Optimal Unilateral Carbon Policy*

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Abstract

We consider climate policy in a world with international trade where one region imposes a climate policy and the rest of the world does not. Regional climate policy may generate inefficient shifts in the location of extraction, production, and consumption, an effect known as leakage. We derive the optimal unilateral policy and show how it can be implemented through taxes. The optimal policy involves (i) a tax on extraction at a rate equal to the marginal harm from emissions, (ii) a border adjustment on the import and export of energy and on the import, but not the export, of goods, with the border adjustment at a different (usually lower) rate than the extraction tax rate, and (iii) an export policy designed to expand the export margin. The optimal policy controls leakage by controlling the price of energy and exploits international trade to expand the reach of the climate policy. We calibrate and simulate the model to illustrate how the optimal policy compares to more traditional policies.

Keywords: carbon taxes, border adjustments, leakage
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1 Introduction

Global negotiations have given up trying to achieve a uniform approach to climate change, such as a harmonized global carbon tax. Instead, current negotiations focus on achieving uniform participation, with each country pursuing its own approach and its own level of emissions reductions. As a result, policies to control emissions of carbon dioxide vary widely by country, and are likely to continue to do so for the indefinite future.

Widely varying carbon policies potentially affect patterns of trade, the location of extraction, production and consumption, the effectiveness of the policies, and the welfare of people in various countries or regions. These effects are of critical importance to the design of carbon policy and to its political feasibility. For example, trade and location effects were central to the design of the European Union Emissions Trading System, the Regional Greenhouse Gas Initiative, and California's carbon pricing system. One of the reasons that the United States did not ratify the Kyoto Protocol was concern about the lack of emissions policies in developing countries and the resulting trade effects. Unless concerns about the effects of differential carbon prices are addressed, it may be difficult to achieve significant reductions in global emissions.

To address this problem, we develop an analytic general equilibrium model of carbon pricing and trade, where one region imposes a carbon policy and the rest of the world does not. The model is a mix of Markusen (1975) and Dornbusch, Fisher, and Samuelson (1977; henceforth DFS). Our solution strategy borrows from Costinot, Donaldson, Vogel, and Werning (2015; henceforth CDVW). We solve for the outcomes that are optimal for the region imposing the policy and then show how those outcomes can be implemented in a decentralized equilibrium using taxes and subsidies.

Our solution to the model suggests a novel approach to the problem of unilateral carbon pricing, one where increasing the extent of trade actually improves outcomes rather than making them worse. The approach involves imposing a domestic carbon tax on the extraction of fossil fuels at a rate equal to the marginal harm from emissions, along with: (i) what we call partial or imperfect border adjustments and (ii) an export policy designed to expand low-carbon exports from the carbon-pricing region to the rest of the world. The partial border adjustment is a tax on imports of fossil fuels, a rebate of prior taxes paid on exports of fossil fuels, and a tax on imported goods based on the energy used in their production. It is partial in that the border adjustment rate is not the same as (and is typically lower than) the rate of the underlying extraction tax. Furthermore, the border adjustment does not apply to the energy embodied in exported goods. Rather than rebating prior taxes paid, as in a conventional border adjustment, the export policy expands exports via fixed subsidies not determined by prior taxes. In effect, the taxing region maximizes the reach of its carbon tax to include all goods produced domestically (regardless of where they are consumed), all goods consumed domestically (regardless of where they are produced), and, moreover, expands the scope of its exports to further broaden the tax base. As the extent of trade increases, the taxing region is able to expand the tax base further, generating better outcomes.

To understand the quantitative implications of our analysis, we calibrate the model and solve it numerically. In our core calibration, we assume that the OECD countries impose a carbon price and the rest of the world does not. We compare the optimal policy to more conventional policies: (i) a tax only on the extraction of fossil fuels (as suggested by Metcalf and Weisbach (2009)); (ii) a tax only on the use of energy in production (which is how most cap and trade systems work); and (iii) a tax on production combined with conventional border adjustments (which is the structure of many recent carbon tax proposals). Conventional border adjustments shift a tax on the use of energy in domestic production to a tax on the energy embodied in domestic consumption, so we think of (iii) as a tax only on implicit consumption of carbon. We also examine the three pairwise combinations of the conventional policies: a hybrid of an extraction tax and a production tax, a hybrid of an extraction tax and a consumption tax, and a hybrid of a production tax and a consumption tax, in each case choosing the optimal mix of the two taxes.

Two of the hybrid taxes perform particularly well in our simulations, the hybrids involving an extraction tax. These two policies do not perform quite as well as the optimal tax, but considerably outperform the more standard production and consumption taxes. The combination of an extraction tax and a production tax would also be much simpler to impose than the other choices. It could be imposed with a nominal tax on extraction combined with border adjustments (at a lower rate) on the imports and exports of energy, but not goods. As suggested by Metcalf and Weisbach (2009), an extraction tax would be easy to impose because there are a relatively small number of large extractors who would need to remit taxes. Border adjustments on energy would also be easy to impose because imports and exports of energy are already carefully tracked. As a result, the simulations suggest that the combination of an extraction tax and a production is a promising policy to explore.[1]

To understand why these hybrids and the optimal tax perform so much better than conventional taxes on emissions from production, we look at the effects of various taxes on the price of energy. Taxes on emissions from production or those same taxes with border adjustments reduce the demand for energy, lowering the global price. This reduction in the global price of energy creates incentives to expand activities in non-taxing regions. The two extraction tax hybrids and the optimal tax moderate the change in the price of energy by combining demand-side taxes with a tax on the supply of energy. This allows the taxing region to control the effects of its policies on the activities in the non-taxing region.

Our core model does not include renewable energy, and stimulating renewables is often seen as a central goal of carbon pricing. To examine this issue, we extend the analysis to show that including renewables only requires modest adjustments to the optimal policy. Not surprisingly, renewables are exempt from the tax on extraction. If they can be sold in the market at the same price as fossil fuels, this exemption stimulates the production of renewables. In addition, while the optimal policy attempts to limit increases in fossil fuel extraction in other countries, it does not do so for renewables because the additional use of renewables in other countries does not generate harm.

The paper proceeds as follows. The remainder of this section provides additional motivation and reviews the relevant literature. Those eager to get to the foundations of our analysis may skip to Section 2, which lays out the basic elements of the model and characterizes the competitive equilibrium absent taxes.

Section 3 solves the problem of a planner designing an optimal carbon policy for one region with the other region behaving as in the competitive equilibrium. In Section 4 we derive a set of taxes and subsidies that implement the optimal policy, which we take to be the policy recommendations of this analysis. We explore the quantitative implications of the optimal policy in Section 5, using a calibrated version of the model. Section 6 extends the analysis to include a renewable energy sector. Section 7 concludes.

[1] While we do not address legal issues, it is also likely that the extraction/production hybrid raises fewer concerns about WTO compatibility than do the optimal tax or conventional border adjustments imposed on goods.

1.1 Background

As noted, the possibility that varying carbon prices in different regions might affect patterns of trade, the location of extraction, production, or consumption, and the effectiveness of carbon prices has been a central concern in the design of carbon policy. Our central motivation is to understand these effects and the optimal response to them. A related motivation is that there is a virtual zoo of possible responses, and, while there has been extensive work analyzing some of the most prominent responses, so far it has not been clear how to pick among the full range. That is, the question is not simply whether adding border adjustments to a conventional production tax is desirable, which is the focus of much of the literature. Instead, there are a wide variety of policies, and we need a method of picking among them. To illustrate this latter problem, we describe the range of possibilities here, along the way defining terms that will be useful in understanding the optimal, decentralized solution.

For simplicity, we assume (here and throughout the paper) that the price on carbon is imposed via a tax rather than a cap and trade system. Although there may be differences between taxes and cap and trade systems (e.g., Weitzman (1974)), these differences are not relevant for our purposes. We also only focus on carbon emissions from fossil fuels, which have been the central focus of existing and proposed carbon prices, ignoring agriculture and deforestation, two other major sources of emissions. For the most part, we also set aside administration concerns, including the problems of imposing border adjustments discussed in Kortum and Weisbach (2017). Doing so allows us to understand the shape of the optimal policy, which is a necessary step to designing administrable policies.

Normally, Pigouvian taxes need to be imposed directly on the externality-causing activity rather than on imperfect proxies. There is, however, an almost one-for-one relationship between fossil fuel inputs into the economy and eventual emissions. That is, almost all carbon molecules that enter the economy as fossil fuels are eventually emitted as CO_2 through combustion. This fact means that in a closed economy we can tax carbon at any, or multiple, stages of production without losing accuracy.

Metcalf and Weisbach (2009) exploit this fact to suggest imposing a carbon tax upstream on the extraction of fossil fuels (or very nearly so). They reasoned that there are a small number of large, sophisticated extractors, compared to a much larger number of manufacturers using fossil fuels and a vastly larger number of consumers of products made using fossil fuels. They estimated that the United States could tax essentially all domestic extraction of fossil fuels by taxing just 2,500 entities, compared to, say, the roughly 250 million vehicle tailpipes, among many other items, that would have to be taxed with a direct tax on consumers.

In a closed economy, a tax on extraction would be the same as a tax anywhere else in the chain of production (but for administrative costs). A tax on extraction would be embedded in the price of the fuel, causing manufacturers and consumers (as well as extractors) to internalize climate externalities. This is not true, however, in an open economy. Extraction taxes increase the pre-tax price of fossil fuels. If t_e is the extraction tax and p_e the pre-tax price of energy, extractors receive $p_e - t_e$ after tax. Unless the tax is entirely borne by extractors, p_e will go up. Because the price of fossil fuels goes up, extraction taxes cause foreign extractors, not subject to the tax, to increase extraction, generating what we call extraction leakage. Extraction leakage reduces the effectiveness of an extraction tax. To the extent Foreign emissions go up because of extraction leakage, the taxing region suffers harm that it might otherwise have avoided.

While extraction taxes cause a shift in where extraction occurs, on their own they do not shift where production and consumption occur. If there is a global price for energy, all producers and all consumers, globally, see the same higher price for energy generated by the extraction tax in the taxing region. They all adjust their production and consumption accordingly, with no particular differentiation between actors in the taxing region and the non-taxing region.

Actual carbon prices are usually imposed on emissions from domestic production—that is, on the smokestack—rather than on extraction. For example, the European Union Emissions Trading System is on emissions from industrial use of fossil fuels.[2]

With a production tax at rate t_p, producers pay $p_e + t_p$ for energy, increasing the after-tax price of energy. Once again, in a closed economy, the effects of taxing production would be the same as taxing extraction. In an open economy, however, their effects will not be the same. Production taxes lower the global price of energy because demand will go down: producers will

[2]Some recent proposals in the United States require extractors rather than producers to remit taxes. The taxes, however, still fall on domestic extraction because they impose taxes on imports of fossil fuels and rebate taxes on exports of fossil fuels.

shift to cleaner manufacturing techniques and consumers will demand fewer energy-intensive goods.

To the extent there is a global price of energy, all extractors, globally, see a lower price of energy and extract less. There is no extraction leakage with a pure production tax. That is, shifting away from an extraction tax toward a production tax moderates extraction leakage by moderating the price-increasing effect of an extraction tax (an effect we will see in our optimal solution).

Production taxes however, cause production to shift to untaxed regions because they reduce the comparative advantage of producers in the taxed region. This effect is known as production leakage, or because of the predominance of production taxes, often just leakage. Leakage is generally taken as the central measure of the (in)effectiveness of a carbon policy. Fowlie (2009) called it the defining issue in the design of regional climate policies

If there were no trade costs, production taxes would not affect the location of consumption. All consumers, even those abroad, who purchase goods produced in the taxing region would face a higher price for those goods. And all consumers, even those in the taxing region, would see a lower price for goods produced abroad. Production taxes affect where goods are produced but not where they are consumed. With trade costs, however, taxes on production may shift where consumption takes place because trade costs tie production and consumption together to some extent.

Finally, a carbon tax can be imposed directly on consumption. A tax on consumption would be based on the emissions associated with each good when it was produced. For example, if a consumer buys a toaster, the consumer would pay a tax based on the emissions from the production of the toaster. Because of the very large number of products and consumers, and the difficulty of determining the tax, carbon taxes are not normally proposed to be imposed this way. Gasoline taxes, however, might be thought of as a version of a consumption tax, and these are collected at the pump.

These three "pure" taxes, can be combined. For example, a country that wants to impose a \$100/ton tax on emissions of CO_2 could impose a \$50/ton tax on extraction, a \$30/ton tax on production, and a \$20/ton tax on consumption. As we will suggest, the right mix allows the country to moderate the effects of each of the pure taxes. For example, imposing both an extraction tax and a production tax can balance the negative effects on the location of extraction that arise from a pure extraction tax with the negative effects on the location of production from a pure production tax.

The last piece of terminology is "carbon border adjustments" or simply border adjustments.[3] Border adjustments are taxes on imports or rebates of prior taxes paid on exports. They can apply to either fossil fuels or goods, or both. For fossil fuels, the border adjustment is on the carbon content of the fossil fuel. For goods, the border adjustment is on the carbon emissions from the production of the good, what we call the embodied carbon or embodied energy. Kortum and Weisbach (2017) provide a more detailed description of border adjustments.

Border adjustments shift the tax downstream. For example, an extraction tax with border adjustments on the import and export of fuels becomes a tax on domestic production. Any fuel that is extracted domestically but exported has the tax rebated, and any fuel that it extracted abroad but imported has a tax imposed. All fuel used domestically, and only that fuel, bears a tax. Therefore, we can equivalently impose an extraction tax plus a border adjustment or a production tax. They differ only in their nominal description. Similarly, a border adjustment on imports and exports of goods shifts the tax from production to domestic consumption, and we can equivalently impose a production tax plus a border adjustment or a consumption tax.

Full or perfect border adjustments apply equally to imports and exports and are imposed at the same rate as the underlying tax. Border adjustments can be imposed at a different rate than the underlying tax. If the rate is less than the underlying tax, we can think of the border adjustment as shifting that portion downstream. For example, if a nominal extraction tax is imposed at $t_e = \$100$/ton of CO_2, and the border adjustment is at $t_b = \$75$/ton, we can think of this as an effective tax on extraction of $\tilde{t}_e = \$25$/ton tax on extraction and a tax of $\tilde{t}_p = \$75$/ton tax on production. Therefore, we can implement combinations of the three pure taxes via nominal taxes and border adjustments imposed at different rates than the underlying tax.

Border adjustments can also be imperfect because they apply differently to imports and exports. For example, they can be applied to imports but

[3] The term "border adjustment" is most often used in connection with destination-based VATs, widely used throughout the world. Border adjustments in this context are rebates of prior VAT paid when a good is exported and the imposition of VAT when a good is imported.

The term "carbon border adjustment" is a border adjustment based on the carbon content of goods including the carbon emitted during production, rather than their value (as in a VAT). For simplicity, we shorten the term to just "border adjustment" because the usage is unambiguous here.

not exports. A production tax with a border adjustment applied only to imports becomes a tax on all domestic production and on all domestic consumption generating a broader tax base than any of the three pure taxes. More generally, the border adjustment can be applied at different rates to imports and exports (and both those rates might be different than the rate of the underlying tax). We can decompose the effects in the same way as suggested above.[4]

As can be seen, there are a large number of possible taxes. Our goal is to understand the optimal mix of these possibilities for taxing regions.

1.2 Prior Literature

Because of its prominence, there is a voluminous prior literature studying this problem. The overwhelming majority of studies use computable general equilibrium models to simulate carbon taxes and border adjustments. By our count, there are over 50 CGE studies of the general problem of differential carbon prices in the peer-reviewed literature (and many more in the gray literature) and each study considers multiple different scenarios, which means that there are hundreds of simulations of the problem.[5] For example, Branger and Quirion (2014) perform a meta-analysis of 25 studies of differential carbon taxes (20 of which were CGE studies, 5 of which were partial equilibrium studies). These 25 studies, which make up only a portion of the literature, had 310 different modeled scenarios.

CGE studies almost uniformly use leakage as their measure of the effects of differential carbon prices. Leakage is commonly defined as the increase in emissions in non-taxing regions as a percentage of the reduction in emissions in the taxing region. (Hence, 100% leakage means the policy is totally ineffective in reducing global emissions.) Leakage estimates fall within a relatively consistent range. The Branger and Quirion meta-study finds leakage rates between 5% and 25% without border adjustments. They also find that

[4]Border adjustments might apply only to a subset of goods, such as only to goods that are particularly energy intensive. Many border adjustment proposals are limited in this way, in large part to minimize administrative costs. Modern economies import and export a vast number of different goods, and computing accurate border adjustments for each of these goods would be difficult. By imposing border adjustments only where their effects are likely to be large, the administrative costs can be reduced. Because we abstract away from implementation costs, we do not consider this type of border adjustment.

[5]For surveys of the leakage literature, see Droge et al. 2009, Zhang 2012 and Metz et al. 2007

border adjustments reduce leakage by about a third to be within a range of 2% to 12% with a mean value of 8%. Similarly, the Energy Modeling Forum commissioned 12 modeling groups to study the effects of border adjustments on leakage using a common data set and common set of scenarios. Bohringer et al. (2012). They considered emissions prices in the Kyoto Protocol Annex B countries (roughly the OECD) that reduce global emissions by about 9.5%. Without border adjustments, leakage rates were in the range of 5% to 19% with a mean value of 12%. They also find that border adjustments reduce leakage by about a third, with a range between 2% and 12% and a mean value of 8%. Elliott et al (2013) replicated 19 prior studies within their own CGE model, finding leakage rates between 15% and 30% for a tax on Annex B countries that reduced global emissions by about 13%.[6]

We use an analytic general equilibrium model of trade to study the problem. This approach allows us to uncover the underlying economic logic for why some policies perform better than others, although it means that our quantitative analysis is more illustrative than definitive because the model is stripped down. There are a small number of studies that precede us in this approach. The classic study, which we build on, is Markusen (1975). Markusen analyzes a two-country, two-good model in which production of one of the goods generates pollution that harms both countries. Writing before climate change was a widespread concern, he considers a simple pollutant, such as the release of chemicals into Lake Erie by polluters in the United States, which harms Canada (as well as the United States). One of the countries imposes policies to address the pollution; the other is passive. Markusen finds that the optimal tax is a Pigouvian tax on the dirty good combined with a tariff (if the good is imported) or a subsidy (if it is exported). The optimal tariff or subsidy combines terms of trade considerations and considerations related to leakage and is generally lower than the Pigouvian tax.[7]

[6] A smaller number of studies focus on the effects of carbon taxes on particular energy-intensive and trade-exposed sectors. For example, Fowlie et al (2016) consider the effects of a carbon price on the Portland cement industry. They find that a carbon price has the potential to increase distortions associated with market power in that industry. Leakage compounds these costs. They find that border adjustments induce negative leakage because of how industry actors respond, and can generate significant welfare gains at high carbon prices.

[7] Hoel (1996) generalizes Markusen's analysis and produces similar results in the context of climate change and carbon taxes. He also considers the case where the country may not impose tariffs. In this case, the optimal policy will involve carbon taxes that vary by

2 Basic Model

Two countries, Home and Foreign, are endowed with labor, L and L^*, as well as energy deposits E and E^*. The $*$ distinguishes Foreign from Home, whose carbon policy we seek to optimize.

Each country has three sectors: energy e, goods g, and services s. Energy is extracted from deposits using labor, goods are produced by combining labor and energy, and services are provided with labor only. Labor is perfectly mobile across the three sectors within a country.[8] As in DFS, goods come in a continuum, indexed by $j \in [0,1]$.

2.1 Preferences

We denote services consumption by C_s and define an index of goods consumption C_g by:

$$C_g = \left(\int_0^1 c_j^{(\sigma-1)/\sigma} dj \right)^{\sigma/(\sigma-1)},$$

where $\sigma > 0$ is the elasticity of substitution across the individual goods j. Preferences in Home are:

$$U(C_s, C_g, Q_e^W) = C_s + \eta^{1/\sigma} \frac{C_g^{1-1/\sigma} - 1}{1 - 1/\sigma} - \varphi Q_e^W,$$

where η governs Home's overall demand for goods.[9] Home's marginal harm from global emissions is φ, which multiplies global energy extraction, $Q_e^W = Q_e + Q_e^*$.[10]

Preferences in Foreign are the same except with η^* in place of η, σ^* in

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sector (even though the harms from emissions do not vary by sector). There are a number of other analytic models of the problem, including Holladay et al (2018), Hemous (2016), Baylis et al (2014), Jakob, Marschinski and Hubler (2013), Fischer and Fox (2012, 2011), and Hoel (1994).

[8] What we call "labor" can be interpreted as a combination of labor and capital used to extract energy, produce goods, and provide services.

[9] We follow Grossman and Helpman (1994) in adopting quasi-linear preferences, which greatly simplifies the analysis.

[10] Prior to introducing renewable energy in Section 6, we equate energy with fossil fuel, measured by its carbon content.

place of σ, and φ^* in place of φ.[11] Throughout we assume $C_s > 0$ and $C_s^* > 0$, a condition that is easily checked. We restrict $\sigma^* \leq 1$ since inelastic Foreign demand simplifies the solution for Home's optimal exports of individual goods.[12]

2.2 Technology

Energy is deposited in a continuum of fields, characterized by different costs of extraction. The quantity of energy that can be extracted at a unit labor requirement below a is given by $E(a)$ in Home and $E^*(a)$ in Foreign. We assume efficient extraction within each region so that low cost fields are tapped first. The labor L_e employed in Home to extract energy Q_e satisfies:

$$L_e = \int_0^{\bar{a}} a\, dE(a), \tag{1}$$

and

$$Q_e = E(\bar{a}), \tag{2}$$

so that $\bar{a}$ is the highest-cost field that is tapped when Home extracts Q_e. The output of the energy sector is in turn used as an intermediate input by the goods sector.

Goods $j \in [0, 1]$ are produced with input requirement a_j in Home using a Cobb-Douglas combination of labor and energy:

$$q_j = \frac{1}{\nu a_j} L_j^{\alpha} E_j^{1-\alpha}, \tag{3}$$

where L_j is the labor input, E_j is the energy input, α is the output elasticity of labor, and $\nu = \alpha^{\alpha} (1-\alpha)^{1-\alpha}$. The production function in Foreign is the same, but with a_j^* in place of a_j.[13]

Services are provided in both countries with a unit labor requirement.

[11] Since the harm from climate change is inflicted on individuals, we expect the magnitudes of φ and φ^* to be roughly proportional to the populations of Home and Foreign, holding fixed $\varphi^W = \varphi + \varphi^*$.

[12] We can relax this assumption by following the strategy in CDVW. For $\sigma^* = 1$ preferences in Foreign simplify to:

$$U^* = C_s^* + \eta^* \int_0^1 \ln c_j^* dj - \varphi^* Q_e^W,$$

and likewise in Home.

[13] In line with our Ricardian assumptions, we treat α as common across goods and

2.3 International Trade

We assume that energy and services are costlessly traded between Home and Foreign. We take services to be the numéraire, with price 1.[14] Energy is traded at a world price p_e.

Trade in the continuum of manufactured goods follows DFS. Goods are arranged in decreasing order of Home's comparative advantage:

$$\frac{a_j^*}{a_j} = F(j), \tag{4}$$

with $F(j)$ a strictly decreasing continuous function.[15]

Goods are traded subject to iceberg costs on Home's exports $\tau \geq 1$ and on Home's imports $\tau^* \geq 1$. The total input requirement for Home to supply good j to Foreign is thus τa_j and for Foreign to supply good j to Home $\tau^* a_j^*$.

2.4 Labor and Energy Requirements

We now introduce a notation for energy and labor input requirements that will prove convenient throughout the rest of the paper. At a given energy intensity:

$$z_j = E_j / L_j$$

we can invert the production function (3) to get the unit energy requirement for good j:

$$e_j(z_j) = \nu a_j z_j^{\alpha}, \tag{5}$$

in Home, with corresponding unit labor requirement $l_j(z_j) = e_j(z_j)/z_j$. Unit energy and labor requirements in Foreign, $e_j^*(z_j)$ and $l_j^*(z_j)$, are the same but with a_j^* in place of a_j.[16]

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countries. Including the constant ν in the production function simplifies expressions for costs that will appear later. This technology is nearly identical to the production and pollution technology in Shapiro and Walker (2018), although α here is $1 - \alpha$ there. They use it to assess the reduction of air pollution in US manufacturing from 1990-2008.

[14] We will assume that $Q_s^* > 0$ so that, given the unit labor requirement for services, the wage in Foreign is $w^* = 1$. This outcome is guaranteed with a large enough labor endowment in Foreign.

[15] In order to have well defined integrals in what follows, we also assume that a_j and a_j^* can be treated as continuous functions of j.

[16] Our *unit energy requirement*, $e_j(z_j)$, is sometimes called *emissions intensity* in the environmental economics literature, e.g. Shapiro and Walker (2018). We instead use the

So as not to constrain the optimal policy, the energy intensity for good j may depend not only on where the good is produced but also on where it is shipped. To handle that possibility requires some additional notation.

For each good j we distinguish between Home's exports, $x_j \geq 0$ and Home's production for consumption in Home, $y_j = q_j - \tau x_j \geq 0$. We also distinguish between Home's imports, $m_j \geq 0$ and Foreign's production for consumption in Foreign, $y_j^* = q_j^* - \tau^* m_j \geq 0$. (Note that we define exports and imports in terms of the quantity that reaches the destination.) For each good j we allow for the possibility of four different energy intensities z_j^y, z_j^x, z_j^m, and z_j^*, one for each of the four lines of production y_j, x_j, m_j, and y_j^*.

2.5 Carbon Accounting

We take a unit of energy to be a unit of carbon. Energy can be extracted in both countries and Home may may either export or import energy from Foreign. Carbon is released when the energy is used to produce goods. These goods, embodying carbon emissions, may be traded before being consumed by households. We can therefore trace carbon from its extraction through its release into the atmosphere and finally to its implicit consumption.

We define G_e as total intermediate demand for energy by the goods sector in Home and G_e^* by the goods sector in Foreign. Home's net exports of energy, $X_e = Q_e - G_e$, may be positive or negative. Foreign's net energy exports are $X_e^* = -X_e$. These expressions account for the first level of trade in carbon.

