Tracing the Sources of Fiscal Multipliers
Latest version.*

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Abstract

We investigate the impact of fiscal expansions on firm investment through corporate lending by exploiting highly granular Colombian data during 2004-2015. For identification, we focus on firms that have multiple banking relationships and trace the entire loan history across lenders as they change their stock of government bonds. Further, we conduct a localized RDD approach and compare the lending behavior of: (i) banks that barely met and missed the criteria of being a primary dealer, and (ii) barely winners and losers at government auctions. We argue that the lending strategy of banks within the vicinity of the cutoff point is very similar ex ante and can reveal crowding-out effects when confronting liquidity shortages. Our results indicate that during a period of 10 months, a 1 percentage point (pp) increase in banks' bonds-to-assets ratio decreases loans by up to 0.42%. Consistently, our RDD results show that primary dealers reduce their credit to corporates by 10.8% compared to non-primary dealers. As a result, a one standard deviation increase in firms' exposure to lenders with high bond holdings leads to a decline in liabilities, investment, profits, and wages by up to 1.3%, 10.8%, 2.3%, and 5.4%. Our findings are grounded in a quantitative default model with financial and real sectors. It shows that increased government spending limits the amount of available funds to firms and raises sovereign risk. Primary dealers then pass on these costs to local firms. Essentially, the heightened cost of credit lowers private investment.

Keywords: fiscal multipliers, regression discontinuity design, crowding-out channel, financial openness, sovereign default
JEL Codes: E44, F34

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1 Introduction

One of the lessons learned from the financial world crisis of 2008-2009 is that the effects of monetary policy are rather limited, especially in the context of low, zero-bounded, or even negative interest rates. With a decade slow of fragile recovery, the effectiveness of fiscal policy is now at the forefront of macroeconomic debates. However, fiscal expansions (much more politicized than monetary policy according to Alesina and Giavazzi, 2013) are sometimes seen through an overly optimistic lens. In essence, advocates argue that they stimulate economic activity, scaled up by potential multipliers. The bulk of the supporting evidence today has its roots in the seminal papers of Mundell (1963) and Fleming (1962). In turn, critics highlight the dampening effects of lower investment. However, few empirical studies use micro data to support how resources to the private sector can be deterred by the take-up of government bonds (i.e., a crowding-out effect on lending).

In this paper, we investigate the impact of government spending on firm investment through cross-bank liquidity variation on corporate lending. To do so, we focus on firms that have multiple banking relationships. Namely, we trace firms' loan history across lenders as lenders absorb government debt. Similar to Mian and Khwaja (2008), we center on two main operating mechanisms: the bank-lending and firm-borrowing channels. The former responds to bank-specific liquidity shocks, and the latter deals with firms' ability (or inability) to smooth out their debt across different sources of financing. We note, however, that while Mian and Khwaja (2008) use an exogenous event that triggers variation in bank liquidity (unanticipated nuclear tests in Pakistan), we instead focus on the take-up of government debt. This approach allows us to bridge the micro bank-lending literature with that of the macro crowding-out channel that evaluates banks' lending capacity when absorbing domestic public debt (Cook and Yetman, 2012 and Bruno and Shin, 2015).

We recognize that events that trigger changes in the liquidity supply, such as the take-up of government debt, are seldom exogenous and are often linked with changes in investment returns and credit demand. To overcome this endogeneity problem, our estimation strategy consists of two parts. First, we use firm-time and bank fixed effects to compare each firm's loan relationship with its creditor vis-à-vis its other active creditors. Intuitively, while fixed effects allow us to control for firm-time specific changes in the demand for credit, the fact that we observe multiple banking relationships per firm allows us to disentangle the variation in credit supply (i.e., controlling for common supply shocks within the banking system).

Second, we focus particularly on the Colombian primary dealer market, where primary dealers benefit from having a special access to debt issuance from the government. That is, apart from gaining a close relationship to the Ministry of Finance, they trade directly with the government at prices dictated by weekly uniform clearing-price auctions in which they participate. Auction winners are also allowed to participate in non-competitive auctions, at lower prices than secondary markets (similar to a *greenshoe* option). Thus, a dealer has the potential of making significant gains if bond prices increase in the interim. In return, they are required (by regulation) to take on an established amount of government debt (i.e. to underwrite at least 4% - 5% of total debt issuance) and to participate actively in electronic trading platforms.

Hence, our identifying assumption is based on the fact that a part of bond purchases in this market are exogenous (i.e. the amount that would have not otherwise been acquired). These purchases are not readily adjusted in banks' portfolio decisions and are more likely to be passed on as liquidity shortages to firms, dampening their credit lines. For this we also consider general equilibrium effects by evaluating whether firms are able to meet their credit demand from other creditors, once they fail to acquire loans from banks that take up government securities. To thin on our identification strategy, we conduct two types of regression discontinuity design (RDD) exercises. In the first exercise we compare the lending behavior of banks that barely met the criteria of being a primary dealer with those that barely missed the cutoff. Intuitively, we expect the lending behavior of banks in the vicinity of the cutoff to be very similar ex ante. Thus, any change in their private lending can be attributed to government debt issuance. In the second exercise we compare only across primary dealers: barely winners and losers at each auction. In this exercise we use, as running variable, the difference between each bid and the resulting cutoff price. That is, since neighboring bids reveal a similar valuation of government bonds, we exploit the fact that some bids receive a discontinuous treatment (i.e. winning the auction) and thus have fewer resources to lend out than those in the control group (i.e. auction losers).

Our study has two main sets of contributions: empirical and quantitative. On the empirical front, we postulate a crowding-out effect as a function of public debt. That is, we confirm a crowding-out channel to corporates and find that this effect is more pronounced during episodes of high government debt. Further, to investigate real sector effects we propose novel firm-based measures of financial exposure. Specifically, one measure is the firm's share of creditors that qualify as primary dealers over its total number of creditors, and the other measure is the firm's average of its creditors' bond holdings (weighted by loan volume). With these measures, we evaluate the effects on firm's outcomes such as wages, investment, assets, liabilities, and profits.

We use the entire Colombian credit registry to corporates, at a monthly frequency, from 2004 to 2015 (roughly 5.5 million observations). We merge these data with yearly firm-level balance sheet information (roughly 1.5 million observations). Our resulting data, focusing on new loans and adding other bank-level variables such as bond holdings, contain 30 banks, over 32,000 firms, and 730,000 new loans.

Our findings indicate that, during a period of 10 months, a 1 percentage point (pp) increase in banks' bonds-to-assets ratio decreases loans by up to 0.42%. Additionally, we find that the affected firms are only partially able to substitute their loans with other lenders. Our RDD results corroborate these findings: (i) primary dealers reduce their credit to corporates by 10.8% compared to non-primary dealers, and (ii) barely winners at government auctions reduce their credit lines to corporates by 19.3% compared to barely auction losers. As a result, a one standard deviation increase in firms' financial exposure (capturing the extent to which lenders acquire government bonds) leads to a decline in liabilities, investment, profits, and wages by up to 1.3%, 9.5%, 2.3%, and 5.4%, respectively.

Additionally, we use a subperiod of our data, when J.P. Morgan announced the inclusion of several Colombian Treasury bonds in its GBI-EM Global Diversified and GBI-EM Global indices. We document that, immediately after the announcement, primary dealers began off-loading government debt to international investors while significantly increasing their loans to local firms. Thus, the crowding-out effect is weaker when public debt is acquired by foreign investors.

On the quantitative side, we propose an endogenous sovereign default model consistent with the facts presented in our empirical findings. In particular, our model allows for a dynamic interaction between government indebtedness and firms' investment decisions, enriched with primary dealer banks. In our model, domestic and imported foreign investment goods (which are imperfectly substitutable) are used to accumulate capital.

Three key assumptions for the dynamic link between sovereign default risk and firm investment are as follows: (i) firms need working capital financing to fund a subset of foreign investment goods that are necessary for production and to pay for the fraction of wage bills before the production phase; (ii) domestic and imported foreign investment goods are only imperfectly substitutable; and (iii) a positive recovery takes place after default. With these salient ingredients and in response to higher government borrowing, the model yields higher default risks and thus higher funding costs of importing foreign investment goods. According to our model's dynamics, the crowding-out of investment through the issuance of government debt arises from the presence of financial frictions that limit banks' ability to meet the loan demand earmarked to finance firms' working capital (i.e., pay wages) before the production phase.

To further exemplify, consider a situation in which the government increases its borrowing by selling bonds to primary dealers, which in turn are unable to satisfy loan demands of private firms. As firms fail to pay wages, labor and production fall. Additionally, as the government's indebtedness rises, the interest rate on government securities increases and banks pass on these costs to firms. Heightened interest rates on loans increase wages and reduce labor demand. Firms then become more reluctant to import foreign investment goods and thus turn to local ones. This substitution entails an efficiency loss because imported foreign investment goods and domestic counterparts are imperfect substitutes. Finally, the positive recovery assumption during exclusion periods facilitates a positive interest rate, which is used by local banks to charge firms for their working capital loan requirements. This chain of events implies that if the government exceeds issuing a threshold level of debt, government borrowing crowds out private investment.

Our investigation relates to two main strands of literature. The first strand evaluates banks' lending capacity when absorbing public debt. While quite ample, this literature has yet to reach a general consensus. For instance, studies such as Herrera et al. (2013) and Gadanecz et al. (2014) argue that banks adjust their asset composition in relation to a long term ratio of investment-to-lending. Others, such as Cook and Yetman (2012) and Bruno and Shin (2015), favor loan substitution for securities. The second strand of literature centers on government spending and fiscal multipliers. Ilzetzki et al. (2013), for instance, show that multipliers vary according to country specifics such as the exchange rate regime, public indebtedness, level of development, and openness to trade.

Our study, to the best of our knowledge, is the first to establish a causal link (using micro data) wherein resources to the private sector are deterred by the take-up of government debt, which in turn leads to lower investment. Throughout our investigation, we stress the importance of primary dealers' take-up of sovereign bonds. In this sense, perhaps the papers closest to our investigation are Evrin et al. (2019), Clancy et al. (2019), and Williams (2018), who find that, as the share of foreign-held public debt increases, available credit to firms also increases. Our study also relates to Jiménez et al. (2014) in that we direct attention to the triple interaction term between banks' holdings of government bonds, primary dealer banks, and total public debt. In contrast, Jiménez et al. (2014) document the interaction between interest rates, bank capital, and firms' credit risk.

Our quantitative default model builds on the theoretical framework of Eaton and Gersovitz (1981) and the quantitative models of Aguiar and Gopinath (2006) and Arellano (2008). Later on, Hatchondo and Martinez (2009) and Chatterjee and Eyigungor (2012) introduce long-term debt and show gains in terms of matching debt, spread, and default moments. Mendoza and Yue (2012) introduce endogenous business cycles into the canonical sovereign default model. Our framework, in contrast, is enriched with investment, financial sectors and long-term debt.

Our paper proceeds as follows. In Section 2 we provide a detailed view of our case study, describe the data, and provide intuition for our main identification strategy. In Section 3 we present the empirical methodology and report our findings. In Sections 4 and 5 we present our theoretical model of sovereign debt, present calibrations, and report quantitative results. Finally, Section 6 concludes.

2 The Colombian Case

2.1 Matching Firm- and Bank-level data

In our empirical exercises, we use highly granular data, comprising the entire Colombian credit registry, at a monthly frequency from 2004 to 2015. This database, from the Financial Superintendency (*Superintendencia Financiera de Colombia, Formato 341*), contains over 5.5 million observations, with information on all loans extended to corporates, such as interest rate, loan amount, maturity, issuance date, expiration date, delinquency rate, and ex-ante credit rating. We merge these data with yearly firm-level balance sheet information from the Corporate Superintendency (*Superintendencia de Sociedades*) in order to include firm-specific variables such as asset size, indebtedness, profits, wages, investment, and equity. After merging these two sources, we match 1.5 million observations, which include a total of 30 private banks and 32,000 firms.

Given that our unit of measurement consists of new loans disbursed from bank j to firm i, we observe 730,000 new loans. Also, as a fundamental part of the study, we use private banks' total bond holdings of sovereign debt (*Títulos de deuda pública — TES*).

To give some initial context, an average Colombian bank has 3,400 new loans with different firms per month. However, large banks, with assets in the top 75th percentile of the banking system, account for 44% of the bank-corporate relationship. Additionally, a large bank has, on average, \$2.4 trillion COP in government bonds, whereas the banking average reports an amount of \$1.78 trillion.

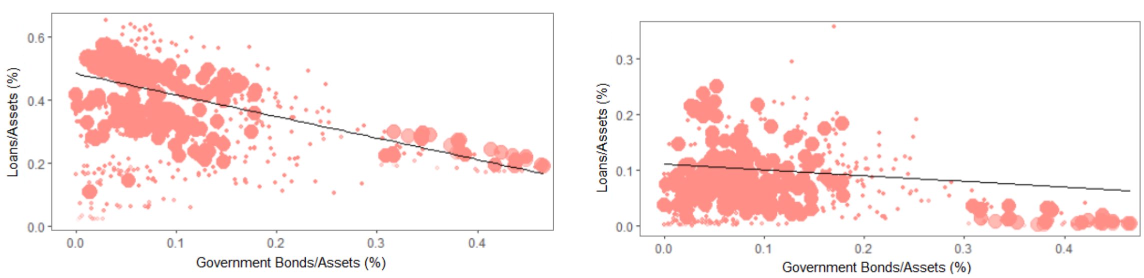
2.2 Contextual characteristics

Figure 1 shows a seemingly negative relationship between banks' take-up of government bonds and their loan portfolio (panel (a) shows the stock of loans along its y-axis and panel (b) shows the amount of new loans; both are displayed as a share of total assets). Note that this is the main relationship that we evaluate in our investigation (in Section 3, we claim causal evidence of government bonds on corporate loans). Notably, the figure displays some heterogeneity in the take-up of government bonds as a function of bank size.

We also shed light on the incremental effect brought forth by the country's total debt, that is, the overall impact that fiscal debt can exert on the crowding-out channel. Figure 2 shows that, during our sample period, the Colombian government debt (as a share of GDP) oscillated between 20% and 40%. This approach allows us to observe various debt levels with sufficient variation. Periods of high government debt took place at the onset of the millennium and after the year 2013.

Table 1 provides descriptive statistics of the banking sector variables employed in our panel exercises, broken down by primary and non-primary dealer banks. We also include our loan-level dependent variable: the volume of new loans to corporates, although aggregated at the bank level, for readability purposes.

