Dark Knights: The Rise in Firm Intervention by CDS Investors

András Danis Andrea Gamba*

August 28, 2020

ABSTRACT

We document an increase in cases where credit default swap (CDS) investors intervene in the restructuring of a distressed firm. In our theoretical analysis, we show that—contrary to popular belief—intervention by CDS investors is not necessarily reducing firm value. While the equilibrium CDS spread seems excessive for the protection buyer, that cost is offset by the reduced probability of liquidation. Ex ante borrowing costs go down, and investment and firm value both increase. Under certain assumptions, investment reaches first-best. Our results suggest that the empty creditor problem could be at least partially solved by CDS investor intervention.

Keywords: credit default swaps, CDS, empty creditor, bankruptcy, hedge fund activism
JEL classification: G33, G34

*Danis is at the Scheller College of Business, Georgia Institute of Technology, and can be reached at andras.danis@scheller.gatech.edu. Gamba is at Warwick Business School, University of Warwick, and can be reached at Andrea.Gamba@wbs.ac.uk. We thank conference participants at the EFA Meeting 2019, Canadian Derivatives Institute (CDI) 2019, WFA 2020, IDC 2020, and the MFA 2020 for their feedback. We also thank Patrick Augustin, Philip Bond, Sudheer Chava, Jonathan Cohn, Matthias Efing, Vincent Glode, Martin Oehmke, Heather Tookes, Zhe Wang, Daniel Weagley, and Adam Zawadowski as well as seminar participants at Georgia Tech, the Office of the Comptroller of the Currency, University of Maryland, Tilburg University, Erasmus University, ESCP Paris, University of Luxembourg, ESSEC, EDHEC, and Copenhagen Business School for their comments. We thank Yue Hu for excellent research assistance.

Norske Skog, a company based in Norway and one of the world's leading paper producers, was suffering from years of declining sales leading up to 2016. In March of that year, the company announced that it had raised both debt and equity financing from two hedge funds, Blackstone's GSO Capital Partners and Cyrus Capital Partners. The interesting part of the deal was that these hedge funds had previously sold Credit Default Swap (CDS) protection on the firm's debt. Because of their CDS positions, they had an economic incentive to support the distressed firm and to avoid bankruptcy. The deal was controversial because some investors had speculated on the failure of Norske Skog. Among these investors was BlueCrest Capital Management, who had purchased CDSs on the underlying firm.[1]

This is not the only case where CDS investors interfered with a corporate restructuring. Table 1 summarizes several cases that have occurred in recent years. The underlying firms include Codere, Caesars Entertainment, Forest Oil, RadioShack, Matalan, Hovnanian, McClatchy, Supervalu, Windstream, Neiman Marcus, and Thomas Cook. In some of these cases, the protection seller actively intervenes in a distressed firm's restructuring to avoid bankruptcy, while in others it is the protection buyer who tries to trigger a credit event to receive a payoff from a CDS contract. What the cases have in common is that a CDS investor with a large position tries to actively influence corporate restructuring. Also, all these cases have occurred recently, between 2013 and 2019.

There is anecdotal evidence, based on an article in the Wall Street Journal, that these cases are just examples of a broader shift in how hedge funds participate in the CDS market.[2] The Wall Street Journal argues that activist investors are increasingly using long or short positions in the CDS market to affect corporate deals. Starting in recent years, the authors argue, CDS trading activity has shifted to smaller firms, where it is easier to buy a large position in CDSs and use it together with a controlling position in stocks or bonds to influence firms.

[1] See Financial Times, "Hedge funds spar over Norske Skog debt restructuring", February 25, 2016.

[2] The Wall Street Journal, "Credit-Default Swaps Get Activist New Look", December 23, 2014.

Following the recent cases of CDS intervention, several commentators have called for a reform of the CDS market. Some even suggest to shut down the CDS market completely.[3] Regulators are becoming concerned, too. The Commodity Futures Trading Commission (CFTC), in an unprecedented move, issued a public warning in 2018 that certain activities of CDS investors could be viewed as market manipulation.[4] In a more recent public announcement in 2019, the CFTC acknowledged that it is concerned about CDS activism and that this is a recent and growing phenomenon: "The earliest of these activities date back to 2006. During the following 10 years, we have observed a total of six instances of similar strategies. In the last two and a half year period, we have observed 14 strategies, seven of which have occurred in just the last six months."[5]

We examine the question of whether the recent rise in CDS investor intervention is beneficial or harmful. In particular, we are interested in the effect on the underlying firm, and use firm value as a proxy for aggregate welfare. We look at two types of CDS intervention. First, we examine what happens if a firm's lender is allowed to purchase CDS protection, which creates an incentive to push the firm into bankruptcy. Second, we analyze the effect of allowing a protection seller to intervene in the restructuring of a distressed firm as well. We use a theoretical model to show that the popular conclusion is not necessarily valid. We find that under symmetric information, intervention by a protection buyer together with CDS seller intervention increases firm value instead of destroying it.