The second level of trade in carbon is embodied in goods. Table 1 depicts the bilateral flows, with rows indicating the location of consumption and columns the location of production. For example, Home's implicit consumption of carbon C_e (in the upper right) is the sum of carbon released by producers in Home serving the local market, C_e^{HH}, and carbon released by Foreign producers in supplying Home's imports, C_e^{HF}.

2.6 Competitive Equilibrium

Before turning to the planning problem, from which we derive the optimal carbon policy, we present the key elements of the competitive equilibrium, with no taxes. It serves as our business-as-usual (BAU) baseline in simulations and it describes the behavior of Foreign under the unilateral policy

term *energy intensity* for energy per worker, z_j, by analogy to the common use of *capital intensity* for capital per worker.

Table 1: Carbon Accounting Matrix

	Home	Foreign	Total
Home	$C_e^{HH} = \int_0^1 e_j(z_j^y) y_j dj$	$C_e^{HF} = \tau^* \int_0^1 e_j^*(z_j^m) m_j dj$	$C_e = C_e^{HH} + C_e^{HF}$
Foreign	$C_e^{FH} = \tau \int_0^1 e_j(z_j^x) x_j dj$	$C_e^{FF} = \int_0^1 e_j^*(z_j^*) y_j^* dj$	$C_e^* = C_e^{FH} + C_e^{FF}$
Total	$G_e = C_e^{HH} + C_e^{FH}$	$G_e^* = C_e^{HF} + C_e^{FF}$	$G_e^W = C_e^W = Q_e^W.$

that we will consider. We therefore focus on results for carbon flows and outcomes in Foreign.[17]

In this BAU scenario the model collapses to a simple version of DFS, with all producers facing a common wage $w = w^* = 1$ and a common energy price p_e. Each good is produced at the common, cost minimizing, energy intensity:

$$z = z^* = \frac{1-\alpha}{\alpha p_e}. \tag{6}$$

Foreign's unit energy requirement, substituting into (5), is therefore:

$$e_j^*(z^*) = (1-\alpha) a_j^* p_e^{-\alpha}, \tag{7}$$

so that its cost of producing good j is:

$$p_j^* = l_j^*(z^*) + p_e e_j^*(z^*) = a_j^* p_e^{1-\alpha}. \tag{8}$$

Home's cost of producing good j is the same except with a_j in place of a_j^*.

Home exports goods for which its cost, including transport, is below Foreign's, or goods $j < \bar{j}_x$. This extensive margin of exports, applying (4), satisfies:

$$F(\bar{j}_x) = \tau. \tag{9}$$

Likewise, Home imports goods $j > \bar{j}_m$ where the extensive margin of imports satisfies:

$$F(\bar{j}_m) = \frac{1}{\tau^*}. \tag{10}$$

[17] The full solution is provided in Appendix A, as a corollary of the global planner's problem.

These extensive margins are invariant to the energy price.

The intensive margins of demand for goods, $y_j = \eta p_j^{-\sigma}$, $x_j = \eta^*(\tau p_j)^{-\sigma^*}$, $m_j = \eta(\tau^* p_j^*)^{-\sigma}$, and $y_j^* = \eta^*(p_j^*)^{-\sigma^*}$, are governed by the substitution elasticities, σ and σ^*. The elasticity of demand for embodied energy with respect to the energy price is $\epsilon_D = \alpha + (1-\alpha)\sigma$ in Home and in Foreign:

$$\epsilon_D^* = \alpha + (1-\alpha)\sigma^*, \tag{11}$$

capturing both the elasticity of the unit energy requirement in production α as well as the elasticity of the price of the good with respect to the energy price, $1-\alpha$, multiplied by σ^*. Integrating the implied demands for embodied energy over the appropriate ranges, Table 2 shows the carbon accounting matrix in our BAU competitive equilibrium. The sum of all four expressions gives global energy demand, $C_e^W(p_e)$.

Table 2: Carbon Matrix for BAU

	Home	Foreign
Home	$\frac{C_e^{HH}}{1-\alpha} = \eta p_e^{-\epsilon_D} \int_0^{\bar{j}_m} a_j^{1-\sigma} dj$	$\frac{C_e^{HF}}{1-\alpha} = \eta p_e^{-\epsilon_D} \int_{\bar{j}_m}^1 \left(\tau^* a_j^*\right)^{1-\sigma} dj$
Foreign	$\frac{C_e^{FH}}{1-\alpha} = \eta^* p_e^{-\epsilon_D^*} \int_0^{\bar{j}_x} \left(\tau a_j\right)^{1-\sigma^*} dj$	$\frac{C_e^{FF}}{1-\alpha} = \eta^* p_e^{-\epsilon_D^*} \int_{\bar{j}_x}^1 \left(a_j^*\right)^{1-\sigma^*} dj$

On the energy supply side, since the wage is 1, energy is extracted from all fields with $a \leq p_e$. The energy supply curve in Home is thus $Q_e = E(p_e)$ and in Foreign:

$$Q_e^* = E^*(p_e). \tag{12}$$

Unlike the constant Foreign demand elasticity, ϵ_D^*, the Foreign supply elasticity:

$$\epsilon_S^* = \frac{dE^*}{dp_e}\frac{p_e}{E^*}. \tag{13}$$

may vary with the energy price. The equilibrium energy price equates global supply and demand for energy, $E(p_e) + E^*(p_e) = C_e^W(p_e)$. This energy price determines the size of the goods, energy, and services sectors.[18]

[18]The size of the services sector is determined by the quantity of labor not employed in

3 Home's Planning Problem

We consider a planner that allocates all resources in Home (Q_e, X_e, and p_e as well as energy intensities, output, exports, and imports of individual goods) so as to maximize Home's welfare. Foreign outcomes are determined as a competitive equilibrium given the choices made by the planner. After solving the optimization in this section, in the following section we deduce a set of taxes on extraction, production, consumption, and trade that implement Home's optimal carbon policy.

3.1 Preliminaries

We start by considering the various constraints faced by the planner, the first of which involves prices.

3.1.1 Prices and International Trade

Home and Foreign trade services at a price of 1. They trade energy at a price p_e, which is chosen by the planner (a higher price means a policy that reduces Foreign energy demand while a lower price means a policy that reduces Foreign energy supply).

For each good j the planner dictates the quantities y_j, x_j, and m_j. It also dictates the corresponding energy intensities of production z_j^y, z_j^x, and z_j^m (although Home's imports are produced in Foreign, the planner can set the energy intensity for how they are produced). The energy intensity of Foreign, and its price, in serving its own consumers, given by (7) and (8), are outside the reach of the planner.

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the production of goods (with labor share α) or the extraction of energy:

$$Q_s = L - L_g - L_e = L - \alpha G_e/(1-\alpha) - \int_0^{p_e} a\, dE(a).$$

The same applies in Foreign with Q_s^* in place of Q_s, G_e^* in place of G_e, and E^* in place of E. We assume that L and L^* are large enough so that Q_s and Q_s^* are both strictly positive. Services are costlessly traded to achieve trade balance:

$$C_s - Q_s = p_e(Q_e - G_e) + \int_0^{\bar{j}_x} \tau p_j x_j dj - \int_{\bar{j}_m}^1 \tau^* p_j^* m_j dj.$$

When Home imports from Foreign, the price is determined by Foreign's cost of production. This cost depends on both labor cost and energy cost in a combination determined by the energy intensity z_j^m that the planner dictates. It also depends on the iceberg cost τ^*. The price that Home pays for imports of good j, if $m_j > 0$, is thus:

$$p_j^m = \tau^* \left(l_j^*(z_j^m) + p_e e_j^*(z_j^m) \right). \tag{14}$$

When Home exports to Foreign, the price is constrained to be weakly below both the cost at which Foreign producers can make good j themselves and the marginal utility of good j to Foreign consumers:

$$p_j^x \leq \min \left\{ p_j^*, \left(x_j/\eta^* \right)^{-1/\sigma^*} \right\}.$$

While we don't denote it explicitly, $e_j^*(z^*)$, p_j^*, p_j^m, and p_j^x each depend on the energy price p_e. With these prices we can evaluate trade balance.

3.1.2 Trade Balance Constraint

The value of Home's net exports of services is $X_s = Q_s - C_s$ and the value of its net exports of goods is $X_g = V_g^{FH} - V_g^{HF}$, where gross exports and imports are:

$$V_g^{FH} = \int_0^1 p_j^x x_j dj, \quad \text{and} \quad V_g^{HF} = \int_0^1 p_j^m m_j dj.$$

The trade balance constraint is:

$$X_s = -X_g - p_e X_e. \tag{15}$$

3.1.3 Energy Constraints

The constaints on each country's extraction, use, and export of energy are:

$$Q_e = G_e + X_e \tag{16}$$

$$Q_e^* = G_e^* - X_e. \tag{17}$$

Here Q_e and X_e are chosen by the planner, Q_e^* is given by (12), and expressions for G_e and G_e^* are in Table 1. Foreign production of good j for the domestic market, which appears in the integral for G_e^*, is given by:

$$y_j^* = \begin{cases} \max \left\{ \eta^* \left(p_j^* \right)^{-\sigma^*} - x_j, 0 \right\} & \text{if } p_j^x = p_j^* \\ 0 & \text{if } p_j^x < p_j^*. \end{cases}$$

Together (16) and (17) imply the global energy constraint:

$$Q_e^W = G_e + G_e^*. \tag{18}$$

3.1.4 Labor Constraint

Combining (1) and (2), the labor L_e required to extract a quantity of energy Q_e is:

$$L_e(Q_e) = \int_0^{E^{-1}(Q_e)} a\, dE(a). \tag{19}$$

The labor required to provide services is simply $L_s = Q_s$. Accounting for labor used in goods production, L_g, to supply domestic consumers and to export, Home's labor constraint is:

$$Q_s = L - L_e(Q_e) - \int_0^1 \left(l_j(z_j^y)y_j + \tau l_j(z_j^x)x_j\right) dj. \tag{20}$$

3.2 The Planner's Lagrangian

The planner's objective is to maximize Home welfare:

$$U = Q_s - X_s + \frac{\eta^{1/\sigma}}{1 - 1/\sigma} \int_0^1 \left((y_j + m_j)^{1-1/\sigma} - 1\right) dj - \varphi\left(Q_e + Q_e^*\right),$$

subject to four constraints: (i) Home's labor constraint (20), (ii) trade balance (15), (iii) Home's energy constraint (16), and (iv) Foreign's energy constraint (17).

We can simply substitute the first two constraints into the objective (eliminating $Q_s - X_s$ from the objective). For the third and fourth it's easier to apply Lagrange multipliers λ_e and λ_e^*. The resulting Lagrangian is (dropping constants such as L):

$$\begin{aligned}\mathcal{L} = &\frac{\eta^{1/\sigma}}{1-1/\sigma}\int_0^1 (y_j+m_j)^{1-1/\sigma}\,dj - \varphi\left(Q_e+Q_e^*\right)\\ &- L_e(Q_e) - \int_0^1 \left(l_j(z_j^y)y_j + \tau l_j(z_j^x)x_j\right)dj\\ &- \int_0^1 \tau^*\left(l_j^*(z_j^m)+p_e e_j^*(z_j^m)\right)m_j dj + \int_0^1 p_j^x x_j dj + p_e X_e\\ &- \lambda_e\left(\int_0^1 \left(e_j(z_j^y)y_j + \tau e_j(z_j^x)x_j\right)dj - Q_e + X_e\right)\\ &- \lambda_e^*\left(\int_0^1 \left(e_j^*(z^*)y_j^* + \tau^* e_j^*(z_j^m)m_j\right)dj - Q_e^* - X_e\right).\end{aligned}$$

The terms in the Lagrangian are, line by line: (i) Home's welfare less services consumption, (ii) what remains of Home's labor constraint after substituting out the provision of services, (iii) what remains of Home's trade-balance constraint after substituting out services exports, (iv) Home's energy constraint, and (v) Foreign's energy constraint. We want to maximize this Lagrangian by the optimal choice of $\{y_j\}$, $\{x_j\}$, $\{m_j\}$, $\{z_j^y\}$, $\{z_j^x\}$, $\{z_j^m\}$, Q_e, X_e, and p_e. (When we get to the point of optimizing over p_e we will be explicit about how Q_e^*, p_j^x, $e_j^*(z^*)$, and y_j^* each depend on p_e.)

We solve this problem, starting with what CDVW call the *inner problem*, involving optimality conditions for an individual good given values for Q_e, X_e, λ_e, λ_e^*, and p_e. We then evaluate the optimality conditions for Q_e, X_e, and p_e. The Lagrange multipliers λ_e and λ_e^* will be solved in the process, once we clear the energy market.

3.3 Inner Problem

Solving the inner problem consists of first order conditions with respect to the variables that are specific to some good j: y_j, x_j, m_j, z_j^y, z_j^x, and z_j^m. These first order conditions, and their implications given Q_e, X_e, λ_e, λ_e^*, and p_e, can be considered one good at time. We therefore define a Lagrangian for good j:

$$\begin{aligned}
\mathcal{L}_j = & \frac{\eta^{1/\sigma}}{1-1/\sigma}\left(y_j+m_j\right)^{1-1/\sigma} \\
& -\nu a_j\left(y_j\left(z_j^y\right)^{\alpha-1}+\tau x_j\left(z_j^x\right)^{\alpha-1}\right) \\
& -\nu a_j^*\tau^*\left(\left(z_j^m\right)^{\alpha-1}+p_e\left(z_j^m\right)^{\alpha}\right)m_j+p_j^x x_j \\
& -\lambda_e\nu a_j\left(y_j\left(z_j^y\right)^{\alpha}+\tau x_j\left(z_j^x\right)^{\alpha}\right) \\
& -\lambda_e^*\nu a_j^*\left(\max\left\{\eta^*\left(p_j^*\right)^{-\sigma^*}-x_j,0\right\}\left(z^*\right)^{\alpha}+\tau^* m_j\left(z_j^m\right)^{\alpha}\right),
\end{aligned}$$

where we have substituted in the expressions for unit input requirements (5) in Home (as well as their analogs in Foreign). We consider the variables relevant to Home consumers first, then turn to those relevant to Foreign consumers. Details are relegated to Appendix B.

3.3.1 Goods for Home Consumers

The first order condition for z_j^y implies:

$$z_j^y = z^y = \frac{1-\alpha}{\alpha\lambda_e}.$$

The planner requires all Home producers serving the domestic market to produce at the same energy intensity. Similarly, the first order condition for z_j^m implies:

$$z_j^m = z^m = \frac{1-\alpha}{\alpha\left(p_e+\lambda_e^*\right)}.$$

All Foreign producers serving consumers in Home also produce at the same energy intensity, but potentially different from producers in Home.

Due to the inherent corner solutions, the first order conditions for y_j and m_j are more intricate. Yet their implications are easy to distill by defining the good $\bar{j}_m$ for which both first order conditions hold with equality. Applying (4), this cutoff good satisfies:

$$F(\bar{j}_m) = \frac{1}{\tau^*}\left(\frac{\lambda_e}{p_e+\lambda_e^*}\right)^{1-\alpha}, \tag{21}$$

so that $\bar{j}_m$ is increasing in the iceberg cost τ^* as in the standard DFS model.

1. For any good $j < \bar{j}_m$ Home has a comparative advantage, which leads it to produce for itself:
$$y_j = \eta \left(a_j \lambda_e^{1-\alpha}\right)^{-\sigma},$$
while importing nothing, $m_j = 0$.

2. For any good $j > \bar{j}_m$ Foreign has a comparative advantage, which leads Home to import:
$$m_j = \eta \left(\tau^* a_j^* \left(p_e + \lambda_e^*\right)^{1-\alpha}\right)^{-\sigma},$$
while producing nothing for itself, $y_j = 0$.

3. We can ignore the pattern of trade for the measure-zero cutoff good $j = \bar{j}_m$.

3.3.2 Goods for Foreign Consumers

We now turn to Home's exports to Foreign. The first order condition for z_j^x implies:
$$z_j^x = z^x = \frac{1-\alpha}{\alpha \lambda_e}.$$
All Home producers serving the export market produce at the same energy intensity, the same as their energy intensity when serving the domestic market.

From the derivative with respect to x_j, we first obtain an optimality condition that:
$$p_j^x = p_j^*, \tag{22}$$
when $x_j > 0$. This result is driven by our assumption that Foreign demand is inelastic.[19]

Imposing (22), so that p_j^x is held fixed, the first order condition for x_j gives:
$$\frac{\partial \mathcal{L}_j}{\partial x_j} = -\nu a_j \tau \left(z_j^x\right)^{\alpha-1} - \lambda_e \nu a_j \tau \left(z_j^x\right)^{\alpha} + p_j^x + \lambda_e^* \nu a_j^* \left(z^*\right)^{\alpha},$$

[19]Suppose instead that:
$$p_j^x = \left(x_j/\eta^*\right)^{-1/\sigma^*} < p_j^*.$$
It follows that $y_j^* = 0$ and:
$$\frac{\partial p_j^x}{\partial x_j} = -\frac{p_j^x}{\sigma^* x_j}.$$

for $x_j < \eta^* \left(p_j^*\right)^{-\sigma^*}$. The terms on the right-hand side are: (i) the marginal value of Home's labor used to produce more of x_j, (ii) the value of the energy used, (iii) the value of the revenue obtained, and (iv) the value of reducing Foreign's energy use. Substituting in the solution for z_j^x, z^*, and p_j^x and then combining the first two terms, this derivative simplifies to:

$$\frac{\partial \mathcal{L}_j}{\partial x_j} = -\tau a_j \lambda_e^{1-\alpha} + a_j^* p_e^{1-\alpha} + a_j^* \lambda_e^* \left(1-\alpha\right) p_e^{-\alpha}. \tag{23}$$

The last term plays a novel role in the solution. It represents the value that the planner places on the energy Foreign would use to produce an additional unit of good j for itself.

To distill results about consumption in Foreign, define the good $\bar{j}_x$ such that

$$\left.\frac{\partial \mathcal{L}_j}{\partial x_j}\right|_{j=\bar{j}_x} = 0.$$

Applying (23) and (4), this cutoff good satisfies:

$$F(\bar{j}_x) = \frac{\tau \left(\frac{\lambda_e}{p_e}\right)^{1-\alpha}}{1 + \left(1-\alpha\right)\frac{\lambda_e^*}{p_e}}. \tag{24}$$

While $\bar{j}_x$ is decreasing in the iceberg cost τ (as in the standard DFS model) the denominator brings in new elements, which we will return to later.

1. For any good $j < \bar{j}_x$ Home has comparative advantage, which leads it to export:
$$x_j = \eta^* \left(a_j^* p_e^{1-\alpha}\right)^{-\sigma^*},$$
while Foreign produces $y_j^* = 0$ for itself. (Note that Home's export quantity for any such good is at a corner solution, which will become relevant later when we consider optimal p_e.)

In this case, under our assumption that $\sigma^* \leq 1$:

$$\frac{\partial \mathcal{L}_j}{\partial x_j} = -\nu a_j \tau \left(z_j^x\right)^{\alpha-1} - \lambda_e \nu a_j \tau \left(z_j^x\right)^{\alpha} - \left(\frac{1-\sigma^*}{\sigma^*}\right) p_j^x < 0.$$

As a consequence of this negative derivative, the planner would never choose $x_j > 0$ if $p_j^x < p_j^*$.

Table 3: Production and Distribution of a Good

	Home		Foreign	
Home	$y_j = \eta \left(a_j \lambda_e^{1-\alpha}\right)^{-\sigma}$	$j < \bar{j}_m$	$m_j = \eta \left(\tau^* a_j^* \left(p_e + \lambda_e^*\right)^{1-\alpha}\right)^{-\sigma}$	$j > \bar{j}_m$
Foreign	$x_j = \eta^* \left(a_j^* p_e^{1-\alpha}\right)^{-\sigma^*}$	$j < \bar{j}_x$	$y_j^* = \eta^* \left(a_j^* p_e^{1-\alpha}\right)^{-\sigma^*}$	$j > \bar{j}_x$

2. For any good $j > \bar{j}_x$ Foreign has a comparative advantage, which leads it to produce for itself:

$$y_j^* = \eta^* \left(a_j^* p_e^{1-\alpha}\right)^{-\sigma^*},$$

with $x_j = 0$.

3. We can ignore the pattern of trade for the measure-zero cutoff good $j = \bar{j}_x$.

Table 3 compares the terms for each of the four quantities of good j. As in Table 1, the rows indicate the location of consumption while the columns indicate the location of production.

These terms are as expected except for Home's exports, x_j: (i) exports of good j reflect the global price of energy p_e rather than Home's shadow price λ_e, (ii) although produced in Home, they reflect Foreign productivity a_j^* rather than Home's productivity a_j, and (iii) they do not reflect the iceberg costs of export τ. That is, $x_j \neq \eta^* \left(\tau a_j \lambda_e^{1-\alpha}\right)^{-\sigma}$ as one would expect. The reason is that Home is constrained to price exports no higher than Foreign's cost, and it is never optimal for Home to export at a price below Foreign's cost.

3.4 Outer Problem

We now turn to the optimality conditions for X_e, Q_e, and p_e, rewriting the Lagrangian in terms of aggregate magnitudes:

$$\mathcal{L} = \frac{\eta^{1/\sigma}}{1-1/\sigma} C_g^{1-1/\sigma} - \varphi\left(Q_e + Q_e^*\right) - L_e(Q_e) - L_g + X_g + p_e X_e \\ - \lambda_e\left(G_e - Q_e + X_e\right) - \lambda_e^*\left(G_e^* - Q_e^* - X_e\right). \tag{25}$$

We consider how Q_e^*, L_g, X_g, G_e, and G_e^* depend on the energy price once we get to the first order condition for p_e.

3.4.1 Energy Exports

The first order condition with respect to Home's exports of energy, X_e, is:

$$\frac{\partial \mathcal{L}}{\partial X_e} = p_e - \lambda_e + \lambda_e^* = 0,$$

which implies:

$$\lambda_e = p_e + \lambda_e^*. \tag{26}$$

Thus, the optimal energy export policy requires the shadow value of energy in Home, λ_e, to be the same as the shadow value of energy faced by producers in Foreign that serve customers in Home, $p_e + \lambda_e^*$.

Combined with the results from the inner problem, an implication is that optimal energy intensity is the same for all producers serving consumers in Home (whether the producers are in Home or Foreign) and for all producers in Home (whether serving consumers in Home or Foreign):

$$z^y = z^x = z^m = z = \frac{1-\alpha}{\alpha\lambda_e}, \tag{27}$$

where from the inner problem we established that these energy intensities do not vary by good. The exception to equalization is z^*, the common energy intensity for any goods produced in Foreign for consumption there.

We now have the full set of optimal unit energy requirements to accompany (7) (for when Foreign serves its own consumers). For Home production, whether for the domestic or export market:

$$e_j(z) = (1-\alpha)a_j\lambda_e^{-\alpha}.$$

When Foreign serves consumers in Home:

$$e_j^*(z) = (1-\alpha) a_j^* \lambda_e^{-\alpha}.$$

The corresponding optimal unit labor requirements are $l_j(z) = e_j(z)/z$, $l_j^*(z) = e_j^*(z)/z$, and (for when Foreign serves its own consumers) $l_j^*(z^*) = e_j^*(z^*)/z^*$.

Another implication is that the extensive margin for Home's imports, that is, which goods are produced locally and which are imported, is the same as in the BAU equilibrium: $F(\bar{j}_m) = 1/\tau^*$. For the goods that Home imports, the price (14) simplifies to:

$$p_j^m = l_j^*(z) + p_e e_j^*(z) = \alpha \tau^* a_j^* \lambda_e^{1-\alpha} + (1-\alpha)\, p_e \tau^* a_j^* \lambda_e^{-\alpha}. \tag{28}$$

Note that this is the price at the port, appropriate for computing the trade balance.

3.4.2 Energy Extraction

The first order condition with respect to Q_e is:

$$\frac{\partial \mathcal{L}}{\partial Q_e} = -\varphi - \frac{\partial L_e}{\partial Q_e} + \lambda_e = 0.$$

The extra labor in Home to extract a bit more energy is the labor needed to exploit the marginal energy deposit, $E^{-1}(Q_e)$.[20] Hence, applying (26), the first order condition simplifies to:

$$Q_e = E\left(p_e + \lambda_e^* - \varphi\right). \tag{29}$$

This condition is the same as Home's energy-extraction supply curve in the BAU scenario, but with $p_e + \lambda_e^* - \varphi$ in place of p_e.[21]

[20] Integrating (19) by parts, it becomes:

$$L_e(Q_e) = E^{-1}(Q_e) Q_e - \int_0^{E^{-1}(Q_e)} E(a) da.$$

Differentiating it in this form:

$$\frac{\partial L_e}{\partial Q_e} = E^{-1}(Q_e) + Q_e \frac{\partial E^{-1}}{\partial Q_e} - E(E^{-1}(Q_e)) \frac{\partial E^{-1}}{\partial Q_e} = E^{-1}(Q_e).$$

[21] If φ is large enough it's possible that $\partial \mathcal{L}/\partial Q_e < 0$ even at $Q_e = 0$. The corner solution is then $Q_e = 0$ and $\lambda_e \leq \varphi$.

We've now determined global energy supply given λ_e^* and p_e, with Home extraction being lower when marginal damages φ are high. A more definitive statement requires solving for λ_e^* and p_e, which we turn to next.

3.4.3 Energy Price

The first order condition with respect to p_e is:

$$\frac{\partial \mathcal{L}}{\partial p_e} = -\varphi \frac{\partial Q_e^*}{\partial p_e} - \frac{\partial L_g}{\partial p_e} + \frac{\partial X_g}{\partial p_e} + X_e - \lambda_e \frac{\partial G_e}{\partial p_e} - \lambda_e^* \left(\frac{\partial G_e^*}{\partial p_e} - \frac{\partial Q_e^*}{\partial p_e} \right) = 0,$$

which, imposing the global energy constraint (18), implies:

$$\lambda_e^* \left(\frac{\partial Q_e^*}{\partial p_e} - \frac{\partial G_e^*}{\partial p_e} \right) = \varphi \frac{\partial Q_e^*}{\partial p_e} + Q_e^* - G_e^* - \frac{\partial X_g}{\partial p_e} + \frac{\partial L_g}{\partial p_e} + \lambda_e \frac{\partial G_e}{\partial p_e}, \tag{30}$$

To make sense of this condition requires that we clarify how the five aggregate variables Q_e^*, G_e^*, G_e, L_g, and X_g appearing in (25) depend on p_e.