In turn, Table 2 shows descriptive statistics of our yearly firm-level variables. Note that in most of our empirical exercises, we use firm-time and bank fixed effects. Hence, several variables individually wash out of the regressions. Notwithstanding, when evaluating the effects on the real sector, we use firm-level data (assets, investment, wages, profits, liabilities, and equity) as dependent variables.



(a) Stock of All Loans/Assets (b) Flow of New Loans/Assets

Figure 1: Sovereign securities (x-axis) vs loans (y-axis). The panels show the seemingly negative relationship between banks' government bonds holdings-to-assets ratio and loans-to-asset ratio (i.e. loans to corporates). The left panel shows the stock of loans in its y-axis while the right panel displays the amount of new loans, both as a share of total assets. The circle sizes are weighted according to bank size (larger circles correspond to larger banks).

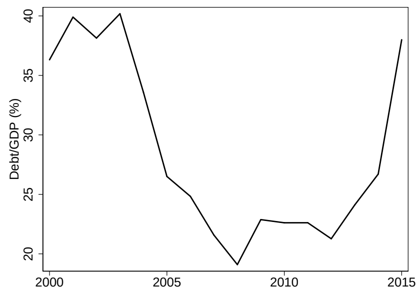


Figure 2: The figure displays the quarterly evolution of Colombian sovereign debt-to-GDP ratio.

Table 1: Bank-level Descriptive Statistics

Variable	**Mean**		**Median**		**St. Dev.**		**St. Dev. Between**		**St. Dev. Within**	
	PD	**NPD**	**PD**	**NPD**	**PD**	**NPD**	**PD**	**NPD**	**PD**	**NPD**
Dependent Variable										
New Loans [a]	1.69	1.13	0.22	0.20	10.20	4.60	8.03	4.43	7.19	2.88
Covariates										
Government Bonds/assets	0.15	0.07	0.13	0.06	0.12	0.08	0.1	0.1	0.05	0.04
Total assets [b]	18.27	5.59	11.74	3.54	17.68	6.06	12.13	5.63	11.96	3.20
Equity/assets	0.12	0.15	0.11	0.12	0.04	0.12	0.04	0.12	0.02	0.07
Liquidity	1.14	1.53	1.13	1.13	0.05	5.64	0.06	3.47	0.02	5.24
Deposits/assets	0.71	0.69	0.72	0.74	0.08	0.16	0.08	0.16	0.05	0.07

Authors' calculations. PD (NPD) denotes primary and non-primary dealers.[a] Variables are in billion COP (10^9) and [b] are in trillion COP (10^{12}). Liquidity is defined as assets over liabilities.

Table 2: Firm-level Descriptive Statistics

Variable	**Mean**	**Median**	**St. Dev.**	**St. Dev. Between**	**St. Dev. Within**
Dependent Variable					
Assets	16.14	3.96	63.92	42.41	40.96
Investment [a]	3.04	0.13	75.99	103.40	20.81
Wages	0.07	0.02	0.33	0.22	0.19
Liabilities	17.51	3.50	93.90	78.06	45.30
Profits	8.75	2.03	52.31	38.81	31.69
Equity	16.98	2.26	139.25	157.80	47.18

Authors' calculations. Total assets, investment, wages, liabilities, profits, and equity are in billion COP (10^9). [a] Firm investment includes shares, quotas, securities, corporate papers, and any other negotiable document acquired temporarily or on a permanent basis, with the purpose of maintaining a secondary liquidity reserve, establishing economic relations with other entities, or to meet legal or regulatory provisions.

2.3 Identification

For expositional purposes, we refer to a crowding-out effect when government expenditure fails to boost aggregate demand due to a similar fall in private sector spending and investment (displayed as the movement from point B to point A in Figure 3). Intuitively, when the private sector lends money to the government, the resources available for private investment funding fall. However, if the economy is below its full capacity (point C), then the increased spending does not necessarily lead to a crowding-out effect.

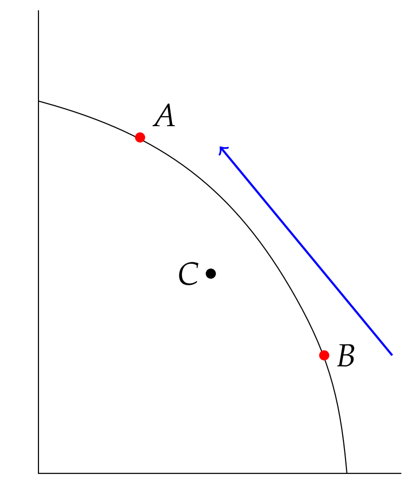


Figure 3: The production possibilities curve

A key challenge for identification is that the banking sector optimally balances its portfolio mix between government securities and corporate lending. In such an environment, our identification relies on firms' borrowing from multiple banks, one of which is a primary dealer bank. These primary dealer banks have privileged access to participate in government bond auctions. In return, they are required (by regulation) to take on a predetermined amount of government debt (i.e., to underwrite at least 4% - 5% of total government debt issuance.[1] These restriction is largely binding: meeting the required amount in 87% of cases, and losing their primary dealer status in the remaining 13% of cases. Our identification strategy exploits this feature. We test whether primary dealers are more adversely affected during government spending booms.

A potential concern is that primary dealers off-load their government securities in a secondary market, in order to reduce their risk exposure. However, off-loading can be

[1] The exact amount (between 4% - 5%) changes every year, which adds an additional source of variability.

costly in terms of both time and price uncertainty (e.g., bond prices can change between the time banks purchase bonds at an auction and the time they sell bonds in a close-to-centralized secondary market). Further, primary dealers must show at least a 4.5% intake of total debt to avoid being penalized by the Financial Regulatory Authority (*Superintendencia Financiera de Colombia*).

Figures 4 and 5 investigate whether primary dealers are in fact off-loading securities. In the Colombian case, government auctions (primary market) are issued on two different days of the week, almost every week. Figure 4 depicts the net purchases of bonds (negative values for sales) by primary and non-primary dealer banks, each day relative to the auction day, at $t = 0$. Hence, period 1 is the day after each auction and period -1 is the day before (the figure stacks all auctions together). In essence, the figure shows that (i) primary dealers acquired more bonds during auction dates (attributable to auctions), and (ii) bond trading before and after the auction was similar for both primary and non-primary dealers. Visually, it does not appear that primary banks purchased government bonds at auctions only to dispose of them in the secondary market.

Similarly, Figure 5 (left panel for total bonds and right panel for bonds/assets) displays the evolution of primary and non-primary dealers' share of government debt. It shows that primary dealers hold higher government debt and that the purchase amount difference relative to non-primary banks is relatively constant through time. A potential concern is whether banks pledge these government securities to borrow from the Central Bank's discount window in order to increase lending to corporates (i.e. thorough repurchasing agreements -REPOs). But this does not seem to be the case because the discount window facility is meant to help banks manage their short-term liquidity shortages, usually overnight, while corporate lending is conducted in longer-term maturities.

Finally, we recognize that primary dealers may differ systematically from the rest of the banking system. After all, Table 1 shows larger balances for primary dealer banks. This difference can be potentially unsettling if the reasons why they differ are also correlated with a stronger or weaker portfolio rebalancing after acquiring government debt. To rule out this concern, in Section 3.2 we conduct a localized approach using a regression discontinuity design (RDD), where we compare the behavior of banks that barely met the criteria to be primary dealers, with those that barely missed the cutoff. Further, we note that primary dealers are designated every year, which adds an additional source of exogeneity to our exercises.

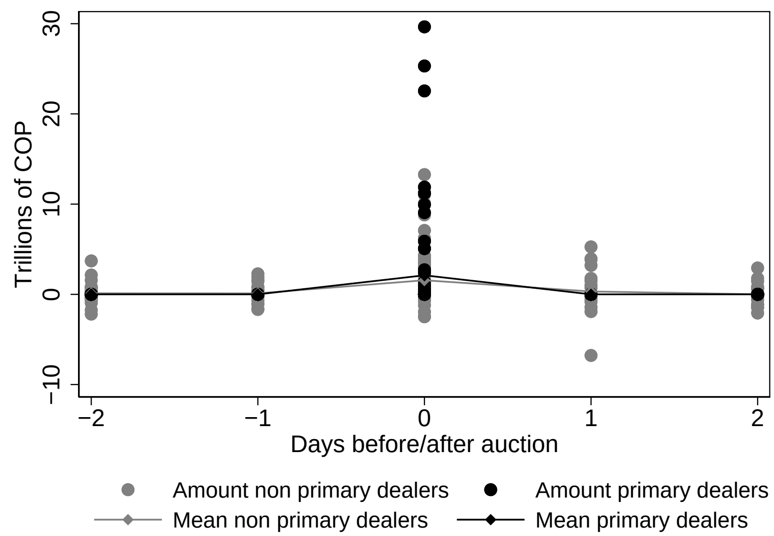
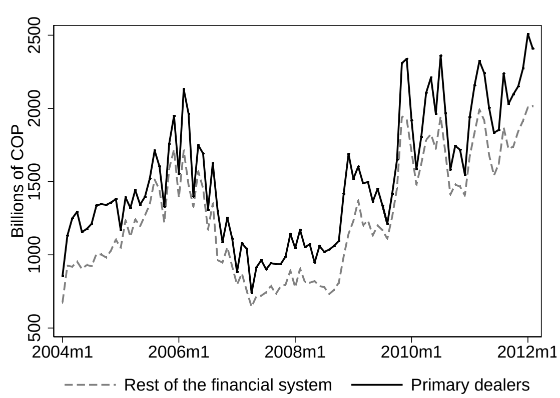
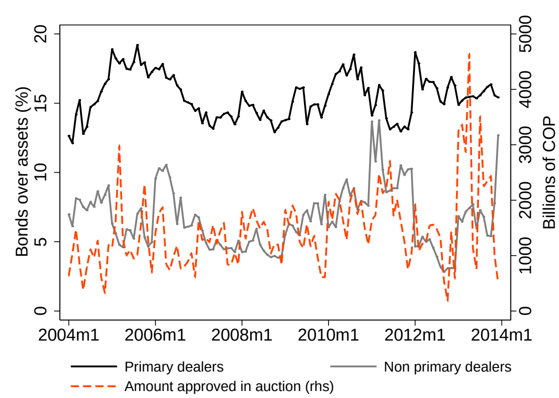


Figure 4: The figure plots the net daily amount of government bonds purchased by primary dealers and by the rest of the financial system. The diamonds represent averages.



(a) Amount in COP



(b) Share Bonds/Assets

Figure 5: The figure displays the monthly evolution of primary dealers' and non-primary dealers' amount of government debt (panel a) and as a share of banks' assets (panel b) over time. The dotted orange-red line in panel (b) presents the amount of government auctions along the right axis.

3 Empirical Exercises and Results

In the empirical exercises that follow, we use a monthly panel dataset containing information on 32,000 firms, 30 private banks, and 730,000 new loans during the period of 2004-2015. Similar to Mian and Khwaja (2008), we restrict our attention to firms that have multiple banking relationships in order to trace the entire loan history across lenders, as they change their stock of bond holdings. This approach allows us to observe different levels of government securities across banks (our treatment variable) for each firm.

Our estimation strategy consists of three main parts. First, in Section 3.1 we cover the entire financial system and compare each firm's loan relationship with its creditor vis-Ãă-vis its other active creditors. For these exercises we use firm-time and bank fixed effects to control for changes in the demand for credit and for any common supply shocks within the banking system. We also cluster standard errors at the bank level.

Second, in Section 3.2 we narrow in on the primary dealer market and employ a localized RDD approach. More specifically, in Section 3.2.1 we compare the lending behavior of banks that barely met the criteria of being a primary dealer with those that barely missed the cutoff. In section 3.2.2 we restrict our focus to only primary dealers, and compare institutions which barely won and lost at each government auction.

To further corroborate our results, in Section 3.3 we examine an episode in Colombia where foreign demand for government bonds experienced an exogenous positive shock. Through a difference-in-difference estimation, we evaluate J.P. Morgan's unanticipated announcement in March 2014 of the inclusion of several Colombian Treasury bonds in its investable indices.

Finally, in Section 3.4 we map our analysis to the real economy. Namely, to investigate real sector effects we propose two firm-based measures of financial exposure. Using time, industry, and time-industry fixed effects, we then evaluate their impact on firm's outcomes such as: wages, investment, assets, liabilities, equity, and profits.

3.1 Dampening of corporate credit in financial sector

We begin by evaluating the effects of banks' sovereign bond holdings on corporate loans by using the entire banking credit registry. Following the work of Hofmann et al. (2019), we focus on new loans (credit flows) as opposed to credit stocks, since it allows for a clearer identification by filtering out pre-existing loans that would not be expected to react. Formally, we estimate the following regression model:

$$\begin{aligned} Loan_{i,j,t} =& \alpha_{j,it} + \theta Bonds_{j,t-1} + \gamma Primary_{j,t-1} + \varphi ColDebt_{t-1} + \rho(Bonds * Primary)_{j,t-1} \\ &+ \delta(Bonds * ColDebt)_{j,t-1} + \nu(Bonds * Primary * ColDebt)_{j,t-1} + \epsilon_{i,j,t}, \end{aligned} \tag{1}$$

where $Loan_{i,j,t}$ corresponds to the value (in logs) of all new loans from bank j to firm i, in month t. The variable $Bonds_j$ denotes the bank's stock of government bonds as a share of its assets. $Primary_j$ indicates the amount of bonds purchased in the primary dealer market by bank j, also as a share of its assets (non-primary dealer banks take a zero value). The term $\alpha_{j,it}$ indicates bank and firm-time fixed effects. Finally, $ColDebt_t$ is the macroeconomic variable denoting total government debt over GDP, which individually washes out of the regressions because of the time fixed effects.

To evaluate the duration of the effects in equation (1) we estimate implied impulse response functions (IRFs) in the spirit of Jordá's (2005) method of local projections. Essentially, we estimate sequential regressions in which loans are shifted forward each month. Specifically, we estimate equation (1) for a time horizon of $k = 1 - 12$ months, as follows:

$$\arg\min_{\omega^k} \sum_{k=1}^{K} \sum_{ij=1}^{IJ} \sum_{t=2}^{T-k} (Loan_{i,j,t+k} - \alpha^k_{j,it} - X'_{i,j,t}\omega^k)^2 \tag{2}$$

where $\omega = (\omega_1, ..., \omega_K)'$ are the impulse-response coefficients that capture the impact of bond holdings on loans k-months after treatment (i.e., changes in the bond holdings of bank j). For notational purposes, we include all firm-bank relationships in the second summation of equation (2). That is, since the regression is estimated at the loan level, we exploit all banking relationships for each firm.

