How Large are Pre-Default Costs of Financial Distress? Estimates from a Dynamic Model*

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September 2020

Abstract

We estimate the costs of financial distress prior to default (pre-default costs) separately from the loss incurred at default (the loss given default) using a dynamic trade-off model of capital structure. We document that pre-default costs are on average equal to 6.5% of firm value per year. We show that pre-default costs account for a large fraction of total distress costs, approximately 68.5%. Last, we show that the expected pre-default costs of financial distress vary significantly across industries with a range between 3.75% and 8.75%, and are higher for firms with highly tangible assets.

Keywords: Dynamic Capital Structure, Financial Distress, Structural Estimation

JEL Classification: G30, G32, G33

*We would like to thank Alex Corhay, Tom McCurdy, Kristian R. Miltersen (discussant), Jincheng Tong, Fabrice Tourre (discussant), Wenyu Wang (discussant), and seminar participants at the University of Toronto, European Finance Association, Midwest Finance Association, Northern Finance Association meetings for helpful comments and suggestions. All errors are our own.

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1 Introduction

How much value do firms lose because of financial distress? Davydenko et al. (2012) and Korteweg (2010) show that the average distress costs are approximately 15-30% for firms that are in or near default.[1] However, firms experience costs of financial distress prior to default (pre-default costs) as discussed in Titman (1984).[2] Elkamhi, Ericsson, and Parsons (2012) use a calibration exercise to show that pre-default costs can greatly increase the net present value of financial distress costs, and help match the observed leverage ratios. Despite the fact that pre-default costs have been shown to be important, there is limited empirical evidence on their magnitude. Some studies analyzed ex-post costs of financial distress documenting, for example, a drop in the sale price of used car when the carmaker's CDS spread increases (Hortaçsu et al., 2013), fire sales of aircrafts (Pulvino, 1998) and inability to respond to a competitor's entry in the casino business (Cookson, 2017). However, it has been proven to be difficult to find comparable empirical designs to quantify the ex-ante (expected) costs of financial distress for the average firm, and assess the cross-sectional variation amongst industries and firms with different characteristics. Our goal is to fill this gap.

Specifically, the purpose of this paper is to (1) quantify pre-default costs *jointly* with the loss given default (i.e. the expected loss at the time of default), (2) analyze the bias in the estimates of costs of financial distress if pre-default costs are omitted, (3) analyze how such costs vary amongst different industries and firm's characteristics, and (4) provide an alternative view to the selection bias discussed in Glover (2016). To answer the above questions, we first develop a dynamic trade-off model that accounts for pre-default costs and dynamic leverage.[3] We then fit our model to the data. To the best of our knowledge, this is the first study that jointly estimates both pre-default costs of financial distress and the loss given default.

[1] Davydenko et al. (2012) estimate that the average costs of default are approximately 21.7% using a sample of 175 firms that defaulted between 1997 and 2010. Korteweg (2010) uses a structural estimation approach to recover the net benefits of leverage for 290 firms between 1994 and 2004, and finds that costs of financial distress are between 15% and 30%. Also, Andrade and Kaplan (1998) estimate that, in a sample of 31 leveraged buyout firms, the default costs are between 10-23%.

[2] For example, there is anecdotal evidence that customers might not buy the products of a highly levered firm because they fear that the firm will not be able to honour warranty, or managers might not focus on the core business because they have to deal with creditors when the firm is in distress. We refer to Hotchkiss et al. (2008) and Senbet and Wang (2012) for excellent reviews of the costs of financial distress.

[3] Our model is similar in spirit to Goldstein et al. (2001) and Elkamhi et al. (2012).

Including pre-default costs of financial distress into a dynamic model of capital structure is important for at least 3 reasons. First, pre-default costs have different implications for corporate policies and probabilities of default compared to the loss given default. For example, Hortaçsu et al. (2013) shows that car makers suffer from a loss of customers if they are perceived as financially distressed. Second, as shown by Chen, Hackbarth, and Strebulaev (2019), including pre-default costs of financial distress allows a trade-off model of capital structure to explain two apparently contradicting empirical puzzles: the negative relationship between probability of default and stock returns (Campbell et al., 2008) and the positive distress risk premium (Friewald et al., 2014). Third, as we discuss below, the inclusion of pre-default costs in trade-off models provides an alternative explanation on the selection bias discussed in Glover (2016).

In our model, we solve for the value of contingent claims in a setting where the firm can experience pre-default costs. Pre-default costs are captured by letting the firm "leak" a percentage of its value during times of financial distress.[4] We estimate our model by fitting it to observed financing choices as well as actual default rates using Simulated Method of Moments (SMM). The identification of our model relies on the different effect that pre-default costs and loss given default have on the behavior of leverage and default probabilities.[5] On the one hand, increasing the loss given default raises the present value of financial distress costs, thus leading the firm to choose a lower optimal leverage each time they issue debt. The lower optimal leverage also implies a *lower* probability of default. On the other hand, an increase in pre-default costs reduces the optimal leverage but it affects the probability of default through the "value leak" which quickly pushes the value of assets towards the default boundary. This leads to a *higher* probability of default, since the firm will be losing value (e.g. loss of customers, loss of talented employees, managers distracted from the core business, etc.) during times of financial distress.

We show that the average pre-default cost expressed as a percentage of cash flows lost during

[4]Elkamhi et al. (2012) calibrate a similar model to show the qualitative material effect of pre-default costs of financial distress on trade-off models of capital structure. Our aim in this paper is to estimate jointly the loss given default and pre-default costs of financial distress. The dynamic of leverage is modeled similarly to Goldstein et al. (2001), Chen (2010) and Chen et al. (2019).

[5]In our model, probability of default is equal to default rate. We will use these two terms interchangeably but it is worth noting that our empirical moment is calibrated to default rates.

times of financial distress is 6.5% per year. The estimated average loss given default is 22.4%, a value that is close to empirical evidence (Korteweg, 2010; Davydenko et al., 2012). We find that pre-default costs account 68.5% of the total costs of financial distress (loss given default plus pre-default).

Using the estimates from our model, we provide an alternative explanation for the selection bias discussed in Glover (2016), which states that defaulted firms have a significantly lower loss given default (LGD) than the average firm (i.e. LGD for defaulted firms is 25% vs. LGD of 45% for non-defaulted firms). Our results show that the timing of financial distress costs also causes a selection bias because firms that end up defaulting have already experienced pre-default costs of financial distress compared to non-defaulted ones. Therefore, measuring financial distress costs on a sample of defaulted firms leads to a bias: we would only measure the loss given default and would not capture the (large) pre-default costs that firms experienced before defaulting. Our alternative explanation does not invalidate the channel proposed by Glover (2016) but it highlights an additional mechanism that can co-exist with it. To this end, we provide simulations of our model and show that, when we take into account for pre-default costs of financial distress, the selection bias still exists but its magnitude is more moderate. Specifically, we show that the LGD for defaulted firms is slightly lower than the LGD for non-defaulted firms (i.e. LGD for defaulted is 18.05% vs. LGD of 22.44% for non-defaulted firms).

Next, our findings shed light on the bias of estimating the loss given default in a model that ignores the costs of financial distress experienced prior to default. Understanding this bias has practical importance because of the widespread practice of calibrating trade-off models to gauge insights of firms' responses to a policy change or to evaluate the behaviour of credit spreads, leverage and probability of default.[6] To quantify this bias, we estimate the model again, but we fix the pre-default costs to zero. We then compare the results from this exercise to the ones from the model that includes pre-default costs. While the model with pre-default costs requires an average loss given default of 22.4%, the same model where we fix pre-default costs to zero needs an average

[6]Examples of papers that use a calibration and/or estimation of structural models to study various phenomena include Huang and Huang (2012), Schaefer and Strebulaev (2008), Du, Elkamhi, and Ericsson (2018), Feldhütter and Schaefer (2018) and Bai, Goldstein, and Yang (2020).

loss given default of 55.4% in order to fit both leverage and default probabilities.[7] Without pre-default costs, the model needs a loss at default which is approximately 2.5 times higher than in the model with pre-default costs. This difference suggests that omitting pre-default costs from trade-off models can lead to a significant bias when one calibrates them to the empirically observed values of loss at default.[8]

In addition to measuring the average pre-default costs of financial distress, we also study cross-sectional variation of pre-default costs across industries and firms with different asset's tangibility. We find that there is a large cross-sectional variation in the estimates of pre-default costs and loss given default across industries. The estimate of pre-default costs ranges from 3.75% to 8.75% while the loss given default varies between 10.3% and 36.5%. We find that the average pre-default cost amongst industries is 6.35%, which is close to the average value of 6.5% that we estimated for the average firm. We also estimate our model on sub-samples of firms split by the tangibility of assets. Consistent with the intuition that tangible firms should recover more than intangible firms in case of default (Elkamhi, Jacobs, and Pan, 2014), we find that firms with highly tangible assets have a loss given default that is considerably lower than intangible firms (12.826% vs. 54.335%). Our results also show that intangible firms have lower pre-default costs compared to tangible firms (2.137% vs. 7.352%).

Empirically we do not observe when firms start experiencing pre-default costs of financial distress. In our main estimations, both for the average firm and for sub-samples split by industries, we assume that firms start incurring pre-default costs when their cash flows fall below their required coupon payments.[9] It is reasonable to expect that firms experience some sort of financial distress when their cash flows are not enough to cover interest expenses, and our choice is consistent with

[7]Glover (2016) estimates the expected loss given default to be approximately 45% without requiring the model to match default probabilities. In our estimation, we require our model to simultaneously match the volatility of equity, leverage and probability of default. To compare our estimation results to those in Glover (2016), we estimate a version of our model where (i) pre-default costs are omitted and (ii) we fit it only to leverage and volatility of equity (not the probability of default). In this specification, our estimated loss given default is 49.818%, a value that is consistent with Glover (2016).

[8]We also explore the effect that pre-default costs have on credit spreads. When fitted to the same empirical data (including leverage and objective default probabilities), a model with pre-default costs generates a credit spread that is 16.8% larger than in a model without pre-default costs.

[9]Since leverage (and hence coupon payments) is dynamic in our model, the model produces dynamic distress boundaries.

the assumptions made in previous studies (Andrade and Kaplan, 1998; Titman and Tsyplakov, 2007). However, it is possible to specify alternative definitions. For example, firms may start incurring pre-default costs from the moment they get downgraded to a speculative grade status. There is even anecdotal evidence that some subindustries (e.g. insurance companies) are concerned about loss of customers following a one notch downgrade even when their current rating is in the A's category. As a robustness test, we study to what extent our assumption affects the results by estimating our model under alternative assumptions regarding the onset of financial distress.[10] Specifically, we analyze how the ratio of pre-default costs over total distress costs varies across different specifications. Our results show that the percentage of pre-default costs as a fraction of the total (henceforth denoted by $\%\,PDC$) is approximately the same if we assume that financial distress starts when cash flows are twice as large as coupon payments. Indeed, we estimate our model under this assumption and we find that the $\%\,PDC$ is 65.97%, which is very close to the 68.5% which we estimated under the assumption that financial distress starts when cash flows fall below required coupon payments.

Methodologically, this paper is close to the structural estimation literature that uses SMM to study various questions in corporate finance.[11] For example, Hennessy and Whited (2005) and Hennessy and Whited (2007) rely on this methodology to estimate a discrete-time dynamic capital structure model and show that it is able to generate several empirical facts (e.g. path-dependency of leverage and its relation with liquidity). Nikolov and Whited (2014) also use the same methodology to estimate a dynamic model and show that agency conflicts between managers and shareholders enhance the ability of the model to explain the dynamics of cash. We contribute to this literature by estimating the costs of financial distress incurred prior to default separately from the costs incurred at the time of default.

The rest of the paper is structured as follows. Section 2 develops a trade-off model of capital

[10]It would also be informative to jointly estimate the "distress boundary" (i.e. time when financial distress starts) to understand when firms expect to incur pre-default costs and whether there is cross-sectional variation between industries and firms with different characteristics. However, our model cannot simultaneously identify both the distress boundary and the level of pre-default costs.

[11]See Strebulaev and Whited (2012) for a comprehensive review of papers that use a structural estimation approach in Corporate Finance.

structure which allows for pre-default costs of financial distress. Section 3 presents the comparative statics of the model, while Section 4 presents the details of the identification and the structural estimation methodology. Section 5 discusses the results of the estimation and robustness tests. Section 6 concludes.

2 Model

This section presents a dynamic trade-off model that incorporates costs of financial distress incurred before the firm defaults. Firms in our model are exposed to both systematic and idiosyncratic risks. We let the aggregate economy's operating cash flows (i.e. EBIT) follow a Geometric Brownian Motion

$$\frac{dX_{At}}{X_{At}} = \mu_A^{\mathcal{P}} dt + \sigma_A dB_t^{A,\mathcal{P}} \tag{1}$$

where $\mu_A^{\mathcal{P}}$ is the expected growth rate of the economy under the physical probability space, σ_A is the volatility parameter,and $dB_t^{A,\mathcal{P}}$ is a standard Brownian Motion. A common assumption in the literature (Leland, 1994; Abel, 2018) is that firms experience a deadweight loss at the moment they default. Creditors receive a fraction $1 - \alpha$ of the continuation value of the firm in the event of default, so the total social cost is a fraction α of the continuation value, where $\alpha \in [0, 1]$.

We do not include time-varying macroeconomic conditions in the estimation of our model which, in calibration exercises, have been shown to be an important component of this class of models to explain observed average leverage ratios and credit spreads (Bhamra et al., 2009; Chen, 2010; Bhamra et al., 2010; Chen et al., 2009; Elkamhi et al., 2020). Our choice is driven by the need to keep our model parsimonious in order to estimate it to the data. Furthermore, our aim is not to generate reasonable covariance between cash flows and pricing kernel and hence large default and risk premia. Our goal is to provide a robust estimate of pre-default costs of financial distress using a parsimonious model.[12]

Following Elkamhi et al. (2012), we relax the assumption that default costs are incurred exclu-

[12]We compare our model with a model that includes macroeconomic risk in Appendix and provide further details in Section 5 where we discuss the results of our estimation.

sively as a lump sum when firms default. In our model, firms experience a deadweight loss at the moment they default (i.e. a proportion α of firm's assets is lost at default), and "leak" a fraction of their value when EBIT drops below the distress boundary X_D.[13] More specifically, a firm's EBIT is governed under the physical probability measure $\mathcal{P}$ by the process

$$\frac{dX_t}{X_t} = \begin{cases} \left(\mu + \beta(\mu_A^{\mathcal{P}} - r)\right) dt + \beta\sigma_A dB_t^{A,\mathcal{P}} + \sigma_F dB_t^F & \text{for } X_B \leq X_D \leq X_t \\ \underbrace{\left(\mu + \beta(\mu_A^{\mathcal{P}} - r)\right) dt}_{\substack{\text{Expected growth} \\ \text{rate without distress}}} + \underbrace{\beta\sigma_A dB_t^{A,\mathcal{P}} + \sigma_F dB_t^F}_{\substack{\text{Systematic and} \\ \text{Idiosyncratic shocks}}} \underbrace{-\gamma dt}_{\substack{\text{Pre-default} \\ \text{Costs}}} & \text{for } X_B < X_t < X_D \end{cases} \tag{2}$$

where μ is the firm specific (risk-neutral) expected growth rate of EBIT, β is the exposure to market risk, γ is the constant rate at which the firm loses value when it is in financial distress (i.e. $X_B < X_t < X_D$), r is the constant risk-free rate, dB_t^F is a standard Brownian Motion (independent from $dB_t^{A,\mathcal{P}}$) that governs the idiosyncratic firm-specific shocks, σ_F is the idiosyncratic volatility of the firm-specific shocks, X_D and X_B are the distress and default thresholds, respectively. As it will be explained in Section 2.3, both the default and distress thresholds are endogenous and time-varying.

In Figure 1 we plot two EBIT paths using Equation (2). The two paths have been generated using the same random seeds and parameters except for the level of pre-default costs γ. The blue solid line depicts the EBIT path for a firm that does not suffer any pre-default costs (i.e. $\gamma = 0$). The red dashed line shows the EBIT path for a firm that suffers pre-default costs equal to a loss of 2% per year of the value of its assets during financial distress (i.e. $\gamma = 2\%$).

[Insert Figure 1 here]

The key intuition from Figure 1 is that pre-default costs strongly affect the probability of default. In Section 4.2 we confirm that this intuition holds even when we allow the firm to choose its optimal leverage given the parameters of the model. More specifically, we show that two firms that differ

[13] Equation (2) shows that pre-default costs are proportional to the value of X_t. Since the value of assets is a monotonically increasing function of X_t (as shown in Equation (4)) then pre-default are also proportional to the value of assets.

only in the parameter values for α and γ, and optimally choose identical leverage would exhibit different probabilities of default.

Under the risk-neutral measure $\mathcal{Q}$, the firm's EBIT is governed by the process

$$\frac{dX_t}{X_t} = \begin{cases} \mu\, dt + \sigma_X dB_t & \text{for } X_B \leq X_D \leq X_t \\ (\mu - \gamma)dt + \sigma_X dB_t & \text{for } X_B < X_t < X_D \end{cases} \tag{3}$$

where $\sigma_X = \sqrt{(\beta\sigma_A)^2 + \sigma_F^2}$ is the total volatility of the firm. We provide the derivation of Equation (3) in Appendix A.

Earnings are taxed at a constant rate τ^c. Therefore, firms have an incentive to issue debt to benefit from its tax-shield. As in Leland (1994), we ensure a time-homogeneous setting by assuming that firms issue an infinitely lived debt which pays a continuous flow of coupons C. To allow for dynamic leverage, debt is callable and issued at par. Following Goldstein et al. (2001), firms can adjust their capital structure upwards by incurring a proportional cost λ but they cannot reduce their debt downward. The firm's initial debt structure remains fixed until either the firm goes default or calls its debt at par and restructures with newly issued debt. The proceeds from debt issuance are distributed proportionally to shareholders[14]. The personal tax rate on dividends is τ^e and coupon payments' tax rate is τ^d. We define $\tau = 1 - (1 - \tau^c)(1 - \tau^e)$ as the effective tax rate including both corporate and personal taxes. All investors face the same tax rates.

The value of (after-tax) unlevered assets is given by

$$V(X_t) = \mathbb{E}^{\mathcal{Q}}\left[\int_t^{\infty} (1-\tau)X_{is}e^{-rs}ds\right] = (1-\tau)\frac{X_t}{r-\mu} \tag{4}$$

2.1 Pricing of Debt and Equity

Before discussing optimal capital structure decisions, we compute the value of debt and equity for fixed levels of coupon (C), distress (X_D) and default (X_B) thresholds as well as the restructuring

[14] This is a standard assumption when our goal is not to examine the dynamics of cash while it would be restrictive when studying precautionary saving and cash dynamics as shown, for example, in Hennessy and Whited (2005) and Nikolov and Whited (2014).

boundary (X_U). The value of EBIT at the time when the firm makes its decision if X_0. When the firm's EBIT reaches X_U, the firm retires its previously issued debt at par and it issues new one. In Section 2.2, we discuss the optimal capital structure decision made by the firm conditional on the limited liability constraint. In Section 2.3 we describe the endogenous distress boundary X_D.