We start with a model where the firm's lender can purchase CDS protection but without the ability of the protection seller to intervene in a debt restructuring. This model, which builds on the well-known theory in Bolton and Oehmke (2011), allows us to understand cases like Caesars Entertainment, Windstream, and Neiman Marcus in Table 1. The economy consists of a firm, a lender, and a protection seller. The firm's owner faces multiple frictions

[3] Financial Times, "Time to wipe out the absurd credit default swap market", May 11, 2018.

[4] The Wall Street Journal, "How Regulators Averted a Debacle in Credit Default Swaps", July 8, 2018.

[5] CFTC public webcast titled "Credit Derivatives", published on July 10, 2019 at `https://youtu.be/Qqo9KR6JXaM`.

like the inability to commit to repaying the debt in the future, taxes, and bankruptcy costs. These frictions constrain the firm and prohibit it from investing at the first-best level.

In our first set of results, we show that the existence of a CDS market, without an active protection seller, has both negative and positive effects on firm value. On the one hand, a lender who is hedged with credit derivatives is better off in bankruptcy than in an out-of-court debt restructuring. This increases the probability of costly liquidation, which increases borrowing costs and reduces firm value ex ante. On the other hand, the lender's payoff in an out-of-court restructuring is higher compared to a world without a CDS market, which alleviates the commitment problem between equity holders and the lender. This decreases upfront borrowing costs and increases firm value. The net effect on firm value can be either positive or negative and depends on parameter values.

We extend the active CDS buyer model by allowing the CDS seller to intervene in the firm's debt restructuring. This model allows us to understand cases like Norske Skog and RadioShack in Table 1. The protection seller can reduce the firm's debt by injecting equity capital. The results of this small change in the model are remarkable, as firm investment increases substantially. The reason is that the probability of liquidation drops because the protection seller injects enough equity to keep the firm alive. This reduces the cost of borrowing ex ante, which allows the firm to invest more. Remarkably, under certain assumptions on parameter values, the probability of liquidation drops to zero and investment reaches the first-best level.

Another result is that even though the probability of liquidation is extremely low, the debt holder purchases a large amount of CDS contracts. This initially counter-intuitive outcome is an important feature of the equilibrium: It is precisely *because* the lender purchases a lot of protection that the CDS seller has a strong incentive to save the firm if it is in distress. The lender understands this and buys more CDS protection ex ante.

A related result is that the protection seller charges a positive CDS spread upfront, even though a credit event is never triggered in equilibrium. At first sight, this might seem counter-intuitive and unfair. A casual observer might complain that the protection buyer pays an excessively high insurance premium. But by assumption, the protection seller makes zero profit in expectation. The insurance premium is just fair compensation for the protection seller for saving the firm from liquidation. Actually, the seller is providing a valuable service to society, by avoiding costly liquidation.

Our results on the effect of intervention by a protection seller on firm value might seem—at first sight—to be an implication of the Coase Theorem. After all, should we not expect firm value to increase if we allow all the protection sellers to sit at the bargaining table? We show that this analogy is flawed. The Coase Theorem is about how ex post bargaining between two parties can avoid inefficient outcomes. However, some of our most interesting results are not the efficient outcome ex post, but what happens ex ante. And even what happens ex post in our model is different from the simple Coasian prediction.

To see this, note that we refer to ex post as everything that happens after the firm's final profit shock realizes. If the lender is hedged with a CDS contract, and if the realization of the profit shock is sufficiently low, then the lender's outside option in debt renegotiation is very high relative to the firm's asset value. This makes debt renegotiation infeasible and so the firm is liquidated. The lender's tough stance in debt renegotiation effectively pushes the firm into bankruptcy. To avoid a negative cash flow shock, the protection seller makes a payment to the owner—not to the lender—to avoid liquidation. This is already different from the Coasian setup because it is the lender who effectively causes the deadweight cost of liquidation, but the protection seller makes a payment to a different party—the owner—to avoid the inefficiency. Additionally, our results on the ex ante effects on the lender and the firm further differentiate our findings from the Coase Theorem.

Our model also provides multiple novel testable predictions. For example, we predict that following an exogenous increase in the number of protection sellers, the probability of bankruptcy and CDS spreads will increase, while investment and firm value will drop. We do not explicitly test these predictions, because the opacity of the CDS market makes it difficult to empirically determine the number of protection sellers, let alone to find an exogenous shock to that number. In the future, as data availability improves, these predictions should become directly testable.