Finally, it remains to show whether firms are able to meet their loan demand by seeking credit from other banks i.e., once they fail to find resources from a primary dealer bank. This would shed light on general equilibrium effects of government spending on the banking sector's entire lending capacity. For that, we explore whether firms that borrow from primary dealer banks can substitute their loans from non-primary dealer banks. More generally, this determination sheds light on whether the economy is lying on point C of Figure 3, i.e. if firms' financing can be sourced from non-primary dealer banks at no cost, then this would suggest that the economy is operating below capacity. As such, we estimate a similar version of equation (2) but now using as a dependent variable all other loans of firm i (excluding loans with bank j), as follows:

$$\arg\min_{\bar{\omega}^k} \sum_{k=1}^{K} \sum_{ij=1}^{IJ} \sum_{t=2}^{T-k} (Loan_{i,\text{-}\mathbf{j},t+k} - \alpha^k_{j,it} - X'_{i,j,t}\bar{\omega}^k)^2. \tag{3}$$

Essentially, $\bar{\omega}$ in equation (3) evaluates the degree of loan substitution: an increase in bank j's bond holdings (which decreases its credit line with firm i) forces the firm to look for additional credit. Hence, if the amount of loans deterred (captured by ω) is greater in absolute value than new loans acquired with other creditors (captured by $\bar{\omega}$), then we can infer that the firm is only partially able to substitute its loans. To statistically assess the overall effect, in Appendix B we also consider all loans of firm j $(Loan_{i,j} + Loan_{i,-j})$ as dependent variable.

Results are reported in Table 3. As shown, the negative effect of banks' bond holdings on loans is significant and lasts for 10 months before it subsides. Specifically, in period 7 (peak month) we find that a 1 percentage point (pp) increase in banks' bonds-to-assets ratio decreases loans to firms by 0.42%. Similarly, bond holdings have a negative incremental effect when primary dealers take on more government bonds and when the government issues more debt (i.e., the triple interaction term $Bonds * Primary * ColDebt$). Results in period 7 show that a 1 pp increase in the triple interaction term decreases loans by 0.46%.

Additionally, firms with other lenders have a positive but smaller substitution effect that lasts for 10 months. For period 7, columns 3 and 4 in Table 3 show that a 1 pp increase in a lender's bonds-to-assets ratio leads the firm to acquire credit with other lenders by at most 0.4%. The incremental effect of $Bonds * Primary * ColDebt$ is also smaller, showing an increase in loans with other lenders by up to 0.27%. In Table A1 of Appendix B we evaluate the net substitution effect when considering all loans. It shows that firms only partially substitute out their debt: firms reduce their net credit in up to 0.05%, with an incremental net effect of 0.15%.[2]

Finally, in Appendix B we investigate the effects when the economy operates below its full capacity, that is, when the variable of total public debt is below its 25^{th} percentile ($D_{Low\_Debt} = 1$). With this change in mind, we report the same set of exercises of Table 3 in Table A2. We report that for low levels of debt, a 1 pp increase in banks' bonds-to-assets ratio decreases loans by 0.35% as expected, but with a positive incremental effect of $Bonds * Primary * D_{Low\_Debt}$ of 0.03%. Put differently, results also show a crowding-out effect on loans (negative impact of bonds), but in lesser magnitude.

[2]Our results remain robust to the inclusion of interest rates as a control variable as well. Results are presented in Table A3.

Table 3: IRFs of banks' bond holdings on corporate loans (in %)

	$Loan_{i,j,t+k}$ (Bank j)		$Loan_{i,\text{-}\mathbf{j},t+k}$ (Other Bank $-j$)	
Periods	*Bonds*	$Bonds * Primary * ColDebt$	*Bonds*	$Bonds * Primary * ColDebt$
1	-0.18	0.10	0.14	-0.16
	(0.17)	(0.10)	(0.13)	(0.092)
2	-0.28	0.12	0.19	-0.12
	(0.18)	(0.11)	(0.13)	(0.084)
3	-0.30*	-0.17	0.23*	0.16
	(0.16)	(0.15)	(0.12)	(0.11)
4	-0.34**	-0.048	0.24**	0.085
	(0.14)	(0.15)	(0.10)	(0.11)
5	-0.32**	-0.22*	0.27**	0.20**
	(0.16)	(0.12)	(0.11)	(0.086)
6	-0.41***	-0.19	0.36***	0.083
	(0.13)	(0.13)	(0.10)	(0.13)
7	-0.42***	-0.46***	0.40***	0.27**
	(0.14)	(0.14)	(0.086)	(0.12)
8	-0.28**	-0.41***	0.24**	0.23*
	(0.14)	(0.13)	(0.12)	(0.13)
9	-0.37***	-0.55**	0.28***	0.40**
	(0.13)	(0.18)	(0.10)	(0.15)
10	-0.18	-0.42**	0.12	0.30*
	(0.19)	(0.17)	(0.16)	(0.16)
11	-0.12	-0.13	0.042	0.046
	(0.15)	(0.14)	(0.12)	(0.14)
12	-0.12	-0.24	0.070	0.18
	(0.18)	(0.18)	(0.13)	(0.14)
Clustered by bank	Y	Y	Y	Y
Firm-time fixed effects	Y	Y	Y	Y
Bank fixed effects	Y	Y	Y	Y

Authors' calculations. The sample includes all months from December 2004 to December 2015. Each listed coefficient results from a separate regression following equations (2) and (3). Rows denote outcomes k-months after treatment. $Loan_{i,j,t+k}$ corresponds to the value (in logs) of all new loans from bank j to firm i, in month $t+k$. *Bonds* denotes the bank's stock of government bonds as a share of its assets. *Primary* indicates the amount of bonds purchased in the primary dealer market, also as a share of its assets. ***, **, and * denote statistical significance at the 1%, 5%, and 10% level, respectively. For all regressions, the average R^2 is 0.80 with 60,000 observations.

3.2 Localized approach using a Regression Discontinuity Design

3.2.1 Comparing primary dealers with non-primary dealers

We first compare the behavior of banks that barely met the criteria to be primary dealers, with those that barely missed the cutoff. Specifically, The Ministry of Finance publishes the rankings of financial sector participants that compete every year to be part of the "market makers" program for public debt securities. Hence, similar financial institutions (mostly the largest banks in the country) are sorted according to a continuous score. Given limited membership, only the institutions ranked 10^{th} or above become primary dealers.[3] Our intuition is that the lending strategy of these banks within the vicinity of cutoff point is very similar ex ante. Thus, bank differences in the amount of loans to corporates can be attributed to government spending.

We exploit this discontinuity in treatment, by comparing entities in the vicinity of the cutoff point. Formally, the assignment of treatment, $\hat{D}_{it}$, is deterministically determined by the Ministry's rankings ($Rank_{it}$), for which we use as running variable, as follows:

$$\hat{D}_{it} = \mathbf{1}\left\{Rank_{it} \geq r\right\} \tag{4}$$

where $\mathbf{1}$ denotes an indicator function and r denotes the 10^{th} ranked entity.[4] We then estimate a similar specification as that of equation (2), only now set locally around the eligibility criteria to be a primary dealer:

$$\underset{\theta^k}{\arg\min} \sum_{k=1}^{K} \sum_{ij=1}^{IJ} \sum_{t=2}^{T-k} \left[Loan_{i,j,t+k} - \alpha^k - \theta_j \hat{D}_{it} - b_j\left(Rank_{it} - r\right) - \tau_j \hat{D}_{it}\left(Rank_{it} - r\right)\right]^2 K\left(\frac{Rank_{it} - r}{h}\right) \tag{5}$$

where $\theta = (\theta_1, \theta_2, \theta_3, ..., \theta_J)'$ accounts for the average treatment effect, i.e. the effect of loans due to being a primary dealer, and $K(\cdot)$ is a triangular kernel with bandwidth h. The inclusion of the term $\hat{D}_{it}\left(Rank_{it} - r\right)$ allows for different specifications of how the running variable affects the outcome, at either side of the cutoff point.

In our estimations (using a yearly panel dataset) we consider optimal bandwidth choices as described in Imbens and Kalyanaraman (2012), and also report bandwidth sizes twice as optimal (2x). Also, we consider bandwidths from pooled years as well as by year. Results are reported in Table 4 and show that loan values pertaining to primary dealer banks are in fact lower. Specifically, primary dealers reduce their credit to corporates

[3]In some years, the single state-owned bank ends up being the 10^{th} bank in ranking. Nonetheless, as part of our robustness checks, we run our the same analysis but taking out the state-owned bank and obtain very similar results.

[4]While we do not observe the exact bank score, we do observe its rank position and cutoff point.

by 10.8%. Additionally, a 1 pp increase in banks' bonds-to-assets ratio has a negative incremental effect on loans of close to 0.03%.

Table 4: Effect of *barely* being a Primary Dealer

	Loan-level RDD	
	Log loans	$Bonds_j$ interaction term
Optimal Bandwidth (pooled)		
	-0.108***	-0.0224***
	(0.019)	(0.005)
Optimal Bandwidth (by year)		
	-0.108***	-0.0354***
	(0.019)	(0.005)
2x Optimal Bandwidth (pooled)		
	-0.851***	-0.122***
	(0.031)	(0.003)
2x Optimal Bandwidth (by year)		
	-0.382***	-0.142***
	(0.025)	(0.003)
Obs bw (pooled)	54,139	53,170
Obs 2x bw (pooled)	107,693	88,462
Obs bw (by year)	54,139	53,170
Obs 2x bw (by year)	154,450	129,830

Authors' calculations. The sample covers 2004-2015. The dependent variable is the value (in logs) of all new loans from bank j to firm i, in year t. The interaction term is between the primary dealer treatment dummy variable and the bank's stock of government bonds as a share of its assets. Reported estimates correspond to equation (5) using an RDD methodology. Bandwidth choices (optimal and 2x optimal) are based on Imbens and Kalyanaraman (2012). We also consider bandwidths from pooled years as well as by year.

3.2.2 Comparing only primary dealers: winners and losers at auctions

We now restrict our focus to primary dealers, and compare institutions which barely won and lost at each weekly government auction. In essence, government auctions operate under a uniform clearing price structure, i.e. the government sells bonds to primary dealers with the highest price bids (i.e. lowest interest rate bids), until the announced quota is filled. The resulting cutoff price is the lowest price bid (highest interest rate bid) among all dealers from whom the government sells a positive amount.[5]

Similar to the exercise in Section 3.2.1, we sustain that neighboring bids reveal a similar valuation of government bonds and only differ in actually winning or losing the auction.

[5] Auction winners are also allowed to participate in non-competitive auctions 3 working days after the auction (similar to a *greenshoe* option).

Hence, auction winners exhibit a liquidity decrease in the amount of government bonds purchased. We exploit this discontinuity in bond holdings as a function of the price bids.

We observe weekly data from all government auctions, where bids, denoted in interest rate yields, are sorted in ascending order. The government then sells bonds in the amount offered by each bidder, starting from the lowest yield, until the quota is filled. Consequently, banks bidding above (below) the cutoff are winners (losers). The bank whose bid lies exactly at the cutoff is in all likelihood a partial winner, since it purchases the remaining amount of the quota, but which in turn is lower or equal than its offered amount to purchase. Since the auction is uniform, the government pays the cutoff rate to all winners regardless of their bid.[6]

We construct our running variable as the cutoff rate minus each bid, so that non-negative numbers are winning bids. We note, however, that there are multiple auctions per day and that each bank can register multiple bids per auction. As such, for each auction winner we compute an aggregated daily bid, defined as the average of its bids, weighted by volume purchased. Alternatively, for each auction loser (i.e. losing in all auctions during the day) we compute its aggregated daily bid as the average of its bids, weighted by volume offered. The resulting number of winners and loser are depicted in Figure 6.

Results using a similar RDD estimation as in equation (5) are presented in Table 5, only now with a different running variable (i.e. cutoff rate minus bank's bid). Given the similarity of institutions (all primary dealer banks) and the narrowness of optimal bandwidth sizes, we only present the marginal effect of winning the auction. We nonetheless report the optimal bandwidth, again as in Imbens and Kalyanaraman (2012) as well as twice the optimal size. Results indicate that primary dealer banks that barely win at government auctions reduce their credit lines to corporates in 19.3%

[6]Auction regulations are provided by the Ministry of Finance under regulations 2822 of 2002, 3766 of 2009, and 3781 of 2009. More information on the structure of the auction is found at: https://www.superfinanciera.gov.co/jsp/16127.

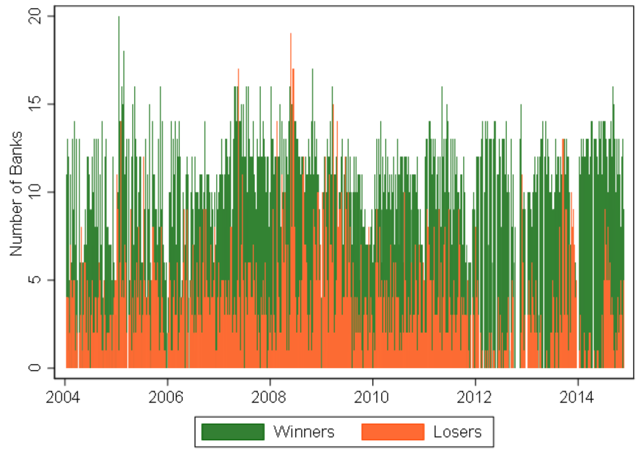


Figure 6: The figure plots the number of primary banks which won and lost at government auctions.

Table 5: Effect of *barely* winning a government auction

Loan-level RDD	Log loans
Optimal Bandwidth	
	-0.193***
	(0.00956)
2x Optimal Bandwidth	
	-0.219***
	(0.00715)
Obs bw	185,716
Obs 2x bw	328,778

Authors' calculations. The sample covers 2004-2015. The dependent variable is the value (in logs) of all new loans from bank j to firm i, in year t. Reported estimates correspond to equation (5) using an RDD methodology but using, as running variable, the government auction cutoff rate minus each bank's bid. Bandwidth choices (optimal and 2x optimal) are based on Imbens and Kalyanaraman (2012).