2.1.1 Net Income

We start by computing the value of a claim on net income, which represents the cash flows continuously accruing to shareholders at each time t, $(1-\tau)(X_t - C)$. We define a refinancing cycle as the period of time over which the firm's capital structure does not change. That is, the period of time when X_t, remains between X_U and X_B. Formally, let $T = \min\{T^U, T^B\}$ where $T^U = \inf\{t > 0 : X_t \geq X_U \text{ and}, \forall\, s < t\,, X_s > X_B\}$ is the first time EBIT reaches the restructuring boundary conditional on not having hit the default threshold, and $T^B = \inf\{t > 0 : X_t \leq X_B$ and $\forall\, s < t\,, X_s < X_U\}$ is the first time X_t reaches the default threshold conditional on not having hit the restructuring boundary. The refinancing cycle is defined as the period of time until time T, $\{t : 0 < t < T\}$.

Consider a claim over net income over one refinancing cycle.[15] This claim depends on the level of EBIT, X_t, and the coupon paid on the firm's outstanding debt. Intuitively, this claim is simply the expected net present value of the cash flows accrued to shareholders between time t and T (the first time the firm changes its capital structure either by defaulting or restructuring its debt). Its value is

$$\mathbf{n}(X_t, C) = \begin{cases} \mathbb{E}^{\mathcal{Q}}\left[\int_t^T (1-\tau)(X_s - C)e^{-r(s-t)}ds \mid X_D \leq X_t \leq X_U\right] \\ \mathbb{E}^{\mathcal{Q}}\left[\int_t^T (1-\tau)(X_s - C)e^{-r(s-t)}ds \mid X_B < X_t < X_D\right] \end{cases} \tag{5}$$

Equation (5) considers two cases: for $X_D \leq X_t \leq X_U$, the firm does not incur pre-default costs and its EBIT X_t is governed by the process described in the top expression in Equation (2); for $X_B < X_t < X_D$, the firm experiences pre-default costs and it loses a fraction γ of its EBIT per period of time as described in the bottom expression in Equation (2).

[15] We follow a notation similar to Morellec et al. (2012).

To simplify the notation, we denote the state when the firm is not distressed, $X_D \leq X_t \leq X_U$, as ND (No Distress state) and the state when it is distressed, $X_B < X_t < X_D$, as DS (Distressed State). Let $\mathbf{p}^U_{ND}(X_t)$ and $\mathbf{p}^U_{DS}(X_t)$ be the present value of \$1 to be received at the time of restructuring, contingent on restructuring occurring before default when the state is ND and DS, respectively. Similarly, let $\mathbf{p}^B_{ND}(X_t)$ and $\mathbf{p}^B_{DS}(X_t)$ be the present value of \$1 to be received at the time of default, contingent on default occurring before restructuring when the state is ND and DS, respectively. It follows that the solutions to Equation (5) can be written as follows

$$\mathbf{n}(X_t, C) = \begin{cases} (1-\tau)\left[\frac{X_t}{r-\mu} - \frac{C}{r} - \mathbf{p}^U_{ND}(X_t)\left(\frac{X_U}{r-\mu} - \frac{C}{r}\right) - \mathbf{p}^B_{ND}(X_t)\left(\frac{X_B}{r-\mu} - \frac{C}{r}\right)\right] \\ (1-\tau)\left[\frac{X_t}{r-\mu} - \frac{C}{r} - \mathbf{p}^U_{DS}(X_t)\left(\frac{X_U}{r-\mu} - \frac{C}{r}\right) - \mathbf{p}^B_{DS}(X_t)\left(\frac{X_B}{r-\mu} - \frac{C}{r}\right)\right] \end{cases} \tag{6}$$

where the expressions for $\mathbf{p}^U_{ND}(X_t)$, $\mathbf{p}^B_{ND}(X_t)$, $\mathbf{p}^U_{DS}(X_t)$, and $\mathbf{p}^B_{DS}(X_t)$ are provided in Appendix B.

To calculate the value of a claim on net income over all refinancing cycles we need an intermediate result. We need to show that, at the time of restructuring T_U, all claims are scaled up by the same proportion $\rho = X_U/X_0$. This feature of the model is known as the scaling property, and it is widely used in this class of models.[16]

The value of net income over all refinancing cycles is equal to the expected net present value of cash flows accrued to shareholders over the entire life of the firm which we can write as follows

$$\mathbf{NI}(X_t, C) = \begin{cases} \mathbf{n}(X_t, C) + \mathbf{p}^U_{ND}(X_t) \cdot \mathbf{NI}(X_U, C_U) & \text{for } X_D \leq X_t \leq X_U \\ \underbrace{\mathbf{n}(X_t, C)}_{\substack{\text{Net Income} \\ \text{over 1 cycle}}} + \underbrace{\mathbf{p}^U_{DS}(X_t) \cdot \mathbf{NI}(X_U, C_U)}_{\substack{\text{NPV of net income} \\ \text{for future cycles}}} & \text{for } X_B < X_t < X_D \end{cases} \tag{7}$$

where X_U is the restructuring boundary that defines when the firm issues new debt and retires the existing one and C_U is the new coupon paid by the firm after having restructured its debt. The scaling property implies that $\mathbf{NI}(X_U, C_U) = \rho\mathbf{NI}(X_0, C)$ and $C_U = \rho C$ where $\rho = X_U/X_0$. At the

[16] See, for example, Goldstein et al. (2001) and Morellec et al. (2012). A formal proof that shows the validity of the scaling property in this type of models can be found in Goldstein et al. (2001), pages 509-511.

time of debt issuance, the value of net income over all refinancing cycles simplifies to

$$\mathbf{NI}(X_0, C) = \begin{cases} \frac{\mathbf{n}(X_0,C)}{1-\rho\cdot\mathbf{p}^U_{ND}(X_0)} & \text{for } X_D \leq X_0 \leq X_U \\ \frac{\mathbf{n}(X_0,C)}{1-\rho\cdot\mathbf{p}^U_{DS}(X_0)} & \text{for } X_B < X_0 < X_D \end{cases} \tag{8}$$

2.1.2 Debt

The value of a claim on coupon payments over one refinancing cycle includes the expected present value of all coupons to be received until the firm either restructures its debt or goes default plus the recovery value in the event of default. Accounting for taxes, we can write the value of this claim as follows

$$\mathbf{d}(X_t, C) = \begin{cases} \mathbb{E}^{\mathcal{Q}}\left[\int_t^T (1-\tau^d)Ce^{-r(s-t)}ds + (1-\alpha)\cdot V(X_B)\cdot e^{-rT_B} \mid X_D \leq X_t \leq X_U\right] \\ \mathbb{E}^{\mathcal{Q}}\left[\underbrace{\int_t^T (1-\tau^d)Ce^{-r(s-t)}ds}_{\substack{\text{NPV of coupons} \\ \text{over 1 cycle}}} + \underbrace{(1-\alpha)\cdot V(X_B)\cdot e^{-rT_B}}_{\substack{\text{NPV of unlevered} \\ \text{assets at default}}} \mid X_B < X_t < X_D\right] \end{cases} \tag{9}$$

where α represents the default costs which are proportional to the value of the unlevered assets. Similar to the analysis for net income, Equation (9) contains two cases: the expression above evaluates the value of debt over one refinancing cycle when the firm is not in financial distress ($X_D \leq X_t \leq X_U$, see top expression in Equation (2)) while the one below provides the value of debt when the firm is experiencing pre-default costs ($X_B < X_t < X_D$, see bottom expression in Equation (2)).

We can write the solution to Equation (9) as follows

$$\mathbf{d}(X_t, C) = \begin{cases} \left(1 - \mathbf{p}^U_{ND}(X_t) - \mathbf{p}^B_{ND}(X_t)\right)\frac{(1-\tau^d)C}{r} + \mathbf{p}^B_{ND}(X_t)(1-\alpha)\cdot V(X_B) \\ \left(1 - \mathbf{p}^U_{DS}(X_t) - \mathbf{p}^B_{DS}(X_t)\right)\frac{(1-\tau^d)C}{r} + \mathbf{p}^B_{DS}(X_t)(1-\alpha)\cdot V(X_B) \end{cases} \tag{10}$$

Debt is retired at par when the firm restructures its debt. Assuming that the last time the firm issued debt was at time $t = 0$, the value of the outstanding debt, $\mathbf{D}(X_t, C)$, is equal to the value of debt over one refinancing cycle $\mathbf{d}(X_t, C)$ (provided in Equation (10)) plus the expected net present value of the repayment when the firm retires the debt at par. Since debt is issued at par, the face value of debt is equal to $\mathbf{D}(X_0, C)$. We can write the value of the outstanding debt as follows

$$\mathbf{D}(X_t, C) = \begin{cases} \mathbf{d}(X_t, C) + \mathbf{p}^U_{ND}(X_t) \cdot \mathbf{D}(X_0, C) & \text{for } X_D \leq X_t \leq X_U \\ \mathbf{d}(X_t, C) + \underbrace{\mathbf{p}^U_{DS}(X_t) \cdot \mathbf{D}(X_0, C)}_{\substack{\text{NPV of repayment when} \\ \text{debt is retired at par}}} & \text{for } X_B < X_t < X_D \end{cases} \tag{11}$$

Equation (11) holds for any $X_t \in [X_B, X_U]$ therefore at the time of issuance, when the value of EBIT is X_0, the value of debt is

$$\mathbf{D}(X_0, C) = \begin{cases} \frac{\mathbf{d}(X_0)}{1-\mathbf{p}^U_{ND}(X_0)} & \text{for } X_D \leq X_0 \leq X_U \\ \frac{\mathbf{d}(X_0)}{1-\mathbf{p}^U_{DS}(X_0)} & \text{for } X_B < X_0 < X_D \end{cases} \tag{12}$$

The value of the outstanding debt reflects the value of debt for current debtholders but it does not include the value of debt for future debt issues. To account for new debt issued in the future, we calculate the value of a claim on the coupons that the firm will pay over its entire life (i.e. including the increased coupons following debt restructurings). We denote this claim as $\mathbf{TD}(X_t, C)$. We can write its value as follows

$$\mathbf{TD}(X_t, C) = \begin{cases} \mathbf{d}(X_t, C) + \mathbf{p}^U_{ND}(X_t) \cdot \mathbf{TD}(X_U, C_U) & \text{for } X_D \leq X_t \leq X_U \\ \mathbf{d}(X_t, C) + \mathbf{p}^U_{DS}(X_t) \cdot \mathbf{TD}(X_U, C_U) & \text{for } X_B < X_t < X_D \end{cases} \tag{13}$$

where X_U is the restructuring boundary and C_U is the new coupon paid by the firm after having restructured its debt. The scaling property implies that $\mathbf{TD}(X_U, C_U) = \rho\mathbf{TD}(X_0, C)$ where $\rho = X_U/X_0$. It follows that the value of total debt at X_0 is

$$\mathbf{TD}(X_0) = \begin{cases} \frac{\mathbf{d}(X_0)}{1-\rho\cdot\mathbf{p}^U_{ND}(X_0)} & \text{for } X_D \leq X_0 \leq X_U \\ \frac{\mathbf{d}(X_0)}{1-\rho\cdot\mathbf{p}^U_{DS}(X_0)} & \text{for } X_B < X_0 < X_D \end{cases} \tag{14}$$

2.1.3 Adjustment Costs

We assume that the firm incurs adjustment costs each time it changes its capital structure. These adjustment costs are equal to a percentage λ of the debt being issued. At the time of refinancing, the total value of the adjustment costs are equal to the flotation costs for the debt currently being issued plus the expected adjustment costs that the firm will pay for subsequent debt issues. We can write the value of the adjustment costs at the time of issuance as follows

$$\mathbf{AC}(X_0, C) = \begin{cases} \lambda \mathbf{D}(X_0, C) + \mathbf{p}_{ND}^U(X_0) \cdot \mathbf{AC}(X_U, C_U) & \text{for } X_D \leq X_0 < X_U \\ \underbrace{\lambda \mathbf{D}(X_0, C)}_{\substack{\text{Adjustment cost} \\ \text{for current} \\ \text{debt issued}}} + \underbrace{\mathbf{p}_{DS}^U(X_0) \cdot \mathbf{AC}(X_U, C_U)}_{\substack{\text{NPV of Adjustment} \\ \text{costs for future} \\ \text{debt issues}}} & \text{for } X_B < X_0 < X_D \end{cases} \tag{15}$$

where X_U is the restructuring boundary and C_U is the new coupon paid by the firm after having adjusted its capital structure. As for any claim in our model, Equation (15) differentiates between the time when the firm is not distressed ($X_D \leq X_0 < X_U$) and when it is distressed and is incurring pre-default costs ($X_B < X_0 < X_D$).

By the scaling property, $\mathbf{AC}(X_U, C_U) = \rho \mathbf{AC}(X_0, C)$. We can simplify Equation (15) as follows

$$\mathbf{AC}(X_0, C) = \begin{cases} \frac{\lambda \mathbf{D}(X_0, C)}{1 - \rho \cdot \mathbf{p}_{ND}^U(X_0)} & \text{for } X_D \leq X_0 < X_U \\ \frac{\lambda \mathbf{D}(X_0, C)}{1 - \rho \cdot \mathbf{p}_{DS}^U(X_0)} & \text{for } X_B < X_0 < X_D \end{cases} \tag{16}$$

After having issued debt, the total value of the adjustment costs is equal to the expected adjustment costs that the firm will incur over its entire life which we can write as follows

$$\mathbf{AC}(X_t, C) = \begin{cases} \mathbf{p}_{ND}^U(X_t) \rho \mathbf{AC}(X_0, C) & \text{for } X_D \leq X_t < X_U \\ \mathbf{p}_{DS}^U(X_t) \underbrace{\rho \mathbf{AC}(X_0, C)}_{\substack{\mathbf{AC}(X_U, C_U) \\ \text{NPV of future} \\ \text{adjustment costs}}} & \text{for } X_B < X_t < X_D \end{cases} \tag{17}$$

2.1.4 Firm and Equity Value

At any time t, the levered asset value of the firm, $\mathbf{v}(X_t, C)$, is the sum of the present value of cash flows to shareholders plus cash flows to all debtholders minus the net present value of the adjustment costs. It is given by

$$\mathbf{v}(X_t, C) = \underbrace{\mathbf{NI}(X_t, C)}_{\substack{\text{NPV of claim} \\ \text{on net income}}} + \underbrace{\mathbf{TD}(X_t, C)}_{\substack{\text{NPV of claim} \\ \text{on total debt}}} - \underbrace{\mathbf{AC}(X_t, C)}_{\substack{\text{Adjustment} \\ \text{costs}}} \tag{18}$$

Equity is a residual claim and its value is the difference between the total value of the firm $\mathbf{v}(X_t, C)$ and the value of current debt $\mathbf{D}(X_t, C)$:

$$\mathbf{E}(X_t, C) = \mathbf{v}(X_t, C) - \mathbf{D}(X_t, C) \tag{19}$$

2.2 Optimal Policies

We assume that financing decisions are made by shareholders, and for simplicity we abstract from conflicts of interests between firms' managers and shareholders. When the firm decides its capital structure, it faces a trade-off between the tax benefits of debt and the expected costs of financial distress (both pre-default costs and deadweight loss at the time of default). Since proceeds from debt issuance are distributed proportionally to shareholders and the debt is fairly priced due to complete and arbitrage-free markets, shareholders' objective is to maximize the value of the firm.

Shareholders choose the optimal default threshold X_B by maximizing the value of equity. This is equivalent to applying the smooth-pasting condition as in Leland (1994) to find the optimal default threshold X_B such that

$$\left.\frac{\partial \mathbf{E}(x, C)}{\partial x}\right|_{x=X_B} = 0 \tag{20}$$

The total value of the firm $\mathbf{v}(\cdot)$ is a function of the amount of debt that is issued which affects the coupon level C, and the restructuring boundary X_U which affects how often the firm restructures its debt. At time 0 shareholders choose the coupon C and the restructuring boundary X_U to maximize the ex-ante value of the firm. Formally, shareholders solve the following problem

$$\max_{C,X_U} \mathbf{v}(\cdot) \quad \text{subject to Equation (20)} \tag{21}$$

The above problem can be solved using standard numerical procedures. For any given set of parameters, solving the problem in Equation (21) yields the optimal coupon C^*, the optimal restructuring boundary X_U^* and the optimal default threshold X_B^*.

2.3 Distress Boundary

The role of the distress boundary (X_D) is to act as a trigger start for pre-default costs of financial distress. The higher X_D, the sooner firms experience pre-default costs. Figure 1 shows that firms can emerge from distress when the value of their EBIT improves and becomes higher than the distress threshold X_D. The decision regarding the distress threshold is important. If X_D is set too low (e.g. close to the default threshold X_B), then pre-default costs would not have a strong effect on financing decisions since they are experienced when the probability of default is already extremely high. If X_D is set too high, pre-default costs would play a crucial role in deciding the optimal capital structure for the firm and would overshadow the costs at default α.

Following Titman and Tsyplakov (2007), we define the onset of financial distress to be when X_t drops below the coupon C paid on outstanding debt. We argue that this choice is reasonable. As an example, one of the most used covenants on loans is the minimum interest coverage ratio which imposes a minimum ratio of interest expenses over operating cash flows that firms need to maintain. Covenants on loans are such that the average interest coverage ratio for firms in the U.S. is 2.5 (Greenwald, 2019) while our assumption implies that firms start experiencing distress costs when their interest coverage ratio drops below one.[17] Therefore, our choice of distress boundary is coherent with the assumption that firms start experiencing distress costs when they are already in violation of the interest coverage ratio covenant, and it is reasonable to assume that a firm experiences some financial distress costs when they violate a covenant. For example, Chava and

[17] The average BBB firm has an interest coverage ratio of 4.2 (Kaplan and Zingales, 2000).

Roberts (2008) show a sharp decline in investment following a covenant violation, and Nini, Smith, and Sufi (2012) document a reduction in acquisitions and a decrease in payouts to shareholders.

Last, by linking the distress threshold to required coupon payments, our choice leads to an endogenous distress boundary because the optimal coupon is an endogenous decision that the firm makes as shown in Equation (21). The higher the coupon chosen by the firm, not only the firm will have a higher leverage but it will also start experiencing financial distress earlier. This has strong implications for leverage and default probabilities which we discuss in the next section. In Section 5.3, we estimate our model using different distress boundaries to show the effect that this assumption has on our estimates.

3 Comparative Statics

We now examine the predictions of our model for financing decisions and provide a first look at the importance of pre-default costs in capital structure choice. In Table 1 we report the comparative statics describing the effects of the main parameters of the model on: (i) target leverage which is the optimal leverage chosen by the firm at the time of issuing debt; (ii) the leverage at restructuring, which is informative of the restructuring boundary X_U; (iii) the leverage at the time the firm becomes distressed, which is informative of the distress threshold X_D; (iv) the recovery rate for debtholders at the time of default.

We set the base case parameters as follows. Using the estimates in Graham (1999, 2000), we set the personal tax rate on dividends $\tau^e = 11.6\%$ and the tax rate on interest income $\tau^d = 29.3\%$. The corporate tax rate is set to the maximum marginal tax rate $\tau^c = 35.0\%$. The aggregate earnings growth rate and volatility are set to the average growth rate and volatility of the Net Operating Surplus series from NIPA $\mu_A = 5.84\%$ and $\sigma_A = 9.471\%$ for the period 1990-2013, respectively.[18] The risk-free rate is $r = 2.27\%$. We set the growth rate and idiosyncratic volatility of cash flows to $\mu = 0.5\%$ and $\sigma_F = 14.8\%$. The exposure to market risk parameter is set to $\beta = 0.9$. This calibration implies a total volatility for the firm's EBIT $\sqrt{\beta^2\sigma_A^2 + \sigma_F^2} = 17.08\%$. The loss given

[18] All rates and volatilities reported in this paper are annualized. A detailed description of the data is provided in Section 4.1.

default parameter is $\alpha = 22.44\%$, the pre-default costs are set to $\gamma = 6.5\%$. The proportional adjustment costs is set to $\lambda = 1.0\%$ which is in line with empirical evidence (Kim, Palia, and Saunders, 2008). We set the distress boundary X_D equal to the value of coupon C. We normalize the initial value of operating cash flows $X_0 = \$5.0$. We choose this calibration of our model to match the estimated values that we discuss in Section 5.

