Carrot *and* Stick: A Risk-Sharing Rationale for Fulcrum Fees in Active Fund Management*

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ABSTRACT

We show that risk-sharing considerations rationalize symmetric benchmark-adjusted ("fulcrum") fees in the compensation of privately informed active fund management. By tying fees symmetrically to the appropriate benchmark, investors can tilt a fund portfolio toward their optimal risk exposure and realize almost the full value of the manager's information. Since fulcrum fees do not alter the manager's preferred payoff profile, the optimal contract saves the cost of compensating the managers for exposure to undesired risk and attains near-first-best risk sharing. Under certain conditions, fulcrum fees are not only optimal but necessary to avoid a dominance of active management by passive alternatives.

Keywords: Portfolio delegation, benchmarking, fulcrum fees, asymmetric information, passive management.

JEL Classification: D82, G11, G23.

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1 Introduction

In the past decade, trillions of dollars have left active asset management firms, as investors have become drawn to less-expensive passive alternatives like exchange-traded (ETFs) and index funds. In response to this trend, active managers of prominent fund companies are increasingly advocating a radical shift in their funds' fee structure in favor of so-called "fulcrum" performance fees.[1] Like performance fees used by hedge funds and other segments of the industry, fulcrum fees give managers a proportion of the fund's excess return over the index. Unlike traditional performance fees, fulcrum schemes penalize underperformance—*symmetrically* to the compensation for positive performance—by deducting fees in proportion to the shortfall, eventually bringing overall charges to ETFs' levels. As reasonable as this proposal might look in the eye of the general public, its symmetric and benchmark-adjusted nature, akin to "linear benchmarking," challenges the current academic view on the compensation of portfolio managers, which generally deems pay for relative performance suboptimal. In this paper, we show that for a set of conditions that are likely to be present in the standard portfolio delegation decision, linear benchmarking, including its gradually more popular version of fulcrum fees, achieves near-first-best outcomes for active fund managers and their investors.

The pervasive use of benchmarks in practice has puzzled academics.[2] In an influential article, Admati and Pfleiderer (1997) argue that for the purposes of aligning the managers' portfolios with the optimal portfolio for the investors or, more generally, for efficient risk sharing between the two parties, linear benchmarking is at best irrelevant, and more likely harmful to fund investors. To address this contradiction between industry's practice and academic prescriptions, we focus on two important features of the asset management industry. The first feature is that (at least some) active managers have the potential to add value over a passive portfolio. In particular, we assume that active managers can be more informed than their fund investors—an assumption that encompasses

[1] For example, the Wall Street Journal (November 5, 2018) reports that "a wave of stock-picking firms are stepping up their fight against cheap exchange-traded and index funds with new offerings that dial back fees if they can't beat the market." The article mentions AllianceBernstein Holding LP, Allianz Global Investors and other managers as offering new funds that charge fulcrum fee arrangements. Similarly, the Financial Times (October 26, 2017) quotes Abigail Johnson, chairwoman and chief executive of Fidelity International, as calling for a "fundamental rethink" of the fees charged by asset managers, arguing that fulcrum fees "should be used more widely." This proposal is finding support both overseas (e.g., in UK) and in other segments of the industry such as active ETFs.

[2] BIS (2003) and DelGuercio and Tkac (2002) describe the use of explicit and implicit benchmarking practices, respectively, in the pension fund industry. Elton et al. (2003) document the use of benchmark-adjusted fees in the U.S mutual fund industry, while Chevalier and Ellison (1997) and, more recently, Spiegel and Zhang (2013), characterize empirically the implicit relation between the benchmark-adjusted performance of mutual funds and their future investor flows.

other types of outstanding skills such as superior information processing capabilities. The second feature is a potential misalignment in risk preferences between fund managers and their clients. This implies that the level of portfolio risk that managers prefer, even if based on superior information, can be costly to investors. Extensive evidence suggests that the search for both superior information and risk alignment are core components of investors' portfolio delegation decision.[3] Under these conditions, we show that fee contracts of the type observed between fund companies and their investors should optimally include a fulcrum performance component. We further demonstrate how fulcrum fees help align the preferences of fund managers and investors, attain almost first-best risk sharing, and avoid a dominance of informed active funds by inexpensive passive alternatives.

The portfolio delegation setting we model is as follows. Investors can delegate financial wealth management either to a passive fund (e.g., an index fund or an ETF) or to an active and privately informed asset manager. The passive fund caters perfectly to the risk preferences of the investors but, by definition, adds no value over the corresponding index. The opposite is true with the active fund, which has the potential to outperform the index but also to expose the fund investors to a level of risk different from the desired. The setup approximates a real-life situation in which, absent managers' superior information there would be no extra benefits from delegation to active funds and, absent differences in risk preferences and holding all else equal, active funds run by managers with skill would dominate their passive counterparts.[4]

Following the standard practice in the industry, we assume that passive funds are offered at a fixed fee over assets under management, while active funds can charge an additional fee tied to performance. Motivated by the prevalence of (piecewise) linear contracts in practice, the specific arrangement we consider consists of a linear combination of a proportional asset-based fee (of the

[3] Arguably, investors' belief that managers can produce superior returns, or "alpha," is one of the main reasons behind the existence of the active asset management industry. Several authors document that investors across segments of the money management industry chase fund performance (see, e.g., DelGuercio and Tkac, 2002, Sirri and Tufano, 1998 and Getmansky et al., 2015 for evidence in the pension fund, mutual fund and hedge fund segments, respectively), a behavior that Berk and Green (2004) rationalize as resulting from investors' search for managers with superior investment skills. Moreover, recent research backs the belief that at least certain subgroups of fund managers have superior investment skills (e.g., Kosowski et al., 2006), even if the average manager does not (Carhart, 1997). The extensive use of provisions to limit managers' risk-taking behavior such as tracking error and position limits, leverage and short selling constraints, represents direct evidence of the concern of fund investors about inappropriate levels of risk on their portfolios. See, e.g., BIS (2003) and Almazan et al. (2004) for a description of the use of these constraints in practice.

[4] This tension in the relation between portfolio managers and their institutional clients is broadly recognized. The Financial Times (26 October, 2017) quotes chief executives of large fund companies as recognizing that "a big part of our ability to add value for investors is the skill of our people," but also that the remuneration packages of their managers, based on total shareholder returns, "is a misalignment of interests and it can create tensions." In the same vein, these managers acknowledge that "one of the advantages of index-tracking strategies is that risks are better controlled and incentives are aligned between the asset manager and the client."

type charged by passive funds) and a benchmark-adjusted, fulcrum performance fee. The fulcrum fee is symmetric, in the sense that it rewards outperformance relative to a benchmark just as much as it penalizes underperformance. We also consider the case of asymmetric (nonlinear) performance fees that reward relative outperformance without penalizing underperformance. Funds invest in a risky security and a bond over a finite investment period. At the end of this period, the final value of the actively managed portfolio is split between investors and the manager according to the compensation contract.

We first show, analytically, the optimality of including a benchmarking component in the linear compensation contract of the manager. In designing this contract, investors seek to induce the manager to take advantage of her superior information while keeping the appropriate level of portfolio risk. The solution to this tradeoff is particularly simple when manager and investors exhibit, as assumed by prior literature, constant absolute risk aversion (CARA) preferences. A fixed fee over total assets has the manager invest the right amount of money in the risky asset from the investors' perspective without tying her compensation to undesired risks, so the optimal linear contract includes no benchmark. This prescription, however, does not generalize to other preferences that exhibit more realistic portfolio choice behavior. In particular, under the workhorse power-utility model of portfolio theory, investors can use the benchmark as a risk-aligning device. Under these preferences, the manager will respond to a given realization of the signal by choosing a smaller (respectively, larger) long or short position in the stock than a relatively risk-tolerant (risk-averse) investor would prefer conditional on observing the same signal. The inclusion of the benchmark in the compensation contract tilts the portfolio allocation of the manager toward the portfolio that the investor would choose if endowed with the signal. The opposite side of the coin is that the benchmark composition is decided by the investors conditional on their own, inferior information, and is thus inefficient with respect to the manager's information set. To the extent that linear benchmarking distorts the manager's portfolio towards a conditionally inefficient portfolio, benchmarking is costly to the fund investors. Our main analytical result is that under information asymmetry about asset returns and differences in risk aversion between the manager and the investors, the optimal weight in the fulcrum performance component is non-zero.

We find that the optimal linear contract achieves "near-optimal" risk sharing, in the sense of allowing investors to realize almost the full value of the manager's private information. Under a linear contract, managers do not have to be compensated for choosing a suboptimal—from their perspective—level of risk because they can perfectly hedge their exposure to the benchmark. Thus, the optimal benchmark allows investors to benefit from the manager's private information at low cost. Embedding the standard constant relative risk aversion (CRRA) preferences of the portfolio choice literature into a standard contracting problem allows us to offer a more meaningful quantitative assessment, compared to the typical CARA setting, of the relative merits of benchmarking. In numerical analyses of our model, we find that regardless of the difference in risk aversion between manager and investors, the information advantage of the manager, or the investment horizon, adding a fulcrum fee to the manager's compensation allows investors to enjoy either a substantial fraction or almost all (e.g., 98% or more for some fund investors-manager pairs) of the gains associated with the manager's private information—i.e., the extra certainty equivalent returns that investors could earn if they traded on the manager's signal.

In fact, when the risk-aversion misalignment between manager and investors is large enough, linear benchmarking is not only optimal but also necessary to justify active management. The gains under fulcrum fees relative to asset-based-only fees increase with the risk aversion misalignment between the parties. For sufficiently severe misalignment we show that, unlike in prior studies, linear contracts based only on a fund's total unadjusted returns can be suboptimal and lead to substantial losses. In these situations, if investors are restricted to pay the manager no fulcrum fees, they are better off investing in the passive alternative. This is especially the case when the manager's information advantage is slim, a result that helps explain the increasing preference of investors in recent years for passive over active mutual funds, among which pure proportional fees are the norm.

We then characterize numerically the optimal weight of the benchmark-adjusted fees in the compensation of the manager, and the optimal benchmark composition. On the one hand, the value of linear benchmarking as a portfolio-aligning device increases with the difference in risk aversion between manager and investors. On the other hand, the opportunity cost of tilting the active fund portfolio toward a conditionally inefficient portfolio—the benchmark—increases with the information advantage of the manager. In line with this tradeoff, we find that the optimal fulcrum fee increases with the misalignment in risk preferences between manager and investors but decreases with the information advantage of the manager. This feature might help explain, in a context of highly efficient financial markets and widespread information dissemination (i.e., shrinking information advantage by any party), the recent trend among active management companies toward fulcrum fees.

More surprisingly, the optimal benchmark has either a higher or a lower risk exposure than the unconditionally efficient portfolio households would choose under self-management. Intuitively, from the perspective of relatively risk-tolerant households, the manager underreacts to good news, increasing the weight in the risky asset in her portfolio less than households would prefer. Conversely, the manager overreacts to bad news by lowering the weight in the risky asset in her portfolio more than households would prefer. *Both* under- and overreaction problems can be alleviated by setting a risky benchmark in the managerial contract.[5] Accounting for leverage restrictions, our model further prescribes, much as observed in practice, relatively simple rules (i.e., either all-equity or risk-less portfolios depending on the fund investor's risk profile) for the design of the benchmark specified in the fulcrum fee.

Lastly, we find that, in our setup, linear contracts dominate popular nonlinear variants such as the asymmetric benchmark-adjusted performance fees charged by hedge funds. A simple compensation contract consisting of a fee proportional to total assets plus a fulcrum fee attains higher risk-aversion-aligning benefits at lower cost. Indeed, only under *linear* benchmarking managers can hedge perfectly their exposure to the benchmark. Investors can thus attain portfolio alignment benefits without having to compensate the manager for unwanted exposure to a benchmark, as they would have to under asymmetric performance fees. Hence, although nonlinear benchmarking enlarges the menu of contracts among which the investor can choose, it is irrelevant for the problem we consider. The symmetric nature of fulcrum fees, which increases managerial compensation if the portfolio beats the benchmark but reduces it in the opposite case, is consistent with the requirements of the SEC starting in 1971. The results in this paper contribute to the debate that has emerged since the introduction of this requirement. In particular, our findings alleviate concerns about the welfare losses to investors that the introduction of bans on asymmetric performance fees could entail.

The rest of the paper is organized as follows. Section 2 summarizes our contributions relative to prior literature. Section 3 introduces the economic environment, the information structure and the class of managerial contracts considered. Section 4 describes the contracting problem and characterizes the manager's optimal investment strategy for an arbitrary contract. We establish the optimality of fulcrum fees, quantify the benefits of benchmarking and characterize numerically the optimal linear compensation contract in Section 5. We discuss extensions and practical implications

[5] Similarly, from the perspective of relatively risk-averse households, the manager overreacts to good news by aggressively increasing the weight of the risky asset in the portfolio and underreacts to bad news by reducing too little her position in the risky asset. Both problems can be alleviated by having the manager partially mimic a very conservative benchmark.

of our model in Sections 6. Section 7 contains our conclusions.

2 Related Literature

As we have discussed, the main result of the paper is that the linear benchmarking-based performance evaluation prevalent in practice is optimal under the plausible conditions that the asset manager has superior information (skill) but different risk-aversion than the investor. Therefore, the paper's main contribution is to the literature on optimal compensation in the context of delegated portfolio management. In their influential article, Admati and Pfleiderer (1997) also argue that linear benchmarking is irrelevant either for inducing the manager to exert more effort or for screening out bad managers. In a model featuring CARA preferences, Li and Tiwari (2009) devise a nonlinear benchmarking contract that overcomes effort inducement problems. With the precedent of Gómez and Sharma (2006), who showed that benchmarks become relevant in the presence of short-selling constraints, Dybvig et al. (2010) deviate from the previous work by considering logarithmic utility and making the investment opportunity set of the money manager a central piece of their analysis. They show that the optimal contract to address moral hazard problems when the manager's private information is non-contractible must include both nonlinear benchmark-adjusted fees and portfolio constraints. Similarly, He and Xiong (2013) derive the optimality of investment mandates and tracking error constraints in an institutional asset management industry with agency costs, hidden information, and the availability of negatively skewed bets. Accounting for the value of benchmark-adjusted compensation as a signaling device for superior managers, Das and Sundaram (2002) identify equilibria under which option-like performance fees improve investor welfare relative to fulcrum fees. We demonstrate the effectiveness of *linear* benchmarking, even before accounting for the effects of investment constraints, to improve risk-sharing and to align the portfolio allocation of the manager with the optimal portfolio of investors. Importantly, our CRRA setup allows us to offer a meaningful quantitative comparison of the efficiency gains of linear benchmarking over both no-benchmarking and nonlinear-benchmarking arrangements, while avoiding the well-known drawbacks of CARA preferences (e.g., absence of wealth effects) in the modeling of portfolio choice.

Our work is also related to the recent literature on the optimal compensation of asset managers that accounts for general-equilibrium implications on asset prices. Buffa et al. (2019) study the joint determination of fund managers' contracts and equilibrium asset prices in a symmetric-information setup with CARA preferences and agency frictions. They show that including compensation for relative performance with respect to an exogenous benchmark in the optimal contract improves risk sharing between fund managers and investors, while rewarding absolute performance addresses the agency frictions. Cvitanić et al. (2018) extend Buffa et al. (2019) by generalizing the contract and allowing the manager to invest privately in individual risky assets or an index. They find that the optimal contract involves rewarding the manager for returns in excess of a benchmark, and for quadratic deviation thereof. We demonstrate that risk sharing is at the core of the near optimality of linear benchmarking when the manager is privately informed, even in the absence of agency frictions. Complementing these papers, we show how the endogenously designed benchmark portfolio can differ from the market portfolio or other unconditionally efficient passive alternatives.[6]

Lastly, our paper contributes an analysis of the effect of private information to the literature on portfolio choice and risk sharing under institutional asset management. Under symmetric information between managers and investors, prior studies derive the contract that achieves perfect risk-sharing when the asset manager can exert costly effort to improve the portfolio return (Ou-Yang, 2003; Cadenillas et al., 2007). A different strand of the literature studies the effect of linear and nonlinear benchmarking on the risk-taking of the money manager (Starks, 1987; Basak et al., 2008), on equilibrium prices (Cuoco and Kaniel, 2011; Basak and Pavlova, 2013; Buffa and Hodor, 2018), on information acquisition and market efficiency (Breugem and Buss, 2019; Sockin and Xiaolan, 2019), and on firm's corporate decisions (Kashyap et al., 2018). In the context of a centralized decision maker who hires multiple asset managers and whose compensation depends only on relative performance, van Binsbergen et al. (2008) derive an optimal unconditional linear performance benchmark that aligns risk preferences across managers. Our results show that a linear compensation contract that includes both absolute and relative performance components, similar to those used in practice, can achieve near first-best efficiency under *asymmetric* information about asset returns between fund managers and investors.