3.3 A weaker crowding-out effect when public debt is acquired by foreign investors

We next show that the crowding-out effect is weaker when public debt is acquired by foreign investors. To give some context, in March 2014, J.P. Morgan announced the inclusion of several Colombian Treasury bonds in its GBI-EM Global Diversified index and GBI-EM Global index.[7] As a result, foreign demand for Colombian bonds surged, which affected the portfolio balance of the entire banking system (Ocampo et al. (2019)). Figure 7 shows how foreign participation of domestic bonds increased while participation of the banking system decreased.

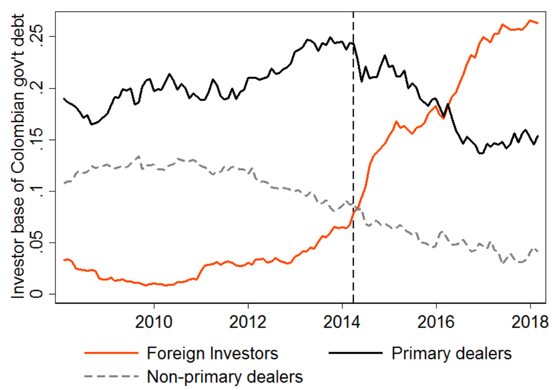


Figure 7: The figure displays the evolution of the investor base of government bond holdings. The vertical line represents J.P. Morgan's announcement of the inclusion of several Colombian Treasury bonds in its investable indices. The series denote the share of government bonds held by foreign investors, primary and non-primary dealer banks.

We proceed by estimating a difference-in-difference estimation (DID), as follows:

$$Loan_{i,j,t} = \alpha_{j,it} + \theta_1 D_t^{Post} + \theta_2 Bonds_j^{2013} + \theta_3 D_t^{Post} Bonds_j^{2013} + \epsilon_{i,j,t}, \tag{6}$$

where D_t^{Post} is a dummy variable switched on from March 2014 until the end of January 2015, and $Bonds_j^{2013}$ are the banks' stock of government bonds as a share of its assets at the end of 2013. Note that, as depicted in Figure 7, banks with high levels of $Bonds_j^{2013}$ were those that mostly sold government securities to non-resident investors and thus experienced a substantial increase in liquidity.

Our difference-in-difference results are reported in Table 6 and show that the inclusion of Colombia in the J.P. Morgan indices had a significant impact on the growth of

[7] The Colombian securities introduced (*TES*) had 2, 4, 8, 10, and 14-year maturities. We extend the exercise documented in Williams (2018) with a highly granular loan-level data.

loans. In other words, as banks sold their government securities to non-residents, they increased their credit lines to firms. Specifically, banks with a 1 percentage point lead in bonds-to-assets over other banks at the end of 2013 (which in turn sold more bonds to foreigners in 2014) saw a surge in liquidity that lead to a 0.17% increase in corporate lending. Additionally, column 2 highlights the positive incremental effect when considering primary dealer banks.[8] It shows that primary dealers increased their lending by 0.4% compared to non-primary dealers. In sum, we find that these results are consistent with our previous findings of Section 3.1. By symmetry, the purchase (sale) of sovereign bonds by the banking sector decreased (increased) corporates' credit.

This analysis implies that the crowding out effect is weaker when the economy is financially more open. We show that banks start lending more to the private firms when public debt is acquired by foreign investors. Turning to Figure 3, this represents an outward shift in the PPF curve.

Table 6: Difference-in-Difference around rebalancing of J.P. Morgan Indices

	Log loans	
	(1)	(2)
D^{Post} x $Bonds_j^{2013}$	0.17**	
	(0.08)	
D^{Post} x $D^{Primary}$		0.004**
		(0.002)
Obs	23,541	12,311
Clustered by bank	X	X
Bank fixed effects	X	X
Firm-time fixed effects	X	X

Authors' calculations. The period used for the analysis was from May-2013 to Jan-2015, (i.e. 10 months before and after JP Morgan's announcement). The average R^2 is 0.7.

[8] Typically, one needs to show that parallel trend assumption should hold in DID analysis. A visual inspection of the parallel's trend assumption and the month-to-month coefficients are shown in Figure 8, with March 2014 established as the time the event occurred.

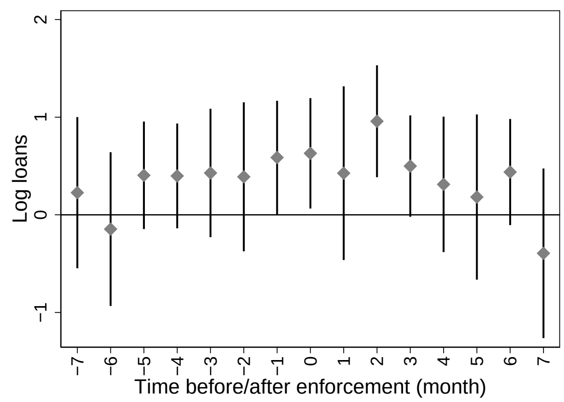


Figure 8: Event analysis for the inclusion of Colombia in the J.P. Morgan indices. Vertical black lines are for confidence intervals, and diamonds are the point estimates for the months before and after the announcement.

3.4 Effects on the real sector

To assess the impact of the overall crowding-out channel on the real sector, we first compute firm-level *financial exposure* variables that capture the extent to which their lenders acquired government bonds. The first measure is the firm's number of creditors that qualify as primary dealers over its total number of creditors. The second measure corresponds to the firm's weighted average of its creditors' bond holdings (weighted by loan volume). These two measures are exemplified as follows:

1. $First\ measure\ of\ Exposure_{i,t} = \frac{1}{J} \sum_j \mathbf{1} \left\{ Primary\_Bank_{i,j,t} \right\}$

2. $Second\ measure\ of\ Exposure_{i,t} = \frac{\sum_j Loans_{i,j,t} * Bonds_{j,t}}{\sum_j Loans_{i,j,t}}$

where again $Loans_{i,j}$ corresponds to the value (in logs) of all new loans from bank j to firm i, $Bonds_j$ is the bank's stock of government bonds as a share of its assets, and $\mathbf{1} \left\{ Primary\_Bank_{i,j} \right\}$ is an indicator function turned on for primary dealers banks. Intuitively, high values of financial exposure implies that the firm is borrowing from liquidity constrained banks.

Next, we use yearly corporate balances from the Corporate Superintendency (*Superintendencia de Sociedades*) in order to include firm-specific outcome variables such as assets, liabilities, investment, profits, and wages. Formally, we estimate the following model:

$$y_{i,t} = \alpha_{t,s} + \beta Financial\_Exposure_{i,t-1} + \epsilon_{i,t}, \tag{7}$$

where the term $\alpha_{t,s,ts}$ accounts for time, industry, and time-industry fixed effects. Results are presented in Table 7 and show that, controlling for time-industry fixed effects, a 1 percentage point increase (pp) in the first measure of financial exposure leads to a decline in liabilities of 0.03%, a decline in investment of 0.24%, a decline in profits of 0.04%, and a decline in wages of 0.12%. We interpret these results as large, since a one standard deviation of the first exposure variable (0.45) is associated with a decline in liabilities of 1.3% (0.03%x45), a decline in investment of 10.8%, a decline in profits of 2.3%, and a decline in wages of 5.4%.

Similarly, controlling for time-industry fixed effects, a 1 pp in the second measure of financial exposure leads to a decline in investment of 0.60%, a decline in profits of 0.13% and a decline in wages of 0.33%. For comparability purposes, a one standard deviation of the second exposure variable (0.06) is associated with a decline in investment of 3.6% (0.60%x6), a decline in profits of 0.8%, and a decline in wages of 2.0%. Consequently, we confirm the negative real sector effects when resources to the private sector are deterred by the take-up of government bonds.

Table 7: Impact of banks' bond holdings on firms' balances

	Growth of assets		Growth of liabilities		Growth of investment		Growth of profits		Growth of wages	
1^{st} $measure\ of\ Exposure_{i,t}$	-0.0187	-0.0184	-0.0320***	-0.0321***	-0.245*	-0.238*	-0.0453**	-0.0426**	-0.118***	-0.120***
	(0.0129)	(0.0132)	(0.0108)	(0.0114)	(0.132)	(0.141)	(0.0200)	(0.0195)	(0.0395)	(0.0380)
2^{nd} $measure\ of\ Exposure_{i,t}$	-0.0279	-0.0369	-0.0477	-0.0553	-0.590*	-0.596*	-0.0868**	-0.129***	-0.342***	-0.330***
	(0.0741)	(0.0734)	(0.0770)	(0.0787)	(0.334)	(0.322)	(0.0406)	(0.0421)	(0.112)	(0.116)
Clustered by industry	yes	yes	yes	yes	yes	yes	yes	yes	yes	yes
Time FE	yes	no	yes	no	yes	no	yes	no	yes	no
Industry FE	yes	no	yes	no	yes	no	yes	no	yes	no
Time-Industry FE	no	yes	no	yes	no	yes	no	yes	no	yes
Obs	17,054	16,989	17,053	16,988	8,350	8,260	16,906	16,841	14,526	14,462
R2	0.033	0.060	0.032	0.054	0.029	0.067	0.023	0.063	0.029	0.052

Authors' calculations. The sample includes all years from 2004 to 2015. ***, **, and * denote statistical significance at the 1%, 5%, and 10% level, respectively. Similar to Berman et al. (2012) we include fixed effects by industry due to the heterogeneity between them in terms of productivity and pricing-to-market. Also, other authors such as Casas (2019) explain the heterogeneity by the difference in relative importance of intermediate inputs in production and Chen and Juvenal (2016) explore the heterogeneity based on the quality differences between industries.

4 A Quantitative Model of Sovereign Debt, Real Sector, and Financial Sector

In this section we formulate a quantitative default model enriched with real and financial sectors in order to rationalize our empirical findings. Our quantitative model explains the following key stylized facts. First, when the government increases its spending, the amount of resources available to the firms decline, so interest rates on government securities increase, and local banks pass on these costs to the firms. Second, private investment declines particularly during high borrowing episodes, highlighting the crowding-out effect. Turning to Figure 3, there are times during which the economy stands at point C and hence, an increase in borrowing does not necessarily crowd out investment. However, a further rise in spending crowds out private investment. To map our empirical results to the quantitative section, we made some simplifications. For instance, we only consider primary dealer banks. Thus, the banking sector has a fixed lending capacity. That is, an increase in government spending reduces the available credit to firms. This is consistent with our empirical findings of Section 3.1, where we show that firms are incapable of meeting their loan demand from another bank once it fails to find resources from a primary dealer bank. In addition, since Colombia is an emerging market economy, an increase in government borrowing entails a default risk. Our framework accounts for that too.

The main departure of our model from the existing quantitative default models resides in the inclusion of banks and a production economy. We mainly model primary dealer banks as intermediaries providing loans to firms (for their salary payments) while at the same time holding on to government securities. We define four types of agents in the economy: households, bankers, producers, and the government. Each of their roles in the economy is described below. The per period time line of main events for the agents in the economy is depicted in Figure 9.

A salient ingredient of our model is that firms require working capital financing to pay for imports of a subset of foreign investment as well as for a fraction of wage bills before production takes place. To make the model tractable and in line with empirical observations, we model the firms' borrowing rate as a one-to-one mapping to the government's borrowing rate.

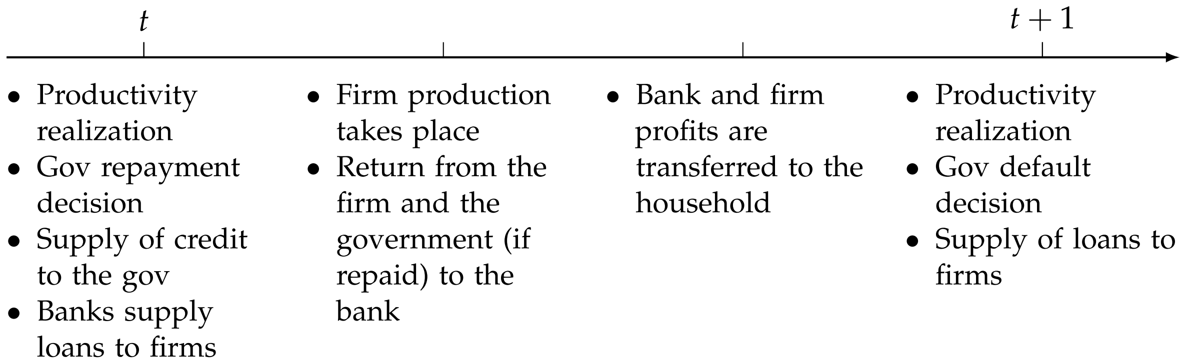


Figure 9: Timing of events in the economy.

4.1 Households

Households choose how much to consume and supply labor to maximize expected discounted utility streams, $E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, L_t)$, where $0 < \beta < 1$ is the subjective discount factor, and c_t and L_t denote consumption and labor, respectively. The period utility function $u(.)$ is continuous, strictly increasing in consumption, strictly decreasing in labor, and strictly concave in both arguments. The expectation operator E_t is conditional on the information set available at period t.