Dependence on the Energy Price Foreign energy extraction depends directly on the energy price via (12), with elasticity given by (13). The dependence on the energy price is more subtle for the other four aggregates. Since Home directly chooses z, $\bar{j}_m$, $\bar{j}_x$, $\{m_j\}$, and $\{y_j\}$, the envelope theorem allows us treat them as fixed when differentiating the Lagrangian with respect to p_e. From the inner problem, each satisfies its own first-order condition with equality.[22] Furthermore, we can take as fixed the unit energy requirement for Home producers, whether supplying the domestic or export market. On the other hand $\{p_j^x\}$, $\{x_j\}$, $\{p_j^m\}$, $\{y_j^*\}$, and $\{e_j^*(z^*)\}$ were either not chosen by the planner or were optimized at a corner solution. They must be considered in the first order condition. We apply (7), (8), (22), (28), and results in the second row of Table 3 to compute the derivatives of the four aggregates.

Energy use by Foreign producers connects to the energy price via:

$$G_e^* = (1-\alpha)\eta^* p_e^{-\epsilon_D^*} \int_{\bar{j}_x}^1 \left(a_j^*\right)^{1-\sigma^*} dj + \tau^* \int_{\bar{j}_m}^1 e_j^*(z) m_j dj,$$

so that:

$$\frac{\partial G_e^*}{\partial p_e} = \frac{\partial C_e^{FF}}{\partial p_e} = -\epsilon_D^* \frac{C_e^{FF}}{p_e} < 0, \tag{31}$$

[22] Thus, C_g in (25) does not appear in (30) since it depends only on terms that were optimized in the inner problem.

where Foreign's demand elasticity ϵ_D^* is from (11). That is, a change in the energy price effects Foreign's use of energy only through its domestic consumption C_e^{FF} and not through its exports of goods to Home C_e^{HF}. Home has chosen and optimized the determinants of C_e^{HF} ($\bar{j}_m$, m_j, and $z^m = z$).

Energy use by Home producers is:

$$G_e = \int_0^{\bar{j}_m} e_j(z) y_j dj + (1-\alpha)\eta^* p_e^{-(1-\alpha)\sigma^*} \int_0^{\bar{j}_x} \tau a_j \left(a_j^*\right)^{-\sigma^*} dj,$$

so that:

$$\frac{\partial G_e}{\partial p_e} = \frac{\partial C_e^{FH}}{\partial p_e} = -(1-\alpha)\sigma^* \frac{C_e^{FH}}{p_e} < 0. \tag{32}$$

Goods-sector employment in Home is tied to energy use via:

$$L_g = G_e/z = \frac{\alpha}{1-\alpha}\lambda_e G_e,$$

so that:

$$\frac{\partial L_g}{\partial p_e} = \frac{\alpha}{1-\alpha}\lambda_e \frac{\partial G_e}{\partial p_e} < 0. \tag{33}$$

Home's net exports are:

$$X_g = \eta^* p_e^{1-\epsilon_D^*} \int_0^{\bar{j}_x} \left(a_j^*\right)^{1-\sigma^*} dj - \alpha \lambda_e^{1-\alpha} \int_{\bar{j}_m}^1 \tau^* a_j^* m_j dj - (1-\alpha)\, p_e \lambda_e^{-\alpha} \int_{\bar{j}_m}^1 \tau^* a_j^* m_j dj,$$

so that:

$$\frac{\partial X_g}{\partial p_e} = \frac{\partial V_g^{FH}}{\partial p_e} - \frac{\partial V_g^{HF}}{\partial p_e} = (1-\epsilon_D^*)\frac{V_g^{FH}}{p_e} - C_e^{HF}. \tag{34}$$

Restatement of the Optimality Condition Using these results, we can rewrite the first order condition as:[23]

$$\lambda_e^* = \frac{\varphi \partial Q_e^*/\partial p_e + \left(Q_e^* - C_e^{FF}\right) - \partial \Pi_g/\partial p_e}{\partial \left(Q_e^* - C_e^{FF}\right)/\partial p_e}, \tag{35}$$

[23] Starting from (30), intermediate steps include:

$$\lambda_e^* \left(\frac{\partial Q_e^*}{\partial p_e} - \frac{\partial G_e^*}{\partial p_e}\right) = \varphi \frac{\partial Q_e^*}{\partial p_e} + Q_e^* - G_e^* - (1-\epsilon_D^*)\frac{V_g^{FH}}{p_e} + C_e^{HF} + \frac{\lambda_e}{1-\alpha}\frac{\partial G_e}{\partial p_e}$$

and

$$\lambda_e^* \left(\frac{\partial Q_e^*}{\partial p_e} - \frac{\partial C_e^{FF}}{\partial p_e}\right) = \varphi \frac{\partial Q_e^*}{\partial p_e} + Q_e^* - C_e^{FF} - \frac{\partial V_g^{FH}}{\partial p_e} + \frac{\lambda_e}{1-\alpha}\frac{\partial C_e^{FH}}{\partial p_e}$$

.

where Π_g is Home's surplus from goods exports:

$$\Pi_g = V_g^{FH} - \frac{\lambda_e}{1-\alpha} C_e^{FH}. \tag{36}$$

Using Foreign's demand and supply elasticities, we can rewrite (35) as:

$$\lambda_e^* = \frac{\varphi \epsilon_S^* Q_e^* + p_e \left(Q_e^* - C_e^{FF} \right) - (1 - \epsilon_D^*) V_g^{FH} - \lambda_e \sigma^* C_e^{FH}}{\epsilon_S^* Q_e^* + \epsilon_D^* C_e^{FF}}.$$

This equality will generate a locus of combinations of p_e and λ_e^*. The global energy constraint (18) will generate another. Their intersection gives us the solution for p_e and λ_e^*.

3.5 Assessment

We can now compute the optimal policy in principle:

1. The inner problem, together with (26), gives G_e, G_e^*, and X_g in terms of p_e and λ_e^*.
2. Equations (12) and (29) give Q_e^* and Q_e as functions of p_e and λ_e^*.
3. Equation (35) and the global energy constraint (18) jointly nail down p_e and λ_e^*.

All other outcomes follow. Before actually computing the solution for a parameterized version of the model, we reinterpret the optimality conditions in terms of a set of taxes and subsidies.

4 Optimal Taxes and Subsidies

We now turn to a set of taxes and subsidies that deliver the optimal outcomes in a competitive equilibrium. The taxes must meet the following conditions:

1. By (27), the energy intensity of production is the same for all goods produced in Home or consumed in Home: $z^y = z^m = z^x$.
2. By (21) together with (26), the import margin is the same as in the BAU case : $F\left(\bar{j}_m\right) = 1/\tau^*$.

3. The energy price faced by Home producers is λ_e, where by (26): $\lambda_e = p_e + \lambda_e^*$.

4. The energy price faced by producers in Foreign serving customers in Home is also $p_e + \lambda_e^*$.

5. Home's energy extraction must satisfy (29): $Q_e = E\left(p_e + \lambda_e^* - \varphi\right)$.

6. Home's export margin is (24): $F\left(\bar{j}_x\right) = \tau \left(\frac{\lambda_e}{p_e}\right)^{1-\alpha} / \left(1 + (1-\alpha)\frac{\lambda_e^*}{p_e}\right)$.

7. Producers in Home selling goods in Foreign do so satisfying (22): $p_j^x = p_j^*$.

8. The energy price, p_e, and λ_e^* satisfy (35).

While these optimal outcomes are unique, the policies that deliver them are not. We focus on a set that is easy to describe.In particular, we consider (1) a nominal tax on extraction, t_e, set at the standard Pigouvian rate φ, (2) a border adjustment, t_b, and (3) an export policy based on Home's comparative advantage in producing goods. The border adjustment is imperfect or partial in that it need not be at the same rate as the underlying extraction tax (and often it will be at a lower rate) and it applies only to imports and exports of energy and imports of goods, but not to the export of goods. Below we illustrate that this set of taxes implements Home's optimal planning outcome.

Our description of the optimal policy uses nominal taxes and border adjustments. An equivalent way to describe the taxes is to use effective taxes. In particular, a nominal tax on extraction of t_e and a border adjustment on energy of t_b can equivalently be described as an effective tax on extraction of $\tilde{t}_e = t_e - t_b$ and an effective tax on production of $\tilde{t}_p = t_b$. We use nominal taxes because the optimal policy is simpler to express in those terms.

4.1 Extraction Tax and Border Adjustments

Consider the following two taxes: a nominal tax on extraction, t_e, equal to marginal harm, φ, and a border adjustment on all imports and exports of energy, and on the energy embodied in imports, all at rate λ_e^*:

$$\begin{aligned} t_e &= \varphi, \\ t_b &= \lambda_e^*. \end{aligned} \tag{37}$$

This set of taxes satisfies conditions 1 through 5.

The border adjustment sets the price of energy in Home to $p_e + t_b = p_e + \lambda_e^* = \lambda_e$. If Home imports energy, the border adjustment adds to the world price p_e, bringing it up to $p_e + t_b$. This price is what producers in Home pay and is also the pre-tax price that Home extractors receive. If Home exports energy the border adjustment is paid to energy exporters so that their pre-tax price remains $p_e + t_b$ even as they sell on the world market at price p_e.

Because the border adjustment applies to the energy embodied in imported goods, Foreign producers of these goods will internalize the cost of energy as being $p_e + t_b$. Their energy intensity of production when shipping to Home is therefore equalized to that of Home producers, giving us $z^y = z^m = z^x$. Moreover, the range of goods that Home imports is invariant to these taxes, so $F\left(\bar{j}_m\right) = \frac{1}{\tau^*}$.

Home extractors receive a pre-tax price of $p_e + t_b$ (whether they sell domestically or export), and they pay an extraction tax, $t_e = \varphi$. The after-tax price received by Home's extractors is thus:

$$p_e + t_b - t_e = p_e + \lambda_e^* - \varphi.$$

Therefore, the energy extraction in Home satisfies (29), inducing the optimal Q_e. While we have not yet determined the optimal level of the border adjustment, we have determined the optimal wedge between the price that Home's producers pay for energy and the price that its extractors receive. That wedge is the extraction tax, which equals the marginal damages from global emissions, φ.[24]

4.2 Export Taxes and Subsidies

Conditions 1, 6, and 7 determine Home's export policy. Condition 1 requires that producers in Home selling in Foreign face the same energy price as when they sell domestically. This means that the border adjustment should not apply to Home's exports. It is optimal for Home to give its exporters the incentive to produce at the same, lower energy intensity that its producers use for domestically consumed goods. A border adjustment on export would remove that incentive.

[24] Markusen (1975) obtains the same result, referring to it as a production tax rather than an extraction tax as we do. His taxes are ad valorem while ours are specific.

To meet conditions 6 and 7, Home provides exporters of each good j with a per unit tax or subsidy, depending on Home's comparative advantage in producing that good. To derive the subsidy and tax, we first solve for the good $j_0 \in (0, \bar{j}_x)$ that Home exports at a cost exactly equal to the price it charges:

$$\tau a_{j_0} \left(p_e + t_b\right)^{1-\alpha} = a^*_{j_0} p_e^{1-\alpha},$$

so that j_0 satisfies:

$$F(j_0) = \tau \left(\frac{p_e + t_b}{p_e}\right)^{1-\alpha}.$$

Exporters could not normally sell any goods, $j \in (j_0, \bar{j}_x]$, because their costs would exceed the price. To meet the optimal export margin, condition 6, Home must subsidize all of these goods in an amount equal to the losses producers would otherwise incur:

$$s^x_j = \tau a_j (p_e + t_b)^{1-\alpha} - a^*_j p_e^{1-\alpha} = \left(\tau \frac{a_j}{a^*_j} \left(\frac{p_e + t_b}{p_e}\right)^{1-\alpha} - 1\right) p^*_j.$$

For goods $j \in [0, j_0)$, Home has stronger comparative advantage. Exporters would normally sell at their cost rather than p^*_j as required. To induce these exporters to sell at p^*_j, condition 7, Home imposes a per-unit tax at a rate of $t^x_j = -s^x_j$.

4.2.1 Net Cost of Taxes and Subsidies

Integrating over all the goods that Home exports, the tax revenue net of the subsidy turns out to be Home's surplus from exporting, as defined in (36):

$$\begin{aligned} &\tau \int_0^{j_0} t^x_j x_j(p_e) dj - \tau \int_{j_0}^{\bar{j}_x} s^x_j x_j(p_e) dj \\ &= \eta^* \int_0^{\bar{j}_x} \left(1 - \tau \frac{a_j}{a^*_j} \left(\frac{p_e + t_b}{p_e}\right)^{1-\alpha}\right) \left(p^*_j\right)^{1-\sigma^*} dj \\ &= V_g^{FH} - (p_e + t_b) \frac{C_e^{FH}}{1-\alpha} \\ &= \Pi_g. \end{aligned} \tag{38}$$

The derivative of this term with respect to the energy price will appear in the expression for the optimal border adjustment below.

4.2.2 Crosshauling

The subsidies for exports create the possibility for crosshauling if iceberg costs are sufficiently low. To see why, note that if $t_b > 0$ (which we discuss below),

$$(p_e + t_b)^{1-\alpha} < p_e^{1-\alpha} + (1-\alpha)\, p_e^{-\alpha} t_b.$$

(The right-hand side is the first-order expansion of the concave function on the left-hand side, around $t_b = 0$.) Rearranging this inequality gives:

$$\frac{\left(\frac{p_e+t_b}{p_e}\right)^{1-\alpha}}{1+(1-\alpha)\,\frac{t_b}{p_e}} < 1.$$

If iceberg costs are modest, this inequality will continue to hold even if the left-hand side is multiplied by the product of the iceberg costs, $\tau\tau^* \geq 1$. In this case:

$$F(\bar{j}_x) < \frac{1}{\tau^*} = F(\bar{j}_m).$$

Since F is monotonically decreasing, it follows that $\bar{j}_m < \bar{j}_x$. Under these conditions of low trade costs, Home simultaneously imports and exports goods $j \in (\bar{j}_m, \bar{j}_x)$. The optimal policy can imply crosshauling.

The economic rationale for crosshauling is that Home controls energy intensity not only for all production in Home, but also for production in Foreign that Home imports. In contrast, goods produced in Foreign, for consumption there, escape Home's climate policy.

Increased trade gives Home more control over the use of energy, helping it to lower global emissions. In particular, if trade costs are low enough, Home is willing to supply a range of goods to Foreign at a price below the shadow price of those goods to Home consumers, which is the rationale for the subsidy s_j^x.

To illustrate, consider a good j for which $a_j = a_j^* = a$, and assume there are no trade costs at all. With $t_b = 0$, Home would be indifferent between exporting this good or having Foreign produce it for itself. But, with $t_b > 0$ global energy use is reduced if the good is produced in Home and exported to Foreign. The energy requirement is $a\,(1-\alpha)\,(p_e + t_b)^{-\alpha}$ which is less than if Foreign produced for itself, with energy requirement $a\,(1-\alpha)\,p_e^{-\alpha}$ per unit produced. On the other hand, Home is indifferent between importing this good or producing it for itself. In either case both the cost of production and the energy content are the same because Home controls the energy content of its imports.

Trade costs mute this effect. With high enough trade costs, $F\,(\bar{j}_x) > F\,(\bar{j}_m)$. The inherent inefficiency of crosshauling overcomes its advantage in reducing global emissions. Yet, even in this case, the optimal policy has broadened the set of goods that Home exports.

4.3 Optimal Border Adjustment

The final component of the tax system is the level of the border adjustment. We can employ equation (35) for that purpose, substituting in $t_b = \lambda_e^*$, and rearranging to get:

$$t_b = \frac{Q_e^* - C_e^{FF}}{\partial\left(Q_e^* - C_e^{FF}\right)/\partial p_e} + \varphi\frac{\partial Q_e^*/\partial p_e}{\partial\left(Q_e^* - C_e^{FF}\right)/\partial p_e} - \frac{\partial \Pi_g/\partial p_e}{\partial\left(Q_e^* - C_e^{FF}\right)/\partial p_e}, \quad (39)$$

The first term captures terms-of-trade manipulation, the second captures Home's response to shifts in where extraction occurs and the third captures the net cost of Home's export policy.

This formula (39) is a generalization of the result in Markusen (1975).[25] We will consider the three terms on the right hand side in turn. It is convenient to first express it in terms of elasticities, as:

$$t_b = \frac{p_e\left(Q_e^* - C_e^{FF}\right)}{\epsilon_S^* Q_e^* + \epsilon_D^* C_e^{FF}} + \varphi\frac{\epsilon_S^* Q_e^*}{\epsilon_S^* Q_e^* + \epsilon_D^* C_e^{FF}} - \frac{(1-\epsilon_D^*)V_g^{FH} + (p_e+t_b)\sigma^* C_e^{FH}}{\epsilon_S^* Q_e^* + \epsilon_D^* C_e^{FF}}.$$

Note that the denominator is always positive.

4.3.1 No Global Externality or Trade in Goods

With no global externality we have $\varphi = t_e = 0$, while we can eliminate all trade in goods by letting the iceberg costs approach infinity, in which case the last term vanishes. Only the first term in (39) remains. Furthermore, with

[25] He refers to it as an optimal trade tax as opposed to a border adjustment. Without trade in differentiated goods, Markusen's formula doesn't have the last term.

no trade in goods, the numerator becomes Foreign's net exports of energy, and the denominator is its derivative:

$$t_b = \frac{X_e^*}{\partial X_e^* / \partial p_e} = \frac{p_e X_e^*}{\epsilon_S^* Q_e^* + \epsilon_D^* C_e^{FF}}.$$

Equation (39) reduces to the classic inverse elasticity formula for the optimal trade tax.

Suppose the numerator is positive so that Home is a net importer of energy. The optimal policy has Home impose a positive border adjustment $t_b > 0$. The tax on energy imports lowers the world price of energy, p_e, improving Home's terms of trade.

If the numerator is negative, so that Home is a net exporter of energy, Home will tax exports with $t_b < 0$, which pushes Home's energy price below p_e. Doing so raises demand for energy in Home (which is not subject to the tax), lowers extraction in Home, and raises the world price, p_e, thus improving Home's terms of trade. With no global externality there is no presumption that the border adjustment is positive.

4.3.2 Global Externality but No Trade in Goods

Now consider marginal damages from the global externality of $\varphi > 0$. The first two terms in (39) remain:

$$t_b = \frac{p_e X_e^*}{\epsilon_S^* Q_e^* + \epsilon_D^* C_e^{FF}} + \varphi \frac{\epsilon_S^* Q_e^*}{\epsilon_S^* Q_e^* + \epsilon_D^* C_e^{FF}}.$$

We focus on the second, having discussed above how the trade tax can be used to improve Home's terms of trade. The second term will determine how the optimal border adjustment depends on the Pigouvian extraction tax of t_e, set equal to marginal damages.

This second term, which multiplies the marginal harm from emissions, φ, is bounded between zero and one. Thus, the externality on its own (treating $X_e^* = 0$) will lead Home to impose a partial border adjustment, in that the rate is lower than the underlying extraction tax:

$$0 \leq t_b = t_e \frac{\epsilon_S^* Q_e^*}{\epsilon_S^* Q_e^* + \epsilon_D^* C_e^{FF}} < t_e.$$

How much t_b is less than t_e depends on the value of $\epsilon_D^* C_e^{FF}$ relative to $\epsilon_S^* Q_e^*$. The $\epsilon_S^* Q_e^*$ term reflects the effect of the extraction tax on Foreign extraction.

Home's tax on extraction increases p_e. In response, Foreign will extract more, as reflected in $\epsilon_S^* Q_e^*$, causing harm to Home of $\varphi \epsilon_S^* Q_e^*$. Increasing t_b moderates this effect because it lowers p_e: a higher t_b gives a higher net-of-tax price to Home's exporters (so expands global energy supply) and makes the use of energy more costly (so lowers global energy demand).

The $\epsilon_D^* C_e^{FF}$ term reflects demand-side effects. As we increase t_b, the price of energy goes down,resulting in an increase in Foreign consumption of energy: $\epsilon_D^* C_e^{FF}$. (Note that the demand side effect only includes C_e^{FF} because C_e^{FH}, C_e^{HF}, and C_e^{HH} are all controlled through Home's policy.) The border adjustment has to balance these effects.

To illustrate, if extraction in Foreign is very responsive to the price of energy while demand is inelastic, Home's optimal border adjustment will approach the level of the Pigouvian extraction tax. The reason is that a higher border adjustment lowers the world energy price, reducing Foreign's extraction of energy, while in this case not inducing much extra demand in Foreign. On the other hand, if Foreign extraction is quite inelastic while demand is elastic, Home's border adjustment will approach 0. The reason is that a lower border adjustment raises the world price of energy, reducing Foreign demand, while in this case not inducing much extra extraction in Foreign.

4.3.3 General Case

Allowing for trade in goods means that we need to take account of all three terms in (39), the last of which captures Home's concern with its surplus in exporting goods. Furthermore, the first two terms no longer depend only on Foreign's net energy exports, but on its energy extraction less the energy it uses in supplying domestic goods consumption.

The third term in (39) reflects Home's market power in exporting goods together with its use of goods exports to broaden the scope of its carbon policy. For example, if $\partial \Pi_g / \partial p_e > 0$, Home's surplus from the export market goes up with a higher energy price, so it lowers t_b.

5 Quantitative Illustration

We now turn to the quantitative implications of the optimal policy. We pursue a strategy, based on Dekle, Eaton, and Kortum (2007), in which we calibrate the BAU competitive equilibrium to data on global carbon flows. We then compute the optimal policy relative to this baseline. Doing so greatly reduces the number of parameters we need to choose.

To proceed we first need to specify the comparative advantage curve $F(j)$ and the energy fields, $E(a)$ and $E^*(a)$. We then calibrate the baseline BAU competitive equilibrium to data on energy extraction and global carbon flows. From this base, we compute the taxes and subsidies that would shift the model economy to the outcomes dictated by the optimal policy. We also compare the BAU and optimal policies to more conventional policies such as pure extraction, production, and consumption taxes. We start by providing some of the details of the simulation (with most of the derivations relegated to the Appendix), and then present our key results.

5.1 Setup

5.1.1 Functional Forms.

To solve the model numerically we employ several convenient functional forms.

Comparative Advantage We parameterize the efficiency of the goods sector in each country by:

$$a_j = \left(\frac{j}{A}\right)^{1/\theta}, \tag{40}$$

$$a_j^* = \left(\frac{1-j}{A^*}\right)^{1/\theta}, \tag{41}$$

where A and A^* determine absolute advantage in either country, and θ determines (inversely) the scope of comparative advantage. Taking the ratio, we obtain a functional form for Home's comparative advantage curve:

$$F(j) = \frac{a_j^*}{a_j} = \left(\frac{A}{A^*}\frac{1-j}{j}\right)^{1/\theta},$$

where $F(j)$ is continuous and strictly decreasing as specified in (4).

This functional form allows us to easily solve for the import and export thresholds in the BAU. In the BAU baseline a country's average spending per good doesn't depend on the source of the good. Since the share of energy in the cost of any good is the same, the baseline value of the import margin is:

$$\bar{j}_m = \frac{C_e^{HH}}{C_e} = \frac{A}{A + (\tau^*)^{-\theta} A^*},$$

while the baseline value of the export margin is:

$$\bar{j}_x = \frac{C_e^{FH}}{C_e^*} = \frac{\tau^{-\theta} A}{\tau^{-\theta} A + A^*}.$$

Energy Supply We parameterize the upper end of the distribution of costs across energy fields, for all $a \geq \underline{a}$, as:

$$E(a) = E a^{\epsilon_S}, \tag{42}$$

$$E^*(a) = E^* a^{\epsilon_S^*}, \tag{43}$$

where E and E^* are parameters governing the quantity of energy in each region while ϵ_S and ϵ_S^* are the constant elasticities of supply at the upper ends of each distribution.[26]

Normalizing the baseline energy price to 1 in the BAU competitive equilibrium, we have $Q_e = E$ and $Q_e^* = E^*$.

5.1.2 Calibration of BAU Scenario.

We calibrate the BAU scenario to carbon accounting data for 2015 from the Trade Embodied in CO_2 (TECO$_2$) database made available by the OECD.[27] Units are gigatonnes of CO_2. Energy extraction data for 2015 is from the International Energy Agency World Energy Statistics Database. We use emissions factors to convert units of energy to units of CO_2.

For most of our results, members of the OECD form the taxing region, Home, and the non-OECD countries are Foreign. Table 4 provides our calibration.

Note that by this CO_2 metric the OECD represents about one-third of the world. It represents a smaller share of extraction and a larger share of implicit consumption, nearly twenty percent of which is imported.

[26] The distributions are unrestricted for $a < \underline{a}$. The threshold $\underline{a}$, however, must be below the after-tax energy price arising under the optimal policy.

[27] The values that we take from TECO$_2$ are broadly consistent with those available from the Global Carbon Project.

Table 4: Baseline Calibration for Home as the OECD

	Home	Foreign	Total
Home	$C_e^{HH} = 11.3$	$C_e^{HF} = 2.5$	$C_e = 13.8$
Foreign	$C_e^{FH} = 0.9$	$C_e^{FF} = 17.6$	$C_e^* = 18.5$
Total	$G_e = 12.2$	$G_e^* = 20.1$	$G_e^W = C_e^W = 32.3$
Extraction	$Q_e = 8.6$	$Q_e^* =23.7$	$Q_e^W = 32.3$

Table 5: Parameter Values

α	ϵ_S	ϵ_S^*	σ	σ^*	θ
0.85	0.5	0.5	1	1	4

In addition to the carbon accounting data, we need values for six parameters: θ, ϵ_S, ϵ_S^*, σ, σ^*, and α, the last three of which determine the demand elasticities, ϵ_D and ϵ_D^*.[28] These parameters remain fixed as we compute the optimal policy. We determine values for them using a variety of sources, described in the Appendix.[29] Table 5 lists our central values for these parameters. (In our simulations, we test for sensitivity to parameter choice, so in some cases, we allow the parameters to vary.) Appendix E provides additional details on our calibration procedure.

[28]The eight other parameters: A, A^*, E, E^*, η, η^*, τ, and τ^* are all subsumed by calibrating to the carbon accounts.