Our findings also have important policy implications. First, the notion that intervention by CDS investors is unfair and reduces welfare—while intuitive—is not necessarily true. With the caveat that we only consider two types of CDS investor intervention, we show that such activism can actually increase firm value. However, we also argue that this result depends on our assumption of symmetric information. If there is a lot of uncertainty about the number of protection sellers for a particular underlying, our results can break down. Therefore, we argue that a more transparent CDS market might help to ensure that underlying firms benefit from the value-enhancing effects of a CDS market. We discuss possible policy measures to improve transparency. For example, similarly to the stock market, where holdings above a certain threshold have to be reported to the SEC, a reporting requirement for protection sellers could lead to lower CDS spreads, fewer bankruptcies, more investment, and higher firm value.

We contribute to multiple strands of the literature. First, our active CDS buyer model examines the positive and negative effects of introducing a CDS market on firm value, without allowing for intervention by the protection seller. The trade-off presented here is similar to the one in Bolton and Oehmke (2011).[6] However, Bolton and Oehmke do not explicitly

[6]Their theoretical framework has since been extended by Kim (2016), Campello and Matta (2016), Bartram, Conrad, Lee, and Subrahmanyam (2018), Colonnello, Efing, and Zucchi (2018), Danis and Gamba (2018), and Wong and Yu (2018), among others. Empirical tests of the theory can be found it Guest, Karampatsas, Petmezas, and Travlos (2017), Bartram, Conrad, Lee, and Subrahmanyam (2018), Batta and Yu (2018), Chang, Chen, Wang, Zhang, and Zhang (2019), Colonnello, Efing, and Zucchi (2018), and Narayanan and Uzmanoglu (2018).

compare the positive and negative effects on firm value and they do not calculate the net value effect. Many empirical papers, such as Subrahmanyam, Tang, and Wang (2014), interpret the Bolton and Oehmke model as one where the CDS market creates negative effects for firms through an increase in the probability of bankruptcy. We show that while this is true, the positive effects of having a lender who is "tough" in renegotiation can outweigh the negative effects, leading to higher investment and firm value.

Our second major contribution is to document the recent empirical trend towards CDS interventions. Some cases of intervention by protection buyers were already documented in Bolton and Oehmke (2011), but other types of CDS intervention seem to be new. In our theoretical analysis, we focus on two types of CDS intervention and show that this activity increases firm value instead of destroying it. To the best of our knowledge, this is the first detailed examination of the value effects of CDS intervention. We show that under certain assumptions on the distribution of the firm's future profitability, the benefits of an active protection seller can be so large that the firm reaches the first-best level of investment. Compared to the theoretical framework of Bolton and Oehmke (2011), this implies that the negative effect of a CDS market, which derives from the so-called empty creditor problem, is reduced to zero, and only the positive effects prevail. In other words, we show that the empty creditor problem can be at least partially solved by CDS intervention, and under certain parameter assumptions, it can be solved completely. We believe that this is a significant contribution to our current understanding of the costs and benefits of having a CDS market.

1. Model with an active CDS buyer

We start with a model where only the CDS buyer can intervene in the underlying firm. We will then extend the model to allow for intervention by the protection seller. A real-life motivation for the first model, where the CDS seller cannot intervene, is that there is a large number of dealers, each with a tiny portion of the CDS contract, which means that none of them have an incentive to save the firm. Another interpretation of this model is a world where the regulator forbids dealers to interfere with the distressed firm.

We present a model that builds on Bolton and Oehmke (2011) and Danis and Gamba (2018). We make several simplifying assumptions, which allow us to derive closed-form solutions and to present the underlying mechanism most transparently. We relax several of these assumptions and derive some of our results in a much more general framework in the internet appendix.

All agents in the model are risk-neutral. We model a single firm, owned by an entrepreneur who makes investment and financing decisions to maximize her expected payoff at the beginning of the period. The main driver of the model is the firm's profit shock, a continuous-state random variable. The probability of the end-of-period shock, z, is determined by the cumulative distribution function $\Gamma(z)$.

We denote by k the capital stock for the period. To finance the investment, the firm issues debt alongside a possible equity injection. The debt contract is an unsecured zero-coupon bond with face value b paid at the end of the period. Both k and b are non–negative. Debt financing is cheaper than equity financing because debt payments reduce the corporate income tax base. The corporate tax rate is $\tau \in [0, 1[$.[7] The risk-free interest rate and equity issuance costs are zero for simplicity. We relax both of these assumptions in the internet appendix.

The firm's profit shock determines the asset value at the end of the period:

$$a(z, k) = zk^{\alpha}, \tag{1}$$

[7]Since the debt is a zero-coupon bond, we assume for simplicity that the whole face value is deductible. Because this overstates the tax benefits of debt compared to the real world, we compensate by choosing a lower parameter value for the tax rate.

where $\alpha \in]0, 1[$ is the return-to-scale parameter.