Households receive a real wage per labor hour w_t, profits paid by the final goods producers π_t^f, and net proceeds obtained from banking activities π_t^b and transfers from the government (T_t). Formally, the household solves the following problem:

$$\max_{c_t, L_t} E_t \left[\sum_{t=0}^{\infty} \beta^t u(c_t, L_t) \right] \tag{8}$$

subject to the period budget constraint

$$c_t = w_t L_t + \pi_t^f + \pi_t^b + T_t. \tag{9}$$

Household preferences are governed by a utility function of the Greenwood et al. (1988) type, which ensures no wealth effect on labor supply.[9] In particular, we use the utility function of the form:

$$u(c_t, L_t) = \frac{\left(c_t - \frac{L_t^{1+\omega}}{1+\omega} \right)^{1-\sigma} - 1}{1-\sigma}, \tag{10}$$

[9] This formulation for preferences removes the wealth effect on labor supply and helps to explain key business cycle facts for small open economies. See, for example, Mendoza and Yue (2012).

where $\sigma > 0$ is the constant relative risk aversion, and $\omega > 0$ governs the (inverse) Frisch elasticity of labor supply. The optimal labor supply is given by

$$L_t^{\omega} = w_t. \tag{11}$$

4.2 Final Goods Producers

We formulate the final goods producers' problem following Aguiar et al. (2009). In particular, these firms maximize profits, π_t, net of depreciation and discounted at the country's borrowing rate:

$$E_0 \left[\sum_t \left(\frac{1}{1+r_t^*} \right) \left(\pi_t^f - p_t^I \left(K_t - (1-\delta) K_{t-1} \right) \right) \right], \tag{12}$$

where p_t^I is the price of the investment good, $I_t \equiv (K_t - (1-\delta)K_{t-1})$ denotes investment, and π_t^f is the output net of labor and working capital costs. As in Neumeyer and Perri (2005) and Uribe and Yue (2006), costs include both wage and working capital costs, which means that the firms need to pay a θ fraction of the wage bills at the beginning of the period. Since the production takes place at the end of the period, firms borrow from the banks at an interest rate of r_t^*.[10] The working capital constraint thus determines producers' demand for loans:

$$\ell_t = \theta w_t L_t. \tag{13}$$

Note that the timing of production is an important ingredient of the model that enables the production economy to be linked with the banking sector. This assumption raises the need for banks as firms need to finance production inputs in advance. Taken together, the profits of the final goods producers are $\pi_t^f = A_t K_t^{\alpha^K} L_t^{\alpha^L} - w_t L_t - \theta(1+r_t^*) w_t L_t$. The production technology is governed by a Cobb-Douglas function given by

$$Y_t = A_t K_t^{\alpha^K} L_t^{\alpha^L}, \tag{14}$$

where α^K is the output elasticity of capital, α^L is the output elasticity of labor, and A_t is the shock to the level of total factor productivity. They all operate under decreasing returns to

[10] This working capital assumption is common in the literature. To name a few examples, see Mendoza and Yue (2012), Perez (2018), Önder (2019) and Mallucci (2015).

scale, that is, $0 < \alpha^K + \alpha^L < 1$.[11] Producers of final goods take factor prices, w_t and p_t^I, and the country borrowing rate, r_t^*, as given and maximize their profits given by equation (12). Multiplying both sides of equation (11) by θL, we can obtain labor as a function of loan supply ℓ:

$$L_t = \left(\frac{\ell_t}{\theta}\right)^{\frac{1}{\omega}}. \tag{15}$$

If loan demand exceeds loan supply, then the employment adjusts (falls) to clear the loan market. This is one of the key frictions linking the banking sector to the real economy.

We next assume that firms perceive the TFP shock as i.i.d., which then renders an optimal level of capital as a function of the country borrowing rate as in Aguiar et al. (2009). That is, for a given r_t^* and a country's borrowing rule, there is a unique and constant level of optimal K_t. The simplification, without loss of generality, is that final goods producers are myopic, and therefore, their maximization problem satisfies $K \equiv K(r^*)$ and $I(r^*) = \delta K(r^*)$. This assumption is not as restrictive as it might appear, since in the sovereign's dynamic problem that we postulate below, r^* fluctuates endogenously from fluctuations in government external borrowing and in TFP, which then results in endogenously time-varying K_t and I_t.[12] Since we assume myopic firms, our problem can be reduced to a static profit maximization problem as in Mendoza and Yue (2012) and Na et al. (2018). With that, there is no precautionary saving from the firm's perspective.
We can now obtain optimal factor demand schedules with profit maximization:

$$\delta P_t^I = \alpha^K A_t K_t^{\alpha^K - 1} L_t^{\alpha^L} \tag{16}$$

$$w_t = \frac{1}{1 + \theta(1 + r_t^*)} \alpha^L A_t K_t^{\alpha^K} L_t^{\alpha^L - 1}. \tag{17}$$

Notice that the wage is equal to the marginal product of labor multiplied by an additional term that encompasses the financial costs of labor. This equation implies that tight conditions in the financial sector are equivalent to an adverse productivity shock.

[11]Under a decreasing returns to scale production technology, the firm makes positive economic profits (which are then transferred to households). Such a transfer of profits helps to better match the consumption dynamics for default episodes.

[12]Our assumption also boils down to the *non-contractibility of investment*, as laid out by Occhino and Pescatori (2010), that is, the firm cannot commit to an exact investment level when signing the contract, and afterward, it is free to choose investment to maximize its own objective function. Or similar in notion to Eggertsson and Krugman (2012), in the face of a realization of the shock in the next period, firms move quickly to bring investment to the new level, given the new endogenous borrowing constraint due to a given (r_{t+1}, A_{t+1}).

Everything else equal, an increase in the cost of working capital translates into higher wages and lower employment. The borrowing cost r_t^* is always away from zero because the bond price that the government faces (q^D) is higher than zero during exclusion periods with positive recovery. To this end, introducing positive recovery into the model facilitates production during exclusion periods, albeit at a higher cost.

4.3 Capital Accumulation and Investment Goods Producers

The importance of foreign investment goods in accumulating capital is well recognized (e.g., Eaton and Kortum (2001) and Alfaro and Hammel (2007)). Along with this literature, we assume that the investment good, I_t, is produced via a CES aggregate of domestic and imported foreign investment goods, I_t^d and I_t^f, respectively. This aggregation satisfies:

$$I_t = \left[\lambda(I_t^d)^\epsilon + (1-\lambda)(I_t^f)^\epsilon\right]^{\frac{1}{\epsilon}}, \tag{18}$$

where $\lambda \in [0,1)$ is the weight of I_t^d, and $\epsilon < 1$ governs the (inverse) elasticity of substitution. Moreover, I_t^f is composed of imperfectly substitutable varieties, for reasons that we discuss below. In particular, I_t^f is a CES aggregate of foreign investment goods varieties:

$$I_t^f = \left(\int_{j\in[0,1]} (I_{i,t}^f)^\nu\right)^{\frac{1}{\nu}}, \tag{19}$$

where $I_{i,t}^f$ is the foreign investment good variety i, and the elasticity of substitution across varieties is given by $|\frac{1}{\nu-1}|$.

Producers of investment goods minimize the cost of producing a certain level of investment $(\overline{I_t})$ by solving the problem:

$$\min_{I_t^d, I_t^f} I_t^d + P_t^f I_t^f \tag{20}$$

subject to $\overline{I_t} = \left[\lambda(I_t^d)^\epsilon + (1-\lambda)(I_t^f)^\epsilon\right]^{\frac{1}{\epsilon}}$.
The optimality conditions for the first stage are:

$$I_t^d = \left(\frac{1-\lambda}{\lambda} \frac{1}{P_t^f} \right)^{\frac{1}{\epsilon-1}} I_t^f \tag{21}$$

$$I_t^f = \left(\lambda \left(\frac{1-\lambda}{\lambda} \frac{1}{P_t^f} \right)^{\frac{\epsilon}{\epsilon-1}} + (1-\lambda) \right)^{-\frac{1}{\epsilon}} I_t \tag{22}$$

and given the zero profit condition, $P_t^I I_t = I_t^d + P_t^f I_t^f$, the price of investment goods satisfies

$$P_t^I = \left(\lambda^{\frac{1}{1-\epsilon}} + (P_t^f)^{\frac{\epsilon}{\epsilon-1}} (1-\lambda)^{\frac{1}{1-\epsilon}} \right)^{\frac{\epsilon-1}{\epsilon}}. \tag{23}$$

At the second stage, given their optimal demand for the aggregate foreign investment good, investment good producers choose their demand for each variety i, subject to a working capital constraint. In particular, they need to finance a θ fraction of their purchase of foreign investment good. One can show that the demand for each variety i satisfies:

$$I_{it}^f = \begin{cases} \left(\frac{p_i^f (1+r_t^*)}{P^f(r_t^*)} \right)^{-\frac{1}{1-\nu}} I_t^f, & \text{for } i \in [0, \theta]; \\ \left(\frac{p_i^f}{P^f(r_t^*)} \right)^{-\frac{1}{1-\nu}} I_t^f, & \text{for } i \in [\theta, 1]. \end{cases} \tag{24}$$

Note that, unlike Mendoza and Yue (2012), we do not assume that producers of investment goods cannot access working capital loans during exclusion periods. Instead, the interest rate that the producers face depends on the price of bonds during exclusion periods, which is captured up by q^D. Thus, the costs rise but do not tend to infinity. Given the CES aggregator for foreign capital (equation 19), and given that a random θ fraction of varieties needs to be financed by working capital loans, then $P^f(r_t^*)$ satisfies the following:

$$P_t^f(r_t^*) = \left[\int_0^\theta (p_i^f (1+r_t^*))^{\frac{\nu}{\nu-1}} di + \int_\theta^1 (p_i^f)^{\frac{\nu}{\nu-1}} di \right]^{\frac{\nu-1}{\nu}} \tag{25}$$

Without a positive recovery assumption, one would have solved the demand function system with the limit of that system as $r_t^* \to \infty$.

4.4 Primary Dealer Banks

Bankers supply loans to firms and provide credit to the government by holding sovereign securities. For simplicity, we say that the timing of events that bankers undertake is divided into two interim periods, morning and afternoon. In the morning, bankers observe the TFP shock A_t with which the government's default decision d_t becomes public. Primary dealer banks meet the government's debt demand by up to B_{t+1}, and with the remaining resources they meet the demands of firms in the form of loans. Thus, the total amount of loans that can be supplied to the firms in the morning is:

$$\ell_t \leq \epsilon_t - \min\left\{\eta\epsilon_t, B_{t+1}(1-d_t)\right\}, \tag{26}$$

where ℓ stands for loans. Thus, the amount of loans that firms can obtain depends on the government's borrowing rule as well as its repayment decision. Notice that as the government increases its borrowing amount, the amount of available resources to the firms decline. So, this effect introduces a friction into the model: during the government's expansionary episodes, the banks may not be able to meet firms' entire loan demand. Recall that firms use these loans to pay a fraction of labor's wage. With the constrained borrowing, the labor market clears at a lower employment level. Notice that even if the government defaults and is left out of the financial markets, firms can still obtain loans from the banking sector.

Endowment ϵ_t is designed to capture bankers' source of income that is free of the government's borrowing and default decisions such as income from mergers and acquisitions, foreign exchange operations, and so on. The parameter η represents a natural limit on the amount of government debt securities a primary bank can hold. This is to ensure that there is a positive amount of loans available to private firms. Thus, primary dealer banks' portfolio includes at most $\eta\epsilon_t$ amount of government securities.

Note that the government's total new debt issuance is given by $b_{t+1} - (1-\delta_b)b_t$ and primary dealer banks can only supply B_{t+1}. International lenders provide the residual demand, that is, $b_{t+1} - (1-\delta_b)b_t - B_{t+1}$. The terms b_{t+1}, b_t, δ_b, d_t and asset price q_t are defined more extensively under the government's problem in Section 4.5. In our simulations, we confirm that the government almost always borrows more than what primary dealers banks can provide which is denoted by B_{t+1}. This is going to be important because the asset price is now going to be determined by the international lenders and primary dealers take it as given.

In the afternoon, bankers receive an interest rate of r_t^* from their loans to the firms and κB_{t+1} is the total amount of coupon receipts after the government's repayment decision accrued from the government's borrowing. The interest rate that the bank receives from holding government debt becomes $\frac{1}{q_t(b_{t+1},A_t)}$. Consequently, we can now write the primary dealers' profit as:

$$\pi_t^b = \overbrace{\epsilon_t - B_{t+1}(1-d_t) - \ell_t}^{morning} + \underbrace{(1+r_t^*)\,\ell_t + \frac{\kappa B_{t+1}\,(1-d_t)}{q_t(b_{t+1},A_t)}}_{afternoon}. \tag{27}$$

Modeling primary dealer banks' profit as in equation (27) provides the following advantages: (i) we do not need to introduce B_t as a state variable when solving the competitive equilibrium, (ii) if one writes the morning lending as $q_t B_{t+1}$ and afternoon receivables as $\kappa B_t\,(1-d_t)$, non-monotonicity arises due to the constraint in 26, that is, price q_t declines as debt issuance B_{t+1} increases.

4.5 Government's problem

This section describes the benevolent government's optimization problem. The government lacks a commitment technology, and thus cannot commit to its future default and borrowing decisions, and it decides how much non-state-contingent long-term debt to issue at each period after repayment.[13] The timing of our paper, as in Eaton and Gersovitz (1981), intends to rule out the multiplicity dynamics.[14]

We introduce an endogenous link between sovereign default and private economic activity. This endogenous link relates to the following three ingredients. First, there is a one-to-one mapping between the government's implied one-period borrowing rate and the firm's short-term working capital loan rate. The channel of that link is as follows: a rise in government debt holdings increases the likelihood of government default, and thus amplifies the bond spreads that are priced by risk-neutral international investors. Since local banks take bond prices as given, they pass on the increased cost of government debt holdings to firms by increasing loan rates. The second ingredient is that domestic and foreign goods are imperfectly substitutable. The intuition is that the higher rates would translate into an efficiency loss for the firms for their production because firms will now try to use more local goods with heightened costs of production. Efficiency loss happens since foreign investment goods and their domestic counterparts are assumed to

[13] Allowing for long-term maturity for sovereign debt helps us to obtain more realistic sovereign bond spreads and default frequencies. Note also that, under certain conditions, long-term debt incorporates short-term debt as a special case (Hatchondo and Martinez (2009), Chatterjee and Eyigungor (2012)).

[14] Önder (2016) documents that the models and extensions of the Eaton-Gersovitz type of sovereign default models are not prone to multiple equilibria.

be imperfectly substitutable. The last ingredient is the positive recovery following default. Without positive recovery, the implied interest rate on the loan would be zero during default episodes. The wages would then tend to infinity, and labor would plummet to zero.

4.5.1 Long-term debt contracts

The government issues long-term-non-state contingent debt in real goods which are traded at a price q_t by infinitely many risk-neutral international lenders. Similar to Hatchondo and Martinez (2009) and Chatterjee and Eyigungor (2012), a debt issued at time t promises a stream of geometrically decreasing coupons κ, which depreciate at a rate $\delta_b \in (0, 1]$. Thus, a government promises to pay $\kappa(1 - \delta_b)^{n-1}$ units of consumption good in period $t + n$ for $n \geq 1$. Coupon payment κ is computed such that in the absence of default risk, the price of non-state-contingent long-term debt is equal to the price of the average one-period debt, denoted as $\frac{r+\delta_b}{1+r}$. This is a common formulation for long-term debt contracts to avoid keeping track of the entire maturity distribution. Hence, the dynamics of the long-term can be represented as follows:

$$b_{t+1} = (1 - \delta_b)b_t + m_t, \tag{28}$$

where b_t and b_{t+1} are the total outstanding debt obligations at the beginning of period t and $t + 1$ and m_t is the amount of debt that is issued or purchased by the government in period t. Note that one-period debt is a special case of long-term debt where $\delta_b = 1$. The budget constraint of a government, conditional on its repayment decision is:

$$c_t = A_t f(K_t, L_t) - I_t^f P_t^f - \kappa b_t + q(b_{t+1}, A_t)m_t. \tag{29}$$

4.5.2 Defaults

A typical assumption in the literature is that the government is excluded from credit markets for a stochastic period of time following its default decision, and gains access to credit markets the next period with constant probability $\psi \in [0, 1]$. Bond contracts often include acceleration and collective action clauses; thus, all debt obligations become due when a government defaults.

