{1}{2}\left[\chi^{+}+\chi^{-}\right]}{\left(R^{b}-\frac{1}{2}\left[\chi_{\theta}^{+}+\chi_{\theta}^{-}\right]\theta_{\mu}\mu\right)}\right) d\log\left(1+i^{m}\right) \in (-R^{m}, 0]$$

$$\frac{d\mathcal{EBP}}{d\log\left(1+i^{m}\right)} = -\left[\frac{\mathcal{EBP}}{R^{m}}\right]^{-1}\left(1-\frac{d\log\left(e\right)}{\log\left(1+i^{m}\right)}\right) \leq 0.$$

Temporary shocks to Dollar Policy Rate. In this case, by symmetry:

$$\log\left(P^{*}\right) = -\frac{R^{*,m}}{\left(R^{*,b}-\frac{1}{2}\left[\chi_{\theta^{*}}^{*,+}+\chi_{\theta^{*}}^{*,-}\right]\theta_{\mu^{*}}^{*}\mu^{*}\right)} d\log\left(1+i^{*,m}\right).$$

Then,

$$d\log\left(e\right) = d\log\left(\mu^{*}\right) = -d\log\left(P^{*}\right) = \frac{R^{*,m}}{\left(R^{*,b}-\frac{1}{2}\left[\chi_{\theta^{*}}^{*,+}+\chi_{\theta^{*}}^{*,-}\right]\theta_{\mu^{*}}^{*}\mu^{*}\right)} \in (0,1]$$

and equal to 1 under satiation. Furthermore:

$$d\mathcal{EBP} = d\mathcal{DLP} = -R^{*,m}\left(d\log\left(1+i^{m}\right) - d\log\left(P^{*}\right)\right) \in [0, R^{m})$$

with equality at satiation.

Permanent Effects. Consider now a permanent effect. Then, expected prices respond with innovation. Thus,

$$d\left(1+i^{m}\right)-\left[\chi_{\theta}^{+}+\chi_{\theta}^{-}\right]\theta_{\mu}\mu d\log P=0.$$

Thus, clearing the condition yields:

$$d\log P=\frac{d\left(1+i^{m}\right)}{\left[\chi_{\theta}^{+}+\chi_{\theta}^{-}\right]\theta_{\mu}\mu}.$$

Multiplying and dividing by $(1+i^{m})$ yields:

$$d\log P=\frac{d\log\left(1+i^{m}\right)}{\left[\chi_{\theta}^{+}+\chi_{\theta}^{-}\right]\theta_{\mu}\mu}>0.$$

Then, for the liquidity ration and the exchange rate:

$$d\log e=d\log P=-d\log\mu.$$

Finally,

$$d\mathcal{EBP}=-d\mathcal{DLP}=R^{m}d\log\left(1+i^{m}\right).$$

Likewise, for dollars:

$$d\log P^{*}=\frac{d\log\left(1+i^{*,m}\right)}{\left[\chi_{\theta^{*}}^{*,+}+\chi_{\theta^{*}}^{*,-}\right]\theta_{\mu^{*}}^{*}\mu^{*}}>0.$$

Then, for the liquidity ration and the exchange rate:

$$d\log e=d\log\mu^{*}=-d\log P^{*}<0.$$

Finally,

$$d\mathcal{EBP}=d\mathcal{DLP}=-R^{m}d\log\left(1+i^{m}\right).$$

B.6 Proofs of Proposition ?? (Open-Market Operations)

Preliminary Observations. We now consider a purchase of loans with issuances of reserve assets. In particular, we let the domestic central bank hold private loans in the amount B_t^G. The central banks' budget constraint in this case is modified to:

$$M_t+T_t+W_{t+1}+\left(1+i_t^{b}\right)\cdot P_{t-1}B_{t-1}^{G}=P_t\cdot B_t^{G}+M_{t-1}(1+i_t^{m})+W_t(1+i_t^{w}).$$

As in earlier proofs, we avoid time subscripts. Next, we study the effects of a one time increase in B^G while holding the path of transfers the same. We evaluate the role of random purchases of loans financed with reserves. We obtain:

$$dM = B^G dP + P dB^G.$$

but any operating losses are finance with transfers. Thus, dividing both sides by P, we obtain:

$$\frac{1}{P}\frac{dM}{dB^G} = \frac{dP}{P}\frac{PB^G}{dB^G} + 1 \rightarrow \frac{M}{P}\frac{dM}{M}\frac{1}{dB^G} = \frac{dP}{P}\frac{B^G}{dB^G} + 1$$

Assuming that a fraction γ central bank's balance liabilities are financed with loans, we have that $\gamma M = PB^G$. Then, we obtain:

$$dM = PB^G\frac{dP}{P} + PB^G\frac{dB^G}{B^g}$$

Dividing both sides by M

$$\frac{dM}{M} = \frac{PB^G}{M}\frac{dP}{P} + PB^G\frac{dB^G}{B^G}.$$

In logarithms, we obtain:

$$d\log M_t = \gamma\left(d\log P_t + d\log B^G\right).$$

The equation has the interpretation that the increase in the money supply needed to finance the open-market operation, in nominal terms has to compensate for the increase in the price level. This is because the operation is defined in real terms.

A temporary Euro open-market operation. From the liquidity premium, we have that:

$$(1+i^m)\frac{P}{\mathbb{E}\left[p\left(X'\right)\right]}\frac{dP}{P} + \frac{1}{2}[\chi^+ + \chi^-]\frac{dP}{P} + \frac{1}{2}\left(\chi_\theta^+ + \chi_\theta^-\right)\theta_\mu d\mu = 0. \tag{47}$$

Now, observe that:

$$d\mu = \mu\left(\frac{dM}{M} - \frac{dP}{P}\right). \tag{48}$$

Collecting terms we obtain:

$$\left(R^m + \frac{1}{2}[\chi^+ + \chi^-] - \frac{1}{2}\left(\chi_\theta^+ + \chi_\theta^-\right)\theta_\mu \mu\right) d\log P + \frac{1}{2}\left(\chi_\theta^+ + \chi_\theta^-\right)\theta_\mu \mu d\log M = 0.$$

Equations (47-48) yields a system of two equations and two unknowns:

$$\left[\begin{array}{cc} \left(R^{m}+\frac{1}{2}[\chi^{+}+\chi^{-}]-\frac{1}{2}\left(\chi_{\theta}^{+}+\chi_{\theta}^{-}\right)\theta_{\mu}\mu\right) & \frac{1}{2}\left(\chi_{\theta}^{+}+\chi_{\theta}^{-}\right)\theta_{\mu}\mu \\ -\gamma & 1 \end{array}\right]\left[\begin{array}{c} d\log P \\ d\log M \end{array}\right]=\left[\begin{array}{c} 0 \\ \gamma d\log B^{G} \end{array}\right].$$

Then, we solve for $d\log M$ and obtain:

$$d\log M=\left[\frac{R^{b}-\frac{1}{2}\left(\chi_{\theta}^{+}+\chi_{\theta}^{-}\right)\theta_{\mu}}{-\frac{1}{2}\left(\chi_{\theta}^{+}+\chi_{\theta}^{-}\right)\theta_{\mu}}\right]d\log P.$$

The relationship is thus positive, but the money supply grows more thane one for one with inflation.

We can now clear the expression for the change in the price level:

$$\left(\frac{R^{b}-\frac{1}{2}\left(\chi_{\theta}^{+}+\chi_{\theta}^{-}\right)\theta_{\mu}}{-\frac{1}{2}\left(\chi_{\theta}^{+}+\chi_{\theta}^{-}\right)\theta_{\mu}}-\gamma\right)d\log P=\gamma d\log B^{G}.$$

Since $\gamma\leq 1$, but $\left(R^{b}-\frac{1}{2}\left(\chi_{\theta}^{+}+\chi_{\theta}^{-}\right)\theta_{\mu}\right)/\left(-\frac{1}{2}\left(\chi_{\theta}^{+}+\chi_{\theta}^{-}\right)\theta_{\mu}\right)\geq 1$, we know that the increase in the price level is less than one for one with the monetary operation.

Hence,

$$\frac{R^{b}-(1-\gamma)\frac{1}{2}\left(\chi_{\theta}^{+}+\chi_{\theta}^{-}\right)\theta_{\mu}}{-\frac{1}{2}\left(\chi_{\theta}^{+}+\chi_{\theta}^{-}\right)\theta_{\mu}}d\log P=\gamma d\log B^{G}.$$

Thus, the operation is inflationary, as seen from the solution.

$$\frac{d\log P}{d\log B^{G}}=\frac{-\frac{1}{2}\left(\chi_{\theta}^{+}+\chi_{\theta}^{-}\right)\theta_{\mu}}{R^{b}-(1-\gamma)\frac{1}{2}\left(\chi_{\theta}^{+}+\chi_{\theta}^{-}\right)\theta_{\mu}\mu}\gamma\in[0,1).$$

From here, we have that the increase in the money supply is:

$$\frac{d\log M}{d\log B^{G}}=\frac{R^{b}-\frac{1}{2}\left(\chi_{\theta}^{+}+\chi_{\theta}^{-}\right)\theta_{\mu}}{R^{b}-(1-\gamma)\frac{1}{2}\left(\chi_{\theta}^{+}+\chi_{\theta}^{-}\right)\theta_{\mu}\mu}\gamma.$$

which necessarily and increase above 1.

