

A Macroeconomic Model with Firm Debt Financing, Bank Lending and Banking Regulations

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Job Market Paper

Non-technical Summary

As an extension of my PhD thesis, this paper investigates the impacts of the Basel-style capital and liquidity requirements on bank lending, firm debt financing, and economic growth. This paper aims to shed new light on the macro-prudential impacts of the banking regulations on the whole economy, which has been less documented in the recent literature. This paper considers bankers (shareholders of banks), entrepreneurs (owners of firms) and savers (investors) and investigates the questions using DSGE modelling. This research reveals that the impacts of the banking regulations might expand to the equilibrium of the whole economy, not only within the banking sector. This research thus indicates that policymakers should be aware that introduction of the future new regulations (for example Basel IV) on banks could affect other sectors (such as non-financial firms) as well and thus some mitigation should be implemented to ensure the stability of the whole economy.

A Macroeconomic Model with Firm Debt Financing, Bank Lending and Banking Regulations

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Abstract

How would banking regulations affect debt financing structure of firms, the borrowers of banks? To deal with this, I develop a general equilibrium model in which firms borrow loans from banks and issue corporate bonds to banks and savers. Savers use their wealth as deposits in the banks and to buy corporate bonds from the firms. The government guarantees the deposits and imposes Basel-style (capital and liquidity) requirements on banks. The model reveals changes in firms' debt financing structure as a result of different requirements and impacts caused to the real economy. Both the requirements result in lower financial fragility, reduce the volatility of the banking system but lower the size of loan lending and output. The requirements also drive up deposit rate, bond rate and loan rate but narrow down spreads between the deposit rate and the other two rates. These effects are more significant when with the liquidity requirements. Due to the increase in the corporate bond rate, firms will not rely on the issuance of corporate bonds to compensate for the reduction in loans. Wealth is redistributed from the owners of banks to savers and the owners of firms. Implementation of the banking regulations might cause sizeable effects on firms and other sectors of the economy.

JEL: G12, G21, G28

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1. Introduction

The Basel committee on Banking Supervision (BCBS) is actively setting up regulatory requirements on banks. Its current version of Basel III raises capital requirements and introduces liquidity requirements. Current literature has revealed that increasing capital requirements will reduce bank lending, and thus depress investment and output. Moreover, there has been criticism that liquidity requirement could impair economic growth by further limiting banks' lending to the real economy.

What is missing in the literature is a quantitative general equilibrium model that embeds financial sectors in a model of macro-economy, and that can reveal the impacts of these requirements not only on the banks' lending but also on the debt financing structure of the firms. My paper builds such a model. In the model, banks lend long-term loans and hold corporate bonds to invest in firms and banks optimally choose the holdings of the loans and the corporate bonds and the dividend to shareholders to maximise the shareholders' wealth.

Firms use the borrowed funds to invest in capital and hire their own and savers' labour for production. Firms default due to an aggregate output and idiosyncratic productivity shocks. Corporate bonds are risky but are entitled to priority in claims if firms default. Savers supply their labour to firms for wages and use the wages and wealth to invest by depositing in banks, receiving a risk-free return, and by buying corporate bonds for a higher return which is risky. Banks collect the residuals from the defaulting firms as loan holders. Government will bail-out banks if they are distressed as a result of the default of the firms. Both the default of the firms and banks will cause bankruptcy costs to the real economy. My model focuses on the measures, regulatory requirements on banks, taken to mitigate the costs caused by the failure of the banks. To capture the regulatory requirements, I follow the Basel-style requirements, and regard them as constraints for each of the requirements in the system of my model. Under the capital requirement, banks are required to hold at least a ratio of capital to their lending. In addition, banks should keep their liquidity coverage ratio higher than a required ratio. The required ratios are the key macro-prudential policy parameters.

My main exercise is to study the macro-prudential impacts of the banking regulatory requirements on the real economy. With the use of the general equilibrium model, I can calculate the results including loan rates, corporate bond rates, (risk-free) deposit rates, probability of default of banks and firms, amount of bank lending and trading volume of corporate bonds. I

compare these results when no requirements are imposed, only capital requirement is introduced, and both of the capital and liquidity requirements are implemented. Principally, capital requirement and liquidity requirements reduce banks' lending and leads to a lower output, but generates a safer and a stable banking system. Implementation of the requirements drives down the credit spread and raises risk-free (deposit) rate. The wealth is then redistributed from bankers to entrepreneurs and savers.

My model matches many features of the data. The results of my model match the risk-free rate (2.20%), corporate bond rate (3.68%) and loan rate (5.60%). My model also captures some key features of the economy. The ratio of capital investment to GDP generated by my model is 26.11%, very close to the data of 21.44%. The standard deviation of this ratio is also very close to that of the data, with 1.52%. The ratio of credit to GDP and the ratio of corporate bond to GDP match the data well. The consumption-to-GDP ratio is 67%, from the data, while the value is 63.9% from the model. Finally, the model generates the firm leverage at around 30.7%, not far from the data of 40%.

My study contributes to the literature by providing a macro-prudential effect on the Basel-style requirements on the real economy. Current literature, such as Repullo and Suarez (2012), De Nicolo *et al.* (2014) and Hugonnier and Morellec (2017), focus exclusively on the banking sector and fail to consider the participation of other sectors, such as firms, the borrowers of the bank loans. The inclusion of savers, who determine the deposit rates, allows me to endogenously analyse the impacts from the perspective of the liability side of banks' balance sheet. The consideration of these sectors captures the impacts of the regulatory requirements on the representative sectors, not restricted to the banking sectors, the effects of which has been extensively documented in the recent literature. Although it is widely recognized that a strengthened requirement will reduce bank lending and thus depress the output, less attention has been paid to the effects on the firms' debt financing structure. Firms will be granted a lower credit, as a result of the reduced bank lending; however, will the firms seek for another source for financing, for example corporate bonds, and how this consideration will affect firms' debt financing are still novel questions. My model indicates that, with the implementation of the requirements, due to the increase in corporate bond rate, bank will not increase the bond issuance to compensate for the reduction in loans.

The second contribution of this model is a separation of risky assets. I consider two forms of the risky assets: loans and risky corporate bonds. Recent literature, for example Elenev *et al.*

(2018), only features bank loans. However, it is estimated, from the data of FRED, that the value of corporate bonds makes up 22% of the GDP of the US from 1995 to 2017, while the value of the bank credit amounts to 65% of the GDP. This estimation indicates that the exclusion of the corporate bonds will result in the loss of generality of the model. To fill this gap, I feature corporate bonds in my model and assign a priority to claims in the case of the default of the firms. Therefore, corporate bonds are less risky than the bank loans but are riskier than the risk-free deposits or government bonds. Corporate bonds serve as a main financing source of the firms, as an alternative to bank loans. With the separation of the risky assets (the consideration of corporate bonds), one can notice how the corporate bond markets are affected by the implementation of the requirements. Prior studies paid more attention to the effects of the requirements on the bank loans, this paper however answers an underdeveloped question: how do the requirements affect the issuance of corporate bonds.

Thirdly, I contribute to the literature by allowing the participation of savers in the corporate bond markets. Although it is revealed that a large portion of households do not hold risky assets, this fact does not imply that, in reality, savers only hold risk-free assets (deposits). It is estimated that around 21% of the wealth of the households is invested in risky assets, such as corporate bonds and stocks. This generalization not only better reflects the reality but also endogenously captures the demand of corporate bonds. This means this model separates the supply and the demand of the corporate bonds by assuming the firms as the suppliers of the corporate bonds, while treating banks and savers as the corporate bond demanders. This assumption provides a channel from the regulatory requirements on the banks to the investment strategies of the savers, in general the households. Thereby, it provides a novel interpretation of the transmission mechanism from the bank regulatory requirements to the wealth redistribution of the savers. I also extend recent literature by revealing the effects of the regulatory requirements on the household sector, not merely restricted to the production sector.

My paper also contributes to the literature in the area of asset pricing. The model features a market for bank loans, corporate bonds and risk-free deposits and government bonds. It not only generates the credit spread but also the spreads between loans and risk-free bonds. With these considerations, I could estimate the impacts of the regulatory requirements on these macro-economic variables. I also contribute to the literature in financial intermediation and banking regulation. The majority of the literature which focus on the evaluation of Basel-style requirements, such as De Nicolo *et al.* (2014), Hugonnier and Morellec (2017), Carletti *et al.* (2018) and Van den Heuvel (2018), merely pay attention to the analysis of the banks

and lose the focus on the other participants of the economy. Moreover, the majority of the current literature assumes an exogenous shock of borrowers' bankruptcy causes the default of banks. I update the literature by following Elenev *et al.* (2018) to consider banks' failure as a result of a productivity shock to firms and the firms' endogenous debt financing structure, which better captures the reality.

The rest of the paper is organized as follows. Section 2 presents the related literature of this paper. Section 3 setup the model, and Section 4 presents the results. Section 5 concludes this paper. All the model derivations and computation procedures are included in the Appendix.

2. Related Literature

This paper connects with several strands of the literature between banking, macro-economics and asset pricing.

A number of recent literature has documented the evaluation of the Basel regulatory requirements, from the perspective of social welfare, financial stability and economic growth. De Nicolo *et al.* (2014) reveal that the inclusion of both of the capital and liquidity requirement will reduce the probability of default of the banks, while the liquidity requirement will reduce bank lending and depress social welfare. Hugonnier and Morellec (2017) and Van den Heuvel (2018) suggest that combining of these two requirements will reduce the default rate and bankruptcy loss. Some studies, such as König (2015), Carletti *et al.* (2018) and Thakor (2018), propose to eliminate liquidity requirements to foster economic growth.

This paper also relates to the asset pricing literature, which specifically focuses on financial intermediaries. Recent literature, such as He and Krishnamurthy (2013), Brunnermeier and Sannikov (2014) and Drechsler *et al.* (2018) consider a typical risky asset and assume household can only invest in risk-free assets¹. I contribute to the above stream of the literature by classifying risky assets into loans and corporate bonds. Since the Basel Accords assign different haircuts for capital and liquidity requirements on different types of the risky assets, this separation of risky assets contributes to the literature by better evaluating the impacts of the Basel requirements on the real economy.

Another stream of literature focuses on the development of the dynamic general equilibrium model, with the emphasis on monetary economics, which is the basis of this paper. Gertler

¹ Except for He and Krishnamurthy (2013), who relaxes the assumption to consider the scenario that households can hold banks' capital.

and Karadi (2011), Gertler, Kiyotaki and Queralto (2012) and Gertler and Kiyotaki (2015) develop an unconventional general equilibrium model in which incorporates households, firms and banks. They develop the model with the aim to reveal the impacts of monetary policy on the real economy, with the consideration of financial crises. Elenev *et al.* (2016), Elenev *et al.* (2018) and Begenau and Landvoigt (2018) develop a dynamic model in which considers firms, intermediaries and savers and solve the model using global solution methods. They use the non-linear equation solver to obtain the results for the system of the equations, which is relevant in the context of this paper, as it will help to generate equilibrium macroeconomic variables with different constraints of regulatory requirements. Three key differences to other work are my focus on the effects of banking regulatory requirements on the other sectors of the economy, the inclusion of corporate bonds as a third asset to better reflect the reality and the savers' holding of risky assets which is less captured in the recent literature.

3. The Model

3.1 Preferences, Technology, Timing

Preferences The model considers two groups of agents: bankers and households. They are modelled as a representative agent with a continuum of members of measure unity. All agents have Epstein-Zin (1989) preferences, represented by U_t^b , and is a function of current utility u_t^b , intertemporal elasticity of substitution v_b , risk aversion σ_b and future utility U_{t+1}^b

$$U_t^b = \left\{ (1 - \beta_b)(u_t^b)^{1-1/v_b} + \beta_b \left(E_t \left[(U_{t+1}^b)^{1-\sigma_b} \right] \right)^{\frac{1-1/v_b}{1-\sigma_b}} \right\}^{\frac{1}{1-1/v_b}} \quad (1)$$

for $b = E, S$, the notation of which will be introduced later. As in Elenev *et al.* (2018), agents derive utility from consumption of the economy, such that $u_t^b = C_t^b$ for $b = E, S$.

Technology Non-financial firms operate the production technology, a firm, which turns capital and labour into aggregate output:

$$Y_t = Z_t K_t^{1-\alpha} L_t^\alpha \quad (2)$$

where K_t is capital, and L_t is labour, and Z_t is total factor productivity (TFP). In addition to the technology for producing consumption goods, firms are able to produce capital at an adjustment cost, as in Gertler and Karadi (2011), Elenev *et al.* (2018). The capital K_t used for

production is funded by loan or bond provided by banks (or financial firms) and wealth owned by firms themselves. Households in-elastically supply labour L_t to firms, which means the amount of labour equals the volume of the households.

3.2 Key Features

In this model, the population of households and bankers are not transferrable, keeping the amount of each occupation constant². Since the population of each occupation is constant, I normalise the total amount of households/labour supply as $\bar{L} = 1$. Figure 1 illustrates the balance sheets of the model's agents and their interactions with each other.

< Insert Figure 1 here >

3.3 Households

There are two types of households: Entrepreneurs (denoted by B) and Savers (denoted by S), both of them in-elastically supply their labour for wages. Entrepreneurs own the firms, and they finance projects from borrowing loans and selling bonds from banks, or from the savers by selling bonds. Bondholders have a priority of claiming residuals in the default of the firms. Savers use their wealth to deposit in the banks or buy the corporate bonds from firms.

3.3.1 Households - Entrepreneurs

Entrepreneurs manage firms and invests in projects for returns. Projects are subject to idiosyncratic productivity shocks $\omega_{i,t} \sim F_t$, and total output for each firm is as:

$$Y_{i,t} = \omega_{i,t} Z_t K_{i,t}^{1-\alpha} L_{i,t}^\alpha \quad (3)$$

The term F_t is a Gamma distribution, and $\omega_{i,t}$ is uncorrelated across firms and time. The unconditional mean of $\omega_{i,t}$ is one across the firms at each time, but its volatility is dependent on the realization of states (There are two states: *Normal times* and *Disasters*). Firms can use partial or all of the consumption (originated from bank borrowing or firms' profit) to produce capital at an additional cost. If the added amount of capital is X_t , the total cost $\Phi(X_t, K_t^F)$ associated with this production is:

² Gertler and Karadi (2011) and Gertler and Kiyotaki (2015) consider a generalised situation where each agents are transferrable to limit the possibility that banks are fully funded by equity. This issue can be avoided, in this model, by a reasonable pay-out policy and endowment of the bankers at each period, which I will discuss later.

$$\Phi(X_t, K_t^F) = X_t + \frac{\varphi}{2} \left(\frac{X_t}{K_t^F} - \delta_K \right)^2 \cdot K_t^F \quad (4)$$

where δ_K is the depreciation rate of the capital of firms, and K_t^F is the amount of capital of each firm, which is the same for all firms before idiosyncratic productivity shocks (Elenev *et al.* 2018). Before idiosyncratic shocks realize, each firm will also receive the same amount of labour, loan outstanding and bond, such that $L_{i,t} = L_t$, $A_{i,t} = A_t$, and $B_{i,t} = B_t$. As in De Nicolo *et al.* (2014), a portion σ of existing stock of loan at t become due at $t + 1$, which means the average maturity of loan is $1/\sigma - 1$. Bond has a maturity of $1/n - 1$, and for each period a portion n of the original bond will become due. In our calibration, $n < \sigma$ indicates corporate bond usually has a longer maturity than a loan. At time t , loan rate is denoted as q_t^A , while bond price³ is p_t^B , which means the return of investing in bond is $q_t^B = 1/p_t^B - 1$.