To regain access to credit markets, the government needs to repay an α fraction of its defaulted debt. This is a simple way of introducing recovery into the model. However, in a model with long-term debt, the government issues an infinite amount of debt prior to its default decision, which may distort the moments in the simulations. To account for that, the assumption is that the government cannot issue bonds with a price lower than $\underline{q}$, as in Hatchondo et al. (2017). In our simulations, we confirm that this constraint never binds. In default, total output is used for consumption and imports of foreign investment goods. Therefore, the budget constraint of an economy in default is denoted as

$$c_t = A_t f(K_t, L_t) - I_t^f P_t^f. \tag{30}$$

4.5.3 Income cost of defaulting

In endogenous quantitative default models, a penalty scheme upon a default decision is required to be able to generate plausible debt-to-income ratios along with spreads. Otherwise, the government would always default, and lenders price it accordingly. Thus, only a limited amount of debt with high spreads can be supported in equilibrium. In our main analysis above, we abstract from assuming an additional income cost and resort to the model's endogenous dynamics. During default episodes, because of the increased cost of firms' working capital financing and the imperfect substitutability of domestic and foreign goods, there is a fall in investment and thus a decline in income. To improve the model's moment-matching success, we additionally assume a quadratic loss function for income during a default episode $\phi(y) = \max\{0, d_0 y + d_1 y^2\}$, as in Chatterjee and Eyigungor (2012).

4.5.4 International lenders

The international risk-neutral lenders invest in non-state-contingent long-term debt, taking decision and borrowing rules as given. Thus, the lenders' supply schedule reflects a no-arbitrage condition with an opportunity cost of funds equal to the world's risk-free rate (r). We denote the pricing schedule of bonds that are repaid as $q_t(b_{t+1}, A_t)$ and for bonds that are in arrears as $q_t^D(b_t, A_t)$. Recall that some portion of the debt is also held by local banks. Thus, the amount of external debt held by international lenders is determined as the residual between the aggregate stock of government debt and the public debt held by banks. In our simulations, we confirm that the entire amount of debt cannot be supplied by the local banks.

4.6 Recursive representation

We now formulate the government's optimization problem recursively. Let V be the value function of the government that has the option of defaulting. The government chooses to repay if the value of repayment V^R is greater than the value of default V^D. The function V satisfies the following functional equation:

$$V(b, A) = \max\left\{V^R(b, A), V^D(b, A)\right\}, \tag{31}$$

where the government's continuation value for repayment is denoted as

$$\begin{aligned} V^R(b, A) &= \underset{b'\geq 0, c\geq 0, L\geq 0, I^f\geq 0, I^d\geq 0}{Max} \left\{u(c, L) + \beta E_A\left[V(b', A')\right]\right\}, \\ \textit{subject to}: & \\ c &= Af(K, L) - I^f P^f(r^*) - b\kappa + q(b', A)\left[b' - (1-\delta_b)b\right], \\ b' &= \hat{b}(b, A), \\ q(b', A) &\geq \underline{q} \text{ if } b' > (1-\delta_b)b, \end{aligned} \tag{32}$$

where $f(K, L) = K_t^{\alpha^K} L_t^{\alpha^L}$. Imported foreign investment goods and the price of these goods are denoted by I^f and $P^f(r^*)$, respectively. The term r^* is the government's implied interest rate on one-period bonds. Note that the benevolent government faces the same allocations of output and factors of production as the agents in the private economy. To be specific, for a given TFP shock, implied short-term borrowing rate r^* and long-term bond price q —which will be obtained below under the price function—, as well as current and next-period government bond holdings, the optimal factor allocations (I^f, I^d, L) chosen by the benevolent government characterize the private sector competitive equilibrium.

The government stays in default at least for one period and regains access to the credit markets with constant probability ψ. The continuation value of default takes into account the positive repayment when the government regains access to the credit markets. We assume that debt in arrears grows at the international risk-free rate r during the sovereign's exclusion from the credit markets. Along these lines, the value of default is

$$\begin{aligned} V^D(b, A) &= \\ & \underset{c\geq 0, L\geq 0, I^f\geq 0, I^d\geq 0}{Max} \left\{u(c, L) + \beta E_A\left[\psi V(\alpha b(1+r), A) + (1-\psi)V^D(b(1+r), A)\right]\right\}, \\ \textit{subject to}: & \\ & c = Af(K, L) - I^f P^f(r^*). \end{aligned} \tag{33}$$

The solution to the government's optimization problem yields decision rules for default $\hat{d}(b, A)$ and borrowing $\hat{b}(b, A)$. A default decision rule is denoted by the binary variable $\hat{d} \in \{0, 1\}$; $\hat{d} = 1$ if the government finds the value of defaulting higher than the value of repayment and 0 otherwise. In equilibrium, defined in Section 4.7, lenders use these decision rules for contract pricing. The price of a bond satisfies:

$$\begin{aligned} q(b', A) &= \frac{1}{1+r} E_A \left[\left(1 - \hat{d}\left(b', A'\right)\right) \left(\kappa + (1-\delta_b)\, q\left(b'', A'\right)\right) + \right. \\ &\quad \left. \hat{d}\left(b', A'\right) q^D\left((1+r)\, b', A'\right) \right], \\ b'' &= \hat{b}(b', A'). \end{aligned} \tag{34}$$

Equation (34) indicates that with the no-arbitrage condition, selling a bond and investing it in a risk-free asset today at an international risk-free rate r (the left-hand side of the equation) is equal to the value of keeping the bond (the right-hand side of the equation). If the lender keeps the bond, she will receive one unit of coupon κ and can sell the unamortized portion of its assets $(1-\delta_b)$ at tomorrow's prevailing price $q\left(b'', A'\right)$ conditional on the government's repayment decision ($\hat{d}\left(b', A'\right) = 0$). If the government defaults, then lenders can still trade their bonds at the secondary market at a price q^D. The value of a bond in default satisfies:

$$q^D(b', A) = \frac{1}{1+r} E_A \left[\begin{array}{l} (1-\psi)\,(1+r) q^D\left((1+r)b', A'\right) + \\ \alpha\psi\hat{d}(b''^{DD}, A') q^D\left(b''^{DD}, A'\right) + \\ \alpha\psi\left(1 - \hat{d}(\alpha(1+r)b', A')\right)\left[\kappa + (1-\delta_b) q\left(b''^{DR}, A'\right)\right] \end{array} \right],$$

where $b''^{DD} = \alpha(1+r)b'$ denotes the next-period end-of-period total debt if the government defaults after exiting the current default and $b''^{DR} = \hat{b}(\alpha b')$ denotes the next-period end-of-period debt obligations chosen by a government that repays after exiting the default.

The short-term interest rate r^* that is used by the investment good producers in the economy is computed by setting $\delta_b = 1$. Notice that when $\delta_b = 1$, equation (34) boils down to the price of one-period debt, which is determined by tomorrow's default probability and the recovery rate.

4.7 Definition of equilibrium

This paper focuses on a Markov perfect equilibrium. The government cannot commit to any future (repayment and borrowing) decisions. Hence, the government's strategies depend only on the payoff-relevant state variables.

Definition 1 (Markov perfect equilibrium) *A Markov perfect equilibrium is characterized by value functions $V(b, A)$, $V^D(b, A)$, $V^R(b, A)$, bond pricing function $q(b', A)$, default rule $\hat{d}$, and borrowing rule $\hat{b}$ such that*

1. *Given the bond pricing function, government policy rules $\left\{\hat{d}, \hat{b}\right\}$ solve the utility maximization problem defined in equations (31), (32), and (34).*

2. *Given government policy rules $\left\{\hat{d}, \hat{b}\right\}$, the pricing function q satisfies condition (34).*

A solution to the Markov perfect equilibrium consists of the solutions to the competitive equilibrium given the government policies. The optimality conditions of the private sector including the factor allocations as well as production with and without access to the credit market, are obtained using Euler equations.

4.8 Numerical Solution

This section briefly sketches the main numerical algorithm, relegating the details of the implementation to the appendix. To solve the model, we take a two-pronged approach. First, the competitive equilibrium, given any government policy rules, is solved using the Euler equations characterized in the text. We obtain the optimal private sector allocations for any productivity shock, the short- and long-term bond price, as well as current and next-period government bond holdings. With these optimal private allocations in hand, we then solve the government's problem with global solution methods. In particular, solving the model relies on iterating the value functions V^R and V^D and price function q as well as an approximation scheme to the private sector's allocation problem. To avoid the potential multiplicity problem outlined in Krusell and Smith (2003), we first solve the equilibrium of the finite-horizon economy. We start with an initial guess for the terminal value and iterate backward until the differences in value and price functions for two subsequent periods are less than 10^{-5}. We then use the obtained values as the equilibrium of the infinite horizon economy.

4.9 Calibration

We calibrate the model economy at a quarterly frequency. For most parameters, we use conventional estimates reported in the literature. For the remaining parameters, we resort to the Colombian data using simulated method of moments (SMM). Table 8 presents the calibrated parameter values.

Table 8: Parameter values

	Parameter	Value	Target
Risk aversion	γ	2	Standard RBC value
Risk-free rate	r	1%	Standard RBC value
Probability of reentry after default	ψ	0.083	Mendoza and Yue (2012), Gelos et al. (2011)
Recovery rate	α	0.55	Cruces and Trebesch (2013)
Bond decay rate	δ_b	0.033	Average duration of 5 years
Lower bound for bond prices	$\underline{q}$	0.5	-
Labor supply curvature parameter	ω	1.455	Frisch wage elasticity (2.2)
Armington weight of domestic investment	λ	0.62	Data
Armington curvature parameter	ϵ	0.62	Mendoza and Yue (2012)
Dixit-Stiglitz curvature parameter	ν	0.59	Mendoza and Yue (2012)
Upper bound to use working capital	θ	0.7	Mendoza and Yue (2012)
Share of capital	α^K	0.45	Zuleta et al. (2009)
Share of labor	α^L	0.45	Bernanke and Gürkaynak (2001); Zuleta et al. (2009)
Depreciation rate of capital	δ	0.035	Data
Limit of govt securities for primary dealer	η	0.15	Colombian Data
Calibrated			
Discount factor	β	0.96	Default frequency (2%)
Income autocorrelation coefficient	ρ	0.97	GDP autocorrelation (0.752)
Standard deviation of innovations	σ_ϵ	2.0%	GDP std. deviation (0.015)
Income cost of defaulting	d_0	-0.99	Spread and Debt
Income cost of defaulting	d_1	1.01	Spread and Debt

The risk aversion parameter is set to 2 and the quarterly world interest rate is set to 1%, the conventional values used in the literature. The labor supply curvature parameter, ω, is set to 1.455, as in Mendoza and Yue (2012). This value yields a Frisch elasticity of labor supply, $\frac{1}{\omega-1}$, equal to 2.2, and is well within the range reported in the literature.[15] The probability of re-entry after default, ψ, is set to 0.083, which implies that the country stays in default about three years before regaining access to international markets (Mendoza and Yue (2012), Gelos et al. (2011)).

The weight of domestic investment goods in the Armington aggregate of investment is based on the OECD Trade-in Value Added (TiVA) database. In particular, about 38% of gross fixed capital formation in Colombia is from imports. We accordingly set λ equal to 0.62. Moreover, we set $1 - \alpha^K - \alpha^L$ equal to 0.1 to have a reasonable profits-to-GDP ratio in the model, consistent with the data. The factors shares α^K and α^L are then set equal to 0.45, in line with Bernanke and Gürkaynak (2001) and Zuleta et al. (2009). The quarterly depreciation rate, δ, is set to 0.035, and is slightly higher than estimates for developed economies. For the remaining parameters for production, ϵ, ν, θ, we follow Mendoza and Yue (2012) and set them equal to 0.62, 0.59, and 0.70, respectively.

[15] See, for example, Rogerson and Wallenius (2009), Christiano et al. (2009), and references therein.

We target the debt-to-GDP ratio of 38.3% using the rich public debt dataset provided by Abbas et al. (2010). The productivity shocks in the model follow an AR(1) process:

$$log(A_t) = \rho log(A_{t-1}) + \varepsilon_t$$

with $|\rho| < 1$ and $\varepsilon_t \sim N\left(0, \sigma_\epsilon^2\right)$. Because of the lack of an actual TFP series, we set ρ and σ_ϵ^2 in the SMM process to target the GDP process of Colombia, using HP-detrended seasonally adjusted quarterly real GDP series for the period 200Q1 to 2017Q4. This is the longest time frame during which the quarterly GDP data are available. The standard deviation coefficient and the autocorrelation coefficient of the cyclical component of estimated GDP is 1.2% and 0.76, respectively. Our SMM procedure spells $\sigma_\epsilon^2 = 0.3\%$ and $\rho = 0.97$.

We have two main calibration exercises, with and without introducing the income cost of defaulting. A penalty scheme is necessary in this class of models because otherwise the government would always default, and lenders would price it accordingly. So in equilibrium, only a limited amount of debt issuance at very high spreads can be generated. We initially resort to our model's dynamics. The endogenous penalty scheme in our model is the decline in investment that is followed by a fall in output. In this exercise, we target the annual debt-to-income ratio of 38.3 percent with $\beta = 0.92$. The discount factor β is smaller than that in regular business cycle models, but it is still much larger than the existing corresponding studies. For instance, Mendoza and Yue (2012) set β as 0.88 and generate a debt-to-GDP ratio of 23%. Similarly, Sosa Padilla (2018) uses a discount factor of 0.80, which helps to generate a debt-to-GDP ratio of 13%. In the next calibration exercise, we introduce an exogenous income cost of defaulting as in typical quantitative default studies to improve the model's moment-matching success. We calibrate the value of the parameters of the income cost of defaulting (d_0 and d_1) targeting the mean levels of debt and sovereign spreads.