3 Model Setup

We consider an economy in which investors, henceforth also referred to as households, delegate their financial wealth w to an investment company (e.g., a mutual fund) over a certain investment

[6] In this sense, our paper is also related to Huddart (1999). Using a model of portfolio misalignment between investors and managers due to *reputation concerns* rather than differences in risk aversion, this author rationalizes linear performance fees relative to an exogenous benchmark as a tool to align the portfolio choices of privately informed managers and their fund investors.

horizon denoted by $[0, T]$. The investment company manages investors' portfolio by allocating their wealth among the available financial assets. During the investment period, no additional funds share purchases or redemptions take place. In return for its services, at $t = T$ households pay the investment company a monetary sum according to a pre-specified (at $t = 0$) fee contract. We introduce this contract in Section 3.2.

The rest of the paper focuses on the relationship between the fund manager in one of such companies, and her fund investors.[7] For most of our analysis, we assume that both the fund manager and households have constant relative risk aversion (CRRA) preferences, with coefficients of relative risk aversion $\gamma > 1$ for the manager and $\gamma_h > 1$ for the investor.

$$u(w) = \begin{cases} \frac{w^{1-\gamma}}{1-\gamma} & \text{if } w > 0 \\ -\infty & \text{if } w \leq 0 \end{cases}, \quad u_h(w) = \begin{cases} \frac{w^{1-\gamma_h}}{1-\gamma_h} & \text{if } w > 0 \\ -\infty & \text{if } w \leq 0 \end{cases}. \tag{1}$$

We note that the power utility assumption (1) departs from the standard literature on delegated portfolio management (e.g., Admati and Pfleiderer, 1997), which for tractability assumes either constant absolute risk aversion (CARA) or log utility for both agents. At least two reasons can justify this departure. First, examining preferences such as power utility that allow simultaneously for differences in risk aversion *and* realistic portfolio behavior—e.g., wealth effects on the demand for assets—is particularly important for the purpose of assessing the *portfolio-alignment* and *risk-sharing* effects of linear benchmarking. Second, power utilities are a workhorse model in the asset management and asset pricing literatures,[8] so our setup should be more easily embedded in either type of problem. Notwithstanding these reasons, we also discuss the implications of assuming either CARA or log preferences in our framework in sections 4 and 5.

3.1 Financial Markets, Information Structure and Fund Companies

Financial markets consist of one riskfree asset and one risky asset, with prices β and S, respectively. The risk-less asset can be a short-term bond or a bank account. The risky asset can be a stock or

[7] Although in practice the manager of a fund advisory company need not be the same person (or team) that manages the fund portfolio, we do not make that distinction and assume that the fund company's compensation accrues entirely to the portfolio manager. See Ibert et al. (2018) for a detailed analysis of the compensation of mutual fund manager. Similarly, we abstract away from the potential for agency conflicts in the relation between the fund advisory company and its portfolio manager, as well as the design of the optimal contract between these two parties. In practice, it is often the case that the compensation of the portfolio manager itself depends on her portfolio's benchmark-adjusted performance, see Farnsworth and Taylor (2006) and Ma et al. (2019).

[8] See, e.g., Campbell (2018)

any portfolio of risky assets (such as the market portfolio or other traded benchmark).[9] All agents are atomistic participants in the asset markets, so they take asset price dynamics as exogenously given. The bond has initial price $\beta_0 = 1$ and pays a constant interest rate r per unit of time. The bond's price dynamics are given by $d\beta_t = r\beta_t dt$.

We let the mean rate of return on the stock, μ, be the realization at $t = 0$ of a random variable $\tilde{\mu}$. We assume that $\tilde{\mu}$ is normally distributed, $\tilde{\mu} \sim \mathbf{N}(r + \sigma\bar{m}, \sigma^2\bar{v}_0)$, for given constants $\bar{m}$ and $\bar{v}_0 \geq 0$, and given stock volatility σ. Equivalently, the market price of risk $\tilde{\eta} \equiv (\tilde{\mu} - r)/\sigma$ follows a normal distribution $\mathbf{N}(\bar{m}, \bar{v}_0)$ with realized value $\eta \equiv (\mu - r)/\sigma$ at $t = 0$. After η is realized, the stock price dynamics are:

$$dS_t^\eta = S_t^\eta(r + \sigma\eta)dt + S_t^\eta \sigma dB_t, \tag{2}$$

where B is a standard Brownian motion process, independent of $\tilde{\eta}$, defined on a filtered probability space $(\Omega, \mathcal{F}, P, \{\mathcal{F}\}_{t \leq T})$ over the interval $0 \leq t \leq T$, and the superscript η makes explicit the dependency of the stock price path on the realized value η of the market price of risk. The stock's initial price is $S_0^\eta = s$ for all realizations of $\tilde{\eta}$.

Arguably, the belief that fund managers can have superior information or investment skills is a key reason behind the existence of active money management. We account for this possibility in our model by assuming that only fund managers, but not their fund investors, observe η, and do so before the investment period starts—i.e., at $t = 0$. This private information represents a potential source of value from delegation, as the manager can choose a portfolio that better suits either poor (low realized η) or good (high realized η) market conditions. We note that the assumption that managers have *perfect* information about η is unrealistic but inessential for our results. In Section 6.3 we assume instead that managers have access to a private *noisy* signal about η, which reduces but does not eliminate the information advantage of managers relative to households.

Households in the model can delegate their financial wealth either to an *actively* managed fund, or to a *passively* managed fund (e.g., an indexed fund or an Exchange-Traded Fund [ETF]).[10] In case they choose to delegate their wealth to an active fund, households offer the manager the performance fee contract that we introduce in the next subsection. Given this contract, the manager chooses the

[9] Our qualitative results do not depend on the existence of a single risky asset, and our examination could in principle be extended to a multiple risky assets. To ease the analysis and interpretation, however, we constrain our setup to the single risky asset case that is standard in the literature (e.g., Dybvig et al., 2010).

[10] That is, we assume that households do not diversify their portfolio between the two types of funds. The assumption is for tractability only and, as we argue in Section 6.2, does not change the main conclusions of our analysis. As we explain below, it allows us to embed the delegation decision into households' participation constraint in the contracting problem of Section 4.

fund portfolio according to her own information and risk preferences.

The passive fund represents the "next best alternative" to the active fund, and its role in our setup is to determine the reservation utility of households in their contracting problem with the active fund manager. It allocates a fixed fraction $\phi^P \in \mathcal{R}$ of the fund's value in the stock, and the remaining fraction $1 - \phi^P$ in the riskfree asset, in exchange for a (minimal) fixed fee $k_m > 0$ over total assets under management (AUM).[11] Importantly, the allocation ϕ^P to the stock is optimally determined by households based on their own (partial) information and risk preferences. As is the case in practice, we assume that the manager of a passive fund "ties her hands," in the sense that she keeps the commitment to invest in the agreed portfolio ϕ^P throughout the investment period.

3.2 Managerial Contract

An influential irrelevance result in the literature is the inability of *linear* benchmarking to achieve optimal risk sharing or to perfectly align the manager's portfolio with the portfolio desired by households. In a model with CARA preferences, Admati and Pfleiderer (1997) show that (i) the use of benchmarks in linear contracts can entail a substantial (even total) loss in the value of a manager's information to households, and (ii) the full value of the manager's information can be realized with linear contracts and no benchmarks, based only on the total unadjusted returns on the manager's portfolio. The increased popularity of fulcrum fees, and the adoption of linear benchmarking practices more broadly (see references in the introduction), raise the question of whether these results extend to other types of preferences guiding investors' and managers' portfolio decisions. In particular, does compensation based only on unadjusted returns still dominate benchmark-adjusted compensation under non-CARA preferences? Are the losses in the utility of non-CARA households associated to linear benchmarking still economically significant?

To address these questions within our CRRA framework, we assume that the compensation that households offer to an active fund manager follows the type of *linear* contracts considered by prior literature (e.g., Admati and Pfleiderer, 1997).[12] Specifically, for given fees $k > 0$ and α, households

[11] This is the standard fee arrangement in the passive fund and ETF industries. Although we take this compensation structure as given, it is easily seen to be optimal in the context we study based on our results in Section 5. Indeed, by construction the passive investment entails perfect portfolio alignment with the fund investor, in which case a pure proportional-only fee schedule is optimal (see Theorem 1).

[12] The assumption that it is households who offer the contract to the manager, and not the converse, is for ease of interpretation of results only and is standard in the literature (see, e.g., Dybvig et al., 2010). All our conclusions remain identical if we assumed instead that the manager offers the contract to the fund investors. See our discussion in Section 4 below.

pay the manager a fee rate

$$f(R_T^W, R_T^Y; k, \alpha) = kR_T^W + k\alpha(R_T^W - R_T^Y) \tag{3}$$

per initial dollar of AUM W_0, where $R_T^W \equiv W_T/W_0$ and $R_T^Y \equiv Y_T/Y_0$ denote, respectively, the fund's absolute performance (with W_T being the end-of-period value of total AUM) and relative performance with respect to a benchmark Y during the investment period $[0, T]$. We refer to kR_T^W as a proportional, total asset-based fee, and to $k\alpha(R_T^W - R_T^Y)$ as a "fulcrum," or relative performance fee.[13] Our goal is to assess whether including a benchmarking component (the fulcrum fee) in a linear contract is optimal—i.e., whether the optimal α is different from 0—and, if so, to quantify the derived gains relative to the optimal linear contract based only on absolute performance ($\alpha = 0$).[14]

The benchmark is the value of a portfolio with *constant* weight ϕ^Y in the risky asset, as determined by the investor, and the remaining $1-\phi^Y$ in the riskfree asset. Since its portfolio composition is fixed over the investment period, the benchmark approximates a *passive* index, as is typically the case in practice (see references in Section 1). Beyond this assumption, we place no restriction on the composition of the benchmark portfolios, so in principle investors can choose ϕ^Y to differ from the passive fund portfolio ϕ^P. For a realization η, the benchmark value process satisfies the self-financing dynamics:

$$dY_t^{\eta;\phi^Y} = Y_t^{\eta;\phi^Y}\left(r + \phi^Y \sigma\eta\right) dt + Y_t^{\eta;\phi^Y} \phi^Y \sigma dB_t, \tag{4}$$

where without loss of generality we normalize the benchmark's initial value $Y_0^{\eta;\phi^Y} = y$ to equal w.

Given the fee contract (3), the manager's compensation is:

$$\begin{aligned} X_T \equiv f_T W_0 &= kW_T + k\alpha(W_T - Y_T) \\ &= f\left(W_T, Y_T; k, \alpha\right). \end{aligned} \tag{5}$$

A management fee contract $\mathscr{C} \equiv (k, \alpha, \phi^Y)$ specifies the proportional and symmetric benchmark-adjusted (fulcrum) fees k and α, and the benchmark composition ϕ^Y. Importantly, since the

[13] We specify the fulcrum fee α as a multiple of the proportional fee k for notational simplicity only. Results do not change if we allow instead separate parameters κ_1 and κ_2 for each of these fees, as we do in our analysis of asymmetric performance fees of Section 6.3.

[14] That is, we seek the rationale of linear benchmarking practices by examining the optimality of a benchmark-adjusted performance component in a linear contract. This goal is different from the more general goal of establishing the optimality of a linear contract.

realization of the manager's signal is unobserved to investors, the contract parameters are restricted to be invariant to realizations of $\tilde{\eta}$. Specification (5) is general enough to encompass existing fees for different type of investment companies (e.g., mutual funds and pension funds).[15] In Section 6, we extend our analysis to contracts that include *asymmetric* (nonlinear) benchmark-adjusted fees, which can accommodate the "two-and-twenty" fee schedule prevalent in the U.S. hedge fund industry.[16]

4 Contracting Problem

Households' problem consists in designing the contract $\mathscr{C}$ that induces the manager to use her private information in their best interest at the lowest possible cost. To this aim, they choose the fee parameters, within the linear class considered in Section 3.2, under which the manager optimally implements the investment policy that maximizes households' expected utility from delegation, the manager agrees to actively manage the fund, and households agree to delegate their wealth to the active fund. We assume that households offer $\mathscr{C}$, and that the manager accepts or declines it, before $\tilde{\eta}$ is realized.[17] Households contracting problem is then to:

$$\max_{\{k,\alpha,\phi^Y\}} E\left[\frac{\left(\hat{W}_T^{\tilde{\eta};\mathscr{C}} - X_T^{\tilde{\eta};\mathscr{C}}\right)^{1-\gamma_h}}{1-\gamma_h}\right] \tag{6}$$

$$s.t. = \begin{cases} \forall \eta: \quad \hat{W}_T^{\eta;\mathscr{C}} = \arg\max_{W_T} E\left[\frac{f\left(W_T, Y_T^{\tilde{\eta};\phi^Y}; k, k\alpha\right)^{1-\gamma}}{1-\gamma}\middle|\tilde{\eta}=\eta\right], & \text{(M's ICC)} \\ E\left[\frac{(X_T^{\tilde{\eta};\mathscr{C}})^{1-\gamma}}{1-\gamma}\right] \geq \bar{U}, & \text{(M's PC)} \\ E\left[\frac{\left(\hat{W}_T^{\tilde{\eta};\mathscr{C}} - X_T^{\tilde{\eta};\mathscr{C}}\right)^{1-\gamma_h}}{1-\gamma_h}\right] \geq U_{h,P}, & \text{(HH's PC)} \end{cases}$$

where expectations are with respect to the joint distribution of $(\tilde{\eta}, B_T)$.

Condition (M's ICC) represents the manager's incentive compatibility constraint: given a con-

[15] For simplicity we do not include a constant component in the compensation arrangement, as doing so would compromise tractability without changing the qualitative results. Nevertheless, we note that a constant payment cannot substitute for the variable components proportional to k. The reason is that *both* principal (households) and agent (the manager) are risk averse in our setup. According to the Borch rule, optimal risk sharing then requires both parties to absorb part of the aggregate risk, for which k cannot be zero at the optimum.

[16] A two-and-twenty fee schedule charges a 2% of total assets under management plus a 20% of profits made in excess of a benchmark, typically specified as a money market rate (e.g., 3-month LIBOR).

[17] We further assume that manager cannot renege on the contracts afterwards.

tract $\mathscr{C} = (k, \alpha, \phi^Y)$, the terminal fund value is the one that maximizes the manager's expected utility for each realization of $\tilde{\eta}$. To preserve the asymmetry in information about asset returns between the manager and households we assume that, up until $t = T$, households do not observe the manager's portfolio choice and the fund value (relative to the benchmark) simultaneously.[18]

Conditions (M's PC) and (HH's PC) represent, respectively, the manager's and households' participation constraints given reservation utilities of $\bar{U}$ and $U_{h,P}$, respectively. The existence of passively managed funds and their roles as (i) the next best investment alternative available to households, and (ii) a feasible investment strategy for the manager, affects both $\bar{U}$ and $U_{h,P}$. The basic tradeoff in households' delegation decision these conditions aim to capture is the following. The active fund offers a valuable information advantage but a costly potential misalignment—to the extent that the contract $\mathscr{C}$ that solves (6) cannot implement perfect portfolio alignment—in portfolio risk. In contrast, the passive fund offers portfolio risk alignment benefits but no information advantage. The utility that households expect to obtain from delegating their money to an active fund can be no lower than the expected utility they derive under passive management of their wealth. Likewise, the utility that a manager obtains from actively managing a fund can be no lower than the utility she derives from offering a passively managed fund. Since households choose the portfolio composition ϕ^P of the passive fund, their risk preferences affect both (HH's PC) and (M's PC). As we show below, the availability of passive funds thus implies that the reservation utilities of the manager and households are not exogenously given but related to one another.[19]

We note that our main results remain the same under the alternative interpretation in which it is the manager who offers the contract to households. Indeed, it is immediate to see that a solution $(\hat{\alpha}, \hat{\phi}^Y)$ to the contracting problem (6) is also a solution to the problem of maximizing the expected utility of the manager subject to the same constraints. Thus, the total surplus generated is the same under the two formulations and how this surplus is split between the contracting parties uniquely determines the optimal total asset-based fee $\hat{k}$ (see Section 5.2 for more details). For economic interpretation and the numeric analysis of our results, however, we henceforth interpret

[18] If households observed both the manager's portfolio choice $\hat{\phi}_t$ and the fund's relative performance at each point in time they could infer the value η from the expression of the optimal portfolio (9) below. Given that professional asset managers disclose their portfolio holdings at a much lower frequency (e.g., only every three months in the case of mutual funds, with lower frequency for other asset management firms like hedge funds) than asset return data is available, the assumption is not particularly unrealistic.