Table 1 shows that an increase in the loss given default α lowers the target leverage, the restructuring boundary and the distress threshold. Consistent with Goldstein et al. (2001), an increase in α also lowers the recovery rate for debtholders. An increase in pre-default costs γ has qualitatively a similar effect on target leverage, restructuring boundary and distress trigger. However, we show in Section 4.2 that γ is positively correlated with the probability of default contrary to α. More specifically, an increase in γ has two opposing effects on the probability of default. On the one hand, it increases the probability of default by reducing the firm's EBIT growth rate when it is distressed. On the other hand, an increase in γ makes debt more costly and less attractive for shareholders, thus leading them to choose a lower optimal leverage. Our model shows that the former effect dominates the latter. Also, γ affects the recovery value only through the change in optimal leverage and lower default threshold. The intuition underlying this result is as follows. A lower optimal leverage implies a lower default threshold which lowers the recovery value. An increase in α also leads to a lower leverage and a lower default threshold. However, in addition to this effect, an increase in α also directly lowers the recovery value $(1 - \alpha)$ for debtholders at the time of default.

The rest of Table 1 shows the effect on leverage and recovery rate of EBIT growth rate μ and volatility σ_F, firm's beta β, and adjustment costs λ. The results are consistent with those previously reported in the literature (see, for example, Strebulaev (2007)).

[Insert Table 1 here]

4 Identification and Structural Estimation

We estimate the model parameters using Simulated Method of Moments (SMM). Before presenting the results of our estimations in Section 5, we discuss the empirical data in Section 4.1, and then proceed to discuss the identification of our model in Section 4.2. A detailed description of the SMM methodology is provided in Appendix C.

4.1 Empirical Data

We collect financial statements from Compustat Fundamentals Quarterly (library "compd", file "fundq"). Following the literature (see, for example, Hennessy and Whited (2005, 2007)), we drop financial firms (SIC codes 6000 - 6999), utilities (4900 - 4999), and public administration firms (9000 - 9999). We gather firms' equity returns from the Center for Research in Security Prices (CRSP). We define a firm's quasi-market leverage for quarter t as the book debt at time t ($dlcq_t + dlttq_t$) divided by the sum of book debt and market value of equity ($prccq \times cshoq$). Since our model is developed based on operating cash flows X_t, we define the operating return on assets (Operating ROA) as the ratio between operating profits and lagged total assets. We get the average default rates by rating and year from the Moody's Default & Recovery Database (Moody's DRD). The Moody's DRD dataset contains the average default rate with a horizon of 5 years for the total market as well as by rating categories. We merge these average default rates to the Compustat data as follows. For each firm i with rating R observed in year y, we assign the average default rate for rating R in year y published by Moody's DRD. A detailed description of the variables is provided in Table 2.

Observations with missing total assets, quasi-market leverage, common shares outstanding, closing price or equity returns for the end of the quarter are excluded. Also, we keep firms with total value of assets of at least \$10 million and a market value of equity of at least \$5 million. We winsorize all variables at the 1% level to avoid the influence of outliers. After applying the aforementioned selection criteria, we obtain a panel data set with $101,032$ firm-quarter observations for $N = 4,603$ individual firms between 1990 and 2013 at the quarterly frequency. Our sample stops in 2013 because one of the moments used in our estimation is the probability of default with a horizon of 5 years. For each year t, Moody's collects information on the actual default rate for the subsequent 5 years for the sample of firms in t. Since our raw data stop in 2018, the latest available data for the default rate with a horizon of 5 years is 2013.

[Insert Table 2 here]

For the calibration of the aggregate economy, we use the quarterly aggregate earnings series from NIPA (Series "Net value added of corporate business: Net operating surplus", Table 1.14, Line 8). We calculate the quarterly returns and calibrate the parameters of the aggregate economy, $\mu_A^{\mathcal{P}}$ and σ_A, to the mean and standard deviation of such returns.

Table 3 provides the descriptive statistics for our sample, which are representative of generic samples from Compustat, except that our sample is slightly biased towards large firms. The median firm assets is $1.97 billion.

[Insert Table 3 here]

4.2 Identification

We estimate 5 parameters: the parameter γ that represents the pre-default costs of financial distress, the loss given default α, the expected (risk neutral) growth rate of EBIT μ, the idiosyncratic volatility σ_F, and the exposure to market risk β. The selection of moments used in the SMM estimation is important to ensure that the parameters of interest are identified. Therefore, it is important to choose moments that are a priori informative about the unknown structural parameters. Intuitively, a moment is informative about an unknown parameter if that moment is sensitive to changes in the parameter.

We use six empirical moments for the identification of our model: the operating ROA, the average equity returns, the quasi-market leverage, the excess return of firm's equity with respect to the risk-free rate, the probability of default within 5 years, the variance of equity returns.[19] We estimate the rest of the model parameters (for example, the risk-free rate r) separately and set them equal to the values of the Base Scenario which are set to the values described in Section 3. For each scenario, we simulate the model 5,000 times for a time period of 150 years. We keep only the last 80 quarters of data to remove the effect of the initial conditions. For each simulation, we calculate model moments and then we average across all simulations.

Every moment is affected by all parameters of our model. However, some parameters are more important than others for a particular moment. In Table 4 we describe which moment is most important to identify each parameter using the elasticities of the model moments with respect to each parameter.[20] The elasticity of moment m with respect to parameters p is defined as $\frac{dm/m}{dp/p}$. The elasticities are calculated at the estimated parameter values discussed in Section 5.

The idiosyncratic volatility σ_F is directly related to the variance of equity returns. This can be seen from the diffusion parameter of the process described in Equation (2) and from the strong effect that σ_F has on the variance equity returns with an elasticity of 8.45 as shown in Table 4. The parameter μ is identified mainly through the Operating ROA. As shown in Table 4, μ has the strongest effect on Operating ROA compared to the other estimated parameters. The exposure to market risk (β) is primarily identified by the equity returns. This is intuitive because we expect high beta stocks to have higher expected returns. To confirm that we are able to separately identify both σ_F and β, in Figure 2 we plot equity returns and and their variance as a function of β and σ_F. This figure shows that σ_F is positively related to both variance of equity and equity returns while β is positively related only to equity returns and has almost no effect on the variance of equity.

[Insert Table 4 and Figure 2 here]

Disentangling pre-default costs of financial distress from the loss given default requires us to separately identify the parameters α and γ. In Table 4, we show that both α and γ negatively

[19]We name our moment "probability of default within 5 years" to refer to the average default rate from Moody's DRD which was described in Section 4.1.

[20]This exercise of checking which moments are most affected by each parameter is similar in spirit to Dou, Taylor, Wang, and Wang (2019).

affect leverage which is consistent with other models in the literature (e.g. Elkamhi et al., 2012). The related result is that α has a negative effect on the probability of default while γ is strongly and positively related to the probability of default. Therefore, the combination of leverage and probability of default at a certain horizon should provide enough information to identify α and γ because the two parameters have different effects on the simultaneous behaviour of leverage and probability of default.[21]

Figure 3 provides additional evidence for the identification of α and γ. In Panel A, we show how leverage and probability of default are affected by α. Specifically, we simulate the model 5,000 times for a horizon of 150 years. We keep only the last 80 quarters of data to remove the effect of the initial conditions. For each simulation, we calculate the average Quasi-Market Leverage and Probability of Default at 5 years. We then average across all simulations. We plot Quasi-Market Leverage (left y-axis) and the Probability of Default (right y-axis) as a function of α for a fixed level of $\gamma = 6.5\%$, and the other model parameters are set to the Base Case scenario described in Table 1. Panel A shows that increasing α leads to lower leverage (blue solid line) and lower probability of default (red dashed line). In Panel B we repeat the same exercise but we vary γ for a fixed level of $\alpha = 22\%$. Panel B shows that increasing γ decreases leverage (blue solid line) and sharply increases the probability of default (red dashed line). This is different from the implication that α had on leverage and probability of default (Panel A of Figure 3).

In Panel C of Figure 3, we choose combinations of α and γ that yield the same leverage ratio and we check whether the probability of default changes. For each value of α, we look for the level of γ that keeps Quasi-Market Leverage constant at 32.0%. The blue solid line in Panel C plots the pair of α (left y-axis) and γ (x-axis) that yield a constant leverage ratio of 32%. As expected, if α decreases, the model needs a higher γ in order to keep leverage constant. The red dashed line in Panel C shows the probability of default at 5 years (right y-axis) corresponding to the various pairs of α and γ that keep leverage constant at 32%. The red dashed line shows a strong positive relation

[21] For the probability of default we choose a horizon of 5 years in order to match Moody's actual default rates. Leland (2004) shows that dynamic trade-off models (without jumps) can match the empirical observed default rates for longer horizons. Even if jumps have been shown to be an important component for a model to match short-term (1 year) default probabilities (Du et al., 2018), we decide not to include them in order to keep our model parsimonious and limit the number of parameters to be estimated.

between γ and the probability of default. This implication stems from the fact that both α and γ negatively affect leverage because an increase in any of these two parameters increases the overall cost of debt (sum of the net present values of pre-default costs and loss given default). However, while an increase in α leads to a lower optimal leverage and lower probability of default, the effect of γ on leverage and probability of default is different. An increase in γ lowers the optimal leverage, which in turn, has a negative effect on the probability of default. However, at the same time, it increases the pre-default costs, thus increasing the probability of default. Panel C of Figure 3 shows that the latter effect dominates the former and leads to a higher probability of default for an increase in γ.

[Insert Figure 3 here]

In Section 2.3, we described our choice of distress boundary. Here we discuss what the implications of such a choice are for the estimation of the pre-default cost parameter γ and the net present value of pre-default costs. We define the onset of financial distress to be the level when the operating cash flow X_t drops below the coupon C paid on outstanding debt. How does the choice of the distress boundary affect the estimation of γ? On the one hand, if X_D is set too low (e.g. close to the default threshold X_B), then pre-default costs would not have a strong effect on financing decisions since they are experienced when the probability of default is already very high. On the other hand, if X_D is set too high, pre-default costs would play a crucial role in deciding the optimal capital structure for the firm, and they would overshadow the loss given default α.

As explained in Section 2.3, it is reasonable to expect that firms experience some sort of financial distress cost when their operating cash flows are lower than the required coupon payment. For example, when cash flows are lower than required coupon payments, most firms that issued a loan with covenants would be in violation of at least one covenant, and there is evidence that violating covenants is costly (Chava and Roberts, 2008; Nini et al., 2012). There is also evidence that firms experience a loss of customers (Hortaçsu et al., 2013) and reduced access to trade credit (Sautner and Vladimirov, 2018) when they are close to default. In addition, our choice of distress boundary implies that financial distress begins when firms have high leverage as their operating cash flows have already substantially dropped. This is consistent with empirical evidence suggesting that rarely investment grade firms become financially distressed, and that the majority of firms are already speculative grade when they start experiencing financial distress costs.[22] As a robustness test, in Section 5.3 we re-estimate our model with various distress boundaries.

5 SMM Estimation Results

We first present the results of the estimation of our model using SMM on the entire sample under the assumption that financial distress starts when the operating cash flows of the firm are lower than the required coupon payments. Next, we present the results for sub-samples split by industries and tangibility. We then show the results of the estimation under alternative definitions of financial distress and calculate the net present value of expected financial distress costs.

5.1 Estimations for the entire sample

We estimate three different specifications of our model. We label them (1) *Model With Pre-Default Costs*, (2) *Model Without Pre-Default Costs*, and (3) *Not Matching Prob. of Default*. The *Model With Pre-Default Costs* estimates 5 parameters using the moments described in details in Section 4.2. This model includes both parameter γ (pre-default costs) and parameter α (loss given default). Table 5 shows the model fit. We report the empirical moments (column "Data") as well as the simulated moments from the model. The t-statistics for the difference between empirical and simulated moments are reported in parenthesis. The *Model With Pre-Default Costs* fits the data well. The simulated leverage and probability of default within 5 years are 34.949% and 5.370%, respectively.[23] As confirmed by the low t-statics, these values are not statistically different from their empirical counterparts which are 34.980% for leverage and 5.428% for the probability of default. Table 5 also confirms that the other four simulated moments are not statistically different

[22]Using a sample of 175 firms defaulted between 1997 and 2010, Davydenko et al. (2012) show that only 12.9% of the defaulted firms were rated investment grade at the time of debt issuance.

[23]For brevity, in this section we write "probability of default" to refer to the "probability of default within 5 years".

from their empirical counterparts.

[Insert Table 5 here]

Table 6 presents the estimated parameters for the estimation of the *Model With Pre-Default Costs*. The estimated parameter β (exposure of the firm's cash flows to market risk) is approximately equal to 0.897 .[24] The parameter μ (risk-neutral expected growth rate of the firm) is 0.517% and the volatility of the expected earnings' growth rate σ_F is estimated at 14.818%.

The two main parameters of interest for our study are γ and α. We find that $\gamma = 6.531\%$ and $\alpha = 22.444\%$. Our estimate of γ implies that, on average, when firms are in financial distress they are expected to lose approximately 6.5% of their value per year. This value is large and, as we explain further below, is such that pre-default costs of financial distress constitute a large proportion of the total costs of financial distress.[25]

In addition, our estimated value corroborates the findings of Hortaçsu et al. (2013) related to the automotive sector. The authors find that during the 2008 financial crisis General Motors would have lost approximately 6% of its operating margins if it were to become distressed.[26] Our estimate of γ is also consistent with the empirical evidence from the casino industry in Cookson (2017). The author studies a specific type of pre-default costs, namely the inability of highly levered casinos to respond to the entry of a new competitor. His evidence suggest that such pre-default costs can account for approximately 5% of firm value.

The estimated parameter α implies that firms are expected to lose 22.444% at the time of default. Such a value is consistent with empirical values estimated by Davydenko et al. (2012)

[24] We report the parameters multiplied by 100 in our tables for ease of readability.

[25] In order to keep the model parsimonious, we abstract from macroeconomic risk. However, we run robustness tests in Appendix D to evaluate the effect of macroeconomic risk on the parameters α and γ. Accounting for changes in macroeconomic conditions increases the net present values of both the loss given default and pre-default costs because they are likely to happen in bad states of the economy, hence smaller values of α and γ are required in order to match leverage ratios and probabilities of default. Consistent with this intuition, we show that a model with macroeconomic risk requires a value of α and γ equal to 15% and 4.3%, respectively, in order to match leverage ratios. These values are approximately 2/3 of the estimates from our model without macroeconomic risk. Importantly for our research, our results show that pre-default costs of financial distress are an important determinant for the choice of capital structure regardless of the presence of macroeconomic risk.

[26] They define General Motors to be distressed if it experiences a 3,000 CDS point increase relative to Ford. Such a loss in operating margins could imply a loss in firm value of up to 10%. Please see Hortaçsu et al. (2013) for more details.

and Korteweg (2010) that estimate the average costs to be approximately 15-30% of firm value. Glover (2016) shows that firms that have lower expected default costs use more leverage and, consequently, they end up defaulting more often. Even if the average distress costs are higher for the average firm in the economy, when calculated for the sub-sample of defaulted firms distress costs are approximately 25%, a value consistent with empirical evidence (Andrade and Kaplan, 1998; Davydenko et al., 2012).

Our results provide an alternative explanation that can co-exist with the one in Glover (2016). In addition to the argument proposed by Glover, studying a sample of defaulted firms might lead to a bias in the estimate of distress costs because defaulted firms lose only the loss given default α and they have already experienced (large) pre-default costs of financial distress. Therefore, a selection bias still exists but the source can be in the timing of when distress costs are experienced. Indeed, our model shows that even if our firms are all equal ex-ante, there would be a bias in the measurement of financial distress costs if they are measured only at default. Only the loss given default would be captured while the (large) pre-default costs would be omitted. In Appendix E we use simulations from our model to provide evidence that defaulted firms are likely to have lower loss given default than the non-defaulted ones but the magnitude of the difference between defaulted and non-defaulted firms is smaller when we account for pre-default costs of financial distress. Specifically, in Glover, the defaulted firms have an average loss given default (LGD) which is approximately 44% lower than the average firm (LGD of 25% for defaulted vs. LGD of 45% for non-defaulted firms). Our results show that, when we take into account for pre-default costs of financial distress, the LGD for defaulted firms is approximately 19.56% lower than non-defaulted firms (LGD of 18.05% for defaulted vs. LGD of 22.44% for non-defaulted firms).

Next, we address the bias caused by the omission of pre-default costs of financial distress. The standard procedure to calibrate trade-off models consists of using the empirical estimates of tax benefits (Graham, 2000) and default costs (Andrade and Kaplan, 1998; Davydenko et al., 2012). However, in order to calculate empirical estimates of default costs, previous studies estimate the change in firm value that happens around the time of default, when the firm has (potentially) already incurred pre-default costs. Therefore, applying such estimates to firms that are not close to default would underestimate the effective costs of financial distress (Elkamhi et al., 2012). To measure the magnitude of this underestimation, we estimate a modified version of our model which we label *Model Without Pre-Default Costs*. In this specification, we exogenously set $\gamma = 0$ so the costs of financial distress are experienced only at the time of default when the firm is expected to lose a portion α of its value. We fit this specification to the data using the same six moments that we used to estimate the *Model With Pre-Default Costs*. Comparing the estimated α in the *Model Without Pre-Default Costs* with the one from the *Model With Pre-Default Costs* provides an estimate of the factor by which empirical estimates should be multiplied.[27]

In column *Model Without Pre-Default Costs* of Table 5, we discuss the results for the specification where pre-default costs are omitted and we fit the model to the same six moments discussed above. The overall fit of the model is not as good as it was for the *Model With Pre-Default Costs*. For example, the t-statistic for the difference between the empirical moment of the variance of equity and its simulated counterpart is 4.216.[28] Table 6 also shows that that the estimated loss given default α in the *Model Without Pre-Default Costs* is 55.44%. This value is approximately 2.5 times higher than the 22.444% that we had estimated with the *Model With Pre-Default Costs*. This suggests that there is a large bias. Researchers should not only be aware of such bias but they should also be using a higher value of loss given default than the empirical estimates (Davydenko et al., 2012) in the calibration of their models if they omit pre-default costs.

[Insert Table 6 here]

Our estimate of $\alpha = 55.44\%$ in the *Model Without Pre-Default Costs* is also higher than the one presented by Glover (2016) when he fitted a dynamic trade-off model to observed leverage choices but not probabilities of default. To understand whether this difference is driven by our choice to fit the model also to default probabilities in addition to leverage, we estimate a version of our model

[27] We use the default rate as one of our moments, in Appendix F we show that our results are robust to the use of a model implied PD.

[28] The *Model With Pre-Default Costs* is instead able to generate a variance of equity that is not statistically different from the empirical counterpart.

which we label *Not Matching Prob. of Default*. In this specification, we exogenously set $\gamma = 0$ as in the *Model Without Pre-Default Costs* but we exclude the Probability of Default moment (i.e. deviation from the empirical probability of default are not penalized in this specification) to make it comparable to Glover (2016). The results of this estimation are presented in column *Not Matching Prob. of Default* of Table 6. In this specification the estimated loss given default is $\alpha = 49.818\%$. While still slightly higher then the value of 45% estimated by Glover (2016), such a small difference can easily be attributed to the difference in the samples used. Indeed, while Glover estimates his model on a panel of 2,505 firms between 1990 and 2010, we estimate our model on a panel of 4,603 firms for the period 1990 and 2013.[29] As stated above Glover (2016) accounts for macroeconomic risk, which increases the present value of the loss given default implying that the model needs slightly lower α in order to match the data compared to a model without macroeconomic risk.