After the idiosyncratic productivity shocks, firms' total value should be sufficient to pay households wages $\sum_b w_t^b L_t^b$, pay banks the matured loan (with interest), denoted by $\sigma(1 + q_t^A)A_t^F$ and to honour remaining debt $(1 - \sigma)A_t^F + B_t^F$. Thereby, the value $\pi_{i,t}^F$ of individual producer i is:

$$\pi_{i,t}^F = \omega_{i,t} Z_t (K_t^F)^{1-\alpha} L_t^\alpha - \sum_b w_t^b L_t^b - (1 + q_t^A)A_t^F - B_t^F \quad (5)$$

Denote a minimum level of the value $\underline{\pi}$ as the threshold for default, and firms with value $\pi_{i,t} < \underline{\pi}$ will default. Thus, the threshold of a productivity shock ω_t^* is

$$\omega_t^* = \frac{\underline{\pi} + \sum_b w_t^b L_t^b + (1 + q_t^A)A_t^F + B_t^F}{Z_t (K_t^F)^{1-\alpha} L_t^\alpha} = \frac{\underline{\pi} + \sum_b w_t^b L_t^b + (1 + q_t^A)A_t^F + B_t^F}{Y_t} \quad (6)$$

Observe that Equation (5) and (6) use the definition that the unconditional mean of aggregate output $Y_t = Z_t (K_t^F)^{1-\alpha} L_t^\alpha$. Denote $\Omega_D(\omega_t^*)$ as an indicator for the fraction of surviving firms:

$$\Omega_D(\omega_t^*) = \Pr[\omega_{i,t} \geq \omega_t^*] \quad (7)$$

Note that upon default, bond has priority of being claimed with loan. The total profit of firms are subject to a corporate profit tax with rate τ_F . The average productivity of the output of the firms that do not default is as

³ Elenev *et al.* (2018) consider a more generalized situation by incorporating a coupon payment of the existing bond. In this paper, the bond with zero coupon.

$$\Omega_E(\omega_t^*) = \Pr[\omega_{i,t} \geq \omega_t^*] E[\omega_{i,t} | \omega_{i,t} \geq \omega_t^*] \quad (8)$$

In addition, I introduce $\Omega_B(\omega_t^\#)$ to calibrate the expected claim of bond:

$$\begin{aligned} \Omega_B(\omega_t^\#) = & \Pr[\omega_{i,t} \geq \omega_t^\#] \\ & + \left\{ \Pr[\omega_{i,t} \leq \omega_t^\#] E[\omega_{i,t} | \omega_{i,t} \leq \omega_t^\#] Z_t(K_t^F)^{1-\alpha} L_t^\alpha - \underline{\pi} - \sum_b w_t^b L_t^b \right\} / B_t^F \end{aligned} \quad (9)$$

where $\omega_t^\# = \frac{\underline{\pi} + \sum_b w_t^b L_t^b + B_t^F}{Y_t}$

Observe that after the productivity shock, loan and bond will mature. Thereby, the taxable profit Π_t^F is the output revenue net of labour expenses, capital depreciation and interest payments to banks:

$$\Pi_t^F = \Omega_E(\omega_t^*) Z_t(K_t^F)^{1-\alpha} L_t^\alpha - \Omega_D(\omega_t^*) [\delta_K p_t K_t^F + A_t^F + B_t^F + \sum_b w_t^b L_t^b] \quad (10)$$

Equation (10) shows that tax shield will benefit firms by deducting the existing debt $A_t^F + B_t^F$ from the taxable profit. The entrepreneurs' problem is to choose consumption C_t^E , net capital investment for next period X_t , new loan investment A_{t+1}^F , bond amount B_{t+1}^F , capital amount K_{t+1}^F and labour input L_t to maximize utility function in Equation (1), subject to the budget constraint in (11) and a leverage constraint in (12) when operating firms (following Kiyotaki and Moore, 1997).

$$\begin{aligned} C_t^E + \Phi(X_t, K_t^F) + (1 + q_t^A) \Omega_D(\omega_t^*) A_t^F + \Omega_B(\omega_t^\#) B_t^F + p_t K_{t+1}^F + \Omega_D(\omega_t^*) \sum_b w_t^b L_t^b + \\ \tau_F \Pi_t^F = \Omega_E(\omega_t^*) Z_t(K_t^F)^{1-\alpha} L_t^\alpha + (1 - \tau_W^E) w_t^E \bar{L}_t^E + p_t [X_t + \Omega_D(\omega_t^*) (1 - \delta_K) K_t^F] + \\ p_t^B B_{t+1}^F + (1 + q_t^A) A_{t+1}^F + H_t \end{aligned} \quad (11)$$

$$F_A A_{t+1}^F + F_B B_{t+1}^F \leq \text{hp}_t [1 - (1 - \tau_F) \delta_K] \Omega_D(\omega_t^*) K_t^F \quad (12)$$

Entrepreneurs receive investment revenue $\Omega_E(\omega_t^*) Z_t(K_t^F)^{1-\alpha} L_t^\alpha$, after-tax wage income, net value of selling capital $p_t [X_t + \Omega_D(\omega_t^*) (1 - \delta_K) K_t^F]$, obtain fund from banks by selling bond B_{t+1}^F at the price of p_t^B , outstanding loan A_{t+1}^F valued at the market price of $1 + q_t^A$ and the claim from the defaulting proceedings H_t , where

$$H_t = (1 - \text{lv})(1 - c^F)\{[1 - \Omega_D(\omega_t^*)](1 - \delta_K)p_t K_t^F + [1 - \Omega_E(\omega_t^*)]Z_t(K_t^F)^{1-\alpha}L_t^\alpha\} - [1 - \Omega_D(\omega_t^*)]\sum_b w_t^b L_t^b \quad (13)$$

in which c^F denotes the bankruptcy cost of firms, and $1 - \text{lv}$ is the fraction that firms can claim from the default, while the rest of the fraction will be paid to banks, as their roles of the borrowers of loans. All these incomes are paid for consumption C_t^E , deposit capital production cost $\Phi(X_t, K_t^F)$, outstanding loan and bond $(1 + q_t^A)\Omega_D(\omega_t^*)A_t^F + \Omega_B(\omega_t^\#)B_t^F$, new capital outstanding $p_t K_{t+1}^F$, wages $\Omega_D(\omega_t^*)\sum_b w_t^b L_t^b$ and corporate tax $\tau_F \Pi_t^F$. The leverage constraint limits firms' ability of borrowing, where $F_A > F_B$ captures a heavier haircut for the loan borrowing than the bond issuance, and the term lv denotes the Loss Given Default (LGD). The total borrowing should not exceed a fraction of the market value of the capital of the firms, after adjusting for the tax-shield effect of the depreciated capital.

3.3.2 Households - Savers

Savers can deposit in the banks to enjoy a risk-free return or buy the corporate bonds issued by the firms for a credit spread. For simplicity, savers cannot have access to investments in government bonds⁴. They in-elastically supply their labour \bar{L}_t^S for the wage of w_t^S . The savers' problem is to choose consumption C_t^S , deposits D_{t+1}^S and corporate bonds B_{t+1}^S to maximize utility U_t^S in (1), subject to the budget constraint:

$$C_t^S + (q_t^f + \tau_D r_t^f)D_{t+1}^S + p_t^B B_{t+1}^S \leq W_t^S + (1 - \tau_W)w_t^S \bar{L}_t^S \quad (14)$$

where $W_t^S = D_t^S + [1 - \tau_B n(1 - p_t^B)]\Omega_B(\omega_t^\#)B_t^S$ representing savers' initial wealth, which is from their investment incomes of deposits and corporate bonds from the last period.

Budget constraint in (14) shows that savers use their initial wealth and after-tax wage to pay for their consumption, deposits (with deposit interest tax) and corporate bonds. The deposit interest rate (risk-free) is the yield on deposits, i.e. $r_t^f = 1/q_t^f - 1$.

3.3.3 Bankers

Bankers finance their investments by borrowing from savers, in the form of deposits, and by using their own retained earnings, in the form of equity. Bankers can lend loans (A_t^B) to the firms or buy corporate bonds (B_t^B). Loans are risky and will return a higher expected return.

⁴ Since the government bond is also risk-free and the deposits can be regarded as a perfect substitute in our model, which means this assumption of simplicity does not lose generality.

Corporate bonds are less riskier and exist for a longer lifetime. Issuing or liquidating loans is subject to an adjustment cost, which can be summarized in a quadratic function:

$$M(I_t^B) = m|I_t^B| + \frac{\kappa}{2} \left(\frac{I_t^B}{A_{t-1}^B} - \sigma \right)^2 A_{t-1}^B + \chi_{\{I_t^B < 0\}} \cdot \frac{(I_t^B)^2}{2A_{t-1}^B}. \quad (15)$$

where A_{t-1}^B is the amount of existing loans and I_t^B the amount of adjusted loans; χ is an indicator of negative loan adjustment amount; m is the unit adjustment cost; κ is an additional cost introduced to match the average loan rate. This assumption incorporates the cost of fire sales as in Diamond and Rajan (2011). The taxable profit of banks is $\Pi_t^B = \Omega_D(\omega_t^*)\sigma q_t^A A_t^B + n(1 - p_t^B)\Omega_B(\omega_t^\#)B_t^B - r_t^f D_{t+1}^B$, and banks' asset value is:

$$\pi_t^B = (1 + q_t^A)\Omega_D(\omega_t^*)A_t^B + p_t^B \Omega_B(\omega_t^\#)B_t^B$$

To make dividend at d_t^B , the total cost will be $d_t^B + \Xi(d_t^B)$, where

$$\Xi(d_t^B) = \frac{\vartheta}{2} [d_t^B - \bar{d}^B]^2 \quad (16)$$

The term \bar{d}^B denotes the target level of dividend of bankers. The bankers' problem is to choose dividend d_t^B , loan investment A_{t+1}^B and bond amount B_{t+1}^B to maximize utility function in (17) below:

$$V^B(A_t^B, B_t^B, D_t^B, s_t) = \max_{\{d_t^B, A_{t+1}^B, B_{t+1}^B\}} d_t^B + E_t[\Lambda_{t,t+1}^B V^B(A_{t+1}^B, B_{t+1}^B, D_{t+1}^B, s_{t+1})] \quad (17)$$

where $\Lambda_{t,t+1}^B$ denotes the Stochastic Discount Factor (SDF) for banks, which I will introduce later. Banks' optimal choice is subject to the budget constraint in (18) and a leverage constraint.

$$d_t^B + \Xi(d_t^B) + (1 + q_t^A)A_{t+1}^B + M(I_{t+1}^B) + p_t^G B_t^G + p_t^B B_{t+1}^B + D_t^B \leq \pi_t^B - \tau_B \Pi_t^B + B_{t-1}^G + q_t^f D_{t+1}^B + HF_t \quad (18)$$

Where $I_{t+1}^B = A_{t+1}^B - (1 - \sigma)\Omega_D(\omega_t^*)A_t^B$. Equation (18) indicates that bankers pay d_t^B with an extra cost $\Xi(d_t^B)$, raise fund for new loans $(1 + q_t^A)A_{t+1}^B$, pay an adjustment fee of $M(I_{t+1}^B)$, purchase government bond $p_t^G B_t^G$, pay for newly purchased bond $p_t^B B_{t+1}^B$ and repay depositors D_t^B . These funds originate from selling the assets of the banks $\pi_t^B - \tau_B \Pi_t^B$, from matured government bond B_{t-1}^G (the income of which is tax-free) and from the new deposits $q_t^f D_{t+1}^B$, and a claim HF_t paid by the defaulting firms, where

$$HF_t = \text{h}(1 - c^F) \{ [1 - \Omega_D(\omega_t^*)](1 - \delta_K)p_t K_t^F + [1 - \Omega_E(\omega_t^*)]Z_t(K_t^F)^{1-\alpha}L_t^\alpha \} \\ - [1 - \Omega_D(\omega_t^*)] \sum_b w_t^b L_t^b$$

In addition, bankers will also be subject to capital and liquidity requirement constraints. The capital and liquidity requirements are defined in feasible choice sets below:

$$\Theta(D_{t+1}^B) = \{(A_{t+1}^B, B_{t+1}^B) \mid (1 - k)A_{t+1}^B + (1 - k_B)B_{t+1}^B \geq D_{t+1}^B\} \quad (19)$$

$$\mathcal{E}(D_{t+1}^B) = \{(A_{t+1}^B, B_{t+1}^B) \mid \bar{h}_A A_{t+1}^B + \bar{h}_B B_{t+1}^B + lD_w \geq lD_{t+1}^B\} \quad (20)$$

$$Y(D_{t+1}^B) = \{(A_{t+1}^B, B_{t+1}^B) \mid A_{t+1}^B + B_{t+1}^B \leq F_D D_{t+1}^B\} \quad (21)$$

where \bar{h}_A and \bar{h}_B in (20) is the haircut for loans and corporate debt securities and the deduction of which is included in the Appendix. Equation (21) shows the leverage constraint for banks, which means the adjusted amount of assets will not exceed a F_D portion of the debt. The term r_w^d is the worst-case deposit rate; p_w^B is the worst-case price of bond; D_w represents the worst-case deposit amount; and q_w^A the worst-case loan rate. Unregulated banks will operate within the feasible set $Y(D_{t+1}^B)$, while banks that are under capital requirements only will follow the feasible set $Y(D_{t+1}^B) \cap \Theta(D_{t+1}^B)$ and banks that are subject to both capital and liquidity requirements will be confronted within the feasible set $Y(D_{t+1}^B) \cap \Theta(D_{t+1}^B) \cap \mathcal{E}(D_{t+1}^B)$.

3.3.4 Government

Government is responsible for paying for the defaulting banks. The total cost the government is responsible for paying G_t is:

$$G_t = -\{1 - \chi_B[\Omega_D(\omega_t^*)]\} \cdot \{(1 - c^B)[(1 + q_t^A)\Omega_D(\omega_t^*)A_t^B + \Omega_B(\omega_t^*)B_t^B] - D_t^B\} \quad (22)$$

where $G_t^B < 0$ representing the negative payoff due to the default of banks, c^B denotes the bankruptcy cost of banks. Government has to finance its expenditure, G_t , from taxation T_t and from issuing government debt B_t^G . Thereby, the government is subject to the following constraint:

$$B_{t-1}^G + G_t + G_t^w = T_t + p_t^G B_t^G \quad (23)$$

where G_t^w is the exogenous government spending and T_t is defined as:

$$T_t = \tau_D r_t^d D_t^F + \tau_F \Pi_t^F + \tau_B \Pi_t^B + \tau_W \sum_b w_t^b L_t^b \quad (24)$$

Tax is levied from risk-free debt interest revenue tax, firm and bank profit tax, together with labour income tax.

3.3.5 Equilibrium

Given a sequence of productivity shocks $\{Z_t, \omega_{i,t}\}$ and a set of regulatory constraints as in (12), (19), (20) and (21). I solve this model by allocating a competitive equilibrium with optimal choice $\{C_t^E, X_t, A_{t+1}^F, B_{t+1}^F, K_{t+1}^F, L_t\}$ for entrepreneurs, $\{C_t^S, D_{t+1}^S, B_{t+1}^S\}$ for savers and $\{d_t^B, A_{t+1}^B, B_{t+1}^B\}$ for bankers. The market clearing conditions are:

$$\text{Loans:} \quad A_{t+1}^F = A_{t+1}^B \quad (25)$$

$$\text{Default-able bonds:} \quad B_{t+1}^F = B_{t+1}^B + B_{t+1}^S \quad (26)$$

$$\text{Capital:} \quad K_{t+1}^F = (1 - \delta_K) \Omega_D(\omega_t^*) K_t^F + X_t \quad (27)$$

$$\text{Labour:} \quad L_t^b = \bar{L}_t^b \text{ for all } b = E, S \quad (28)$$

$$\text{Deposits:} \quad D_{t+1}^B = D_{t+1}^S = D_{t+1} \quad (29)$$

GDP:

$$Y_t = \sum_b C_t^b + d_t^B + \Xi(d_t^B) + \Phi(X_t, K_t^F) + m(I_{t+1}^B) + G_t + G_t^W \quad (30)$$

Equation (30) indicates the resource constraint of economy. It states that total output plus the endowments to bankers equals the sum of consumption including additional cost, investment together with capital adjustment and equity issuance cost and refinancing cost, government expenditure G_t^W and aggregate loss, due to bankruptcies.

4. Calibration

The model is calibrated at an annual frequency. The value of the parameters are chosen to match and fit in the economy, and the sources of these values are empirical data or existing literature. In this section, I present these values in Table 1 and provide a brief interpretation, along with the targets or the sources for each of these parameters.

Exogenous Shocks The persistence and volatility of TFP is from the GDP growth of 1995 to 2017. The standard deviation of the growth of GDP is 1.73%, and the persistence of the GDP growth, following an AR(1) process, is 0.73. I thus adopt them as $\rho_Z = 0.7$ and $\sigma_A = 2.0\%$,

both of which are very close to the estimates of Elenev *et al.* (2018). The calibration of uncertainty of idiosyncratic shocks is originally from Bloom *et al.* (2018) and was presented by Elenev *et al.* (2018). I use this estimation as well to make my simulation consistent. The transition probability across business cycles is from Repullo and Suarez (2012), who estimate that the expected duration of booms and recessions are 5 and 2.8 years.