[29]We choose $\alpha = 0.85$ based on the ratio of the value of energy used in production to value added. (In our model that ratio is $(1 - \alpha)/\alpha$.) Values for ϵ_S and ϵ_S^* are estimated using data in Asker, Collard-Wexler, and De Loecker (2018), by fitting the slope of $E(a)$ and $E^*(a)$ among oil fields with costs above the median. Appendix E provides more details. We take $\theta = 4$ based on the preferred estimate in Simonovska and Waugh (2014). The values for σ and σ^* are interim.

5.1.3 From BAU to Optimal

For any endogenous variable x we denote the BAU baseline value by x and the value under the optimal policy by $x(p_e, t_b)$. In the baseline $t_b = 0$, $t_e = 0$, and the energy price is 1. The magnitude of the tax rates under the optimal policy can be interpreted in the ad-valorem sense, relative to the initial energy price. The optimal policy requires that we set the extraction tax to $t_e = \varphi$.

For example, under the optimal policy, the import margin remains fixed while the export margin changes to:

$$\bar{j}_x(p_e, t_b) = \frac{\tau^{-\theta} A \left(p_e + (1-\alpha)\, t_b\right)^{\theta}}{\tau^{-\theta} A \left(p_e + (1-\alpha)\, t_b\right)^{\theta} + A^* \left(p_e^{\alpha} \left(p_e + t_b\right)^{1-\alpha}\right)^{\theta}}$$

$$= \frac{\left(p_e + (1-\alpha)\, t_b\right)^{\theta} C_e^{FH}}{\left(p_e + (1-\alpha)\, t_b\right)^{\theta} C_e^{FH} + \left(p_e^{\alpha} \left(p_e + t_b\right)^{1-\alpha}\right)^{\theta} C_e^{FF}}.$$

The second line shows what we achieve by calibrating to the carbon accounts. Energy extraction, for $p_e + t_b - \varphi \geq \underline{a}$, is:

$$Q_e(p_e, t_b) = \left(p_e + t_b - \varphi\right)^{\epsilon_S} Q_e.$$

Using similar reasoning we can express the values under the optimal policy for each component of energy demand and for the value of goods trade. See Appendix D for details.

We express the change in welfare relative to Home's baseline spending on goods:

$$W = \frac{U(p_e, t_b) - U}{V_g}.$$

5.1.4 Constrained Optimal Policies

To understand the optimal policy, it is useful to compare it to more conventional policies. We will consider three conventional policies, a pure extraction tax, a pure production tax, and a pure consumption tax, and two hybrids of these policies, an optimal mix of extraction and production taxes and an optimal mix of production and consumption taxes. In each case, we find Home's optimal policy assuming it is constrained to a particular choice (e.g., we find Home's optimal extraction tax conditional on Home being constrained to choosing only an extraction tax).

Pure Extraction Tax To obtain an optimal pure extraction tax, we constrain Home's planner to choose only Q_e, X_e, and p_e, with all other outcomes determined in a decentralized equilibrium. We solve the Lagrangian for this problem and reinterpret the outcome as a decentralized equilibrium. The optimal extraction tax in this case is:

$$t_e = \varphi \frac{\epsilon_D C_e + \epsilon_D^* C_e^*}{\epsilon_S^* Q_e^* + \epsilon_D C_e + \epsilon_D^* C_e^*} - \frac{p_e \left(Q_e^* - C_e^*\right)}{\epsilon_S^* Q_e^* + \epsilon_D C_e + \epsilon_D^* C_e^*}. \tag{44}$$

Ignoring the second term, this rate is below the value of $t_e = \varphi$ in the optimal policy. How much below turns on the value of $\epsilon_S^* Q_e^*$. If Foreign is a major energy extractor and if its price elasticity of supply is high, then Home will want to choose a lower extraction tax because of concerns about the effects of a high extraction tax on Foreign extraction

Turning to the second term, note that the numerator is the value of Foreign's net exports of energy based on its implicit consumption of embodied energy. Its use of energy in production doesn't matter here. If Foreign is an exporter in this sense then Home wants a lower extraction tax to improve its terms of trade. For the same reason, it will choose a higher extraction tax if Foreign is a large net importer in this sense.

Pure Consumption Tax For a pure consumption tax, we follow the same procedure except now the planner is constrained to choose only: $\left\{z_j^y\right\}$, $\left\{z_j^m\right\}$, $\{y_j\}$, $\{m_j\}$, X_e, and p_e, with all other outcomes determined as in a decentralized competitive equilibrium. Expressed as an effectivd tax (i.e., a tax directly only consumption), the optimal consumption tax is:

$$\tilde{t}_c = \varphi \frac{\epsilon_S Q_e + \epsilon_S^* Q_e^*}{\epsilon_S Q_e + \epsilon_S^* Q_e^* + \epsilon_D^* C_e^*} + \frac{p_e \left(Q_e^* - C_e^*\right)}{\epsilon_S Q_e + \epsilon_S^* Q_e^* + \epsilon_D^* C_e^*}. \tag{45}$$

Expressed in nominal terms, Home would impose an extraction tax of $t_e = \tilde{t}_c$ and border adjustments on both energy and goods at that same rate, $t_b = \tilde{t}_c$.

As with an extraction tax, the first term, multiplying φ is less than one, but the amount it is less than one now depends on the relative value of $\epsilon_D^* C_e^*$. If Foreign is a major energy consumer and it its price elasticity of demand is high, then Home will want to choose a lower consumption tax because of concerns that a high consumption tax will produce a corresponding shift in Foreign consumption.[30]

[30]We can equivalently impose a consumption tax by imposing an extraction tax at

Optimal Hybrid In our hybrid tax, we allow Home to combine extraction taxes and consumption taxes. Solving for the optimal mix is the same as for a pure consumption tax, replacing the competitively determined Q_e with the planner's choice. The optimal tax in this case is:

$$t_e = \varphi$$

together with a border tax of:

$$t_b = \varphi \frac{\epsilon_S^* Q_e^*}{\epsilon_S^* Q_e^* + \epsilon_D^* C_e^*} + \frac{p_e \left(Q_e^* - C_e^*\right)}{\epsilon_S^* Q_e^* + \epsilon_D^* C_e^*}. \tag{46}$$

Since it is a partial border adjustment, unlike the pure consumption tax, the net-of-tax price received by Home's energy extractors is $p_e + t_b - \varphi$

The equivalent effective taxes are:

$$\tilde{t}_e = \varphi \frac{\epsilon_D^* C_e^*}{\epsilon_S^* Q_e^* + \epsilon_D^* C_e^*} - \frac{p_e \left(Q_e^* - C_e^*\right)}{\epsilon_S^* Q_e^* + \epsilon_D^* C_e^*}$$

and a consumption tax, $\tilde{t}_c = t_b$ as determined in (46). Expressed this way, these terms have obvious parallels with the pure extraction and consumption tax expressions, (44) and (45).

Pure Production Tax and Hybrids We find the optimal pure production tax and hybrids involving production taxes numerically.

5.1.5 Solving for Equilibrium Values

Returning to the optimal policy, the equilibrium energy price p_e clears the market:

$$C_e^{HH}(p_e, t_b) + C_e^{FH}(p_e, t_b) + C_e^{FF}(p_e, t_b) + C_e^{HF}(p_e, t_b) = Q_e(p_e, t_b) + Q_e^*(p_e), \tag{47}$$

while the optimal border adjustment t_b satisfies:

$$t_b = \frac{\varphi \epsilon_S^* Q_e^*(p_e) - p_e C_e^{FF}(p_e, t_b) - (1 - \epsilon_D^*) V_g^{FH}(p_e, t_b) - (p_e + t_b)\sigma^* C_e^{FH}(p_e, t_b)}{\epsilon_S^* Q_e^*(p_e) + \epsilon_D^* C_e^{FF}(p_e, t_b)}. \tag{48}$$

$t_e = t_c$, and a full border adjustment, $t_b = t_e$, on all imports and exports of energy and goods. Unlike the optimal policy, the border adjustment applies to exports of goods, which means these goods bear no tax.

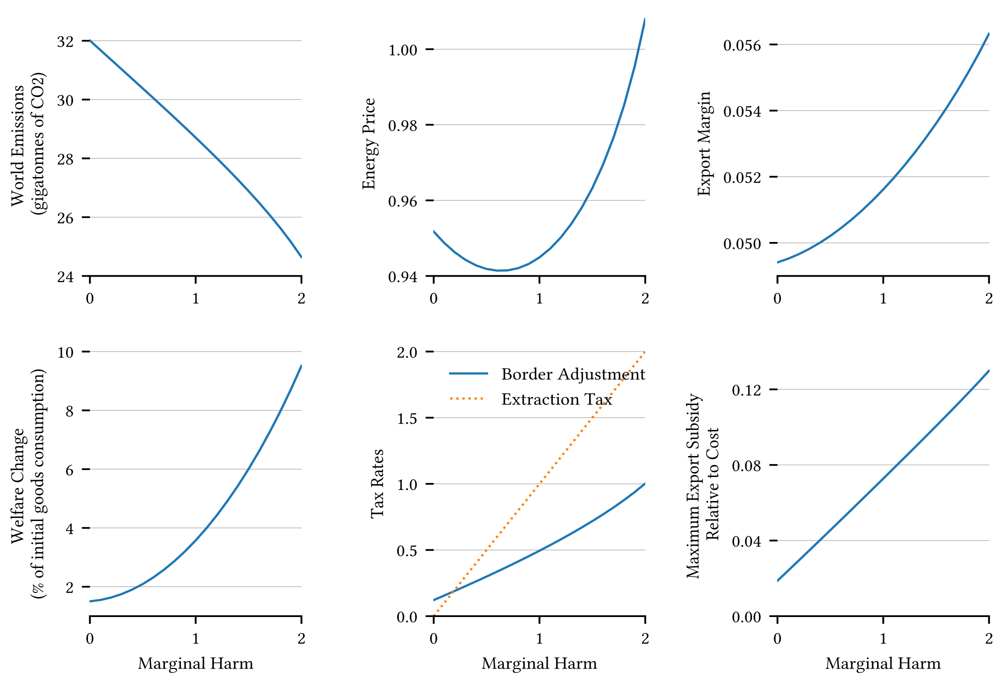
We iterate between (47) and (48) until we find a pair (p_e, t_b) that satisfies both. We can then evaluate any outcome of the model at this pair to explore the optimal policy. We use a similar procedure for the optimal constrained policies.

Our script is in Matlab, and we use the solving procedure described above rather than a built-in solver. Our code is available at `https://github.com/dweisbach/Optimal-Unilateral-Carbon-Policy`.

5.2 Simulations

5.2.1 Optimal Policy

Figure 1: Optimal Policy in the OECD



We begin with a simulation of the optimal policy in the OECD (Figure 1). We illustrate the policy for the marginal harm ranging from $\varphi = 0$ to

$\varphi = 2$. We show the emissions reductions, the change in welfare, the change in the price of energy, and the tax rates under the optimal policy.

Global emissions go down by about 1/4 with $\varphi = 2$. This is a substantial reduction given that emissions in the OECD are only about 1/3 of global emissions (as reflected in the value of G_e in Table 4). Note that while the OECD reduces its extraction to near zero, this does not mean that it reduces its emissions to near zero. As we will discuss, some of the reductions arise outside the OECD because of how the optimal policy expands the carbon price to trading partners.

Notably, the OECD would choose to impose a significant carbon policy even when the rest of the world does not. For $\varphi = 2$, the optimal carbon policy reduces global emissions by 7.6 Gt CO_2. That the OECD would choose these policies on its own may have important implications for the design of climate negotiations: even if one or more countries hold out, it makes sense for the remaining countries to impose a substantial carbon price.

The extraction tax rate (bottom middle) is always equal to φ. Recalling that the tax rate can be interpreted in the ad-valorem sense, the optimal tax rates range from 0 to up to twice the initial price of energy.

For values of φ close to zero, the border adjustment is still positive and actually exceeds the extraction tax rate. This policy arises because the OECD has a deficit in energy ($Q_e = 8.6$ and $G_e = 12.2$). The border adjustment hits energy imports, raising the price faced by energy users in the OECD, stimulating energy extraction there, and lowering the price of energy on the world market. The price of energy (pre-border adjustment) falls from 1 to 0.95, an improvement in the OECD's terms of trade. For the same reason, welfare goes up even when there is no need for a climate policy, at $\varphi = 0$.

As φ and the extraction tax go up, the two lines cross. When $\varphi = 2$ the border adjustment is just half the value of φ. Energy extractors are thus bearing a large share of the carbon tax. The OECD's policy, however, still pushes the energy price (top middle) below 1 until φ and the extraction tax approach 2. At this extreme, the net price received by energy extractors in the OECD, $p_e + t_b - t_e$, approaches zero.

Examining the two graphs on the right-hand column of Figure 1, we can see that Home expands its export margin as marginal damages increase. By expanding its export margin, Home is able to broaden the application of its carbon policy, which becomes more important as the marginal harm from emissions increases. This feature of the policy comes at a cost that rises with φ.

To further examine the features of the optimal policy, we present four simulations that vary different elements of Home's policy.

5.2.2 Coalition Size

A key factor in global climate negotiations is the set of countries that will agree to emissions reductions. As noted, one of the major criticisms of the Kyoto Protocol was that it left some large emitters out of the emissions reduction coalition. The Paris Agreement was, in part, designed to address this criticism by trying to achieve universal participation.

To examine the effects of coalition size, Figure 2 shows the effects on global emissions of optimal policies with five increasingly larger coalitions, starting with just the EU and eventually going up to a globally harmonized tax.[31] Tables 6, 7 and 8 provide the calibrations for the three new scenarios. All other parameters remain the same across each case. For example, we do not adjust the energy supply elasticities based on which extractors are in the taxing coalition.

The left-hand graph shows the welfare cost (excluding harms from climate change) of achieving a given percentage reduction in emissions from the 2015 level (32.3 Gt CO_2). Reading horizontally, it gives the effective price of emissions reductions for different taxing coalitions.[32]

The right-hand graph shows the emissions reductions that each of the coalitions would achieve with the optimal policy for a given level of marginal harm. We assume that the value of φ is the same for each coalition rather than trying to scale it for population size. This means that its effective value is lower as the coalition gets larger because the same harm is spread over more people.

Both figures show a consistent story, which is that there are substantial gains from expanding the taxing coalition. The EU alone has almost no power to reduce emissions. Adding the United States or the rest of the OECD countries helps significantly. Adding China to the taxing coalition

[31] We treat the global case as the limit of our two-region model as Foreign becomes infinitesimally small. The EU is treated as having 28 members as it had, prior to Brexit, in 2015.

[32] Note that for coalitions other than the global coalition, the price is positive rather than negative for low levels of emissions reductions. This is because under the optimal policy, Home manipulates its terms of trade. The global coalition cannot do this, so those gains do not appear in the global tax case.

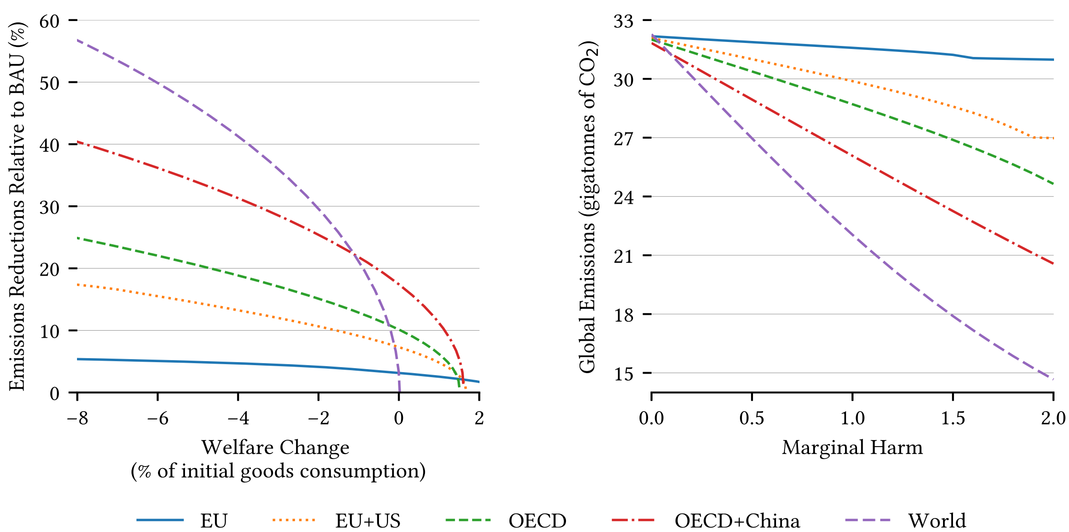
Table 6: Calibration for the European Union

	Home	Foreign	Total
Home	$C_e^{HH} = 3.0$	$C_e^{HF} = 1.0$	$C_e = 4.0$
Foreign	$C_e^{FH} = 0.5$	$C_e^{FF} = 27.8$	$C_e^{*} = 28.3$
Total	$G_e = 3.5$	$G_e^{*} = 28.8$	$G_e^{W} = C_e^{W} = 32.3$
Extraction	$Q_e = 1.0$	$Q_e^{*} =31.3$	$Q_e^{W} = 32.3$

Table 7: Calibration for the EU and the United States

	Home	Foreign	Total
Home	$C_e^{HH} = 7.7$	$C_e^{HF} = 2.0$	$C_e = 9.8$
Foreign	$C_e^{FH} = 0.7$	$C_e^{FF} = 21.8$	$C_e^{*} = 22.5$
Total	$G_e = 8.5$	$G_e^{*} = 23.8$	$G_e^{W} = C_e^{W} = 32.3$
Extraction	$Q_e = 5.4$	$Q_e^{*} =26.9$	$Q_e^{W} = 32.3$

Figure 2: Choice of Pricing Coalition



further reduces the costs of any given level of reductions and increases the willingness of the coalition to reduce emissions (even without attempting to adjust φ).

Looking at the calibration tables, we can see that the size of the extraction base is the key difference between the EU and the coalition of the EU and the United States. Production and consumption roughly double, reflecting the relative size of the two economies, but extraction goes up by a factor of more than 5. With almost no extraction, the EU on its own is unable to take advantage of the extraction tax portion of the optimal policy, which means that acting alone, it is ineffective at reducing emissions. Adding the United States expands the extraction base and makes the policy more effective.

The key difference when the coalition increases to include the entire OECD and China (other than its shear increase in size) is that energy use in production and consumption in the taxing coalition are now about equal. The coalition is still a net importer of energy, however. Extraction in the coalition is half of global extraction while production and consumption are about 2/3 of the global values.

Table 8: Calibration for the OECD plus China

	Home	Foreign	Total
Home	$C_e^{HH} = 20.1$	$C_e^{HF} = 1.7$	$C_e = 21.8$
Foreign	$C_e^{FH} = 1.4$	$C_e^{FF} = 9.1$	$C_e^* = 10.5$
Total	$G_e = 21.5$	$G_e^* = 10.8$	$G_e^W = C_e^W = 32.3$
Extraction	$Q_e = 16.24$	$Q_e^* =16.1$	$Q_e^W = 32.3$

5.2.3 Choice of tax

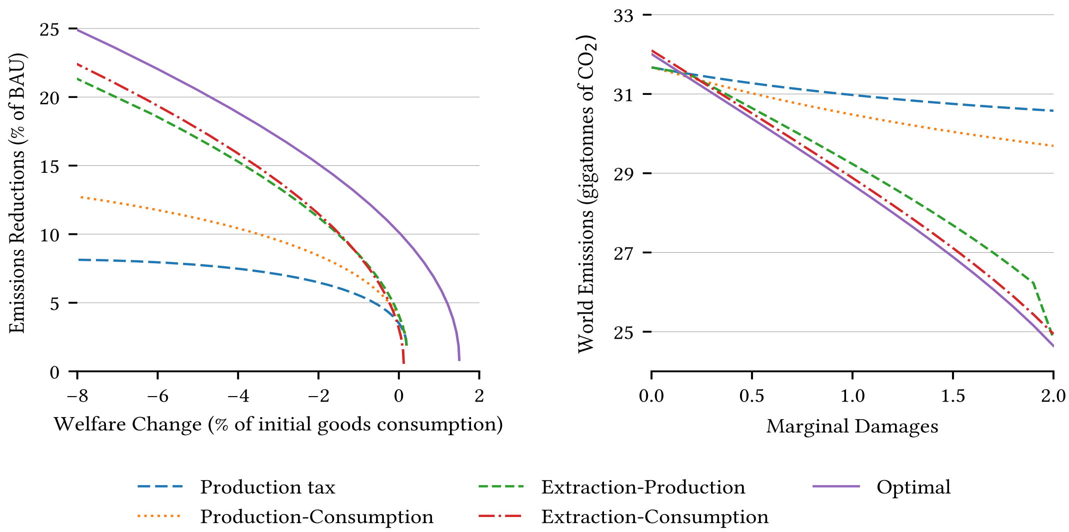
Most analyses of carbon taxes and trade examine the effects of adding border adjustments to production taxes, shifting the tax base to domestic consumption. Returning to the assumption that the OECD is the taxing coalition, Figure 3 examines the effects of border adjustments, comparing a production tax and a production tax with optimal border adjustments to the optimal policy and to two hybrid taxes: an extraction/production hybrid and an extraction/consumption hybrid (as determined in (46)).

As can be seen, the pure production taxes do poorly whether measured by their cost per unit of emissions reduction or the level of emissions reductions that they optimally achieve. Adding border adjustments improves their performance, but only modestly. The two hybrid taxes perform much better. They reduce emissions almost as much as the optimal tax (right hand figure) but do so at a somewhat greater cost (left hand figure).

The reason we suspect that the extraction tax hybrids perform relatively well is how they control the price of energy. Figure 4 illustrates. It shows the change in p_e for the three "pure" taxes (extraction, production, and consumption) the optimal tax, and the two extraction tax hybrids. Both the production and consumption taxes act as demand-side taxes, reducing demand and, therefore, the price of energy transmitted to Foreign. The extraction tax acts as a tax on the supply of energy. By reduce Home's supply of energy, it increases p_e.

The extraction/production hybrids combine demand and supply-side taxes,

Figure 3: Effects of different taxes on emissions



moderating the effects on p_e. As we will explore immediately below, the result is more moderate incentives in Foreign to make adjustments that offset Home's tax. The optimal tax is similar, producing a pattern very close to the extraction/consumption hybrid, although with slightly lower values of p_e. In all three cases, p_e eventually increases as the value of φ gets high.

As seen in Figure 3, the two extraction tax hybrids perform roughly the same. The extraction/production hybrid, however, only requires border adjustments on energy while the extraction/consumption hybrid requires border adjustments on goods as well. Border adjustments on energy would be simple to implement while border adjustments on all goods would be extremely complex to implement. As a result, a clear implication of this comparison is that if the taxing coalition is constrained to pick among the conventional taxes (for example, because the optimal tax is too complex to administer or because it might run afoul of international trade law), it should choose the extraction/production hybrid. This combination produces better results than a production tax with border adjustments and roughly equivalent results to an extraction tax with border adjustments on goods, yet is easier to implement than either. The extraction/production hybrid would also be easier to implement than a conventional production tax with border adjustments on goods. As a result, the extraction/production hybrid should be

Figure 4: Effects on p_e

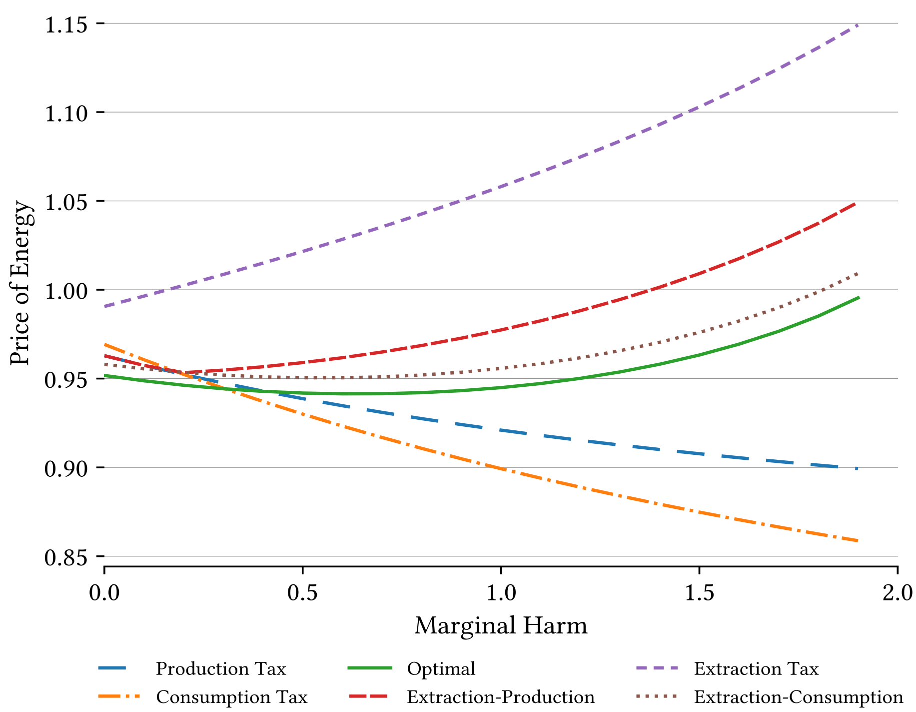
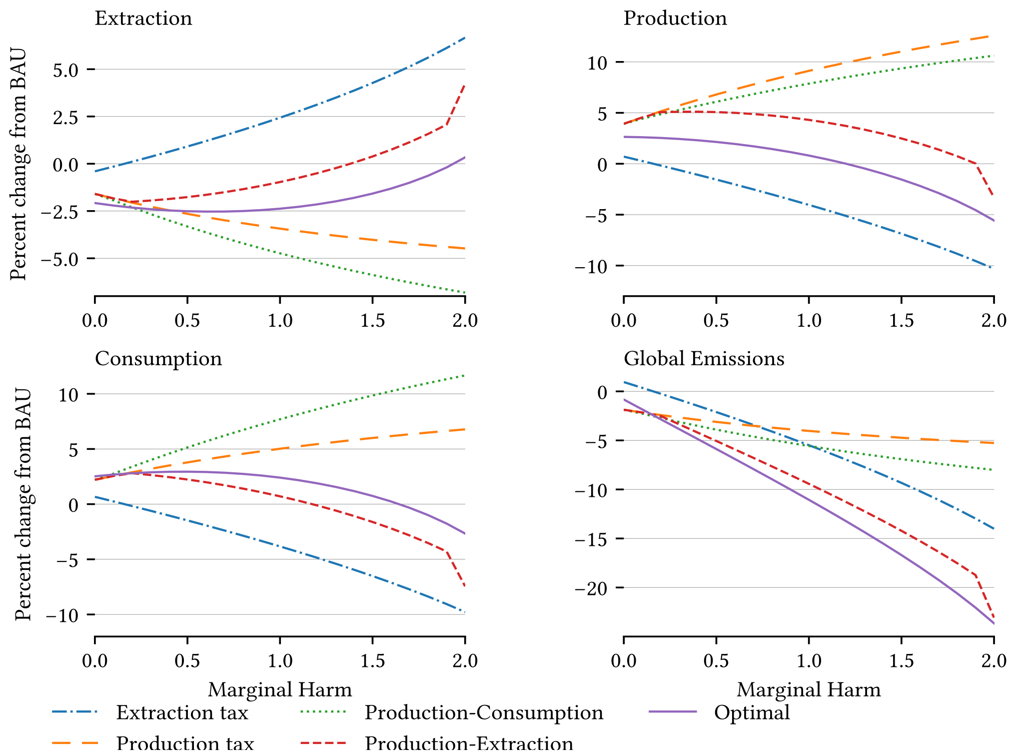


Figure 5: Effects on Foreign Activities



strongly preferred to the conventional alternative. It both performs better and is simpler to implement.