[19] Households' contracting problem is essentially *static*, in the sense that the compensation contract $\mathscr{C}$ is decided at $t = 0$ and kept fixed until the end of the investment period T. Our assumption of *dynamic* trading in the financial markets follows from its analytical tractability under CRRA preferences and log-normally distributed returns. We believe the assumption is particularly realistic in the context of *active* fund management.

the contract as being "offered by households to the manager."

Problem (6) defines a contracting model similar to the ones considered by Ou-Yang (2003) and Cadenillas et al. (2007), among others. Like in these studies, a solution to (6) aims to achieve optimal risk sharing between households and the manager when the parties face a potential misalignment in risk preferences but no moral hazard or adverse selection problems.[20] Our innovation is to allow for information asymmetry about asset returns between the parties, while at the same time abstracting away from the choice of managerial effort or the potential costs of selecting projects. This focus allows us to examine the role of superior investment ability and risk misalignment, intrinsic to the choice between active and passive management, in understanding the recent preference for fulcrum fees among active managers (see Section 1).

To approach problem (6) we proceed in several steps. For any given fee contract $\mathscr{C}$ and realized market price of risk η, we first characterize the solution $\hat{W}_T^{\eta;\mathscr{C}}$ to the manager's optimization problem that determines her incentive compatibility constraint (M's ICC). We next solve for households' optimal passive portfolio and characterize the reservation utilities $\bar{U}$ and $U_{h,P}$ that determine the participation constraints (HH's PC) and (M's PC) for both parties. We complete the characterization of the optimal linear contract in Section 5.

4.1 Incentive Compatibility and Optimal Active Fund Portfolio

After observing the realization of the market price of risk η at $t = 0$, and for a given fee contract $\mathscr{C}$, the manager chooses a dynamic investment policy ϕ_t^η representing the fraction of the fund's wealth W_t^η allocated in the risky asset over the investment period $t \in [0, T]$. The investment policy seeks to maximize her expected utility conditional on η over end-of-period compensation $X_T^{\eta;\mathscr{C}}$:

$$E\left[\left.\frac{(X_T^{\tilde{\eta};\mathscr{C}})^{1-\gamma}}{1-\gamma}\right| \tilde{\eta} = \eta\right], \tag{7}$$

subject to the self-financing constraint:

$$dW_t^\eta = W_t^\eta \left(r + \phi_t \sigma \eta\right) dt + W_t^\eta \phi_t \sigma dB_t, \tag{8}$$

and initial wealth $W_0^\eta = w$.

[20] To focus on the portfolio alignment and risk-sharing aspects of the portfolio delegation problem we assume no moral hazard or adverse selection frictions. Our findings should thus be interpreted as applying in addition to any effect stemming from either of these other frictions..

Markets are complete in our setup (a single risky asset S driven by a single Brownian motion B). Absent arbitrage opportunities, and given the realization η of $\tilde{\eta}$, the manager sees financial markets as driven by a unique state-price deflator $\pi_t^\eta = \exp\{-(r+\eta^2/2)t - \eta B_t\}$. For a given compensation contract $\mathscr{C}$, the manager's optimal fund value $\hat{W}^\eta$ and investment strategy $\hat{\phi}^\eta$ are characterized explicitly in the following:

Proposition 1. *For an arbitrary contract $\mathscr{C} \equiv (k, \alpha, \phi^Y)$ and realization η of the market price of risk, the manager's optimal risk exposure at time $t \in [0, T]$ is given by:*

$$\hat{\phi}_t^{\eta;\alpha,\phi^Y} = \frac{\eta}{\gamma\sigma} + \frac{\alpha}{1+\alpha}\frac{1}{\hat{Z}_t^{\eta;\alpha,\phi^Y}}\left(\phi^Y - \frac{\eta}{\gamma\sigma}\right), \tag{9}$$

where $\hat{Z}_t^{\eta;\alpha,\phi^Y} \equiv \hat{W}_t^{\eta;\alpha,\phi^Y}/Y_t^{\eta;\phi^Y}$ is the fund's time-t (optimal) relative performance.

The time-t ($t \in [0, T]$) optimal fund value and the manager's end-period compensation are given by:

$$\hat{W}_t^{\eta;\alpha,\phi^Y} = \frac{w}{1+\alpha} g(t;\eta,\gamma)(\pi_t^\eta)^{-\frac{1}{\gamma}} + \frac{\alpha}{1+\alpha} Y_t^{\eta;\phi^Y}. \tag{10}$$

$$\hat{X}_T^{\eta;k} = kwg(t;\eta,\gamma)(\pi_T^\eta)^{-\frac{1}{\gamma}}, \tag{11}$$

where $g(t;a,b) \equiv \exp((1-1/b)(r + a^2/(2b))t)$.

The manager's investment policy combines a standard mean-variance portfolio $\eta/(\gamma\sigma)$ with an additional component. The mean-variance portfolio reflects the manager's optimal risk exposure absent benchmarking ($\alpha = 0$). It implies that, if trading for her own account, the manager would choose the constant allocation $\eta/(\gamma\sigma)$ to the stock over the entire investment period (i.e., the Merton, 1971 portfolio). Importantly, this allocation reflects not only her superior information η but also her own relative risk aversion γ.

The difference between the manager's optimal policy and the mean-variance portfolio has the usual interpretation as a *hedging* demand for the stock. In the presence of benchmarking, this component hedges the manager's exposure to the benchmark Y, and varies over time according to her (optimal) relative performance $\hat{Z}_t$. Depending on whether the stock allocation of the mean-variance portfolio is greater or less than the allocation ϕ^Y of the benchmark, the hedging demand will either decrease or increase the portfolio weight in the stock relative to the mean-variance portfolio. Thus, the manager's portfolio partially *mimics* the benchmark, the more so the higher the relative performance fee α.

Eq. (11) shows that the optimal policy fully hedges the manager's exposure to the benchmark. By levering up or down the stock in the portfolio, the manager's trading policy can undo any unwanted asset exposure induced by the fulcrum fee α or the composition of the benchmark ϕ^Y. The result follows from the symmetric nature of fulcrum fees, which penalize underperformance as much as they reward outperformance, and is standard in prior studies of linear benchmarking (e.g., Stoughton, 1993). In our setup, it implies that households can calibrate the fulcrum fee and the benchmark composition to maximize utility of end-of-period after-fee wealth $W_T^{\eta;\alpha,\phi^Y} - X_T^{\eta;k}$ without having to compensate the manager for inducing an undesired level of risk in her portfolio. This feature reduces the compensation cost of linear benchmarking relative to more complex, nonlinear contracts that expose the manager to either excessive or insufficient risk, and is a main reason for the optimality of fulcrum fees that we establish in Section 5.

In contrast to α and ϕ^Y, the proportional fee k affects the manager's compensation but not the optimal portfolio compositions $\hat{\phi}_t$. Unlike in a CARA-normal setup, fees tied to total fund returns have no role in potentially distorting the manager's portfolio in the direction of either increasing or decreasing the allocation to the stock. Eq. (9) shows that, as we depart from the CARA-normal framework, fulcrum fees α and the benchmark's stock allocation ϕ^Y can subsume this role. In particular, a higher α increases the manager's concern relative to the benchmark, while a greater ϕ^Y increases the fraction of the manager's portfolio invested in the stock.

4.2 Reservation Utilities

Alternatively to delegating their money to the active manager, households in our setup can opt to invest in a passively managed fund, or ETF. In this case, households select the ETF with stock allocation $\phi^P \in \Phi = \mathcal{R}$ to maximizes expected utility of their after-fees wealth, $(1-k_m)W_{P,T}$, where $W_{P,T}$ is the terminal value (total AUM) of the ϕ^P-ETF. In reality, different choices for the portfolio weight $\phi^P \in \Phi$ could correspond, for instance, to different passive investment styles (e.g., "balanced portfolio") available in the market. Importantly, since the choice of ETF is made by households its portfolio composition reflects no private information.

As is standard in the literature, we assume that the manager is endowed with no initial wealth. However, she can set up and offer a passively managed fund at no cost. In particular, she can offer the ϕ^P-ETF that households would select if they decided to delegate their wealth to a passive fund. If so, the manager collects fees $k_m W_{P,T}$ at the end of the period. The expected utility over these fees determines the manager's reservation utility $\bar{U}$.[21]

We characterize the ETF optimal composition ϕ^P, households' derived utility $U_{h,P}$ from delegation to the passive fund, and the manager's reservation utility $\bar{U}$, in terms of households' prior $\bar{m}$ for the market price of risk and associated uncertainty $\bar{v}_0$ in the following:

Lemma 1. *When households delegate their wealth to a passively managed fund, they choose an ETF with fixed weight ϕ^P in the stock as given by:*

$$\phi^P = \frac{1}{\gamma_h + (\gamma_h - 1)\bar{v}_0 T} \frac{\bar{m}}{\sigma} \tag{12}$$

The ex-ante utility of households under passive portfolio management (households' reservation utility) is:

$$U_{h,P}(w) \equiv E\left[\frac{(W_{P,T})^{1-\gamma_h}}{1-\gamma_h}\right] = \frac{((1-k_m)w)^{1-\gamma_h}}{1-\gamma_h} \times \exp\left\{-(\gamma_h - 1)\left(r + \frac{1}{\gamma_h + (\gamma_h - 1)\bar{v}_0 T}\frac{\bar{m}^2}{2}\right)T\right\}, \tag{13}$$

whereas the ex-ante utility of the manager (the manager's reservation utility) is:

$$\bar{U}(w) \equiv E\left[\frac{(W_{P,T})^{1-\gamma}}{1-\gamma}\right] = \frac{(k_m w)^{1-\gamma}}{1-\gamma} \times \exp\left\{-(\gamma - 1)\left(r + \frac{2\gamma_h - \gamma + (2\gamma_h - \gamma - 1)\bar{v}_0 T}{(\gamma_h + (\gamma_h - 1)\bar{v}_0 T)^2}\frac{\bar{m}^2}{2}\right)T\right\}. \tag{14}$$

According to Lemma 1, households' reservation utility depends positively on the expected market conditions $\bar{m}$ and negatively on the level of uncertainty $\bar{v}_0$.[22] Intuitively, the better the expected market conditions the lower the impact that a given level of information disadvantage will have on households' direct passive investment, and the higher the performance that they will demand from the active manager. By contrast, an actively managed fund is not affected by prior uncertainty, so households would be willing to pay more for active management as this uncertainty worsens.

Lemma 1 also shows that, in the contracting problem (6), the reservation utilities of households and the manager are linked through the ETF portfolio composition that households select. Since

[21] Alternatively, we could assume an exogenous, arbitrary value for the manager's reservation utility, without significantly affecting our main conclusions. Our formulation has the advantage of endogenizing the utility accruing to the passive fund manager within a parsimonious framework and reduce the number of free parameters in the model.

[22] Note that $U_{h,P}$ is negative for our assumption that $\gamma_h > 1$, so a smaller argument of the exponential function on the RHS of (13) increases households' reservation utility.

the compensation of the manager under passive management depends on the risk preferences and uncertainty of households, both γ_h and $\bar{v}_0$ affect her reservation utility $\bar{U}$. For instance, a higher prior uncertainty renders the information advantage that a passive manager foregoes more valuable, i.e., it increases the manager's opportunity cost of managing the passive fund. Therefore, the reservation utility $\bar{U}$ of the manager falls with $\bar{v}_0$.

5 The Optimal Linear Contract

In this section we establish the main result in the paper, namely the optimality of benchmarking in the contracting problem (6) between households and active fund managers (subsection 5.2). Before doing so, we characterize the benchmark scenario in which the manager's private information—i.e., the realization of $\tilde{\eta}$—can be contracted upon (subsection 5.1). Since the information asymmetry that motivates the contracting problem (6) is absent in this scenario, we refer to it interchangeably as the complete information (CI) or "first-best" case, and use it to quantify the value of the manager's information to households. Finally, we analyze numerically the optimal contract parameters and the associated losses in households' utility relative to the first-best case (subsection 5.3).

5.1 Value of the Manager's Information to Households

Under the first-best scenario, at $t = 0$ the manager reveals the realized value η of her signal $\tilde{\eta}$ to households conditional on receiving (state-dependent) payment $k_{CI}(\eta)w$. Upon completing the transaction, *both* households and the manager have complete information about the stock fundamentals over the investment period $[0, T]$. Conditional on η, the portfolio that maximizes households' utility solves the following problem:

$$\max_{\{\phi_t\}_{t\in[0,T]}} E_0 \left[\frac{(W_T^{\tilde{\eta}})^{1-\gamma_h}}{1-\gamma_h} \middle| \tilde{\eta} = \eta \right], \tag{15}$$

subject to the self-financing constraint (8) and initial wealth $W_0^\eta = (1 - k_{CI}(\eta))w$. The solution to this problem is the standard Merton (1971) portfolio:

$$\phi_{CI}^\eta = \frac{\eta}{\gamma_h \sigma}, \tag{16}$$

with associated wealth process, for $t \in [0, T]$, equal to:

$$W^{\eta}_{CI,t} = (1 - k_{CI}(\eta))wg(t; \eta, \gamma_h)(\pi^{\eta}_t)^{-\frac{1}{\gamma_h}}, \tag{17}$$

where $g(\cdot)$ is as in Proposition 1. For each realization of $\tilde{\eta}$ at $t = 0$, the first-best portfolio (16) is constant over time. This implies that households can implement the first-best investment policy by choosing, after observing η, the ETF that allocates the fraction $\phi^P = \phi^{\eta}_{CI}$ to the stock. For simplicity, we assume that households can invest in ETFs at no cost in this idealized case. Their end-of-period wealth is $W^{\eta}_{CI,T}$, for W^{η}_{CI} as characterized by (17) at $t = T$.

In turn, the manager collects the amount $k_{CI}(\eta)w$ at $t = 0$ from households and invests this amount over the investment period $[0, T]$ according to her own preferences. Her end-of-period wealth is then $k_{CI}(\eta)\hat{W}^{\eta;0,\phi^Y}_T$, for $\hat{W}^{\eta;0,\phi^Y}$ as characterized by (10) for $\alpha = 0$. Setting $k_{CI}(\eta)$ to equate the manager's utility under the CI and passive management cases *state-by-state*, we obtain the following:

Proposition 2. *In the first-best scenario, households can acquire the manager's private signal at cost*

$$k_{CI}(\eta)w = k_m \exp\left\{-\frac{1}{2\gamma}\left(\eta - \frac{\gamma\bar{m}}{\gamma_h + (\gamma_h - 1)\bar{v}_0 T}\right)^2 T\right\} w. \tag{18}$$

By acquiring the manager's signal and allocating their remaining wealth in a ϕ^{η}_{CI}-ETF at zero cost, households attain an ex-ante utility of

$$U_{h,CI}(w) \equiv E\left[\frac{(W^{\tilde{\eta}}_{CI,T})^{1-\gamma_h}}{1-\gamma_h}\right] = \frac{w^{1-\gamma_h}}{1-\gamma_h}\int_{-\infty}^{+\infty}(1 - k_{CI}(x))^{1-\gamma_h} e^{-(\gamma_h - 1)\left(r + \frac{x^2}{2\gamma_h}\right)T}\mathbf{n}_{\eta}(x)dx \tag{19}$$

where $\mathbf{n}_{\eta}(\cdot)$ is the probability density function of $\tilde{\eta}$, while the manager attains ex-ante utility of

$$U_{CI}(w) \equiv E\left[\frac{\left(k_{CI}(\tilde{\eta})\hat{W}^{\tilde{\eta};0,\phi^Y}_T\right)^{1-\gamma}}{1-\gamma}\right] = \bar{U}(w), \tag{20}$$

where expectations are with respect to the joint distribution of $(\tilde{\eta}, B_T)$.