Parameters for entrepreneurs and savers Capital adjustment cost is set at $\varphi = 2$, while the annual depreciation rate is adopted at $\delta_K = 8\%$. Both of these parameters, together with labour share in production $\alpha = 0.71$, are from Elenev *et al.* (2018). The amount of deposit in each period is modelled as:

$$\frac{D_t}{Y_t} = \bar{D} \times e^{d_i(g_t - \bar{g})} \quad (31)$$

where g_t denotes the GDP (excludes net exports) growth rate of year t , and \bar{g} is the averaged GDP growth rate. I use the deposit volume of US chartered banks and the GDP from 1995 to 2017 from Federal Reserve Bank report. I set $\bar{D} = \exp(-0.911) = 0.40$ and $d_i = -3.7$ to match the regression result of log deposit-to-GDP on a constant and GDP growth. The labour income share of entrepreneurs and savers $\{\gamma^E, \gamma^S\}$ are $\{0.40, 0.60\}$ to capture the labour income share of savers of 60%, as in Elenev *et al.* (2018), which is also the source of the population of labour supply of $\{0.31, 0.69\}$.

Table 1: Calibration

Parameters	Description	Value	Target or Source
Exogenous Shocks			
ρ_Z	Persistence of TFP	0.70	AR(1) GDP growth, 95-17
σ_A	Volatility of TFP	2.0%	Vol. of GDP growth, 95-17
$\sigma_{\omega,L}$	Idiosyn. shock uncer. in <i>Normal Times</i>	0.095	Elenev <i>et al.</i> (2018)
$\sigma_{\omega,H}$	Idiosyn. shock uncer. in <i>Disasters</i>	0.175	Elenev <i>et al.</i> (2018)
$\{p_{LL}^\omega, p_{HH}^\omega\}$	Transition probability	$\{0.80, 0.64\}$	Repullo and Suarez (2012)
Parameters for Entrepreneurs and Savers			
φ	Capital adjustment cost	2	Elenev <i>et al.</i> (2018)
α	Labour share in production	0.71	Elenev <i>et al.</i> (2018)
δ_K	Capital depreciation rate	8%	Elenev <i>et al.</i> (2018)
\bar{D}	Base of deposit-GDP ratio	0.40	Deposits to GDP ratio, 95-17
d_i	Slope of deposit-GDP growth ratio	-3.7	Deposits to GDP growth, 95-17
$\{\gamma^E, \gamma^S\}$	Inc. share of entrepreneurs and savers	$\{0.40, 0.60\}$	Elenev <i>et al.</i> (2018)
$\{L^E, L^S\}$	Labour supply and population	$\{0.31, 0.69\}$	Elenev <i>et al.</i> (2018)
Parameters for Bankers			
σ	Annual percentage of matured loans	20%	De Nicolo <i>et al.</i> (2014)
n	Annual percentage of returned bonds	12.5%	Expected duration of 7 years
$\{\bar{d}^B, \vartheta\}$	Target Dividend and cost of bankers	$\{1\%, 5\}$	Credit spread of 1.48%, 95-17
m	Unit loan adjustment cost	0.01	Loan rate of 5.6%, 95-17
κ	Marginal loan adjustment cost	0.2	Ratio of credit-GDP of 65%, 95-17
Preferences			
β_E	Discount factor of entrepreneurs	0.94	Cap. Inv.-GDP of 20.74%, 95-17

β_S	Discount factor of savers	0.985	Risk-free rate of 2.2%, 95-17
$\sigma_E = \sigma_S$	Risk avers. of entrepreneurs and savers	1	Standard value
$v_E = v_S$	IES of entrepreneurs and savers	1	Standard value
Marco-Prudential Dynamic and Government Policy			
$\underline{\pi}$	Default threshold of firms	0.04	Firm leverage of 40%, 95-17
$\{F_A, F_B\}$	Leverage constraint for loans and bonds	{1.04, 1.01}	Basel III, Giesecke <i>et al.</i> (2011)
F_D	Reserve requirement constraint	1.2	Ratio of loans to GDP 65%, 95-17
lv	Loss given default	0.45	'Foundation IRB' of Basel II
τ_W	Labour income tax rate	25%	Indiv. tax. to GDP of 9.62%, 95-17
$\tau_F = \tau_B$	Corporate tax rate	20%	Cor. tax rev. to GDP of 1.76%, 95-17
τ_D	Interest income (deposit) tax rate	0%	Risk-free rate of 2.2%, 95-17
\bar{G}	Base of Gov. Inv.-GDP ratio	18%	Gov. Inv. to GDP ratio, 95-17
go_i	Slope of Gov. Inv.-GDP growth ratio	-1.56	Gov. Inv. to GDP growth ratio, 95-17
c^F	Bankruptcy cost of firms	0.10	Hennessy and Whited (2007)
c^B	Bankruptcy cost of banks	0.30	Mendicino <i>et al.</i> (2018)
$\{k, k_B\}$	Capital requirement for loans and bonds	{8%, 1.5%}	Basel III, Giesecke <i>et al.</i> (2011)
l	Liquidity requirement	20%	De Nicolo <i>et al.</i> (2014)
$\{\bar{h}_A, \bar{h}_B\}$	Haircut of loans and corporate bonds	{50%, 85%}	BIS (2013)

Parameters for Bankers The annual matured rate of loans is set at $\sigma = 20\%$, as in Repullo and Suarez (2012), whose estimated duration of loans is of 4 years. The rate of reimbursed bond is at $n = 12.5\%$ to match the average duration of corporate bond of 7 years, as advised by FDIC. The loan adjustment cost is at $m = 0.01$, which is introduced to target the average bank prime loan rate of 5.6%, and loan adjustment cost $\kappa = 0.2$ is set to target the average ratio of bank credit to non-financial institutions to GDP (Net exports excluded) of 65% from 1995 to 2017⁵.

Preferences The time discount factor of entrepreneurs is set at $\beta_E = 0.931$ to target the average percentage of capital investment to GDP from 1995 to 2017 at 20.74%⁶.

The discount rate of bankers and savers is $\beta_S = 0.985$ to target the average risk-free rate of 2.2%. The risk aversions of entrepreneurs and savers are set at $\sigma_E = \sigma_S = 1$ as a standard value. The intertemporal substitution elasticity is set at $v_E = v_S = 1$, which also is commonly used in asset-pricing literature⁷.

Marco-prudential Dynamic and Government Policy Bankruptcy threshold for firms is at $\underline{\pi} = 0.04$, which is set to target the non-financial business leverage of 40%, which is expressed as the percentage of debt to the market value of the equities, from 1995 to 2017. The

⁵ The ratio of corporate bonds in non-financial business to GDP (net exports excluded) is 22%, which means in my model the total debt of firm is 97% of GDP.

⁶ Given the fact the rate of capital depreciation $\delta_K = 8\%$ in my calibration, the average ratio of capital-to-GDP in equilibrium is $0.2074/0.08 = 2.59$, which is very close to 2.24, the estimate of Elenev *et al.* (2018). This estimate also correlates to my model: Given the average leverage of firms is 40%, the total debt of firms is $2.59 \times 40\% = 104\%$ of GDP, which is close to 97%, my estimate of debt of firms considering the sum of bank credit and corporate bonds.

⁷ Such as Elenev *et al.* (2018) and Elenev (2019).

value for leverage constraint, pioneered by Kiyotaki and Moore (1997), who use risk-free rate as the value for that parameter, is set at $F_A = 1.04$ and $F_B = 1.01$ for loans and risky bonds, to mimic the expected return for these assets respectively. The parameter for reserve requirement constraint is at $F_D = 1.20$ to target the ratio of loans-to-GDP of 65%, which means the banks in my model is leveraged. This consideration closely captures the macroeconomic situation of the US, from 1995 to 2017. The level for Loss Given Default (LGD) is at $lv = 0.45$, according to Basel II Accords. The tax rate of labour income is set at $\tau_W = 25\%$ to target the average tax revenue on individual income of individuals of 9.62%, and $\tau_F = \tau_B = 20\%$ targets the average tax revenue on profits of corporates of 1.76%. The tax rate of deposits is set at $\tau_D = 0\%$ to target the risk-free rate at 2.2%, which is represented by the one-year US Treasury constant maturity rate from 1995 to 2017. Similar to (31), the exogenous government consumption and investment can be modelled as $G_t^w/Y_t = \bar{G} \exp[go_i(g_t - \bar{g})]$, where \bar{G} and go_i are estimated from the data of 1995 to 2017. The values of the parameters are $\bar{G} = \exp(-1.702) = 18\%$ and $d_i = -1.56$ from the result of log Government Spending-to-GDP on a constant and GDP growth. The bankruptcy costs are $c^F = 0.10$ and $c^B = 0.30$ for firms and banks respectively. Both these estimations are close to the respective streams of existing literature, such as Hennessy and Whited (2007) and Mendicino *et al.* (2018). I use the value $k = 8\%$ for the capital requirement for loans, as in Basel III, and use $k_B = 1.5\%$ as a ‘haircut’ for the investment in risky corporate bonds. This value for the haircut is very similar to Giesecke *et al.* (2011), who present an unconditional mean default rate of 1.517% of corporate bonds within United States, from 1866 to 2008. I pin down the ratio of liquidity requirement at $l = 20\%$ from De Nicolo *et al.* (2014), who use this value as their baseline parameter. The haircuts for loans and corporate bonds for the measurement of liquidity is set at $\bar{h}_A = 50\%$ and $\bar{h}_B = 85\%$ as in BIS (2013).

5. Results

In this section, I present the results of my model, with different regulation regimes. Then, I illustrate the impulse responses of the exogenous shocks. Finally, I show some computational errors of each equation of the nonlinear system of the simulated path.

5.1 Quantitative Results

I report the results for the scenarios when banks are with no requirements, with capital requirements only and with capital and liquidity requirements. I report the means and standard

deviations from a simulation of the model (2,000 periods/years) for each of the scenarios. For the simulation, I firstly insert an initial guess (same for all the scenarios) and continue the simulation procedures for 2,050 periods. To avoid the bias due to the dependence on the initial guess, I discard the first 50 periods. I also keep the series of the simulated exogenous shocks for each of the scenario to preserve comparability across scenarios. To assess the quality of my model in capturing the mechanism of the real economy, I compare some of the macroeconomic variables from the data and generated by my model. For comparison, I use the results from the scenario when no requirements are imposed. The results are presented in Table 2.

Table 2: Macroeconomic variables: Data vs. Model (in %)

Variables	Notation	Data	Model
Risk-free rate	r^f	2.20	2.19
Standard deviation of risk-free rate	$\sigma(r^f)$	2.24	3.57
Loan rate	q^A	5.60	5.45
Corporate bond rate	$1/p^B - 1$	3.68	3.54
Credit spread	$1/p^B - 1 - r^f$	1.48	1.35
Capital Investment-GDP ratio	I^B/Y	21.44	26.11
Standard deviation of cap. In. GDP ratio	$\sigma(I^B/Y)$	1.52	1.68
Credit-GDP ratio	A^F/Y	65	65.8
Corporate bond-GDP ratio	B^F/Y	22	12.9
Standard deviation of dividend-GDP ratio	$\sigma(d^B/Y)$	0.14	13.1
Consumption-GDP ratio	C/Y	67	63.9
Firm leverage	$(A^F + B^F)/K$	40	30.7

This table compares the key macroeconomic variables from data and from the simulated results. The results presented to compare is from the scenario with no requirements. All figures are in %.

From Table 2, one can notice that the majority of the variables from the simulated results match the data well, except for the standard deviation of dividend-GDP ratio, which seems to deviate from the data largely. Additional improvements can be done to remedy this issue by considering a more appropriate set of parameters to eliminate the deviation of this variable. Overall, the results from Table 2 imply that my model matches the data well, and captures the majority of the mechanism of the real economy.

I present the results of the impacts of different requirements on the banking system and the real economy. The results are presented for the scenarios when banks are under no requirements, are under the capital requirements only and are under both of the capital and liquidity requirements. Table 3 presents the results.

Table 3: Quantitative results: Different requirements (in %)

	No Requirements		Capital Requirements		Capital & Liquidity Requirements	
	mean	std. dev.	mean	std. dev.	mean	std. dev.
Entrepreneurs and Firms						
1. Capital	196.43	23.92	194.32	27.43	192.15	23.29
2. Cop. Bonds	9.56	8.66	5.97	3.41	6.26	3.71
3. Entre. Consp.	9.49	11.18	6.06	11.28	6.99	12.43
4. Default of firms	2.63	2.08	2.62	2.14	2.72	2.12
5. Loss in def.	4.03	3.12	3.96	3.22	4.13	3.20
6. Entrpre. value	134.63	21.92	138.23	18.51	138.43	26.72
Bankers and Banks						
7. Bankruptcy	1.56	2.11	0.05	0.40	0.05	0.42
8. Loans	49.32	6.11	48.94	5.90	47.12	5.63
9. Bank Bond	8.14	9.23	4.88	3.67	5.11	4.25
10. Dividends	6.92	9.55	10.68	4.55	11.32	2.74
11. Bankers value	16.85	25.11	16.54	7.70	14.58	9.08
Savers						
12. Savr. Consp.	17.62	1.39	17.65	1.51	17.82	0.99
13. Saver Bond	1.35	1.82	1.09	1.69	1.15	1.97
14. Savers value	31.92	2.62	32.11	1.93	32.59	2.14
Macro-economy and Prices						
15. GDP	74.91	3.36	74.59	3.79	74.40	3.19
16. Loan rate	5.45	1.86	5.51	1.24	5.94	1.59
17. Bond rate	3.54	1.98	3.86	1.75	3.87	2.26
18. Risk-free rate	2.20	3.57	3.25	2.66	3.84	3.99
19. Credit spread	1.35	3.73	0.61	2.61	0.03	3.55
20. Loan-bond spread	1.91	2.78	1.65	3.10	2.07	5.29

The table reports the quantitative results of the macro-economy when banks are with no requirements, are only with capital requirements, and are with both of the capital and liquidity requirements. The parameters set for the requirements are in Table 1. All values are in %.

Table 3 compares the quantitative results for different requirements. I will discuss the results for each sector and the macro-economy and prices in the subsequent sections.

5.1.1 Entrepreneurs and Firms

The capital stock declines from 196.43% (with no requirements) to 192.15% (with capital and liquidity requirements), with the volatility stays around 24%. Corporate bonds show an inverted U-shaped relationship between the requirements. The amount of corporate bonds is 9.56%, and this value reduces to 6.06% when with the capital requirements, but raises to 6.99% when both of the capital and liquidity requirements are imposed. This trend is partially due to the fact that the bond prices raises (row 17), which makes banks reduce the issuance of the corporate bonds. This result implies, as a result of the increase in the cost of bond issuance, that firms might not increase the issuance of the corporate bonds to compensate for their reduced borrowing from the banks. The value of entrepreneurs raises from 134.63% to 138.43%, with the implementation of the requirements, partially due to the reduction in debts (loans and bond issuances) and the reductions in consumption (row 3).

5.1.2 Bankers and Banks

The probability of bankruptcy reduces from 1.56% (no requirements) to 0.05% (with requirements), and the volatility of the bankruptcy reduces, as a result. Bank lending (the amount of loans) is reduced as well, from 49.32% to 47.15%, when the requirements are imposed.

Banks pay a higher dividend with the implementation of the requirements, while the value of the bankers reduces. One can also notice that, generally speaking, banks are less volatile (reduction in the standard deviations in rows 7-11) with the implementations of the requirements. This fact implies that the banking requirements (both of the capital and liquidity requirements) will help to stabilize the banking system.

5.1.3 Savers

Savers consume more when the requirements are imposed. Their consumption raises from 17.62% (with no requirements) to 17.82% (with capital and liquidity requirements). As a result, they invest less in the corporate bonds, although the expected return of buying corporate bonds is higher (as in row 17). These changes can be explained by two reasons. First, firms reduce the amount of corporate bonds, which leaves less corporate bonds for savers to purchase. Second, the increase in the risk-free rate (row 18) and the reduction in credit spread (row 19), which makes corporate bonds a less attractive investment for the savers, because of their risk-aversions. The value of the savers increases from 31.92% (with no requirements) to 32.52% (with capital and liquidity requirements). This value increase is primarily owing to the increase in the risk-free (deposit) rate, which increases savers' revenues from their deposits.