5.2.4 Location

Figure 5 explores the effects of taxes on leakage and other shifts in location, focusing on how activities in Foreign change in response to Home's taxes. It illustrates five of the taxes considered in Figure 4 (dropping the extraction/consumption hybrid). It shows the percent changes in Q_e^*, G_e^*, and C_e^* relative to their values with no tax. We show, for reference in the bottom right, the change in global emissions for each of these taxes.

Changes to extraction (top left) are consistent with the changes to p_e seen in Figure 4. Extraction taxes drive up p_e and as a result, cause Foreign to increase its extraction. Production and consumption taxes drive p_e

down, causing Foreign to reduce its extraction. The optimal tax and the extraction/production hybrid moderate the effects on Foreign extraction.

The opposite occurs for G_e^* and C_e^* (top right, bottom left). Because production and consumption taxes drive p_e down, G_e^* and C_e^* both go up when Home imposes those taxes. Correspondingly, Foreign production and consumption both go down when Home imposes an extraction tax. And once again, the optimal and the extraction/production hybrid operate in the middle.

A clear implication of Figures 4 and 5 is that Home's strategy in choosing its carbon tax policy should be to choose a mix of taxes that moderate the effects on p_e. Doing so allows Home to reduce shifts in location that offset domestic emissions reductions. This involves mixing an extraction tax, which raises p_e and either a production or consumption tax, which lowers it.

The extraction/production hybrid, in particular, has similar effects to the optimal tax but may be far simpler to implement. As a result, it is a promising avenue for further exploration.

6 Renewable Energy

Up to this point we have assumed that all energy is from fossil fuels. We now consider how the optimal policy changes in the presence renewable energy, which we take to be carbon free and a perfect substitute for users.

With renewables, Home's supply of energy is now the sum of fossil fuels, Q_f, and renewables, Q_r:

$$Q_e = Q_f + Q_r,$$

Endowments of fossil fuels are still summarized by $E(a)$ while the quantity of renewable energy that can be generated at a cost below a in Home is $R(a)$. Foreign is endowed with $E^*(a)$ and $R^*(a)$. Foreign's energy supply curve is thus:

$$Q_e^* = E^*(p_e) + R^*(p_e),$$

while their extraction of fossil fuels is:

$$Q_f^* = E^*(p_e).$$

Global emissions are:

$$Q_f^W = Q_f + Q_f^*.$$

We assume that renewable energy is nontradable, so net exports of energy are the same as net exports of fossil fuel: $X_e = X_f$. We also assume that Home continues to use fossil fuels, thus ruling out a situation in which Home chooses $Q_f = 0$ (if marginal damages are very high) while also importing no energy, $X_e = 0$ (if Home's renewable sector is very efficient).

A final assumption, which we explored in Kortum and Weisbach (2017), is that Home cannot influence the size of the renewables sector in Foreign through its import policy.[33] Home may find it impossible to verify the type of energy used in Foreign to produce goods. If it cannot verify the source of energy used, it cannot condition imports on the use of renewables. Moreover, even if Home can verify the type of energy used in production, if Foreign's renewable sector is sufficiently large, it could simply use renewables for its exports to Home and fossil fuels for its domestically consumed goods. In this case, a requirement that Foreign's exports to Home be produced with renewable energy would be futile because it would not change Foreign's, or global, emissions.

Under these assumptions introducing renewables into the planner's optimization turns out to be rather simple. Given the price and shadow values of energy (p_e, λ_e, and λ_e^*) the inner problem, which concerns the production and allocation of an individual good j, is unchanged. The outer problem is the same as in our base case (25), except that it must account separately for labor used in the fossil fuels and renewables sectors.

This difference generates only two changes to the first order conditions and the resulting taxes. First, while Home's optimal supply curve for fossil fuels remains the same

$$Q_f = E(p_e + \lambda_e^* - \varphi),$$

the first order condition for Q_r does not include the marginal damages parameter because renewables do not cause harm:

$$Q_r = R(\lambda_e).$$

As a result, while the extraction tax $t_e = \varphi$ continues to apply to Home's fossil-fuel extractors, producers of renewable energy are not taxed at all. They receive the domestic energy price in Home of $p_e + t_b$ for the energy that

[33] Home can influence Foreign's use of renewables via p_e. The share of renewables in Foreign's energy mix, however, could go either up or down with p_e depending on the shape of $E^*(a)$ and $R^*(a)$ in the relevant range around $a = p_e$.

they produce. This implicit subsidy to renewables is hidden from users of energy who pay p_e+t_b for either type of energy. Similarly, the energy intensity of goods producers is computed as before, without regard to which type of energy they use. We break the connection between carbon and energy.

Second, in the expression for the optimal border adjustment, the term reflecting marginal harm from Foreign extraction applies only to Q_f^* because Foreign's use of renewables does not generate harm in Home:

$$t_b = \frac{Q_e^* - C_e^{FF}}{\partial\left(Q_e^* - C_e^{FF}\right)/\partial p_e} + \varphi \frac{\partial Q_f^*/\partial p_e}{\partial\left(Q_e^* - C_e^{FF}\right)/\partial p_e} - \frac{\partial \Pi_g/\partial p_e}{\partial\left(Q_e^* - C_e^{FF}\right)/\partial p_e}.$$

The interpretation of the border adjustment remains effectively the same as for (39).

The global market-clearing condition for energy becomes:

$$C_e^W = E(p_e + t_b - \varphi) + R\left(p_e + t_b\right) + E^*(p_e) + R^*(p_e).$$

The demand for energy (the left-hand side) remains unchanged by the addition of renewables, given the price of energy and the border adjustment.

7 Conclusion

While the model in this paper is highly stylized, its simplicity yields analytical insights into the features of an optimal unilateral carbon policy. The main new finding is the extent to which international trade can be exploited to broaden the reach of carbon policy. The optimal carbon policy uses trade to expand carbon pricing and to lower energy use outside the narrow borders of the taxing region.

To see whether such effects are of first-order importance, it is critical to push the analysis in a more quantitative direction, extending it to multiple countries and perhaps to multiple periods of time as well. For the first extension, the multi-country model of Eaton and Kortum (2002) retains the Ricardian structure of trade in goods used here, Farrokhi and Lashkaripour (2020) consider optimal policy in a multi-country world, and the model of Larch and Wanner (2019) contains a natural multi-country extension of the energy sector. On the second extension, the dynamic analysis in Golosov, Hassler, Krusell, and Tsyvinski (2014) appears amenable to nesting within a multi-country world.

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A Global Planner's Problem

Consider a planner seeking to maximize global welfare:

$$U^W = C_s + C_s^* + \frac{\eta^{1/\sigma}}{1-1/\sigma}\int_0^1 \left((y_j+m_j)^{1-1/\sigma} - 1\right) dj + \frac{(\eta^*)^{1/\sigma^*}}{1-1/\sigma^*}\int_0^1 \left(\left(y_j^* + x_j\right)^{1-1/\sigma^*} - 1\right) dj - \varphi^W\left(Q_e + Q_e^*\right).$$

Here

$$\varphi^W = \varphi + \varphi^*$$

is marginal global damages from global emissions.[34] The planner is constrained by the endowments of labor in each country:

$$L_s + L_e + L_g \leq L$$

and

$$L_s^* + L_e^* + L_g^* \leq L^*,$$

as well as by a global energy constraint:

$$C_e + C_e^* \leq Q_e + Q_e^*.$$

Since $C_s + C_s^* = L_s + L_s^*$ we can substitute the two labor constraints into the objective while applying a Lagrange multiplier λ_e^W to the global energy constraint. The resulting Lagrangian, after dropping L, L^*, and other constants in the objective, is:

$$\begin{aligned}\mathcal{L} = &\frac{\eta^{1/\sigma}}{1-1/\sigma}\int_0^1 (y_j+m_j)^{1-1/\sigma}\, dj + \frac{(\eta^*)^{1/\sigma^*}}{1-1/\sigma^*}\int_0^1 \left(y_j^*+x_j\right)^{1-1/\sigma^*} dj - \varphi^W\left(Q_e+Q_e^*\right)\\ &- L_e(Q_e) - L_e^*(Q_e^*)\\ &- \int_0^1 \left(l_j(z_j^y)y_j + \tau l_j(z_j^x)x_j + l_j^*(z_j^*)y_j + \tau^* l_j^*(z_j^m)m_j\right) dj\\ &- \lambda_e^W\left(\int_0^1 \left(e_j(z_j^y)y_j + \tau e_j(z_j^x)x_j + e_j^*(z_j^*)y_j^* + \tau^* e_j^*(z_j^m)m_j\right) dj - (Q_e+Q_e^*)\right),\end{aligned}$$

where $L_e^*(Q_e^*)$ is the Foreign analog of (19). The planner chooses Q_e, Q_e^*, $\{y_j\}$, $\{y_j^*\}$, $\{x_j\}$, $\{m_j\}$, $\{z_j^y\}$, $\{z_j^*\}$, $\{z_j^x\}$, and $\{z_j^m\}$ to maximize $\mathcal{L}$.

[34]Since welfare is linear in consumption of services, transfers between countries (as long as both countries still consume services) do not alter global welfare. This indeterminacy has no implications for our objective of determining optimal global energy extraction as well as production and consumption of manufactured goods in each country.

A.1 Solution

Following CDVW, we first solve the inner problem, involving conditions for an individual good given λ_e^W. We then turn to the outer problem, optimizing over Q_e and Q_e^* while solving for λ_e^W.

A.1.1 Inner Problem

Solving the inner problem consists of evaluating first order conditions with respect to the variables that are specific to some good j: y_j, y_j^*, x_j, m_j, z_j^y, z_j^*, z_j^x, and z_j^m. The Lagrangian for good j is:

$$\begin{aligned}\mathcal{L}_j = & \frac{\eta^{1/\sigma}}{1-1/\sigma}\left(y_j+m_j\right)^{1-1/\sigma}+\frac{(\eta^*)^{1/\sigma^*}}{1-1/\sigma^*}\left(y_j^*+x_j\right)^{1-1/\sigma^*} \\ & -\nu a_j\left(y_j\left(z_j^y\right)^{\alpha-1}+\tau x_j\left(z_j^x\right)^{\alpha-1}\right)-\nu a_j^*\left(y_j^*\left(z_j^*\right)^{\alpha-1}+\tau^* m_j\left(z_j^m\right)^{\alpha-1}\right) \\ & -\lambda_e^W\left(\nu a_j\left(y_j\left(z_j^y\right)^{\alpha}+\tau x_j\left(z_j^x\right)^{\alpha}\right)+\nu a_j^*\left(y_j^*\left(z_j^*\right)^{\alpha}+\tau^* m_j\left(z_j^m\right)^{\alpha}\right)\right).\end{aligned}$$

The first order conditions for energy intensities of production imply:

$$z_j^y = z_j^x = z_j^* = z_j^m = z = \frac{1-\alpha}{\alpha\lambda_e^W}.$$

The unit energy requirement in Home is thus:

$$e_j(z) = (1-\alpha)a_j\left(\lambda_e^W\right)^{-\alpha},$$

while in Foreign:

$$e_j^*(z) = (1-\alpha)a_j^*\left(\lambda_e^W\right)^{-\alpha}.$$

The FOC for y_j implies:

$$\left(\left(y_j+m_j\right)/\eta\right)^{-1/\sigma} \leq a_j\left(\lambda_e^W\right)^{1-\alpha},$$

with equality if $y_j > 0$. The FOC for m_j implies:

$$\left(\left(y_j+m_j\right)/\eta\right)^{-1/\sigma} \leq a_j^*\tau^*\left(\lambda_e^W\right)^{1-\alpha},$$

with equality if $m_j > 0$. The good $\bar{j}_m$ at which the FOC's for y_j and m_j both hold with equality satisfies:

$$F(\bar{j}_m) = \frac{1}{\tau^*}.$$

Thus, if $j < \bar{j}_m$:

$$y_j = \eta \left(a_j \left(\lambda_e^W\right)^{1-\alpha} \right)^{-\sigma}$$

and $m_j = 0$ while for $j > \bar{j}_m$:

$$m_j = \eta \left(a_j^* \tau^* \left(\lambda_e^W\right)^{1-\alpha} \right)^{-\sigma}$$

and $y_j = 0$.

The FOC for y_j^* implies:

$$\left(\left(y_j^* + x_j \right) / \eta^* \right)^{-1/\sigma^*} \leq a_j^* \left(\lambda_e^W\right)^{1-\alpha},$$

with equality if $y_j^* > 0$. The FOC for x_j implies:

$$\left(\left(y_j^* + x_j \right) / \eta^* \right)^{-1/\sigma^*} \leq a_j \tau \left(\lambda_e^W\right)^{1-\alpha},$$

with equality if $x_j > 0$. The good $\bar{j}_x$ at which the FOC's for y_j^* and x_j both hold satisfies:

$$F(\bar{j}_x) = \tau.$$

Since F is monotonically decreasing, it follows that $\bar{j}_x < \bar{j}_m$. For $j < \bar{j}_x$:

$$x_j = \eta^* \left(a_j \tau \left(\lambda_e^W\right)^{1-\alpha} \right)^{-\sigma^*}$$

and $y_j^* = 0$ while for $j > \bar{j}_x$:

$$y_j^* = \eta^* \left(a_j^* \left(\lambda_e^W\right)^{1-\alpha} \right)^{-\sigma^*}$$

and $x_j = 0$.

A.1.2 Implications for Aggregates

Aggregating these results from the inner problem:

$$C_e(\lambda_e^W) = (1-\alpha)\,\eta \left(\int_0^{\bar{j}_m} a_j^{1-\sigma} dj + (\tau^*)^{1-\sigma} \int_{\bar{j}_m}^1 \left(a_j^*\right)^{1-\sigma} dj \right) \left(\lambda_e^W\right)^{-\epsilon_D},$$

$$C_e^*(\lambda_e^W) = (1-\alpha)\eta^* \left(\tau^{1-\sigma^*} \int_0^{\bar{j}_x} a_j^{1-\sigma^*} dj + \int_{\bar{j}_x}^1 \left(a_j^*\right)^{1-\sigma^*} dj \right) \left(\lambda_e^W\right)^{-\epsilon_D^*},$$

$$L_g(\lambda_e^W) = \alpha\eta \left(\int_0^{\bar{j}_m} a_j^{1-\sigma} dj \right) \left(\lambda_e^W\right)^{1-\epsilon_D} + \alpha\eta^* \left(\int_0^{\bar{j}_x} \left(\tau a_j\right)^{1-\sigma} dj \right) \left(\lambda_e^W\right)^{1-\epsilon_D^*},$$

$$L_g^*(\lambda_e^W) = \alpha\eta \left(\int_{\bar{j}_m}^1 (\tau^* a_j^*)^{1-\sigma} dj \right) \left(\lambda_e^W\right)^{1-\epsilon_D} + \alpha\eta^* \left(\int_{\bar{j}_x}^1 \left(a_j^*\right)^{1-\sigma} dj \right) \left(\lambda_e^W\right)^{1-\epsilon_D^*},$$

$$C_g(\lambda_e^W) = \eta \left(\int_0^{\bar{j}_m} a_j^{1-\sigma} dj + \int_{\bar{j}_m}^1 \left(a_j^* \tau^*\right)^{1-\sigma} dj \right)^{\sigma/(\sigma-1)} \left(\lambda_e^W\right)^{-(1-\alpha)\sigma},$$

and

$$C_g^*(\lambda_e^W) = \eta^* \left(\int_0^{\bar{j}_x} \left(\tau a_j\right)^{1-\sigma} dj + \int_{\bar{j}_x}^1 \left(a_j^*\right)^{1-\sigma} dj \right)^{\sigma^*/(\sigma^*-1)} \left(\lambda_e^W\right)^{-(1-\alpha)\sigma^*}.$$

These six terms are fully determined by λ_e^W.

A.1.3 Outer Problem

We now turn to the optimality conditions for Q_e and Q_e^* while choosing λ_e^W to clear the global energy market. We can rewrite the Lagrangian in terms of aggregate magnitudes as:

$$\begin{aligned}
\mathcal{L} = & \frac{\eta^{1/\sigma}}{1-1/\sigma} C_g^{1-1/\sigma} + \frac{\left(\eta^*\right)^{1/\sigma^*}}{1-1/\sigma^*} \left(C_g^*\right)^{1-1/\sigma^*} - \varphi^W \left(Q_e + Q_e^*\right) \\
& - \left(L_e(Q_e) + L_e^*(Q_e^*) + L_g + L_g^*\right) \\
& - \lambda_e^W \left(\left(C_e + C_e^*\right) - \left(Q_e + Q_e^*\right)\right).
\end{aligned}$$

The first order condition with respect to Home energy extraction implies:

$$Q_e = E(\lambda_e^W - \varphi^W).$$

Likewise for Foreign energy extraction:

$$Q_e^* = E^*(\lambda_e^W - \varphi^W).$$

The global energy constraint determines the Lagrange multiplier as the solution to:

$$C_e(\lambda_e^W) + C_e^*(\lambda_e^W) = E\left(\lambda_e^W - \varphi\right) + E^*\left(\lambda_e^W - \varphi\right).$$

A.2 Decentralized Global Optimum

We can interpret the solution in terms of a decentralized economy with a price of energy:

$$p_e = \lambda_e^W.$$

The global externality can be solved with an extraction tax in both countries equal to global damages:

$$t_e = t_e^* = \varphi^W.$$

Thus, energy extractors in both countries receive $p_e - \varphi^W$.

A.3 Competitive Equilibrium

In a competitive equilibrium agents ignore the global externality. All outcomes other than global welfare are the same as if we simply set $\varphi^W = 0$ in the decentralized global optimum above. We treat this case as our business-as-usual baseline.

B Home Planner's Problem (Missing Steps)

Here we provide missing steps from Section 3 of the text, which derives the optimal policy. We focus on the inner problem.

The Lagrangian for good j, repeated here for convenience, is:

$$\begin{aligned}
\mathcal{L}_j = & \frac{\eta^{1/\sigma}}{1-1/\sigma}\left(y_j + m_j\right)^{1-1/\sigma} \\
& - \nu a_j \left(y_j \left(z_j^y\right)^{\alpha-1} + \tau x_j \left(z_j^x\right)^{\alpha-1}\right) \\
& - \nu a_j^* \tau^* \left(\left(z_j^m\right)^{\alpha-1} + p_e \left(z_j^m\right)^{\alpha}\right) m_j + p_j^x x_j \\
& - \lambda_e \nu a_j \left(y_j \left(z_j^y\right)^{\alpha} + \tau x_j \left(z_j^x\right)^{\alpha}\right) \\
& - \lambda_e^* \nu a_j^* \left(\max\left\{c_j^* - x_j, 0\right\} \left(z^*\right)^{\alpha} + \tau^* m_j \left(z_j^m\right)^{\alpha}\right).
\end{aligned}$$

We want to maximize by choice of $\{y_j\}$, $\{x_j\}$, $\{m_j\}$, $\{z_j^y\}$, $\{z_j^x\}$, $\{z_j^m\}$. We consider the variables relevant to Home consumers first, then turn to those relevant to Foreign consumers.

B.1 Goods for Home Consumers

The first order condition for y_j is:

$$\left(\left(y_j + m_j\right)/\eta\right)^{-1/\sigma} \leq \nu a_j \left(z_j^y\right)^{\alpha-1} + \lambda_e \nu a_j \left(z_j^y\right)^{\alpha} = \nu a_j \left(z_j^y\right)^{\alpha-1} \left(1 + \lambda_e z_j^y\right).$$

Substituting in the optimal $z_j^y = z^y$, this condition reduces to:

$$\left(\left(y_j + m_j\right)/\eta\right)^{-1/\sigma} \leq a_j \lambda_e^{1-\alpha}, \tag{49}$$

with equality if $y_j > 0$.

The first order condition for m_j is:

$$\left(\left(y_j + m_j\right)/\eta\right)^{-1/\sigma} \leq \nu \tau^* a_j^* \left(z_j^m\right)^{\alpha-1} \left(1 + \left(p_e + \lambda_e^*\right) z_j^m\right).$$

Substituting in the optimal $z_j^m = z^m$ this condition reduces to:

$$\left(\left(y_j + m_j\right)/\eta\right)^{-1/\sigma} \leq \tau^* a_j^* \left(p_e + \lambda_e^*\right)^{1-\alpha}, \tag{50}$$

with equality if $m_j > 0$.

For good $j = \bar{j}_m$ the right hand sides of (49) and (50) are equal:

$$a_{\bar{j}_m} \lambda_e^{1-\alpha} = \tau^* a_{\bar{j}_m}^* \left(p_e + \lambda_e^*\right)^{1-\alpha},$$

yielding, via (4) as in the text:

$$F(\bar{j}_m) = \frac{1}{\tau^*} \left(\frac{\lambda_e}{p_e + \lambda_e^*}\right)^{1-\alpha}.$$

Since F is strictly decreasing, for $j < \bar{j}_m$ we get (49) holding with equality and (50) as a strict inequality. Thus $m_j = 0$ with y_j satisfying (49). For for $j > \bar{j}_m$ we get (50) holding with equality and (49)as a strict inequality. Thus $y_j = 0$ with m_j satisfying (50). This reasoning confirms the results asserted in the text.

B.2 Goods for Foreign Consumers

The derivative with respect to x_j is:

$$\frac{\partial \mathcal{L}_j}{\partial x_j} = -\nu a_j \tau \left(z_j^x\right)^{\alpha-1} - \lambda_e \nu a_j \tau \left(z_j^x\right)^{\alpha} + p_j^x(p_e) + \lambda_e^* \nu a_j^* \left(z^*(p_e)\right)^{\alpha}.$$

Substituting in the optimal $z_j^x = z^x$, this derivative simplifies to:

$$\frac{\partial \mathcal{L}_j}{\partial x_j} = -a_j \tau \lambda_e^{1-\alpha} + a_j^* p_e^{1-\alpha} + a_j^* \lambda_e^* (1-\alpha) p_e^{-\alpha},$$

whose sign determines the solution for x_j. If

$$a_j \tau \lambda_e^{1-\alpha} < a_j^* p_e^{1-\alpha} + a_j^* \lambda_e^* (1-\alpha) p_e^{-\alpha} \tag{51}$$

then $x_j > 0$. In that case x_j is pushed to the maximum quantity that Foreign will be willing to buy at price p_j^x. That maximum is reached when the marginal utility of Foreign equals that price (which is the price at which Foreign could supply the good for itself):

$$(x_j/\eta^*)^{-1/\sigma^*} = a_j^* p_e^{1-\alpha}.$$

If inequality (51) is reversed then $x_j = 0$ and y_j^* solves:

$$\left(y_j^*/\eta^*\right)^{-1/\sigma^*} = a_j^* p_e^{1-\alpha}.$$

In this case Foreign produces all of the good j that it consumes.

The good $j = \bar{j}_x$ for which the inequality is () is replacehese results about consumption in Foreign, define the good $\bar{j}_x$ such that

$$\left.\frac{\partial \mathcal{L}_j}{\partial x_j}\right|_{j=\bar{j}_x} = 0.$$

Applying (4), this cutoff good will satisfy:

$$F(\bar{j}_x) = \frac{\tau \left(\frac{\lambda_e}{p_e}\right)^{1-\alpha}}{1 + (1-\alpha)\frac{\lambda_e^*}{p_e}}.$$

1. For any good $j < \bar{j}_x$ Home has comparative advantage, which leads it to export:
$$x_j = \eta^* \left(a_j^* p_e^{1-\alpha}\right)^{-\sigma^*},$$
while Foreign produces nothing for itself, $y_j^* = 0$. (Home's export quantity for any such good is at a corner solution with $u^{*\prime}(x_j) = p_j^*$.)

2. For any good $j > \bar{j}_x$ Foreign has a comparative advantage, which leads it to produce for itself:
$$y_j^* = \eta^* \left(a_j^* p_e^{1-\alpha}\right)^{-\sigma^*},$$
while demanding no exports from Home, $x_j = 0$. (Foreign's production of any such good is determined by Foreign demand at price p_j^*.)

C Constrained-Optimal Policies

Here we derive constrained-optimal policies that focus exclusively on either: (i) Home's extraction of energy, (ii) Home's implicit consumption of energy, or (iii) a combination of both. We do so by considering a planner whose menu of choice variables is limited in particular ways. All variables not on the menu are determined as in a competitive equilibrium. After solving each planner's problem, we show how it can be implemented with taxes in a decentralized equilibrium, and we present formulas for the optimal tax rates.

In each case the planner's objective is to maximize Home's welfare:

$$U = C_s + \frac{\eta^{1/\sigma}}{1-1/\sigma} \int_0^1 \left(c_j^{1-1/\sigma} - 1 \right) dj - \varphi \left(Q_e + Q_e^* \right), \tag{52}$$

subject to four constraints: (i) Home's labor constraint (20), (ii) trade balance (15), (iii) Home's energy constraint (16), and (iv) Foreign's energy constraint (17). While it involves some redundancy, maintaining four constraints facilitates comparison to the optimal policy.

C.1 Optimal Pure Extraction Tax

Suppose the planner is constrained to choose only Q_e, X_e, and p_e, with all other outcomes determined in a decentralized competitive equilibrium. We use this problem to derive the constrained-optimal pure extraction tax.