The certainty equivalent of (19) in excess of the certainty equivalent of the reservation utility $U_{h,P}$ gives us the *maximum value that households can extract from the manager's information.*

5.2 Optimality of Benchmark-Adjusted Fees in a Linear Contract

When the manager's signal (the realization of $\tilde{\eta}$) is not observable, it cannot be verified and no contract can be written on it.[23] In this second-best scenario, households choose the contract parameters $\mathscr{C}$ to solve problem (6).

We start by characterizing the optimal proportional fee $\hat{k}$. According to Eqs. (10) and (11), k does not affect the total wealth $\hat{W}_T^{\eta;\alpha,\phi^Y}$ but only how this wealth is split between households and the manager. Since the after-fee wealth $W_T - X_T$ of households is decreasing in k, the optimal proportional fee $\hat{k}$ satisfies the manager's participation constraint (M's PC) with equality:

Lemma 2. *The proportional fee $\hat{k}$ in the optimal linear fee contract in the class (3) is independent of the benchmark-adjusted fee α and the benchmark composition ϕ^Y:*

$$\hat{k} = \frac{k_m}{\sqrt{\left(1+\left(1-\frac{1}{\gamma}\right)\bar{v}_0 T\right)^{\frac{1}{\gamma-1}}}} \times \exp\left\{\left(\frac{2\gamma_h - \gamma + (2\gamma_h - \gamma - 1)\bar{v}_0 T}{(\gamma_h + (\gamma_h - 1)\bar{v}_0 T)^2} - \frac{1}{\gamma + (\gamma - 1)\bar{v}_0 T}\right)\frac{\bar{m}^2}{2}T\right\}. \tag{21}$$

To analyze the potential value of fulcrum fees to households, we first abstract away from the problem of compensating the manager and assess the *portfolio-alignment* role of the linear fee contract (3). Comparing the optimal active fund portfolio under $\mathscr{C}$ (9) and the complete-information portfolio of households (16) we obtain the following:

Lemma 3. *When households and the manager have identical risk aversion ($\gamma_h = \gamma$), a pure proportional fee contract $\{(k, 0, \phi^Y) : \phi^Y \in \mathbb{R}\}$ implements the CI portfolio ϕ^η_{CI} for all realizations of $\tilde{\eta}$. When the manager's and households' risk aversion coefficients differ ($\gamma_h \neq \gamma$), no linear fee contract implements the CI portfolio ϕ^η_{CI} for all realizations of $\tilde{\eta}$.*

In the knife-edge case in which the manager and households have the same coefficient of risk aversion, there is no portfolio-alignment role for fulcrum fees. However, when the parties' coefficients of risk aversion differ, a fee proportional to total returns cannot align the manager's allocation with the CI portfolio. From investors' point of view, *both* pure proportional and benchmark-adjusted linear fees induce the manager to choose a suboptimal portfolio. The inability of pure proportional fees to perfectly align portfolios in Lemma 3 contrasts with the portfolio-alignment role of this

[23] Households cannot rely on the manager truthfully revealing her signal either. Indeed, her expected utility is increasing in $k_{CI}(\eta)$, so the manager finds it optimal to report $\hat{\eta} = \gamma\bar{m}/(\gamma_h + (\gamma_h - 1)\bar{v}_0 T)$ across all realizations of $\tilde{\eta}$.

type of fees in the CARA-normal setup, and follows exclusively from our assumption of non-CARA preferences. Indeed, we show in Appendix B that when both the manager and households display CARA preferences (possibly, with different risk aversion coefficients) perfect portfolio alignment can still be attained with pure proportional fees.

Note, however, that perfect portfolio alignment between the actively managed and CI portfolios will generally not solve the risk-sharing problem posed by the contracting model (6). The reason is that in a risk-sharing problem not only the payoffs to households matter but also the *compensation of the manager*. In fact, it is straightforward to see that, to perfectly align the objective function of the manager with their own, households can offer the manager the nonlinear contract $s(W_T) = (cW_T)^{(1-\gamma_h)/(1-\gamma)}$, for some constant c. The contract ensures that the manager selects the CI portfolio (16) but at the same time distorts her compensation away from the preferred payoff profile (11). To ensure the manager's participation, households then need to offer her extra pay in compensation for the undesired risk she has to bear, potentially rendering perfect portfolio alignment inefficient in the first place.[24]

To see why accounting for risk-sharing considerations can overturn the irrelevance of linear benchmarking under CRRA preferences, note that Eqs. (9)-(11) suggests a potential portfolio alignment role for fulcrum fees *at low cost*. Consider the following tradeoff associated with tying the manager's compensation to the benchmark. On the one hand, increasing the fulcrum fee allows households to improve portfolio risk alignment without the need to compensate the manager for unwanted risk. When the fulcrum fee is set to zero (pure-proportional fee arrangement), the manager's portfolio choice in Eq. (9) fully reflects her private (superior) information but also her own risk preferences. In particular, for a given realization of $\tilde{\eta}$ the manager chooses a smaller (respectively, larger) long or short position in the stock than relatively risk tolerant (averse) investors would desire conditional on the same signal. The role of fulcrum fees consists in tilting the portfolio allocation of the manager closer to the portfolio that investors would choose if endowed with the same signal. Due to its symmetric nature, fulcrum fees do not alter the manager's preferred payoff profile (11), so linear benchmarking plays this role at *no cost of compensating the manager* for exposure to undesired risk. Larger fulcrum fees should then increase the ex-ante value of the manager's information that households can realize, the more so the larger the difference between

[24] Indeed, for CRRA preferences with $\gamma_h \neq \gamma$ and symmetric information about asset returns, this nonlinear compensation function differs from the efficient sharing rule characterized by Cadenillas et al. (2007). The inefficiency might explain the nonexistence of this type of contracts in practice.

their risk aversion and the risk aversion of the manager.

On the other hand, increasing the fulcrum fee distorts the manager's asset allocation (9) towards a conditionally *inefficient* portfolio. In contrast to the manager, households can exploit no private information in the design of the benchmark portfolio. For each realization of $\tilde{\eta}$ at $t = 0$, the benchmark portfolio will thus be different from the conditionally efficient CI portfolio ϕ^{η}_{CI} that households would choose if they observed the manager's signal.[25] Once again, the manager need not be compensated for the exposure of her pay to the inefficient benchmark because she can fully hedge this exposure. However, the tilt toward the conditionally inefficient portfolio prevents households from realizing the full value of the manager's signal. Larger fulcrum fees should then reduce the ex-ante value of the manager's information that households can realize, the more so the more valuable this information—i.e., the greater the uncertainty $\bar{v}_0$ that households face—is.

We next show that, in exploiting this tradeoff, both relatively risk tolerant and risk averse households will always include a benchmarking component in the fund manager's linear compensation contract:

Theorem 1. *Assume a solution $(\hat{a}, \hat{b})$ exists to the following system of equations in (a, b):*

$$E\left[\left(\hat{W}_T^{\tilde{\eta};a,b} - \hat{X}_T^{\tilde{\eta};\hat{k}}\right)^{-\gamma_h}\left(Y_T^{\tilde{\eta};b} - wg(t;\tilde{\eta},\gamma)(\pi_T^{\tilde{\eta}})^{-\frac{1}{\gamma}}\right)\right] = 0 \tag{22}$$

$$E\left[\left(\hat{W}_T^{\tilde{\eta};a,b} - \hat{X}_T^{\tilde{\eta};\hat{k}}\right)^{-\gamma_h} aY_T^{\tilde{\eta};b}\big(\sigma(\tilde{\eta} - b\sigma)T + \sigma B_T\big)\right] = 0, \tag{23}$$

where expectations are taken with respect to the joint distribution of $(\tilde{\eta}, B_T)$, and $g(\cdot)$ is as in Proposition 1. Then the optimal fulcrum fee and benchmark portfolio solving the linear contracting problem (6) are $\hat{\alpha} = \hat{a}$ and $\hat{\phi}^Y = \hat{b}$. In particular,

(i) When households and the manager have identical risk aversion coefficients ($\gamma_h = \gamma$), a pure proportional fee contract ($\alpha = 0, \phi^Y \in \mathbb{R}$) is optimal.

(ii) When the risk aversion coefficients of the manager and households differ ($\gamma_h \neq \gamma$), a linear fee contract that includes benchmark-adjusted performance fees ($\hat{\alpha} \neq 0$) increases households' expected utility from delegation to an active manager with respect to a pure proportional ($\alpha = 0$) fee.

Not surprisingly, since pure proportional fees do not expose the manager to unwanted relative

[25] Except for the zero-probability event in which $\eta = \bar{m}/(1 + (1 - 1/\gamma_h)\bar{v}_0 T)$, see Eqs. (12) and (16).

risk there is no risk sharing role for linear benchmarking under perfect risk-aversion alignment. In particular, fulcrum fees are irrelevant in the optimal contract between households and the fund manager when both parties exhibit log preferences (i.e., $\gamma_h = \gamma = 1$). This result is standard in delegated portfolio management models in which the principal (households, in our setup) both dictates the terms of the managerial contract and delegates the management of her entire portfolio to the agent (Cadenillas et al., 2007; Cuoco and Kaniel, 2011).[26] A similar reason explains why, even when the risk preferences of households and the manager are not aligned in the CARA case of Appendix B, fulcrum fees do not improve risk-sharing—and actually worsen it—over a linear fee contract based only on total returns.

More interestingly, whenever the risk aversions of households and the manager differ, Theorem 1 shows that the proportional fee-only contract is suboptimal. The inclusion of fulcrum fees in the fee contract increases the value of the manager's information that investors can realize while simultaneously addressing risk-sharing considerations. Theorem 1 then implies that the irrelevance of benchmarks in prior literature breaks down in the case of CRRA preferences for both the manager and households. Given any pure proportional fee, including a benchmark-adjusted performance component in the linear fee contract increases households' expected utility from delegation without increasing the cost of compensating the manager. Whenever a solution to Eqs. (22)-(23) exists, it has *non-zero fulcrum fees* ($\hat{\alpha} \neq 0$).

We highlight that the rationale and nature of linear benchmarking in our framework differ from those in Ou-Yang (2003). In that paper, both households and the manager have CARA preferences and symmetric (complete) information about asset returns. The benchmark irrelevance is overturned by enlarging the set of admissible benchmarks to include not only passive portfolios but also "active indexes" whose number of shares invested in each asset varies over time. The optimal contract is shown to consists of a fixed fee plus a fraction of total assets plus a symmetric benchmark-adjusted fee. Precisely because they are as informed as the manager, households in Ou-Yang (2003) can choose an efficient portfolio as the benchmark so as to induce the manager to invest the right amount of money in the assets at each point in time. In contrast, households in our setup do not observe the efficient portfolio. They are constrained to optimize risk sharing by trading off the benefits and costs of introducing concerns relative to an inefficient benchmark

[26] In a setting where neither of these conditions are met, Cuoco and Kaniel (2011) show that a linear sharing rule can be suboptimal even when both the manager and households have CRRA preferences with the same relative risk aversion coefficient.

in the objective of the manager. Theorem 1 shows that including fulcrum fees in a linear contract is optimal even when the benchmark specified in these fees is inefficient. In fact, our numerical analysis in the next section indicates that the benchmark portfolio that households optimally select is generally inefficient also with respect to their own, inferior information.

5.3 Numerical Analysis

We next solve for the optimal benchmark-adjusted fee $\hat{\alpha}$ and benchmark composition $\hat{\phi}^Y$ of Theorem 1 numerically, and assess quantitatively the benefits of linear benchmarking.[27] Our baseline and alternative parameterizations are as follows. We identify the risk-less asset β with the 3-month U.S. Treasury bill, and the stock S with a broad-based market portfolio. We set the real risk-less interest rate r equal to 3%. For the prior value $\bar{m}$ of the market price of risk and for the market volatility σ, we use historical estimates during the sample period January 1980-December 2006. This corresponds to a relatively recent and long period over which the hypothesis of normality of annual returns cannot be rejected.[28] Following Brennan and Xia (2001), we set the prior value for the market excess return $\mu - r$ equal to the sample mean return of the Fama and French (1996) market portfolio during the period, 8.1%. The corresponding standard deviation of the market portfolio, σ, equals 15.8%. We set the baseline prior variance $\bar{v}_0$ equal to the square of the standard error of the sample mean market price of risk. This standard error equals 0.19 for the period 1980-2006 and corresponds to a standard error for the mean return of 3%, in line with baseline values used in the literature. Our alternative parameterizations of $\bar{v}_0$ consider both low ($\bar{v}_0 = 0.02$) and high ($\bar{v}_0 = 0.4$) levels of uncertainty. We note that the higher the uncertainty $\bar{v}_0$ about $\tilde{\eta}$, the greater the information asymmetry about asset returns between households and the manager. Therefore, we interpret $\bar{v}_0$ (equivalently, $\sqrt{\bar{v}_0}$) as the manager's *information advantage* over households in our analysis below.

We consider investment horizons from $T = 1$ to $T = 5$ years. The middle of the range, $T = 3$, agrees with the average performance evaluation period for fulcrum fees in the US mutual fund industry (Cuoco and Kaniel, 2011), so we take this as our baseline value. Based on data from the Investment Company Institute, we set the management fee k_m for passive funds (ETFs) to 15 basis points (bps). In line with standard practice in the industry (see, e.g., Elton et al., 2003), we constrain

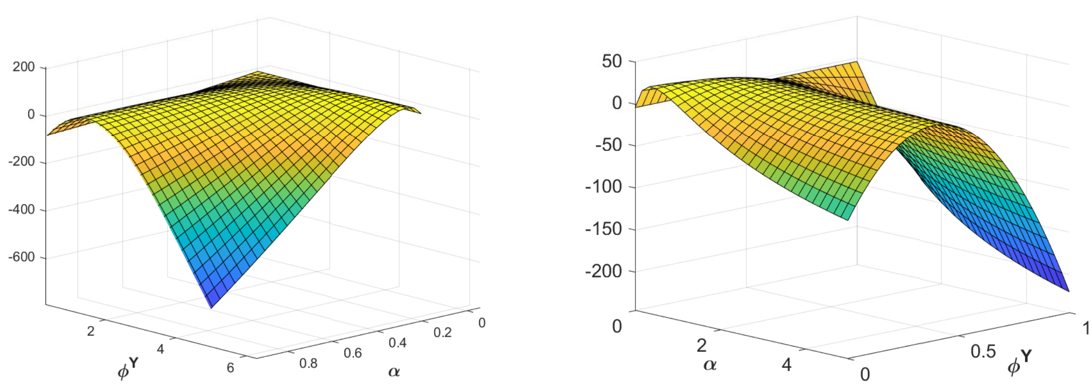
[27] In general, the expectations in (22)-(23) cannot be computed explicitly. However, because they are defined with respect to two independent normally distributed variables, they can be approximated with arbitrary precision using standard numerical integration methods.

[28] The Jarque-Bera test of normality results in a p-value of 16.6% during this period.

the benchmark-linked performance fee to be non-negative ($\alpha \geq 0$) across all parameterizations.

We assume a coefficient of relative risk aversion γ for the manager of 5 in order to approximately match the mean estimate of Koijen (2014) in a setup similar to ours. Following the dispersion in the relative risk aversion coefficients of households that Kimball et al. (2008) extract from the Health and Retirement Survey, we consider both relatively risk tolerant ($\gamma_h < 5$) and risk averse ($\gamma_h > 5$) fund investors.