From the liquidity ratio, (48), is given by:

$$d\log\mu=d\log M-d\log P=\frac{\gamma R^{b}}{R^{b}-(1-\gamma)\frac{1}{2}\left(\chi_{\theta}^{+}+\chi_{\theta}^{-}\right)\theta_{\mu}\mu}\in[\gamma,1].$$

The exchange rate, follows:

$$\frac{d\log e}{d\log B^{G}}=\frac{d\log P}{d\log B^{G}}=\frac{-\frac{1}{2}\left(\chi_{\theta}^{+}+\chi_{\theta}^{-}\right)\theta_{\mu}}{R^{b}-(1-\gamma)\frac{1}{2}\left(\chi_{\theta}^{+}+\chi_{\theta}^{-}\right)\theta_{\mu}\mu}\gamma\in[0,1).$$

As in previous cases, we obtain:

$$d\log e = d\log P$$

The dollar liquidity premium is:

$$d\mathcal{DLP} = R^m d\log e.$$

Hence,

$$\frac{d\mathcal{DLP}}{d\log B^G} = R^m \frac{d\log P}{d\log B^G} = -\gamma \frac{R^m}{2}\left(\chi_\theta^+ + \chi_\theta^-\right)\theta_\mu \mu > 0.$$

and the Euro excess bond premium is given by:

$$d\mathcal{EUBP} \quad = d\mathcal{DLP} = \quad -R^m d\log e.$$

A temporary Dollar open-market operation. Now consider a purchase in the US. By symmetry, the increase in dollar prices is:

$$1 > \frac{d\log P^*}{d\log B^{*,G}} = \frac{-\frac{1}{2}\left(\chi_{\theta^*}^{*,+} + \chi_{\theta^*}^{*,-}\right)\theta_{\mu^*}^*}{R^{*,b} - (1-\gamma^*)\frac{1}{2}\left(\chi_{\theta^*}^{*,+} + \chi_{\theta^*}^{*,-}\right)\theta_{\mu^*}^*\mu^*}\gamma^* \geq 0.$$

and the increase in the dollar money supply is given by:

$$\frac{d\log M^*}{d\log B^{*,G}} = \frac{R^b - \frac{1}{2}\left(\chi_{\theta^*}^{*,+} + \chi_{\theta^*}^{*,-}\right)\theta_{\mu^*}^*}{R^{*,b} - (1-\gamma^*)\frac{1}{2}\left(\chi_{\theta^*}^{*,+} + \chi_{\theta^*}^{*,-}\right)\theta_{\mu^*}^*\mu^*}\gamma^*.$$

and the dollar liquidity ratio is:

$$\frac{d\log \mu^*}{d\log B^{*,G}} = \frac{d\log M^*}{d\log B^{*,G}} - \frac{d\log P^*}{d\log B^{*,G}} = \frac{\gamma^* R^b}{R^b - (1-\gamma^*)\frac{1}{2}\left(\chi_{\theta^*}^{*,+} + \chi_{\theta^*}^{*,-}\right)\theta_{\mu^*}^*\mu^*} \in [0,1].$$

Now, in the exchange rate in this case follows:

$$d\log e = -d\log P^*.$$

Finally, the excess-bond premium and the dollar liquidity premium is:

$$d\mathcal{EBP} = d\mathcal{DLP} = R^{*,m} d\log\left(e^*\right) \in [0, R^m).$$

A permanent Euro open-market operation. From the liquidity premium, we have that:

$$\frac{1}{2}\left(\chi_{\theta}^{+}+\chi_{\theta}^{-}\right)\theta_{\mu}d\mu=0. \tag{49}$$

Now, observe that:

$$d\mu=\mu\left(\frac{dM}{M}-\frac{dP}{P}\right). \tag{50}$$

Combining (49-50) we have that:

$$d\log P=d\log M.$$

We combine this restriction with the open-market operation and obtain:

$$\begin{bmatrix} -1 & 1 \\ -\gamma & 1 \end{bmatrix}\begin{bmatrix} d\log P \\ d\log M \end{bmatrix}=\begin{bmatrix} 0 \\ \gamma d\log B^{G} \end{bmatrix}.$$

Then, we solve for $d\log M$ and obtain:

$$d\log M=d\log P=\frac{\gamma}{1-\gamma}d\log B^{G}.$$

Naturally,

$$d\log\mu=0.$$

The exchange rate, follows:

$$\frac{d\log e}{d\log B^{G}}=\frac{d\log P}{d\log B^{G}}=\frac{\gamma}{1-\gamma}.$$

and

$$d\mathcal{EUBP}\quad=d\mathcal{DLP}=\quad 0.$$

A permanent US open-market operation. In this case, we obtain:

$$\frac{d\log M^{*}}{d\log B^{*,G}}=\frac{d\log P^{*}}{d\log B^{*,G}}=-\frac{d\log e}{d\log B^{*,G}}=-\frac{\gamma}{1-\gamma}.$$

In terms of the liquidity ratio, we obtain

$$\frac{d\log\mu^{*}}{d\log B^{*,G}}=0.$$

and

$$d\mathcal{EUBP}\quad=d\mathcal{DLP}=\quad 0.$$

B.7 Proofs of Proposition 2 (Open-Market Operations)

Preliminary Observations. Next, we derive the effects of a foreign direct intervention. We now consider a purchase of dollar reserves by the domestic country, with issuances of domestic reserves. In particular, we let the domestic central bank hold dollar reserves in the amount M_t^{**}. The central banks' budget constraint in this case is modified to:

$$M_t + T_t + W_{t+1} + (1 + i_t^{*,m}) \cdot e_t \cdot M_{t-1}^{**} = e_t \cdot M_t^{**} + M_{t-1}(1 + i_t^m) + W_t(1 + i_t^w).$$

Implicit in this budget is the idea that the domestic central bank has access to the interest on reserves as does any other bank. Next, we study the effects of a one time increase in M_t^{**} while holding the path of transfers the same.

Thus, the money market condition is now modified to:

$$M_t^* = M_t^{**} + \mu_t^* D_t^*$$

We evaluate the role of random foreign exchange interventions in this economy, assuming M_t^{**} is an i.i.d. random variable. We note that:

$$-D^* d\mu^* = dM^{**} \rightarrow d\log \mu^* = -\mathcal{C} d\log M^{**},$$

where $\mathcal{A}$ represents the size of dollar holdings by the domestic central bank vis-a-vis the private holdings:

$$\mathcal{C} = \frac{M^{**}}{\mu_t^* D_t^*}.$$

Within the domestic central bank's balance sheet, we have an equivalent increase between domestic reserves and domestic foreign reserves. Thus, the following relation hold,

$$dM = M^{**} de + e dM^{**} \rightarrow d\log M = \mathcal{F}\left(d\log e + d\log M^{**}\right).$$

where

$$\mathcal{F} \equiv \frac{M^{**}}{M}.$$

represents the amount of foreign-currency backing of the domestic liabilities. We obtain the following results.

Temporary Foreign Exchange Intervention. From the euro excess bond premium, we have that:

$$R^m \frac{dP}{P} + \frac{1}{2}[\chi^+ + \chi^-]\frac{dP}{P} + \frac{1}{2}\left[\chi_\theta^+ + \chi_\theta^-\right]\theta_\mu \left(\frac{d\mu}{dM} dM - \frac{d\mu}{dP} dP\right) = 0. \quad (51)$$

and using:

$$\frac{d\mu}{dM}dM = \frac{1}{P}dM = \frac{M}{P}\frac{dM}{M} = \mu d\log M.$$

and

$$\frac{d\mu}{dP} = \frac{d\frac{M}{P}\frac{1}{D}}{dP} = -\mu d\log P.$$

Thus, we obtain:

$$\left[R^b - \frac{1}{2}\left(\chi_\theta^+ + \chi_\theta^-\right)\theta_\mu \mu\right] d\log P + \frac{1}{2}\left(\chi_\theta^+ + \chi_\theta^-\right)\theta_\mu \mu d\log M = 0.$$

If we follow the same steps, from the dollar excess bond premium, we obtain a

$$R^b d\log P^* + \frac{1}{2}\left(\chi_{\theta^*}^{*,+} + \chi_{\theta^*}^{*,-}\right)\theta_{\mu^*}^* \mu^* d\log \mu^* = 0.$$

The equation above and (xxx) yields a system of two equations and two unknowns:

$$\begin{bmatrix} R^b - \frac{1}{2}\left(\chi_\theta^+ + \chi_\theta^-\right)\theta_\mu \mu & \frac{1}{2}\left(\chi_\theta^+ + \chi_\theta^-\right)\theta_\mu \mu & 0 & 0 \\ -\mathcal{F} & 1 & \mathcal{F} & 0 \\ 0 & 0 & R^b & \frac{1}{2}\left(\chi_{\theta^*}^{*,+} + \chi_{\theta^*}^{*,-}\right)\theta_{\mu^*}^* \mu^* \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} d\log P \\ d\log M \\ d\log P^* \\ d\log \mu^* \end{bmatrix} = \begin{bmatrix} 0 \\ \mathcal{F} \\ 0 \\ -\mathcal{C} \end{bmatrix} d\log M^{**}.$$

We use the method of Gaussian elimination. Combining the ultimate, we obtain that:

$$d\log \mu^* = -\mathcal{C} d\log M^{**} < 0.$$

From the penultimate row, we have that

$$d\log P^* = \frac{\mathcal{C}}{2}\frac{\frac{1}{2}\left(\chi_{\theta^*}^{*,+} + \chi_{\theta^*}^{*,-}\right)\theta_{\mu^*}^* \mu^*}{R^b} d\log M^{**} < 0.$$