Energy intensities and the intensive and extensive margins of trade are as in a competitive equilibrium, given p_e. Similarly, spending on goods by Home's consumers, the term relevant to their welfare, becomes:

$$V_g = \eta^{1/\sigma} C_g{}^{1-1/\sigma} = \eta p_e^{(1-\alpha)(1-\sigma)} \left(\int_0^{\bar{j}_m} a_j^{1-\sigma} dj + \int_{\bar{j}_m}^1 \left(\tau^* a_j^* \right)^{1-\sigma} dj \right). \tag{53}$$

Furthermore, this spending term is tightly linked to Home's consumption of embodied energy:

$$p_e C_e = (1-\alpha) V_g,$$

a result that we exploit in what follows.[35] We can also exploit the connection

[35] In the absence of border adjustments on goods imports, as is the case with a pure extraction tax, we get the equation: $V_g = V_g^{HH} + V_g^{HF}$. Border adjustments introduce a wedge between the price of goods at the port, relevant for V_g^{HF}, and the price paid by consumers, relevant for V_g. Hence this equation no longer holds.

between spending on labor and energy by goods producers in Home :

$$L_g = \frac{\alpha}{1-\alpha} p_e G_e.$$

The use and implicit consumption of energy are connected to the value of Home's net exports of goods via:

$$p_e G_e - p_e C_e = (1-\alpha) X_g.$$

Recall that the trade balance constraint is:

$$X_s = -X_g - p_e X_e.$$

Home's labor constraint is:

$$Q_s = L - L_e(Q_e) - L_g,$$

where $L_e(Q_e)$ is from (19). Home's energy constraint is:

$$Q_e - G_e - X_e = 0.$$

Foreign's energy constraint is:

$$Q_e^* - G_e^* + X_e = 0.$$

C.1.1 The Planner's Lagrangian

We substitute the first two constraints into the objective (52) to eliminate C_s while attaching Lagrange multipliers λ_e and λ_e^* to the energy constraints. The resulting Lagrangian (dropping constants) is:

$$\begin{aligned}
\mathcal{L} = & \frac{\sigma}{\sigma-1} V_g - \varphi \left(Q_e + Q_e^*\right) \\
& - L_e(Q_e) - L_g + X_g + p_e X_e \\
& - \lambda_e \left(G_e - Q_e + X_e\right) - \lambda_e^* \left(G_e^* - Q_e^* - X_e\right).
\end{aligned}$$

This Lagrangian is similar to the outer problem for the planner choosing the optimal policy after maximizing the inner problem. The difference is that, by not maximizing the inner problem, we can't invoke the envelope condition. All aggregates, except the three that are directly chosen, depend on the energy price.

C.1.2 Solution

The first order condition for X_e is:

$$\lambda_e = p_e + \lambda_e^*,$$

as in the optimal policy. The first order condition for Q_e also matches the optimal policy:

$$Q_e = E(p_e + \lambda_e^* - \varphi).$$

The first order condition for p_e is different. First, exploiting (53) and the equation beneath it:

$$\frac{\partial V_g}{\partial p_e} = \frac{(1-\alpha)(1-\sigma)}{p_e} V_g = (1-\sigma) C_e.$$

Hence, we can write the first order condition as:

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial p_e} = & -\sigma C_e - \varphi \frac{\partial Q_e^*}{\partial p_e} - \frac{\partial L_g}{\partial p_e} + \frac{\partial X_g}{\partial p_e} + X_e \\ & - \lambda_e \frac{\partial G_e}{\partial p_e} - \lambda_e^* \left(\frac{\partial G_e^*}{\partial p_e} - \frac{\partial Q_e^*}{\partial p_e} \right) = 0.\end{aligned}$$

Substituting in the first order condition for X_e and

$$\frac{\partial L_g}{\partial p_e} = \frac{\alpha}{1-\alpha} \left(G_e + p_e \frac{\partial G_e}{\partial p_e} \right),$$

we get:

$$\lambda_e^* \left(\frac{\partial G_e}{\partial p_e} + \frac{\partial G_e^*}{\partial p_e} - \frac{\partial Q_e^*}{\partial p_e} \right) = -\sigma C_e - \varphi \frac{\partial Q_e^*}{\partial p_e} + \frac{\partial X_g}{\partial p_e} + X_e - \frac{\alpha}{1-\alpha} \left(G_e + p_e \frac{\partial G_e}{\partial p_e} \right) - p_e \frac{\partial G_e}{\partial p_e}$$

Finally, we can substitute in:

$$\frac{\partial X_g}{\partial p_e} = \frac{1}{1-\alpha} \left(G_e + p_e \frac{\partial G_e}{\partial p_e} \right) + (\sigma - 1) C_e,$$

to get:

$$\lambda_e^* \left(\frac{\partial G_e}{\partial p_e} + \frac{\partial G_e^*}{\partial p_e} - \frac{\partial Q_e^*}{\partial p_e} \right) = -\varphi \frac{\partial Q_e^*}{\partial p_e} + Q_e - C_e.$$

We can rewrite this expression as:

$$\lambda_e^* = \varphi \frac{\partial Q_e^* / \partial p_e}{\partial \left(Q_e^* - C_e^W \right) / \partial p_e} + \frac{Q_e^* - C_e^*}{\partial \left(Q_e^* - C_e^W \right) / \partial p_e}.$$

In terms of elasticities, we have:

$$\lambda_e^* = \varphi \frac{\epsilon_S^* Q_e^*}{\epsilon_S^* Q_e^* + \epsilon_D C_e + \epsilon_D^* C_e^*} + \frac{p_e \left(Q_e^* - C_e^* \right)}{\epsilon_S^* Q_e^* + \epsilon_D C_e + \epsilon_D^* C_e^*}.$$

C.1.3 Decentralization

In a decentralized equilibrium we can set the extraction tax as $t_e = \varphi - \lambda_e^*$ so that:

$$Q_e = E(p_e - t_e).$$

Using the expression above for λ_e^* we get the optimal value for the extraction tax:

$$t_e = \varphi \frac{\epsilon_D C_e + \epsilon_D^* C_e^*}{\epsilon_S^* Q_e^* + \epsilon_D C_e + \epsilon_D^* C_e^*} - \frac{p_e \left(Q_e^* - C_e^*\right)}{\epsilon_S^* Q_e^* + \epsilon_D C_e + \epsilon_D^* C_e^*}. \tag{54}$$

Ignoring the second term, this rate is below the value of $t_e = \varphi$ in the optimal policy. How much below turns on the value of $\epsilon_S^* Q_e^*$. If Foreign is a major energy extractor and if its price elasticity of supply is high, then Home will want to choose a lower extraction tax, reducing extraction leakage.

Turning to the second term, note that the numerator is the value of Foreign's net exports of energy based on its implicit consumption of embodied energy. Its use of energy in production doesn't matter here. If Foreign is an exporter in this sense then Home chooses a lower extraction tax to improve its terms of trade. Likewise, Home will improve its terms of trade by choosing a higher extraction tax if Foreign is a large net importer in this sense.

C.2 Optimal Pure Consumption Tax

Suppose the planner is constrained to choose only: $\left\{z_j^y\right\}$, $\left\{z_j^m\right\}$, $\{y_j\}$, $\{m_j\}$, X_e, and p_e, with all other outcomes determined as in a decentralized competitive equilibrium. We use this problem to derive a constrained-optimal pure consumption tax.

Any good j consumed in Foreign, whether made in Home or Foreign, is produced at energy intensity:

$$z_j^x = z_j^* = z^* = \frac{1-\alpha}{\alpha p_e}.$$

If Foreign produces it the price is:

$$p_j^* = a_j^* p_e^{1-\alpha}$$

while if Home exports it:

$$p_j^x = \tau a_j p_e^{1-\alpha}.$$

Seeking the lowest price, Foreign consumers will import any good $j \leq \bar{j}_x$ and will purchase locally any good $j > \bar{j}_x$, where the cutoff satisfies:

$$F(\bar{j}_x) = \tau.$$

Using these results we can compute some aggregates which depend only on the global energy price. In particular, the value of Home's goods exports is:

$$V_g^{FH} = \eta^* p_e^{1-\epsilon_D^*} \int_0^{\bar{j}_x} (\tau a_j)^{1-\sigma^*} \, dj.$$

A fraction α of this value is paid to labor employed in Home to produce these exports, with the rest used to purchase the energy input:

$$p_e C_e^{FH} = (1-\alpha) V_g^{FH}$$

Foreign's revenue from domestic sales is:

$$V_g^{FF} = \eta^* p_e^{1-\epsilon_D^*} \int_{\bar{j}_x}^{1} \left(a_j^*\right)^{1-\sigma^*} \, dj,$$

with:

$$p_e C_e^{FF} = (1-\alpha) V_g^{FF}.$$

Energy extraction in each country is determined as in the competitive equilibrium.

C.2.1 The Planner's Lagrangian

We again substitute the labor and trade balance constraints into the objective (52) to eliminate C_s while attaching Lagrange multipliers λ_e and λ_e^* to the energy constraints. The resulting Lagrangian is (dropping constants):

$$\begin{aligned}
\mathcal{L} = {} & \frac{\eta^{1/\sigma}}{1-1/\sigma} \int_0^1 (y_j + m_j)^{1-1/\sigma} \, dj - \varphi \left(Q_e + Q_e^*\right) \\
& - L_e(Q_e) - \int_0^1 l_j(z_j^y) y_j dj - \alpha V_g^{FH} \\
& - \int_0^1 \tau^* \left(l_j^*(z_j^m) + p_e e_j^*(z_j^m)\right) m_j dj + V_g^{FH} + p_e X_e \\
& - \lambda_e \left(\int_0^1 e_j(z_j^y) y_j dj + C_e^{FH} - Q_e + X_e\right) \\
& - \lambda_e^* \left(C_e^{FF} + \int_0^1 \tau^* e_j^*(z_j^m) m_j dj - Q_e^* - X_e\right).
\end{aligned}$$

We want to maximize this Lagrangian by the optimal choice of $\{z_j^y\}$, $\{z_j^m\}$, $\{y_j\}$, $\{m_j\}$, X_e, and p_e.

C.2.2 Solution

We start with the *inner problem*, involving conditions for an individual good j given values for X_e, λ_e, λ_e^*, and p_e. We then evaluate the optimal conditions for X_e and p_e while solving for λ_e and λ_e^*.

Inner Problem Solving the inner problem consists of first order conditions with respect to y_j, m_j, z_j^y, and z_j^m. These first order conditions, and their implications given X_e, λ_e, λ_e^*, and p_e, can be considered one good at time. We therefore define a Lagrangian for good j:

$$\begin{aligned}\mathcal{L}_j = & \frac{\eta^{1/\sigma}}{1-1/\sigma}\left(y_j+m_j\right)^{1-1/\sigma} \\ & -\nu a_j y_j \left(z_j^y\right)^{\alpha-1} - \nu a_j^* \tau^* \left(\left(z_j^m\right)^{\alpha-1} + p_e \left(z_j^m\right)^{\alpha}\right) m_j \\ & -\lambda_e \nu a_j y_j \left(z_j^y\right)^{\alpha} - \lambda_e^* \nu a_j^* \tau^* m_j \left(z_j^m\right)^{\alpha},\end{aligned}$$

where we have substituted in the expressions for unit input requirements (5) in Home (as well as their analogs in Foreign).

The first order condition for z_j^y implies:

$$z_j^y = z^y = \frac{1-\alpha}{\alpha\lambda_e}.$$

Unlike for the optimal policy or for the case of a pure extraction tax, Home uses a different energy intensity for serving consumers in Home and Foreign.

The FOC for y_j is:

$$\left(\left(y_j+m_j\right)/\eta\right)^{-1/\sigma} \leq \nu a_j \left(z^y\right)^{\alpha-1} + \lambda_e \nu a_j \left(z^y\right)^{\alpha} = \nu a_j \left(z^y\right)^{\alpha-1}\left(1+\lambda_e z^y\right).$$

Substituting in the solution for z^y this FOC reduces to:

$$\left(\left(y_j+m_j\right)/\eta\right)^{-1/\sigma} \leq a_j \lambda_e^{1-\alpha},$$

with equality if $y_j > 0$. If this FOC holds with a strict inequality then $y_j = 0$ and Home imports the good.

The first order condition for z_j^m implies:

$$z_j^m = z^m = \frac{1-\alpha}{\alpha\left(p_e + \lambda_e^*\right)}.$$

All producers serving consumers in Home, whether domestic or foreign, produce at the same energy intensity.

The FOC for m_j is:

$$\left(\left(y_j + m_j\right)/\eta\right)^{-1/\sigma} \leq \nu a_j^* \tau^* \left(z^m\right)^{\alpha-1} \left(1 + \left(p_e + \lambda_e^*\right) z^m\right).$$

Substituting in the solution for z^m this FOC reduces to:

$$\left(\left(y_j + m_j\right)/\eta\right)^{-1/\sigma} \leq a_j^* \tau^* \left(p_e + \lambda_e^*\right)^{1-\alpha},$$

with equality if $m_j > 0$.

To distill these results about consumption in Home, define the good $\bar{j}_m$ at which the FOC for y and m both hold. Applying (4), this cutoff good will satisfy:

$$F(\bar{j}_m) = \frac{1}{\tau^*}\left(\frac{\lambda_e}{p_e + \lambda_e^*}\right)^{1-\alpha}.$$

1. For any good $j < \bar{j}_m$ Home has a comparative advantage, which leads it to produce for itself:

$$y_j = \eta\left(a_j \lambda_e^{1-\alpha}\right)^{-\sigma},$$

while importing nothing, $m_j = 0$.

2. For any good $j > \bar{j}_m$ Foreign has a comparative advantage, which leads Home to import:

$$m_j = \eta\left(a_j^* \tau^* \left(p_e + \lambda_e^*\right)^{1-\alpha}\right)^{-\sigma},$$

while producing nothing for itself, $y_j = 0$.

Outer Problem We now turn to the optimality conditions for X_e and p_e while finding the Lagrange multipliers that clear the global energy market. First, we collect results from the inner problem. The price Home pays for imports is:

$$p_j^m = \tau^* l_j^*(z^m) + p_e \tau^* e_j^*(z^m).$$

Since this price depends directly on p_e, so does Home's spending on imported goods:

$$V_g^{HF} = \int_{\bar{j}_m}^{1} p_j^m m_j dj = \int_{\bar{j}_m}^{1} \tau^* l_j^*(z^m) m_j dj + p_e \int_{\bar{j}_m}^{1} \tau^* e_j^*(z^m) m_j dj.$$

By the envelope condition we ignore the dependence of m_j and z^m on p_e, so that the appropriate derivative is:

$$\frac{\partial V_g^{HF}}{\partial p_e} = \int_{\bar{j}_m}^{1} \tau^* e_j^*(z^m) m_j dj = C_e^{HF}.$$

Energy use by Home's producers serving the domestic market is completely determined by the inner problem:

$$C_e^{HH} = \int_0^{\bar{j}_m} e_j(z^y) y_j dj,$$

as is the labor employed:

$$L_g^{HH} = \int_0^{\bar{j}_m} l_j(z^y) y_j dj.$$

We therefore rewrite the Lagrangian in terms of aggregate magnitudes:

$$\begin{aligned}
\mathcal{L} = {} & \frac{\eta^{1/\sigma}}{1 - 1/\sigma} C_g^{1-1/\sigma} - \varphi\left(Q_e(p_e) + Q_e^*(p_e)\right) \\
& - L_e(Q_e) - L_g^{HH} - \alpha V_g^{FH}(p_e) \\
& - V_g^{HF} + V_g^{FH} + p_e X_e \\
& - \lambda_e \left(C_e^{HH} + C_e^{FH} - Q_e + X_e\right) \\
& - \lambda_e^* \left(C_e^{FF} + C_e^{HF} - Q_e^* - X_e\right).
\end{aligned}$$

We now turn to the first order conditions for maximizing $\mathcal{L}$ with respect to X_e and p_e.

Home Energy Exports The first order condition with respect to X_e gives:

$$\lambda_e = p_e + \lambda_e^*,$$

as in the optimal policy. Combined with the inner problem, we get results familiar from the optimal policy. For $j \leq \bar{j}_m$ Home buys:

$$y_j = \eta \left(a_j \lambda_e^{1-\alpha}\right)^{-\sigma}$$

from domestic producers and for $j > \bar{j}_m$ Home imports:

$$m_j = \eta \left(a_j^* \tau^* \lambda_e^{1-\alpha}\right)^{-\sigma},$$

where the import margin satisfies:

$$F(\bar{j}_m) = \frac{1}{\tau^*}.$$

Finally, we have:

$$L_g = \int_0^{\bar{j}_m} l_j(z_j^y) y_j dj + \int_0^{\bar{j}_x} l_j(z_j^x) x_j dj = \alpha \eta \lambda_e^{1-\epsilon_D} \int_0^{\bar{j}_m} a_j^{1-\sigma} dj + \alpha V_g^{FH}.$$

Optimal Energy Price The first order condition with respect to p_e is:

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial p_e} &= -\varphi \left(\frac{\partial Q_e}{\partial p_e} + \frac{\partial Q_e^*}{\partial p_e}\right) - \frac{\partial L_e}{\partial Q_e}\frac{\partial Q_e}{\partial p_e} - \alpha \frac{\partial V_g^{FH}}{\partial p_e} - \frac{\partial V_g^{HF}}{\partial p_e} + \frac{\partial V_g^{FH}}{\partial p_e} \\ &\quad + X_e - \lambda_e \frac{\partial C_e^{FH}}{\partial p_e} + \lambda_e \frac{\partial Q_e}{\partial p_e} - \lambda_e^* \left(\frac{\partial C_e^{FF}}{\partial p_e} - \frac{\partial Q_e^*}{\partial p_e}\right) = 0. \end{aligned}$$

Substituting in the first order condition for X_e we get:

$$\begin{aligned} 0 &= -\varphi \left(\frac{\partial Q_e}{\partial p_e} + \frac{\partial Q_e^*}{\partial p_e}\right) - \frac{\partial L_e}{\partial Q_e}\frac{\partial Q_e}{\partial p_e} + (1-\alpha) \frac{\partial V_g^{FH}}{\partial p_e} - \frac{\partial V_g^{HF}}{\partial p_e} \\ &\quad + X_e - p_e \frac{\partial C_e^{FH}}{\partial p_e} + p_e \frac{\partial Q_e}{\partial p_e} - \lambda_e^* \left(\frac{\partial C_e^{FF}}{\partial p_e} + \frac{\partial C_e^{FH}}{\partial p_e} - \frac{\partial Q_e}{\partial p_e} - \frac{\partial Q_e^*}{\partial p_e}\right). \end{aligned}$$

Noting that:

$$\frac{\partial L_e}{\partial Q_e} = p_e,$$

and grouping terms, we can simplify to:

$$\begin{aligned} 0 &= (\lambda_e^* - \varphi) \frac{\partial Q_e^W}{\partial p_e} + (1-\alpha) \frac{\partial V_g^{FH}}{\partial p_e} - \frac{\partial V_g^{HF}}{\partial p_e} \\ &\quad + X_e - p_e \frac{\partial C_e^{FH}}{\partial p_e} - \lambda_e^* \left(\frac{\partial C_e^{FF}}{\partial p_e} + \frac{\partial C_e^{FH}}{\partial p_e}\right). \end{aligned}$$

To make further progress, from $p_e C_e^{FH} = (1-\alpha) V_g^{FH}$ we have:

$$C_e^{FH} + p_e \frac{\partial C_e^{FH}}{\partial p_e} = (1-\alpha) \frac{\partial V_g^{FH}}{\partial p_e}.$$

Substituting into the first order condtion, together with $\partial V_g^{HF} / \partial p_e = C_e^{HF}$, we get:

$$\begin{aligned} 0 &= (\lambda_e^* - \varphi) \frac{\partial Q_e^W}{\partial p_e} + C_e^{FH} - C_e^{HF} + X_e - \lambda_e^* \frac{\partial C_e^*}{\partial p_e} \\ &= (\lambda_e^* - \varphi) \frac{\partial Q_e^W}{\partial p_e} + Q_e - C_e - \lambda_e^* \frac{\partial C_e^*}{\partial p_e}. \end{aligned}$$

Applying the global energy constaint, we finally arrive at:

$$0 = \varphi \frac{\partial Q_e^W}{\partial p_e} + (Q_e^* - C_e^*) - \lambda_e^* \left(\frac{\partial Q_e^W}{\partial p_e} - \frac{\partial C_e^*}{\partial p_e} \right),$$

or:

$$\lambda_e^* = \varphi \frac{\partial Q_e^W / \partial p_e}{\partial Q_e^W / \partial p_e - \partial C_e^* / \partial p_e} + \frac{Q_e^* - C_e^*}{\partial Q_e^W / \partial p_e - \partial C_e^* / \partial p_e}.$$

C.2.3 Decentralization

We can decentralize this outcome by simply imposing an extraction tax, and an equal border tax of:

$$t_e = t_b = \varphi \frac{\partial Q_e^W / \partial p_e}{\partial Q_e^W / \partial p_e - \partial C_e^* / \partial p_e} + \frac{Q_e^* - C_e^*}{\partial Q_e^W / \partial p_e - \partial C_e^* / \partial p_e}.$$

To operationalize the formula, we can rewrite it in terms of elasticites:

$$t_e = t_b = \varphi \frac{\epsilon_S Q_e + \epsilon_S^* Q_e^*}{\epsilon_S Q_e + \epsilon_S^* Q_e^* + \epsilon_D^* C_e^*} + \frac{p_e (Q_e^* - C_e^*)}{\epsilon_S Q_e + \epsilon_S^* Q_e^* + \epsilon_D^* C_e^*}. \tag{55}$$

The border adjustment is applied to Home's energy imports raising the price of energy in Home to $p_e + t_b$. It is applied to Home's exporters of energy so that they receive a pre-tax price is $p_e + t_b$ wherever they sell. Their net-of-tax price, after paying the extraction tax, is always p_e. The border adjustment is also applied to the energy content of Home's goods imports and it is removed on the energy content of Home's goods exports.

C.3 Optimal Hybrid

Now consider augmenting the pure consumption tax by allowing the planner to choose energy extraction in Home. Doing so should give us a hybrid of the pure extraction tax and the pure consumption tax. We need only tweak the pure consumption case solved above by replacing the competitively determined Q_e with an optimally chosen Q_e.

C.3.1 The Planner's Lagrangian

We can jump directly to the outer problem as the inner problem is unchanged from the pure consumption tax case. The Lagrangian for the outer problem becomes:

$$\begin{aligned}\mathcal{L} = & \frac{\eta^{1/\sigma}}{1-1/\sigma} C_g^{1-1/\sigma} - \varphi\left(Q_e + Q_e^*\right) \\ & - L_e(Q_e) - L_g^{HH} - \alpha V_g^{FH} \\ & - V_g^{HF} + V_g^{FH} + p_e X_e \\ & - \lambda_e \left(C_e^{HH} + C_e^{FH} - Q_e + X_e\right) \\ & - \lambda_e^* \left(C_e^{FF} + C_e^{HF} - Q_e^* - X_e\right).\end{aligned}$$

We now turn to the first order conditions for maximizing $\mathcal{L}$ with respect to Q_e, X_e and p_e.

Home Energy Exports The first order condition with respect to X_e remains:

$$\lambda_e = p_e + \lambda_e^*,$$

as in the optimal policy.

Optimal Home Energy Extraction The first order condition for Q_e remains as in the optimal policy:

$$Q_e = E(p_e + \lambda_e^* - \varphi).$$

Optimal Energy Price The first order condition with respect to p_e is now:

$$\frac{\partial \mathcal{L}}{\partial p_e} = -\varphi \frac{\partial Q_e^*}{\partial p_e} - \alpha \frac{\partial V_g^{FH}}{\partial p_e} - \frac{\partial V_g^{HF}}{\partial p_e} + \frac{\partial V_g^{FH}}{\partial p_e} + X_e - \lambda_e \frac{\partial C_e^{FH}}{\partial p_e} - \lambda_e^* \left(\frac{\partial C_e^{FF}}{\partial p_e} - \frac{\partial Q_e^*}{\partial p_e} \right) = 0.$$

Substituting in the FOC for energy exports and grouping terms, we can write the FOC as:

$$0 = -\varphi \frac{\partial Q_e^*}{\partial p_e} + (1-\alpha) \frac{\partial V_g^{FH}}{\partial p_e} - \frac{\partial V_g^{HF}}{\partial p_e} + X_e - p_e \frac{\partial C_e^{FH}}{\partial p_e} - \lambda_e^* \left(\frac{\partial C_e^*}{\partial p_e} - \frac{\partial Q_e^*}{\partial p_e} \right).$$

As in the pure consumption case, we have:

$$C_e^{FH} + p_e \frac{\partial C_e^{FH}}{\partial p_e} = (1-\alpha) \frac{\partial V_g^{FH}}{\partial p_e},$$

which together with $\partial V_g^{HF} / \partial p_e = C_e^{HF}$, gives:

$$0 = -\varphi \frac{\partial Q_e^*}{\partial p_e} + C_e^{FH} - C_e^{HF} + X_e - \lambda_e^* \left(\frac{\partial C_e^*}{\partial p_e} - \frac{\partial Q_e^*}{\partial p_e} \right).$$

Using the global energy constraint:

$$0 = -\varphi \frac{\partial Q_e^*}{\partial p_e} + C_e^* - Q_e^* - \lambda_e^* \left(\frac{\partial C_e^*}{\partial p_e} - \frac{\partial Q_e^*}{\partial p_e} \right),$$

which we can rewrite as:

$$\lambda_e^* = \varphi \frac{\partial Q_e^* / \partial p_e}{\partial \left(Q_e^* - C_e^* \right) / \partial p_e} + \frac{Q_e^* - C_e^*}{\partial \left(Q_e^* - C_e^* \right) / \partial p_e}.$$

C.3.2 Decentralization

We can decentralize this outcome by imposing an extraction tax:

$$t_e = \varphi$$

together with a border tax of:

$$t_b = \varphi \frac{\partial Q_e^*/\partial p_e}{\partial \left(Q_e^* - C_e^*\right)/\partial p_e} + \frac{Q_e^* - C_e^*}{\partial \left(Q_e^* - C_e^*\right)/\partial p_e}.$$

To operationalize this formula for the border tax, we can rewrite it as:

$$t_b = \varphi \frac{\epsilon_S^* Q_e^*}{\epsilon_S^* Q_e^* + \epsilon_D^* C_e^*} + \frac{p_e \left(Q_e^* - C_e^*\right)}{\epsilon_S^* Q_e^* + \epsilon_D^* C_e^*}. \tag{56}$$

The border adjustment is applied to Home's energy imports raising the price of energy in Home to $p_e + t_b$. It is applied to Home's exporters of energy so that they receive a pre-tax price is $p_e + t_b$ wherever they sell. Their net-of-tax price is always $p_e + t_b - \varphi$. The border adjustment is also applied to the energy content of Home's goods imports and it is removed on the energy content of Home's goods exports.