5.1.4 Macro-economy and Prices

The output (GDP) reduces from 74.91% (with no requirements) to 74.40% (with capital and liquidity requirements), due to a lower level of capital invested by the firms (in row 1). The reduced capital stock of the firms is primarily as a result of the reduced loan lending (row 8) and the cut in corporate bond issuance (row 2). Loan rate (row 16), bond rate (row 17) and risk-free rate (row 18) raise with the implementations of the requirements. The increase in risk-free rate is more significant, thereby driving down the credit spread. The volatility of the macro-economy has no noticeable changes across the scenarios, which means the introductions of the banking requirements has limited effects in stabilizing the real economy, although

the requirements are effective in creating a more stable banking system (as revealed in Section 5.1.2). Finally, the wealth is redistributed from bankers to entrepreneurs and savers (as in rows 6, 11, 14).

5.2 Impulse Response

I use this section to present the impulse responses of some key variables to the TFP shocks. I collect the generated series of each scenario (with no requirements, with only capital requirements, with capital and liquidity requirements) and calculate the impulse response using Cholesky method (degree of freedom adjusted). I will present the results in the following figures.

Figure 2 Impulse Response: Assets

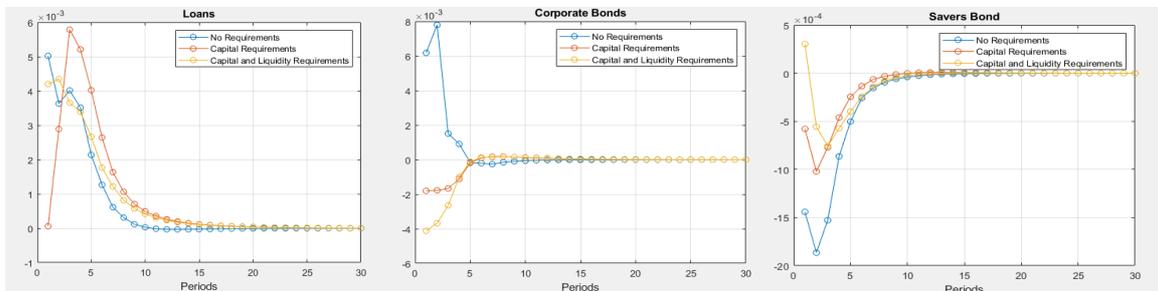


Figure 2 shows the impulse response of assets of the economy. The responses of corporate bonds recover more quickly, and the effect dies away after 10 periods of the impulse. The effect on the loans takes a longer period, around 15 periods after the impulse. One can also notice that the requirements stabilizes the amount of corporate bonds (including savers' bonds) by showing a less deviation from the steady states.

Figure 3 Impulse Response: Prices

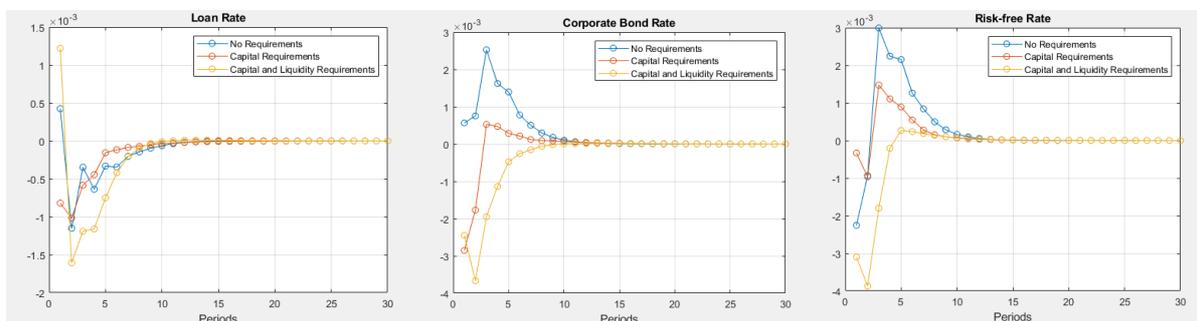


Figure 3 shows the impulse responses of the prices. It takes around 10 periods for the prices to normalize the impacts of the impulse. The requirements have little effects in stabilizing the prices as the maximum (absolute) value of the deviations is not reduced with the implementation of the requirements.

Figure 4 Impulse Response: Dynamics of banks

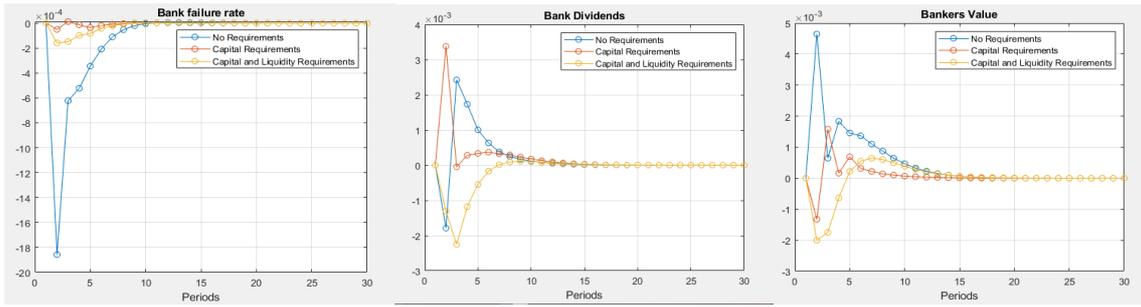


Figure 4 presents the impulse response of the mechanisms of the banks. One can notice that with the implementation of the requirements, banks' defaults are more stable when facing an impulse. Although the effects are not significant for bank dividends and bankers' value, the results from Figure 4, to some degree, indicate that the bank requirements help to stabilise the banking system. The impacts of the impulse die away within 10 periods on the bank failure rate and bank dividends, while it takes a longer time (approximately 15 periods) for the bankers' value to recover.

5.3 Computational Errors

To measure the accuracy of the numerical solution, I report the computational errors of the results that I presented in Section 5.1. A detailed description of the calculation of the errors is in Appendix C.2. For illustration, I report the computational errors for the case when banks are under capital and liquidity requirements. The errors are reported for each equation of the systems. I report median, 75th percentile, 95th percentile, 99th percentile and maximum absolute value errors of each state space point of the 2,000 simulated periods. The results are in Table 4.

Table 4: Computational errors

Equation	Percentile				
	Median	75th	95th	99th	Max
E1	0.0009	0.0022	0.0041	0.0056	0.0282
E2	0.0009	0.0018	0.0042	0.0074	0.0341
E3	0.0009	0.0012	0.0026	0.0031	0.0331
E4	0.0008	0.0041	0.0071	0.0107	0.0140
E5	0.0008	0.0035	0.0061	0.0102	0.0135
E6	0.0038	0.0058	0.0140	0.0178	0.0324
E7	0.0071	0.0084	0.0225	0.0290	0.0719
E8	0.0028	0.0037	0.0090	0.0098	0.0893
E9	0.0008	0.0023	0.0028	0.0043	0.0372
E10	0.0020	0.0037	0.0075	0.0102	0.0278
E11	0.0070	0.0080	0.0190	0.0201	0.0608
E12	0.0073	0.0101	0.0188	0.0206	0.0607
E13	0.0063	0.0091	0.0148	0.0186	0.0317
E14	0.0093	0.0201	0.0343	0.0362	0.0645

AE1 (1)	0.0082	0.0205	0.0514	0.0609	0.0707
AE1 (2)	0.0083	0.0205	0.0459	0.0579	0.0825
AE2 (1)	0.0083	0.0223	0.0388	0.0412	0.1044
AE2 (2)	0.0092	0.0280	0.0356	0.0393	0.0799

The table reports median, 75th percentile, 95th percentile, 99th percentile and maximum (absolute) value errors of the state spaces of the 2,000 simulated periods of the scenario with capital and liquidity requirements. Each rows contains the errors for the respective equation, the detailed description of the equations is in the Step 2 of the Appendix C.1.

From the table 3, one can see the accuracy of the computation is acceptable since the errors are within a tolerant boundary. The maximum (absolute) value of the errors is 0.1044, occurred in (AE2). The errors can be reduced if more state points are adopted and a higher function evaluation limit of the iteration is set. However, both of these treatments will increase the computational time considerably; otherwise more computation resources will be required. There is a trade-off between the accuracy of the computation and the computational time (and computing resources).

6. Conclusions

This paper examines the impacts of Basel-style regulatory requirements on the real economy. I develop a general equilibrium model using the global solution method and consider three agents: entrepreneurs, bankers and savers. Entrepreneurs borrow in the form of loans and corporate bonds to produce output. Bankers operate the banks, which are under regulatory requirements, lend in the form of loans and corporate bonds and receive deposits from savers. Savers use their wealth to invest in deposits (risk-free assets) and corporate bonds (risky assets).

This paper contributes to the recent literature by providing a novel investigation of the regulatory requirements. I consider its impact on the firms' debt financing structure, which has been less documented in recent literature. This impact transmits from banking system to the rest of the economy, for example savers, who can buy corporate bonds from the firms. I also update the current literature by separating risky assets in loans and corporate bonds, which are assigned with different riskiness and liquidity by the recent Basel Accord. This separation thus enables to me to reveal a more detailed evaluation of the Accord. Previous studies assumed that risk-averse savers only hold risk free assets. However I relax this assumption and allow their holdings to also comprise (risky) corporate bonds.

I find that the implementations of the Basel-style requirements, including the capital and liquidity requirements, reduce the probability of bank failure and reduce the volatility of the

banking system, this sacrificing (leading to a lower) loan lending, bank size and the output. Firms will not rely on the issuance of corporate bonds to compensate for the reduction in bank lending, due primarily to the increase in the bond rate. Wealth is redistributed from bankers to entrepreneurs and savers with the introduction of the requirements. Recent requirements on banks might cause a sizable effect on other sectors of the economy, which seems to be underestimated.

Future research could explore several extensions of this model. One could consider an endogenous labour supply, which could better capture the effects of the requirements on the real economy. Modelling heterogeneity within the banking system could be appealing, for example splitting banks into systemically important and non-systemically important banks. Finally, future research could consider the roles of the governmental bailout. My model assumes the governmental bailout will be taken whenever banks fail. Comparing scenarios in which a governmental bailout is anticipated, is not anticipated and is conditionally provided could reveal insightful suggestions to policy makers.

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Appendix A. Proofs and Figure

A.1. Deduction of Liquidity Requirement

As in Basel III (BIS, 2013), a mandatory *liquidity coverage ratio* is introduced to mitigate potential liquidity stress. Banks are required to hold a stock of liquid assets such that the predicted net cash outflows over a 30-day time period is lower than a certain fraction of that stock. Suppose the certain fraction level is defined as l , then at the end of time t the requirement is required as follows:

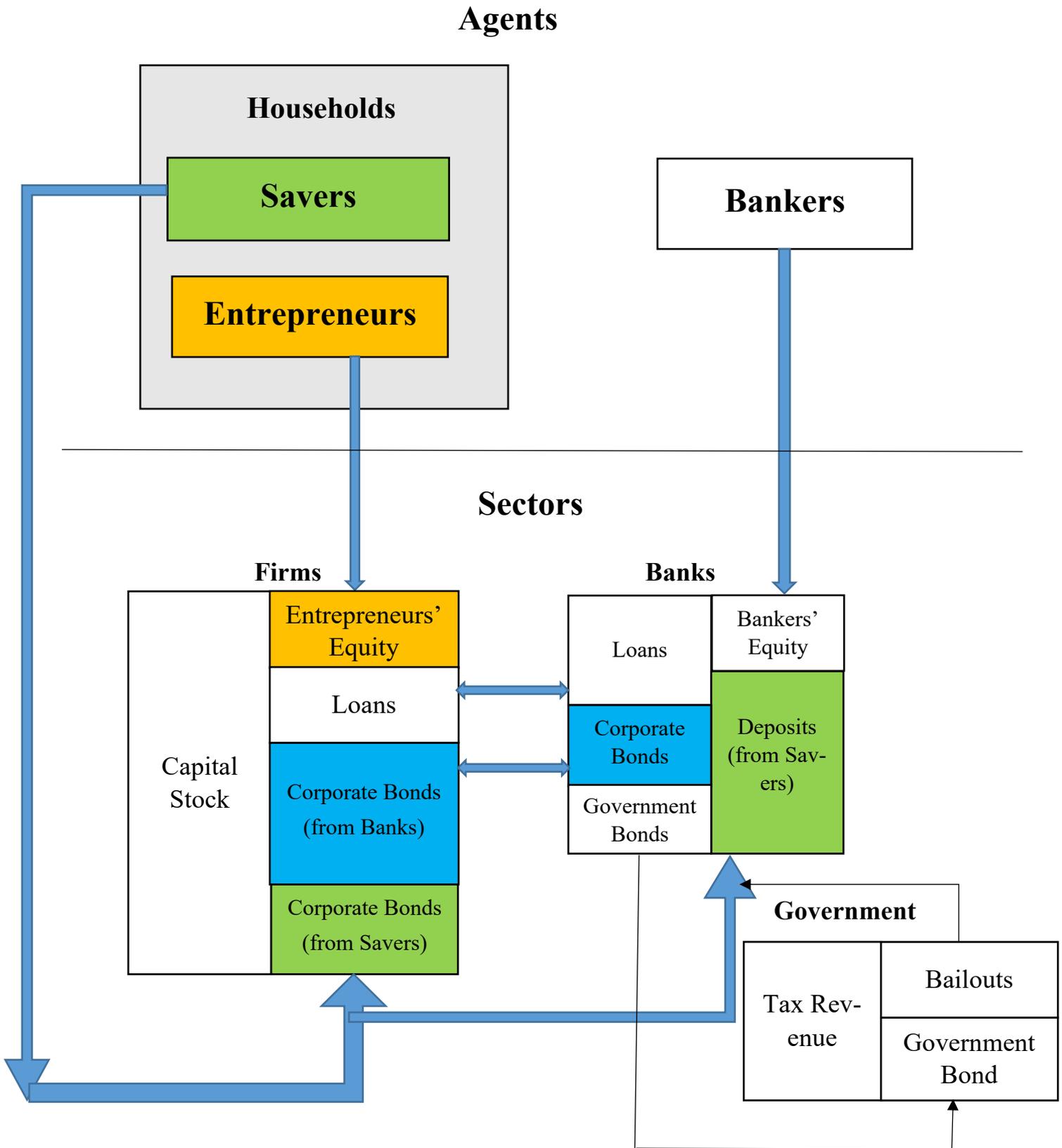
$$\frac{\bar{h}_A A_{t+1}^B + \bar{h}_B B_{t+1}^B}{D_{t+1}^B - D_w} \geq l$$

where D_w the worst-case (minimum) deposit amount. Transforming the above equation will yield the feasible set for banks under liquidity requirements:

$$\mathcal{E}(D_{t+1}^B) = \{(A_{t+1}^B, B_{t+1}^B) \mid \bar{h}_A A_{t+1}^B + \bar{h}_B B_{t+1}^B + l D_w \geq l D_{t+1}^B\}$$

The above equation is thus the liquidity requirements for regulated banks as in (20).

Figure 1: Overview of Balance Sheets of Model Agents



Appendix B. Model Solutions

B.1. Entrepreneur problem

B.1.1. Definitions

I start by introducing some definitions for my analysis. The TFP Z_t follows the law of motion as in:

$$\log Z_t = (1 - \rho_Z) \log \bar{Z} + \rho_Z \log Z_{t-1} + \epsilon_t^A$$

where $\epsilon_t^A \sim iid N(0, \sigma_A)$. Idiosyncratic productivity shocks $\omega_{i,t}$ follow a Gamma distribution Γ , such that

$$\omega_{i,t} \sim \Gamma(\chi_{0,s}, \chi_{1,s})$$

Parameter $\chi_{0,s}$ and $\chi_{1,s}$ are chosen to ensure they map into mean and variance of the distribution as

$$\chi_{0,s} = \mu_{\omega,s} / \chi_{1,s}$$

$$\chi_{1,s} = \sigma_{\omega,s}^2 / \mu_{\omega,s}$$

where I adopt $\mu_{\omega,s} = 1$ for all states. The transition of state variables s is governed by a transition matrix T^ω , which calibrates *Normal Times* (denoted by $s = L$) and *Disasters Times* (denoted by $s = H$):

$$T^\omega = \begin{pmatrix} p_{LL}^\omega & p_{LH}^\omega \\ p_{HL}^\omega & p_{HH}^\omega \end{pmatrix}$$

B.1.2. Preliminary calculations

I use this section to conduct some preliminary calculations to be used for later use.