Substituting this outcome into the second column, we obtain

$$\begin{bmatrix} R^b - \frac{1}{2}\left(\chi_\theta^+ + \chi_\theta^-\right)\theta_\mu \mu & \frac{1}{2}\left(\chi_\theta^+ + \chi_\theta^-\right)\theta_\mu \mu \\ -\mathcal{F} & 1 \end{bmatrix} \begin{bmatrix} d\log P \\ d\log M \end{bmatrix} = \begin{bmatrix} 0 \\ \mathcal{F}\left(1 - \frac{\mathcal{C}}{2}\frac{\frac{1}{2}\left(\chi_{\theta^*}^{*,+} + \chi_{\theta^*}^{*,-}\right)\theta_{\mu^*}^* \mu^*}{R^b}\right) \end{bmatrix} d\log M^{**}.$$

Inverting the matrix on the right, we have:

$$\begin{bmatrix} \frac{dP}{P} \\ \frac{dM}{M} \end{bmatrix} = \begin{bmatrix} 1 & -\frac{1}{2}\left(\chi_\theta^+ + \chi_\theta^-\right)\theta_\mu \mu \\ \mathcal{F} & R^b - \frac{1}{2}\left(\chi_\theta^+ + \chi_\theta^-\right)\theta_\mu \mu \end{bmatrix} \begin{bmatrix} 0 \\ \mathcal{F}\left(1 - \frac{\mathcal{C}}{2}\frac{\frac{1}{2}\left(\chi_{\theta^*}^{*,+} + \chi_{\theta^*}^{*,-}\right)\theta_{\mu^*}^* \mu^*}{R^b}\right) \end{bmatrix} d\log M^{**}.$$

Hence, the solution is:

$$\begin{bmatrix} \frac{dP}{P} \\ \frac{dM}{M} \end{bmatrix} = \frac{\begin{bmatrix} -\frac{1}{2}\left(\chi_{\theta}^{+}+\chi_{\theta}^{-}\right)\theta_{\mu}\mu \\ R^{b}-\frac{1}{2}\left(\chi_{\theta}^{+}+\chi_{\theta}^{-}\right)\theta_{\mu}\mu \end{bmatrix}}{R^{b}-\frac{(1-\mathcal{F})}{2}\left(\chi_{\theta}^{+}+\chi_{\theta}^{-}\right)\theta_{\mu}\mu}\mathcal{F}\left(1-\frac{\mathcal{C}}{2}\frac{\frac{1}{2}\left(\chi_{\theta^{*}}^{*,+}+\chi_{\theta^{*}}^{*,-}\right)\theta_{\mu^{*}}^{*}\mu^{*}}{R^{b}}\right)\frac{dM^{**}}{M^{**}} \geq 0.$$

Hence, both terms are greater than zero. Thus we have that:

$$\frac{d\log e}{d\log M^{**}} = \frac{d\log P}{d\log M^{**}} - \frac{d\log P^{*}}{d\log M^{**}} = \Gamma\mathcal{F}\left(1-\mathcal{C}\cdot\Gamma^{*}\right)-\mathcal{C}\Gamma^{*} \geq 0.$$

where:

$$\Gamma^{*} \equiv \frac{\frac{1}{2}\frac{d[\chi^{*,+}+\chi^{*,-}]}{d\theta^{*}}\frac{d\theta^{*}}{d\mu^{*}}\mu^{*}}{R^{b}},$$

is the sensitivity of dollar prices to a change in the liquidity ratio and

$$\Gamma \equiv \frac{-\frac{1}{2}\left(\chi_{\theta}^{+}+\chi_{\theta}^{-}\right)\theta_{\mu}\mu}{R^{b}-(1-\mathcal{F})\frac{1}{2}\left(\chi_{\theta}^{+}+\chi_{\theta}^{-}\right)\theta_{\mu}\mu},$$

the sensitivity of Euro prices.

— STOPPED HERE...

The liquidity ratio follows:

$$\frac{d\log\mu}{d\log M^{**}} = \frac{dM}{M} - \frac{dP}{P} = \frac{R^{b}\cdot\left(1-\Gamma^{*}\cdot\mathcal{M}\cdot\mathcal{A}\right)}{R^{b}-(1-\mathcal{F})\frac{1}{2}\left(\chi_{\theta}^{+}+\chi_{\theta}^{-}\right)\theta_{\mu}\mu} \geq 0.$$

From here we obtain that:

$$\frac{d\mathcal{DLP}}{d\log M^{**}} = R^{m}\Gamma\left(1-\mathcal{M}\cdot\mathcal{A}\cdot\Gamma^{*}\right)-R^{*,m}\mathcal{A}\Gamma^{*} \geq 0.$$

and that:

$$\frac{d\mathcal{EBP}}{d\log M^{**}} = -R^{*,m}\mathcal{A}\Gamma^{*} > 0.$$

Sterilized Intervention.

B.8 Proof of Proposition (Dollar Dominance)

B.8.1 The case with $s^{*}_{[s^{*}>0]}+s_{[s<0]}>0$.

Consider now the case where dollars can be partially used to settle euro positions. We also let volatility in payments be the same in both countries, $\delta=\delta^{*}$. The corresponding tightness in Euros is given by:

$$\theta=\frac{\frac{1}{4}\left(\delta D-M\right)}{\frac{1}{2}\left(\delta D+M\right)-\frac{1}{4}\left(\delta D-M\right)}=\frac{\delta-\mu}{\delta+\mu}.$$

For the dollar, the tightness is given by:

$$\theta^{*}=\frac{1/2\left(\delta D^{*}-M^{*}\right)}{\frac{1}{4}\delta D^{*}+M^{*}+\frac{1}{4}\left(\delta D^{*}+M^{*}-e\left(\delta D-M\right)\right)}=\frac{\delta-\mu^{*}}{\delta+\mu^{*}-\frac{1}{2}\nu\left(\delta-\mu\right)}.$$

where $\nu=D/P/D^{*}/P^{*}$. For convenience, we derive the following derivatives:

We obtain the following derivatives:

$$\theta_{\mu}=-\frac{(1+\theta)}{\delta+\mu}\text{ and }\theta_{\delta}=\frac{(1-\theta)}{\delta+\mu}>0.$$

and

$$\theta_{\mu^{*}}^{*}=-\frac{(1+\theta^{*})}{\delta+\mu^{*}-\nu\left(\delta-\mu\right)},\ \theta_{\mu}^{*}=-\theta^{*}\frac{\frac{1}{2}\nu}{\delta+\mu^{*}-\nu\left(\delta-\mu\right)},\text{ and }\theta_{\delta}^{*}=\frac{(1-\theta^{*})}{\delta+\mu^{*}-\nu\left(\delta-\mu\right)}>0.$$

Also, consider a situation where tightness in both currencies is the same. Then,

$$\frac{\delta-\mu}{\delta+\mu}=\frac{\delta-\mu^{*}}{\delta+\mu^{*}-\frac{1}{2}\nu\left(\delta-\mu\right)}.$$

If $\mu=\mu^{*}$, we have that $\theta^{*}>\theta$. Thus, since we know the that the dollar tightness decreases with μ^{*}, it must be that $\mu^{*}>\mu$ if $\theta=\theta^{*}$. Then for the symmetric case:

$$\theta_{\mu^{*}}^{*}<\theta_{\mu}\text{ and }\theta_{\delta}^{*}¿\theta_{\delta}$$

Temporary shocks to Payment volatility: Now, consider the effect of an increase in δ, in both countries, under the assumption that $\delta=\delta^{*}$. The excess bond premium in dollars is,

$$R^{b}=\left(1+i^{*,m}\right)i\frac{P^{*}}{\mathbb{E}\left[p^{*}\left(X'\right)\right]}+\frac{1}{2}\left(\chi^{*,+}+\chi^{*,-}\right).\tag{52}$$

The excess bond premium in Euros is,

$$R^{b}=\left(1+i^{m}\right)\frac{P}{\mathbb{E}\left[p\left(X'\right)\right]}+\frac{1}{2}\chi^{+}+\frac{1}{4}\chi^{-}+\frac{1}{4}\chi^{*,+}.\tag{53}$$

Taking total differentials:

$$
\begin{aligned}
0 &= \left(1+i^{*,m}\right) \frac{P^*}{\mathbb{E}\left[p^*\left(X'\right)\right]} \frac{dP^*}{P^*} + \frac{1}{2}\left(\chi^{*,+} + \chi^{*,-}\right) \frac{dP^*}{P^*} \\
&+ \frac{1}{2}\left(\chi_{\theta^*}^{*,+} + \chi_{\theta^*}^{*,-}\right) \cdot \left(\theta_{\mu^*}^* d\mu^* + \theta_{\mu}^* d\mu + \theta_{\delta^*}^* d\delta^*\right).
\end{aligned}
$$

Likewise for the Euro:

$$
\begin{aligned}
0 &= \left(1+i^{m}\right) \frac{P}{\mathbb{E}\left[p\left(X'\right)\right]} \frac{dP}{P} + \frac{1}{2}\left(\chi^{+} + \chi^{-}\right) \frac{dP}{P} \\
&+ \frac{1}{4}\left(\chi_{\theta^*}^{*,+}\right) \cdot \left(\theta_{\mu^*}^* d\mu^* + \theta_{\mu}^* d\mu + \theta_{\delta^*}^* d\delta\right) \\
&+ \left(\frac{1}{2}\chi_{\theta}^{+} + \frac{1}{4}\chi_{\theta}^{-}\right) \cdot \left(\theta_{\mu} d\mu + \theta_{\delta} d\delta\right).
\end{aligned}
$$