5.2 Splits by Tangibility and Industries

It is known that firms with more pledgeable (tangible) assets take on more leverage (Almeida and Campello, 2007) and they are also able to recover more in case of default (Elkamhi et al., 2014). Therefore, we split our sample based on the tangibility of their assets to evaluate how loss given default and pre-default costs change with the tangibility of assets. Intuitively, for a given level of leverage, we should expect firms with more tangible assets to recover more than intangible firms since creditors will have physical assets to claim. Therefore the loss given default should be lower. Using Compustat Quarterly, we define tangibility as "Property Plant and Equipment - Total (Net)" divided by Total Assets. For every year and quarter, we group firms into "High Tangibility" (above median) and "Low Tangibility" (below median).

Table 8 shows the model fit for the estimation of the *Model With Pre-Default Costs* on the subsample of firms split by tangibility. We present the estimated parameters and their standard errors (in parentheses) based on the splits by tangibility in Panel A. The loss given default α for

[29]The differences in our samples are due to the different methodologies used. Following Hennessy and Whited (2005, 2007) we study the average firm in the economy. Glover (2016) estimates his model firm-by-firm and limits his sample to firms that have been in Compustat for at least 5 years. We do not need firms to be in Compustat for at least 5 years using our methodology.

high and low tangibility are 12.826% and 54.335%, respectively. This large difference in loss given default between the two sub-samples confirms the intuition that more tangible firms are expected to recover more (i.e. have a lower loss at default) than intangible firms. In addition, our estimation shows that there is a clear difference not only for the loss given default but also for the pre-default costs' parameter γ. The parameter γ is equal to 7.354% for highly tangible firms and 2.137% for low tangibility firms, thus showing that highly tangible firms are expected to suffer more from pre-default costs of financial distress. This result challenges the literature documenting that distressed firms with low tangible (high intangible) capital lose more human capital than high tangible (low intangible) capital (Babina, 2020; Gortmaker et al., 2019; Baghai et al., 2017).[30]

Panel B of Table 8 shows the empirical moments, the simulated moments, and the t-statistics for the difference between empirical and simulated moments. The model fits both low and high tangibility firms well since none of the t-statistics display a statistically significant difference between empirical and simulated moments.

[Insert Table 8 here]

To further evaluate how pre-default costs vary cross-sectionally, we estimate our model on samples splits by industry. We define an industry according to the 2-digit SIC code, which results in 8 different industries. We present the parameter estimates in Table 9 while Table 10 displays the model fit for each industry. There is a large variation in the estimated parameters across industries. The pre-default costs vary from a minimum of 3.75% to a maximum of 8.75%. Industries such as Mining, Retail Trade and Services show considerably higher estimated values of pre-default costs γ compared to Construction, Manufacturing and Wholesale Trade. The loss given default α also displays large cross-sectional variation. The estimated loss given default varies from 10.3% to a maximum of 35.6%. The last row of Table 9 shows the average parameters of the estimations by

[30] Babina (2020) shows that employees in sectors with more intangible assets are considerably more likely to leave their (distressed) firms and start new companies. Gortmaker et al. (2019) and Baghai, Silva, Thell, and Vig (2017) show that talented employees are more likely to leave the firm after a deterioration in a firm's credit quality. In Appendix G we classify firms into producers of durable vs. non-durable goods. Evaluating the durability of a firm's product is challenging from an empirically point of view and we follow Gomes et al. (2009). We then estimate our model on the split of durable vs. non-durable firms and show that non-durable firms have lower pre-default costs than durable firms.

industries. The average pre-default cost is 6.325%, a value that is close to the estimated value for the average firm of 6.531% presented in Table 6. The average loss given default amongst industries is 24.972% which is also close to the value estimated for the average firm (22.444%).

Our results show that the average α and γ by industry are not only varying when considered independently but there is also a cross-sectional variation in the combined levels. That is, for some industries such as Mining, the estimates of both α and γ are above average. For some others (e.g. Transportation & Public Utilities) the estimate of α is above average while the estimate of γ is below average while it is the opposite for for industries like Retail Trade which exhibits an estimate of α below average and an estimate of γ above average.

[Insert Table 9 and Table 10 here]

Overall, the model is able to fit the various industries quite well. The simulated moments match the empirical moments as shown by t-statistics that are mostly below two with a few exceptions. There is one industry for which the model struggles: Construction. For this industry, even if the model is able to match the probability of default, the operating ROA and the variance of Equity, it struggles to simultaneously match the empirical leverage of 52.452% and the equity returns of 9.589%.

5.3 Alternative Distress Boundaries and Credit Spreads

All the results presented in the previous sections use the assumption that firms start to experience pre-default costs of financial distress when operating cash flows are lower than the required coupon payments. This modeling choice has been used by other authors (Titman and Tsyplakov, 2007; Chen et al., 2019). Even if we discussed several reasons that make this a sensible choice in Section 2.3, we recognize that this is ultimately an arbitrary assumption. Therefore we provide evidence of how such assumption impacts our estimation. Intuitively, if we allowed firms to experience pre-default costs of financial distress sooner (e.g. when EBIT is twice as high as coupon payments) and our model is identified (e.g. we are able to separate γ from α), we would expect that the our model needs a lower γ in order to fit the data. This is because the firm "leaks" less value but it does so when firm value is higher.

We proceed as follows. The endogenous distress boundary is defined as X_D, and it represents the level of EBIT below which the firm experiences pre-default costs of financial distress. In Table 11 we estimate three different versions of the *Model With Pre-Default Costs* with various distress boundaries: $X_D = k \times C$ for $k \in \{2.0, 1.0, 0.5\}$. Panel A reports the estimated model parameters with standard errors in parentheses. Consistent with our intuition, γ is inversely related to the level of the distress boundary X_D. When $X_D = 2 \times C$ our model estimates a $\gamma = 3.718\%$. When $X_D = 1 \times C$ our model estimates a $\gamma = 6.531\%$, while the estimated $\gamma = 14.308\%$ for $X_D = 0.5 \times C$. Panel B shows the model fit of the various estimations. Overall, the choice of distress boundary does not have a strong effect on the ability of the model to fit the data except for the moment of variance of equity. Only the model with $X_D = 1 \times C$ is able to match the variance of equity while the other choices of distress boundary generate simulated moments of variance of equity that are statistically different from the empirical counterparts as shown by the t-statistics which are higher than 6 (six). This confirms that choosing $X_D = 1$ is not only supported by empirical evidence and reasonable as discussed in Section 2.3, but it is also the choice that allows the model to best fit the data.

[Insert Table 11 here]

Next, we evaluate how the choice of the distress boundary affects our results. We are mostly interested in understanding how choosing a distress boundary above the required coupon payments ($X_D > C$) affects the composition of pre-default costs of financial distress. For the case $X_D < C$, it is expected that pre default costs might decrease their importance because, intuitively, if we set the distress boundary to be equal to the default boundary ($X_D = X_B$), the pre-default costs of financial distress will be zero in net present value terms as the firm never experiences them (i.e. it defaults as soon as it hits the distress boundary).[31]

[31] This is mechanical because if the firm defaults as soon as it becomes distressed than the net present value of pre-default costs is zero.

Table 12 shows the value of pre-default costs and loss given default as a percentage of total distress costs.[32] Specifically, the column $\%\,PDC$ presents the percentage of pre-default costs as a fraction of the total distress costs (loss given default plus pre-default costs); the column $\%\,LGD$ presents the same percentage for the loss given default. For $X_D = 2.0 \times C$, the percentage of pre-default costs as a fraction of the total is 66.5% while for $X_D = 1.0 \times C$ the same percentage is 68.5%. This result shows that pre-default costs of financial distress are not only large compared to the loss given default but also that the composition of total distress costs ($\%\,PDC$ vs. $\%\,LGD$) does not vary much if we increase the distress boundary from $X_D = 1.0 \times C$ to $X_D = 2.0 \times C$. Indeed, these values are close considering that they are the result of estimating the model for very different distress boundaries.

For $X_D = 0.5 \times C$, our results show that the composition of total distress costs varies significantly. Indeed, the percentage of pre-default costs as a fraction of the total drops to 30.2% in this case. This result is not surprising because, as we argued above, in the extreme where we set $X_D = X_B$ the percentage of pre-default costs $\%\,PDC$ is zero. We followed previous studies and chose a value $X_D = 1.0 \times C$ for our main estimation, and we acknowledge that lowering the distress boundary below the required coupon payments would diminish the effects of pre-default costs of financial distress.

[Insert Table 12 and Table 13 here]

Finally, as a practical application of the material decomposition of α and γ in our setting, we address how credit spreads vary across the various estimations of our model. We answer this question in Table 13 which presents the credit spreads implied by our model at the time of debt issuance. We define the credit spread for the perpetual bond in our model as the required coupon payment divided by the value of debt minus the risk-free rate $(C/D - r)$. The *Model With Pre-Default Costs* implies a credit spread of 2.35% while the *Model Without Pre-Default Costs* shows a credit spread of 2.01%. Both models are fit to the same empirical data and they differ only for the

[32] We provide the expressions to value pre-default costs and loss given default in Appendix H.

presence of pre-default costs. Therefore, the difference in credit spreads between the two models captures the ability of pre-default costs to increase the credit spread for a fixed level of leverage and probability of default, following Huang and Huang (2012). We match the default probabilities and our loss given default leads to reasonable recovery rates as shown in Table 1. Our results imply that a model with pre-default costs generates higher credit spreads than a model without pre-default costs when calibrated to the same empirical data. While the focus of this paper is not the analysis of credit spreads, our findings suggest that models of credit risk that try to match both probabilities of default and credit spreads would benefit from the inclusion of pre-default costs.

6 Conclusion

This paper quantifies the expected costs of financial distress that firms experience prior to default (pre-default costs) separately from the loss given default. We develop a dynamic model of capital structure which includes pre-default costs and we fit it to the data. We find that pre-default costs are large and on average equal to 6.5% of firm value per year during times of financial distress. We show that there is a large cross sectional variation of pre-default costs across industries, from 3.75% to 8.75%. Also, our estimates show that firms with more tangible assets have a lower loss given default and higher pre-default costs than intangible firms.

Our study has implications for academics using calibrated dynamic capital structure models to gauge insights of the firms' responses to a policy change or to evaluate the behaviour of credit spreads, leverage and probability of default. We also show an alternative explanation to the selection bias discussed in Glover (2016) based on the timing of financial distress. While Glover (2016) demonstrate that there exists a selection bias because firms are ex-ante different and that defaulted firms have lower distress costs than the average firm, we show that measuring distress costs from defaulted firms would lead to a bias because we would not capture the (large) pre-default costs experienced prior to default. Our rationale does not invalidate the argument in Glover (2016) but can co-exists with it. Furthermore, we document that omitting pre-default costs leads to a large bias in the estimates of loss at default when using dynamic models of capital structure. While the average loss given default in our model with pre-default costs of financial distress is 22.44%, the same model where we exogenously fix pre-default costs to zero needs a loss given default of 55.44% in order to match both leverage and probability of default or, alternatively, a loss given default of 49.18% in order to match leverage alone.

Lastly, we do not "micro-found" the parameter that measures pre-default costs of financial distress (γ) therefore our results are silent on the consequences that pre-default costs of financial distress have on investment, dividend payouts, customer/supplier relations, etc. These are all important questions that highlight a number of new avenues that could be explored in future studies.

References

Abel, Andrew B, 2018, Optimal debt and profitability in the trade-off theory, *The Journal of Finance* 73, 95–143.

Almeida, Heitor, and Murillo Campello, 2007, Financial constraints, asset tangibility, and corporate investment, *The Review of Financial Studies* 20, 1429–1460.

Andrade, Gregor, and Steven N Kaplan, 1998, How costly is financial (not economic) distress? Evidence from highly leveraged transactions that became distressed, *The Journal of Finance* 53, 1443–1493.

Babina, Tania, 2020, Destructive creation at work: How financial distress spurs entrepreneurship, *The Review of Financial Studies* 33, 4061–4101.

Baghai, Ramin, Rui Silva, Viktor Thell, and Vikrant Vig, 2017, Talent in distressed firms: Investigating the labor costs of financial distress, *Working Paper* .

Bai, Jennie, Robert S Goldstein, and Fan Yang, 2020, Is the credit spread puzzle a myth?, *Journal of Financial Economics* .

Bazdresch, Santiago, R Jay Kahn, and Toni M Whited, 2017, Estimating and testing dynamic corporate finance models, *The Review of Financial Studies* 31, 322–361.

Bhamra, Harjoat S, Lars-Alexander Kuehn, and Ilya A Strebulaev, 2009, The levered equity risk premium and credit spreads: A unified framework, *The Review of Financial Studies* 23, 645–703.

Bhamra, Harjoat S, Lars-Alexander Kuehn, and Ilya A Strebulaev, 2010, The aggregate dynamics of capital structure and macroeconomic risk, *The Review of Financial Studies* 23, 4187–4241.

Bharath, Sreedhar T, and Tyler Shumway, 2008, Forecasting default with the merton distance to default model, *The Review of Financial Studies* 21, 1339–1369.

Campbell, John Y, Jens Hilscher, and Jan Szilagyi, 2008, In search of distress risk, *The Journal of Finance* 63, 2899–2939.

Chava, Sudheer, and Michael R Roberts, 2008, How does financing impact investment? the role of debt covenants, *The Journal of Finance* 63, 2085–2121.

Chen, Hui, 2010, Macroeconomic conditions and the puzzles of credit spreads and capital structure, *The Journal of Finance* 65, 2171–2212.

Chen, Long, Pierre Collin-Dufresne, and Robert S Goldstein, 2009, On the relation between the credit spread puzzle and the equity premium puzzle, *The Review of Financial Studies* 22, 3367–3409.

Chen, Zhiyao, Dirk Hackbarth, and Ilya A Strebulaev, 2019, A unified model of distress risk puzzles, *Working Paper* .

Cookson, J Anthony, 2017, Leverage and strategic preemption: Lessons from entry plans and incumbent investments, *Journal of Financial Economics* 123, 292–312.

Davydenko, Sergei A, Ilya A Strebulaev, and Xiaofei Zhao, 2012, A market-based study of the cost of default, *The Review of Financial Studies* 25, 2959–2999.

Dou, Winston, Lucian A Taylor, Wei Wang, and Wenyu Wang, 2019, Dissecting bankruptcy frictions, *Available at SSRN 3383837* .

Du, Du, Redouane Elkamhi, and Jan Ericsson, 2018, Time-Varying Asset Volatility and the Credit Spread Puzzle, *The Journal of Finance* .

Elkamhi, Redouane, Jan Ericsson, and Christopher A Parsons, 2012, The cost and timing of financial distress, *Journal of Financial Economics* 105, 62–81.

Elkamhi, Redouane, Kris Jacobs, and Xuhui Pan, 2014, The cross section of recovery rates and default probabilities implied by credit default swap spreads, *Journal of Financial and Quantitative Analysis* 193–220.

Elkamhi, Redouane, Chanik Jo, and Marco Salerno, 2020, Business cycles and the bankruptcy code: a structural approach, *Working Paper* .

Erickson, Timothy, and Toni M Whited, 2002, Two-step GMM estimation of the errors-in-variables model using high-order moments, *Econometric Theory* 18, 776–799.

Feldhütter, Peter, and Stephen M Schaefer, 2018, The myth of the credit spread puzzle, *The Review of Financial Studies* 31, 2897–2942.

Friewald, Nils, Christian Wagner, and Josef Zechner, 2014, The cross-section of credit risk premia and equity returns, *The Journal of Finance* 69, 2419–2469.

Glover, Brent, 2016, The expected cost of default, *Journal of Financial Economics* 119, 284–299.

Goldstein, Robert, Nengjiu Ju, and Hayne Leland, 2001, An EBIT-based model of dynamic capital structure, *The Journal of Business* 74, 483–512.

Gomes, Joao F, Leonid Kogan, and Motohiro Yogo, 2009, Durability of output and expected stock returns, *Journal of Political Economy* 117, 941–986.

Gortmaker, Jeff, Jessica Jeffers, and Michael Junho Lee, 2019, Labor reactions to financial distress: Evidence from linkedin activity, *Available at SSRN* .

Gourieroux, Christian, and Alain Monfort, 1996, *Simulation-based Econometric Methods* (Oxford university press).

Graham, John R, 1999, Do personal taxes affect corporate financing decisions?, *Journal of Public Economics* 73, 147–185.

Graham, John R, 2000, How big are the tax benefits of debt?, *The Journal of Finance* 55, 1901–1941.

Greenwald, Daniel, 2019, Firm debt covenants and the macroeconomy: The interest coverage channel, *MIT Working Paper* .

Harrison, J Michael, and David M Kreps, 1979, Martingales and arbitrage in multi-period securities markets, *Journal of Economic Theory* 20, 381–408.

Hennessy, Christopher A, and Toni M Whited, 2005, Debt dynamics, *The Journal of Finance* 60, 1129–1165.

Hennessy, Christopher A, and Toni M Whited, 2007, How costly is external financing? Evidence from a structural estimation, *The Journal of Finance* 62, 1705–1745.

Hortaçsu, Ali, Gregor Matvos, Chad Syverson, and Sriram Venkataraman, 2013, Indirect costs of financial distress in durable goods industries: The case of auto manufacturers, *The Review of Financial Studies* 26, 1248–1290.

Hotchkiss, Edith S, Kose John, Robert M Mooradian, and Karin S Thorburn, 2008, Bankruptcy and the resolution of financial distress, in *Handbook of Empirical Corporate Finance*, 235–287 (Elsevier).

Huang, Jing-Zhi, and Ming Huang, 2012, How much of the corporate-treasury yield spread is due to credit risk?, *The Review of Asset Pricing Studies* 2, 153–202.

Kaplan, Steven N, and Luigi Zingales, 2000, Investment-cash flow sensitivities are not valid measures of financing constraints, *The Quarterly Journal of Economics* 115, 707–712.

Kim, Dongcheol, Darius Palia, and Anthony Saunders, 2008, The impact of commercial banks on underwriting spreads: Evidence from three decades, *Journal of Financial and Quantitative Analysis* 43, 975–1000.

Korteweg, Arthur, 2010, The net benefits to leverage, *The Journal of Finance* 65, 2137–2170.

Leland, H, 2004, Prediction of expected default frequencies in structural models of debt, *Journal of Investment Management* 2.

Leland, Hayne E, 1994, Corporate debt value, bond covenants, and optimal capital structure, *The Journal of Finance* 49, 1213–1252.

Loken, Chris, Daniel Gruner, Leslie Groer, Richard Peltier, Neil Bunn, Michael Craig, Teresa Henriques, Jillian Dempsey, Ching-Hsing Yu, Joseph Chen, et al., 2010, Scinet: lessons learned from building a power-efficient top-20 system and data centre, in *Journal of Physics: Conference Series*, 012026, IOP Publishing.

Morellec, Erwan, Boris Nikolov, and Norman Schürhoff, 2012, Corporate governance and capital structure dynamics, *The Journal of Finance* 67, 803–848.

Nikolov, Boris, and Toni M Whited, 2014, Agency conflicts and cash: Estimates from a dynamic model, *The Journal of Finance* 69, 1883–1921.