Basic Calculations

$$\Omega_D(\omega_t^*) = 1 - F_\omega(\omega_t^*) \quad (32)$$

$$\Omega_E(\omega_t^*) = \int_{\omega_t^*}^{\infty} \omega dF_\omega(\omega_t^*) \quad (33)$$

$$\Omega_B(\omega_t^\#) = 1 - F_\omega(\omega_t^\#) + \frac{\int_{-\infty}^{\omega_t^\#} \omega dF_\omega(\omega_t^\#) Z_t (K_t^F)^{1-\alpha} L_t^\alpha - \underline{\pi} - \sum_b w_t^b L_t^b}{B_t^F} \quad (34)$$

$$\frac{\partial \Omega_D(\omega_t^*)}{\partial \omega_t^*} = -f_\omega(\omega_t^*) \quad (35)$$

where $f_\omega(\omega_t^*)$ denotes the corresponding *Probability Density Function* (PDF) of $F_\omega(\omega_t^*)$.

$$\frac{\partial \Omega_E(\omega_t^*)}{\partial \omega_t^*} = -\omega_t^* f_\omega(\omega_t^*) \quad (36)$$

$$\frac{\partial \Omega_B(\omega_t^\#)}{\partial \omega_t^\#} = -f_\omega(\omega_t^\#) + \omega_t^\# f_\omega(\omega_t^\#) \frac{Y_t}{B_t^F} = f_\omega(\omega_t^\#) \frac{\underline{\pi} + \sum_b w_t^b L_t^b}{B_t^F} \quad (37)$$

where (37) use the definition $\omega_t^\# = \frac{\underline{\pi} + \sum_b w_t^b L_t^b + B_t^F}{Y_t}$.

Calculation of $\frac{\partial C_t^F}{\partial \omega_t^*}$

$$\begin{aligned}\frac{\partial C_t^E}{\partial \omega_t^*} &= (1 - \tau_F)Y_t \frac{\partial \Omega_E(\omega_t^*)}{\partial \omega_t^*} \\ &\quad + \frac{\partial \Omega_D(\omega_t^*)}{\partial \omega_t^*} \left\{ -(1 + q_t^A)A_t^F - (1 - \tau_F) \sum_b w_t^b L_t^b + \tau_F(A_t^F + B_t^F) \right. \\ &\quad \left. + [1 - (1 - \tau_F)\delta_K]p_t K_t^F \right\}\end{aligned}$$

which can be simplified to:

$$\frac{\partial C_t^E}{\partial \omega_t^*} = -f_\omega(\omega_t^*)Y_t \left\{ \frac{[1 - (1 - \tau_F)\delta_K]p_t K_t^F - \tau_F q_t^A A_t^F + B_t^F}{Y_t} \right\} = -f_\omega(\omega_t^*)Y_t \mathcal{A}_t \quad (38)$$

where

$$\mathcal{A}_t = \frac{[1 - (1 - \tau_F)\delta_K]p_t K_t^F - \tau_F q_t^A A_t^F + B_t^F}{Y_t}.$$

Calculation of $\frac{\partial C_t^E}{\partial \omega_t^\#}$

$$\frac{\partial C_t^E}{\partial \omega_t^\#} = -\frac{\partial \Omega_B(\omega_t^\#)}{\partial \omega_t^\#} B_t^F$$

which means

$$\frac{\partial C_t^E}{\partial \omega_t^\#} = -f_\omega(\omega_t^\#) \sum_b w_t^b L_t^b = -f_\omega(\omega_t^\#)Y_t \mathcal{C}_t \quad (39)$$

where

$$\mathcal{C}_t = \frac{\sum_b w_t^b L_t^b}{Y_t}.$$

Calculation of $\frac{\partial \omega_t^*}{\partial L_t^i}$ and $\frac{\partial \omega_t^\#}{\partial L_t^i}$ Defining $\gamma^E = 1 - \gamma^S$, the aggregate amount of labour employed for production is:

$$L_t = \prod_{b=E,S} (L_t^b)^{\gamma^b}$$

I can obtain:

$$\begin{aligned}\frac{\partial \omega_t^*}{\partial L_t^b} &= \frac{w_t^b Y_t - \alpha \gamma^b \omega_t^* Y_t Z_t \frac{L_t}{L_t^b} \left(\frac{K_t^F}{L_t} \right)^{1-\alpha}}{(Y_t)^2} = \left(\frac{w_t^b}{Y_t} - \frac{\alpha \gamma^b \omega_t^*}{L_t^b} \right), \\ \frac{\partial \omega_t^\#}{\partial L_t^b} &= \frac{w_t^b Y_t - \alpha \gamma^b \omega_t^\# Y_t Z_t \frac{L_t}{L_t^b} \left(\frac{K_t^F}{L_t} \right)^{1-\alpha}}{(Y_t)^2} = \left(\frac{w_t^b}{Y_t} - \frac{\alpha \gamma^b \omega_t^\#}{L_t^b} \right).\end{aligned}$$

B.1.3. Optimization problem

Entrepreneurs make the optimal set of choice $\{C_t^F, X_t, A_{t+1}^F, B_{t+1}^F, K_{t+1}^F, L_t\}$ subject to exogenous state variables $s_t = (Z_t, \omega_{i,t})$. The optimization problem for entrepreneurs is thus as:

$$\begin{aligned}
V^E(K_t^F, A_t^F, B_t^F, D_t^F, S_t) &= \max_{\{C_t^F, X_t, A_{t+1}^F, B_{t+1}^F, K_{t+1}^F, L_t\}} \left\{ (1 - \beta_E)(C_t^E)^{1-1/v_E} \right. \\
&\quad \left. + \beta_E \left(E_t \left[\left(V^E(K_{t+1}^F, A_{t+1}^F, B_{t+1}^F, D_{t+1}^F, S_{t+1}) \right)^{1-\sigma_E} \right] \right)^{\frac{1-1/v_E}{1-\sigma_E}} \right\}^{\frac{1}{1-1/v_E}}
\end{aligned} \tag{40}$$

subject to

$$\begin{aligned}
C_t^E + \Phi(X_t, K_t^F) + (1 + q_t^A)\Omega_D(\omega_t^*)A_t^F + \Omega_B(\omega_t^\#)B_t^F + p_t K_{t+1}^F + \Omega_D(\omega_t^*) \sum_b w_t^b L_t^b + \tau_F \Pi_t^F = \\
\Omega_E(\omega_t^*)Z_t(K_t^F)^{1-\alpha}L_t^\alpha + (1 - \tau_W^E)w_t^E \bar{L}_t^E + p_t [X_t + \Omega_D(\omega_t^*)(1 - \delta_K)K_t^F] + p_t^B B_{t+1}^F + (1 + q_t^A)A_{t+1}^F
\end{aligned} \tag{41}$$

$$F_A A_{t+1}^F + F_B B_{t+1}^F \leq \text{hp}_t [1 - (1 - \tau_F)\delta_K] \Omega_D(\omega_t^*) K_t^F \tag{42}$$

Together with:

$$\Pi_t^F = \Omega_E(\omega_t^*)Z_t(K_t^F)^{1-\alpha}L_t^\alpha - \Omega_D(\omega_t^*)[\delta_K p_t K_t^F + A_t^F + B_t^F + \sum_b w_t^b L_t^b]$$

B.1.4. First-order conditions

Investment The FOC for capital investment X_t is:

$$\left[1 + \varphi \left(\frac{X_t}{K_t^F} - \delta_K \right) - p_t \right] (1 - \beta_E)(C_t^E)^{1/v_E} (V_t^E)^{1/v_E} = 0 \tag{43}$$

which can be simplified to

$$p_t = 1 + \varphi \left(\frac{X_t}{K_t^F} - \delta_K \right) \tag{44}$$

Loans The FOC for loans A_{t+1}^F is:

$$\lambda_t^{AB} F_A = (1 - \beta_E)(1 + q_t^A) \left(\frac{V_t^E}{C_t^E} \right)^{1/v_E} + \beta_E E_t [(V_{t+1}^E)^{1-\sigma_E}]^{\frac{\sigma_E-1/v_E}{1-\sigma_E}} E_t \left[(V_{t+1}^E)^{-\sigma_E} \frac{\partial V_{t+1}^E}{\partial A_{t+1}^F} \right] (V_t^E)^{1/v_E} \tag{45}$$

where λ_t^{AB} is the Lagrange multiplier on the condition (42).

Bonds The FOC for bonds B_{t+1}^F is:

$$\lambda_t^{AB} F_B = p_t^B (1 - \beta_E) \left(\frac{V_t^E}{C_t^E} \right)^{1/v_E} + \beta_E E_t [(V_{t+1}^E)^{1-\sigma_E}]^{\frac{\sigma_E-1/v_E}{1-\sigma_E}} E_t \left[(V_{t+1}^E)^{-\sigma_E} \frac{\partial V_{t+1}^E}{\partial B_{t+1}^F} \right] (V_t^E)^{1/v_E} \tag{46}$$

where λ_t^{AB} is the Lagrange multiplier on the condition (42).

Capital The FOC for capital K_{t+1}^F is:

$$p_t (1 - \beta_E) \left(\frac{V_t^E}{C_t^E} \right)^{1/v_E} = \beta_E E_t [(V_{t+1}^E)^{1-\sigma_E}]^{\frac{\sigma_E-1/v_E}{1-\sigma_E}} E_t \left[(V_{t+1}^E)^{-\sigma_E} \frac{\partial V_{t+1}^E}{\partial K_{t+1}^F} \right] (V_t^E)^{1/v_E} \tag{47}$$

Labour The FOC for labour L_t^j is:

$$\begin{aligned}
\left[-(1 - \tau_F)w_t^b \Omega_D(\omega_t^*) + (1 - \tau_F)\alpha \gamma^b \Omega_E(\omega_t^*) \frac{Y_t}{L_t^b} + \frac{\partial C_t^E}{\partial \omega_t^*} \frac{\partial \omega_t^*}{\partial L_t^b} + \frac{\partial C_t^F}{\partial \omega_t^\#} \frac{\partial \omega_t^\#}{\partial L_t^b} \right] (1 \\
- \beta_E)(C_t^E)^{1/v_E} (V_t^E)^{1/v_E} = 0
\end{aligned}$$

(48)

which can be simplified to:

$$\begin{aligned} (1 - \tau_F)\Omega_E(\omega_t^*)ML_t^b \\ = (1 - \tau_F)\Omega_D(\omega_t^*)w_t^b + f_\omega(\omega_t^*)(w_t^b - \omega_t^*ML_t^b)\mathcal{A}_t + f_\omega(\omega_t^\#)(w_t^b - \omega_t^\#ML_t^b)\mathcal{C}_t \end{aligned} \quad (49)$$

where I denote $ML_t^b = \alpha\gamma^b \frac{Y_t}{L_t^b}$.

B.1.5. Function Deductions

I use this section to derive the Euler Equations to be used for the simulation using the first-order conditions obtained from B.1.4.

Loans The problem of obtaining the derivative of value function V_{t+1}^E with respect to A_{t+1}^F can be reduced to find $\partial V_t^E / \partial A_t^F$ and forward one period. After obtaining $\partial V_{t+1}^E / \partial A_{t+1}^F$, I can write the Euler Equation for loans. The term $\partial V_t^E / \partial A_t^F$ can be written as:

$$\frac{\partial V_t^E}{\partial A_t^F} = (1 - \beta_E) \left(\frac{V_t^E}{C_t^E} \right)^{1/v_E} \left\{ -\Omega_D(\omega_t^*)(1 - \tau_F + q_t^A) + \frac{\partial C_t^E}{\partial \omega_t^*} \frac{\partial \omega_t^*}{\partial A_t^F} + \frac{\partial C_t^E}{\partial \omega_t^\#} \frac{\partial \omega_t^\#}{\partial A_t^F} \right\}$$

which can be simplified to:

$$\frac{\partial V_t^E}{\partial A_t^F} = -(1 - \beta_E) \left(\frac{V_t^E}{C_t^E} \right)^{1/v_E} [(1 - \tau_F + q_t^A)\Omega_D(\omega_t^*) + (1 + q_t^A)f_\omega(\omega_t^*)\mathcal{A}_t] \quad (50)$$

Equation (50) uses the fact that $\frac{\partial \omega_t^*}{\partial A_t^F} = \frac{1+q_t^A}{Y_t}$ and $\frac{\partial \omega_t^\#}{\partial A_t^F} = 0$.

Bonds The term $\partial V_t^E / \partial B_t^F$ can be written as:

$$\frac{\partial V_t^E}{\partial B_t^F} = (1 - \beta_E) \left(\frac{V_t^E}{C_t^E} \right)^{1/v_E} \left\{ -\Omega_B(\omega_t^\#) + \tau_F\Omega_D(\omega_t^*) + \frac{\partial C_t^E}{\partial \omega_t^*} \frac{\partial \omega_t^*}{\partial B_t^F} + \frac{\partial C_t^E}{\partial \omega_t^\#} \frac{\partial \omega_t^\#}{\partial B_t^F} \right\}$$

which can be simplified to:

$$\frac{\partial V_t^E}{\partial B_t^F} = -(1 - \beta_E) \left(\frac{V_t^E}{C_t^E} \right)^{1/v_E} \left\{ \Omega_B(\omega_t^\#) - \tau_F\Omega_D(\omega_t^*) + f_\omega(\omega_t^*)\mathcal{A}_t + f_\omega(\omega_t^\#)\mathcal{C}_t \right\} \quad (51)$$

Equation (51) uses the fact that $\frac{\partial \omega_t^*}{\partial B_t^F} = \frac{\partial \omega_t^\#}{\partial B_t^F} = \frac{1}{Y_t}$.

Capital The term $\partial V_t^E / \partial K_t^F$ can be written as:

$$\begin{aligned} \frac{\partial V_t^E}{\partial K_t^F} = (1 - \beta_E) \left(\frac{V_t^E}{C_t^E} \right)^{1/v_E} & \left\{ (1 - \tau_F)(1 - \alpha)\Omega_E(\omega_t^*)Z_t(K_t^F)^{-\alpha}L_t^\alpha + \tau_F\delta_K p_t\Omega_D(\omega_t^*) \right. \\ & + p_t\Omega_D(\omega_t^*)(1 - \delta_K) + \frac{\varphi}{2} \left[\left(\frac{X_t}{K_t^F} \right)^2 - \delta_K^2 \right] \\ & \left. + \tilde{\lambda}_t^{AB} \text{hp}_t [1 - (1 - \tau_F)\delta_K] \left[\Omega_D(\omega_t^*) + \frac{\partial \Omega_D(\omega_t^*)}{\partial \omega_t^*} \frac{\partial \omega_t^*}{\partial K_t^F} K_t^F \right] + \frac{\partial C_t^E}{\partial \omega_t^*} \frac{\partial \omega_t^*}{\partial K_t^F} + \frac{\partial C_t^E}{\partial \omega_t^\#} \frac{\partial \omega_t^\#}{\partial K_t^F} \right\} \end{aligned}$$

where $\tilde{\lambda}_t^{AB} = \lambda_t^{AB}(1 - \beta_E)^{-1} \left(\frac{V_t^E}{C_t^E} \right)^{-1/v_E}$ and above equation can be simplified to:

$$\frac{\partial V_t^F}{\partial K_t^F} = (1 - \beta_E) \left(\frac{V_t^E}{C_t^E} \right)^{1/v_E} \left[p_t \Omega_D(\omega_t^*) [1 - (1 - \tau_F) \delta_K] (1 + \text{h}\tilde{\lambda}_t^{AB}) + (1 - \tau_F) (1 - \alpha) \Omega_E(\omega_t^*) Z_t \left(\frac{K_t^F}{L_t} \right)^{-\alpha} + \frac{\varphi}{2} \left[\left(\frac{X_t}{K_t^F} \right)^2 - \delta_K^2 \right] + (1 - \alpha) \left\{ Z_t \left(\frac{K_t^F}{L_t} \right)^{-\alpha} \mathcal{D}_t + [1 - (1 - \tau_F) \delta_K] \omega_t^* f_\omega(\omega_t^*) \text{h}\tilde{\lambda}_t^{AB} p_t \right\} \right] \quad (52)$$

where $\mathcal{D}_t = \omega_t^* f_\omega(\omega_t^*) \mathcal{A}_t + \omega_t^\# f_\omega(\omega_t^\#) \mathcal{C}_t$. Equation (52) uses the fact $\frac{\partial \omega_t^*}{\partial K_t^F} = -\frac{\omega_t^*}{Y_t} (1 - \alpha) Z_t (K_t^F)^{-\alpha} L_t^\alpha = -\frac{(1-\alpha)\omega_t^*}{K_t^F}$ and $\frac{\partial \omega_t^\#}{\partial K_t^F} = -\frac{\omega_t^\#}{Y_t} (1 - \alpha) Z_t (K_t^F)^{-\alpha} L_t^\alpha = -\frac{(1-\alpha)\omega_t^\#}{K_t^F}$.