We also know that:

$$
d\mu = -\mu d\log P \text{ and } d\mu^* = -\mu^* d\log P^*.
$$

Thus, the premia become:

$$
\begin{aligned}
0 &= \left(R^b - \frac{1}{2}\left(\chi_{\theta^*}^{*,+} + \chi_{\theta^*}^{*,-}\right)\theta_{\mu^*}^* \mu^*\right) d\log P^* \\
&- \frac{1}{2}\left(\chi_{\theta^*}^{*,+} + \chi_{\theta^*}^{*,-}\right)\theta_{\mu}^* \mu d\log P \\
&+ \frac{1}{2}\left(\chi_{\theta^*}^{*,+} + \chi_{\theta^*}^{*,-}\right) \cdot \left(\theta_{\delta^*}^* \delta d\log \delta\right).
\end{aligned}
$$

Likewise, we have that:

$$
\begin{aligned}
0 &= \left(R^b - \left(\frac{1}{2}\chi_{\theta}^{+} + \frac{1}{4}\chi_{\theta}^{-}\right)\theta_{\mu}\mu - \frac{1}{4}\left(\chi_{\theta^*}^{*,+}\right)\theta_{\mu}^* \mu\right) d\log P \\
&- \frac{1}{4}\chi_{\theta^*}^{*,+} \cdot \theta_{\mu^*}^* \cdot \mu \cdot d\log P^* \\
&+ \left(\frac{1}{2}\chi_{\theta}^{+} + \frac{1}{4}\chi_{\theta}^{-} + \frac{1}{4}\chi_{\theta^*}^{*,+}\right) \cdot c d\log \delta.
\end{aligned}
$$

We stack both equations to obtain:

$$
\mathcal{A}\begin{bmatrix} d\log P^* \\ d\log P \end{bmatrix} = \mathcal{C} \cdot d\log \delta.
$$

where

$$\mathcal{A} = \begin{bmatrix} \left(R^b - \frac{1}{2}\left(\chi_{\theta^*}^{*,+} + \chi_{\theta^*}^{*,-}\right)\theta_{\mu^*}^*\mu^*\right) & -\frac{1}{2}\left(\chi_{\theta^*}^{*,+} + \chi_{\theta^*}^{*,-}\right)\theta_{\mu}^*\mu \\ -\frac{1}{4}\chi_{\theta^*}^{*,+} \cdot \theta_{\mu^*}^* \cdot \mu & \left(R^b - \left(\frac{1}{2}\chi_\theta^+ + \frac{1}{4}\chi_\theta^-\right)\theta_\mu\mu - \frac{1}{4}\left(\chi_{\theta^*}^{*,+}\right)\theta_\mu^*\mu\right) \end{bmatrix}$$

and

$$\mathcal{C} = \begin{bmatrix} \frac{1}{2}\left(\chi_{\theta^*}^{*,+} + \chi_{\theta^*}^{*,-}\right)\theta_\delta^* \\ \left(\left(\frac{1}{2}\chi_\theta^+ + \frac{1}{4}\chi_\theta^-\right)\theta_\delta + \frac{1}{4}\chi_{\theta^*}^{*,+}\theta_\delta^*\right) \end{bmatrix}.$$

Note that:

$$\mathcal{A}^{-1} = \frac{\begin{bmatrix} \left(R^b - \left(\frac{1}{2}\chi_\theta^+ + \frac{1}{4}\chi_\theta^-\right)\theta_\mu\mu - \frac{1}{4}\left(\chi_{\theta^*}^{*,+}\right)\theta_\mu^*\mu\right) & \frac{1}{2}\left(\chi_{\theta^*}^{*,+} + \chi_{\theta^*}^{*,-}\right)\theta_\mu^*\mu \\ \frac{1}{4}\chi_{\theta^*}^{*,+} \cdot \theta_{\mu^*}^* \cdot \mu & \left(R^b - \frac{1}{2}\left(\chi_{\theta^*}^{*,+} + \chi_{\theta^*}^{*,-}\right)\theta_{\mu^*}^*\mu^*\right) \end{bmatrix}}{\det A}.$$

where

$$\begin{aligned} \det A &= \left(R^b - \frac{1}{2}\left(\chi_{\theta^*}^{*,+} + \chi_{\theta^*}^{*,-}\right)\theta_{\mu^*}^*\mu^*\right) \\ &\times \left(R^b - \left(\frac{1}{2}\chi_\theta^+ + \frac{1}{4}\chi_\theta^-\right)\theta_\mu\mu - \frac{1}{4}\left(\chi_{\theta^*}^{*,+}\right)\theta_\mu^*\mu\right) \\ &- \left(\frac{1}{4}\chi_{\theta^*}^{*,+} \cdot \theta_{\mu^*}^* \cdot \mu\right)\frac{1}{2}\left(\chi_{\theta^*}^{*,+} + \chi_{\theta^*}^{*,-}\right)\theta_\mu^*\mu. \end{aligned}$$

Next, observe that:

$$\begin{bmatrix} d\log P^* \\ d\log P \end{bmatrix} = \begin{bmatrix} \left(R^b - \left(\frac{1}{2}\chi_\theta^+ + \frac{1}{4}\chi_\theta^-\right)\theta_\mu\mu - \frac{1}{4}\left(\chi_{\theta^*}^{*,+}\right)\theta_\mu^*\mu\right)\frac{1}{2}\left(\chi_{\theta^*}^{*,+} + \chi_{\theta^*}^{*,-}\right)\theta_\delta^* + \frac{1}{2}\left(\chi_{\theta^*}^{*,+} + \chi_{\theta^*}^{*,-}\right)\theta_\mu^*\mu\left(\left(\frac{1}{2}\chi_\theta^+ + \frac{1}{4}\chi\right.\right. \\ \frac{1}{4}\chi_{\theta^*}^{*,+} \cdot \theta_{\mu^*}^* \cdot \mu\frac{1}{2}\left(\chi_{\theta^*}^{*,+} + \chi_{\theta^*}^{*,-}\right)\theta_\delta + \left(R^b - \frac{1}{2}\left(\chi_{\theta^*}^{*,+} + \chi_{\theta^*}^{*,-}\right)\theta_{\mu^*}^*\mu^*\right)\left(\left(\frac{1}{2}\chi_\theta^+ + \frac{1}{4}\chi_\theta^-\right)\theta_\delta + \right.\right. \end{bmatrix}$$

Subtracting the first row from the second:

$$\begin{aligned} \frac{d\log e}{d\log\delta} &= \left(R^b - \left(\chi_{\theta^*}^{*,+} + \chi_{\theta^*}^{*,-}\right)\theta_{\mu^*}^*\mu^*\right)\left(\left(\frac{1}{2}\chi_\theta^+ + \frac{1}{4}\chi_\theta^-\right)\theta_\delta^* + \frac{1}{4}\chi_{\theta^*}^{*,+}\theta_\delta^*\right) \\ &\quad -\frac{1}{2}\left(R^b - \left(\frac{1}{2}\chi_\theta^+ + \frac{1}{4}\chi_\theta^-\right)\theta_\mu\mu - \frac{1}{4}\chi_{\theta^*}^{*,+}\theta_\mu^*\mu\right)\left(\chi_{\theta^*}^{*,+} + \chi_{\theta^*}^{*,-}\right)\theta_\delta^*. \end{aligned}$$

Then, $d\log e > 0$ if and only if:

$$\frac{\left(\frac{1}{2}\chi_\theta^+\theta_\delta + \frac{1}{4}\chi_\theta^-\theta_\delta + \frac{1}{4}\chi_\theta^+\theta_\delta^*\right)}{\left(R^b - \left(\frac{1}{2}\chi_\theta^+ + \frac{1}{4}\chi_\theta^-\right)\theta_\mu\mu - \frac{1}{4}\left(\chi_\theta^+\right)\theta_\mu^*\mu\right)} > \frac{\frac{1}{2}\left(\chi_\theta^+ + \chi_\theta^-\right)\theta_\delta^*}{R^b - \frac{1}{2}\left(\chi_\theta^+ + \chi_\theta^-\right)\theta_{\mu^*}^*\mu^*}..$$

Hence, the sign is clearly ambiguous because both terms are positive in absolute value.

Provided we start with a symmetric case where $\theta = \theta^*$, we obtain that $d\log e > 0$ if

$$\frac{\left(\frac{1}{2}\chi_\theta^+\theta_\delta + \frac{1}{4}\chi_\theta^-\theta_\delta + \frac{1}{4}\chi_\theta^+\theta_\delta^*\right)}{\left(R^b - \left(\frac{1}{2}\chi_\theta^+ + \frac{1}{4}\chi_\theta^-\right)\theta_\mu\mu - \frac{1}{4}\left(\chi_\theta^+\right)\theta_\mu^*\mu\right)} > \frac{\frac{1}{2}\left(\chi_\theta^+ + \chi_\theta^-\right)\theta_\delta^*}{R^b - \frac{1}{2}\left(\chi_\theta^+ + \chi_\theta^-\right)\theta_{\mu^*}^*\mu^*}.$$

Notice that when $\theta = \theta^*$, we have that:

$$\frac{\theta_\delta}{\theta_\delta^*} = \frac{\theta_\mu}{\theta_{\mu^*}^*} = \frac{\delta + \mu^* - \nu\left(\delta - \mu\right)}{\delta + \mu} \text{ and } \frac{\theta_\mu^*}{\theta_{\mu^*}^*} = \frac{1}{2}\frac{\theta^*}{(1+\theta^*)}\nu$$

We obtain the desired inequality if conditions #1

$$\frac{1}{2}\chi_\theta^+\theta_\delta + \frac{1}{4}\chi_\theta^-\theta_\delta + \frac{1}{4}\chi_\theta^+\theta_\delta^* > \frac{1}{2}\left(\chi_\theta^+ + \chi_\theta^-\right)\theta_\delta^*$$

and #2 hold together:

$$\frac{1}{2}\left(\chi_\theta^+ + \chi_\theta^-\right)\theta_{\mu^*}^*\mu^* < \left(\frac{1}{2}\chi_\theta^+ + \frac{1}{4}\chi_\theta^-\right)\theta_\mu\mu - \frac{1}{4}\left(\chi_\theta^+\right)\theta_\mu^*\mu.$$

Condition #1 can be re-written to obtain:

$$\frac{\theta_\delta}{\theta_\delta^*} > \frac{\frac{1}{4}\chi_\theta^+ + \frac{1}{2}\chi_\theta^-}{\left(\frac{1}{2}\chi_\theta^+ + \frac{1}{4}\chi_\theta^-\right)} > 1.$$

Thus, we need that be sufficiently large:

$$\frac{\theta_\delta}{\theta_\delta^*} = \frac{\delta + \mu^* - \nu\left(\delta - \mu\right)}{\delta + \mu} > 1,$$

hence, ν be sufficiently large.