Nini, Greg, David C Smith, and Amir Sufi, 2012, Creditor control rights, corporate governance, and firm value, *The Review of Financial Studies* 25, 1713–1761.

Ponce, Marcelo, Ramses van Zon, Scott Northrup, Daniel Gruner, Joseph Chen, Fatih Ertinaz, Alexey Fedoseev, Leslie Groer, Fei Mao, Bruno C Mundim, et al., 2019, Deploying a top-100 supercomputer for large parallel workloads: The niagara supercomputer, *Working Paper* 1–8.

Pulvino, Todd C, 1998, Do asset fire sales exist? An empirical investigation of commercial aircraft transactions, *The Journal of Finance* 53, 939–978.

Sautner, Zacharias, and Vladimir Vladimirov, 2018, Indirect costs of financial distress and bankruptcy law: Evidence from trade credit and sales, *Review of Finance* 22, 1667–1704.

Schaefer, Stephen M, and Ilya A Strebulaev, 2008, Structural models of credit risk are useful: Evidence from hedge ratios on corporate bonds, *Journal of Financial Economics* 90, 1–19.

Senbet, Lemma W, and Tracy Yue Wang, 2012, Corporate financial distress and bankruptcy: A survey, *Foundations and Trends® in Finance* 5, 243–335.

Strebulaev, Ilya A, 2007, Do tests of capital structure theory mean what they say?, *The Journal of Finance* 62, 1747–1787.

Strebulaev, Ilya A, and Toni M Whited, 2012, Dynamic models and structural estimation in corporate finance, *Foundations and Trends® in Finance* 6, 1–163.

Titman, Sheridan, 1984, The effect of capital structure on a firm's liquidation decision, *Journal of Financial Economics* 13, 137–151.

Titman, Sheridan, and Sergey Tsyplakov, 2007, A dynamic model of optimal capital structure, *Review of Finance* 11, 401–451.

Figures

Figure 1
Model Intuition

This figure shows the impact that pre-default costs have on the path of EBIT. EBIT follows a geometric brownian motion as described in Equation (2):

$$\frac{dX_t}{X_t} = \begin{cases} \left(\mu + \beta(\mu_A^{\mathcal{P}} - r)\right) dt + \beta\sigma_A dB_t^{A,\mathcal{P}} + \sigma_F dB_t^F & \text{for } X_B \leq X_D \leq X_t \\ \underbrace{\left(\mu + \beta(\mu_A^{\mathcal{P}} - r)\right) dt}_{\substack{\text{Expected growth} \\ \text{rate without distress}}} + \underbrace{\beta\sigma_A dB_t^{A,\mathcal{P}} + \sigma_F dB_t^F}_{\substack{\text{Systematic and} \\ \text{Idiosyncratic shocks}}} \underbrace{-\gamma dt}_{\substack{\text{Pre-default} \\ \text{Costs}}} & \text{for } X_B < X_t < X_D \end{cases}$$

The figure shows two scenarios. The blue solid line shows the path of the firm's EBIT if it had no pre-default costs of financial distress ($\gamma = 0$) while the red dashed line shows the scenario when the firm's has pre-default costs ($\gamma = 2\%$). The rest of the parameters are the same in the two scenarios. Both paths start from X_0 and they evolve according to the law of motion described above. When X_t is below X_D, the firm experiences pre-default costs. When X_t reaches X_B the firm defaults.

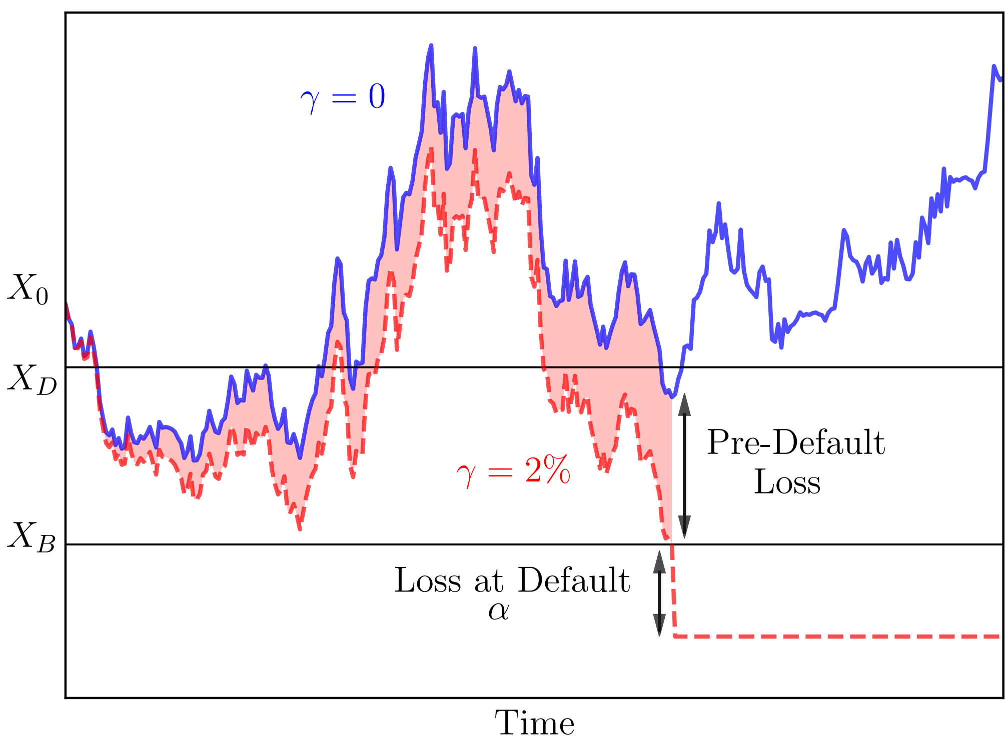


Figure 2

Identification β and σ_F

This figure depicts the relation between the firm's exposure to market risk β and the idiosyncratic volatility of EBIT σ_F with respect to the moments of Variance of Equity (Panel A and Panel B), and Equity Return (Panel C and Panel D). The data used to create these figures have been generate as follows. We simulate the model 5,000 times for a time period of 150 years. We keep only the last 80 quarters of data to remove the effect of the initial conditions, and we calculate the Variance of Equity and the Equity Return. The model parameters are set to the base case scenario described in Table 1. Panel A and Panel B plot the Variance of Equity as a function of σ_F and β, respectively. Panel C and Panel D plot the Equity Return as a function of σ_F and β, respectively. The model parameters are set to the base case scenario described in Table 1.

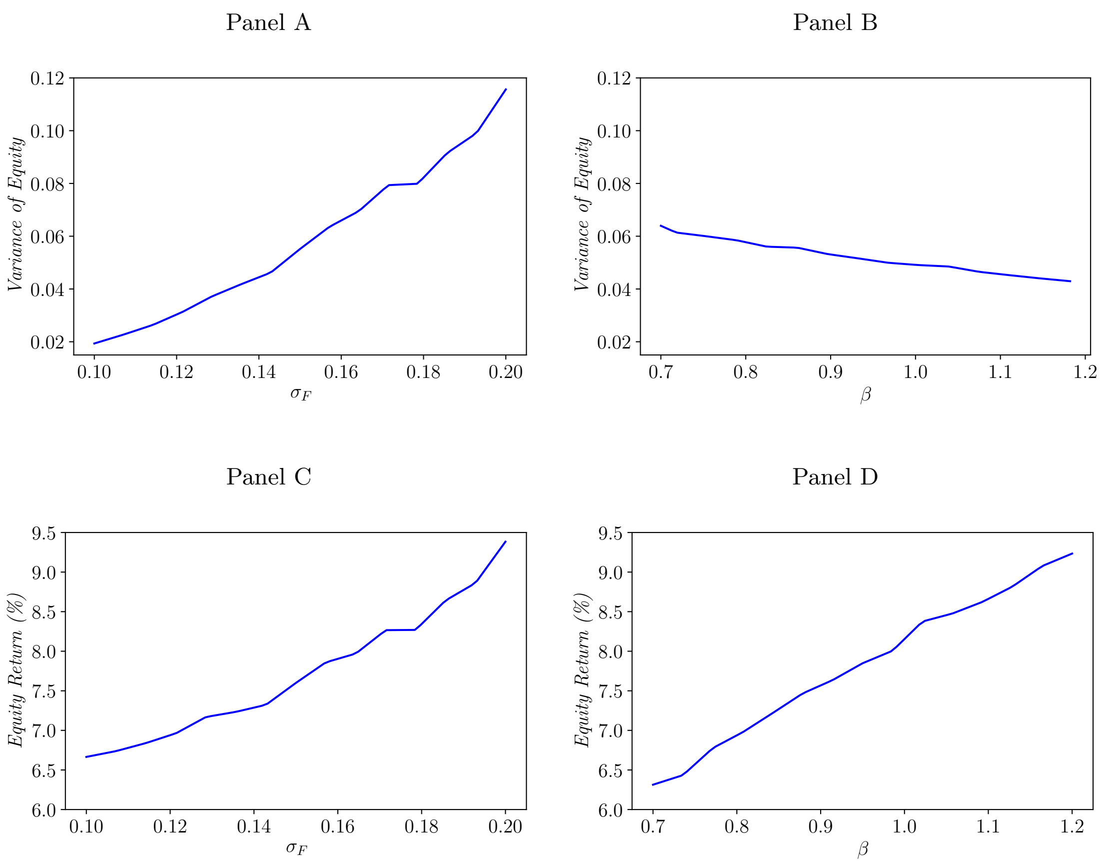
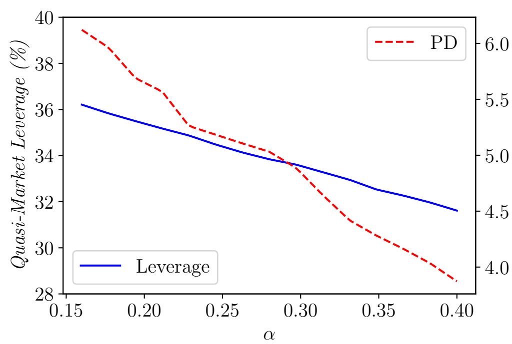


Figure 3

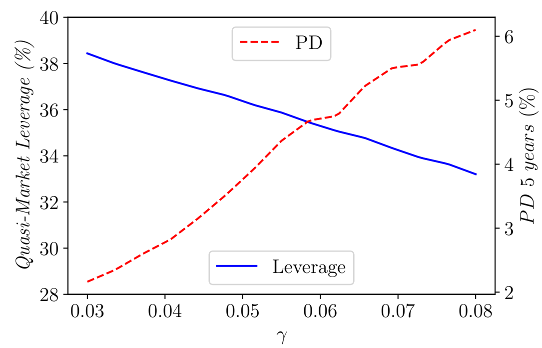
Identification of α and γ

This figure depicts the relation between the default-loss α, the pre-default costs γ, and the moments of leverage and probability of default. The data used to create these figures have been generate as follows. We simulate the model 5,000 times for a time period of 150 years. We keep only the last 80 quarters of data to remove the effect of the initial conditions. For each simulation, we calculate the Quasi-Market Leverage and Probability of Default at 5 years. We then average across all simulations. Panel A plots the Quasi-Market Leverage (left y-axis) and the Probability of Default (right y-axis) as a function of α for a fixed level of $\gamma = 6.5\%$. Panel B plots Quasi-Market Leverage (left y-axis) and the Probability of Default (right y-axis) as a function of γ for a fixed level of $\alpha = 22\%$. The other model parameters are set to the base case scenario described in Table 1. Panel C plots the probability of default for various combinations of α and γ that yield a constant Quasi-Market Leverage. The blue solid line in Panel A plots the value of α (left y-axis) and γ (x-axis) that yield a constant Quasi-Market Leverage. Specifically, for each value of α, we look for the level of γ that keeps Quasi-Market Leverage constant at 32.0%. The red dashed line plots the probability of default (right y-axis) corresponding to the pair of α and γ that yield a constant leverage of 32%.

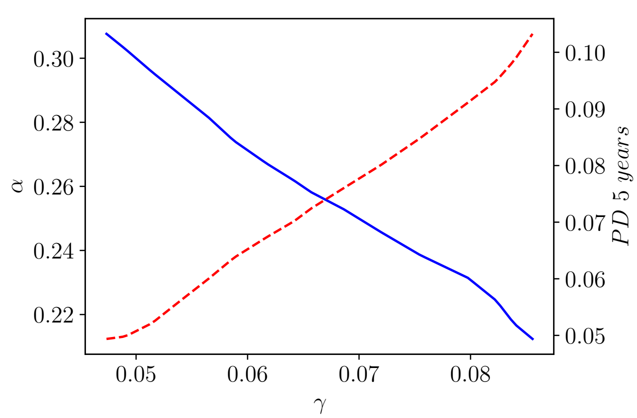
Panel A



Panel B



Panel C



Tables

Table 1
Comparative Statics

This table presents the comparative statics of the model with regards to financing decisions. We set the base case parameters as follows. The personal tax rate on dividends $\tau^e = 11.6\%$, the tax rate on interest income $\tau^d = 29.3\%$, the corporate tax rate $\tau^c = 35.0\%$, the aggregate earnings growth rate and volatility are $\mu_A = 5.84\%$ and $\sigma_A = 9.471\%$, the risk-free rate is $r = 2.27\%$, the growth rate and volatility of cash flows are $\mu = 0.5\%$ and $\sigma_F = 14.8\%$, the exposure to market risk parameter is set to $\beta = 0.9$, the loss given default parameter is $\alpha = 22.44\%$, the pre-default costs are set to $\gamma = 6.5\%$, the proportional adjustment costs is set to $\lambda = 1.0\%$. We normalize the initial value of operating cash flows $X_0 = 5.0$ and set the distress boundary X_D equal to the value of coupon C.

		Quasi-Market Leverage (%) at			
Scenario	X_B/X_0	Distress	Target	Restructuring	Recovery Rate (%)
Base	0.092	46.92	32.56	16.46	35.27
Pre-Default Costs (Base: $\gamma = 6.5\%$)					
γ =8.0%	0.078	44.11	30.99	15.72	31.59
γ =5.0%	0.112	50.19	34.63	17.40	40.14
Costs at Default (Base: $\alpha = 22.44\%$)					
α =34.0%	0.086	46.54	31.00	15.59	29.47
α =12.0%	0.099	47.23	34.70	17.28	40.74
Expected Growth Rate of EBIT (Base: $\mu = 0.5\%$)					
μ =0.8%	0.085	41.97	30.05	13.43	35.29
μ =0.26%	0.104	50.87	36.01	18.37	35.78
Idiosyncratic Volatility (Base: $\sigma_F = 14.8\%$)					
σ_F =18.0%	0.085	45.11	30.03	14.36	35.33
σ_F =12.0%	0.099	48.33	35.11	18.68	35.07
Exposure to market risk (Base: $\beta = 0.9$)					
β =1.35	0.086	45.33	30.30	14.58	35.33
β =0.5	0.097	47.84	34.14	17.83	35.16
Cost of debt issuance (Base: $\lambda = 1.0\%$)					
λ =1.5%	0.094	47.02	33.51	15.17	35.23
λ =0.5%	0.089	46.80	31.13	18.26	35.29

Table 2
Variables Definition

This table presents a description of the empirical variables.

Variables	**Definition**
Compustat	
Book Debt	Debt in Current Liabilities (dlcq) + Long Term Debt (dlttq)
Market Value	Market price (prccq) × Common Shares Outstanding (cshoq)
Quasi-Market Leverage	Book Debt / (Book Debt + Market Value)
Operating Profit	Operating Income Before Depreciation (oibdpq) if oibdpq missing, Sales (saleq) - Operating Expenses (xoprq) if saleq - xoprq missing, Revenues (revtq) - Operating Expenses (xoprq)
Operating ROA	Operating Profit / Lagged Total Asset (atq)
CRSP and FED	
Risk free rate	10-Year Constant Maturity Government Bond Yield
Equity Return	CRSP Total Return (trt1m)
Excess Return	Equity Returns - Risk-Free Rate
Moody's	
Prob. of Default 5 Years	Average Probability of Default at 5 years by year and rating

Table 3
Descriptive Statistics

This table presents descriptive statistics for the variables used in the estimation. The sample is based on financial statements from Compustat quarterly Industrial Files and returns from CRSP. The Probability of Default are taken from the average default rates provided by Moody's. Table 2 provides a detailed definition of the moments.

	Mean	St.Dev	25th	Median	75th
Total Assets (Billions)	8.23	23.94	0.70	1.97	5.93
Operating ROA (%)	12.18	13.85	8.10	13.30	18.95
Quasi-Market Leverage (%)	34.98	25.97	13.26	34.38	54.16
Prob. of Default 5 years (%)	5.43	8.21	0.00	1.72	7.57
Excess Returns (%)	4.85	91.39	-45.69	4.47	54.37
Equity Returns (%)	9.57	91.37	-40.91	9.13	58.97

Table 4

Elasticity of Moments to Parameters

This table shows the elasticity of model-implied moments (in columns) with respect to model parameters (in rows). The elasticity of moment m with respect to parameters p is defined as $\frac{dm/m}{dp/p}$. The elasticities are calculated at the estimated parameter values from Table 6. Parameter σ_F is the idiosyncratic EBIT volatility, μ is the risk-neutral expected growht rate of EBIT, β is the firm's exposure to market risk, γ is the parameter that captures pre-default costs (i.e. constant rate at which the firm loses value when it is in financial distress but has not defaulted yet), α is the parameter that captures the expected loss given default. A description of the moments is provided in Table 2.

	Moments					
	Variance of Equity	Operating ROA	Equity Returns	Probability of Default 5 years	Quasi-Market Leverage	Excess Returns
σ_F	**8.45**	0.02	1.23	11.40	0.42	1.85
μ	1.62	**0.58**	0.21	-0.49	0.29	0.35
β	-1.41	-0.01	**1.72**	-4.83	-0.60	2.51
γ	1.12	0.00	-0.30	**1.35**	-0.49	-0.40
α	-4.26	0.00	-0.73	-0.48	**-0.27**	-1.07

Table 5

Simulated Moments Estimation

The estimation is conducted via Simulated Method of Moments (SMM) which searches for the model parameters that minimize the distance between the empirical moments and the moments calculated from a simulated panel of firms. This table shows the empirical moments (column Data) as well as the simulated moments. The t-statistics for the difference between empirical and simulated moments are reported in parenthesis. We estimate three different specifications of our model: *Model With Pre-Default Costs* includes both the parameter γ to capture pre-default costs of financial distress and the parameter α that captures the expected loss given default, *Model Without Pre-Default Costs* exogenously sets $\gamma = 0$ so the costs of financial distress are experienced only at the time of default when the firm is expected to lose a portion α of its value, *Not Matching Prob. of Default* estimates the same parameters as the *Model Without Pre-Default Costs* but it excludes the Probability of Default moment (i.e. deviation from the empirical probability of default are not penalized in this specification). The *Not Matching Prob. of Default* allows for comparability with previous studies such as Glover (2016).