B.1.6. Simulation Equations

This section presents equations, deduced from B.1.5 and B.1.6 for entrepreneurs, to be used for simulation procedure. I denote

$$E_t[(V_{t+1}^E)^{1-\sigma_E}]^{\frac{\sigma_E-1/v_E}{1-\sigma_E}} = CEE_{t+1}^{\sigma_E-1/v_E} \quad (53)$$

In addition, I introduce the Stochastic Discount Factor (SDF) $\Lambda_{t,t+1}^E$ for entrepreneurs from t to $t+1$ as:

$$\Lambda_{t,t+1}^E = \beta_E \left(\frac{V_{t+1}^E}{CEE_{t+1}} \right)^{1/v_E - \sigma_E} \left(\frac{C_{t+1}^E}{C_t^E} \right)^{-1/v_E} \quad (54)$$

Loans Forwarding $\partial V_t^E / \partial A_t^F$ obtained from (50), inserting in (45) and using the SDF defined in (54), I get:

$$p_t^A = \tilde{\lambda}_t^{AB} F_A + E_t \left\{ \Lambda_{t,t+1}^E [(1 - \tau_F + q_{t+1}^A) \Omega_D(\omega_{t+1}^*) + (1 + q_{t+1}^A) f_\omega(\omega_{t+1}^*) \mathcal{A}_{t+1}] \right\} - 1 \quad (55)$$

where $\tilde{\lambda}_t^{AB} = \lambda_t^{AB} (1 - \beta_E)^{-1} \left(\frac{V_t^E}{C_t^E} \right)^{-1/v_E}$.

Bonds Forwarding $\partial V_t^E / \partial B_t^F$ obtained from (51), inserting in (46) and using the SDF defined in (54), I get:

$$p_t^B = \tilde{\lambda}_t^{AB} F_B + E_t \left\{ \Lambda_{t,t+1}^E [\Omega_B(\omega_{t+1}^\#) - \tau_F \Omega_D(\omega_{t+1}^*) + f_\omega(\omega_{t+1}^*) \mathcal{A}_{t+1} + f_\omega(\omega_{t+1}^\#) \mathcal{C}_{t+1}] \right\} \quad (56)$$

where $\tilde{\lambda}_t^{AB} = \lambda_t^{AB} (1 - \beta_E)^{-1} \left(\frac{V_t^E}{C_t^E} \right)^{-1/v_E}$.

Capital Forwarding $\partial V_t^E / \partial K_t^F$ obtained from (52), inserting in (47) and using the SDF defined in (54), I get:

$$p_t = E_t \left\{ \Lambda_{t,t+1}^E \left[p_{t+1} \Omega_D(\omega_{t+1}^*) [1 - (1 - \tau_F) \delta_K] (1 + \text{h}\tilde{\lambda}_{t+1}^{AB}) + (1 - \tau_F) (1 - \alpha) \Omega_E(\omega_{t+1}^*) Z_{t+1} \left(\frac{K_{t+1}^F}{L_{t+1}} \right)^{-\alpha} + \frac{\varphi}{2} \left[\left(\frac{X_{t+1}}{K_{t+1}^F} \right)^2 - \delta_K^2 \right] + (1 - \alpha) \left\{ Z_{t+1} \left(\frac{K_{t+1}^F}{L_{t+1}} \right)^{-\alpha} \mathcal{D}_{t+1} + [1 - (1 - \tau_F) \delta_K] \omega_{t+1}^* f_\omega(\omega_{t+1}^*) \text{h}\tilde{\lambda}_{t+1}^{AB} p_{t+1} \right\} \right] \right\}$$

(57)

B.2. Saver problem

The maximization problem for savers is as follows:

$$V^S(B_t^S, D_t^S, s_t) = \max_{\{C_t^S, D_{t+1}^S, B_{t+1}^S\}} \left\{ (1 - \beta_S)(C_t^S)^{1-1/v_S} \right. \\ \left. + \beta_S \left(E_t \left[\left(V^S(B_{t+1}^S, D_{t+1}^S, s_{t+1}) \right)^{1-\sigma_S} \right] \right)^{\frac{1-1/v_S}{1-\sigma_S}} \right\}^{\frac{1}{1-1/v_S}} \quad (58)$$

subject to:

$$C_t^S + (q_t^f + \tau_D r_t^f) D_{t+1}^S + p_t^B B_{t+1}^S \leq D_t^S + [1 - \tau_B n(1 - p_t^B)] \Omega_B(\omega_t^\#) B_t^S + (1 - \tau_W) w_t^S \bar{L}_t^S \quad (59)$$

$$B_{t+1}^S \geq 0 \quad (60)$$

B.2.1. First-order conditions

Deposits The FOC for deposits D_{t+1}^S is:

$$-(1 - \beta_S)(q_t^f + \tau_D r_t^f) \left(\frac{V_t^S}{C_t^S} \right)^{1/v_S} + \beta_S E_t \left[(V_{t+1}^S)^{1-\sigma_S} \right]^{\frac{\sigma_S-1/v_S}{1-\sigma_S}} E_t \left[(V_{t+1}^S)^{-\sigma_S} \frac{\partial V_{t+1}^S}{\partial D_{t+1}^S} \right] (V_t^S)^{1/v_S} = 0 \quad (61)$$

Bonds The FOC for deposits B_{t+1}^S is:

$$\lambda_t^S - (1 - \beta_S) p_t^B \left(\frac{V_t^S}{C_t^S} \right)^{1/v_S} + \beta_S E_t \left[(V_{t+1}^S)^{1-\sigma_S} \right]^{\frac{\sigma_S-1/v_S}{1-\sigma_S}} E_t \left[(V_{t+1}^S)^{-\sigma_S} \frac{\partial V_{t+1}^S}{\partial B_{t+1}^S} \right] (V_t^S)^{1/v_S} = 0 \quad (62)$$

where λ_t^S is the Lagrange multiplier on the condition (60).

B.2.2. Function Deductions

I use this section to derive the Euler Equations to be used for the simulation using the first-order conditions obtained from B.2.1. I denote:

$$E_t \left[(V_{t+1}^S)^{1-\sigma_S} \right]^{\frac{\sigma_S-1/v_S}{1-\sigma_S}} = CES_{t+1}^{\sigma_S-1/v_S}. \quad (63)$$

In addition, I introduce the Stochastic Discount Factor (SDF) $\Lambda_{t,t+1}^E$ for entrepreneurs from t to $t + 1$ as:

$$\Lambda_{t,t+1}^S = \beta_S \left(\frac{V_{t+1}^S}{CES_{t+1}^S} \right)^{1/v_S - \sigma_S} \left(\frac{C_{t+1}^S}{C_t^S} \right)^{-1/v_S}. \quad (64)$$

Deposits The problem of obtaining the derivative of value function V_{t+1}^S with respect to D_{t+1}^S can be reduced to find $\partial V_t^S / \partial D_t^S$ and forward one period. After obtaining $\partial V_{t+1}^S / \partial D_{t+1}^S$, I can write the Euler Equation for deposits. The term $\partial V_t^S / \partial D_t^S$ can be written as:

$$\frac{\partial V_t^S}{\partial D_t^S} = (1 - \beta_S) \left(\frac{V_t^S}{C_t^S} \right)^{1/v_S}. \quad (65)$$

Forwarding $\partial V_t^S / \partial D_t^S$ obtained from (65), inserting in (61) and using the SDF defined in (64), I get:

$$q_t^f + \tau_D r_t^f = E_t[\Lambda_{t,t+1}^S]. \quad (66)$$

Bonds The problem of obtaining the derivative of value function V_{t+1}^S with respect to B_{t+1}^S can be reduced to find $\partial V_t^S / \partial B_t^S$ and forward one period. After obtaining $\partial V_{t+1}^S / \partial B_t^S$, I can write the Euler Equation for deposits. The term $\partial V_t^S / \partial B_t^S$ can be written as:

$$\frac{\partial V_t^S}{\partial B_t^S} = (1 - \beta_S) \left(\frac{V_t^S}{C_t^S} \right)^{1/v_S} \left\{ \Omega_B(\omega_t^\#) + \frac{\partial C_t^S}{\partial \omega_t^\#} \frac{\partial \omega_t^\#}{\partial B_t^S} + \frac{\partial C_t^S}{\partial \omega_t^\#} \frac{\partial \omega_t^\#}{\partial B_t^S} \right\}$$

Using the fact that $\partial C_t^S / \partial \omega_t^\# = 0$, $\partial C_t^S / \partial \omega_t^\# = f_\omega(\omega_t^\#)(\underline{\pi} + \sum_b w_t^b L_t^b) B_t^S / B_t^F$ and $B_t^F = B_t^S + B_t^B$, the above equation can be rewritten as:

$$\frac{\partial V_t^S}{\partial B_t^S} = (1 - \beta_S) \left(\frac{V_t^S}{C_t^S} \right)^{1/v_S} [\Omega_B(\omega_t^\#) + f_\omega(\omega_t^\#) \mathcal{G}_t]. \quad (67)$$

where $\mathcal{G}_t = \frac{(\underline{\pi} + \sum_b w_t^b L_t^b) B_t^S}{Y_t B_t^F}$.

Forwarding $\partial V_t^S / \partial B_t^S$ obtained from (69), inserting in (62) and using the SDF defined in (64), I get:

$$p_t^B = \tilde{\lambda}_t^S + E_t\{\Lambda_{t,t+1}^S [\Omega_B(\omega_{t+1}^\#) + f_\omega(\omega_{t+1}^\#) \mathcal{G}_{t+1}]\}. \quad (68)$$

where $\tilde{\lambda}_t^S = \lambda_t^S (1 - \beta_S)^{-1} \left(\frac{V_t^S}{C_t^S} \right)^{-1/v_S}$.

B.3. Banker problem

The maximization problem for bankers can be rewritten as follows:

$$V^B(W_t^B, s_t) = \max_{\{d_t^B, A_{t+1}^B, B_{t+1}^B\}} d_t^B + E_t[\Lambda_{t,t+1}^B V^B(W_{t+1}^B, s_{t+1})] \quad (69)$$

subject to

$$d_t^B + \exists(d_t^B) + (1 + q_t^A) A_{t+1}^B + M(I_{t+1}^B) + p_t^B B_{t+1}^B \leq W_t^B \quad (70)$$

$$W_{t+1}^B = [1 + (1 - \tau_B \sigma) q_{t+1}^A] \Omega_D(\omega_{t+1}^*) A_{t+1}^B + [1 - \tau_B n(1 - p_t^B)] \Omega_B(\omega_{t+1}^\#) B_{t+1}^B + H_{t+1} - D_{t+1}^B \quad (71)$$

where I denote $W_t^B = \pi_t^B - \tau_B \Pi_t^B + q_t^f D_{t+1}^B - D_t^B + H F_t + T_t - G_t - G_t^W$ to represent banks' disposable wealth for expenditure. Note that combining with the definition of W_t^S , I can express:

$$W_t^B + W_t^S = (1 + \tilde{q}_t^A) \Omega_D(\omega_t^*) A_t^B + (1 - \tilde{p}_t^B) \Omega_B(\omega_t^\#) B_t^F + q_t^f D_t^B + T_t - G_t$$

where $\tilde{q}_t^A = (1 - \tau_B \sigma) q_t^A$ and $\tilde{p}_t^B = \tau_B n(1 - p_t^B)$. Together with the collateral constraint and requirement regimes:

$$\Theta(D_{t+1}^B) = \{(A_{t+1}^B, B_{t+1}^B) \mid (1-k)A_{t+1}^B + (1-k_B)B_{t+1}^B \geq D_{t+1}^B\} \quad (72)$$

$$\Xi(D_{t+1}^B) = \{(A_{t+1}^B, B_{t+1}^B) \mid \bar{h}_A A_{t+1}^B + \bar{h}_B B_{t+1}^B + lD_w \geq lD_{t+1}^B\} \quad (73)$$

$$\Upsilon(D_{t+1}^B) = \{(A_{t+1}^B, B_{t+1}^B) \mid A_{t+1}^B + B_{t+1}^B \leq F_D D_{t+1}^B\} \quad (74)$$

where

$$\exists(d_t^B) = \frac{\vartheta}{2} [(d_t^B) - \bar{d}^B]^2$$

$$\pi_t^B = (1 + q_t^A)\Omega_D(\omega_t^*)A_t^B + p_t^B \Omega_B(\omega_t^\#)B_t^B$$

$$H_t = (1 - c^F)\{[1 - \Omega_D(\omega_t^*)](1 - \delta_K)p_t K_t^F + [1 - \Omega_E(\omega_t^*)]Z_t(K_t^F)^{1-\alpha}L_t^\alpha\} \\ - [1 - \Omega_D(\omega_t^*)] \sum_b w_t^b L_t^b$$

$$\Pi_t^B = \Omega_D(\omega_t^*)\sigma q_t^A A_t^B + n(1 - p_t^B)\Omega_B(\omega_t^\#)B_t^B - r_t^f D_t^B$$

B.3.1. Preliminary Calculations

I use this section to conduct some preliminary calculations which will be used for later use. Using equation (70), I can eliminate B_{t+1}^B to rewrite W_{t+1}^B as:

$$W_{t+1}^B = [1 + (1 - \tau_B \sigma)q_{t+1}^A]\Omega_D(\omega_{t+1}^*)A_{t+1}^B + \frac{\tilde{B}}{p_t^B} [W_t^B - d_t^B - \exists(d_t^B) - (1 + q_t^A)A_{t+1}^B - M(I_{t+1}^B)] \\ + HF_{t+1} - D_{t+1}^B$$

where I define $\tilde{B} = [1 - \tau_B n(1 - p_t^B)]\Omega_B(\omega_{t+1}^\#)$

The constraints in (72), (73) and (74) will be:

$$\Theta(D_{t+1}^B) = \{(A_{t+1}^B, d_t^B) \mid (1-k)A_{t+1}^B + (1-k_B)\frac{W_t^B - d_t^B - \exists(d_t^B) - (1+q_t^A)A_{t+1}^B - M(I_{t+1}^B)}{p_t^B} \geq D_{t+1}^B\}$$

$$\Xi(D_{t+1}^B) = \{(A_{t+1}^B, d_t^B) \mid \bar{h}_A A_{t+1}^B + \bar{h}_B \frac{W_t^B - d_t^B - \exists(d_t^B) - (1+q_t^A)A_{t+1}^B - M(I_{t+1}^B)}{p_t^B} + lD_w \geq lD_{t+1}^B\}$$

$$\Upsilon(D_{t+1}^B) = \{(A_{t+1}^B, d_t^B) \mid A_{t+1}^B + \frac{W_t^B - d_t^B - \exists(d_t^B) - (1+q_t^A)A_{t+1}^B - M(I_{t+1}^B)}{p_t^B} \leq F_D D_{t+1}^B\}.$$

B.3.2. First-order conditions

Dividend The FOC for new loan investment d_t^B is:

$$\frac{1}{1+\exists'(d_t^B)} = \lambda_t^{\theta EY} + \frac{1}{p_t^B} E_t [A_{t,t+1}^B \tilde{B} V_{W,t+1}^B] \quad (75)$$

where $\lambda_t^{\theta EY} = \frac{1}{p_t^B} [(1 - k_B)\lambda_t^\theta + \bar{h}_B \lambda_t^\Xi - \lambda_t^\Upsilon]$ are Lagrange multiplier for regulatory constraints. I also

denote $V_{W,t+1}^B = \frac{\partial V^B(w_{t+1}^B, s_{t+1})}{\partial w_{t+1}^B}$ and $\exists'(d_t^B) = \frac{\partial \exists(d_t^B)}{\partial d_t^B}$.