Condition #2, we obtain that:

$$\frac{1}{2}\left(\chi_\theta^+ + \chi_\theta^-\right) < \left(\frac{1}{2}\chi_\theta^+ + \frac{1}{4}\chi_\theta^-\right)\frac{\theta_\mu\mu}{\theta_{\mu^*}^*\mu^*} + \frac{1}{4}\left(\chi_\theta^+\right)\frac{\theta_\mu^*\mu}{\theta_{\mu^*}^*\mu^*}.$$

Thus, we again need a high value for $\frac{\theta_\mu}{\theta_\mu^*}$ and for $\frac{\theta_\mu^*}{\theta_{\mu^*}^*}$. Thus,

$$\frac{\theta_\mu^*}{\theta_{\mu^*}^*} = \frac{1}{2}\nu\frac{\theta^*}{(1-\theta^*)}.$$

Thus, in both cases, we need a sufficiently high value for ν.

B.8.2 The case with $s^*_{[s^*>0]} + s_{[s<0]} < 0$.

Preliminary Considerations. Consider now the case where dollars can be partially used to settle euro positions. We also let volatility in payments be the same in both countries, $\delta = \delta^*$. In particular, we let a fraction α of Euros be used for settlements. The corresponding tightness in Euros and Dollars are given by:

$$\theta = -\frac{\frac{1}{4}(\delta D - M) + \frac{1}{4}(\delta D - M - e\alpha M^*)}{\frac{1}{2}(\delta D + M)} = -\frac{1}{2} \cdot \left(\frac{\delta - \mu}{\delta + \mu} + \frac{\delta - \mu - \alpha \cdot \mu^* \cdot \nu}{\delta + \mu}\right).$$

where $\nu = D^*/P^*/D/P$. For convenience, we derive the following derivatives:

$$\theta_\mu = \frac{1+\theta}{\delta+\mu} > 0, \; \theta_{\mu^*} = \frac{1}{2} \cdot \frac{\alpha\nu}{\delta+\mu} > 0, \text{ and } \theta_\delta = -\frac{1-\theta}{\delta+\mu} < 0.$$

For the dollar, the tightness is given by:

$$\theta = -\frac{(\delta D^* - M^*)}{\frac{1}{4}(\delta D^* + M^*) + \frac{1}{4}(\delta D^* + (1-\alpha) M^*)} = -2\left(\frac{\delta - \mu^*}{\delta + \mu^*} + \frac{\delta - \mu^*}{\delta + (1-\alpha)\mu^*}\right).$$

We obtain the following derivatives:

$$\theta^*_{\mu^*} = 2\left(\frac{1}{\delta+\mu^*} + \frac{1}{\delta + (1-\alpha)\mu^*}\right) + 2\left(\frac{\delta-\mu^*}{\delta+\mu^*}\frac{1}{\delta+\mu^*} + \frac{\delta-\mu^*}{\delta+(1-\alpha)\mu^*}\frac{(1-\alpha)}{\delta+(1-\alpha)\mu^*}\right) > 0,$$

and

$$\theta^*_{\delta} = 2\left(\frac{1}{\delta+\mu^*} + \frac{1}{\delta + (1-\alpha)\mu^*}\right) + 2\left(\frac{\delta-\mu^*}{\delta+\mu^*}\frac{1}{\delta+\mu^*} + \frac{\delta-\mu^*}{\delta+(1-\alpha)\mu^*}\frac{1}{\delta+(1-\alpha)\mu^*}\right) > 0.$$

Key Equilibrium Conditions: case with dollar surplus less than Euro deficit Now, consider the effect of an increase in δ, in both countries, under the assumption that $\delta = \delta^*$. The excess bond premium in dollars is,

$$R^b = (1+i^{*,m})\frac{P^*}{\mathbb{E}[p^*(X')]} + \frac{1}{4}\left(\chi^{*,+} + \chi^-\right) + \frac{1}{2}\chi^{*,-}. \tag{54}$$

The excess bond premium in Euros is,

$$R^b = (1+i^m)\frac{P}{\mathbb{E}[p(X')]} + \frac{1}{2}\left(\chi^- + \chi^+\right). \tag{55}$$

We compute the total differential for small changes in δ in both excess bond premia. For dollars, we obtain:

$$\begin{aligned}0 &= (1+i^{*,m})\frac{P^*}{\mathbb{E}\left[p^*(X')\right]}\frac{dP^*}{P^*}+\frac{1}{2}\left(\frac{1}{2}\chi^{*,+}+\chi^{*,-}\right)\frac{dP^*}{P^*}\\ &+ \frac{1}{4}\chi^-\frac{dP}{P}+\frac{1}{2}\left(\frac{1}{2}\chi^{*,+}_{\theta^*}+\chi^{*,-}_{\theta^*}\right)\cdot\left(\frac{\partial\theta^*}{\partial\mu^*}d\mu^*+\frac{\partial\theta^*}{\partial\delta^*}d\delta^*\right)+\frac{1}{4}\chi^-_\theta\frac{\partial\theta}{\partial\mu^*}d\mu^*.\end{aligned}$$

Re-arranging and collecting terms, and substituting (xxx, FX.logdifferential), expressing terms in their log differential forms, we obtain:

$$\begin{aligned}0 &= \left[(1+i^{*,m})\frac{P^*}{\mathbb{E}\left[p^*(X')\right]}+\frac{1}{2}\left(\frac{1}{2}\chi^{*,+}+\chi^{*,-}\right)\right]d\log P^*\\ &+ \frac{1}{4}\chi^-d\log P\\ &+ \frac{1}{2}\left(\frac{1}{2}\chi^{*,+}_{\theta^*}+\chi^{*,-}_{\theta^*}\right)\cdot\left(\frac{\partial\theta^*}{\partial\mu^*}\mu^*d\log\mu^*+\frac{\partial\theta^*}{\partial\delta^*}\delta^*d\log\delta^*\right)+\frac{1}{4}\chi^-_\theta\left(\frac{\partial\theta}{\partial\mu^*}\mu^*d\mu^*+\frac{\partial\theta}{\partial\delta}\delta d\log\delta\right).\end{aligned}$$

Thus, we obtain:

$$\begin{aligned}0 &= \left(R^b-\frac{1}{4}\chi^-\right)d\log P^*\\ &+ \frac{1}{4}\chi^-d\log P\\ &+ \frac{1}{2}\left(\frac{1}{2}\chi^{*,+}_{\theta^*}+\chi^{*,-}_{\theta^*}\right)\cdot\frac{\partial\theta^*}{\partial\mu^*}\mu^*d\log\mu^*+\frac{1}{4}\chi^-_\theta\cdot\frac{\partial\theta}{\partial\mu^*}\mu^*d\log\mu^*\\ &+ \left(\frac{1}{2}\left(\frac{1}{2}\chi^{*,+}_{\theta^*}+\chi^{*,-}_{\theta^*}\right)\frac{\partial\theta^*}{\partial\delta^*}\delta^*+\frac{1}{4}\chi^-_\theta\frac{\partial\theta}{\partial\delta}\delta\right)d\log\delta.\end{aligned} \tag{56}$$

Then, for domestic EBP, we obtain:

$$0=(1+i^m)\frac{P}{\mathbb{E}\left[p(X')\right]}\frac{dP}{P}+\frac{1}{2}\left(\chi^++\chi^-\right)\frac{dP}{P}+\frac{1}{2}\left(\chi^+_\theta+\chi^-_\theta\right)\cdot\left(\frac{\partial\theta}{\partial\mu}d\mu+\frac{\partial\theta}{\partial\mu^*}d\mu^*\frac{\partial\theta}{\partial\delta}d\delta\right).$$

Re-arranging terms and expressing the differentials in their logistic version, we obtain:

$$0=R^bd\log P+\frac{1}{2}\left(\chi^+_\theta+\chi^-_\theta\right)\cdot\left(\frac{\partial\theta}{\partial\mu}\mu d\log\mu+\frac{\partial\theta}{\partial\mu^*}\mu^*d\log\mu^*+\frac{\partial\theta}{\partial\delta}\delta d\log\delta\right).$$

Thus, collecting terms we obtain:

$$\begin{aligned} 0 &= R^b d\log P \\ &+ \frac{1}{2}\left(\chi_\theta^+ + \chi_\theta^-\right)\frac{\partial\theta}{\partial\mu}\mu d\log\mu \\ &+ \frac{1}{2}\left(\chi_\theta^+ + \chi_\theta^-\right)\frac{\partial\theta}{\partial\mu^*}\mu^* d\log\mu^* \\ &+ \frac{1}{2}\left(\chi_\theta^+ + \chi_\theta^-\right)\frac{\partial\theta}{\partial\delta}\delta d\log\delta. \end{aligned} \tag{57}$$