Figure 1 plots the mapping of the choice variables (α, ϕ^Y) to households' excess certainty equivalent returns from delegation to an active manager. The resulting surface is smooth and typically has a unique maximum. These features makes the numerical search for the optimal contract very fast and reliable.



(a) Risk-Tolerant Investors ($\gamma_h = 3$)

(b) Risk-Averse Investors ($\gamma_h = 7$)

Figure 1: **Excess CE Return (in bps) from Delegation to an Active Manager.**
The figure plots households' annualized excess certainty equivalent returns (CER) from delegation to an active manager under a linear contract $(\hat{k}, \alpha, \phi^Y)$. Excess CER are computed with respect to households' CER from delegation to a passively managed fund. The model parameters are: $T = 3, r = 3\%, \sigma = 0.158, \bar{m} = 0.513, \bar{v}_0 = 0.037, \gamma = 5$.

5.3.1 Value of Benchmark-Adjusted Fees

According to Lemma 3, when $\gamma_h \neq \gamma$ linear fees specified as proportions of either total or benchmark-adjusted returns distort the active fund portfolio away from the CI portfolio. Thus, a linear contract implies a loss in the ex-ante value that households can extract from the manager's information. In this subsection, we assess this loss quantitatively and study whether fulcrum fees can improve on pure proportional fees.

For different levels $|\gamma_h - \gamma|$ of risk aversion misalignment between households and the manager,

Fig. 2 plots households' certainty equivalent (CE) rate of returns in excess of the return to a passive strategy under the optimal linear contract (red solid line) and under the optimal pure proportional fee (blue dotted line). We present the excess CE returns as a percentage of the excess CE returns under our reference CI case or, equivalently, as the (%) fraction of the *ex-ante value that households extract from the manager's information.* For example, when $\gamma_h = 3$ ($|\gamma_h - \gamma| = 2$), households earn an ex-ante CE return of 7.01% on the optimally designed passive fund. This return would increase instead to 7.61% if they could purchase the private signal of the manager (CI case), leading to an excess CE return of 60 bps. The figure illustrates the relative merits of the different contract types for the active manager as a percentage of this CI excess CER. A negative number means that households lose money by delegating their wealth to an active manager that is subject to the corresponding fee contract, instead of investing it in their preferred passively managed fund (ETF).

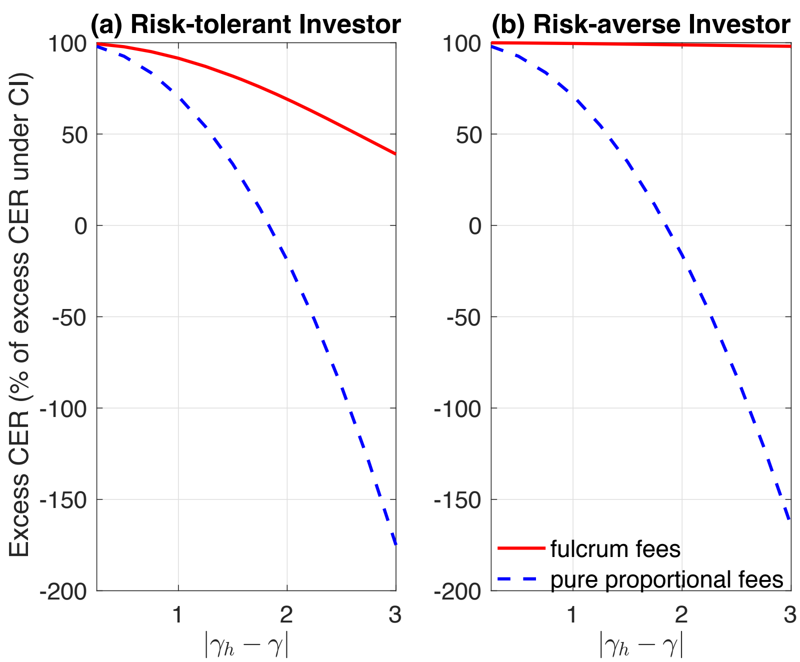


Figure 2: **Benefits of Active Management under Optimal Benchmarking.**
The figure plots the excess certainty equivalent returns (CER) from delegation under the optimal linear benchmark-adjusted contract (red solid line) and under the pure proportional fee contract (blue dashed line), for relatively risk-tolerant ($\gamma_h < 5$, Panel (a)) and risk-averse ($\gamma_h > 5$, Panel (b)) fund investors. Excess CER are computed with respect to households' CER from delegation to a passively managed fund and reported as a percentage of the excess CE returns under the reference CI case. The rest of the model parameters are as in Fig. 1.

A first result is that, in contrast to prior literature, linear benchmarking can be highly valuable to delegating investors in our framework. Both relatively risk-tolerant ($\gamma_h < 5$) and risk-averse ($\gamma_h > 5$) households realize a higher fraction of the ex-ante value of the manager's private information under optimal linear benchmarking than under the optimal pure proportional contract. As anticipated from our discussion in Section 5.2, the gains of adding fulcrum fees to a pure proportional fee contract increase with the risk aversion misalignment $|\gamma_h - \gamma|$. For example, these gains equal 20.8 and 28.6 percentage points, respectively, for households with RRA coefficients of 4 (relatively risk tolerant) and 6 (relatively risk averse). For even lower or higher coefficients of RRA for households, equal to 3 or 7, these gains increase to 88.2 and 115.1 percentage points, respectively.

A second result is that, by including a fulcrum fee with respect to an appropriately designed benchmark in the fee contract, households can extract almost the entire ex-ante value of the active manager's information. This is especially the case for relatively risk-averse investors. Following the result in Lemma 3, the fraction of this value that investors of the active fund realize under *any* linear contract falls with the misalignment in risk aversion with the fund manager. This fraction falls very steeply with $|\gamma_h - \gamma|$ under the pure proportional fee contract. However, it falls at a much lower pace when the optimal benchmark-adjusted fee is added to the contract. This implies that relatively risk-tolerant investors with RRA coefficient of 2 or higher realize more than 39% of the ex-ante value of the manager's information. Relatively risk-averse investors with RRA coefficient of 8 or lower, on the other hand, extract at least 98% of this value by using the optimal linear contract.[29] In this sense, the optimal linear contract achieves "near-first best" risk sharing between conservative fund investors and their manager.

Finally, we observe that a pure proportional fee linear contract can entail substantial losses to the active fund investors. Indeed, sufficiently risk tolerant ($\gamma_h \leq 3$) and sufficiently risk averse ($\gamma_h \geq 7$) households both earn negative CE returns in excess of passive investing when delegating to active managers with pure proportional fees. Intuitively, from these investors' viewpoints the manager can take either insufficient or excessive risks in response to absolute-performance compensation, and the costs of these risks can more than offset the benefits they derive from the manager's private signal. This type of fund investors is better off by investing their wealth in the passive strategy. Equivalently, benchmarking is not only optimal but necessary for the delegation of their wealth to active funds to occur, even if the information of the manager has a potentially high value. This result is in stark contrast with prior literature arguing that relative compensation (fulcrum fees), but not absolute compensation (pure proportional fees) in a linear contract can inflict losses to

[29] Under symmetric information, moral hazard problems and portfolio constraints, Dybvig et al. (2010) also argue that linear contracts entail little losses relative to the optimal contract between an investor and a manager with CRRA preferences. Our result in this section extend theirs to a delegation context of asymmetric information about asset returns between the fund manager and her investors.

active fund investors.

γ_h	T	$\sqrt{\bar{v}_0}$	Excess CER in CI case (bps)	Excess CER (% of Excess CI CER) Pure Proportional Fee Contract (1)	Optimal Linear Contract (2)	(2) - (1)
3	1	0.02	1.5	< -300	46.2	> 100.0
		0.2	99.6	-17.2	72.0	89.2
		0.4	385.6	61.6	78.9	17.3
	3	0.02	1.5	< -300	37.4	> 100.0
		0.2	97.7	-10.3	69.8	80.0
		0.4	359.2	67.6	80.4	12.9
	5	0.02	1.5	< -300	33.9	> 100.0
		0.2	95.8	-4.0	69.5	73.5
		0.4	337.1	71.7	82.1	10.4
7	1	0.02	0.4	< -300	89.2	> 100.0
		0.2	32.9	-16.2	99.5	> 100.0
		0.4	125.5	62.6	99.7	37.1
	3	0.02	0.4	< -300	80.1	> 100.0
		0.2	31.9	-7.4	98.9	> 100.0
		0.4	112.5	69.6	99.5	29.9
	5	0.02	0.4	< -300	74.7	> 100.0
		0.2	31.0	0.3	98.5	98.2
		0.4	102.4	74.2	99.5	25.3

Table 1: **Value of Active Management.** This table reports annualized excess certainty equivalent returns (CER) from delegation under the complete information (CI) reference case, the pure proportional fee and the optimal linear contract that solves problem (6). Values are reported for different combinations of household's risk aversion, time horizons T, and degrees of manager's information advantage $\sqrt{\bar{v}_0}$. Excess CER are computed with respect to households' CER from delegation to a passively managed fund. The rest of the model parameters are as in Fig. 1.

Table 1 shows that these findings generalize to different investment horizons T and degrees of information advantage $\bar{v}_0$ for the manager. It also indicates that the ex-ante value that households extract from the manager's information under the optimal linear contract increases with the information advantage of the manager, and is substantial (70% or more for risk-tolerant investors, close to 100% for risk-averse investors) when the advantage is sufficiently large (e.g., $\sqrt{\bar{v}_0} > 0.02$ in the Table). At the same time, the benefits of fulcrum fees relative to pure proportional fees (last column) decrease with the information advantage of the manager. This was anticipated in Section 5.2, where we argued that distorting the fund portfolio towards an inefficient benchmark becomes increasingly costly as the information advantage of the manager widens.

5.3.2 Optimal Linear Contract Parameters

Figure 3 plots the optimal contract as a function of the risk aversion misalignment $|\gamma_h - \gamma|$ between households and the manager. Panels (a) and (b) depict the optimal fulcrum fee $\hat{\alpha}$ (top row) and benchmark allocation to the stock $\hat{\phi}^Y$ (bottom row) of, respectively, relatively risk-tolerant and risk-averse fund investors. For comparison with the benchmark portfolio, the bottom row also displays the optimal allocation ϕ^P of the passive fund to the stock. As described above, ϕ^P corresponds to the fixed-weight portfolio that households would choose if they traded on their own (inferior) information, and provides a meaningful reference against which to compare the benchmark portfolio that households implement in the optimal linear contract. The figure illustrates our main result stated in Theorem 1: under risk aversion misalignment $\gamma \neq \gamma_h$, the optimal fulcrum fee is always non-zero—in this case, positive, regardless of whether fund investors are relatively more or less risk averse than the fund manager.

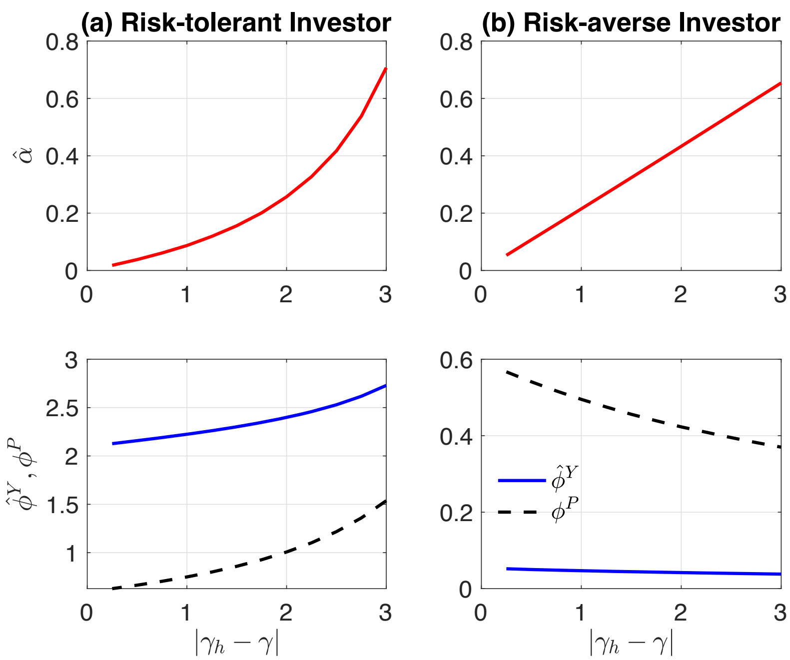


Figure 3: **Benefits of Active Management under Optimal Benchmarking.**

The figure plots the fulcrum fee $\hat{\alpha}$ (red solid line, top row) and benchmark allocation in the stock $\hat{\phi}^Y$ (blue solid line, bottom row) of the optimal contract for relatively risk-tolerant ($\gamma_h < 5$, Panel (a)) and risk-averse ($\gamma_h > 5$, Panel (b)) fund investors. The bottom row also displays the optimal allocation ϕ^P of the passive fund to the stock (black dashed line). The rest of the model parameters are as in Fig. 1.

Other novel patterns are evident from Fig. 3. First, the optimal fulcrum fee $\hat{\alpha}$ increases in the risk aversion misalignment $|\gamma - \gamma_h|$ across investor types. This result follows the intuition in Section

5.2, whereby a greater misalignment in risk aversion raises the benefits of tilting the manager's portfolio toward the allocation preferred by households, even if this allocation is conditionally inefficient.

Second, the optimal benchmark portfolio in the fee contract is unconditionally *inefficient* from households' viewpoint. The result departs from those in prior models with no information asymmetry about asset returns (e.g. Dybvig et al., 2010), where the prescribed benchmark is an uninformed optimum portfolio. In our setup, households' uninformed optimal portfolio is represented by ϕ^P, the stock allocation of the ETF that households would select under passive management of their wealth. The bottom panels of Fig. 2 show that the benchmark allocation $\hat{\phi}^Y$ in the stock of the optimal linear contract is either significantly greater or significantly lower than ϕ^P depending on whether active fund investors are, respectively, relatively risk tolerant or risk averse.

The optimal portfolio of the active manager and the CI portfolio of households that we present in sections 4.1 and 5.1 provide the rationale for this result. From the perspective of relatively risk-tolerant fund investors ($\gamma_h < \gamma$), the manager underreacts to good news ($\eta > \bar{m} > 0$), as she increases the weight in the risky asset in the fund portfolio less than investors would prefer. Conversely, the manager overreacts to bad news ($0 < \eta < \bar{m}$) by lowering the weight of the risky asset in the fund portfolio more than the fund investors would prefer. Both under- and overreaction problems can be alleviated by setting a risky benchmark in the managerial contract. Similarly, from the perspective of relatively risk-averse fund investors ($\gamma_h > \gamma$) the manager overreacts to good news by aggressively increasing the weight of the risky asset in the portfolio and underreacts to bad news by reducing too little her position in the risky asset. Both problems can be alleviated by having the manager partially mimic a very conservative benchmark—a low value of $\hat{\phi}^Y$.

These results are robust to alternative calibrations of the model. Panel A of Table 2 reports the optimal contract parameters for the model parameterizations of Table 1. Across the different investment horizons T and degrees of information advantage $\sqrt{\overline{v}_0}$ of the active manager, the fulcrum fee $\hat{\alpha}$ is positive and the benchmark allocation $\hat{\phi}^Y$ in the stock differs substantially from the unconditionally efficient allocation ϕ^P, in line with our observations above. In Section 6.1, we show that the results are also robust to constraining the benchmark to take no levered positions in the assets ($0 \leq \phi^Y \leq 1$), as typically observed in practice.