C.4 Optimal Pure Production Tax

To derive the optimal pure production tax we let the planner choose only z, X_e, and p_e, with the constraint that $z_j^m = z_j^y = z$ for all j. All other outcomes are determined as in a competitive equilibrium, with the price of energy for producers in Home determined by the value of z:

$$\tilde{p}_e(z) = \frac{1-\alpha}{\alpha z}.$$

Since there is a one-to-one mapping between $\tilde{p}_e(z)$ and z, it is equivalent to think of the planner choosing $\tilde{p}_e$ instead. Since that formulation is more convenient, in what follows the planner chooses: X_e, $\tilde{p}_e$, and p_e

We immediately have four key prices for goods:

$$p_j^* = a_j^* p_e^{1-\alpha},$$

$$p_j^m = \tau^* p_j^*,$$

$$p_j = a_j \tilde{p}_e^{1-\alpha},$$

and

$$p_j^x = \tau p_j,$$

We get the export margin by equating p_j^x with p_j^* at $j = \bar{j}_x$:

$$F(\bar{j}_x) = \tau \left(\frac{\tilde{p}_e}{p_e}\right)^{1-\alpha},$$

which can be inverted by applying the functional form for comparative advantage:

$$\bar{j}_x = \frac{A\tau^{-\theta}\tilde{p}_e^{-\theta(1-\alpha)}}{A\tau^{-\theta}\tilde{p}_e^{-\theta(1-\alpha)} + A^* p_e^{-\theta(1-\alpha)}}.$$

For any good $j < \bar{j}_x$ the quantity of Home exports demanded by Foreign is:

$$x_j = \eta^*(\tau a_j \tilde{p}_e^{1-\alpha})^{-\sigma^*}$$

while $y_j^* = 0$. For any good $j > \bar{j}_x$ the quantity demanded by Foreign from its local producers is:

$$y_j^* = \eta^*(a_j^* p_e^{1-\alpha})^{-\sigma^*}$$

while $x_j = 0$. We have fully characterized Foreign's consumption of goods, as a function of p_e and $\tilde{p}_e$.

We get the import margin by equating p_j^m with p_j at $j = \bar{j}_m$:

$$F(\bar{j}_m) = \frac{1}{\tau^*}\left(\frac{\tilde{p}_e}{p_e}\right)^{1-\alpha},$$

which can be inverted by applying the functional form for comparative advantage:

$$\bar{j}_m = \frac{A\tilde{p}_e^{-\theta(1-\alpha)}}{A\tilde{p}_e^{-\theta(1-\alpha)} + A^*(\tau^*)^{-\theta} p_e^{-\theta(1-\alpha)}}.$$

For any good $j > \bar{j}_m$, Home imports:

$$m_j = \eta(\tau^* a_j^* p_e^{1-\alpha})^{-\sigma}$$

while $y_j = 0$. For any good $j < \bar{j}_m$ Home purchases:

$$y_j = \eta(a_j \tilde{p}_e^{1-\alpha})^{-\sigma}$$

from local producers, while $m_j = 0$. We have fully characterized Home's consumption of goods, as a function of $\tilde{p}_e$ and p_e.

Using these results we can compute aggregates that depend only on $\tilde{p}_e$ and p_e. In particular, the energy embodied in the goods that Home exports is:

$$C_e^{FH} = \int_0^1 \tau e_j(z) x_j dj = (1-\alpha)\eta^* \tilde{p}_e^{-\epsilon_D^*} \int_0^{\bar{j}_x} (\tau a_j)^{1-\sigma^*} dj.$$

The energy embodied in goods that Home produces for itself is:

$$C_e^{HH} = \int_0^1 \tau e_j(z) y_j dj = (1-\alpha)\eta \tilde{p}_e^{-\epsilon_D} \int_0^{\bar{j}_m} a_j^{1-\sigma} dj.$$

Home uses a quantity of labor:

$$L_g = \frac{\alpha}{1-\alpha} \tilde{p}_e (C_e^{FH} + C_e^{HH})$$

to produce these goods. The quantity of energy used in Foreign to produce goods for its domestic consumption is:

$$C_e^{FF} = (1-\alpha)\eta^* p_e^{-\epsilon_D^*} \int_{\bar{j}_x}^1 \left(a_j^*\right)^{1-\sigma^*} dj,$$

while for imports to Home:

$$C_e^{HF} = (1-\alpha)\eta p_e^{-\epsilon_D} \int_{\bar{j}_m}^1 \left(\tau^* a_j^*\right)^{1-\sigma} dj.$$

The value of Home's net exports of goods is:

$$X_g = V_g^{FH} - V_g^{HF} = \frac{1}{1-\alpha} (\tilde{p}_e C_e^{FH} - p_e C_e^{HF}).$$

The energy sector is competitive, responding to the global energy price p_e. Thus $Q_e = E(p_e)$, $Q_e^* = E^*(p_e)$, and:

$$\frac{dL_e}{dQ_e} = E^{-1}(Q_e) = p_e.$$

C.4.1 The Planner's Lagrangian

We again substitute the labor and trade balance constraints into the objective (52) to eliminate C_s while attaching Lagrange multipliers λ_e and λ_e^* to the energy constraints. The resulting Lagrangian is (dropping constants):

$$\begin{aligned}\mathcal{L} = & \frac{\eta^{1/\sigma}}{1-1/\sigma}\int_0^1 (y_j+m_j)^{1-1/\sigma}\, dj - \varphi\left(Q_e+Q_e^*\right) \\ & - L_e - L_g + X_g + p_e X_e \\ & - \lambda_e\left(G_e - Q_e + X_e\right) - \lambda_e^*\left(G_e^* - Q_e^* - X_e\right).\end{aligned}$$

We want to maximize this Lagrangian by the optimal choice of X_e, $\tilde{p}_e$, and p_e.

To make it more explicit:

$$\int_0^1 (y_j+m_j)^{1-1/\sigma}\, dj = \int_0^{\bar{j}_m} y_j^{(\sigma-1)/\sigma} dj + \int_{\bar{j}_m}^1 m_j^{(\sigma-1)/\sigma} dj$$

so that:

$$\frac{\eta^{1/\sigma}}{1-1/\sigma}\int_0^1 (y_j+m_j)^{1-1/\sigma}\, dj = \frac{\sigma}{(\sigma-1)(1-\alpha)}(\tilde{p}_e C_e^{HH} + p_e C_e^{HF}).$$

We can now rewrite the Lagrangian using four key terms, C_e^{HH}, C_e^{HF}, C_e^{FH}, and C_e^{FF}:

$$\begin{aligned}\mathcal{L} = & \frac{\sigma}{(\sigma-1)(1-\alpha)}(\tilde{p}_e C_e^{HH} + p_e C_e^{HF}) - \varphi\left(Q_e+Q_e^*\right) \\ & - L_e - \frac{\alpha}{1-\alpha}\tilde{p}_e(C_e^{HH}+C_e^{FH}) + \frac{1}{1-\alpha}(\tilde{p}_e C_e^{FH} - p_e C_e^{HF}) + p_e X_e \\ & - \lambda_e\left(C_e^{HH} + C_e^{FH} - Q_e + X_e\right) - \lambda_e^*\left(C_e^{FF} + C_e^{HF} - Q_e^* - X_e\right).\end{aligned}$$

To simplify the problem we introduce a new variable $r_e = \tilde{p}_e/p_e$ and then change variables from $(\tilde{p}_e, p_e)$ to (r_e, p_e). The advantage is that $\bar{j}_m$ and $\bar{j}_x$ depend only on r_e. After some rearranging, the Lagrangian becomes:

$$\begin{aligned}\mathcal{L} = & \frac{\sigma}{(\sigma-1)(1-\alpha)}p_e(r_e C_e^{HH} + C_e^{HF}) - \varphi\left(Q_e+Q_e^*\right) \\ & - L_e - \frac{\alpha}{1-\alpha}p_e(r_e C_e^{HH} + C_e^{HF}) + p_e(r_e C_e^{FH} - C_e^{HF}) + p_e X_e \\ & - \lambda_e\left(C_e^{HH} + C_e^{FH} - Q_e + X_e\right) - \lambda_e^*\left(C_e^{FF} + C_e^{HF} - Q_e^* - X_e\right).\end{aligned}$$

There is no inner problem. We can immediately compute the first order condition with respect to X_e, which, as always, gives us:

$$\lambda_e^* = \lambda_e - p_e.$$

The first order conditions with respect to r_e and with respect to p_e are more involved, so we begin by computing some derivatives of the objects in the Lagrangian.

C.4.2 Key Derivatives

First consider the trade margins, which are functions only of r_e:

$$\frac{\partial \bar{j}_m}{\partial r_e} = -\frac{\theta(1-\alpha)}{r_e}\bar{j}_m(1-\bar{j}_m)$$

and

$$\frac{\partial \bar{j}_x}{\partial r_e} = -\frac{\theta(1-\alpha)}{r_e}\bar{j}_x(1-\bar{j}_x).$$

Now consider the four terms in the carbon accounting matrix, starting with the energy embodied in goods produced in Home for consumption in Home. The partial derivative with respect to the global energy price is simply:

$$\frac{\partial C_e^{HH}}{\partial p_e} = -\epsilon_D \frac{C_e^{HH}}{p_e}.$$

The partial derivative with respect to r_e is more involved:

$$\frac{\partial C_e^{HH}}{\partial r_e} = -\epsilon_D \frac{C_e^{HH}}{r_e} + (1-\alpha)\eta(p_e r_e)^{-\epsilon_D} a_{\bar{j}_m}^{1-\sigma} \frac{\partial \bar{j}_m}{\partial r_e}.$$

To simplify more, note that:

$$\frac{a_{\bar{j}_m}^{1-\sigma}}{\int_0^{\bar{j}_m} a_j^{1-\sigma} dj} = \frac{1+(1-\sigma)/\theta}{\bar{j}_m},$$

so that:

$$\frac{\partial C_e^{HH}}{\partial r_e} = -\epsilon_D \frac{C_e^{HH}}{r_e} - ((1-\alpha)\theta + 1 - \epsilon_D)\frac{C_e^{HH}}{r_e}(1-\bar{j}_m).$$

Substituting in the import margin:

$$\frac{\partial C_e^{HH}}{\partial r_e} = -\frac{C_e^{HH}}{r_e}\left(\epsilon_D + \tilde{\alpha}\frac{C_e^{HF}}{r_e C_e^{HH} + C_e^{HF}}\right),$$

where:

$$\tilde{\alpha} = (1-\alpha)\theta + 1 - \epsilon_D.$$

For energy embodied in Home's imports:

$$\frac{\partial C_e^{HF}}{\partial p_e} = -\epsilon_D \frac{C_e^{HF}}{p_e}$$

and

$$\frac{\partial C_e^{HF}}{\partial r_e} = \tilde{\alpha}\frac{C_e^{HF}}{r_e}\bar{j}_m = \frac{C_e^{HF}}{r_e}\tilde{\alpha}\frac{r_e C_e^{HH}}{r_e C_e^{HH} + C_e^{HF}}.$$

Combining terms, the partial derivatives of Home's total embodied consumption of energy are:

$$\frac{\partial C_e}{\partial p_e} = -\epsilon_D \frac{C_e}{p_e}$$

and

$$\frac{\partial C_e}{\partial r_e} = -\frac{C_e^{HH}}{r_e}\left(\epsilon_D + \tilde{\alpha}(1-r_e)\frac{C_e^{HF}}{r_e C_e^{HH} + C_e^{HF}}\right)$$

Define the value of Home's consumption of embodied energy as:

$$V_e = p_e(r_e C_e^{HH} + C_e^{HF}).$$

Its partial derivatives are:

$$\frac{\partial V_e}{\partial p_e} = (1-\epsilon_D)\frac{V_e}{p_e} = (1-\epsilon_D)(r_e C_e^{HH} + C_e^{HF})$$

and

$$\frac{\partial V_e}{\partial r_e} = (1-\epsilon_D)p_e C_e^{HH}.$$

For energy embodied in Home's exports:

$$\frac{\partial C_e^{FH}}{\partial p_e} = -\epsilon_D^* \frac{C_e^{FH}}{p_e}$$

and

$$\frac{\partial C_e^{FH}}{\partial r_e} = -\epsilon_D^*\frac{C_e^{FH}}{r_e} - \tilde{\alpha}^*\frac{C_e^{FH}}{r_e}(1-\bar{j}_x) = -\frac{C_e^{FH}}{r_e}\left(\epsilon_D^* + \tilde{\alpha}^*\frac{C_e^{FF}}{r_e C_e^{FH} + C_e^{FF}}\right),$$

where:

$$\tilde{\alpha}^* = (1-\alpha)\theta + 1 - \epsilon_D^*.$$

For energy embodied in Foreign production for Foreign consumers:

$$\frac{\partial C_e^{FF}}{\partial p_e} = -\epsilon_D^* \frac{C_e^{FF}}{p_e}$$

and

$$\frac{\partial C_e^{FF}}{\partial r_e} == \tilde{\alpha}^* \frac{C_e^{FF}}{r_e} \bar{j}_x = \frac{C_e^{FF}}{r_e} \tilde{\alpha}^* \frac{r_e C_e^{FH}}{r_e C_e^{FH} + C_e^{FF}}.$$

Combining terms:

$$\frac{\partial C_e^*}{\partial r_e} = -\frac{C_e^{FH}}{r_e} \left(\epsilon_D^* + \tilde{\alpha}^* (1 - r_e) \frac{C_e^{FF}}{r_e C_e^{FH} + C_e^{FF}} \right)$$

C.4.3 First Order Conditions

Using the new notation and combining like terms, the Lagrangian is:

$$\begin{aligned} \mathcal{L} = & \frac{\epsilon_D}{\epsilon_D - 1} V_e - \varphi Q_e^W - L_e \\ & + p_e (r_e C_e^{FH} - C_e^{HF}) + p_e X_e \\ & - \lambda_e \left(C_e^{HH} + C_e^{FH} - Q_e + X_e \right) - \lambda_e^* \left(C_e^{FF} + C_e^{HF} - Q_e^* - X_e \right). \end{aligned}$$

We already have the FOC with respect to X_e. We now turn to the FOC's for r_e and then p_e.

The FOC with respect to r_e is:

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial r_e} = & \frac{\epsilon_D}{\epsilon_D - 1} \frac{\partial V_e}{\partial r_e} + p_e C_e^{FH} + p_e r_e \frac{\partial C_e^{FH}}{\partial r_e} - p_e \frac{\partial C_e^{HF}}{\partial r_e} \\ & - \lambda_e \left(\frac{\partial C_e^{HH}}{\partial r_e} + \frac{\partial C_e^{FH}}{\partial r_e} \right) - \lambda_e^* \left(\frac{\partial C_e^{FF}}{\partial r_e} + \frac{\partial C_e^{HF}}{\partial r_e} \right) = 0. \end{aligned}$$

Substituting in the partial derivative for V_e computed above as well as the FOC for X_e:

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial r_e} = & -\epsilon_D p_e C_e^{HH} + p_e C_e^{FH} + p_e r_e \frac{\partial C_e^{FH}}{\partial r_e} - p_e \frac{\partial C_e^{HF}}{\partial r_e} \\ & + p_e \frac{\partial C_e^{FF}}{\partial r_e} + p_e \frac{\partial C_e^{HF}}{\partial r_e} - \lambda_e \left(\frac{\partial C_e}{\partial r_e} + \frac{\partial C_e^*}{\partial r_e} \right) = 0. \end{aligned}$$

Cancelling terms:

$$\frac{\partial \mathcal{L}}{\partial r_e} = -\epsilon_D p_e C_e^{HH} + p_e C_e^{FH} + p_e r_e \frac{\partial C_e^{FH}}{\partial r_e} + p_e \frac{\partial C_e^{FF}}{\partial r_e} - \lambda_e \left(\frac{\partial C_e}{\partial r_e} + \frac{\partial C_e^*}{\partial r_e} \right) = 0.$$

Substituting in the other derivatives computed above:

$$\epsilon_D p_e C_e^{HH} + (\epsilon_D^* - 1) p_e C_e^{FH} = -\lambda_e \left(\frac{\partial C_e}{\partial r_e} + \frac{\partial C_e^*}{\partial r_e} \right).$$

Solving for $\tilde{p}_e = r_e p_e$:

$$\tilde{p}_e = \lambda_e \frac{(\epsilon_D + \tilde{\alpha}(1 - r_e)(1 - \bar{j}_m)) C_e^{HH} + (\epsilon_D^* + \tilde{\alpha}^*(1 - r_e)(1 - \bar{j}_x)) C_e^{FH}}{\epsilon_D C_e^{HH} + (\epsilon_D^* - 1) C_e^{FH}}$$

As a reality check, suppose iceberg costs are infinite so that there is no trade in goods. Thus $\bar{j}_m = 1$, $\bar{j}_x = 0$, and $C_e^{FH} = 0$. The expression collapses to:

$$\tilde{p}_e = \lambda_e \frac{\epsilon_D C_e^{HH}}{\epsilon_D C_e^{HH}} = \lambda_e,$$

as we would expect in a closed economy.

The FOC with respect to p_e is:

$$\frac{\partial \mathcal{L}}{\partial p_e} = \frac{\epsilon_D}{\epsilon_D - 1} \frac{\partial V_e}{\partial p_e} - \varphi \frac{\partial Q_e^W}{\partial p_e} - p_e \frac{\partial Q_e}{\partial p_e} + (r_e C_e^{FH} - C_e^{HF}) + p_e r_e \frac{\partial C_e^{FH}}{\partial p_e} - p_e \frac{\partial C_e^{HF}}{\partial p_e} + X_e - \lambda_e \left(\frac{\partial C_e^{HH}}{\partial p_e} + \frac{\partial C_e^{FH}}{\partial p_e} - \frac{\partial Q_e}{\partial p_e} \right) - \lambda_e^* \left(\frac{\partial C_e^{FF}}{\partial p_e} + \frac{\partial C_e^{HF}}{\partial p_e} - \frac{\partial Q_e^*}{\partial p_e} \right) = 0.$$

Substituting in the partial derivatives computed above as well as the FOC for X_e:

$$\frac{\partial \mathcal{L}}{\partial p_e} = -\epsilon_D (r_e C_e^{HH} + C_e^{HF}) - \varphi \frac{\partial Q_e^W}{\partial p_e} - p_e \frac{\partial Q_e}{\partial p_e} + (r_e C_e^{FH} - C_e^{HF}) + p_e r_e \frac{\partial C_e^{FH}}{\partial p_e} - p_e \frac{\partial C_e^{HF}}{\partial p_e} + X_e + p_e \frac{\partial C_e^{FF}}{\partial p_e} + p_e \frac{\partial C_e^{HF}}{\partial p_e} - p_e \frac{\partial Q_e^*}{\partial p_e} - \lambda_e \left(\frac{\partial C_e}{\partial p_e} + \frac{\partial C_e^*}{\partial p_e} - \frac{\partial Q_e^W}{\partial p_e} \right) = 0.$$

Simplifying:

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial p_e} &= -\epsilon_D(r_e C_e^{HH} + C_e^{HF}) - (\varphi + p_e)\frac{\partial Q_e^W}{\partial p_e} \\ &\quad + (r_e C_e^{FH} - C_e^{HF}) + p_e r_e \frac{\partial C_e^{FH}}{\partial p_e} + X_e \\ &\quad + p_e \frac{\partial C_e^{FF}}{\partial p_e} - \lambda_e \left(\frac{\partial C_e}{\partial p_e} + \frac{\partial C_e^*}{\partial p_e} - \frac{\partial Q_e^W}{\partial p_e} \right) = 0.\end{aligned}$$

Substituting in the other derivatives:

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial p_e} &= -\epsilon_D \frac{V_e}{p_e} - (\varphi + p_e)\frac{\partial Q_e^W}{\partial p_e} \\ &\quad + r_e C_e^{FH} - C_e^{HF} - r_e \epsilon_D^* C_e^{FH} + X_e \\ &\quad - \epsilon_D^* C_e^{FF} - \lambda_e \left(\frac{\partial C_e}{\partial p_e} + \frac{\partial C_e^*}{\partial p_e} - \frac{\partial Q_e^W}{\partial p_e} \right) = 0.\end{aligned}$$

Simplifying:

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial p_e} &= -\epsilon_D \frac{V_e}{p_e} - (\varphi + p_e)\frac{\partial Q_e^W}{\partial p_e} \\ &\quad + (1 - \epsilon_D^*)\frac{V_e^*}{p_e} - C_e^{HF} - C_e^{FF} + X_e \\ &\quad - \lambda_e \left(\frac{\partial C_e}{\partial p_e} + \frac{\partial C_e^*}{\partial p_e} - \frac{\partial Q_e^W}{\partial p_e} \right) = 0.\end{aligned}$$

Simplifying again:

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial p_e} &= -\epsilon_D \frac{V_e}{p_e} - (\varphi + p_e)\frac{\partial Q_e^W}{\partial p_e} \\ &\quad - (\epsilon_D^* - 1)\frac{V_e^*}{p_e} - Q_e^* \\ &\quad - \lambda_e \left(\frac{\partial C_e}{\partial p_e} + \frac{\partial C_e^*}{\partial p_e} - \frac{\partial Q_e^W}{\partial p_e} \right) = 0.\end{aligned}$$

Solving for λ_e:

$$\lambda_e = \frac{(\varphi + p_e)(\epsilon_S Q_e + \epsilon_S^* Q_e^*) + \epsilon_D V_e + (\epsilon_D^* - 1)V_e^* + p_e Q_e^*}{\epsilon_D C_e + \epsilon_D^* C_e^* + \epsilon_S Q_e + \epsilon_S^* Q_e^*}.$$

D Solutions for Quantitative Illustration

Here we provide a list of equations for the parameterized version of the model that we use for the quantitative results in Section 5 of the paper. For each outcome, we start with the BAU competitive equilibrium that we calibrate the model to. We then show how to express the optimal outcomes in terms of these BAU outcomes. To distinguished the two, we express outcomes under the optimal policy as functions of p_e and t_b, while leaving off these arguments to represent a BAU outcomes. Throughout we impose (40), (41), (42), and (43).

D.1 Expressions to Compute the Optimal Policy

Under the optimal policy, unit energy requirements are:

1. for production in Home

$$e_j(z) = (1-\alpha)a_j(p_e + t_b)^{-\alpha},$$

2. for production in Foreign to serve consumers in Home

$$e_j^*(z) = (1-\alpha)a_j^*(p_e + t_b)^{-\alpha},$$

3. for production in Foreign to serve consumers in Foreign

$$e_j^*(z^*) = (1-\alpha)a_j^* p_e^{-\alpha}.$$

These expressions apply to BAU as well by setting $p_e = 1$ and $t_b = 0$. Now we start to show both BAU and unilateral optimal outcomes for competitive equilibrium.