Since deposits and the money supplies are fixed, we have that $d\log P^* = -d\log\mu^*$ and $d\log P = -d\log\mu$, we can write (56) and (57) as:

$$\begin{aligned} 0 &= \left(R^b - \frac{1}{4}\left(\chi^- + \chi_\theta^- \cdot \frac{\partial\theta}{\partial\mu^*}\mu^*\right) - \frac{1}{2}\left(\frac{1}{2}\chi_{\theta^*}^{*,+} + \chi_{\theta^*}^{*,-}\right)\cdot\frac{\partial\theta^*}{\partial\mu^*}\mu^*\right) d\log P^* \\ &+ \frac{1}{4}\chi^- d\log P \\ &+ \left(\frac{1}{2}\left(\frac{1}{2}\chi_{\theta^*}^{*,+} + \chi_{\theta^*}^{*,-}\right)\frac{\partial\theta^*}{\partial\delta^*}\delta^* + \frac{1}{4}\chi_\theta^-\frac{\partial\theta}{\partial\delta}\delta\right) d\log\delta. \end{aligned} \tag{58}$$

and

$$\begin{aligned} 0 &= \left(R^b - \frac{1}{2}\left(\chi_\theta^+ + \chi_\theta^-\right)\frac{\partial\theta}{\partial\mu}\mu\right) d\log P \\ &- \frac{1}{2}\left(\chi_\theta^+ + \chi_\theta^-\right)\frac{\partial\theta}{\partial\mu^*}\mu^* d\log P^* \\ &+ \frac{1}{2}\left(\chi_\theta^+ + \chi_\theta^-\right)\frac{\partial\theta}{\partial\delta}\delta d\log\delta. \end{aligned} \tag{59}$$

We express (59) and (59) in matrix form,

$$\mathcal{A}\begin{bmatrix} d\log P^* \\ d\log P \end{bmatrix} = \mathcal{C}\cdot d\log\delta.$$

where

$$\mathcal{A} = \begin{bmatrix} \left(R^b - \frac{1}{4}\left(\chi^- + \chi_\theta^-\cdot\frac{\partial\theta}{\partial\mu^*}\mu^*\right) - \frac{1}{2}\left(\frac{1}{2}\chi_{\theta^*}^{*,+} + \chi_{\theta^*}^{*,-}\right)\cdot\frac{\partial\theta^*}{\partial\mu^*}\mu^*\right) & \frac{1}{4}\chi^- \\ -\frac{1}{2}\left(\chi_\theta^+ + \chi_\theta^-\right)\left(\frac{\partial\theta}{\partial\mu^*}\mu^*\right) & \left(R^b - \frac{1}{2}\left(\chi_\theta^+ + \chi_\theta^-\right)\frac{\partial\theta}{\partial\mu}\mu\right) \end{bmatrix}$$

and

$$\mathcal{C} = -\frac{1}{2}\left[\begin{array}{c} \left(\frac{1}{2}\left(\frac{1}{2}\chi_{\theta^*}^{*,+} + \chi_{\theta^*}^{*,-}\right)\frac{\partial\theta^*}{\partial\delta^*}\delta^* + \frac{1}{4}\chi_\theta^- \frac{\partial\theta}{\partial\delta}\delta\right) \\ \frac{1}{2}\left(\chi_\theta^+ + \chi_\theta^-\right)\frac{\partial\theta}{\partial\delta}\delta \end{array}\right].$$

Thus,

$$\left[\begin{array}{c} d\log P^* \\ d\log P \end{array}\right] = \mathcal{A}^{-1}\mathcal{C}\cdot d\log\delta.$$

Note that:

$$\mathcal{A}^{-1} = \frac{\left[\begin{array}{cc} \left(R^b - \frac{1}{2}\left(\chi_\theta^+ + \chi_\theta^-\right)\frac{\partial\theta}{\partial\mu}\mu\right) & -\frac{1}{4}\chi^- \\ \frac{1}{2}\left(\chi_\theta^+ + \chi_\theta^-\right)\left(\frac{\partial\theta}{\partial\mu^*}\mu^*\right) & \left(R^b - \frac{1}{4}\left(\chi^- + \chi_\theta^- \cdot \frac{\partial\theta}{\partial\mu^*}\mu^*\right) - \frac{1}{2}\left(\frac{1}{2}\chi_{\theta^*}^{*,+} + \chi_{\theta^*}^{*,-}\right)\cdot\frac{\partial\theta^*}{\partial\mu^*}\mu^*\right) \end{array}\right]}{\det A}.$$

where

$$\begin{aligned} \det A &= \left(R^b - \frac{1}{2}\left(\frac{1}{2}\chi_{\theta^*}^{*,+} + \chi_{\theta^*}^{*,-}\right)\cdot\frac{\partial\theta^*}{\partial\mu^*}\mu^* - \frac{1}{4}\left(\chi^- + \chi_\theta^- \cdot \frac{\partial\theta}{\partial\mu^*}\mu^*\right)\right) \\ &\times \left(R^b - \frac{1}{2}\left(\chi_\theta^+ + \chi_\theta^-\right)\frac{\partial\theta}{\partial\mu}\mu\right) \\ &+ \frac{1}{4}\chi^- \frac{1}{2}\left(\chi_\theta^+ + \chi_\theta^-\right)\frac{\partial\theta}{\partial\mu^*}\mu^*. \end{aligned}$$

Then,

$$\left[\begin{array}{c} d\log P^* \\ d\log P \end{array}\right] = -\frac{1}{2}\left[\begin{array}{cc} \left(R^b - \frac{1}{2}\left(\chi_\theta^+ + \chi_\theta^-\right)\frac{\partial\theta}{\partial\mu}\mu\right)\left(\chi_\theta^+ + \chi_\theta^-\right)\frac{\partial\theta}{\partial\delta}\delta & -\frac{1}{4}\chi^-\left(\frac{1}{2}\chi_{\theta^*}^{*,+} + \chi_{\theta^*}^{*,-} + \frac{1}{4}\chi \right. \\ \frac{1}{2}\left(\chi_\theta^+ + \chi_\theta^-\right)\frac{\partial\theta}{\partial\mu^*}\mu^*\left(\chi_\theta^+ + \chi_\theta^-\right)\frac{\partial\theta}{\partial\delta}\delta & \left(R^b - \frac{1}{4}\chi^- - \frac{1}{2}\left(\frac{1}{2}\chi_{\theta^*}^{*,+} + \chi_{\theta^*}^{*,-} + \frac{1}{4}\chi_\theta^-\right)\cdot\frac{\partial\theta^*}{\partial\mu^*}\mu^*\right) \end{array}\right.$$

To obtain the effect on the exchange rate, we note that:

$$d\log e = d\log P - d\log P^*$$

and thus:

$$d\log e = \frac{1}{2}R^b\left(\chi_\theta^+ + \chi_\theta^-\right)\frac{\partial\theta}{\partial\delta}\delta - \frac{1}{2}\left(R^b - \frac{1}{2}\left(\frac{1}{2}\chi_{\theta^*}^{*,+} + \chi_{\theta^*}^{*,-} + \frac{1}{4}\chi_\theta^-\right)\cdot\frac{\partial\theta^*}{\partial\mu^*}\mu^*\right)\left(\frac{1}{2}\chi_{\theta^*}^{*,+} + \chi_{\theta^*}^{*,-} + \frac{1}{4}\chi_\theta^-\right)\frac{\partial\theta^*}{\partial\delta}\delta$$

and thus:

$$d\log e = R^b\left(\frac{1}{2}\left(\chi_\theta^+ + \chi_\theta^-\right)\frac{\partial\theta}{\partial\delta}\delta - \left(\frac{1}{4}\chi_{\theta^*}^{*,+} + \frac{1}{2}\chi_{\theta^*}^{*,-} + \frac{1}{4}\chi_\theta^-\right)\frac{\partial\theta^*}{\partial\delta}\delta\right)$$
$$+ \frac{1}{4}\left(\frac{1}{2}\chi_{\theta^*}^{*,+} + \chi_{\theta^*}^{*,-} + \frac{1}{4}\chi_\theta^-\right)^2 \cdot \frac{\partial\theta^*}{\partial\mu^*}\mu^*\left(\frac{1}{2}\chi_{\theta^*}^{*,+} + \chi_{\theta^*}^{*,-} + \frac{1}{4}\chi_\theta^-\right)\frac{\partial\theta^*}{\partial\delta}\delta.$$

Reduced	Θ^x	ϵ^x	Θ^b	ϵ^b
Structural	$\bar{X}_t \beta^{1/\gamma^x}$	$\frac{1}{\gamma^x} - 1$	$(\alpha A_{t+1})^{-\left(\frac{\nu+1}{\alpha-(\nu+1)}\right)}$	$\left(\frac{\nu+1}{\alpha-(\nu+1)}\right)$

Table 6B: Structural to Reduced form Parameters

C Microfoundations for Deposit Supplies and Loan Demands

The equivalence table from the structural parameters to the reduced form paratemeters is:

Household Problem. Define the household net worth $e^h = \left(1+i_t^d\right) D + \left(1+i_t^G\right) G + \left(q_t + r_t^h\right) \Sigma - T_t^h$, as the right-hand side of its budget constraint, excluding labor income. Then, substitute c^h from the budget constraint and employ the definition e^h. We obtain the following value function:

$$V_t^h\left(G^h, D, \Sigma\right) = \max_{\left\{c^d, c^g, h, G', D', \Sigma'\right\}} U^d\left(c^d\right) + U^g\left(c^g\right) - \frac{h^{1+\nu}}{1+\nu} + e^h + \frac{z_t h - \left(P_t c^g + P_t c^d + D' + G' + q_t \Sigma'\right)}{P_t}$$
$$+\beta V_{t+1}^h\left(G', D', \Sigma'\right)$$

subject to $c^d \leq \left(1+i_t^d\right) \frac{D}{P_t}$ and $c^g \leq \frac{D}{G_t}$.