Loans The FOC for loans A_{t+1}^B is:

$$\begin{aligned}
& -[1 + q_t^A + M'(I_{t+1}^B)]\lambda_t^{\theta\Xi Y} + E_t\{\Lambda_{t,t+1}^B V_{W,t+1}^B [1 + (1 - \tau_B \sigma)q_{t+1}^A]\Omega_D(\omega_{t+1}^*)\} \\
& = -\hat{\lambda}_t^{\theta\Xi Y} + [1 + q_t^A + M'(I_{t+1}^B)]\frac{1}{p_t^B} E_t[\Lambda_{t,t+1}^B \tilde{B} V_{W,t+1}^B]
\end{aligned}$$

where $\hat{\lambda}_t^{\theta\Xi Y} = (1 - k)\lambda_t^\theta + \bar{h}_A \lambda_t^\Xi - \lambda_t^Y$. Using (75), I can express the above equation as:

$$\frac{1 + q_t^A + M'(I_{t+1}^B)}{1 + \exists'(d_t^B)} = \hat{\lambda}_t^{\theta\Xi Y} + E_t\{\Lambda_{t,t+1}^B V_{W,t+1}^B [1 + (1 - \tau_B \sigma)q_{t+1}^A]\Omega_D(\omega_{t+1}^*)\} \quad (76)$$

B.3.3. Function Deductions

I use this section to derive the Euler Equations to be used for the simulation using the first-order conditions obtained from B.3.2.

Use the envelope condition, I can get:

$$V_{W,t}^B = \lambda_t^{\theta\Xi Y} + \frac{1}{p_t^B} E_t[\Lambda_{t,t+1}^B \tilde{B} V_{W,t+1}^B]$$

Combining the above equation with (75), I can get:

$$V_{W,t}^B = \frac{1}{1 + \exists'(d_t^B)}$$

Forward the above to one period to get $V_{W,t+1}^B$ and insert it into (75) and (76), I can obtain the simulation equations for dividend and loan investment by defining:

$$\tilde{\Lambda}_{t,t+1}^B = \Lambda_{t,t+1}^B \frac{1 + \exists'(d_t^B)}{1 + \exists'(d_{t+1}^B)} \quad (77)$$

The equations for simulation in terms of dividend and loan investment are:

$$1 = \tilde{\lambda}_t^{\theta\Xi Y} + \tilde{p}_t^B E_t\{\tilde{\Lambda}_{t,t+1}^B \Omega_B(\omega_{t+1}^*)\} \quad (78)$$

where $\tilde{\lambda}_t^{\theta\Xi Y} = [1 + \exists'(d_t^B)]\lambda_t^{\theta\Xi Y}$ and $\tilde{p}_t^B = \frac{1 - \tau_B n(1 - p_t^B)}{p_t^B}$;

$$q_t^A = \check{\lambda}_t^{\theta\Xi Y} + E_t\{\tilde{\Lambda}_{t,t+1}^B [1 + (1 - \tau_B \sigma)q_{t+1}^A]\Omega_D(\omega_{t+1}^*)\} - M'(I_{t+1}^B) - 1 \quad (79)$$

where $\check{\lambda}_t^{\theta\Xi Y} = [1 + \exists'(d_t^B)]\hat{\lambda}_t^{\theta\Xi Y}$ and $M'(I_{t+1}^B) = m + \kappa \left(\frac{I_{t+1}^B}{A_t^B} - \sigma \right) + \frac{X_{\{I_{t+1}^B < 0\}}}{A_t^B} |I_{t+1}^B|$.

Appendix C. Simulation Steps

The simulation steps for this paper are established by approximating a path to satisfy an equilibrium, which is designed by different scenarios. I use the global projection method, pioneered by Judd (1998), to generate results. This method, as pointed out by Begeau and Landvoigt (2018), outperforms the Perturbation-based solution method in terms of the better quality of approximation for non-linear dynamic models with constraints. The global projection method is conducted by determining an appropriate state space for exogenous shocks and endogenous state variables, which will be introduced later, approximating the results according to the state space and iterating the model until convergence. I will introduce these steps in details below.

C.1. Solution Procedure

This projection-based solution developed for this paper comes with three main steps.

Step 1. Construct matrices for discretized variables. To simulate the model, I will firstly discrete exogenous shocks and endogenous state variables, store them in matrices and calculate the pre-computed values for each entry of the matrices, which will be used for later iterations. This step involves the usage of MATLAB library to solve a system of nonlinear multivariate functions.

Step 2. Iterate to solve unknown functions until convergence. Once the matrices in Step 1 are constructed, I will iterate the model given a first guess for the variables until the convergence of the model. This step involves the implementation of Markov chain for the transition of exogenous shocks and a dampening process for variables to reduce oscillation during the iterations.

Step 3. Simulate the model for a given length of period following the simulated steady state. After obtaining the steady state, the convergence, of the model from Step 2, I will simulate the model for a given period and collect the results associated with the state variables. This step involves the adjustment for the grids of the discretized variables, as in Step 1, in case of the hitting of boundaries or the unneglectable simulation errors.

I will provide a more detailed description of the above three steps.

Step 1.

The state space consists of:

Two exogenous state variables $[Z_t, s_t]$, and

Six endogenous state variables $[A_t^B, B_t^F, B_t^B, W_t^S, K_t^F, B_t^G]$.

I first discretize Z_t into a N^Z -state Markov chain following Rouwenhorst (1995), which will result in a $N^Z \times N^Z$ transition matrix. When considering the realization of financial situations, captured by s_t , the dimension of the matrix doubles, and I take the overall exogenous transition matrix as $\Pi_x = \Pi_{N^Z} \otimes \Pi_s$, which is governed by the two exogenous shocks.

Regarding the endogenous state variables, I need to discretize them for approximation purpose. Assume these variables can take on values in a continuous and convex subset of the reals, and each of the variables is within $[\bar{S}_l, \bar{S}_u]$. Note that two endogenous state variable B_t^B and B_t^G can be eliminated by using other four variables. Thus, I can rewrite the sets of endogenous variables as in $\mathcal{S}_n = \prod_i [\bar{S}_{i,l}, \bar{S}_{i,u}]$, where S_j represents for $A_k^B, B_l^B, W_m^S, K_n^F$ respectively. Choose an appropriate number of grids for each endogenous variable for the simulation, which will result in $\mathcal{S}_n = \{A_k^B\}_{k=1}^{N_A} \times \{B_l^F\}_{l=1}^{N_F} \times$

$\{W_m^S\}_{m=1}^{N_S} \times \{K_n^F\}_{n=1}^{N_K}$. These grids are chosen to ensure each grid covers the ergodic distribution of the economy and to minimize simulation errors. Denote the number of state for exogenous variables as \mathcal{S}_x , thus the total variable space is $\mathcal{S} = \mathcal{S}_x \times \mathcal{S}_n$, and the total number of the points is $N_S = N_x \cdot N_A \cdot N_B \cdot N_S \cdot N_K$, where N_x represents the total number of exogenous variables.

Step 2.

Given an initial guess $\bar{\mathcal{C}}_C^m$, where m denotes the number of iterations and $m = 0$ representing the first iteration. The guess $\bar{\mathcal{C}}_C^m$ is set for each point of $S_j \subseteq \mathcal{S}$, where $j = 1, 2 \dots N_S$ and for every possible future realization of each point x_i , where $i = 1, 2 \dots N_x$. The variables to guess include current dividend payment d_j^B , consumptions $\{C_j^E, C_j^S\}$, loan rate q_j^A , bond price p_j^B , wages w_j^b , risk-free rate (deposit rate) r_j^f and future realizations of endogenous variables $\{A_{i,j}^B, B_{i,j}^F, W_{i,j}^S, K_{i,j}^F\}$ to obtain forecast variables $f_{i,j}^m = \mathcal{C}_F^m(S_j, x_i)$ for each combination of current state S_j and future exogenous state x_i . This means $\bar{\mathcal{C}}_C^m = \{d_j^B, C_j^E, C_j^S, q_j^A, p_j^B, w_j^b, r_j^f, A_{i,j}^B, B_{i,j}^F, W_{i,j}^S, K_{i,j}^F\}$.

The forecast variables include loan rate $q^A(\mathcal{S})$, bond price $p^B(\mathcal{S})$, capital investment $X(\mathcal{S})$, wages $w^b(\mathcal{S})$, new loan investment $I(\mathcal{S})$, saver's investment in corporate bonds $B^S(\mathcal{S})$, bank dividend $d^B(\mathcal{S})$, consumptions $C^b(\mathcal{S})$, Lagrange multiplier for firm leverage $\lambda^{AB}(\mathcal{S})$ and the value of bankers, entrepreneurs and savers $V^B(\mathcal{S}), V^E(\mathcal{S}), V^S(\mathcal{S})$. Thus, I need to construct a $N_x \times N_S$ matrix \mathbf{F}^m with each entry being a vector

$$f_{i,j}^m = \{q_{i,j}^A, p_{i,j}, w_{i,j}^E, w_{i,j}^S, d_{i,j}^B, C_{i,j}^E, C_{i,j}^S, \lambda_{i,j}^{AB}, V_{i,j}^B, V_{i,j}^E, V_{i,j}^S\}$$

of the pre-computed values to be used for the calculation in Step 3. To obtain $f_{i,j}^m$, insert the initial guess $\bar{\mathcal{C}}_C^m$ and solve for the following system of equations:

$$\bar{q}_j^A = \lambda_{i,j}^{AB} F_A + \beta_E (C_{i,j}^E / \bar{C}_j^E)^{-1/v_E} [(1 - \tau_F + q_{i,j}^A) \Omega_D(\bar{\omega}_{i,j}^*) + (1 + q_{i,j}^A) f_\omega(\bar{\omega}_{i,j}^*) \mathcal{A}_{i,j}] - 1 \quad (\text{F1})$$

$$\bar{p}_j^B = \lambda_{i,j}^{AB} F_B + \beta_E (C_{i,j}^E / \bar{C}_j^E)^{-1/v_E} [\Omega_B(\bar{\omega}_{i,j}^*) - \tau_F \Omega_D(\bar{\omega}_{i,j}^*) + f_\omega(\bar{\omega}_{i,j}^*) \mathcal{A}_{i,j} + f_\omega(\bar{\omega}_{i,j}^*) C_{i,j}] \quad (\text{F2})$$

$$p_{i,j} = 1 + \varphi [(\bar{K}_{i,j}^F - \Omega_D(\bar{\omega}_{i,j}^*) \bar{K}_j^F) / \bar{K}_j^F - \delta_K] \quad (\text{F3})$$

$$(1 - \tau_F) \Omega_E(\bar{\omega}_{i,j}^*) \bar{M} \bar{L}_{i,j}^E = (1 - \tau_F) \Omega_D(\bar{\omega}_{i,j}^*) w_{i,j}^E + f_\omega(\bar{\omega}_{i,j}^*) (w_{i,j}^E - \bar{\omega}_{i,j}^* \bar{M} \bar{L}_{i,j}^E) \mathcal{A}_{i,j} + f_\omega(\bar{\omega}_{i,j}^*) (w_{i,j}^E - \bar{\omega}_{i,j}^* \bar{M} \bar{L}_{i,j}^E) C_{i,j} \quad (\text{F4})$$

$$(1 - \tau_F) \Omega_E(\bar{\omega}_{i,j}^*) \bar{M} \bar{L}_{i,j}^S = (1 - \tau_F) \Omega_D(\bar{\omega}_{i,j}^*) w_{i,j}^S + f_\omega(\bar{\omega}_{i,j}^*) (w_{i,j}^S - \bar{\omega}_{i,j}^* \bar{M} \bar{L}_{i,j}^S) \mathcal{A}_{i,j} + f_\omega(\bar{\omega}_{i,j}^*) (w_{i,j}^S - \bar{\omega}_{i,j}^* \bar{M} \bar{L}_{i,j}^S) C_{i,j} \quad (\text{F5})$$

$$1 = \bar{p}_j^B \mathcal{O}_{i,j}^B \Omega_B(\bar{\omega}_{i,j}^*) \quad (\text{F6})$$

$$(\ln p_{i,j} [1 - (1 - \tau_F) \delta_K] \Omega_D(\bar{\omega}_{i,j}^*) \bar{K}_{i,j}^F - F_A \bar{A}_{i,j}^B - F_B \bar{B}_{i,j}^F) \lambda_{i,j}^{AB} = 0 \quad (\text{F7})$$

In the above equations, the variables with a bar ($\bar{\cdot}$) indicates that they are direct functions of the guessed variables $\bar{\mathcal{C}}_C^m$, and thus are known before the calculation. Note that from the above set of equations, I can obtain $\hat{f}_{i,j}^m = \{q_{i,j}^A, p_{i,j}, w_{i,j}^E, w_{i,j}^S, d_{i,j}^B, C_{i,j}^E, \lambda_{i,j}^{AB}\}$ as part of the entry of $f_{i,j}^m$. The remaining entry of $f_{i,j}^m$, namely $\{V_{i,j}^E, V_{i,j}^S, V_{i,j}^B, C_{i,j}^S\}$ can be obtained from the variables in $\hat{f}_{i,j}^m$. $\{V_{i,j}^E, V_{i,j}^S, V_{i,j}^B\}$ can be determined using definitions in (40), (58) and (69), respectively, while $C_{i,j}^S$ can be determined using the constraint of savers in (14):

$$C_{i,j}^S = \bar{W}_{i,j}^S + (1 - \tau_W) w_{i,j}^S \bar{L}_{i,j}^S - (\bar{q}_j^f + \tau_D \bar{r}_j^f) \bar{D}_{i,j}^S - \bar{p}_j^B \bar{B}_{i,j}^S$$

Note that the regulatory constraints and no-shorting constraint on savers' bond holdings are not imposed here as the constraints are automatically satisfied in my guessed variables. (E3) and (E4) use the fact that $\bar{X}_{i,j} = \bar{K}_{i,j}^F - \Omega_D(\bar{\omega}_{i,j}^*)\bar{K}_{i,j}^F$. In (E8) and (E9), I denote $\mathcal{O}_{i,j}^B = (C_{i,j}^E/\bar{C}_{i,j}^E)^{-1/\nu_E} (1 + \exists'(d_{i,j}^B)) / (1 + \exists'(d_{i,j}^B))$, and $\bar{I}_{i,j}^B = \bar{A}_{i,j}^B - \Omega_D(\bar{\omega}_{i,j}^*)\bar{A}_{i,j}^B$. Using the definition of $\bar{W}_{i,j}^S$, I can express $\bar{B}_{i,j}^S = (\bar{W}_{i,j}^S - \bar{D}_{i,j}^S) / \Omega_B(\bar{\omega}_{i,j}^\#)$.

Once the completion of the construction of F^m , I will proceed to the Step 3.

Step 3.