Lastly, Panel A of Table 2 shows that, keeping the risk aversion misalignment $|\gamma - \gamma_h|$ fixed, the optimal fulcrum fee *falls* while the distance between the benchmark and the passive fund portfolios $|\hat{\phi}^Y - \phi^P|$ *increases* with the information advantage $\sqrt{\overline{v}_0}$ of the manager. Intuitively, when investors

				Panel A: Unconstrained Contract			Panel B: Constrained Contract		
γ_h	T	$\sqrt{\bar{v}_0}$	ϕ^P	$\hat{k}$ (in bps)	$\hat{\alpha}$	$\hat{\phi}^Y$	$\hat{k}$ (in bps)	$\hat{\alpha}$	$\hat{\phi}^Y$
3	1	0.02	1.08	14.8	1.47	1.37	14.8	$>$ 20.0	1.00
		0.2	1.05	14.8	0.19	3.08	14.8	1.33	1.00
		0.4	0.98	14.6	0.09	4.56	14.6	0.01	1.00
	3	0.02	1.08	14.5	2.63	1.24	14.5	$>$ 20.0	1.00
		0.2	1.00	14.3	0.24	2.44	14.3	1.22	1.00
		0.4	0.82	14.0	0.09	3.51	14.0	0.09	1.00
	5	0.02	1.08	14.1	3.55	1.20	14.1	$>$ 20.0	1.00
		0.2	0.95	13.9	0.27	2.20	13.9	1.12	1.00
		0.4	0.71	13.4	0.09	3.08	13.4	0.12	1.00
7	1	0.02	0.46	15.0	0.78	0.23	15.0	0.78	0.23
		0.2	0.45	14.9	0.41	0.02	14.9	0.41	0.02
		0.4	0.41	14.7	0.40	0.01	14.7	0.40	0.01
	3	0.02	0.46	14.9	1.08	0.29	14.9	1.08	0.29
		0.2	0.42	14.7	0.43	0.04	14.7	0.43	0.04
		0.4	0.33	14.3	0.40	0.02	14.3	0.40	0.02
	5	0.02	0.46	14.8	1.29	0.32	14.8	1.29	0.32
		0.2	0.40	14.6	0.44	0.05	14.6	0.44	0.05
		0.4	0.27	14.0	0.40	0.02	14.0	0.40	0.02

Table 2: **Optimal Managerial Contract.** This table reports the optimal allocation ϕ^P of the passive fund to the stock and the optimal contract $(\hat{k}, \hat{\alpha}, \hat{\phi}^Y)$ for different combinations of household's risk aversion, time horizons T, and degrees of manager's information advantage $\sqrt{\bar{v}_0}$. In Panel A, we impose no additional constraint on the contract space beyond those introduced in Section 4. In Panel B, we constrain the benchmark to adopt a long-only position in both the riskfree and risky assets ($0 \leq \phi^Y \leq 1$). The rest of the model parameters are as in Fig. 1.

are relatively less informed, fulcrum fees deviate the *informed* investment allocation—the active manager's—towards a highly inefficient portfolio $\hat{\phi}^Y$. In this situation, the costs in terms of forgone excess returns on the fund portfolio outweigh the benefits of risk alignment. Investors then reduce the rewards and penalties for relative out- and underperformance by lowering the weight of fulcrum fees in the optimal contract. In turn, they choose a benchmark with either a very low or very high, respectively, exposure to the risky asset so as to tilt the manager's portfolio towards either the conservative or the risky asset allocation that a risk-averse or a risk-tolerant investor prefers.

6 Extensions and Practical Implications

6.1 Unlevered Benchmark

In practice, the benchmarks stipulated in performance fee schedules typically represent an unlevered position either in a market index such as the S&P 500, or in a money market instrument such as the 3-month T-bill rate. To account for this empirical regularity, in Panel B of Table 2 we solve for the optimal linear contract under the constraint that the benchmark includes no short positions ("long-only") in either the riskfree or risky assets ($0 \leq \phi^Y \leq 1$). Since this constraint is not binding for $\gamma_h = 7$ in Panel A, we discuss the resulting optimal constrained contract for $\gamma_h = 3$ only.

Importantly, fulcrum fees remain optimal ($\hat{\alpha} > 0$) and their benefits over proportional-only fees sizable (non-tabulated) in this constrained case. The optimal constrained benchmark, which allocates 100% weight in the stock, fits well the type of benchmarks commonly observed, for example, in the mutual fund industry. Comparing the optimal contract parameters across panels, we observe that under the constrained contracting problem fund investors substitute a higher weight of the fulcrum fees in the optimal contract for a lower benchmark allocation in the stock. This is especially the case for relatively lower levels of the managerial information advantage $\sqrt{\bar{v}_0}$, under which the portfolio-alignment benefits of benchmarking exceed the portfolio (conditional) inefficiency cost that it brings about.

In fact, as the value of active management falls down to zero, the active manager's optimal constrained compensation is almost entirely based on the fund relative performance. This is shown in Panel B of Table 2 by optimal fulcrum fees $\hat{\alpha}$ in excess of 20 times the asset-based fee $\hat{k}$ for $\sqrt{\bar{v}_0} = 0.02$. Intuitively, there is no purpose in encouraging the manager to trade on her signal by rewarding absolute performance when this signal adds little value to the fund investors. In such cases, however, there can be no value of delegation to an active manager in the first place—even if compensated optimally, and fund investors might be better off investing their wealth in the passive alternative.[30] Thus, assuming that active managers have access to superior information is crucial for the optimality of linear benchmarking as a risk-sharing device when the availability of a passive funds is accounted for. Indeed, as we approach the symmetric-information case, the role of linear benchmarking vanishes with the value of active management in our setup.

[30] Indeed, when $\sqrt{\bar{v}_0} = 0.02$ the risk-tolerant fund investors ($\gamma_h = 3$) of Panel B earn *negative* excess CE returns over the optimal ETF under the optimal linear contract (not reported).

6.2 Larger Investment Opportunity Set for Households

In our setup, households make a binary decision to allocate their entire wealth to either the active or the passive funds. One could think that, if households were allowed to invest simultaneously in active *and* passive funds, they could replicate the portfolio alignment role of benchmarking using their own portfolio rather than the manager's compensation contract. If that were the case, they could potentially achieve a better risk-sharing outcome without the need to benchmark the active fund manager. The argument would be reinforced, in principle, if households were additionally allowed to invest a fraction of their wealth in the riskfree asset.

By contrast, we note that expanding the investment opportunity set of households in this way will not qualitatively affect our results. In particular, it will not unravel the optimality of benchmark-adjusted fees in the linear managerial contract. First, in choosing the optimal ETF composition ϕ^P, households directly decide the fraction of their wealth to allocate in the riskfree asset. Second, and more importantly, allowing households to mix up their active and passive fund holdings will not perfectly substitute the role of the optimal managerial compensation in solving the portfolio alignment and risk-sharing problems that we study. The reason for this is that, precisely because of the asymmetry in information about asset returns between households and the manager, the former cannot anticipate the signal observed by the latter. Thus, the allocation of households in an active fund with no benchmarked-adjusted fees may (and will likely) be too low for certain realizations of this signal, and too high for others.

This can be clearly seen in Fig. 2 and Table 1, where for reasonable parameterizations of the model there is no possible combination of a pure-proportional fee active fund and a passive fund under which households achieve higher utility (CE returns) than under the optimal linear contract. For example, when delegating to an active fund manager with a short horizon and moderate information advantage ($T = 1, \sqrt{\overline{v}_0} = 0.2$), the risk-tolerant households ($\gamma_h = 3$) of Table 1 earn a negative CE return in excess of the optimal ETF alternative (column (1)). Thus, the problem of allocating these households' wealth between an active fund subject to no benchmarking and the ETF has a corner solution with a 100% allocation to the ETF. However, households could do better by allocating their entire wealth to the benchmarked active fund (column (2)) instead, where they would earn positive excess CE returns relative to their 100%-ETF allocation. Thus, the optimality of benchmarking in our setup does not depend on the particular investment opportunity set for households that we assume.

6.3 Asymmetric Performance Fees and Imperfect Signals

In an accompanying Internet Appendix we generalize our setup of Section 3 to (i) allow the manager to have *partial* (instead of *complete*) information about the return fundamental $\tilde{\eta}$, and (ii) enlarge the managerial contract space to include an *asymmetric* benchmark-adjusted performance fee.

Specifically, we assume that, at $t = 0$ the manager has access to a noisy signal $\tilde{\theta} = \tilde{\eta} + \tilde{\epsilon}$ about the market price of risk $\tilde{\eta}$, where $\tilde{\epsilon}$ is an independent and normally distributed noise term: $\tilde{\epsilon} \sim \mathbf{N}(0, \sigma_\epsilon^2)$. As long as $\sigma_\epsilon < \infty$ the realization of the private signal θ provides the manager with a more accurate initial assessment, relative to the common prior $\mathbf{N}(\bar{m}, \bar{v}_0)$ she shares with households, of the realized market price of risk η. From then on, at any $t \in [0, T]$ the manager updates her estimate of η based on the information flow $\mathcal{F}_t^S \equiv \sigma \{S_u, 0 \leq u \leq t\}$ according to Bayes' rule.

We specify the fee rate f_T as a potentially nonlinear function tying the manager's compensation to performance according to $f_T = f(R_T^W, R_T^Y; \kappa_1, \kappa_2, \bar{\kappa}_3, \bar{\delta})$, with $R_T^W \equiv W_T/W_0$, $R_T^Y \equiv Y_T/Y_0$, $\kappa_1, \kappa_2, \bar{\kappa}_3, \bar{\delta} > 0$,

$$f(x, y; \kappa_1, \kappa_2, \kappa_3, \delta) = \kappa_1 x + \kappa_2 (x - y) + \bar{\kappa}_3 (x - \bar{\delta} y)^+, \tag{24}$$

and $x^+ \equiv \max(x, 0)$. Fee contract (24) is similar to the specification of Cuoco and Kaniel (2011) and consists of three components: a *proportional* fee $\kappa_1 R_T^W$, a fulcrum performance fee $\kappa_2 (R_T^W - R_T^Y)$, and an *asymmetric* benchmark-linked performance fee $\bar{\kappa}_3 (R_T^W - \bar{\delta} R_T^Y)^+$. The proportional and fulcrum fee rates κ_1 and κ_2 are identical to the fee rates k and $k\alpha$, respectively, of Section 3.2. The asymmetric performance fee rate $\bar{\kappa}_3$ rewards the fund's excess performance over the benchmark, scaled by a threshold $\bar{\delta}$. Relative to the class of fee contracts considered above, (24) encompasses also the type of incentives fees vastly used in the hedge fund industry ("two-and-twenty" fee schemes), as well as implicit asymmetric incentives (e.g., investors' flows as a function of past relative performance) between a fund manager and the fund owners (see the literature review in Section 1). Letting $\delta \equiv \bar{\delta} W_0 / Y_0$, total managerial compensation can be expressed as:

$$\begin{aligned} X_T \equiv f_T W_0 &= \kappa_1 W_T + \kappa_2 (W_T - \delta Y_T) + \bar{\kappa}_3 (W_T - \delta Y_T)^+ \\ &= f(W_T, Y_T; \kappa_1, \kappa_2, \bar{\kappa}_3, \delta). \end{aligned} \tag{25}$$

A management fee contract $\mathscr{C}$ now specifies the different fee rates κ_1, κ_2 and $\bar{\kappa}_3$, the benchmark's stock allocation ϕ^Y and the threshold δ.

In the Internet Appendix we provide a detailed exposition of the equivalent formulation of the contracting problem under this extension of our basic setup, solve for the manager's optimal investment policy in response to the nonlinear incentives (25), and redefine the ex-ante value of the manager's information when this consists (as we assume in this subsection) of a noisy signal. Within this framework, we analyze numerically the optimal enlarged contract $\{\kappa_1, \kappa_2, \bar{\kappa}_3, \phi^Y, 1\}$, where for ease of computation and comparability with other studies (e.g., Li and Tiwari, 2009) we fix the performance threshold δ to equal 1. To account for a commonly adopted investment constraint among institutional investor, as in Section 6.1 we constrain the benchmark choice to be an unlevered position in the assets.

Our findings under this setup further strengthen the argument for fulcrum fees in asset management. First, the optimal contract is *always linear*, as it includes a positive fulcrum fee for both relatively risk-tolerant and risk-averse investors but *no asymmetric performance fees.* Since linear benchmarking serves the purpose of risk-alignment better than nonlinear benchmarking and at lower cost, the margin of outperformance increases with the risk aversion misalignment $|\gamma_h - \gamma|$.[31] While dominated by fulcrum fees, we find that the optimal asymmetric performance fee schedule can dominate proportional fee-only contracts, especially among risk-averse investors.

Second, although the benefits of fulcrum fees increase with the precision of the manager's information, the optimality of these fees does not depend on the assumption of complete information for the manager. The optimal fulcrum fee contract allows risk-averse fund investors to realize almost the full value of the manager's information even when this information is imperfect. It also delivers positive excess CE returns from (sufficiently informed) active management to risk-tolerant investors, even though in their case the long-only constraint on the benchmark is binding.

6.4 Practical Implications

Our findings imply that active fund managers should optimally include a fulcrum component in their fee schedules, whose size will depend on both their information advantage (investment skills) and the risk profile of the investors to which the fund is catered. In particular, (i) as the information advantage shrinks the weight of the fulcrum component in the optimal fee schedule should increase; and (ii) for a sufficiently small information advantage, passive funds eventually dominate active funds for any misalignment in risk aversion between manager and investors. In a context of highly

[31] Under symmetric information between the manager and fund investors, Cuoco and Kaniel (2011) find instead that, when investors do not internalize the effect of their decisions on fund fees, asymmetric performance fees can dominate both proportional and fulcrum fees.

efficient financial markets and widespread information dissemination (i.e., shrinking information advantage by any party), these results help explain two recent trends in the asset management industry. The first is the adoption of fulcrum fees by several prominent asset management companies in the last few years that motivates this paper (see Section 1). The second is the increasing preference of investors for passive over active funds in the mutual fund space, where pure proportional fees are the norm, in the last decade (see, e.g., the Investment Company Institute 2019 Factbook).[32]

Accounting for leverage restrictions, our results prescribe relatively simple rules for the design of the benchmark specified in the fulcrum fee, namely an all-equity index (e.g., the S&P 500 Index) for investors with aggressive investment profile and a risk-less asset (e.g., a LIBOR benchmark) for more conservative investors. Besides aligning with the type of benchmarks commonly found in practice, this prescription is robust to the specific level of risk aversion of the investors, as variations in their risk profiles do not substantially change the optimal composition of the benchmark portfolio.

The implications of our findings go beyond the analysis of explicit fulcrum fees in the compensation of asset management companies. There is broad consensus that the annual flow of funds into mutual funds displays an increasing profile around fund performance relative to a benchmark.[33] Most mutual funds charge a fixed annual percentage of the funds under management, so the total revenue of the fund resembles the profile of the flow of money. Clearly, this profile fits the type of benchmark-adjusted performance fees that we address in this paper, and strengthens the importance of analyzing the relative merits of benchmark-adjusted contracts.

This line of research is also relevant to the ongoing debate on acceptable incentives for fund managers. The use of performance fees by mutual funds has been allowed only recently in many European countries (e.g., in 2004 in the U.K.) and has attracted scrutiny ever since.[34] In the U.S., prior to 1971 mutual funds could use either fulcrum or "bonus" (asymmetric) performance fees. In 1971, the SEC ruled that if investment companies use performance-based compensation contracts, the contracts have to be symmetric—the use of bonus performance contracts was prohibited. Our findings suggest that the prohibition does not entail welfare loss necessarily, and most likely might be welfare-improving, for delegating investors.

Our results do not support the use of option-like fees for the purposes of either aligning the

[32] Available at the Investment Company Institute's website: https://www.ici.org/

[33] See footnote 2 for references.

[34] See, e.g., the renewed call among European authorities for a performance fee ban: http://www.ft.com/cms/s/0/630fefc4-2fd9-11e2-ae7d-00144feabdc0.html#axzz4IhH7AVDY.

optimal portfolios of fund managers and investors or optimal risk-sharing. However, we cannot rule out that they still help resolve moral hazard problems between the two parties, from which our model abstracts away. When investors feature either CARA or log preferences, Li and Tiwari (2009) and Dybvig et al. (2010) show that the inclusion of nonlinear incentive components linked to the performance of a benchmark—along with, potentially, restrictions on the set of allowable strategies—can induce managers to exert the optimal level of effort. The question of whether nonlinear fees can help resolve moral hazard problems also under CRRA preferences remains open.

7 Conclusions

The recent adoption of symmetric benchmark-adjusted fees in active fund management has found limited support in the portfolio delegation literature. In particular, prior research has demonstrated the irrelevance of linear benchmarking, under a particular type of preferences, to align the optimal portfolios of fund managers and their investors or to improve risk sharing between the two parties. In this paper we show that these results are overturned in situations in which an active fund manager has better information or investment skills than her fund investors, a misalignment in risk aversion between the two parties exists, and passive fund alternatives are available to the investors.