	Data	Model With Pre-Default Costs	Model Without Pre-Default Costs	Not Matching Prob. of Default
Quasi Market Leverage	34.980	34.949	35.662	34.852
		(0.001)	(0.301)	(0.011)
Probability of Default (5Y)	5.428	5.370	5.307	2.382
		(0.018)	(0.078)	
Operating ROA	12.178	12.198	12.303	12.169
		(0.011)	(0.439)	(0.002)
Variance of Equity	5.218	5.298	6.722	5.174
		(0.012)	(4.216)	(0.004)
Equity Returns	9.570	7.562	9.154	8.951
		(0.853)	(0.037)	(0.081)
Excess Returns	4.855	5.314	6.889	6.687
		(0.045)	(0.876)	(0.71)

Table 6
Parameter Estimates

This table reports the structural parameters estimated via Simulated Method of Moments (SMM). Standard errors are in parentheses. The parameter γ captures pre-default costs of financial distress (i.e. constant rate at which the firm loses value when it is in financial distress but has not defaulted yet), μ captures a firm fixed effect for the expected earnings' growth rate, β is the exposure of the firm's cash flows to market risk, σ_F is a firm fixed effect for the volatility of the expected earnings' growth rate, α is the parameter that captures the expected loss given default. We estimate three different specifications of our model: *Model With Pre-Default Costs* includes both the parameter γ to capture pre-default costs of financial distress and the parameter α that captures the expected loss given default, *Model Without Pre-Default Costs* exogenously sets $\gamma = 0$ so the costs of financial distress are experienced only at the time of default when the firm is expected to lose a portion α of its value, *Not Matching Prob. of Default* estimates the same parameters as the *Model Without Pre-Default Costs* but it excludes the Probability of Default moment (i.e. deviation from the empirical probability of default are not penalized in this specification). The *Not Matching Prob. of Default* allows for comparability with previous studies such as Glover (2016).

Parameter	Description	Model With Pre-Default Costs	Model Without Pre-Default Costs	Not Matching Prob. of Default
$\gamma \times 100$	Pre-default costs	6.531		
		(0.013)		
$\alpha \times 100$	Loss at-default	22.444	55.440	49.818
		(1.289)	(0.697)	(2.319)
$\beta \times 100$	Exposure to market risk	89.729	66.309	80.773
		(0.23)	(0.184)	(23.801)
$\mu \times 100$	Idyiosyncratic component	0.517	0.508	0.523
		(0.002)	(0.002)	(0.005)
$\sigma_F \times 100$	Volatility of EBIT	14.818	20.251	18.231
		(0.016)	(0.017)	(2.802)

Table 7

Simulation of firms with various α and γ

This table reports the average parameters' for defaulted firms. We simulate the firms according to the following methodology. We fix the parameters μ, β and σ_F to the estimated parameters from Table 6. We let α and γ be distributed according to a bivariate Normal distribution with means m_α and m_γ equal to the estimated values of α and γ from Table 6. A detailed explanation is provided in **??**. We consider three different cases for the correlation between α and γ: zero correlation, correlation of 0.5 (positive) and correlation of -0.5 (negative). We set the standard deviations of α and γ equal to $\iota \times m_\alpha$ and $\iota \times m_\gamma$, respectively. We consider three different values of ι: $\iota = 0.6$ in Panel A, $\iota = 0.5$ in Panel B and $\iota = 0.4$ in Panel C. The column "$\bar{\alpha}_{Def}$" ("$\bar{\gamma}_{Def}$") reports the average loss given default (pre-default costs) for defaulted firms except for the row "Average Firm" in which case we report the parameter estimated in Table 6. The column "% chg α" ("% chg γ") reports the percentage change of the "$\bar{\alpha}_{Def}$" ("$\bar{\gamma}_{Def}$") for the defaulted firms with respect to the α (γ) for the Average Firm.

Panel A: $\iota = 0.6$

	Correlation α and γ	$\bar{\alpha}_{Def}$	**% chg α**	$\bar{\gamma}_{Def}$	**% chg γ**
Average Firm		22.444%		6.531%	
Defaulted Firms	**Zero**	20.253%	-9.761%	8.073%	23.604%
Defaulted Firms	**Positive**	22.379%	-0.289%	7.634%	16.888%
Defaulted Firms	**Negative**	18.054%	-19.560%	8.477%	29.792%

Panel B: $\iota = 0.5$

	Correlation α and γ	$\bar{\alpha}_{Def}$	**% chg α**	$\bar{\gamma}_{Def}$	**% chg γ**
Average Firm		22.444%		6.531%	
Defaulted Firms	**Zero**	20.619%	-8.132%	7.694%	17.804%
Defaulted Firms	**Positive**	22.293%	-0.674%	7.375%	12.918%
Defaulted Firms	**Negative**	18.852%	-16.001%	8.003%	22.530%

Panel C: $\iota = 0.4$

	Correlation α and γ	$\bar{\alpha}_{Def}$	**% chg α**	$\bar{\gamma}_{Def}$	**% chg γ**
Average Firm		22.444%		6.531%	
Defaulted Firms	**Zero**	21.056%	-6.181%	7.330%	12.223%
Defaulted Firms	**Positive**	22.253%	-0.849%	7.110%	8.858%
Defaulted Firms	**Negative**	19.850%	-11.559%	7.524%	15.199%

Table 8

Simulated Moments Estimation: Splits by Tangibility

This table describes the results for firms split by the tangibility. We define tangibility as "Property Plant and Equipment - Total (Net)" over Total Assets. Panel A reports the structural parameters estimated via Simulated Method of Moments (SMM). Standard errors are in parentheses. The parameter γ captures pre-default costs of financial distress (i.e. constant rate at which the firm loses value when it is in financial distress but has not defaulted yet), μ captures the risk-neutral expected earnings' growth rate, β is the exposure of the firm's EBIT to market risk, σ_F is the idiosyncratic volatility of the expected earnings' growth rate, α is the parameter that captures the expected loss given default. Panel B shows the empirical moments, the simulated moments, and the t-statistics for the difference between empirical and simulated moments. A detailed description of the moments is provided in Table 2.

Panel A: Parameter Estimates by Tangibility

	$\gamma \times 100$	$\alpha \times 100$	$\beta \times 100$	$\mu \times 100$	$\sigma_F \times 100$
Low Tang	2.137	54.335	64.781	0.752	18.904
	(0.215)	(0.461)	(8.745)	(0.001)	(0.624)
High Tang	7.354	12.826	111.559	0.247	15.148
	(0.401)	(0.629)	(6.520)	(0.020)	(0.510)

Panel B: Model Fit by Tangibility

		Quasi-Market Leverage	Prob. of Def. at 5 years	Operating ROA	Variance of Equity	Equity Returns	Excess Returns
Low Tang	Data	33.609	5.353	10.463	5.419	9.182	4.484
	Model	34.173	5.397	10.588	6.153	8.065	5.808
	t-stat	0.205	0.011	0.438	1.004	0.264	0.370
High Tang	Data	37.298	5.663	14.127	4.976	10.016	5.318
	Model	37.296	5.628	14.076	5.141	8.588	6.331
	t-stat	0.000	0.006	0.071	0.051	0.431	0.217

Table 9

Parameter Estimates: Industry Splits

This table reports the structural parameters estimated via Simulated Method of Moments (SMM) for various industries. Standard errors are in parentheses. The parameter γ captures pre-default costs of financial distress (i.e. constant rate at which the firm loses value when it is in financial distress but has not defaulted yet), μ captures a firm fixed effect for the expected earnings' growth rate, β is the exposure of the firm's cash flows to market risk, σ_F is a firm fixed effect for the volatility of the expected earnings' growth rate, α is the parameter that captures the expected loss given default.

	$\gamma \times 100$	$\alpha \times 100$	$\beta \times 100$	$\mu \times 100$	$\sigma_F \times 100$
Agriculture, Forestry, & Fishing	7.500	17.750	50.400	0.400	10.909
	(0.008)	(1.16)	(0.133)	(0.002)	(0.004)
Construction	4.100	10.300	20.300	0.963	9.273
	(0.008)	(0.507)	(0.127)	(0.002)	(0.015)
Manufacturing	4.625	35.600	110.200	0.600	19.273
	(0.01)	(0.783)	(0.202)	(0.002)	(0.01)
Mining	8.120	36.500	115.100	0.275	17.545
	(0.026)	(1.86)	(0.224)	(0.002)	(0.007)
Retail Trade	8.750	17.500	121.100	0.000	14.455
	(0.003)	(0.564)	(0.168)	(0.002)	(0.007)
Services	8.129	29.998	79.608	0.511	14.767
	(0.012)	(0.863)	(0.204)	(0.002)	(0.008)
Transportation & Public Utilities	5.625	30.625	75.600	0.600	15.727
	(0.004)	(0.928)	(0.19)	(0.002)	(0.009)
Wholesale Trade	3.750	21.500	57.500	0.625	13.909
	(0.002)	(0.656)	(0.192)	(0.002)	(0.021)
Average	6.325	24.972	78.726	0.497	14.482

Table 10

Simulated Moments Estimation: Industry Splits

This table shows the empirical moments and the simulated moments for different industries. The t-statistics for the difference between empirical and simulated moments are also reported. A detailed description of the moments is provided in Table 2.

		Quasi-Market Leverage	Probability of Default at 5 years	Operating ROA	Variance of Equity	Equity Returns	Excess Returns
Agriculture, Forestry, & Fishing	Data	40.287	6.555	12.274	3.903	1.185	-3.668
	Model	40.717	6.483	13.012	3.633	3.575	1.306
	t-stat	0.119	0.028	15.182	0.136	1.208	5.233
Construction	Data	52.452	8.568	8.954	5.835	9.589	4.891
	Model	45.676	8.954	9.104	5.534	2.996	0.760
	t-stat	29.701	0.802	0.636	0.168	9.192	3.611
Manufacturing	Data	34.443	4.465	11.727	4.755	10.801	6.043
	Model	31.744	4.431	11.638	5.803	9.840	7.583
	t-stat	4.711	0.006	0.223	2.045	0.195	0.502
Mining	Data	31.556	6.580	13.554	5.686	9.314	4.783
	Model	31.801	6.316	13.895	5.910	8.973	6.718
	t-stat	0.039	0.376	3.254	0.093	0.025	0.792
Retail Trade	Data	37.144	6.027	15.514	4.936	10.671	5.831
	Model	36.790	5.843	15.785	5.302	8.689	6.435
	t-stat	0.081	0.182	2.046	0.250	0.831	0.077
Services	Data	32.456	6.518	12.212	6.116	7.411	2.842
	Model	33.142	6.502	12.244	6.050	6.527	4.278
	t-stat	0.304	0.001	0.029	0.008	0.165	0.436
Transportation & Public Utilities	Data	35.538	6.070	11.517	5.893	7.494	2.763
	Model	35.659	6.018	11.631	5.805	7.048	4.805
	t-stat	0.009	0.015	0.364	0.015	0.042	0.882
Wholesale Trade	Data	42.821	5.561	11.468	4.630	8.497	3.687
	Model	41.599	5.452	11.452	4.928	6.292	4.047
	t-stat	0.966	0.064	0.007	0.165	1.028	0.027

Table 11

Estimation with Different Distress Boundaries

This table reports the structural parameters and the model fit for the estimation of the *Model With Pre-Default Costs* for different distress boundary. The *Model With Pre-Default Costs* includes both the parameter γ to capture costs of financial distress experienced prior to bankruptcy and the parameter α that captures the expected costs of default. The distress boundary is denoted with X_D, and it represents the level of EBIT below which the firm experiences pre-default costs of financial distress. In this table we estimate four different versions of the *Model With Pre-Default Costs* with various distress boundaries: $X_D = k \times C$ for $k \in \{2.0, 1.0, 0.5\}$. Panel A reports the estimated model parameters with standard errors in parentheses. The parameter γ captures pre-default costs of financial distress (i.e. constant rate at which the firm loses value when it is in financial distress but has not defaulted yet), μ captures a firm fixed effect for the expected earnings' growth rate, β is the exposure of the firm's cash flows to market risk, σ_F is a firm fixed effect for the volatility of the expected earnings' growth rate, α is the parameter that captures the expected loss given default. Panel B shows the empirical moments, the simulated moments, and the t-statistics for the difference between empirical and simulated moments. A detailed description of the moments is provided in Table 2.

Panel A: Parameter Estimates

	$\gamma \times 100$	$\alpha \times 100$	$\beta \times 100$	$\mu \times 100$	$\sigma_F \times 100$
$X_D = 2.0 \times C$	3.718	25.511	83.785	0.455	15.948
	(0.005)	(0.702)	(0.131)	(0.002)	(0.015)
$X_D = 1.0 \times C$	6.531	22.444	89.729	0.517	14.818
	(0.013)	(1.289)	(0.23)	(0.002)	(0.016)
$X_D = 0.5 \times C$	14.308	33.106	110.816	0.455	18.637
	(0.017)	(0.838)	(0.191)	(0.002)	(0.005)

Panel B: Model Fit

		Quasi-Market Leverage	Prob. of Def. at 5 years	Operating ROA	Variance of Equity	Equity Returns	Excess Returns
$X_D = 2.0 \times C$	Data	34.980	5.428	12.178	5.218	9.570	4.855
	Model	36.589	5.499	12.654	5.883	7.322	5.071
	t-stat	1.674	0.027	6.334	0.825	1.069	0.010
$X_D = 1.0 \times C$	Data	34.980	5.428	12.178	5.218	9.570	4.855
	Model	34.949	5.370	12.198	5.298	7.562	5.314
	t-stat	0.001	0.018	0.011	0.012	0.853	0.045
$X_D = 0.5 \times C$	Data	34.980	5.428	12.178	5.218	9.570	4.855
	Model	35.012	5.472	12.646	5.116	8.875	6.608
	t-stat	0.001	0.011	6.114	0.020	0.102	0.650

Table 12

Composition of Financial Distress Costs

This table presents a decomposition of total financial distress costs into pre-default costs and loss given default. The column $\% \, PDC$ presents the percentage of pre-default costs over the total distress costs (loss given default plus pre-default costs); the column $\% \, LGD$ presents the same percentage for the loss given default. We provide the expressions to value pre-default costs and loss given default in Appendix H. We present the results for different specifications of the distress boundary. The distress boundary is denoted with X_D, and it represents the level of EBIT below which the firm experiences pre-default costs of financial distress. We consider three different distress boundaries: $X_D = k \times C$ for $k \in \{2.0, 1.0, 0.5\}$.

	$\% \, PDC$	$\% \, LGD$	Total
$X_D = 2 \times C$	66.5%	33.5%	100%
$X_D = 1 \times C$	68.5%	31.5%	100%
$X_D = 0.5 \times C$	30.2%	69.8%	100%

Table 13
Credit Spreads

This table compares the Credit Spread in various estimations of our model. The *Model With Pre-Default Costs* includes both the parameter γ to capture pre-default costs of financial distress and the parameter α that captures the expected loss given default, *Model Without Pre-Default Costs* exogenously sets $\gamma = 0$ so the costs of financial distress are experienced only at the time of default when the firm is expected to lose a portion α of its value, *Not Matching Prob. of Default* estimates the same parameters as the *Model Without Pre-Default Costs* but it excludes the Probability of Default moment (i.e. deviation from the empirical probability of default are not penalized in this specification). The distress boundary is denoted with X_D, and it represents the level of EBIT below which the firm experiences pre-default costs of financial distress. We assume $X_D = 1 \times C$ for the models in the first three rows. In the last two rows, we report two different versions of the *Model With Pre-Default Costs* with distress boundaries: $X_D = 2 \times C$ and $X_D = 0.5 \times C$, respectively.

		Simulated Moments		Estimated Parameters of Distress Costs	
Model	Credit Spread	Leverage (%)	Prob. of Default (%)	α	γ
With Pre-Default Costs	2.35%	34.949	5.370	22.444	6.531
Without Pre-Default Costs	2.01%	35.662	5.307	55.440	
Not Matching Prob. of Default	1.91%	34.852	2.382	49.818	
$X_D = 2 \times C$	2.31%	36.589	5.499	25.511	3.718
$X_D = 0.5 \times C$	2.76%	35.012	5.472	33.106	14.308

Appendix A Pricing Kernel

We prove that under the risk neutral measure $\mathcal{Q}$, the firm's EBIT process is governed by the expression in Equation (3). Recall that the firm's EBIT under the physical probability space $\mathcal{P}$ follows the process defined in Equation (2). Let the (exogenous) pricing kernel be

$$\frac{d\xi_t}{\xi_t} = -rdt - \varphi dB_t^{A,\mathcal{P}} \tag{A.1}$$

where $\varphi = (\mu_A^{\mathcal{P}} - r)/\sigma_A$ is the market Sharpe ratio, and $B_t^{A,\mathcal{P}}$ is a standard Brownian Motion under the physical probability space.

We define the density process for the risk-neutral measure as

$$\nu_t = \mathbf{E}_t \left[\frac{d\mathcal{Q}}{d\mathcal{P}} \right]$$

Following Harrison and Kreps (1979), the density process and the pricing kernel are related as follows

$$\nu_t = \xi_t e^{\int_0^t r ds} = \xi_t e^{rt}$$

Applying Ito's lemma we have

$$d\nu_t = e^{rt} d\xi_t + \xi_t r e^{rt} dt \tag{A.2}$$

Substituting Equation (A.1) in Equation (A.2) and recalling that $\xi_t = \nu_t / e^{rt}$

$$d\nu_t = -\varphi \xi_t e^{rt} dB_t^{A,\mathcal{P}} \implies \frac{d\nu_t}{\nu_t} = -\varphi dB_t^{A,\mathcal{P}}$$

Applying the First Fundamental Theorem of Asset Pricing, we have

$$dB_t^{A,\mathcal{Q}} = dB_t^{A,\mathcal{P}} + \varphi dt \tag{A.3}$$

Substituting Equation (A.3) in Equation (2) we obtain that the firm's EBIT under the risk-neutral

$\mathcal{Q}$ measure is governed by the process

$$\frac{dX_t}{X_t} = \begin{cases} \mu dt + \beta\sigma_A dB_t^{A,\mathcal{Q}} + \sigma_F dB_t^F & \text{for } X_B < X_D < X_t \\ (\mu - \gamma)dt + \beta\sigma_A dB_t^{A,\mathcal{Q}} + \sigma_F dB_t^F & \text{for } X_B < X_t < X_D \end{cases} \tag{A.4}$$

The total firm volatility is

$$\sigma_X = \sqrt{(\beta\sigma_A)^2 + \sigma_F^2}$$

therefore we can re-write firm's EBIT process under the risk-neutral measure $\mathcal{Q}$ more compactly as follows

$$\frac{dX_t}{X_t} = \begin{cases} \mu dt + \sigma_X dB_t & \text{for } X_B < X_D \leq X_t \\ (\mu - \gamma)dt + \sigma_X dB_t & \text{for } X_B < X_t < X_D \end{cases} \tag{A.5}$$

where

$$dB_t = \frac{\beta\sigma_A}{\sigma_X} dB_t^{A,\mathcal{Q}} + \frac{\sigma_F}{\sigma_X} dB_t^F$$

is a Standard Brownian Motion under the risk-neutral measure. Equation (A.5) is exactly the firm's EBIT process under the risk neutral measure $\mathcal{Q}$ described in Equation (3).