After the realization of z, the owner decides whether to pay b in full, to renegotiate the debt by paying an amount b_r, or to file for bankruptcy and liquidate the firm. The owner cannot commit not to default on the debt in the future. For simplicity, the whole firm value is lost in liquidation. In the internet appendix, we relax this assumption and allow for an arbitrary proportional liquidation cost.

We assume a competitive market for insuring against credit risk. In particular, the debt holder can purchase a CDS from a dealer (protection seller) at the time the debt contract is issued. The lender (protection buyer) chooses the fraction h of the debt exposure covered by the CDS contract. The dollar amount, or notional amount, of insured debt is therefore hb. After observing h, the protection seller sets the CDS spread (the insurance premium) accordingly. The CDS spread is endogenously determined and the protection seller has rational expectations: he understands that selling CDS protection to the debt holder may change both the probability of default and the debt payoff in default and adjusts the CDS spread accordingly.

The debt holder chooses the hedge ratio, h, to maximize his expected payoff. Because we assume that the credit risk market is competitive, the CDS spread is fair (and the transaction has zero–NPV for the protection seller). In the first part of this section, h will be an arbitrary hedge ratio. We discuss later how the optimal hedge ratio is determined.[8]

The sequence of events is as follows. The firm owner chooses a capital level k and a face value of debt b. The debt holder observes the outcome of these decisions and chooses a hedge ratio h. At the end of the period, nature chooses a profitability shock z, and the

[8]There are no speculators in the CDS market in our model, and therefore no so-called naked CDS positions. The only agents who trade in the CDS market are the lender and the dealer. However, speculators would not change much in this framework. By definition, they do not own the debt of the underlying firm, so they cannot participate in any debt restructuring. Therefore, they would not have any effect on the probability of default or on the recovery rate in default.

owner decides between repaying the debt, renegotiating with the debt holder, or liquidating the firm. The timeline of events is shown in Figure 1.

We next describe the payoffs to equity and debt as a function of the owner's default decision. The payoff to equity at the end of the period is $a - b$ if the debt is repaid, $a - b_r$ if the debt is successfully renegotiated, and 0 if the firm is liquidated. The corresponding payoff to debt is b when it is repaid in full, and b_r in case of a successful renegotiation. In the case of liquidation, the debt holder does not receive anything from the firm, due to liquidation costs, but he receives hb from the protection seller.

If the debt is renegotiated, the equity holder makes a take-it-or-leave-it offer to the bondholder. This assumption is relaxed in the internet appendix, where we solve a Nash bargaining game with an arbitrary distribution of bargaining power between the two parties.

The bargaining problem has two constraints, $a - b_r \geq 0$ and $b_r \geq hb$. The first constraint states that the owner's payoff after successful renegotiation, $a - b_r$, must be at least as large as her outside option. Similarly, the second constraint makes sure that the debt holder's renegotiation payoff b_r is not below his outside option. The bargaining problem, together with the constraints, is important for the results of the model. If the hedge ratio h is sufficiently high, then the outside option of the debt holder hb is so large that there is no b_r that can satisfy the two constraints. The result is that renegotiation is not feasible. And even if renegotiation is feasible, a higher hedge ratio increases the payoff to the debt holder, because the b_r that solves the bargaining game is increasing in the debt holder's outside option. Because of the simplifying assumption on the distribution of bargaining power, it is easy to see that the bargaining game is feasible if and only if $hb < a$. The solution, if it exists, is $b_r = hb$.

In the following, we assume that the optimal hedge ratio is $h^* \in [0, 1]$. We prove that this is true even in a much more general setting in the internet appendix.

It is relatively easy to derive the owner's optimal default decision: If the firm's asset value a is above the threshold $a_R = hb$, it is optimal to renegotiate the debt. The new face value of debt in these states is $b_r = hb$. If the asset value is below a_R, it is optimal to liquidate the firm.

Interestingly, it is never optimal to repay the debt in full. This is because—due to our simplifying assumptions—there is no cost associated with renegotiation. Also, since $h^* \in [0, 1]$ and $b_r = hb$, we have that $b_r \leq b$, i.e., the renegotiated face value of debt is always below the full face value. Therefore, renegotiation always dominates repayment. In the internet appendix, we allow for renegotiation costs and show that in very high states of the world debt repayment is optimal. The main results, however, are similar to the simple model.