We set δ_b at 3.3%. With that value, the maturity of long-term debt becomes 5 years, which is roughly the average bond duration estimated for emerging economies documented by Cruces et al. (2002)[16]. The definition of duration in Macaulay (1938) is standard in calculating the long-term debt duration. Duration D is the weighted average maturity of future cash flows. A bond issued at time t makes periodic payments κ for the subsequent periods with a geometrically decay rate δ_b.[17] Observe that equation (36) is 1 for $\delta = 1$. The

[16]Cruces et al. (2002) report an average debt duration of 4.8 years with a standard deviation of 1.5 in the year 2000 for emerging economies.

[17]Duration D satisfies,

$$D(\kappa) = \frac{1}{q}\left(\sum_{j=1}^{J} j\frac{\kappa(1-\delta_b)^{j-1}}{(1+i)^j}\right)$$

sovereign spread r_s is defined as the difference between yield i and the risk-free rate r. The annualized spread reported in the tables is computed as

$$1 + r_s = \left(\frac{1+i}{1+r}\right)^4.$$

The debt levels obtained from the simulations are equivalent to the present value of future debt obligations, which are discounted at the risk-free rate and computed as $\frac{b(1+r)}{\delta+r}$.

5 Quantitative Results

This section presents our baseline results. Table 9 compares the data moments from Colombian data with the one obtained from the model. The model features plausible moments and matches both the sovereign debt statistics as well as the business cycle moments reasonably well. We undertake two calibration exercises: (i) one that does not introduce an exogenous income shock during defaults and resorts to the endogenous decline in investment during exclusion episodes, and (ii) one that engineers exogenous income cost of defaulting.

The model without exogenous income cost of defaulting generates debt-to-GDP ratio of 31 percent, on average. This result is mainly a byproduct of an endogenous decline in investment during default episodes which increases the cost of defaulting. Part of the reason why our study does a better job in matching debt-to-GDP ratio and the spreads, lies in the modeling choice. Inclusion of long-term maturity of sovereign debt and partial recovery after default are important ingredients. The role of long-term debt in quantitative

where i is the periodic yield an investor would earn if the bond is held to maturity without any defaults and it satisfies

$$q = \sum_{j=1}^{\infty} \frac{\kappa(1-\delta)^{j-1}}{(1+i)^j},$$

and the periodic yield i reads as

$$i = \frac{\kappa}{q} - \delta_b$$

with which D becomes

$$D = \frac{1}{q(1-\delta_b)} \lim_{J\to\infty} \sum_{j=1}^{J} j \left(\frac{1-\delta}{1+i}\right)^j \quad (35)$$

$$= \frac{1+i}{i+\delta_b}. \quad (36)$$

sovereign default models are thoroughly discussed in Hatchondo and Martinez (2009) and Chatterjee and Eyigungor (2012).

As expected and in line with the data, the model generates negative correlation between interest rate spreads and GDP. Quantitative models of sovereign default as well as the business cycles in emerging markets yield countercyclical interest rates. However, in the former one output is an exogenous process and in the latter one the interest rate spreads are exogenous. In contrast, this paper introduces an endogenous income process and endogenously determined interest rate spreads both of which are influenced from each other. We should note that Mendoza and Yue (2012) also generate, even though it is weak, a negative correlation between interest rate spreads and income.

Turning to our second calibration which engineers an exogenous income cost of defaulting, third column of Table 9 shows that the model generates positive correlation between investment-to-GDP ratio and debt-to-GDP ratio. However, as our empirical model shows, the negative feedback between investment and GDP intensifies when the government debt ratio is high. To account for that, we also report the correlation of these two moments for the instances where debt-to-GDP ratio exceeds its 75th percentile. The model now produces significant crowding-out channel. This result hinges on the inter-linkages between private sector and the sovereign risk. There are essentially two forces working in the same direction. First, higher government indebtedness limits the primary dealers' ability to provide loans to firms as they are primarily required to supply credit to the government. Due to this supply crunch, both labor and investment falls. Second, higher government debt induces a rise in default risk resulting in higher bond spreads. Primary dealers then pass on these costs to firms. This chain of events leads a decline in the private sector's investment levels.

Figure 10 displays the dynamics of debt-to-GDP and investment-to-GDP ratios. From the figure it is visible that investment-to-GDP is not responsive for low levels of debt-to-GDP. This follows because the government's implied short-term borrowing rate does not move much so that the banks do not pass their costs to firms. Furthermore, availability of funds from banks to the private sectors is not necessarily hindered. The country risk is almost zero for low enough debt levels (see Figure 11). Recall that the private sector's investment also depends on the imperfectly substitutable foreign investment goods whose cost depends on the country's short-term risk. As the government's total borrowing increases, the default risk and thus the short-term borrowing rate as well as the price of foreign investment goods rise. This, in turn, reduces per period equilibrium investment and explains the intuition of why crowding-out kicks in for higher levels of sovereign indebtedness. In addition, the model also features a negative correlation between sovereign

Table 9: Long-run Statistical Moments

	(1) Data	(2) Model	(3) Income cost
Long-term debt statistics[a] (%)			
Mean debt-to-GDP	38.3	31	36.3
Mean r_s	2.1	0.2	2.1
Default rate	2	0.5	2.07
Debt duration (years)	5	5	5.8
Mean investment-to-GDP	22.0	26.4	26.6
Inv decline during exclusions(%)	35	25	2.5
Volatilities (%)			
$\sigma(c)/\sigma(GDP)$	0.92	1.1	1
$\sigma(tb/GDP)$	0.86	1.71	0.7
$\sigma(r_s)$	0.69	0.41	0.01
Correlations			
$\rho(c, GDP)$	0.91	0.98	0.7
$\rho(tb/GDP, GDP)$	0.37	0.72	0.35
$\rho(r_s, GDP)$	-0.51	-0.83	-0.08
$\rho(inv, GDP)$	0.86	0.97	0.97
$\rho(r_s, inv)$	-0.54	-0.80	-0.77
$\rho(r_s, tb/GDP)$	-0.11	0.94	0.92
$\rho(inv/GDP, b/GDP)$	0.38	0.63	0.63
$\rho(inv/GDP, b/GDP)^a$	-0.25	-0.60	-0.88

Long-term statistics are annualized. The third column represents the moments obtained with income cost of defaulting. Consumption (c), investment (inv), trade balance (tb) are in real terms. GDP excludes government expenditures (inline with the model). All series are deseasonalized (X13), and HP filtered with a smoothing parameter of 1600. EMBIs (r_s) are quarterly averaged. Public debt (b) is reported annual, so the correlations that involves b are based on annual data. The model moments are calculated using the averages of 250 simulated sample paths of 100 quarters (25 years) without any default and each sample path starts at least five years after a default incident. The business cycle moments are detrended with HP-filter using a smoothing parameter of 1600. If otherwise noted, source of the data is Central Bank of Colombia. $\sigma(h)$ denotes the standard deviation of a variable h. $\rho(h, m)$ denotes the coefficient correlation between variables h and m.
a Computes the correlations of instances where b/GDP is higher than its 75th percentile.

spreads and private investment. Intuitively, the banks pass on the costs of government's borrowing to firms by requiring higher loan rates which reduces investment.

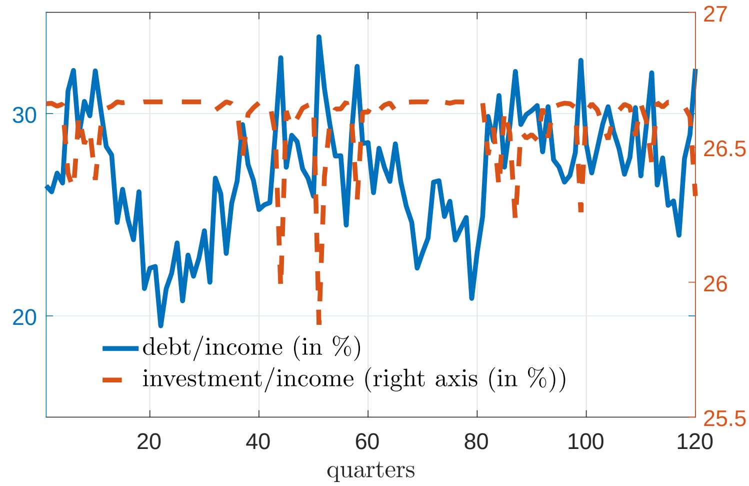


Figure 10: Dynamics of debt-to-GDP and investment-to-GDP in the model without the samples of default. Blue solid line represents debt-to-income ratio and dashed line represents invest-to-income ratio on the right-y axis.

It is visible in Figure 11 that there is one-to-one mapping between investment-to-GDP ratio and the short-term borrowing rate. As noted in the model, firms and investment goods producers use within period working capital loans and therefore, only the short-term country risk matters. To better gauge the dynamics of the model, we will enforce firms to borrow from the long-term rate in an exercise below.

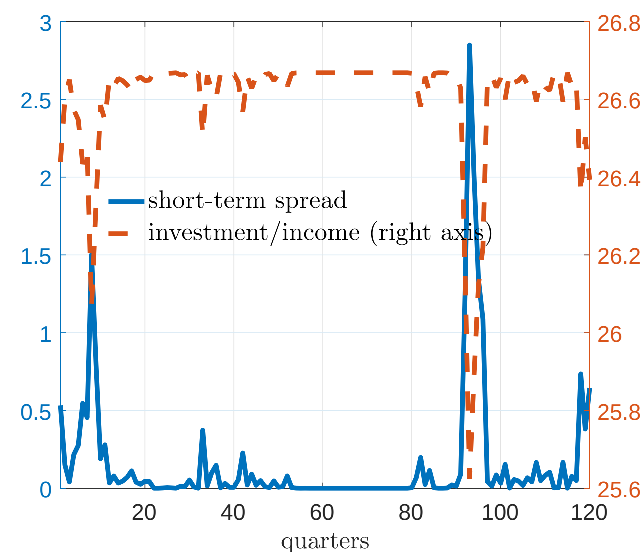


Figure 11: Dynamics of short term country risk spread and investment-to-GDP in the model without the samples of default.

Crucially, the dynamics of one-period debt and long-term debt are not identical. In contrast to the pricing of short-term debt, the price of long-term debt depends not only on the current borrowing amount but also on the future borrowing and default decisions. This paper also adds to our understanding of quantitative endogenous sovereign default models with long-term debt. Figure 12 plots the spreads for long-term debt and the implied spreads for one-period debt. Observe that long-term debt spreads never fall to zero while one-period debt spreads do. This follows with the feedback mechanism inherently present with long-term debt: future default and borrowing decisions influence the current price dynamics. This is important because in a model where price of the foreign goods depends on long-run borrowing rate, the implications would be boosted since spreads under one period debt are significantly smaller than the one with long-term debt.

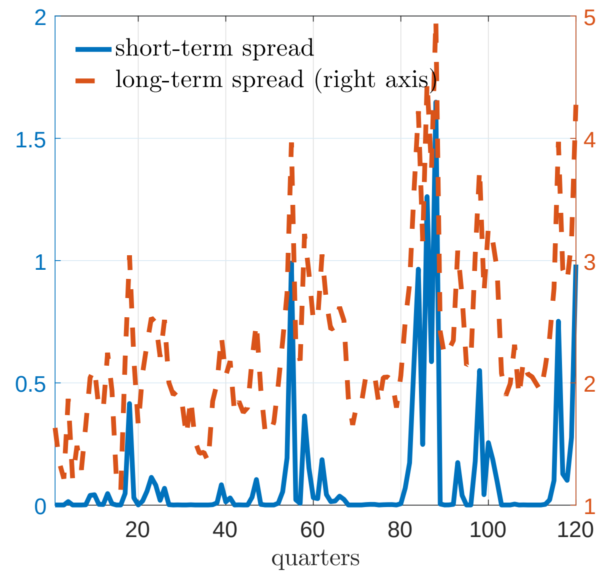


Figure 12: Dynamics of long-term (left-axis) and implied short-term (right-axis) borrowing rates without the samples of default.

Our model also accounts for the sharp fall in GDP, consumption, labor and investment goods during government's exclusion from the credit markets that are consistent with the existing empirical literature on sudden stop events. For the vast majority of the sudden stop literature, these dynamics are generated as an outcome of the exogenous shock; in contrast, in our model sudden stop occurs with an endogenous default following a decline in firm investment. Upon government's default decision, the cost of production jumps for firms because of high interest rates on loans for foreign investment goods which is an imperfect substitute for production. Our model slightly underestimates the decline in investment during financial exclusion. Total investment drops 25 percent in our model whereas this value corresponds to 35 percent in the data.

Fiscal Multipliers In Figure 13 we plot how income, investment and labor responds when the government borrows slightly more than its optimal borrowing rule $\hat{b}(b, A)$. Simply, we plot how the change in optimal borrowing rule $(\Delta\hat{b}(b, A))$ affects the change in output (Δy). Notice that an increase in the optimal borrowing rule results in a decline in the government's income. The black dot on the blue lines of the figure represents the debt level b observed in the simulations for mean TFP shock. The right panel of the figure decomposes the source of the decline in income. It has two major elements: decline in investment and decline in labor. Investment declines because primary dealer banks pass on their cost of holding government debt to local firms. Labor mainly declines because financial costs of labor following an increase in wage bills (derived in equation (17). The intuition relies on the fact that the firms are now more reluctant to hire labor because cost of working capital rises; portion of the wage bills is paid upfront prior to the production phase.

Notice in Figure 13 that change in income as well as change in investment and labor are not responsive to change in to debt levels for relatively lower levels of debt. This mainly relies on the cost of working capital being modeled as one period loan rate. Thus, next period's cost of defaulting for lower levels of debt is zero which makes one period loan rate to be risk-free rate. In Figure 14, however, we enforce the firms to borrow from the government's long-run borrowing rate. The figure now shows that change in income, investment and labor are all responsive to a change in smaller debt levels. This is because long-run borrowing rate never becomes zero as it depends on future payment streams.