Appendix B Arrow-Debreu securities

For ease of notation, let $V(X) \equiv V$ where $V(X)$ is the unlevered asset value defined in Equation (4). Denote the state when the firm is not distressed, $X_B < X_D \leq X_t \leq X_U$, as ND (No Distress state) and denote the state when it is distressed, $X_B < X_t < X_D < X_U$, as DS (Distressed State). Let $\mathbf{p}_{ND}^U(X)$ and $\mathbf{p}_{DS}^U(X)$ be the present value of \$1 to be received at the time of restructuring, contingent on restructuring occurring before default when the state is ND and DS, respectively. Using the standard no-arbitrage argument, these claims must satisfy the following system of partial differential equations (PDEs)

$$\begin{cases} \frac{\sigma^2}{2}V^2\frac{\partial^2 \mathbf{p}^U_{ND}(\cdot)}{\partial V^2} + \mu V\frac{\partial \mathbf{p}^U_{ND}(\cdot)}{\partial V} - r\mathbf{p}^U_{ND}(\cdot) = 0 & \text{for } X_{i,B} < X_{i,D} \leq X_{it} \\ \frac{\sigma^2}{2}V^2\frac{\partial^2 \mathbf{p}^U_{DS}(\cdot)}{\partial V^2} + (\mu-\gamma) V\frac{\partial \mathbf{p}^U_{DS}(\cdot)}{\partial V} - r\mathbf{p}^U_{DS}(\cdot) = 0 & \text{for } X_{i,B} < X_{it} < X_{iD} \end{cases}$$

The general solution to this system of PDEs is

$$\begin{cases} \mathbf{p}^U_{ND}(X) = H_{1,ND}V^{\beta_{1,ND}} + H_{2,ND}V^{\beta_{2,ND}} \\ \mathbf{p}^U_{DS}(X) = H_{1,DS}V^{\beta_{1,DS}} + H_{2,DS}V^{\beta_{2,DS}} \end{cases}$$

The constants $\beta_{1,ND}$, $\beta_{2,ND}$, $\beta_{1,DS}$, and $\beta_{2,DS}$ are

$$\begin{aligned} \beta_{1,ND} &= \frac{1}{\sigma^2}\left[-(\mu-0.5\sigma^2) + \sqrt{(\mu-0.5\sigma^2)^2 + 2r\sigma^2)}\right] \\ \beta_{2,ND} &= -\frac{1}{\sigma^2}\left[(\mu-0.5\sigma^2) + \sqrt{(\mu-0.5\sigma^2)^2 + 2r\sigma^2)}\right] \\ \beta_{1,DS} &= \frac{1}{\sigma^2}\left[-(\mu-\gamma-0.5\sigma^2) + \sqrt{(\mu-\gamma-0.5\sigma^2)^2 + 2r\sigma^2)}\right] \\ \beta_{2,DS} &= -\frac{1}{\sigma^2}\left[(\mu-\gamma-0.5\sigma^2) + \sqrt{(\mu-\gamma-0.5\sigma^2)^2 + 2r\sigma^2)}\right] \end{aligned}$$

The constants $H_{1,ND}$, $H_{2,ND}$, $H_{1,DS}$, and $H_{2,DS}$ are solved by imposing the following boundary conditions

$$\begin{aligned} &\mathbf{p}^U_{ND}(X_U) = 1 && \mathbf{p}^U_{DS}(X_B) = 0 \\ &\mathbf{p}^U_{ND}(X_D) = \mathbf{p}^U_{DS}(X_D) && \left.\frac{\partial \mathbf{p}^U_{ND}(X)}{\partial X}\right|_{X=X_D} = \left.\frac{\partial \mathbf{p}^U_{DS}(X)}{\partial X}\right|_{X=X_D} \end{aligned}$$

Re-writing the above conditions in matrix form yields the following solution

$$\begin{bmatrix} H_{1,ND} & H_{2,ND} & H_{1,DS} & H_{2,DS} \end{bmatrix}' = \mathbf{M}^{-1} \cdot \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}' \tag{B.1}$$

where

$$
\mathbf{M} = \begin{bmatrix} V_U^{\beta_{1,ND}} & V_U^{\beta_{2,ND}} & 0 & 0 \\ 0 & 0 & V_B^{\beta_{1,DS}} & V_B^{\beta_{2,DS}} \\ V_D^{\beta_{1,ND}} & V_D^{\beta_{2,ND}} & -V_D^{\beta_{1,DS}} & -V_D^{\beta_{2,DS}} \\ \beta_{1,ND}V_D^{\beta_{1,ND}} & \beta_{2,ND}V_D^{\beta_{2,ND}} & -\beta_{1,DS}V_D^{\beta_{1,DS}} & -\beta_{2,DS}V_D^{\beta_{2,DS}} \end{bmatrix} \tag{B.2}
$$

Let $\mathbf{p}_{ND}^B(X)$ and $\mathbf{p}_{DS}^B(X)$ be the present value of \$1 to be received at the time of default, contingent on default occurring before refinancing when the state is ND and DS, respectively. Using the standard no-arbitrage argument, these claims must satisfy the following PDEs

$$
\begin{cases} \frac{\sigma^2}{2}V^2\frac{\partial^2\mathbf{p}_{ND}^B(\cdot)}{\partial V^2} + \mu V\frac{\partial\mathbf{p}_{ND}^B(\cdot)}{\partial V} - r\mathbf{p}_{ND}^B(\cdot) = 0 & \text{for } X_{it} \geq X_{i,D} > X_{i,B} \\ \frac{\sigma^2}{2}V^2\frac{\partial^2\mathbf{p}_{DS}^B(\cdot)}{\partial V^2} + (\mu-\gamma)V\frac{\partial\mathbf{p}_{DS}^B(\cdot)}{\partial V} - r\mathbf{p}_{DS}^B(\cdot) = 0 & \text{for } X_{i,D} > X_{it} > X_{i,B} \end{cases}
$$

The general solution to this pair of PDEs is

$$
\begin{cases} \mathbf{p}_{ND}^B(X) = J_{1,ND}V^{\beta_{1,ND}} + J_{2,ND}V^{\beta_{2,ND}} \\ \mathbf{p}_{DS}^B(X) = J_{1,DS}V^{\beta_{1,DS}} + J_{2,DS}V^{\beta_{2,DS}} \end{cases}
$$

The constants $J_{1,ND}$, $J_{2,ND}$, $J_{1,DS}$, and $J_{2,DS}$ are solved by imposing the following boundary conditions

$$
\begin{aligned} &\mathbf{p}_{ND}^B(X_U) = 0 && \mathbf{p}_{DS}^B(X_B) = 1 \\ &\mathbf{p}_{ND}^B(X_D) = \mathbf{p}_{DS}^B(X_D) && \left.\frac{\partial\mathbf{p}_{ND}^B(X)}{\partial X}\right|_{X=X_D} = \left.\frac{\partial\mathbf{p}_{DS}^B(X)}{\partial X}\right|_{X=X_D} \end{aligned}
$$

Re-writing the above conditions in matrix form yields the following solution

$$
\begin{bmatrix} J_{1,ND} & J_{2,ND} & J_{1,DS} & J_{2,DS} \end{bmatrix}' = \mathbf{M}^{-1} \cdot \begin{bmatrix} 0 & 1 & 0 & 0 \end{bmatrix}' \tag{B.3}
$$

Appendix C Simulated Method of Moments

For each firm i, we calculate a vector of empirical moments, $h(Y_i)$, using the empirical data $Y_i = [y_{i,1}, y_{i,2}, \cdots, y_{i,T_i}]$ where T_i is the empirical sample length for firm i. We use six empirical moments: the operating ROA, the quasi-market leverage, the excess return of firm's equity with respect to the risk-free rate, the probability of default at 5 years, the variance and returns of equity. We estimate the parameters of the model, $\theta = [\alpha, \gamma, \mu, \sigma_F, \beta]$, using the Simulated Method of Moments (SMM) (Gourieroux and Monfort, 1996). The SMM searches for the vector of parameters to "fit" the simulated moments to their empirical counterparts. More specifically, we search for the vector of parameters that minimizes the weighted distance between the simulated and empirical moments, $\Lambda(\theta)$:

$$\theta^* = \arg\min_{\theta} \Lambda(\theta) \tag{C.1}$$

where

$$\Lambda(\theta) = g(\theta)'\hat{W}g(\theta)$$
$$g(\theta) = h(Y_i) - \frac{1}{S}\sum_{s=1}^{S} h\left(Y_s(\theta)\right)$$

where S is the number of simulations, θ is the vector of parameters, $Y_k(\theta)$ is the vector of the simulated data for the k-th simulation given parameters θ, and $\hat{W}$ is a positive definite weighting matrix. This estimator is known to be asymptotically normal for fixed S; for $T_i \to \infty$ the estimator's asymptotic distribution is

$$\sqrt{T_i}(\theta^* - \theta) \xrightarrow{d} \mathcal{N}\left(0, \mathrm{Var}(\theta^*)\right) \tag{C.2}$$

where

$$\mathrm{Var}(\theta^*) = \left(1 + \frac{1}{S}\right)\left[D' \cdot W \cdot D\right]^{-1}$$
$$D = \left.\frac{\partial g(\theta)}{\partial \theta'}\right|_{\theta=\theta^*}$$

W is the optimal weighting matrix. The optimal weighting matrix is chosen as to place greater weights on more precisely estimated moments (i.e. moments with lower variance):

$$\hat{W} = \left[\hat{\text{Var}}(h(Y_i))\right]^{-1} \tag{C.3}$$

The estimated variance-covariance matrix, $\hat{\text{Var}}(h(Y_i)$, is calculated using the influence function approach described in Erickson and Whited (2002) which has better finite sample properties as shown in Bazdresch, Kahn, and Whited (2017). This methodology ensures that $\hat{\text{Var}}(h(Y_i) \xrightarrow{p} \text{Var}(h(Y_i)$.

We conduct the estimation in three steps. First, we build a fine grid containing 20 equally spaced points for each parameter which implies evaluating the model on 3,200,000 points.[33] Second, we use the minimum found in the previous step as the mid-point and we build another grid of 20 equally spaced points for each parameter. Third, we use the results from the previous step as the starting values of a local minimization algorithm (Nelder-Mead) to achieve a more precise minimum.

Appendix D Macroeconomic Risk

In this appendix we provide robustness results using a model that includes macroeconomic risk (Bhamra et al. (2009), Bhamra et al. (2010), Chen (2010), Chen et al. (2019)). Specifically, we show that, in order for the model to match leverage ratio, a model with macroeconomic risk requires slightly lower values of *both* loss given default α *and* pre-default costs γ in order to match observed leverage ratios.

We use the model in Chen et al. (2019) which includes both macreconomic risk and pre-default costs of financial distress. We calibrate it as follows. We set the tax rate to 0.2 which is an approximation of the statutory corporate tax rate relative to personal taxes (Nikolov and Whited, 2014). As in Bhamra et al. (2010), we set the market volatility in the good state to be higher ($\sigma_M(G) = 0.12$) than the market volatility in the bad state ($\sigma_M(B) = 0.14$). The idiosyncratic volatility is set to

[33]Computations were performed on the Niagara supercomputer at the SciNet HPC Consortium (Ponce et al., 2019; Loken et al., 2010). SciNet is funded by: the Canada Foundation for Innovation; the Government of Ontario; Ontario Research Fund - Research Excellence; and the University of Toronto.

0.14 following We set the exposure to market risk equal to 1 in both the good and the bad state ($\beta(G) = \beta(B) = 1$). The idiosyncratic volatility is set to $\sigma_F(G) = \sigma_F(B) = 0.14$ in both the good and bad state. The total volatility of the firm in state i is $\sigma_X(i) = \sqrt{\sigma_F^2(i) + \beta^2(i) \times \sigma_M^2(i)}$.

Following the literature on macroeconomic risk in trade-off models of capital structure (Bhamra et al., 2009; Chen, 2010), we assume that the price of risk is higher in the bad state ($\theta(B) = 0.5$) compared to the good state ($\theta(G) = 0.4$).[34] The physical growth rate in state i when the firm is healthy is equal to $\widehat{\mu}(i) = \mu(i) + \beta(i) \times \sigma_M(i) \times \theta(i)$. We set the firm risk-neutral growth rates of EBIT equal to $\mu(G) = 0.02$ in the good state and $\mu(B) = -0.12$ in the bad state. This choice implies that the physical growth rates are equal to $\widehat{\mu}(G) = 0.068$ for the good state and $\widehat{\mu}(B) = -0.05$ for the bad state. We let probability of transitioning from the good to the bad state be $p_G = 0.1$ and the probability of transitioning from the bad to the good state be $p_B = 0.3$. We set the loss given default α to be the same between the two states and equal to 15%, which is approximately 2/3 of the 22.444% value estimated in Section 5. Similarly, we set the pre-default cost parameter γ to be equal to 4.3% which is approximately 2/3 of the 6.53% value estimated in Section 5.

We solve the model and look for the optimal leverage chosen by the firm given the above parameters. Table D.1 shows the optimal coupons, default and restructuring thresholds, and optimal leverage. Specifically, $C(i)$ indicates the optimal coupon if the firm refinances in state i, $X_B(i)$ is the optimal default threshold, $X_U(i)$ is the optimal restructuring threshold, $QML(i)$ indicates the optimal leverage (i.e. debt / (debt + market value of equity)) at the time of refinancing assuming that the firm refinanced its debt in the good state.

Panel A of Table D.1 shows that given the calibration of the model with macroeconomic risk, the optimal leverage is 0.3228 if the firm refinances in the good state and 0.4074 if it refinances in the bad state. Since the transition probabilities are $p_G = 0.1$ and $p_B = 0.3$, it implies that the average duration of a good state is 3 years while the bad one is 1 year. If we take the average optimal leverage weighted by the probabilities of being in each state, the optimal leverage is 0.344, a value that is very close to the one used in our main estimation in Section 5. It follows from Panel

[34] The excess market risk premium in a given economic state is equal to the market price of risk times the market volatility.

A of Table D.1 that using parameters α and γ that are approximately 2/3 as large as those used in our main estimation, we can obtain leverage ratios that are similar to those observed empirically. We choose not to estimate this model but elect to use a simpler version because it provides a better identification. In particular, estimating a model with macroeconomic risk requires the calibration of more parameters and it would be harder to identify compared to the model analyzed in this paper (see Section 2). This is because for each parameter, e.g. the growth rate of the economy, we have to calibrate/estimate at minimum two values (one for the good and one for the bad state) rather than one as in our model.

Table D.1

The optimal leverage, default boundaries, coupons and restructuring thresholds in a model with macroeconomic risk. We solve the model and look for the solution that maximizes firm value given the parameters described in Appendix D. For each state of the economy $i \in \{G, B\}$, $X_B(i)$ is the optimal default threshold, $X_U(i)$ is the optimal restructuring threshold, $QML(i)$ indicates the optimal leverage (i.e. debt / (debt + market value of equity)) at the time of refinancing. $C(i)$ is the optimal coupon if the firm refinances in state i.

$C(G)$	$C(B)$	$X_B(G)$	$X_B(B)$	$X_U(G)$	$X_U(B)$	$QML(G)$	$QML(B)$
0.287	0.246	0.177	0.218	15.683	31.010	0.3228	0.4074

Appendix E Variation in α and γ for defaulted firms

In this appendix, we introduce heterogeneity in our model and we analyze what the average α and γ are for defaulted firms. Glover (2016) shows that the average loss given default is approximately 45% for the firms in his sample. However, in a simulation exercise, he also shows that the average loss given default for defaulted firms is much lower (approximately equal to 25%), which means that there is a large selection bias between defaulted and non-defaulted firms. Here, we investigate an alternative explanation for the selection bias through the lens of pre-default costs of financial distress, which are omitted for simplicity in Glover (2016). As we explain below, our results show that accounting for pre-default costs of financial distress can potentially mitigate the selection bias. In Glover, the defaulted firms have an average loss given default (LGD) which is approximately 44% lower than the average firm (25% LGD for defaulted vs. 45% LGD for non-defaulted firms). Our results show that when we take into account for pre-default costs of financial distress the LGD for defaulted firms is 19.56% lower than non-defaulted firms (18.05% LGD for defaulted vs. 22.44% LGD for non-defaulted firms).

Our estimation in Section 5.1 provides the estimated parameters for the average firms in the economy but it does not give information on parameters' heterogeneity amongst firms. In order to evaluate whether defaulted firms differ from non-defaulted firms, we elect to make a distributional assumption on the distribution of the parameters α (loss given default) and γ (pre-default costs). We are not aware of studies that describe how these two parameters should be related empirically therefore we consider several scenarios that we detail below.

We simulate our model by fixing the parameters μ, β and σ_F to the estimated parameters for the average firm from Table 6. We let α and γ be distributed according to a bivariate Normal distribution with means m_α and m_γ equal to the estimated values of α and γ from Table 6. In addition to the means of α and γ, we need also their (unknown) variance-covariance matrix (i.e. correlation plus standard deviations). As for the standard deviations, we consider several cases to understand how this assumption affects the results. We set the standard deviations of α and γ equal to $\iota \times m_\alpha$ and $\iota \times m_\gamma$, respectively. We consider three different values of ι. In Table 7, we set $\iota = 0.6$ for Panel A, $\iota = 0.5$ for Panel B and $\iota = 0.4$ for Panel C. As for the correlation between α and γ, we consider three different cases: zero correlation, correlation of 0.5 (positive) and correlation of -0.5 (negative).

We simulate a total of $S = 10,000$ firms with heterogeneous α and γ. Specifically, for each firm i we draw the parameters α_i and γ_i from the multivariate normal distribution described above. We generate $N = 1,000$ paths for the EBIT of firm i using the parameters α_i and γ_i. From these 1,000 paths, we count the number of times that the firm would have defaulted (Def_i) and then calculate the default rate as $DefRate_i = Def_i/N$. We calculate the average parameters for defaulted firms by averaging over the $S = 10,000$ simulated firms. Specifically, we calculate the weighted averages for α and γ as follows

$$\bar{\alpha}_{Def} = \frac{1}{S}\sum_{i=1}^{S}\alpha_i \times \underbrace{\frac{DefRate_i}{\sum_{i=1}^{S} DefRate_i}}_{\substack{\text{Firm } i\text{'s weight given} \\ \text{its default rate}}} \qquad \bar{\gamma}_{Def} = \frac{1}{S}\sum_{i=1}^{S}\gamma_i \times \underbrace{\frac{DefRate_i}{\sum_{i=1}^{S} DefRate_i}}_{\substack{\text{Firm } i\text{'s weight given} \\ \text{its default rate}}}$$

where $\bar{\alpha}_{Def}$ is the average loss given default for defaulted firms, and $\bar{\gamma}_{Def}$ is the average pre-default costs parameter for defaulted firms. The weight for each firm i is calculated with respect to the default rate, which is a function of the firm specific parameters α_i and γ_i. The intuition is that for parameters values that lead to higher default rates, the weight is higher. We divide by the sum of the total default rates to ensure that the weights sum up to one.

We report the results of our simulations in Table 7. In each panel, the column "$\bar{\alpha}_{Def}$" ("$\bar{\gamma}_{Def}$") reports the average loss given default (pre-default costs) for defaulted firms except for the row "Average Firm" in which case we report the parameter estimated in Table 6. The column "% chg α" ("% chg γ") reports the percentage change of the "$\bar{\alpha}_{Def}$" ("$\bar{\gamma}_{Def}$") for the defaulted firms with respect to the α (γ) for the Average Firm. The average firm has α and γ equal to 22.444% and 6.531%, respectively, as shown in Table 6. In Panel A, our simulations show that, for $\iota = 0.6$ and zero correlation between α_i and γ_i, the average loss given default for defaulted firms $\bar{\alpha}_{Def}$ is 20.253%, a value that is 9.761% lower than the loss given default for the average firm. The percentage drop in the loss given default in our model (9.761%) is much more modest than that observed by Glover

(2016) in a model that omits pre-default costs ($\approx 44.4\%$). Also, we find that the average pre-default costs' parameter for defaulted firms $\bar{\gamma}_{Def}$ is 8.073%, a value that is 23.604% grater than the pre-default costs for the average firm. The increase in the pre-default costs parameter is much larger in percentage terms compared to the decrease in the loss given default confirming that defaulted firms have high pre-default costs and only slightly lower loss given default compared to the average firm. Panel A of Table 7 also confirms that these results for $\bar{\alpha}_{Def}$ and $\bar{\gamma}_{Def}$ hold not only for the case of zero correlation between α and γ but also for the case of positive and negative correlations. In the case of positive correlation $\bar{\alpha}_{Def}$ drops by 0.289% while $\bar{\gamma}_{Def}$ increases by 16.888% grater than the pre-default costs for the average firm. In the case of negative correlation $\bar{\alpha}_{Def}$ drops by 19.560% while $\bar{\gamma}_{Def}$ increases by 29.792% grater than the pre-default costs for the average firm. Regardless of the correlation, the decrease in $\bar{\alpha}_{Def}$ is always smaller than the increase in $\bar{\gamma}_{Def}$.