Figure 2 shows the optimal default decision graphically. Low asset values a lead to liquidation, while high asset values trigger renegotiation. Figure 2 also summarizes nicely the positive and negative effects of adding a CDS market to the economy together with the protection buyer's ability to actively participate in bargaining with the firm. On the one hand, if the hedge ratio increases from $h = 0$ to $h = h^*$, the payoff to debt holder in a future renegotiation, b_r, increases. This is beneficial to the firm as well because the ex ante cost of borrowing goes down. On the other hand, the increase in the hedge ratio from $h = 0$ to $h = h^*$ pushes the threshold a_R up, which makes liquidation more likely and renegotiation less likely, reducing the expected payoff to the debt holder. Anticipating this outcome, the bondholder adjusts credit spreads upwards when the debt is issued.

The model encompasses also the case without a CDS market. Debt renegotiation is always feasible in that world, because, as we have seen before, the only thing that would make it infeasible is the bondholder's hedging activity. For the same reason, the bondholder's outside option is zero, and since he has no bargaining power, the bargaining outcome is always $b_r = 0$. This leads to an extreme equilibrium where the debt is always renegotiated to $b_r = 0$, which implies that the debt holder never receives any payments ex post. The result is that the bondholder never wants to lend to the firm ex ante. The commitment problem between equity holders and the lender is so strong that the debt market breaks down completely.

The reason for this extreme outcome is the combined assumption of very high liquidation costs and very low bargaining power of the lender. In a debt renegotiation, the owner can credibly threaten to liquidate the firm, in which case the lender would receive a payoff of zero. Since any debt renegotiation is a take-it-or-leave-it offer, the lender can do nothing but accept a debt payment of $b_r = 0$. As shown in the internet appendix, the outcome is a bit less extreme in a more general model, where the bondholder receives a positive payoff in most states of the world.

We now turn to the owner's ex ante decision. The owner maximizes the cum-dividend value of equity (i.e., of the firm), defined as

$$\begin{aligned} V(k^*, b^*, h(k^*, b^*)) &= \max_{k,b} V(k, b, h(k, b)) \\ &= \max_{k,b} \left\{ m(k, b) - k + (1 - \tau) \int_{z_R}^{\infty} (a(z, k) - b_r) d\Gamma(z) \right\}, \end{aligned} \tag{2}$$

where $m(k, b)$ denotes the equilibrium price of debt, which we will derive later on. The dividend $m(k, b) - k$ can have either sign; if it is negative, it is the amount of injected equity capital. The lower limit of the integral is $z_R = a_R/k^\alpha$. While a_R and b_r are a function of (b, h), in what follows we suppress these dependencies for notational convenience.

The debt holder maximizes his expected payoff, denoted by M, by choosing the hedge ratio:

$$m(k, b) = \max_h \; M(k, b, h). \tag{3}$$

The solution of the above program, $h = h(k, b)$, is the state-contingent optimal hedge ratio that is considered in the owner's program in (2).

To find M, the expected payoff to the debt holder, we first derive the fair price of the CDS contract. The credit event that triggers the CDS payment is bankruptcy/liquidation. An out-of-court debt restructuring does not trigger a CDS payment, in line with the Standard North American Contract (SNAC) of the International Swaps and Derivatives Association (e.g., Bolton and Oehmke, 2011). The price of credit protection for a given hedge ratio $h \in [0, 1]$ is the expectation of the net compensation from the protection seller:

$$C(k, b, h) = \int_0^{z_R} hb \; d\Gamma(z). \tag{4}$$

The expected payoff to the debt holder, including the payment from the protection seller in case of a credit event, but excluding the insurance premium $C(k, b, h)$, is

$$\psi(k, b, h) = \int_0^{z_R} hb \; d\Gamma(z) + \int_{z_R}^{\infty} b_r \; d\Gamma(z). \tag{5}$$

The expected value of the debt for a given hedge ratio, h, net of the cost of the CDS is

$$M(k, b, h) = \psi(k, b, h) - C(k, b, h).$$

After simplifying, the price of debt and the optimal h are found by solving the program

$$m(k, b) = \max_h \int_{z_R}^{\infty} b_r \; d\Gamma(z). \tag{6}$$

We assume that the profitability shock z follows a uniform distribution, $z \sim U[0, Z]$, which allows us to solve the model in closed form. Under this assumption, one can show that the optimal hedge ratio is $h^* = 1$.[9]

[9] It is sufficient to show that $\partial M/\partial h$ is positive for all $h \in [0, 1]$ because of our assumption that $h^* \in [0, 1]$. The derivative of Equation (6) with respect to h is $b(Zk^\alpha - 2hb)/Zk^\alpha$. This is trivially positive at $h = 0$. At $h = 1$, it is positive if and only if $b \leq Zk^\alpha/2$. We conjecture that $b \leq Zk^\alpha/2$ and verify that it is true below, in Equation (8), for all $\tau \in [0, 1]$. Under this assumption, the derivative $\partial M/\partial h$ is also positive for all $h \in [0, 1]$.