Step 1 - deposit and bond-goods demand. The step is to take the first-order conditions for $\left\{c^d, c^g\right\}$. Since $\{G, D\}$ enter symmetrically into the problem, we express the formulas in terms of $x \in \{d, g\}$, an index that corresponds to each asset. From the first-order conditions with respect to $\frac{D}{P_t}$ and $\frac{G}{P_t}$, we obtain that:

$$c^x(X, t) = \min\left\{\left(U_{c^x}^x\right)^{-1}(1), R_t^x \cdot \frac{X}{P_{t-1}}\right\} \text{ for } x \in \{d, g\}. \tag{60}$$

The expression shows that the deposit- and bond-in-advance constraints bind if the marginal utility associated with their consumption is less than one. Note that

$$U_{c^x}^x\left(\bar{X}\right) = \left(\bar{X}\right)^{\gamma^x} x^{-\gamma^x} \text{ for } x \in \{d, g\}, \tag{61}$$

marginal utility is above 1, for $X/P_t < \bar{X}$. Then, the marginal consumption as a function of real balances is:

$$c_{X/P_t}^x(X, t) = \begin{cases} R_t^x & X/P_t < \bar{X} \\ 0 & \text{otherwise} \end{cases} \text{ for } x \in \{d, g\}$$

We return to this conditions below to derive the demand for deposits and bonds by the non-financial sector.

Step 2 - labor supply. The first-order condition with respect to labor supply yields a labor supply that only depends on the real wage:

$$h_t^{\nu} = z_t/P_t. \tag{62}$$

Step 3 - deposit and bond demand. Next, we the derive deposit demand and T-Bill demand. By taking first-order conditions with respect to D'/P_t and G'/P_t, the real balances of deposits and bonds.

$$1 \quad = \quad \beta \frac{\partial V_{t+1}^h}{\partial (X'/P_t)} = \beta \left[\frac{\partial U^x}{\partial c^x} \cdot \frac{\partial c^x}{\partial (X'/P_t)} + \frac{\partial U^h}{\partial c^h} \cdot \frac{\partial c^h}{\partial (X'/P_t)} \right] \text{ for } x \in \{d, g\}.$$

The first equality follows directly from the first-order condtion and the second uses the envolope Theorem and the solution for the optimal consumption rule. If we shift the period in (60), by one period, the first-order condition then becomes:

$$\frac{1}{\beta} = \begin{cases} \frac{\partial U^x}{\partial c^x} R_t^x & X/P_t < \bar{X} \\ R_t^x & \text{otherwise} \end{cases} \text{ for } x \in \{d, g\}.$$

Finally, once we employe the definition of marginal utility, we obtain:

$$\frac{1}{\beta} = \begin{cases} \left(\bar{X}\right)^{\gamma^x} \left(R_t^x X/P_t\right)^{-\gamma^x} R_t^x & X/P_t < \bar{X} \\ R_t^x & \text{otherwise} \end{cases} \text{ for } x \in \{d, g\}.$$

Inverting the condition yields:

$$X/P_t = \begin{cases} \bar{X} \beta^{1/\gamma^x} \left(R_t^x\right)^{\frac{1}{\gamma^x} - 1} & R_t^x < 1/\beta \\ [\bar{X}, \infty) & R_t^x = 1/\beta \end{cases} \text{ for } x \in \{d, g\}.$$

Thus, we have that

$$\Theta_t^x = \bar{X}_t \beta^{1/\gamma^x} \text{ and } \epsilon^x = \frac{1}{\gamma^x} - 1 \text{ for } x \in \{d, g\}.$$

Next, we move to the firm's problem to obtain the demand for loans.

Firm Problem. In the appendix, we allow the firm to save in deposits whatever it doesn't spend in wages. From firm's problem, if we substitute the production function into the objective we obtain:

$$P_{t+1} r_{t+1}^h = \max_{B_{t+1}^d \geq 0, x_{t+1}, h_t \geq 0} P_{t+1} A_{t+1} h_t^{\alpha} - \left(1 + i_{t+1}^b\right) B_{t+1}^d + \left(1 + i_{t+1}^d\right) \left(B_{t+1}^d - z_t h_t\right)$$

subject to $z_t h_t \leq B^d_{t+1}$. Observe that

$$P_{t+1}A_{t+1}h_t^{\alpha} - \left(1+i^b_{t+1}\right)B^d_{t+1} + \left(1+i^d_{t+1}\right)\left(B^d_{t+1} - z_t h_t\right)$$
$$= P_{t+1}A_{t+1}h_t^{\alpha} - z_t h_t - \left(i^b_{t+1} - i^d_{t+1}\right)\left(B^d_{t+1} + z_t h_t\right).$$

Step 4 - loans demand. Since $i^b_{t+1} \geq i^d_{t+1}$, then it is wihout without loss of generality, that the working capital constraint is binding, $z_t h_t = B^d_{t+1}$. Thus, the objective is

$$P_{t+1}A_{t+1}h_t^{\alpha} - \left(1+i^b_{t+1}\right)z_t h_t.$$

The first-order condition in h_t yields

$$P_{t+1}\alpha A_{t+1}h_t^{\alpha} = \left(1+i^b_{t+1}\right)z_t h_t.$$

Dividing both sides by P_t, we obtain

$$\frac{P_{t+1}}{P_t}\alpha A_{t+1}h_t^{\alpha} = \left(1+i^b_{t+1}\right)\frac{z_t}{P_t}h_t.$$

Next, we use the labor supply function (62), to obtain the labor demand as a function of the loans rate:

$$\frac{P_{t+1}}{P_t}\alpha A_{t+1}h_t^{\alpha} \quad = \quad \left(1+i^b_{t+1}\right)h_t^{\nu+1} \rightarrow R^b_t = \frac{\alpha A_{t+1}h_t^{\alpha}}{h_t^{\nu+1}}. \tag{63}$$

Once we have the wage bill, and the fact that the working capital constraint is biding,

$$\frac{B^d_{t+1}}{P_t} \quad = \quad h_t\frac{z_t h_t}{P_t} = h_t^{\nu+1} \rightarrow h_t = \left(\frac{B^d_{t+1}}{P_t}\right)^{\frac{1}{\nu+1}}. \tag{64}$$

Thus, we can combine (63) and (64) to obtain the demand for loans:

$$R^b_t \quad = \quad \alpha A_{t+1}\left(\frac{B^d_{t+1}}{P_t}\right)^{-1}\left(\frac{B^d_{t+1}}{P_t}\right)^{\frac{\alpha}{\nu+1}} \rightarrow \frac{B^d_{t+1}}{P_t} = \Theta_t\left(R^b_{t+1}\right)^{\epsilon^b} \tag{65}$$

Thus, the coefficients of the loans demand are

$$\Theta^b_t \quad = \quad \left(\alpha A_{t+1}\right)^{-\epsilon^b} \text{ and } \epsilon^b = \left(\frac{\nu+1}{\alpha-(\nu+1)}\right).$$

Step 5 - deposit and bond demand. We replace the loans demand (65) into (64), to obtain the labor market equilibrium:

$$h_t = \left(\frac{1}{\alpha A_{t+1}}\right)^{\frac{1}{\alpha-(\nu+1)}} \left(R^b_{t+1}\right)^{\frac{1}{\alpha-(\nu+1)}}.$$

We replace (64) into the production function to obtain:

$$y_{t+1} = A_{t+1}\left(\frac{1}{\alpha A_{t+1}}\right)^{\frac{\alpha}{\alpha-(\nu+1)}} \left(R^b_{t+1}\right)^{\frac{\alpha}{\alpha-(\nu+1)}} \to y_{t+1} = \left(\frac{1}{\alpha}\right)^{\frac{\alpha}{\alpha-(\nu+1)}} A_{t+1}^{\frac{(\nu+1)}{\nu+1-\alpha}} \left(R^b_{t+1}\right)^{\frac{\alpha}{\alpha-(\nu+1)}}.$$

The profit of the firm is given by:

$$r^h_{t+1} = y_{t+1} - R^b_{t+1}B_{t+1} \to r^h_{t+1} = A_{t+1}^{\frac{(\nu+1)}{\nu+1-\alpha}} \left(\alpha^{-\frac{\alpha}{\alpha-(\nu+1)}} - \alpha^{-\frac{\nu+1}{\alpha-(\nu+1)}}\right) \cdot \left(R^b_{t+1}\right)^{\frac{\alpha}{\alpha-(\nu+1)}}.$$

The asset price q_t then is determine as:

$$q_t = \sum_{s\geq 1} \beta^s r^h_s.$$

With this, we conclude that output, hours and the firm price are decreasing in current (and future) loans rate.

Note that throghout the proof we use the labor market clearing condition. Then, clearing in the loans and deposit markets, by Walras's law, implies clearing in the goods market. Once we compute equilibria taking the schedules as exogenous in the bank's problem, it is possible to obtain output and household consumption from the equilibrium rate.