Under these circumstances, linear benchmarking offers investors the potential for alignment in risk tolerances at the cost of distorting the manager's informed investment policy towards an inefficient portfolio. We show that this tradeoff resolves in favor of the inclusion of a fulcrum performance component in the optimal linear fee schedule of fund companies. From investors' perspective, a simple fee proportional to the value of total final wealth leads managers to take either excessive or insufficient risk in their portfolio. Through the inclusion of fulcrum fees, investors can affect the manager's portfolio in a predictable way for any realization of her private information without the need to compensate her for undesired exposure to risk. We show numerically that, by choosing the optimal benchmark-adjusted performance fees and benchmark composition, investors can derive from active management nearly as high utility as if they could trade on the same private information as the manager. In fact, under certain conditions linear benchmarking is not only optimal but also necessary for active management to be a viable alternative to passive investing. We further show that asymmetric benchmark-adjusted performance fees cannot improve over their symmetric counterparts and characterize how the optimal contract depends on the risk-aversion misalignment and the precision of the manager's information.

Our results do not depend on the assumption of a single risky asset in the financial markets. In fact, assuming multiple assets in our setup would allow the optimal contract design to potentially differ depending on whether the manager's superior information relates to a particular set of securities (stock-picking ability) or to the direction of the overall market (market-timing ability). Our results in sections 4.1 and 4 are easily generalizable to the multi-asset case. We anticipate the numerical optimization of Section 5.3 to be more challenging under multiple risky assets and multiple signals for the informed manager. However, we expect a multi-asset analysis to potentially shed light on contract design problems of high practical relevance.

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Appendix

A Proofs

We start by stating the following auxiliary lemma:

Lemma A1. *Let $z \sim \mathbf{N}(0, \sigma_z^2)$, and let $\rho, c, \bar{z} \in \mathbb{R}$. We have:*

(i) $E\left[e^{\rho z} \mathbb{1}_{\{z \leq \bar{z}\}}\right] = e^{\frac{\rho^2 \sigma_z^2}{2}} \mathcal{N}\left(\frac{\bar{z} - \rho\sigma_z^2}{\sigma_z}\right)$,

(ii) $E\left[e^{-\rho(z-c)^2} \mathbb{1}_{\{z \leq \bar{z}\}}\right] = \frac{e^{-\frac{\rho c^2}{1+2\rho\sigma_z^2}}}{\sqrt{1+2\rho\sigma_z^2}} \mathcal{N}\left(\frac{\bar{z} - \frac{2\rho\sigma_z^2}{1+2\rho\sigma_z^2} c}{\sigma_z / \sqrt{1+2\rho\sigma_z^2}}\right)$,

where $\mathcal{N}(.)$ is the standard normal cumulative distribution function.

Proof. Follows from direct integration against the normal density, using the change of variables $\tilde{z} = \frac{z - \rho\sigma_z^2}{\sigma_z}$ for part (i) and $\check{z} = \frac{z - \frac{2\rho\sigma_z^2}{1+2\rho\sigma_z^2}}{\sigma_z/\sqrt{1+2\rho\sigma_z^2}}$, $\check{z}_l = \frac{\bar{z} - \frac{2\rho\sigma_z^2}{1+2\rho\sigma_z^2}}{\sigma_z/\sqrt{1+2\rho\sigma_z^2}}$ for part (ii). □

Proof of Proposition 1. The dynamic self-financing condition (8) can be re-expressed as a static budget constraint (see e.g. Karatzas and Shreve, 2001), so for a given realization η of the manager's signal and contract $\mathscr{C} \equiv (k, \alpha, \phi^Y)$ problem (7)-(8) becomes:

$$\max_{W_T} E\left[u\left(kW_T + k\alpha(W_T - Y_T)\right) \mid \tilde{\eta} = \eta\right], \tag{A1}$$

$$s.t. \qquad E_0\left[\pi_T W_T\right] = w, \tag{A2}$$

where for notational simplicity we omit the superscripts $(\eta; \mathscr{C})$ that associate the processes with the realized value of $\tilde{\eta}$ and with the contract parameters $\mathscr{C}$. In the remainder of this proof we also adopt the convention that conditional expectations as of time $t \in [0, T]$ (e.g., $E_0[\cdot]$) are taken after the realization η (i.e., they are conditional on $\tilde{\eta} = \eta$).

The optimal terminal wealth $\hat{W}_T$ satisfies the first-order condition: $\frac{\partial u(\hat{W}_T)}{\partial W_T} = \lambda\pi_T$ for a Lagrange multiplier λ of the budget constraint (A2) that depends on the realized η. The first-order condition for the manager's portfolio problem leads to:

$$\hat{W}_T = (k(1+\alpha))^{\frac{1}{\gamma}-1}(\lambda\pi_T)^{-\frac{1}{\gamma}} + \frac{\alpha}{1+\alpha} Y_T. \tag{A3}$$

Let Q be the risk-neutral probability and $B_t^Q = B_t + \eta t$ denote the risk-neutral Brownian motion. The manager's budget constrain can be rewritten as

$$e^{-rT} E_0^Q[\hat{W}_T] = w, \tag{A4}$$

where $E^Q[\cdot]$ denotes the expectation under Q. Given that both π_T and Y_T are log-normally distributed with constant mean and variance, the expectation in (A4) can be easily computed to solve for λ as:

$$\lambda = k^{1-\gamma} w^{-\gamma} (1+\alpha) e^{-(\gamma-1)\left(r+\frac{\eta^2}{2\gamma}\right)T}. \tag{A5}$$

Plugging λ back into (A3) we get:

$$\hat{W}_T = \frac{w}{1+\alpha} e^{\left(1-\frac{1}{\gamma}\right)\left(r+\frac{\eta^2}{2\gamma}\right)T} (\pi_T^\eta)^{-\frac{1}{\gamma}} + \frac{\alpha}{1+\alpha} Y_T. \tag{A6}$$

Replacing the optimal terminal wealth (A6) into the manager's compensation (5) and rearranging gives (11).

Since $e^{-rt}W_t$ is a martingale process under Q, we obtain the optimal interim wealth $\hat{W}_t$ as:

$$\begin{aligned}\hat{W}_t =& e^{-r(T-t)} E_t^Q[\hat{W}_T] \\ =& e^{-r(T-t)} \frac{w}{1+\alpha} e^{\left(1-\frac{1}{\gamma}\right)\left(r+\frac{\eta^2}{2\gamma}\right)T} (\pi_t)^{-\frac{1}{\gamma}} E_t^Q\left[\left(\frac{\pi_T}{\pi_t}\right)^{-\frac{1}{\gamma}}\right] + \\ & + \frac{\alpha}{1+\alpha} e^{-r(T-t)} E_t^Q[Y_T].\end{aligned} \tag{A7}$$

Since the discounted benchmark process is a martingale under Q, $e^{-r(T-t)}E_t^Q[Y_T] = Y_t$. Conditional on time-$t$ information, the process π_T/π_t is log-normally distributed with constant mean and variance, so the expectation in the first term of (A7) can be easily computed to yield (10).

To derive the investment policy (9) replicating the optimal portfolio value (10), note that the state-price deflator π_t is a function of t and B_t^Q only. Therefore, the manager's optimal portfolio value can be rewritten as $\hat{W}_t = f(t, B_t^Q)$, for $f \in C^{1,2}$. Applying Itô's Lemma the diffusion term of $d\hat{W}_t$ is:

$$\frac{\eta}{\gamma}\left(\hat{W}_t - \frac{\alpha}{1+\alpha} Y_t\right) + \frac{\alpha}{1+\alpha}\phi^Y \sigma Y_t. \tag{A8}$$

Equating this expression with the diffusion term of dW_t under Q ($\hat{W}_t\hat{\phi}_t\sigma$) leads to the optimal portfolio (9), for $\hat{Z}_t$ as defined in the Proposition. □

Proof of Lemma 1. If $\phi_t = \phi$ for all $t \in [0,T]$ in (8), households' wealth follows a geometric Brownian motion with end-of-period value:

$$W_T^\eta = w \exp\{(r + \phi\sigma\eta - 0.5(\phi\sigma)^2)T + \phi\sigma B_T\}. \tag{A9}$$

Households' ex-ante utility from investing the fixed portfolio ϕ in the stock over the investment period is:

$$E^{\tilde{\eta},B_T}\left[\frac{\left((1-k_m)W_T^{\tilde{\eta}}\right)^{1-\gamma_h}}{1-\gamma_h}\right] = E^{\tilde{\eta}}\left[E_0^{B_T}\left[\frac{\left((1-k_m)W_T^{\tilde{\eta}}\right)^{1-\gamma_h}}{1-\gamma_h}\middle|\tilde{\eta}=\eta\right]\right], \tag{A10}$$

where for clarity we add a superscript in each expectation operator that indicates the variable whose distribution is used to take expectations (e.g., $E^{\tilde{\eta},B_T}$ is the expectation with respect to the joint distribution of $(\tilde{\eta}, B_T)$).

For a given realization η of $\tilde{\eta}$, W_T^{η} is log-normally distributed with fixed mean and variance. This allows the inner expectation in (A10) to be computed as:

$$E_0^{B_T}\left[\frac{\left((1-k_m)W_T^{\tilde{\eta}}\right)^{1-\gamma_h}}{1-\gamma_h}\middle|\tilde{\eta}=\eta\right] = \frac{((1-k_m)w)^{1-\gamma_h}}{1-\gamma_h}\exp\{-(\gamma_h-1)(r+\phi\sigma(\eta-0.5\gamma_h\phi\sigma))T\}. \tag{A11}$$

The RHS of (A11) is log-normally distributed with constant mean and variance, so the expectation in (A11) can be computed as:

$$\begin{aligned} E^{\tilde{\eta},B_T}\left[\frac{\left((1-k_m)W_T^{\tilde{\eta}}\right)^{1-\gamma_h}}{1-\gamma_h}\right] =& \frac{((1-k_m)w)^{1-\gamma_h}}{1-\gamma_h}\times\exp\{-(\gamma_h-1)(r+\phi\sigma\bar{m} \\ &-0.5(\phi\sigma)^2(\gamma_h+(\gamma_h-1)\bar{v}_0T))T\}. \end{aligned} \tag{A12}$$

Households' choice of an ETF ϕ^P can be seen as the choice of the portfolio ϕ^P that maximizes their ex-ante utility (A12), i.e.:

$$\max_{\phi}\frac{((1-k_m)w)^{1-\gamma_h}}{1-\gamma_h}\exp\{-(\gamma_h-1)(r+\phi\sigma\bar{m}-0.5(\phi\sigma)^2(\gamma_h+(\gamma_h-1)\bar{v}_0T))T\}. \tag{A13}$$

The objective function is globally concave. Computing the first-order condition for (A13) and solving for ϕ yields the optimal ETF (12). Replacing (12) back into households' ex-ante utility (A11) gives households' reservation utility (13).

Similarly, the manager's ex-ante utility from the fixed portfolio ϕ is given by:

$$E^{\tilde{\eta},B_T}\left[\frac{\left(k_mW_T^{\tilde{\eta}}\right)^{1-\gamma}}{1-\gamma}\right] = \frac{(k_mw)^{1-\gamma}}{1-\gamma}\exp\{-(\gamma-1)(r+\phi\sigma\bar{m}-0.5(\phi\sigma)^2(\gamma+(\gamma-1)\bar{v}_0T))T\}. \tag{A14}$$

Plugging households' chosen passive portfolio (12) into (A14) gives, after some algebra, the manager's reservation utility (14).

□

Proof of Proposition 2. From Eq. (17), the after-fee end-of-period wealth of households is:

$$W^{\eta}_{CI,T} = (1 - k_{CI}(\eta))we^{-rT+\left(1-\frac{1}{2\gamma_h}\right)\frac{\eta^2}{2\gamma_h}T+\frac{\eta}{\gamma_h}B_T}.$$

Given η, $W^{\eta}_{CI,T}$ is log-normally distributed with constant mean and variance, so:

$$E_0\left[\frac{(W^{\tilde{\eta}}_{CI,T})^{1-\gamma_h}}{1-\gamma_h}\middle|\tilde{\eta}=\eta\right] = \frac{((1-k_{CI}(\eta))w)^{1-\gamma_h}}{1-\gamma_h}\exp\left\{-(\gamma_h-1)\Big(r+\frac{\eta^2}{2\gamma_h}\Big)T\right\}$$

Households' expected utility is then:

$$\begin{aligned}U_{h,CI}(w) \equiv E\left[\frac{(W^{\tilde{\eta}}_{CI,T})^{1-\gamma_h}}{1-\gamma_h}\right] =& E\left[E_0\left[\frac{(W^{\tilde{\eta}}_{CI,T})^{1-\gamma_h}}{1-\gamma_h}\middle|\tilde{\eta}=\eta\right]\right]\\ =& \frac{w^{1-\gamma_h}}{1-\gamma_h}\int_{-\infty}^{+\infty}(1-k_{CI}(x))^{1-\gamma_h}e^{-(\gamma_h-1)\left(r+\frac{x^2}{2\gamma_h}\right)T}\mathbf{n}_{\eta}(x)dx,\end{aligned}$$

where $\mathbf{n}_{\eta}(\cdot)$ is the probability density function of $\tilde{\eta}$.

Following the same reasoning, the manager's end-of-period wealth is:

$$k_{CI}(\eta)\hat{W}^{\eta;0,\phi^Y} = k_{CI}(\eta)we^{-rT+\left(1-\frac{1}{2\gamma}\right)\frac{\eta^2}{2\gamma}T+\frac{\eta}{\gamma}B_T},$$

so

$$E_0\left[\frac{(k_{CI}(\eta)\hat{W}^{\eta;0,\phi^Y})^{1-\gamma}}{1-\gamma}\middle|\tilde{\eta}=\eta\right] = \frac{(k_{CI}(\eta)w)^{1-\gamma}}{1-\gamma}\exp\left\{-(\gamma-1)\Big(r+\frac{\eta^2}{2\gamma}\Big)T\right\}. \tag{A15}$$

From Eqs. (12) and (A11), the manager's utility under passive management in state η is:

$$\frac{(k_m w)^{1-\gamma}}{1-\gamma}\exp\left\{-(\gamma-1)\left(r+\frac{\bar{m}}{\gamma_h+(\gamma_h-1)\bar{v}_0T}\left(\eta-\frac{\gamma\bar{m}}{2(\gamma_h+(\gamma_h-1)\bar{v}_0T)}\right)\right)T\right\} \tag{A16}$$

Equating (A15) to (A16) and solving for $k_{CI}(\eta)$ gives (18). To obtain (20), note that the equality between (A15) and (A16) holds state by state. Therefore, the manager's ex-ante utility in the CI case has to equal his reservation utility (14). □

Proof of Lemma 2. From (11), the only contract parameter that affects the manager's optimal compensation $\hat{X}^{\eta;k}_T$ is the base management fee k. Let U^k be the manager's expected utility for a given fee k, i.e:

$$U^k \equiv E\left[\frac{(\hat{X}^{\tilde{\eta};k}_T)^{1-\gamma}}{1-\gamma}\right]. \tag{A17}$$

Comparing (11) and (17) for $t = T$, we see that $\hat{X}_T^{\tilde{\eta};k} = kW_{CI,T}^{\tilde{\eta}}$. It follows that

$$U^k = h(kw, \gamma), \tag{A18}$$

for $h(\cdot)$ as defined in Proposition 2. The optimal management fee (21) then results from equating U^k and $\bar{U}$ and solving for k. □

Proof of Lemma 3. That pure proportional fees implement households' CI portfolio (16) when $\gamma_h = \gamma$ follows immediately from setting $\alpha = 0$ in (9) and $\gamma_h = \gamma$ in (16), and comparing the two. For $\gamma_h \neq \gamma$, setting $\alpha = 0$ leads the manager to choose a portfolio

$$\hat{\phi}_t^{\eta;0,\phi^Y} = \frac{\eta}{\gamma\sigma} \neq \frac{\eta}{\gamma_h\sigma} = \phi_{CI}^{\eta}. \tag{A19}$$

For $\alpha \neq 0$, perfect alignment of the manager's and households CI portfolio would require choosing a fulcrum fee α and a benchmark composition ϕ^Y such that:

$$\frac{\alpha}{1+\alpha}\frac{1}{\hat{Z}_t^{\eta;\alpha,\phi^Y}}\left(\phi^Y - \frac{\eta}{\gamma\sigma}\right) = (\gamma - \gamma_h)\frac{\eta}{\gamma_h\gamma\sigma}, \tag{A20}$$

i.e.,

$$\phi^Y = \left(\frac{1+\alpha}{\alpha}\hat{Z}_t^{\eta;\alpha,\phi^Y} - \gamma_h\right)\frac{\eta}{\gamma_h\gamma\sigma}. \tag{A21}$$

Condition (A21) is actually a system (more precisely, a continuum) of equations, one per $(\tilde{\eta}, B_t)$-state, with only two unknowns (α, ϕ^Y). Thus, no contract $\mathscr{C}$ can perfectly align the manager's portfolio with households' CI portfolio. □

Proof of Theorem 1. Given the optimal proportional fee $\hat{k}$ characterized by Lemma 2, the optimization program (6) reduces to finding an interior solution $\{\hat{\alpha}, \hat{\phi}^Y\}$ to the following (unconstrained) maximization problem:

$$\max_{\{\alpha,\phi^Y\}} E\left[\frac{\left(\hat{W}_T^{\tilde{\eta};\alpha,\phi^Y} - \hat{X}_T^{\tilde{\eta};\hat{k}}\right)^{1-\gamma_h}}{1-\gamma_h}\right], \tag{A22}$$

and verifying that the solution satisfies the participation constraint of households (HH's PC). In (A22), $\hat{W}_T^{\tilde{\eta};\alpha,\phi^Y}$ and $\hat{X}_T^{\tilde{\eta};\hat{k}}$ are as given by Eqs. (10) and (11), and expectations are with respect to the joint distribution of $(\tilde{\eta}, B_T)$.