Next, we solve the owner's investment and financing problem in Equation (2). Using the optimal hedge ratio and substituting the equation for debt into Equation (2), firm value can be simplified to

$$V(k,b,h)|_{h=1} = -k + \int_{z_R}^{Z} zk^{\alpha} d\Gamma(z) - \tau \int_{z_R}^{Z} (zk^{\alpha} - b) d\Gamma(z), \tag{7}$$

where $z_R = b/k^{\alpha}$. For an arbitrary level of capital, the optimal amount of debt can be found by solving the first-order condition with respect to b, which yields

$$b^*(k) = \frac{Zk^{\alpha}\tau}{1+\tau}. \tag{8}$$

Using the previous result, one can solve the first-order condition with respect to k to find the optimal level of capital:

$$k^* = \left(\frac{\alpha Z}{2(1+\tau)} \right)^{\frac{1}{1-\alpha}}.$$

Note that the first-best level of capital would be

$$k_{FB} = \left(\frac{\alpha Z}{2} \right)^{\frac{1}{1-\alpha}}. \tag{9}$$

The equilibrium level of capital k^* is below the first-best level because of three frictions in the economy: taxes, bankruptcy costs, and the firm's lack of commitment to repaying the debt in the future.

It is informative to compare this equilibrium to a counterfactual outcome in a world without a CDS market and no intervention by a protection buyer. We have explained above that in such a no-CDS world the firm cannot issue any debt, or $b = 0$. It is easy to show that the optimal investment of the firm in this case is

$$k^*_{\text{no-CDS}} = \left(\frac{(1-\tau)\alpha Z}{2} \right)^{\frac{1}{1-\alpha}}. \tag{10}$$

Importantly, one can show that this is always less than the optimal investment in a world with a CDS market. In other words, introducing a CDS market together with one type of CDS intervention allows the firm to move closer to the first-best investment level. This shows that CDS intervention can be beneficial for firms. It can allow the firm to find cheaper debt financing and to invest more. In this simple model, the benefit comes from improving the bondholder's payoff in renegotiation from $b_r = 0$ to $b_r = hb$. While there are also costs created by the introduction of a CDS market, namely through the increased probability of liquidation, the benefits outweigh the costs. The net effect on the firm is always positive.

In a more general model, as shown in the internet appendix, the results are more nuanced. The net effect of a CDS market on investment and firm value can be both positive or negative, depending on parameter values. However, the main message remains true: Allowing for the intervention of a CDS buyer—by having a lender who takes a stronger bargaining position in a future debt renegotiation—has some positive effects on the firm.

In the next section, we extend the model by allowing the protection seller to intervene in the underlying firm as well. As we will show, the results will be very different from the model with only one type of intervention. In particular, having two types of intervention removes the negative effect of a CDS market on firms, so the net effect on firm value is unambiguously positive.

2. Model with an active CDS seller

We extend the previous model by allowing the protection seller to intervene by injecting equity into the firm to reduce the debt. The interpretation of this model is that sometimes there are fewer protection sellers, so they have a stronger incentive to save the firm. Another interpretation is that the regulator allows CDS sellers to intervene in distressed firms. To fix ideas, we will think of the previous model as one with a continuum of protection sellers, and this extension as one with a single CDS seller.

The main change in the model is that after nature has chosen profitability z, the dealer makes a take-it-or-leave-it offer to the owner. The dealer offers to recapitalize the firm by reducing the face value of debt to a new level $b_n \leq b$, through an equity injection in the amount of $b - b_n$. In return, the dealer asks for an equity stake of $\theta \in [0, 1]$ in the restructured firm.[10] The owner can accept or reject the offer. After the debt restructuring, the equity holders[11] make the default decision (i.e., repay/renegotiate/liquidate). For simplicity, we assume that the debt holder can only purchase CDS protection but cannot sell it, or $h \geq 0$. The timeline of events is now slightly different, as depicted in Figure 3.

It might seem inconsistent that the protection seller makes a take-it-or-leave-it offer to the owner, which implies that the dealer has a lot of bargaining power, while at the beginning of the model he sells CDS contracts at a competitive price, which assumes very little bargaining power. We argue that it is plausible that the protection seller's bargaining power can vary across the two settings. After the CDS contract has been sold and after a low profit shock z has realized, the firm is known to be in financial distress. At this point in time, the dealer is the single natural bargaining party for the owner because the dealer has the strongest incentive to bail out the firm. This is different from the ex ante CDS market, where we implicitly assume that the cost of entry is so low that the protection seller has to offer competitive CDS spreads in order to keep other dealers from entering.