[Insert Table 7 here]

Our results provide new evidence on the characteristics of defaulted firms. In Glover (2016) the loss given default α is the only parameter that capture distress costs and, in the sample of defaulted firms, the average α is 44.4% lower than the average α for non-defaulted firms, which leads to conclude that defaulting firms have lower distress costs. Therefore, estimating financial distress costs on samples of defaulted firms might underestimate the ex-ante costs of financial distress. Our results imply a different mechanism. When we include pre-default costs of financial distress, we show that even if all firms have similar *total costs of financial distress*, there is a difference between defaulting and non-defaulting firms in terms of the composition of such total costs of financial distress. Defaulting firms have considerably higher pre-default costs than non-defaulting firms while also having slightly lower loss given default.

Appendix F Estimation using a model based PD

In our main results of Table 5 and Table 6 we used the average probability of default within 5 years using the actual default rates from Moody's. In this appendix we check that the results of our estimation are robust to the use of a model based probability of default. We consider the model in Bharath and Shumway (2008).

In Bharath and Shumway (2008), the probability of default in the year $t+1$ conditional on having survived until year t is $PD(t) = \phi(t) \times \exp(X(t)' \cdot \Gamma)$, where $\phi(t)$ is the baseline hazard rate, $X(t)$ is a vector of firms' characteristics and Γ is a vector of parameters. Bharath and Shumway (2008) consider several models in their paper and we choose the one that can be mapped to our theoretical model. Specifically we choose the model that uses the following covariates: log of equity ($ln(E)$), log of face value of debt ($ln(F)$), the reciprocal of equity's volatility ($1/\sigma_E$), and the market excess return ($r_{i,t-1} - r_{m,t-1}$).[35] These quantities are all generated by our model which allows us to apply this functional form to the simulated data. Empirically, we calculate the market value of equity from CRSP, the face value of debt is set to debt in current liabilities (COMPUSTAT data item dlcq) plus one-half of long-term debt (COMPUSTAT data item dlttq). The volatility of equity for each firm is calculated using daily data from CRSP and a rolling-window of 3-months (90 days). The market excess return is also calculated from CRSP by removing the market return from each firm's return.[36] We call this moment calculated using the functional form of Bharath and Shumway (2008) "Bharath-Shumway PD 1 year".

Table F.1 presents the results for the estimation of our model when we include "Bharath-Shumway PD 1 year" as an additional moment. Panel A shows the model-implied moments as well as the empirical ones. Similarly to our main results in Table 5, the model is able to fit the data reasonably well. In addition, Panel B of Table F.1 shows that the estimated parameters are not different from those estimated in Table 6, which did not use the "Bharath-Shumway PD 1 year" moment. This result confirms that our estimation is robust to the inclusion of a model-implied probability of default.

[35] Please see Model 4 in Table 3 of Bharath and Shumway (2008) for more details.

[36] As for the baseline hazard rate $\phi(t)$, we choose the value of 0.1. This parameter does not affect the estimation since the chosen value of $\phi(t)$ is applied to both empirical and simulated data. That is, if we had chosen a value of $\phi(t) = 0.2$, both the simulated and empirical moments would have been multiplied by 2 compared to our chosen value of 0.1. Since our goal is to minimize the absolute difference between the empirical and simulated moments, choosing any $\phi(t) > 0$ does not affect our results.

Table F.1

Simulated Moments Estimation using model based PD

This table describes the results for the average (same sample as in Table 5 and Table 6) but adding an extra moment based on the probability of default implied by the model of Bharath and Shumway (2008). Panel A shows the empirical moments, the simulated moments. A detailed description of the moments is provided in Table 2. Panel B reports the structural parameters estimated via Simulated Method of Moments (SMM). Standard errors are in parentheses. The parameter γ captures pre-default costs of financial distress (i.e. constant rate at which the firm loses value when it is in financial distress but has not defaulted yet), μ captures the risk-neutral expected earnings' growth rate, β is the exposure of the firm's EBIT to market risk, σ_F is the idiosyncratic volatility of the expected earnings' growth rate, α is the parameter that captures the expected loss given default.

Panel A

	Quasi-Market Leverage	Prob. of Def. at 5 years	Operating ROA	Variance of Equity	Equity Returns	Excess Returns	Bharath-Shumway PD 1 year
Data	34.980	5.428	12.178	5.218	9.570	4.855	1.113
Model	35.006	5.446	12.154	5.122	7.469	5.218	1.372

Panel B

$\gamma \times 100$	$\alpha \times 100$	$\beta \times 100$	$\mu \times 100$	$\sigma_F \times 100$
6.531	21.608	89.129	0.523	14.803
(0.226)	(0.3885)	(5.762)	(0.0523)	(0.0125)

Appendix G Estimation on splits by durability

Financial distress for a supplier of durable goods might compromise its ability to provide warranties, service and spare parts. Customers of such firms factor these risks into their decision making process and either demand a discount or avoid the firm's product all together (Titman, 1984). Either ways the firm suffers a loss that would not have incurred if it did not have debt. Hortaçsu et al. (2013) use a panel data set for used car sales and provide evidence that increases in the CDS spread of a car manufacturer leads to a sharp decrease in the price of the used cars of the same brand. Their paper supports the hypothesis that customers and suppliers are reluctant to buy and sell, respectively, products from a distressed firm because they are worried about the void of warranty, replacement of parts and services.

In order to split the data on the durability of their product, we need to define which industries are subject to distress due to customers being concerned for warranty voidance, etc. We start by looking at the classification of industries by durability of a firm's product in Gomes, Kogan, and Yogo (2009). They categorize the industries in 6 different buckets: Personal consumption expenditures on durable goods (d.), Personal consumption expenditures on nondurable goods (n.d.), Personal consumption expenditures (PCE) on services (s.), Gross private domestic investment (I), Government consumption expenditures and gross investment (G), and Net exports of goods and services (NX). For the purpose of our study, we are only interested in splitting the data into two categories: firms that produce products subject to warranties, services and parts (e.g. a producer of cars and trucks) and those firms whose output is immediately consumed (e.g. a coffee at the cafe' around the corner). We therefore categorize the industries into *Durables* and *Non Durables* by manually looking at the definition of the 4 digit SIC codes.[37] We start from the categories in Gomes et al. (2009) and then adjust them according the our categorization of durables vs. non-durables. After applying our categorization, we obtain a panel of 46,167 firm-quarter observations. The decrease in the number of firms compared to the full sample is due to the fact that not all industries in our full sample are categorized by Gomes et al. (2009).

[37] Our classification is available upon request.

Table G.1 shows the model fit for the estimation of the *Model With Pre-Default Costs* on the subsample of firms split by durability of their output (*Durables* vs. *Non Durables*). Panel A shows the empirical moments, the simulated moments, and the t-statistics for the difference between empirical and simulated moments. The model fits both durables' and non durables' firms well since none of the t-statistics signal a statistically significant difference between empirical and simulated moments. We present the estimated parameters and their standard errors (in parentheses) based on the splits by durability in Panel B of Table G.1. The parameter γ is equal to 6.585% for *Durables* and 6.171% for *Non Durables.* The loss given default for *Durables* and *Non Durables* are 13.375% and 15.187%.[38] The (risk-neutral) expected growth rate μ is higher for *Durables* ($\mu = 0.664\%$) compared to *Non Durables* ($\mu = 0.313\%$) while the estimated β parameters are very similar. The volatilities of EBIT σ_F are similar amongst the two sub-samples: $\sigma_F \approx 16\%$ for *Durables* and $\sigma_F \approx 15.5\%$ for *Non Durables.*

Our intuition is that *Durables*' firms should have higher net present value (NPV) of pre-default costs compared to *Non Durables.* Comparing the estimated values of γ does not provide any information regarding the NPV of pre-default costs since the other parameters (the drift μ, volatility σ_F, etc.) are different between the two sample splits. However, we can calculate the NPV of pre-default costs in our model. We show the results in Panel C. In column Full Model we standardize the value of the firm to \$100.00. In columns $\gamma = 0$ and $\alpha = 0$ we report the increase in firm value that there would have been if the parameters γ and α are set to zero, respectively, while keeping the firm's policies fixed.[39] By keeping the firm's policies fixed, the increases in firm value when we set $\gamma = 0$ and $\alpha = 0$ represent the NPV of pre-default costs and loss given default, respectively. Consistent with the intuition that *Durables* should have higher expected pre-default costs since customers might be worried about services, warranty and replacement of parts, Panel C shows that the NPV of pre-default costs is higher for *Durables* than *Non Durables.* The increase in firm

[38]Note that these values are lower than that of the average firm which is equal to 22.444%. However, as we explained above in this section, our sample of *Durables*' plus *Non Durables*' firms is composed of a panel of 46,167 firm-quarter observations. Such value is considerably lower than the full sample which has 101,032 firm-quarter observations.

[39]Keeping the firm's policies fixed means that we use the same coupon, default boundary and restructuring threshold as in the Full Model.

value when $\gamma = 0$ for *Durables* is 6.61% while the increase for *Non Durables* is more modest at 5.77%. These results confirm empirically our intuition that firms that produce durable products are expected to incur larger pre-default costs compared to *Non Durables*' firms.

Table G.1

Simulated Moments Estimation: Splits by Durability

This table describes the results for firms split by the durability of their output. Panel A shows the empirical moments, the simulated moments, and the t-statistics for the difference between empirical and simulated moments. A detailed description of the moments is provided in Table 2. Panel B reports the structural parameters estimated via Simulated Method of Moments (SMM). Standard errors are in parentheses. The parameter γ captures pre-default costs of financial distress (i.e. constant rate at which the firm loses value when it is in financial distress but has not defaulted yet), μ captures the risk-neutral expected earnings' growth rate, β is the exposure of the firm's EBIT to market risk, σ_F is the idiosyncratic volatility of the expected earnings' growth rate, α is the parameter that captures the expected loss given default. Panel C presents the net present values (NPV) of distress costs which are measured as the increase in firm value that there would have been if the parameters γ and α are set to zero. In column Full Model we standardize the value of the firm to \$100.00. In columns $\gamma = 0$ and $\alpha = 0$ we report the increase in firm value when γ and α are set to zero, respectively, while keeping the firm's policies fixed (coupon, default boundary and restructuring threshold).

Panel A: Model Fit by Durability

		Quasi-Market Leverage	Prob. of Def. at 5 years	Operating ROA	Variance of Equity	Equity Returns	Excess Returns
Durables	Data	33.232	4.665	11.235	5.620	9.952	5.288
	Model	33.781	4.635	11.177	5.479	8.960	6.708
	t-stat	0.195	0.005	0.097	0.037	0.208	0.426
Non Durables	Data	37.118	5.333	13.660	4.586	10.119	5.358
	Model	37.879	5.316	13.617	5.380	8.666	6.412
	t-stat	0.374	0.002	0.051	1.176	0.447	0.235

Panel B: Parameter Estimates by Durability

	$\gamma \times 100$	$\alpha \times 100$	$\beta \times 100$	$\mu \times 100$	$\sigma_F \times 100$
Durables	6.585	13.375	109.304	0.664	16.053
	(0.008)	(0.727)	(0.121)	(0.001)	(0.003)
Non Durables	6.171	15.187	105.484	0.313	15.464
	(0.002)	(0.4876)	(0.0576)	(0.0002)	(0.0043)

Panel C: Net Present Value of Distress Costs

		Full Model	$\gamma = 0$	$\alpha = 0$
Durables	Firm Value	100.00	106.61	100.78
	% Change		6.61%	0.78%
Non Durables	Firm Value	100.00	105.77	101.02
	% Change		5.77%	1.02%

Appendix H NPV of loss given default and pre-default costs

In this section we provide the pricing formulas for the present value of loss given default and pre-default costs. We calculate the value of the securities for the following 2 regions

$$\mathcal{R}_1 : X_B \leq X_t < X_D \tag{H.1}$$

$$\mathcal{R}_2 : X_D \leq X_t < X_U \tag{H.2}$$

where X_B is the default boundary, X_D is the distress threshold which we set equal to the current coupon, and X_U is the restructuring threshold.

NPV of loss given default

Here we provide the general solution as well as the boundary conditions to obtain the net present value of the loss given default. Let us define the loss given default security as $LGD(X_t, s)$ where s denotes whether the firm is distressed D (i.e. it is in region $\mathcal{R}_1$) or healthy H (i.e. it is in region $\mathcal{R}_2$). In the region $\mathcal{R}_1 : X_B \leq X_t < X_D$, the firm is distressed and it incurs pre-default costs. The LGD is realized when EBIT reaches X_B therefore the value function for a claim that pays the loss given default $LGD(X_t, D)$ has to satisfy the following ODE

$$r\, LGD(X_t, D) = (\mu - \gamma) X_t \frac{\partial LGD(X_t, D)}{\partial X_t} + \frac{1}{2} \sigma_X^2 X_t^2 \frac{\partial LGD(X_t, D)}{\partial X_t^2} \tag{H.3}$$

We guess that the functional form for $LGD(X_t, D)$ is

$$LGD(X_t, D) = a_{D1}^L X_t^{\psi_{D1}} + a_{D2}^L X_t^{\psi_{D2}} \tag{H.4}$$

where ψ_{D1} and ψ_{D2} are the positive and negative root of

$$\frac{1}{2} \sigma_X^2 y(y-1) + (\mu - \gamma) y - r = 0 \tag{H.5}$$

In the region $\mathcal{R}_2 : X_D \leq X_t < X_U$, the firm is not distressed and the value function for the $LGD(X_t, H)$ claim is:

$$rLGD(X_t, H) = \mu X_t \frac{\partial LGD(X_t, H)}{\partial X_t} + \frac{1}{2}\sigma_X^2 X_t^2 \frac{\partial LGD(X_t, H)}{\partial X_t^2} \tag{H.6}$$

We guess that the functional form for $LGD(X_t, H)$ is

$$LGD(X_t, H) = e_{H1}^L X_t^{\psi_{H1}} + e_{H2}^L X_t^{\psi_{H2}} \tag{H.7}$$

where ψ_{H1} and ψ_{H2} are the roots of

$$\frac{1}{2}\sigma_X^2 y(y-1) + \mu y - r = 0 \tag{H.8}$$

In order to find the 4 parameters a_{D1}^L, a_{D2}^L, e_{H1}^L, and e_{H2}^L, we use the following boundary conditions.

$$\lim_{X_t \downarrow X_B} LGD(X_t, D) = \alpha \times \frac{X_t}{r - \mu + \gamma} \tag{H.9}$$

$$\lim_{X_t \uparrow X_D} LGD(X_t, D) = \lim_{X_t \downarrow X_D} LGD(X_t, H) \tag{H.10}$$

$$\lim_{X_t \uparrow X_D} \frac{\partial LGD(X_t, D)}{\partial X_t} = \lim_{X_t \downarrow X_D} \frac{\partial LGD(X_t, H)}{\partial X_t} \tag{H.11}$$

$$\lim_{X_t \uparrow X_U} LGD(X_t, H) = \frac{X_U}{X_0} LDG(X_0, H) \tag{H.12}$$

Equation (H.9) implies that when the firm defaults (i.e. X_t reaches X_B) it incurs a deadweight loss proportional to the value of its assets. Equation (H.10) and Equation (H.11) are, respectively, a value-matching and smooth-pasting condition at X_D. Equation (H.12) implies that when firm reaches the restructuring threshold X_U, the value of all securities (including $LGD(\cdot)$) is scaled by the factor X_U/X_0 where X_0 is the value of EBIT at the time of the last refinancing.[40]

[40] See Section 2 for a discussion of the scaling property in this class of models.

NPV of pre-default costs of financial distress

Here we provide the general solution as well as the boundary conditions to obtain the net present value of pre-default costs. Let us define the pre-default cost security as $PDC(X_t, s)$ where s denotes whether the firm is distressed D (i.e. it is in region $\mathcal{R}_1$) or healthy H (i.e. it is in region $\mathcal{R}_2$). In the region $\mathcal{R}_1 : X_B \leq X_t < X_D$, the firm is distressed and it incurs pre-default costs as a loss of EBIT at a rate γ. Upon default, the firm incurs a loss given default but it stops accumulating pre-default costs of financial distress. The value function for such a claim has to satisfy the following ODE

$$r\,PDC(X_t, D) = \gamma X_t + (\mu - \gamma) X_t \frac{\partial PDC(X_t, D)}{\partial X_t} + \frac{1}{2}\sigma_X^2 X_t^2 \frac{\partial PDC(X_t, D)}{\partial X_t^2} \tag{H.13}$$

We guess that the functional form for $PDC(X_t, D)$ is

$$PDC(X_t, D) = \frac{\gamma X_t}{r - \mu + \gamma} + a^P_{D1} X_t^{\psi_{D1}} + a^P_{D2} X_t^{\psi_{D2}} \tag{H.14}$$

where ψ_{D1} and ψ_{D2} are provided in Equation (H.5)

In the region $\mathcal{R}_2 : X_D \leq X_t < X_U$, the firm is not distressed and the value function for the $PDC(X_t, H)$ claim is:

$$rPDC(X_t, H) = \mu X_t \frac{\partial PDC(X_t, H)}{\partial X_t} + \frac{1}{2}\sigma_X^2 X_t^2 \frac{\partial PDC(X_t, H)}{\partial X_t^2} \tag{H.15}$$

We guess that the functional form for $PDC(X_t, H)$ is

$$PDC(X_t, H) = e^P_{H1} X_t^{\psi_{H1}} + e^P_{H2} X_t^{\psi_{H2}} \tag{H.16}$$

where ψ_{H1} and ψ_{H2} are provided in Equation (H.8).

In order to find the 4 parameters a^P_{D1}, a^P_{D2}, e^P_{H1}, and e^P_{H2}, we use the following boundary conditions.

$$\lim_{X_t \downarrow X_B} PDC(X_t, D) = 0 \tag{H.17}$$

$$\lim_{X_t \uparrow X_D} PDC(X_t, D) = \lim_{X_t \downarrow X_D} PDC(X_t, H) \tag{H.18}$$

$$\lim_{X_t \uparrow X_D} \frac{\partial PDC(X_t, D)}{\partial X_t} = \lim_{X_t \downarrow X_D} \frac{\partial PDC(X_t, H)}{\partial X_t} \tag{H.19}$$

$$\lim_{X_t \uparrow X_U} PDC(X_t, H) = \frac{X_U}{X_0} LDG(X_0, H) \tag{H.20}$$

Equation (H.17) implies that upon default (i.e. X_t reaches X_B) the firms stops incurring pre-default costs of financial distress. Equation (H.18) and Equation (H.19) are, respectively, a value-matching and smooth-pasting condition at X_D. Equation (H.20) implies that when firm reaches the restructuring threshold X_U, the value of all securities (including $PDC(\cdot)$) is scaled by the factor X_U/X_0 where X_0 is the value of EBIT at the time of the last refinancing.