From (10), specialized to $t = T$, and (11):

$$\hat{W}_T^{\eta;\alpha,\phi^Y} - \hat{X}_T^{\eta;\hat{k}} = \frac{1-\hat{k}(1+\alpha)}{1+\alpha} we^{\left(1-\frac{1}{\gamma}\right)\left(r+\frac{\eta^2}{2\gamma}\right)T}(\pi_T^{\eta})^{-\frac{1}{\gamma}} + \frac{\alpha}{1+\alpha}Y_T^{\eta;\phi^Y}. \tag{A23}$$

For a given (α, ϕ^Y), define households' indirect utility function V as

$$V(\alpha, \phi^Y) \equiv E\left[\frac{\left(\hat{W}_T^{\tilde{\eta};\alpha,\phi^Y} - \hat{X}_T^{\tilde{\eta};\hat{k}}\right)^{1-\gamma_h}}{1-\gamma_h}\right]. \tag{A24}$$

The first-order conditions for an interior solution $(\hat{\alpha}, \hat{\phi}^Y)$ to the maximization problem (A22) are:

$$\frac{\partial V(\hat{\alpha}, \hat{\phi}^Y)}{\partial \alpha} = \frac{1}{(1+\hat{\alpha})^2} E\left[\left(\hat{W}_T^{\tilde{\eta};\hat{\alpha},\hat{\phi}^Y} - \hat{X}_T^{\tilde{\eta};\hat{k}}\right)^{-\gamma_h}\left(Y_T^{\tilde{\eta};\hat{\phi}^Y} - we^{\left(1-\frac{1}{\gamma}\right)\left(r+\frac{\tilde{\eta}^2}{2\gamma}\right)T}(\pi_T^{\tilde{\eta}})^{-\frac{1}{\gamma}}\right)\right] = 0 \tag{A25}$$

$$\frac{\partial V(\hat{\alpha}, \hat{\phi}^Y)}{\partial \phi^Y} = \frac{1}{1+\hat{\alpha}} E\left[\left(\hat{W}_T^{\tilde{\eta};\hat{\alpha},\hat{\phi}^Y} - \hat{X}_T^{\tilde{\eta};\hat{k}}\right)^{-\gamma_h} \hat{\alpha} Y_T^{\tilde{\eta};\hat{\phi}^Y}\left(\sigma(\tilde{\eta} - \hat{\phi}^Y\sigma)T + \sigma B_T\right)\right] = 0 \tag{A26}$$

Clearly, $\alpha = 0$ satisfies (A26) for any ϕ^Y. To also satisfy (A25) as well, it has to be that:

$$\begin{aligned}
\frac{\partial V(0, \hat{\phi}^Y)}{\partial \alpha} =& E\left[\left(\hat{W}_T^{\tilde{\eta};0,\hat{\phi}^Y} - \hat{X}_T^{\tilde{\eta};\hat{k}}\right)^{-\gamma_h}\left(Y_T^{\tilde{\eta};\hat{\phi}^Y} - we^{\left(1-\frac{1}{\gamma}\right)\left(r+\frac{\tilde{\eta}^2}{2\gamma}\right)T}(\pi_T^{\tilde{\eta}})^{-\frac{1}{\gamma}}\right)\right] \\
=& E\left[\left((1-\hat{k})we^{\left(1-\frac{1}{\gamma}\right)\left(r+\frac{\tilde{\eta}^2}{2\gamma}\right)T}(\pi_T^{\tilde{\eta}})^{-\frac{1}{\gamma}}\right)^{-\gamma_h}\right. \\
&\left.\times\left(Y_T^{\tilde{\eta};\hat{\phi}^Y} - we^{\left(1-\frac{1}{\gamma}\right)\left(r+\frac{\tilde{\eta}^2}{2\gamma}\right)T}(\pi_T^{\tilde{\eta}})^{-\frac{1}{\gamma}}\right)\right] \\
=& 0.
\end{aligned} \tag{A27}$$

According to (A27), $\alpha = 0$ satisfies the first-order conditions if and only if

$$E\left[\left(we^{\left(1-\frac{1}{\gamma}\right)\left(r+\frac{\tilde{\eta}^2}{2\gamma}\right)T}(\pi_T^{\tilde{\eta}})^{-\frac{1}{\gamma}}\right)^{-\gamma_h} Y_T^{\tilde{\eta};\hat{\phi}^Y}\right] = E\left[\left(we^{\left(1-\frac{1}{\gamma}\right)\left(r+\frac{\tilde{\eta}^2}{2\gamma}\right)T}(\pi_T^{\tilde{\eta}})^{-\frac{1}{\gamma}}\right)^{1-\gamma_h}\right] \tag{A28}$$

Using iterated expectations, both sides of (A28) can be computed explicitly by first taking expectations relative to the distribution of B_T for a given η, and then taking expectations with respect to the distribution of $\tilde{\eta}$ (i.e., following the approach in (A10) using the result in Lemma A1). This procedure yields:

$$E\left[\left(we^{\left(1-\frac{1}{\gamma}\right)\left(r+\frac{\tilde{\eta}^2}{2\gamma}\right)T}(\pi_T^{\tilde{\eta}})^{-\frac{1}{\gamma}}\right)^{-\gamma_h} Y_T^{\tilde{\eta};\hat{\phi}^Y}\right] = \gamma\frac{\exp\left\{-\frac{(\gamma-\gamma_h)^2\bar{v}_0 T\frac{(\phi^Y\sigma)^2}{2} + (\gamma-\gamma_h)\gamma\phi^Y\sigma\bar{m} - (2\gamma-\gamma_h-1)\gamma_h\frac{\bar{m}^2}{2}}{\gamma^2+(2\gamma-\gamma_h-1)\gamma_h\bar{v}_0 T}T\right\}}{\sqrt{\gamma^2+(2\gamma-\gamma_h-1)\gamma_h\bar{v}_0 T}}, \tag{A29}$$

and

$$E\left[\left(we^{\left(1-\frac{1}{\gamma}\right)\left(r+\frac{\tilde{\eta}^2}{2\gamma}\right)T}(\pi_T^{\tilde{\eta}})^{-\frac{1}{\gamma}}\right)^{1-\gamma_h}\right] =\gamma\frac{\exp\left\{-\frac{(\gamma_h-1)(2\gamma-\gamma_h)}{\gamma^2+(\gamma_h-\gamma)(2\gamma-\gamma_h)\bar{v}_0 T}\frac{\bar{m}^2}{2}T\right\}}{\sqrt{\gamma^2+(\gamma_h-\gamma)(2\gamma-\gamma_h)\bar{v}_0 T}}. \tag{A30}$$

Note that, for all ϕ^Y:

$$\begin{aligned} V(0,\phi^Y) =& E\left[\frac{\left(\hat{W}_T^{\tilde{\eta};0,\phi^Y}-\hat{X}_T^{\tilde{\eta};\hat{k}}\right)^{1-\gamma_h}}{1-\gamma_h}\right] \\ =& \frac{((1-\hat{k})w)^{1-\gamma_h}}{1-\gamma_h}E\left[\left(we^{\left(1-\frac{1}{\gamma}\right)\left(r+\frac{\tilde{\eta}^2}{2\gamma}\right)T}(\pi_T^{\tilde{\eta}})^{-\frac{1}{\gamma}}\right)^{1-\gamma_h}\right]. \end{aligned} \tag{A31}$$

Equation (A31) shows that $V(0,\phi^Y)$ does not depend on ϕ^Y, so changing ϕ^Y (given that any value of ϕ^Y satisfies (A26) when $\alpha=0$) cannot increase or decrease the expected utility of households. In turn, this implies that if $\alpha=0$ solves problem (A22) then (A28) has to be satisfied for all values of ϕ^Y.[35] Comparing (A29) and (A30), this is the case if and only if $\gamma=\gamma_h$. Thus, $\alpha=0$ is optimal if and only if $\gamma=\gamma_h$. For $\gamma\neq\gamma_h$, (A28) is not satisfied for some values of ϕ^Y (e.g., for $\phi^Y=0$). By continuity of $V(\cdot,\cdot)$ in α and ϕ^Y at $(0,\phi^Y)$, there exists some $\bar{\alpha}\neq 0$ and $\bar{\phi}^Y$ such that $V(\bar{\alpha},\bar{\phi}^Y)>V(0,\phi^Y)$ for all ϕ^Y. Moreover, If a solution $(\hat{\alpha},\hat{\phi}^Y)$ exists to problem (A22), then it has $\hat{\alpha}\neq 0$. □

B Irrelevance of fulcrum fees under CARA preferences

We state and proof the following:

Proposition B1. *When households and the manager have constant absolute risk aversion (CARA) preferences with coefficients γ_h and γ respectively, a pure proportional fee contract $\{(\hat{k}=\gamma_h/\gamma,0,\phi^Y):\phi^Y\in\mathbb{R}\}$ achieves, for each realization η of $\tilde{\eta}$, perfect alignment of the actively managed portfolio with the complete-information portfolio of households. In contrast, non-zero fulcrum fees imply a misalignment between the two portfolios.*

Proof. The utility function of households is:

$$u_h(w)=-\frac{1}{\gamma_h}e^{-\gamma_h w}, \tag{B1}$$

[35] To see this, suppose that (A25) is satisfied for $\hat{\phi}^Y$ but not for some $\bar{\phi}^Y\neq\hat{\phi}^Y$. Without loss of generality, assume $\partial V(0,\bar{\phi}^Y)/\partial\alpha>0$. Since $V(0,\bar{\phi}^Y)=V(0,\hat{\phi}^Y)$, $\partial V(0,\bar{\phi}^Y)/\partial\alpha>0$ implies that there exists some $\epsilon>0$ such that households can improve on their indirect utility relative to $(\alpha,\phi^Y)=(0,\hat{\phi}^Y)$ by choosing $(\alpha,\phi^Y)=(\epsilon,\bar{\phi}^Y)$. Thus, $(\alpha,\phi^Y)=(0,\hat{\phi}^Y)$ cannot maximize (A22). By a similar argument it cannot minimize (A22) either, which means that $(\alpha,\phi^Y)=(0,\hat{\phi}^Y)$ corresponds to a saddle point of $V(\cdot)$.

whereas the utility function of the manager is:

$$u(w) = -\frac{1}{\gamma}e^{-\gamma w}. \tag{B2}$$

Following the approach in the proof of Proposition 1, for each realization η of $\tilde{\eta}$ the manager solves:

$$\max_{W_T} E\left[e^{-\gamma(kW_T + k\alpha(W_T - Y_T))} \middle| \tilde{\eta} = \eta \right], \tag{B3}$$

$$s.t. \qquad e^{-rT}E_0^Q\left[W_T\right] = w, \tag{B4}$$

where for notational simplicity we omit the superscripts $\eta; \mathscr{C}$ that associate the processes with the realized value of $\tilde{\eta}$ and with the contract parameters $\mathscr{C}$. In the remainder of this proof we also adopt the convention that conditional expectations as of time $t \in [0, T]$ (e.g., $E_0[\cdot]$) are taken after the realization η (i.e., they are conditional on $\tilde{\eta} = \eta$).

The manager's optimal terminal wealth $\hat{W}_T$ satisfies the first-order condition: $\frac{\partial u(\hat{W}_T)}{\partial W_T} = \lambda \pi_T$ for a Lagrange multiplier λ of the budget constraint (B4). The first-order condition for the manager's portfolio problem (B3) leads to:

$$\hat{W}_T = \frac{1}{\gamma k(1+\alpha)}(\ln(1+\alpha) - \ln(\lambda \pi_T)) + \frac{\alpha}{1+\alpha}Y_T. \tag{B5}$$

Since $e^{-rt}W_t$ is a martingale process under Q, we obtain the optimal interim wealth $\hat{W}_t$ as:

$$\begin{aligned}\hat{W}_t =& e^{-r(T-t)}E_t^Q[\hat{W}_T] \\ =& \frac{e^{-r(T-t)}}{\gamma k(1+\alpha)}\big(\ln((1+\alpha)/\lambda) + (r - \frac{1}{2}\eta^2)T + \eta B_t^Q\big) + \frac{\alpha}{1+\alpha}Y_t.\end{aligned} \tag{B6}$$

To derive the manager's optimal investment policy $\hat{\varphi}_t^\eta$, note that the manager's optimal portfolio value can be rewritten as $\hat{W}_t = f(t, B_t^Q)$, for $f \in C^{1,2}$. Applying Itô's Lemma the diffusion term of $d\hat{W}_t$ is:

$$\frac{e^{-r(T-t)}}{k(1+\alpha)}\frac{\eta}{\gamma} + \phi^Y \sigma \frac{\alpha}{1+\alpha}Y_t. \tag{B7}$$

Equating this expression with the diffusion term of dW_t under Q ($\hat{\varphi}_t^\eta \sigma$) we obtain the manager's optimal amount $\hat{\varphi}^\eta$ in the risky asset portfolio:

$$\hat{\varphi}_t^\eta = \frac{e^{-r(T-t)}}{k(1+\alpha)}\frac{\eta}{\gamma\sigma} + \frac{\alpha\phi^Y Y_t}{1+\alpha}. \tag{B8}$$

In the CI case of Section 5.1, households solve:

$$\max_{W_T} E\left[e^{-\gamma_h W_T} \middle| \tilde{\eta} = \eta \right], \tag{B9}$$

$$s.t. \qquad e^{-rT} E_0^Q \left[W_T \right] = w. \tag{B10}$$

Note that problem (B9)-(B10) is the same as the manager's problem (B3)-(B4) for $k = \gamma_h / \gamma$ and $\alpha = 0$. Households' CI portfolio can then be computed from (B8) as:

$$\varphi^{\eta}_{CI,t} = e^{-r(T-t)} \frac{\eta}{\gamma_h \sigma}. \tag{B11}$$

This implies that households can achieve perfect portfolio alignment by setting the contract parameters equal to $\hat{k} = \gamma_h / \gamma$ and $\hat{\alpha} = 0$. If $\alpha \neq 0$, then no value of the fulcrum fee α and the benchmark allocation in the stock ϕ^Y can equate Eqs. (B8) and (B11) across all η-states. Hence, non-zero fulcrum fees imply a misalignment between the managed portfolio and the CI portfolio of households. □