[10] One could also assume other types of financing such as debt. We choose equity financing for simplicity. With debt financing, the dealer would want to make sure that the new debt does not trigger a credit event. This could be achieved by providing a debt contract with a maturity that exceeds the maturity of the CDS contract. Alternatively, the protection seller can provide short-term debt, but require the underlying company to issue the debt through a separate legal entity. In either case, the model would become unnecessarily complicated. Empirically, we observe both equity injections (e.g., Norske Skog) and debt injections (e.g., RadioShack, Matalan, or McClatchy) among the cases in Table 1.

[11] We refer to the residual claimants as equity holders because in some states of the world the original owner accepts the offer and the dealer also becomes an equity holder.

It is useful to write down the terminal payoffs of each player in this game, for each possible outcome of the firm's default decision. Table 2 summarizes these payoffs. It shows that under debt repayment and renegotiation the firm's cash flow to equity is split between the initial owner, who receives a fraction $1 - \theta$, and the dealer, who gets θ.

The first step to solve the model is to examine the equity holders' default decision. We know that renegotiation is feasible if there is a b_r such that the two conditions $a - b_r \geq 0$ and $b_r \geq hb$ are simultaneously satisfied, which is equivalent to the condition

$$hb \leq a. \tag{11}$$

Analogously to the active CDS buyer model, the renegotiated debt repayment is $b_r = hb$. Notice that even if debt is reduced from b to b_n, the renegotiation outcome depends on the original face value b. This is because the CDS contract was purchased on the original debt. This feature of b_r is important for many of the results below.

To solve the model, we make the following conjecture: *The debt holder's optimal hedge ratio is $h \leq 1$.* This conjecture greatly simplifies the exposition of the model solution. We prove that this conjecture is correct in Appendix A.

If renegotiation is feasible, the equity holders prefer repayment to renegotiation if $a - b_n \geq a - b_r$, or equivalently, if $b_n \leq hb$. If renegotiation is infeasible, the equity holders prefer repayment to liquidation if $a - b_n \geq 0$, or $b_n \leq a$.

The optimal default decision of the equity holders depends on b_n and hb and is described in the following lemma. The proof is in Appendix B. The default decision is summarized graphically in Figure 4.

Lemma 1. *We can distinguish between two cases. If $b_n \leq hb$, the optimal decision is to liquidate if $a < b_n$ and to repay the debt if $a \geq b_n$. If $b_n > hb$, the optimal decision is to liquidate if $a < hb$ and to renegotiate if $a \geq hb$.*

The intuition for this result is the following. For brevity, we focus on the case where the debt is reduced substantially to $b_n \leq hb$. If the firm's asset value is below the reduced face value of debt, $a < b_n$, the equity holders choose to liquidate the firm and their payoff is zero, due to limited liability. If they repaid the debt instead, their payoff would be less than zero. Renegotiating the debt is not an option, because the debt holder wants to be paid at least hb, which is more than the total asset value of the firm. For high asset values, the owners opt for repaying the debt, because it is better than the prospect of renegotiation. In renegotiation, the debt holders want at least hb, but with repayment, the equity holders only need to pay b_n, with $b_n \leq hb$.

The most important observation about this lemma is that if the debt is reduced a lot, then liquidation occurs only in the states $a < b_n$, whereas if the debt is only reduced a little, then the firm files for bankruptcy in more states, $a < hb$. Since liquidation is socially costly, it is important for ex ante firm value to reduce the probability that it occurs in equilibrium.

Next, we find the owner's optimal decision on whether to accept the protection seller's take-it-or-leave-it offer. In equilibrium, the protection seller will propose an equity stake θ for himself that makes the owner indifferent between accepting and rejecting. As Figure 4 shows, the default decision and the resulting payoffs depend on the region in which a lies. For each case and each region, we determine θ by equating the owner's payoff under acceptance to his payoff under rejection.

The optimal choice of the equity stake θ is summarized in the following lemma. The proof is in Appendix C.

Lemma 2. *If $b_n \leq hb$, then the protection seller's equity stake is*

$$\theta = \begin{cases} 1 - \frac{a-hb}{a-b_n} & \text{if } a \geq hb, \\ 1 & \text{if } a < hb. \end{cases}$$

If $b_n > hb$, *then*

$$\theta = \begin{cases} 0 & \textit{if } a \geq hb, \\ 1 & \textit{if } a < hb. \end{cases}$$

In simple terms, the mechanism behind this result is as follows. For brevity, we only cover the first case, $b_n \leq hb$. If the asset value is high, then the owner has to decide between accepting the equity injection, in which case she will own a smaller share of a firm with a healthier debt load, or rejecting the offer, which would allow her to keep the whole firm, but with a higher debt burden. This tradeoff has an interior solution, which is $\theta = 1 - \frac{a-hb}{a-b_n}$. If the asset value is low, the owner would get zero if she rejected the offer, because the debt load of the firm relative to the asset value would be so high that the firm would have to be liquidated. Therefore, the protection seller can take the whole firm ($\theta = 1$) and get away with it.