1. The import margin is invariant to the optimal policy:

$$\bar{j}_m(p_e, t_b) = \bar{j}_m = \frac{A}{A + (\tau^*)^{-\theta} A^*} = \frac{C^{HH}}{C_e}.$$

2. Export margin:

(a) Under unilateral optimal:

$$\bar{j}_x(p_e, t_b) = \frac{\tau^{-\theta} A \left(p_e + (1-\alpha)\, t_b\right)^{\theta}}{\tau^{-\theta} A \left(p_e + (1-\alpha)\, t_b\right)^{\theta} + A^* \left(p_e^{\alpha} \left(p_e + t_b\right)^{1-\alpha}\right)^{\theta}}.$$

(b) Under BAU:

$$\bar{j}_x = \frac{\tau^{-\theta} A}{\tau^{-\theta} A + A^*} = \frac{C_e^{FH}}{C_e^*};$$

(c) Expressed in terms of BAU:

$$\bar{j}_x(p_e, t_b) = \frac{\left(1 - \bar{j}_x\right)^{-1}}{\left(\left(1 - \bar{j}_x\right)^{-1} - 1\right) + \left(1 + (1-\alpha)\, t_b/p_e\right)^{-\theta} \left(1 + t_b/p_e\right)^{(1-\alpha)\theta}} \bar{j}_x$$

3. Energy used by producers in Home to supply Home consumers:

(a) Under unilateral optimal:

$$\begin{aligned} C_e^{HH}(p_e, t_b) &= \int_0^{\bar{j}_m(p_e,t_b)} e_j(z) y_j dj = \eta\,(1-\alpha)\,(p_e + t_b)^{-\epsilon_D} \int_0^{\bar{j}_m(p_e,t_b)} a_j^{1-\sigma} dj \\ &= \eta\,(1-\alpha)\,(p_e + t_b)^{-\epsilon_D} \frac{A^{(\sigma-1)/\theta}}{1 + (1-\sigma)/\theta} \left(\bar{j}_m(p_e, t_b)\right)^{1+(1-\sigma)/\theta}; \end{aligned}$$

(b) Under BAU:

$$C_e^{HH} = \eta\,(1-\alpha) \frac{A^{(\sigma-1)/\theta}}{1 + (1-\sigma)/\theta} \left(\bar{j}_m\right)^{1+(1-\sigma)/\theta};$$

(c) Expressed in terms of BAU:

$$C_e^{HH}(p_e, t_b) = (p_e + t_b)^{-\epsilon_D}\, C_e^{HH}.$$

4. Energy used by producers in Home to supply exports of Home:

(a) Under unilateral optimal:

$$\begin{aligned} C_e^{FH}(p_e, t_b) &= \tau \int_0^{\bar{j}_x(p_e,t_b)} e_j(z) x_j dj \\ &= \tau\eta^*(1-\alpha) p_e^{(\alpha-1)\sigma^*} (p_e + t_b)^{-\alpha} \int_0^{\bar{j}_x(p_e,t_b)} a_j \left(a_j^*\right)^{-\sigma^*} dj \\ &= \tau\eta^*(1-\alpha) p_e^{(\alpha-1)\sigma^*} (p_e + t_b)^{-\alpha} \frac{(A^*)^{\sigma^*/\theta}}{A^{1/\theta}} B\left(\bar{j}_x(p_e, t_b), \frac{1+\theta}{\theta}, \frac{\theta - \sigma^*}{\theta}\right); \end{aligned}$$

where and $B(x,a,b)$ is the incomplete beta function[36]

(b) Under BAU:

$$C_e^{FH} = \tau^{1-\sigma^*}\eta^*(1-\alpha)\frac{A^{(\sigma^*-1)/\theta}}{1+(1-\sigma^*)/\theta}\left(\bar{j}_x\right)^{1+(1-\sigma^*)/\theta};$$

(c) Expressed in terms of BAU:

$$\begin{aligned}\frac{C_e^{FH}(p_e,t_b)}{C_e^{FH}} &= \tau^{\sigma^*}\left(1+\frac{1-\sigma^*}{\theta}\right)p_e^{-\epsilon_D^*}\left(\frac{p_e+t_b}{p_e}\right)^{-\alpha}\left(\frac{A^*}{A}\right)^{\sigma^*/\theta}\frac{B\left(\bar{j}_x(p_e,t_b),\frac{1+\theta}{\theta},\frac{\theta-\sigma^*}{\theta}\right)}{\bar{j}_x^{1+(1-\sigma^*)/\theta}}\\ &= \left(1+\frac{1-\sigma^*}{\theta}\right)\left(\frac{1-\bar{j}_x}{\bar{j}_x}\right)^{\sigma^*/\theta}p_e^{-\epsilon_D^*}\left(\frac{p_e+t_b}{p_e}\right)^{-\alpha}\frac{B\left(\bar{j}_x(p_e,t_b),\frac{1+\theta}{\theta},\frac{\theta-\sigma^*}{\theta}\right)}{\bar{j}_x^{1+(1-\sigma^*)/\theta}}.\end{aligned}$$

5. Energy used by producers in Foreign to supply Foreign consumers:

(a) Under unilateral optimal:

$$\begin{aligned}C_e^{FF}(p_e,t_b) &= \int_{\bar{j}_x(p_e,t_b)}^1 e_j^*(z^*)y_j^*dj = \eta^*(1-\alpha)p_e^{-\epsilon_D^*}\int_{\bar{j}_x(p_e,t_b)}^1\left(a_j^*\right)^{1-\sigma^*}dj\\ &= \eta^*(1-\alpha)p_e^{-\epsilon_D^*}\frac{\left(A^*\right)^{(\sigma^*-1)/\theta}}{1+(1-\sigma^*)/\theta}\left(1-\bar{j}_x(p_e,t_b)\right)^{1+(1-\sigma^*)/\theta};\end{aligned}$$

(b) Under BAU:

$$C_e^{FF} = \eta^*(1-\alpha)\frac{\left(A^*\right)^{(\sigma^*-1)/\theta}}{1+(1-\sigma^*)/\theta}\left(1-\bar{j}_x\right)^{1+(1-\sigma^*)/\theta};$$

(c) Expressed in terms of BAU:

$$\begin{aligned}C_e^{FF}(p_e,t_b) &= p_e^{-\epsilon_D^*}\left(\frac{1-\bar{j}_x(p_e,t_b)}{1-\bar{j}_x}\right)^{1+(1-\sigma^*)/\theta}C_e^{FF}\\ &= p_e^{-\epsilon_D^*}\left(\frac{C_e^*\left(p_e^\alpha\left(p_e+t_b\right)^{1-\alpha}\right)^\theta}{C_e^{FH}\left(p_e+(1-\alpha)t_b\right)^\theta+C_e^{FF}\left(p_e^\alpha\left(p_e+t_b\right)^{1-\alpha}\right)^\theta}\right)^{1+(1-\sigma^*)/\theta}C_e^{FF}.\end{aligned}$$

[36]The incomplete beta function is:

$$B(x,a,b) = \int_0^x i^{a-1}(1-i)^{b-1}di,$$

for $0 \le x \le 1$, $a > 0$, and $b > 0$. Setting $x = 1$ gives the beta function itself, $B(a,b)$.

6. Energy used by producers in Foreign to supply imports of Home:

   (a) Under unilateral optimal:

$$
\begin{aligned}
C_e^{HF}(p_e,t_b) &= \tau^* \int_{\bar{j}_m(p_e,t_b)}^{1} e_j^*(z) m_j dj \\
&= (\tau^*)^{1-\sigma} \eta(1-\alpha)(p_e+t_b)^{-\epsilon_D} \int_{\bar{j}_m(p_e,t_b)}^{1} \left(a_j^*\right)^{1-\sigma} dj \\
&= (\tau^*)^{1-\sigma} \eta(1-\alpha)(p_e+t_b)^{-\epsilon_D} \frac{(A^*)^{(\sigma-1)/\theta}}{1+(1-\sigma)/\theta} \left(1-\bar{j}_m(p_e,t_b)\right)^{1+(1-\sigma)/\theta};
\end{aligned}
$$

   (b) Under BAU:

$$
C_e^{HF} = (\tau^*)^{1-\sigma} \eta(1-\alpha) \frac{(A^*)^{(\sigma-1)/\theta}}{1+(1-\sigma)/\theta} \left(1-\bar{j}_m\right)^{1+(1-\sigma)/\theta};
$$

   (c) Expressed in terms of BAU:

$$
C_e^{HF}(p_e,t_b) = (p_e+t_b)^{-\epsilon_D} C_e^{HF}.
$$

7. Value of Home exports of goods:

   (a) Under unilateral optimal:

$$
\begin{aligned}
V_g^{FH}(p_e,t_b) &= \int_0^{\bar{j}_x(p_e,t_b)} p_j^x x_j dj = \eta^* p_e^{1-\epsilon_D^*} \int_0^{\bar{j}_x(p_e,t_b)} \left(a_j^*\right)^{1-\sigma^*} dj \\
&= \eta^* p_e^{1-\epsilon_D^*} \frac{(A^*)^{(\sigma^*-1)/\theta}}{1+(1-\sigma^*)/\theta} \left(1-\left(1-\bar{j}_x(p_e,t_b)\right)^{(\theta+1-\sigma^*)/\theta}\right);
\end{aligned}
$$

   (b) Under BAU:

$$
V_g^{FH} = \tau^{1-\sigma^*} \eta^* \frac{A^{(\sigma^*-1)/\theta}}{1+(1-\sigma^*)/\theta} \left(\bar{j}_x\right)^{1+(1-\sigma^*)/\theta};
$$

   (c) Expressed in terms of BAU:

$$
V_g^{FH}(p_e,t_b) = p_e^{1-\epsilon_D^*} \frac{1-\left(1-\bar{j}_x{}'\right)^{1+(1-\sigma^\star)/\theta}}{\bar{j}_x\left(1-\bar{j}_x\right)^{(1-\sigma^\star)/\theta}} V_g^{FH};
$$

Substitute in $V_g^{FH} = \frac{1}{1-\alpha} C_e^{FH}$:

$$V_g^{FH}(p_e, t_b) = p_e^{1-\epsilon_D^*} \frac{1 - \left(1 - \bar{j}_x{}'\right)^{1+(1-\sigma^\star)/\theta}}{\bar{j}_x \left(1 - \bar{j}_x\right)^{(1-\sigma^\star)/\theta}} \frac{1}{1-\alpha} C_e^{FH}.$$

8. Value of Home's imports of goods:

   (a) Under unilateral optimal:

$$\begin{aligned} V_g^{HF}(p_e, t_b) &= \int_{\bar{j}_m(p_e,t_b)}^{1} p_j^m m_j dj \\ &= (\tau^*)^{1-\sigma} \eta \left(p_e + t_b\right)^{1-\epsilon_D} \left(\frac{p_e + \alpha t_b}{p_e + t_b}\right) \int_{\bar{j}_m(p_e,t_b)}^{1} \left(a_j^*\right)^{1-\sigma} dj \\ &= (\tau^*)^{1-\sigma} \eta \left(p_e + t_b\right)^{1-\epsilon_D} \left(\frac{p_e + \alpha t_b}{p_e + t_b}\right) \frac{(A^*)^{(\sigma-1)/\theta}}{1 + (1-\sigma)/\theta} \left(1 - \bar{j}_m(p_e, t_b)\right)^{1+(1-\sigma)/\theta}; \end{aligned}$$

   (b) Under BAU:

$$V_g^{HF} = (\tau^*)^{1-\sigma} \eta \frac{(A^*)^{(\sigma-1)/\theta}}{1 + (1-\sigma)/\theta} \left(1 - \bar{j}_m\right)^{1+(1-\sigma)/\theta};$$

   (c) Expressed in terms of BAU:

$$V_g^{HF}(p_e, t_b) = (p_e + t_b)^{-\epsilon_D} (p_e + \alpha t_b) V_g^{HF}.$$

   Substitute in $V_g^{HF} = \frac{1}{1-\alpha} C_e^{HF}$:

$$V_g^{HF}(p_e, t_b) = (p_e + t_b)^{-\epsilon_D} (p_e + \alpha t_b) \frac{1}{1-\alpha} C_e^{HF}.$$

9. Energy extraction by Home:

   (a) Under unilateral optimal:

$$Q_e(p_e, t_b) = (p_e + t_b - \varphi)^{\epsilon_S} E;$$

   (b) Under BAU:

$$Q_e = E;$$

(c) Expressed in terms of BAU:

$$Q_e(p_e, t_b) = (p_e + t_b - \varphi)^{\epsilon_S} Q_e$$

10. Energy extraction by Foreign:

    (a) Under unilateral optimal:

    $$Q_e^*(p_e) = (p_e)^{\epsilon_S^*} E^*;$$

    (b) Under BAU:

    $$Q_e^* = E^*;$$

    (c) Expressed in terms of BAU:

    $$Q_e^*(p_e) = (p_e)^{\epsilon_S^*} Q_e^*$$

Using these expressions we can return to (47) and (48), searching for the pair (p_e, t_b) that jointly solves them.

D.2 Expressions to Compute Welfare

Having solved for the optimal border adjustment and the corresponding change in the global energy price we can compute all other outcomes as well.
A key outcome is Home's welfare in moving to the optimal unilateral policy from the BAU competitive equilibrium.

Home's Utility (dropping a constant):

1. Under BAU:

$$U = C_s + \frac{\sigma}{\sigma - 1} \eta^{1/\sigma} C_g^{1-1/\sigma} - \varphi Q_e^W = C_s + \frac{\sigma}{\sigma - 1} V_g - \varphi \left(Q_e + Q_e^* \right);$$

2. Under unilateral optimal:

$$U(p_e, t_b) = C_s(p_e, t_b) + \frac{\sigma}{\sigma - 1} V_g(p_e, t_b) - \varphi \left(Q_e(p_e, t_b) + Q_e^*(p_e, t_b) \right);$$

3. The change in moving to optimal unilateral policy from the BAU competitive equilibrium:

$$U(p_e, t_b) - U = C_s(p_e, t_b) - C_s + \frac{\sigma}{\sigma - 1}(V_g(p_e, t_b) - V_g) - \varphi(Q_e^W(p_e, t_b) - Q_e^W);$$

where we denote the value of Home's spending on goods by V_g. Our preferred measure of welfare is normalized by BAU spending on goods:

$$W = \frac{U(p_e, t_b) - U}{V_g}.$$

For the terms in the welfare function above, we show:

1. Consumption of services in Home:

   (a) Under unilateral optimal:

   $$\begin{aligned} C_s(p_e, t_b) &= Q_s(p_e, t_b) - X_s(p_e, t_b) \\ &= L - L_e(p_e, t_b) - L_g(p_e, t_b) + X_g(p_e, t_b) + p_e X_e(p_e, t_b); \end{aligned}$$

   (b) Under BAU:

   $$C_s = Q_s - X_s = L - L_e - L_g + X_g + X_e;$$

2. Intermediate demand for energy (for use in production in Home):

   (a) Under unilateral optimal:

   $$G_e(p_e, t_b) = C_e^{HH}(p_e, t_b) + C_e^{FH}(p_e, t_b);$$

   (b) Under BAU:

   $$G_e = C_e^{HH} + C_e^{FH};$$

3. Consumption demand for embodied energy in Home:

   (a) Under unilateral optimal:

   $$C_e(p_e, t_b) = C_e^{HH}(p_e, t_b) + C_e^{HF}(p_e, t_b);$$

   (b) Under BAU:

   $$C_e = C_e^{HH} + C_e^{HF};$$

4. Home's employment in energy extraction:

   (a) Change from BAU to unilateral optimal:

$$\begin{aligned} L_e(p_e, t_b) - L_e &= \int_1^{p_e+t_b-\varphi} a \, dE(a) \\ &= Q_e \int_1^{p_e+t_b-\varphi} \epsilon_S a^{\epsilon_S} da \\ &= \frac{\epsilon_S}{\epsilon_S + 1}((p_e + t_b - \varphi)^{\epsilon_S+1} - 1)Q_e; \end{aligned}$$

5. Labor employed in production in Home:

   (a) Under unilateral optimal:

$$L_g(p_e, t_b) = \frac{\alpha}{1-\alpha} (p_e + t_b) \, G_e(p_e, t_b);$$

   (b) Under BAU:

$$L_g = \frac{\alpha}{1-\alpha} G_e;$$

   (c) Change from BAU to unilateral optimal:

$$L_g(p_e, t_b) - L_g = \frac{\alpha}{1-\alpha} \left((p_e + t_b) \, G_e(p_e, t_b) - G_e \right);$$

6. The value of Home's net exports of goods:

   (a) Under unilateral optimal:

$$X_g(p_e, t_b) = V_g^{FH}(p_e, t_b) - V_g^{HF}(p_e, t_b);$$

   (b) Under BAU:

$$X_g = V_g^{FH} - V_g^{HF};$$

7. The value of Home's net energy exports is:

   (a) Under unilateral optimal:

$$p_e X_e(p_e, t_b) = p_e \left(Q_e(p_e, t_b) - \left(C_e^{HH}(p_e, t_b) + C_e^{FH}(p_e, t_b) \right) \right);$$

(b) Under BAU:

$$X_e = Q_e - G_e = Q_e - \left(C_e^{HH} + C_e^{FH}\right);$$

8. The value of Home's spending on goods:

   (a) Under unilateral optimal:

$$V_g(p_e, t_b) = \eta^{1/\sigma} C_g(p_e, t_b)^{1-1/\sigma} = \frac{1}{1-\alpha}(p_e + t_b)\left(C_e^{HH}(p_e, t_b) + C_e^{HF}(p_e, t_b)\right)$$

$$= \eta \left(p_e + t_b\right)^{1-\epsilon_D} \frac{\left(A + (\tau^*)^{-\theta} A^*\right)^{(\sigma-1)/\theta}}{1 + (1-\sigma)/\theta};$$

   (b) Under BAU:

$$V_g = \eta \frac{\left(A + (\tau^*)^{-\theta} A^*\right)^{(\sigma-1)/\theta}}{1 + (1-\sigma)/\theta};$$

   (c) Expressed in terms of BAU:

$$V_g(p_e, t_b) = (p_e + t_b)^{1-\epsilon_D} V_g;$$

   Substitue in $V_g = \frac{1}{1-\alpha} C_e$:

$$V_g(p_e, t_b) = (p_e + t_b)^{1-\epsilon_D} \frac{1}{1-\alpha} C_e;$$

9. The term that enters the change in Home's welfare is:

$$\frac{\sigma}{\sigma - 1}\left(V_g(p_e, t_b) - V_g\right) = V_g \frac{\left((p_e + t_b)^{(1-\alpha)(1-\sigma)} - 1\right)}{(\sigma-1)/\sigma};$$

   For the case of $\sigma = 1$ this term reduces to:

$$\lim_{\sigma \to 1} V_g \frac{(p_e + t_b)^{(1-\alpha)(1-\sigma)}}{(\sigma - 1)/\sigma} = -(1-\alpha) V_g \ln(p_e + t_b) = -C_e \ln(p_e + t_b);$$

10. Global emissions:

   (a) Under BAU:

$$Q_e^W = Q_e + Q_e^*;$$

   (b) Under unilateral optimal:

$$Q_e^W(p_e, t_b) = Q_e(p_e, t_b) + Q_e^*(p_e).$$

E Data and Calibration

E.1 Calibration

For our quantitative analysis we calibrate the model to fossil fuel extraction and the energy embodied in trade between the region that, in our model, will enact a carbon policy (Home) and the region that will remain with business as usual (Foreign). Our common unit for energy is gigatonnes of CO_2, based on the quantity released by its combustion.

We consider three scenarios for the regions representing Home and Foreign. In the first, the United States is Home and all other countries are Foreign. The alternative scenarios, respectively, are the European Union prior to Brexit (EU28) as Home (and all other countries as Foreign) and the members of the Organization for Economic Cooperation and Development (OECD37) as Home (and all others as Foreign).

Our data source for energy consumption is The Trade in Embodied CO_2 (TECO2) database from OECD. We use their measure of consumption-based CO_2 emissions embodied in domestic final demand and the country of origin of emissions. This database covers 83 countries and regional groups over the period 2005-2015. Carbon dioxide embodied in world consumption in 2015 is 32.78 gigatonnes. We cross-checked the results with a dataset from the Global Carbon Project. The overall difference is less than ten percent.

Extraction data are from the International Energy Agency (IEA), which provides the World Energy Statistics Database on energy supply from all energy sources, including fossil fuels, biofuels, hydro, geothermal, renewables and waste. This dataset covers 143 countries as well as regional and world totals. The data are provided in units of kilotonnes of oil equivalent (ktoe). In order to keep the units consistent with the energy consumption data (gigatonnes of carbon dioxide), we first convert to terajoules (TJ) (1 ktoe = 41.868 TJ) and then apply emission factors to the five fossil fuel types to calculate CO_2 emissions. The five fossil fuel types considered are coal and coal products, natural gas, peat and peat products, oil products, as well as crude, NGL and feedstocks. The emission factors are default emission factors for stationary combustion from the 2006 IPCC Guidelines for National Greenhouse Gas Inventories. To be specific, we convert 1 TJ of crude, NGL and feedstocks to 73,300 kg CO_2, 1 TJ of natural gas to $56,100$ kg CO_2, and 1 TJ of coal, peat and oil products to $94,600$ kg CO_2. Using this calculation, world extraction is 35.96 gigatonnes of carbon dioxide.

To explain the discrepancy between world consumption and world extraction, note that the OECD data for embodied carbon does not include non-energy use of fossil fuels. In other words, some fossil fuels extracted are not combusted to produce energy. Instead, they are consumed directly or as intermediate goods. For example, petroleum can be used as asphalt and road oil and as petrochemical feedstocks for agricultural land. According to EIA (2018), approximately 8 percent of fossil fuels are not combusted in the United States. Applying this rate to the world extraction, we get a number close to world consumption ($35.96 * 0.92 = 33.08$, vs. 32.78).

Given that combusted energy is the source of CO_2 emissions, non-energy use of fossil fuel extraction is excluded in our analysis. We simply re-scale the world extraction data so that world extraction is equal to world consumption. To be specific, the original extraction data is divided by 1.097 (the ratio of world extraction to world consumption). Tables 4, 6, 7, and 8 display the resulting data we use for our calibration.

E.2 Parameter Values

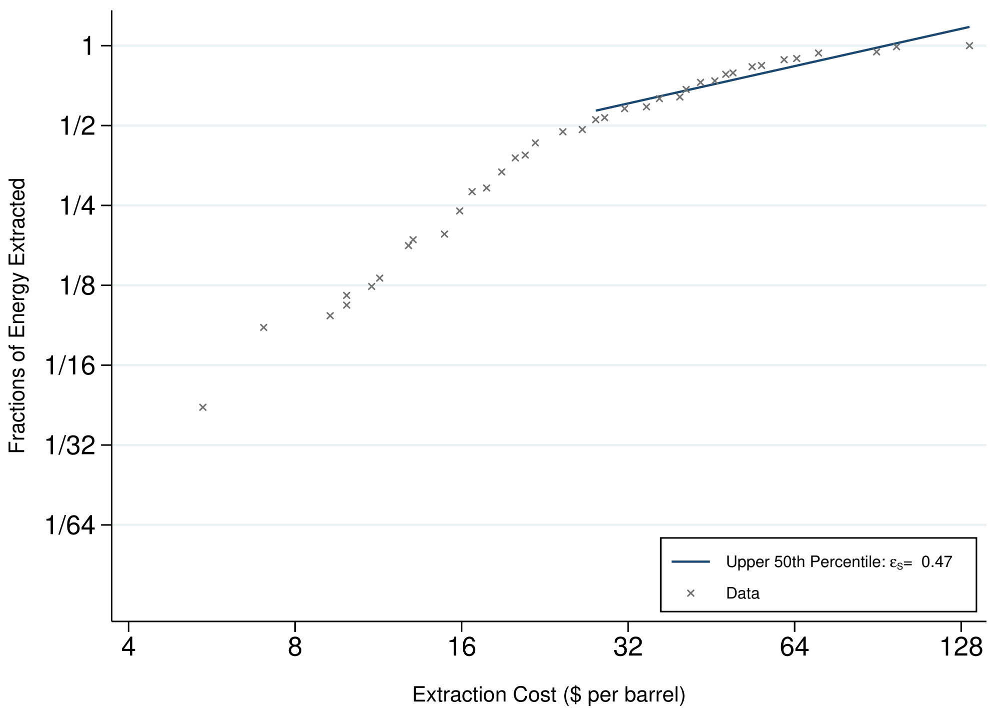
For the key parameter in the goods production function α, the output elasticity of labor, we calibrate $(1-\alpha)/\alpha$ to the value of energy used in production $p_e G_e$ relative to the value added.[37] The data from TECO2 records the carbon emissions embodied by sector and country. We can convert to barrels of oil based on 0.43 metric tons of CO_2 per barrel of crude oil (from EPA, 2019). The price per barrel of oil is taken from the average closing price of West Texas Intermediate (WTI) crude oil in 2015, which is $48.66 per barrel. Value added data comes from OECD Input-Output Tables (2018). We consider three definitions of the goods sector, with both the numerator (value of energy) and the denominator (value added) computed for the same sector definition, either: (i) the manufacturing sector, (ii) manufacturing plus agriculture and construction, and (iii) manufacturing, agriculture, construction, wholesale, retail, and transportation. The values of α that we obtain are, respectively, 0.85, 0.79, and 0.84. Our preferred value is 0.85, very close to two of these three.

For the energy supply elasticities, ϵ_S and ϵ_S^*, we use data from Asker, Collard-Wexler, and De Loecker (2018) on the distribution across oil fields of

[37] We think of value added as the closest proxy to labor cost in the model, since we interpret labor in the model as labor equipped with capital.

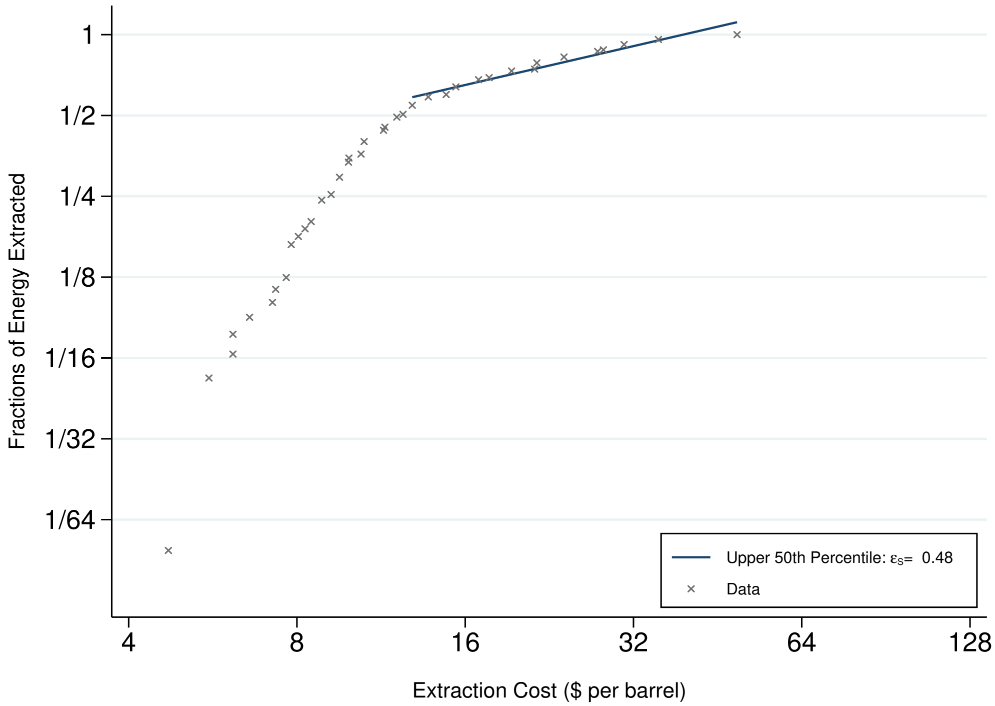
extraction costs. The data come in the form of quantiles (q = 0.05, 0.10, ..., 0.95), separately for the EU, the US, OPEC, and ROW (q% of oil in the US is extracted at a cost below $\$a$ per barrel, for example). We approximate OECD countries by aggregating the EU and US while for the non-OECD region we aggregate OPEC and ROW. To aggregate the quantiles for two regions, we combine them, sort the combination by the cost level, and reassemble after taking account of total oil extraction for each region (available from the IEA). The data are plotted on log scales in Figures 6 and 7, to reveal the supply elasticity as the slope.

Figure 6: Calibration of the Extraction Supply Elasticity in Home



The most costly oil fields in either region would be the first to be abandoned under a carbon policy. Thus, the upper end of the cost distribution is the most relevant for calibrating the supply elasticities. Our baseline values of $\epsilon_S = 0.5$ and $\epsilon_S^* = 0.5$ are close to the slope shown in the figures when we consider only costs above the median. Our alternative value of $\epsilon_S^* = 1$ is

Figure 7: Calibration of the Extraction Supply Elasticity in Foreign



closer to the slope if we were to use the upper 75% of costs or even all the data.

Lacking this distributional data for coal and natural gas fields, we assume that the distribution for oil extraction is representative of all fossil fuels.