D Computational Algorithms

D.1 Algorithm to solve for transitions

We can consider the previous section as a steady-state version of the model, if prices are fixed in both currencies, then policy rates are actual real rates. In this section we consider what happens one period before the steady state, we call that period $t = 0$. Assume that the nominal policy rates are given:

$$(1 + i^{*,a}) \text{ for } a \in \{m, w\},$$

for the US and for the EU:

$$(1 + i^{a}) \text{ for } a \in \{m, w\} \cdot$$

The real rates now satisfy:

$$R^{*,a} = \frac{(1 + i^{*,a})}{(1 + \pi^{*})} \text{ for } a \in \{m, w\},$$

for the US and for the EU:

$$R^{a} = \frac{(1 + i^{a})}{(1 + \pi)} \cdot (1 + \Omega) \text{ for } a \in \{m, w\} \cdot$$

Where now we have that:

$$(1 + \pi^{*}) = \frac{p_{ss}}{p_0}$$

and

$$(1 + \Omega) = \frac{e_{ss}}{e_0}.$$

The values $\{p_{ss}, e_{ss}\}$ are solved from the steady state solution.

Algorithm for T-1 of Economy with Deposit Segmentation **Step 1.** Conjecture $\{R_0^m, R_0^{*,m}, R_0^w, R_0^{*,w}\}$. Solve for the liquidity ratios in Dollars and Euro $\{\mu, \mu^*, R^d, R^{*,d}\}$ using:

$$R^d + \frac{1}{2}\omega\left(\chi^{+}(\mu) - \chi^{-}(\mu)\right) = R^{*,d} + \frac{1}{2}\omega^{*}\left(\chi^{*,+}(\mu^{*}) - \chi^{*,-}(\mu^{*})\right)$$

$$R^m + \frac{1}{2}\left(\chi^{+}(\mu) + \chi^{-}(\mu)\right) = R^{*,m} + \frac{1}{2}\left(\chi^{*,+}(\mu^{*}) + \chi^{*,-}(\mu^{*})\right)$$

$$\Theta^b\left(\left(\upsilon(1-\mu) + (1-\mu^{*})\right)d^{*}\right)^{\epsilon} \quad = \quad R^{*,d} + \frac{1}{2}\omega^{*}\left(\chi^{+}(\mu) - \chi^{-}(\mu)\right)$$

$$R^{m} = R^{*,d} + \frac{1}{2}\omega\left(\chi^{+}(\mu) - \chi^{-}(\mu)\right) - \frac{1}{2}\left(\chi^{+}(\mu) + \chi^{-}(\mu)\right).$$

Step 2. Given the solutions to $\left\{R^{d}, R^{*,d}\right\}$, solve $\{d^{*}, \upsilon\}$ using:

$$d = \left[\frac{R^{d}}{\Theta^{d}}\right]^{1/\varsigma}$$

$$\upsilon = \left[\frac{R^{d}}{\Theta^{d}}\right]^{1/\varsigma}\left[\frac{R^{*,d}}{\Theta^{*,d}}\right]^{-1/\varsigma^{*}}.$$

Step 3. Solve for prices and the exchange rate using the solutions:

$$\mu\upsilon d^{*} = \frac{M}{p_{ss}}$$

$$\mu^{*}d^{*} = \frac{e}{p_{ss}}M^{*}$$

$$p_{ss}^{*} = e^{-1}p_{ss}.$$

Step 4. Update values for real policy rates:

$$R^{*,a} = \frac{(1+i^{*,a})}{(1+\pi^{*})} \text{ for } a \in \{m, w\},$$

for the US and for the EU:

$$R^{a} = \frac{(1+i^{a})}{(1+\pi)} \cdot (1+\Omega) \text{ for } a \in \{m, w\} \cdot$$

Where now we have that:

$$(1+\pi^{*}) = \frac{p_{ss}}{p_{0}}$$

and

$$(1+\Omega) = \frac{e_{ss}}{e_{0}}.$$

D.2 Algorithm to obtain a Global Solution

The algorithm to obtain the global solution to the model follows the algorithm to produce transitions. First, we define $s \in \mathcal{S} = \{1, 2, 3, \ldots, N^{s}\}$ to be a finite set of states. We let s follow a Markov process with transition matrix Q. Thus, $s' \sim Q(s)$. The state now affects the parameters of the model. That is, at each period, $\left\{\delta, \lambda, i^{*,m}, i^{m}, i^{*,w}, i^{w}, M, M^{*}, \Theta^{d}, \Theta^{*,d}, \Theta^{*,m}\right\}$ are all, potentially, functions of the state s.

The algorithm proceeds as follows. We define a "greed" parameter $\Delta^{greed}$and a tolerance parameters ε^{tol}, and construct a grid for $\mathcal{S}$. We conjecture a price-level functions $p_{(0)}(s), p^{*}_{(0)}(s)$ which produces a price levels in both currencies as a function of the state. As an initial guess, we propose to use $p_{(0)}(s) = p^{*}_{ss}$, and $p^{*}_{(0)}(s) = p^{*}_{ss}$ setting the exchange rate to its steady state level in all periods. We proceed by iterations, setting a tolerance count tol to $tol > 2 \cdot \varepsilon^{tol}$.

Outerloop 1: Iteration of price functions. We iterate price functions until they converge. Let n be the $n-th$ step of a given iteration. Given a $p_{(n)}(s), p^{*}_{(n)}(s)$, we produce a new price level functions $p_{(n+1)}(s), p^{*}_{(n+1)}(s)$ if $tol > \varepsilon^{tol}$.

Innerloop 1: Solve for real policy rates. For each s in the grid for $\mathcal{S}$, we solve for

$$\left\{R^{m}(s), R^{*,m}(s), R^{w}(s), R^{*,w}(s)\right\}.$$

Let j be the $j-th$ step of a given iteration. Conjecture values

$$\left\{R^{m}_{(0)}(s), R^{*,m}_{(0)}(s), R^{w}_{(0)}(s), R^{*,w}_{(0)}(s)\right\}$$

—we propose $\{R^{m}_{ss}, R^{*,m}_{ss}, R^{w}_{ss}, R^{*,w}_{ss}\}$ as an initial guess. We then update

$$\left\{R^{m}_{(j)}(s), R^{*,m}_{(j)}(s), R^{w}_{(j)}(s), R^{*,w}_{(j)}(s)\right\}$$

until we obtain convergence:

2.a Given this guess, we solve for the liquidity ratios in Dollars and Euro $\left\{\mu, \mu^{*}, R^{d}, R^{*,d}\right\}$ as a function of the state using:

$$R^{d} + \frac{1}{2}\omega\left(\chi^{+}(\mu) - \chi^{-}(\mu)\right) = R^{*,d} + \frac{1}{2}\omega^{*}\left(\chi^{*,+}(\mu^{*}) - \chi^{*,-}(\mu^{*})\right)$$

$$R^{m} + \frac{1}{2}\left(\chi^{+}(\mu) + \chi^{-}(\mu)\right) = R^{*,m} + \frac{1}{2}\left(\chi^{*,+}(\mu^{*}) + \chi^{*,-}(\mu^{*})\right)$$

$$\Theta^{b}\left(\left(\upsilon(1-\mu) + (1-\mu^{*})\right) d^{*}\right)^{\epsilon} = R^{*,d} + \frac{1}{2}\omega^{*}\left(\chi^{+}(\mu) - \chi^{-}(\mu)\right)$$

$$R^{m} = R^{*,d} + \frac{1}{2}\omega\left(\chi^{+}(\mu) - \chi^{-}(\mu)\right) - \frac{1}{2}\left(\chi^{+}(\mu) + \chi^{-}(\mu)\right).$$

2.b Given the solutions to $\left\{R^d(s), R^{*,d}(s)\right\}$, solve $\{d^*, \upsilon\}$ using:

$$d = \left[\frac{R^d}{\Theta^d}\right]^{1/\varsigma}$$

$$\upsilon = \left[\frac{R^d}{\Theta^d}\right]^{1/\varsigma}\left[\frac{R^{*,d}}{\Theta^{*,d}}\right]^{-1/\varsigma^*}.$$

2.c Given $\{d^*(s), \upsilon(s)\}$ we solve for prices $\{p, p^*, e\}$ using:

$\mu\upsilon d^* = \frac{M}{p}$ $\mu^* d^* = \frac{e}{p}M^*$ $\mathbf{p}^* = e^{-1}p$.

2.d Finally, we update the real policy rates. For that we construct the expected inflation in each currency:

$$\mathbb{E}[\pi^*] = \frac{\sum_{s'\in S} Q(s'|s)\, p^*_{(n)}(s)}{p^*(s)}$$

and

$$\mathbb{E}[\pi] = \frac{\sum_{s'\in S} Q(s'|s)\, p_{(n)}(s)}{p(s)}.$$

We then update the policy rates by:

$$R^{*,a}_{(j+1)} = \frac{(1+i^{*,a})}{(1+\pi^*)} \text{ for } a \in \{m, w\}$$

and

$$R^{a}_{(j+1)} = \frac{(1+i^{a})}{(1+\pi)} \text{ for } a \in \{m, w\}.$$

2.e Repeat steps 2.a-2.d, unless

$$\left\{R^m_{(j)}(s), R^{*,m}_{(j)}(s), R^w_{(j)}(s), R^{*,w}_{(j)}(s)\right\}$$

is close to

$$\left\{R^m_{(j+1)}(s), R^{*,m}_{(j+1)}(s), R^w_{(j+1)}(s), R^{*,w}_{(j+1)}(s)\right\}.$$

If the real policy rates have converged, update prices according to

$$p^*_{(n+1)}(s) = \Delta^{greed}p^* + \left(1-\Delta^{greed}\right)p^*_{(n)}(s)$$

and

$$p_{(n+1)}(s) = \Delta^{greed}p + \left(1-\Delta^{greed}\right)p^*_{(n)}(s)$$

and proceed back to the outerloop.