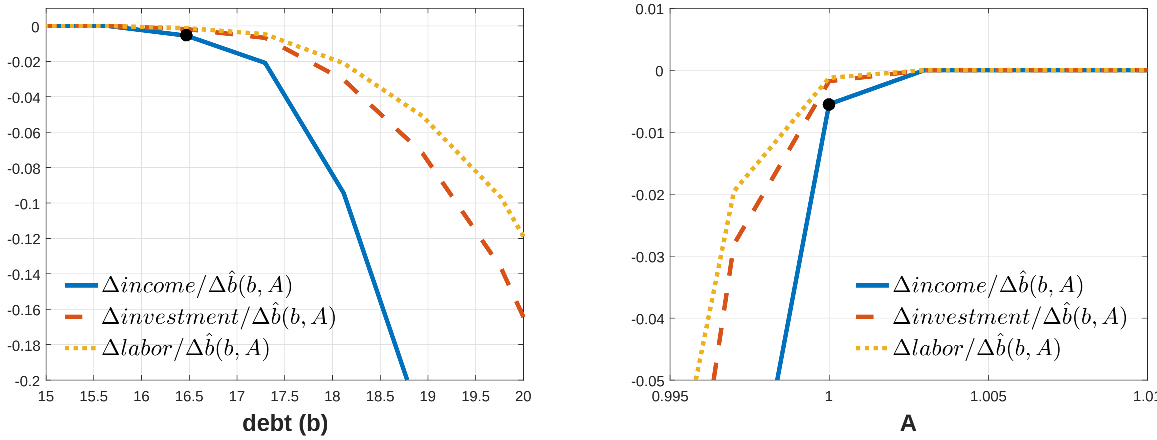


Figure 13: The figure shows the derivative of income, investment and labor to the derivative of optimal next period debt holdings. Left panel plots the derivative for alternative initial outstanding debt holdings for mean income level while the right panel plots for alternative initial tfp shocks for mean income debt level. The black dot on the left and right figures represent the unconditional mean debt level and mean income level obtained in the simulations, respectively.

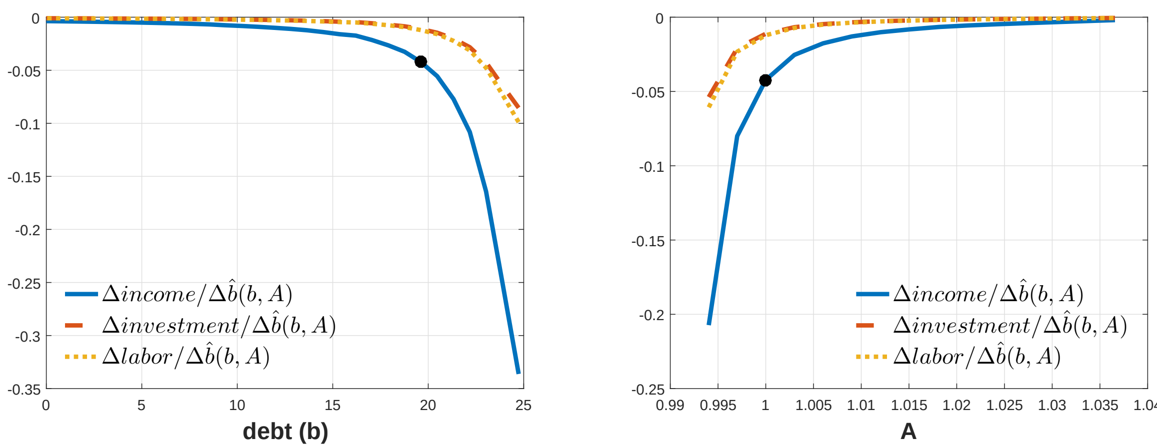


Figure 14: The figure shows the derivative of income, investment and labor to the derivative of optimal next period debt holdings when firms borrow at long-term interest rates instead of the government's implied one-period borrowing rate. Left panel plots the derivative for alternative initial outstanding debt holdings for mean income level while the right panel plots for alternative initial tfp shocks for mean income debt level. The black dot on the left and right figures represent the unconditional mean debt level and mean income level obtained in the simulations, respectively.

6 Conclusions

We investigate the impact of government spending on firm investment, by closely tracing firms with multiple banking relationships. On the empirical front, we postulate a crowding-out effect as a function of public debt. That is, we confirm a crowding-out channel to corporates and find that this effect is more pronounced during episodes of high government debt. Further, to investigate real sector effects we propose novel firm-based measures of financial exposure. With these measures, we evaluate effects on firm's outcomes such as: wages, investments, assets, liabilities, equity, and profits.

All of our results point towards how increased borrowing affects the dynamics of economic activity and crowds out private investment. In particular, we find that an increase in banks' bonds-to-assets ratio decreases loans to corporates. In turn, firm's outcome variables are negatively exposed.

Finally, our findings are grounded in an endogenous sovereign default model with financial and real sectors. In contrast to most of the literature, our framework is enriched with investment, financial sectors and long-term debt. Ultimately, we shows that increased government spending limits the amount of available funds to firms and raises sovereign risk. Primary dealers then pass on these costs to local firms.

Our study, to the best of our knowledge, is the first to establish a causal link (using micro data) wherein resources to the private sector are deterred by the take-up of government debt, which in turn leads to lower investment. Hence, we believe that our findings can better guide fiscal and monetary policy makers, especially in times of spending booms.

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A Computation algorithm

The computational algorithm works through iterating value functions (V^R, V^D) and price function q until a convergence is obtained [18]. As explained in the text, we first set the terminal values of the finite horizon economy and iterate the economy backwards until we obtain convergence.

For a given short-term borrowing rate r_t^*, price q_t, current and any next period borrowing amount b_t, b_{t+1} and TFP shock A_t, one can compute the optimal factor allocations (I_t^f, I_t^d, L_t) with w_t satisfying the shadow prices given by the Lagrange multipliers for labor. To obtain the allocations of the private sector, 100 grid points set for both short-term borrowing rate r_t^* and TFP shock A_t. One can solve this problem separately outside of the value-function iteration procedure and use the equilibrium outcomes of this procedure within the iteration routine using approximation schemes.

The numerical algorithm that is engineered to solve the problem can be explained in steps as follows:

1. Solve the equilibrium factor allocations of the private sector for a given TFP shock A and one-period loan rate r^*, price q, assets b and b_{t+1}.

2. Set the initial guesses for the terminal values of value and pricing functions (V^R, V^D, q, q^D) in the finite-horizon economy as follows:

   - $V^R(b, A) = u(Af(K, L) - I^f P^f - \kappa b, L)$,
   - $V^D(b, A) = u(Af(K, L) - I^f P^f_{aut}, L)$,
   - and $q = 0$, $q^D = 0$.

3. Solve for the optimization problem defined in equation (32) and equation (34) for each grid. We take 40 equally-spaced grid points for b' to obtain the optimal borrowing policy and find the candidate maximizing the optimization problem. We feed this candidate grid point into one-dimensional DUVMIF routine of the IMSL library for Fortran as the initial guess of our optimal b'[19].

4. Iterate the procedure defined in step 3 over equations (32) to (34) until the difference in two subsequent iterations is less than the convergence criterion.

[18] Convergence criteria is 10^-5.

[19] Another alternative would be to increase the grid points for b', however, this methodology is faster than using discrete state space method as outlined in Hatchondo et al. (2010).

After obtaining the converged equilibrium functions, decision rules and factor allocations; I simulate the economy. For that, I fix the number of samples to be 300 and each sample has 1501 periods from which initial 1000 periods are trimmed to eliminate the influence of any arbitrarily chosen initial states. The initial states of A_0 and b_0 are set to be mean A and zero, respectively. It is assumed that the government does not default in the first period. We initially draw sequences of TFP shocks and re-entry probability ψ using random number generator.

B Loan Substitution and Periods of Low Government Debt

Table A1: Net effect of banks' bond holdings on corporate credit lines

	$Loan_{i,j,t+k} + Loan_{i,\text{-j},t+k}$	
Periods	*Bonds*	$Bonds * Primary * ColDebt$
1	-0.038	-0.059
	(0.055)	(0.040)
2	-0.095*	-0.0057
	(0.056)	(0.043)
3	-0.076	-0.011
	(0.049)	(0.056)
4	-0.10**	0.037
	(0.045)	(0.055)
5	-0.044	-0.026
	(0.055)	(0.063)
6	-0.051	-0.10**
	(0.035)	(0.047)
7	-0.018	-0.19***
	(0.055)	(0.050)
8	-0.045	-0.17***
	(0.041)	(0.049)
9	-0.096**	-0.15***
	(0.040)	(0.046)
10	-0.061	-0.12***
	(0.042)	(0.028)
11	-0.079*	-0.083**
	(0.046)	(0.035)
12	-0.050	-0.055
	(0.056)	(0.053)
Cluster by firm	x	x
Firm-year fixed effects	x	x
Bank fixed effects	x	x
Time fixed effects	x	x

Authors' calculations. The sample includes all months from December 2004 to December 2015. Each listed coefficient results from a separate regression following equations (2) and (3). Rows denote outcomes k-months after treatment. $Loan_{i,j,t+k}$ corresponds to the value (in logs) of all new loans from bank j to firm i, in month $t+k$. *Bonds* denotes the bank's stock of government bonds as a share of its assets. *Primary* indicates the amount of bonds purchased in the primary dealer market, also as a share of its assets. ***, **, and * denote statistical significance at the 1%, 5%, and 10% level, respectively. For all regressions, the average R^2 is 0.80 with 60,000 observations.

Table A2: IRFs of bank's bond holdings on corporate credit lines (Periods of Low Debt)

Periods	$Loan_{i,j,t+k}$ (Bank "j")		$Loan_{i,\text{-j},t+k}$ (Other Bank "$-j$")	
	Bonds	$Bonds * Primary * D_{Low\_Debt}$	*Bonds*	$Bonds * Primary * D_{Low\_Debt}$
1	-0.14	0.0035	0.11	0.0019
	(0.18)	(0.010)	(0.14)	(0.0071)
2	-0.23	0.011	0.14	-0.0087
	(0.18)	(0.0084)	(0.14)	(0.0060)
3	-0.27	0.015	0.20	-0.011
	(0.18)	(0.0099)	(0.14)	(0.0077)
4	-0.27*	0.018	0.18	-0.016
	(0.16)	(0.015)	(0.12)	(0.012)
5	-0.23	0.017	0.19*	-0.018**
	(0.16)	(0.010)	(0.11)	(0.0073)
6	-0.36**	0.015*	0.32***	-0.0081
	(0.14)	(0.0072)	(0.11)	(0.0069)
7	-0.35***	0.030**	0.33***	-0.020**
	(0.13)	(0.011)	(0.080)	(0.0080)
8	-0.20	0.030**	0.18	-0.019**
	(0.14)	(0.011)	(0.12)	(0.0089)
9	-0.31**	0.023**	0.23**	-0.017*
	(0.12)	(0.0098)	(0.097)	(0.0086)
10	-0.14	0.034*	0.077	-0.027*
	(0.18)	(0.016)	(0.14)	(0.013)
11	-0.044	0.0093	-0.023	-0.0087
	(0.17)	(0.0093)	(0.12)	(0.0080)
12	-0.059	0.013*	0.015	-0.011
	(0.18)	(0.0073)	(0.12)	(0.0063)
Clustered by bank	Y	Y	Y	Y
Firm-time fixed effects	Y	Y	Y	Y
Bank fixed effects	Y	Y	Y	Y

Authors' calculations. The sample includes all months from December 2004 to December 2015. Each listed coefficient results from a separate regression following equations (2) and (3). Rows denote outcomes k-months after treatment. $Loan_{i,j,t+k}$ corresponds to the value (in logs) of all new loans from bank j to firm i, in month $t+k$. *Bonds* denotes the bank's stock of government bonds as a share of its assets. *Primary* indicates the amount of bonds purchased in the primary dealer market, also as a share of its assets. ***, **, and * denote statistical significance at the 1%, 5%, and 10% level, respectively. For all regressions, the average R^2 is 0.80 with 60,000 observations.

C Robustness - NOT INTENDED FOR PUBLICATION

In this section we provide some robustness tests for our results presented in the text. Table A3 presents the regression output when we include the interest rate on the loan as a control.

Table A3: IRFs of banks' bond holdings on corporate credit lines (interest rate on the loan as a control)

Periods	Loans (Bank "j")		Loans (Bank "$-j$")	
	$\frac{Bonds}{Assets}$	$\frac{Bonds}{Assets} * \frac{Wonamount}{Assets} * \frac{Debt}{GDP}$	$\frac{Bonds}{Assets}$	$\frac{Bonds}{Assets} * \frac{Wonamount}{Assets} * \frac{Debt}{GDP}$
1	-0.029	0.053	0.024	-0.12
	(0.16)	(0.10)	(0.11)	(0.10)
2	-0.12	0.042	0.061	-0.067
	(0.16)	(0.14)	(0.11)	(0.10)
3	-0.18	-0.33	0.13	0.28*
	(0.17)	(0.19)	(0.12)	(0.15)
4	-0.22	-0.15	0.14	0.19
	(0.14)	(0.20)	(0.10)	(0.15)
5	-0.25	-0.21	0.22*	0.22*
	(0.19)	(0.15)	(0.13)	(0.11)
6	-0.35**	-0.25	0.32***	0.15
	(0.14)	(0.16)	(0.12)	(0.14)
7	-0.32**	-0.64***	0.34***	0.41***
	(0.14)	(0.12)	(0.092)	(0.10)
8	-0.20	-0.46**	0.19	0.27*
	(0.14)	(0.16)	(0.12)	(0.15)
9	-0.26**	-0.65**	0.19**	0.48**
	(0.12)	(0.25)	(0.091)	(0.20)
10	-0.023	-0.60**	-0.016	0.49**
	(0.18)	(0.21)	(0.14)	(0.19)
11	-0.020	-0.22	-0.034	0.16
	(0.16)	(0.18)	(0.13)	(0.17)
12	0.024	-0.38	-0.026	0.31
	(0.19)	(0.24)	(0.13)	(0.18)
Cluster by bank	x	x	x	x
Firm-time fixed effects	x	x	x	x
Bank fixed effects	x	x	x	x

Authors' calculations. The sample includes all months from December 2004 to December 2015. Each listed coefficient results from a separate regression following equations (2) and (3). Rows denote outcomes k-months after treatment. $Loan_{i,j,t+k}$ corresponds to the value (in logs) of all new loans from bank j to firm i, in month $t+k$. *Bonds* denotes the bank's stock of government bonds as a share of its assets. *Primary* indicates the amount of bonds purchased in the primary dealer market, also as a share of its assets. ***, **, and * denote statistical significance at the 1%, 5%, and 10% level, respectively. For all regressions, the average R^2 is 0.80 with 60,000 observations.