Once obtaining matrix F^m , I will use it to calculate the expected value conditional on each possible realization of future exogenous shocks, the results of which are the equilibrium values of variables for each point of j :

$$\hat{P}_j = f_j^m = \left\{ \hat{q}_j^A, \hat{p}_j^B, \hat{X}_j, \hat{w}_j^E, \hat{w}_j^S, \hat{l}_j^B, \hat{r}_j^f, \hat{d}_j^B, \hat{C}_j^E, \hat{C}_j^S, \hat{\lambda}_j^{AB}, \hat{\lambda}_j^{Y,1}, \hat{\lambda}_j^{Y,2}, \hat{\lambda}_j^S \right\}$$

The equations for obtaining the results are:

$$\hat{q}_j^A = \hat{\lambda}_j^{AB} F_A + E_{s_{i,j}|s_j} \left\{ \hat{\Lambda}_{i,j}^E \left[(1 - \tau_F + q_{i,j}^A) \Omega_D(\omega_{i,j}^*) + (1 + q_{i,j}^A) f_\omega(\omega_{i,j}^*) \mathcal{A}_{i,j} \right] \right\} - 1 \quad (\text{E1})$$

$$\hat{p}_j^B = \hat{\lambda}_j^{AB} F_B + E_{s_{i,j}|s_j} \left\{ \hat{\Lambda}_{i,j}^E \left[\Omega_B(\omega_{i,j}^\#) - \tau_F \Omega_D(\omega_{i,j}^*) + f_\omega(\omega_{i,j}^*) \mathcal{A}_{i,j} + f_\omega(\omega_{i,j}^\#) \mathcal{C}_{i,j} \right] \right\} \quad (\text{E2})$$

$$\hat{p}_j = E_{s_{i,j}|s_j} \left\{ \hat{\Lambda}_{i,j}^E \left[p_{i,j} \Omega_D(\omega_{i,j}^*) [1 - (1 - \tau_F) \delta_K] (1 + \text{h}\lambda_{i,j}^{AB}) + (1 - \tau_F) (1 - \alpha) \Omega_E(\omega_{i,j}^*) Z_i \left(\frac{K_{i,j}^F}{L_{i,j}} \right)^{-\alpha} + \frac{\varphi}{2} \left[\left(\frac{X_{i,j}}{K_{i,j}^F} \right)^2 - \delta_K^2 \right] + (1 - \alpha) \left\{ Z_i \left(\frac{K_{i,j}^F}{L_{i,j}} \right)^{-\alpha} \mathcal{D}_{i,j} + [1 - (1 - \tau_F) \delta_K] \omega_{i,j}^* f_\omega(\omega_{i,j}^*) \text{h}\lambda_{i,j}^{AB} p_{i,j} \right\} \right] \right\} \quad (\text{E3})$$

$$(1 - \tau_F) \Omega_E(\hat{\omega}_j^*) M L_j^E = (1 - \tau_F) \Omega_D(\hat{\omega}_j^*) \hat{w}_j^E + f_\omega(\hat{\omega}_j^*) (\hat{w}_j^E - \hat{\omega}_j^* M L_j^E) \hat{\mathcal{A}}_j + f_\omega(\hat{\omega}_j^\#) (\hat{w}_j^E - \hat{\omega}_j^\# M L_j^E) \hat{\mathcal{C}}_j \quad (\text{E4})$$

$$(1 - \tau_F) \Omega_E(\hat{\omega}_j^*) M L_j^S = (1 - \tau_F) \Omega_D(\hat{\omega}_j^*) \hat{w}_j^S + f_\omega(\hat{\omega}_j^*) (\hat{w}_j^S - \hat{\omega}_j^* M L_j^S) \hat{\mathcal{A}}_j + f_\omega(\hat{\omega}_j^\#) (\hat{w}_j^S - \hat{\omega}_j^\# M L_j^S) \hat{\mathcal{C}}_j \quad (\text{E5})$$

$$\hat{q}_j^f + \tau_D \hat{r}_j^f = E_{s_{i,j}|s_j} \left[\hat{\Lambda}_{i,j}^S \right] \quad (\text{E6})$$

$$\hat{p}_j^B = \hat{\lambda}_j^S + E_{s_{i,j}|s_j} \left[\hat{\Lambda}_{i,j}^S \left[\Omega_B(\omega_{i,j}^\#) + f_\omega(\omega_{i,j}^\#) \mathcal{G}_{i,j} \right] \right] \quad (\text{E7})$$

$$1 = \hat{\lambda}_j^{Y,1} + \hat{p}_j^B E_{s_{i,j}|s_j} \left[\hat{\Lambda}_{i,j}^B \Omega_B(\omega_{i,j}^\#) \right] \quad (\text{E8})$$

$$\hat{q}_j^A = \hat{\lambda}_j^{Y,2} + E_{s_{i,j}|s_j} \left[\hat{\Lambda}_{i,j}^B \left[1 + (1 - \tau_B \sigma) q_{i,j}^A \right] \Omega_D(\omega_{i,j}^*) \right] - M'(\hat{l}_j^B) - 1 \quad (\text{E9})$$

$$(\text{h}\hat{p}_j [1 - (1 - \tau_F) \delta_K] K_j^F - F_A \hat{A}_j^F - F_B \hat{B}_j^F) \hat{\lambda}_j^{AB} = 0 \quad (\text{E10})$$

$$(F_D D_j^B - \hat{A}_j^B - \hat{B}_j^B) \hat{\lambda}_j^{Y,1} = 0 \quad (\text{E11})$$

$$(F_D D_j^B - \hat{A}_j^B - \hat{B}_j^B) \hat{\lambda}_j^{Y,2} = 0 \quad (\text{E12})$$

$$\hat{B}_j^S \hat{\lambda}_j^S = 0 \quad (\text{E13})$$

$$\hat{B}_j^B + \hat{B}_j^S = \hat{B}_j^F \quad (\text{E14})$$

When in the case of regulations, the following constraints will be imposed:

$$\left((1 - k) \hat{A}_j^B + (1 - k_B) \hat{B}_j^B - D_j^B \right) \hat{\lambda}_j^B = 0 \quad (\text{AE1})$$

$$(\bar{h}_A \hat{A}_j^B + \bar{h}_B \hat{B}_j^B + l D_w - l D_j^B) \hat{\lambda}_j^{\bar{B}} = 0 \quad (\text{AE2})$$

Equation (E1) is the entrepreneurs' FOC of loans in (55) and (E2) of corporate bonds in (56). (E3) is the FOC capital in (57). (E4) and (E5) are the FOCs of labour in (49) for entrepreneurs and savers, respectively. (E6) and (E7) are the savers' FOC of deposits in (66) and corporate bonds in (68), respectively. (E8) and (E9) are bankers' FOC of dividend payment in (78) and loans in (79). (E10) is

the borrowing constraint on entrepreneurs, as in (12). (E11) and (E12) are the reserve requirement on banks in (21). (E13) is no-shorting constraint of corporate bonds on savers in (60). (E14) is the market clearing condition of corporate bonds, as in (26). Note that (E3) uses the fact that $p_{i,j} = 1 + \varphi[X_{i,j}/K_j^F - \delta_K]$ and (E6) uses that $\hat{r}_j^f = 1/\hat{q}_j^f - 1$. (AE1) and (AE2) are constraints for capital requirements (19) and liquidity requirements (20), respectively. When these constraints apply, the multipliers in (E8), (E9) and (E11) will be adjusted. (E1) - (E14) implicitly use the budget constraints of each agent and I insert the budget constraint of savers, as in (14):

$$\hat{B}_j^S = \frac{1}{\hat{p}_j^B} \left[W_j^S + (1 - \tau_W) \hat{w}_j^S \bar{L}_j^S - \hat{C}_j^S - (\hat{q}_j^f + \tau_D \hat{r}_j^f) D_j^S \right]$$

to eliminate \hat{B}_j^S and use the budget constraints of bankers, as in (70):

$$\hat{B}_j^B = \frac{1}{\hat{p}_j^B} \{ W_j^B - \hat{d}_j^B - \exists(\hat{d}_j^B) - (1 + \hat{q}_j^A) [(1 - \sigma) \Omega_D(\hat{\omega}_j^*) A_j^B + \hat{I}_j^B] - M(\hat{I}_j^B) \}$$

to eliminate \hat{B}_j^B to ensure there are 14 equations to solve for 14 unknowns. Once obtaining the results, the value of \hat{B}_j^B can be determined as:

$$\hat{B}_j^F = \frac{1}{\hat{p}_j^B} \left\{ \hat{C}_j^E + \Phi(\hat{X}_j, K_j^F) + (1 + \hat{q}_j^A) \Omega_D(\hat{\omega}_j^*) A_j^F + \Omega_B(\hat{\omega}_j^\#) B_j^F + \hat{p}_j \hat{K}_j^F + \Omega_D(\hat{\omega}_j^*) \sum_b \hat{w}_j^b L_j^b + \tau_F \hat{\Pi}_t^F - \Omega_E(\hat{\omega}_j^*) \hat{Y}_j - (1 - \tau_W^E) \hat{w}_j^E \bar{L}_j^E - \hat{p}_j [\hat{X}_j + \Omega_D(\hat{\omega}_j^*) (1 - \delta_K) K_j^F] - (1 + \hat{q}_j^A) [(1 - \sigma) \Omega_D(\hat{\omega}_j^*) A_j^F + \hat{I}_j^B] \right\}$$

Expectations are computed as weighted sums, with the weights being the probabilities of each possible future realizations x_i , given the current state S_j . Hats ($\hat{\cdot}$) in equations indicates the variables which are the functions of unknowns in \hat{P}_j , which will be calculated by the use of the nonlinear equation solver. The calculation results in a $N_S \times 14$ matrix \mathbf{P}^m with each row the solution vector \hat{P}_j for each point of s_j . From \mathbf{P}^m , I will update the variables for this iteration. The steps are as follows:

$$\hat{V}_j^B = \hat{d}_j^B + E_{s_{i,j}|s_j} [\hat{A}_{i,j}^B V_{i,j}^B] \quad (V1)$$

$$\hat{V}_j^E = \left\{ (1 - \beta_E) (\hat{C}_j^E)^{1-1/v_E} + \beta_E \left(E_{s_{i,j}|s_j} [(V_{i,j}^E)^{1-\sigma_E}] \right)^{\frac{1-1/v_E}{1-\sigma_E}} \right\}^{\frac{1}{1-1/v_E}} \quad (V2)$$

$$\hat{V}_j^S = \left\{ (1 - \beta_S) (\hat{C}_j^S)^{1-1/v_S} + \beta_S \left(E_{s_{i,j}|s_j} [(V_{i,j}^S)^{1-\sigma_S}] \right)^{\frac{1-1/v_S}{1-\sigma_S}} \right\}^{\frac{1}{1-1/v_S}} \quad (V3)$$

Then, update the endogenous state variables of the next period:

$$A_{ij}^B = \hat{A}_j^B = (1 - \sigma) \Omega_D(\hat{\omega}_j^*) A_j^B + \hat{I}_j^B \quad (N1)$$

$$B_{ij}^F = \hat{B}_j^F \quad (N2)$$

$$W_{ij}^S = D_j^S + \Omega_B(\hat{\omega}_j^\#) \hat{B}_j^S \quad (N3)$$

$$K_{ij}^F = (1 - \delta_K) \Omega_D(\hat{\omega}_j^*) K_j^F + \hat{X}_j \quad (N4)$$

Lastly, check the variables $\widehat{\mathbf{C}}_C^{m+1} = \{\hat{d}_j^B, \hat{C}_j^E, \hat{C}_j^S, \hat{q}_j^A, \hat{p}_j^B, \hat{w}_j^b, \hat{r}_j^f, A_{ij}^B, B_{ij}^F, W_{ij}^S, K_{ij}^F\}$ with $\widehat{\mathbf{C}}_C^m$ for each point of j . If $\Delta_C = |\widehat{\mathbf{C}}_C^{m+1} - \widehat{\mathbf{C}}_C^m| \leq Tol_C$, then stop the iteration and take $\widehat{\mathbf{C}}_C^{m+1}$ as approximate solution; otherwise, continue the iteration to period $m + 1$ to update $\widehat{\mathbf{C}}_C^{m+1} = D \times \widehat{\mathbf{C}}_C^m + (1 - D) \times \widehat{\mathbf{C}}_C^{m+1}$ as the initial guess for iteration $m + 1$ and go back to Step 2, until the condition $\Delta_C \leq Tol_C$ is satisfied.

Step 3.

Once obtained the matrices $\widehat{\mathbf{C}}_C^{m+1}$, I can start the simulation by presenting an initial set of endogenous variables $s_0 = \{A_0^B, B_0^F, W_0^S, K_0^F\}$ and a series of generated exogenous shocks $[Z_t, s_t]$ for $T = T_{ini} + T_S$ periods. Record the calculated results using $\widehat{\mathbf{C}}_C^{m+1}$ for each period and proceed the simulation following the generated path of exogenous shocks until reaching the period T . To remove the dependency of the initial state s_0 , I discard the results of the first T_{ini} periods but keep, and report, these of the last T_S periods. I keep the same path of exogenous shocks for all tests to minimise the bias of sampling.

C.2. Simulation Implementation

Accuracy of the solution. I perform two types of checks to assess the quality of my simulation. First, I verify that all the endogenous state variables are within the defined grid bounds. If the simulation exceeds the boundaries, I will expand the grid bounds where they are violated. Second, I compute relative errors of (E1)-(E14) of each computed point j . Take (E1) as an example, the relative error is calculated as:

$$RE_{E1,j} = 1 - \frac{1}{\hat{q}_j^A} \left(\hat{\lambda}_j^{AB} F_A + E_{s_{i,j}|s_j} \{ \hat{\Lambda}_{i,j}^E [(1 - \tau_F + q_{i,j}^A) \Omega_D(\omega_{i,j}^*) + (1 + q_{i,j}^A) f_\omega(\omega_{i,j}^*) \mathcal{A}_{i,j}] \} - 1 \right)$$

These simulation errors are small if the simulated path visits exactly at (or close to) one of the discretized grid points; however, the errors will be large if the simulated path more frequently visits the points that are undefined, i.e. the points that are between the grid points. If the errors of some equations are too large to be ignored, I will add more points to the relevant endogenous variables and repeat the simulation.

Solutions of the system of equations. I solve system of these nonlinear equations for each point in the state space using a nonlinear equation solver (MATLAB's `fsolve`). Regarding the Kuhn-Tucker conditions in the equations, I conduct the following transformation, use (E1) as an example, to minimise the simulation bias from the slackness conditions with constraints.

$$\begin{aligned} \hat{q}_j^A &= \hat{\lambda}_j^{AB} F_A + E_{s_{i,j}|s_j} \{ \hat{\Lambda}_{i,j}^E [(1 - \tau_F + q_{i,j}^A) \Omega_D(\omega_{i,j}^*) + (1 + q_{i,j}^A) f_\omega(\omega_{i,j}^*) \mathcal{A}_{i,j}] \} - 1 \\ (\text{hp}) \hat{p}_j [1 - (1 - \tau_F) \delta_K] \Omega_D(\hat{\omega}_j^*) \hat{K}_j^F - F_A \hat{A}_j^F - F_B \hat{B}_j^F - \hat{\lambda}_j^{AB} &= 0 \end{aligned}$$

Define a variable kt_j and two functions of this variable, such that $\hat{\lambda}_j^{AB,+} = \max\{0, kt_j\}$ and $\hat{\lambda}_j^{AB,-} = \max\{0, -kt_j\}$. Insert these two terms to replace $\hat{\lambda}_j^{AB}$ in the above equations to obtain:

$$\begin{aligned} \hat{q}_j^A &= \hat{\lambda}_j^{AB,+} F_A + E_{s_{i,j}|s_j} \{ \hat{\Lambda}_{i,j}^E [(1 - \tau_F + q_{i,j}^A) \Omega_D(\omega_{i,j}^*) + (1 + q_{i,j}^A) f_\omega(\omega_{i,j}^*) \mathcal{A}_{i,j}] \} - 1 \\ (\text{hp}) \hat{p}_j [1 - (1 - \tau_F) \delta_K] \Omega_D(\hat{\omega}_j^*) \hat{K}_j^F - F_A \hat{A}_j^F - F_B \hat{B}_j^F - \hat{\lambda}_j^{AB,-} &= 0 \end{aligned} \quad (\text{KT})$$

Thus, when $kt_j > 0$, then $\hat{\lambda}_j^{AB,+} > 0$ and $\hat{\lambda}_j^{AB,-} = 0$, which means the constraint binds and thus $\hat{\lambda}_j^{AB,+}$ takes on the value of the Lagrange multiplier. Alternatively, when $kt_j < 0$, then $\hat{\lambda}_j^{AB,+} = 0$ and $\hat{\lambda}_j^{AB,-} = -kt_j$, which means the constraint is non-binding and $\hat{\lambda}_j^{AB,-}$ can take on any value, i.e. kt_j , to make (KT) hold. This transformation method is pioneered by Judd, Kubler and Schmedders (2002) and is followed by Elenev *et al.* (2018). Additionally, there are some parameters being assumed to be

positive, such as \hat{C}_j^E, \hat{C}_j^S . To let the solver try correct values for these parameters, I replace $\log \hat{C}_j^E, \log \hat{C}_j^S$ with their original value to ensure that the positive-defined parameters are only arbitrarily small but not negative.

Grid configuration. The grids of exogenous shocks are simple to be determined following appropriate discretisation method, while grids of endogenous state variables are relatively hard to be pinned down as these grids are expected to cover all the ergodic realization and are finer enough to minimise the simulation errors. The grid points in the simulation are as follows:

Z: [0.96,0.98,1.00,1.02,1.04]. This exogenous shock is discretised into a 5-state Markov chain, using the Rouwenhorst (1995) method, with the transition matrix:

$$\begin{bmatrix} 0.5220 & 0.3685 & 0.0975 & 0.0115 & 0.0005 \\ 0.0921 & 0.5708 & 0.2850 & 0.0493 & 0.0029 \\ 0.0163 & 0.1900 & 0.5875 & 0.1900 & 0.0163 \\ 0.0029 & 0.0493 & 0.2850 & 0.5708 & 0.0921 \\ 0.0005 & 0.0115 & 0.0975 & 0.3685 & 0.5220 \end{bmatrix}$$

s:[0.095, 0.175]

A: [0.30 0.40 0.50 0.60 0.70]

B^F : [-0.20 0.001 0.20 0.25 0.30]

W^S : [0.25 0.33 0.36 0.42 0.48]

K^F : [1.50 1.90 2.10 2.30 2.50]

The transition matrix of states is:

$$T^\omega = \begin{pmatrix} 0.80 & 0.20 \\ 0.36 & 0.64 \end{pmatrix}.$$

This amounts to 6,250 points of the state grids.