Next, we solve the dealer's choice of how much equity to inject, which determines the new face value of debt b_n. The protection seller maximizes the algebraic sum of the expected value of his (negative) CDS payoff in liquidation, of his (positive) payoff as a future equity holder, and his (negative) payoff from injecting equity capital in the amount of $b - b_n$ into the firm. We do not include his cash inflow from the upfront insurance premium because that is sunk at this point.

The dealer's optimal choice of b_n is summarized in the following lemma. For ease of exposition, we assume that the hedge ratio is $h > 1/2$, which is true in equilibrium, as we will show below. The proof, which includes the derivation for an arbitrary hedge ratio, is in Appendix D.

Lemma 3. *For a given hedge ratio that satisfies $h > 1/2$, the new amount of debt is*

$$b_n = \begin{cases} b & \textit{if } a < b - hb, \textit{ followed by liquidation,} \\ a & \textit{if } b - hb \leq a < hb, \textit{ followed by repayment,} \\ b & \textit{if } a \geq hb, \textit{ followed by renegotiation.} \end{cases}$$

If firm value turns out to be very low, $a < b - hb$, then the protection seller does not reduce debt at all. This is because he would need to inject so much equity that it is not worth it for him to avoid the cash outflow associated with a credit event. If the asset value realization is in an intermediate range, $b - hb \leq a < hb$, then the protection seller reduces the debt to $b_n = a$. Interestingly, this is exactly the face value that avoids liquidation later. We know from Lemma 1 that liquidation only occurs if $a < b_n$. In other words, the protection seller injects just enough equity to avoid liquidation. This intuitively makes sense, because a liquidation would create a large cash outflow for the protection seller. He can avoid this large cash outflow by injecting a little bit of equity into the firm.

If the asset value is very high, $a \geq hb$, the protection seller does not inject any equity, which means the principal remains unchanged. The reason is that the protection seller has no incentive to reduce the debt: A debt reduction is costly because he has to inject equity into the firm. The protection seller only gains by acquiring a larger share θ of the firm (Lemma 2 shows that θ is higher if b_n is lower). However, these two effects exactly offset each other, so the protection seller is not better off by injecting equity into such a firm.

A very interesting observation is that the threshold that determines how frequently liquidation will occur in the future, $b - hb$, is decreasing in the hedge ratio h. In other words, if the lender chooses a higher hedge ratio, he can reduce the probability of costly liquidation. This happens because purchasing more CDS protection increases the protection seller's incentive to intervene in the future by injecting equity. This will be important to understand the equilibrium later.

Next, we find the lender's expected payoff at the stage when he chooses the hedge ratio h. We assume that the CDS spread is set so that the protection seller breaks even *including* the contingent equity injection of $b - b_n$.

To derive the market value of debt, we need to take into account several different cash flows: debt repayment by the firm, recapitalization by the dealer, debt payoffs in renegotiation and liquidation (if any), cash inflows from the CDS contract (if any), and the cash outflow of the upfront CDS premium. Similarly to the previous lemma, we only present the case if the hedge ratio is $h > 1/2$, which will be true in equilibrium. The proof, which contains the derivation for an arbitrary hedge ratio h, is in Appendix E.

Lemma 4. *The expected payoff to the debt holder is*

$$M(k,b,h) = \int_0^{z_L} (\underbrace{0}_{\text{bond payoff}} + \underbrace{hb}_{\text{CDS payoff}})d\Gamma(z) + \int_{z_L}^{z_R} [\underbrace{(b-a)}_{\text{recapitalization}} + \underbrace{a}_{\text{debt repaym.}}]d\Gamma(z) + \int_{z_R}^{Z} \underbrace{hb}_{\text{renegotiation}} d\Gamma(z) - \underbrace{\left[\int_0^{z_L} hbd\Gamma(z) + \int_{z_L}^{z_R} (b-a)d\Gamma(z)\right]}_{\text{CDS premium}},$$

where $z_L = (b - hb)/k^\alpha$ and $z_R = hb/k^\alpha$ are the thresholds from Lemma 3. The value of debt can be simplified to

$$M(k,b,h) = \int_{z_L}^{z_R} ad\Gamma(z) + \int_{z_R}^{Z} hbd\Gamma(z). \tag{12}$$

The first integral in the long equation covers the bad states of the world, $a < b - hb$. In these states, as we know from Lemma 3, the debt is not reduced and the firm files for bankruptcy. The second integral is over the intermediate states of the world, $b-hb < a < hb$, where we know from Lemma 3 that the debt b is reduced to $b_n = a$ and the remaining debt is repaid in full. The third integral covers the good states, $a \geq hb$, where Lemma 3 says that the debt is not reduced. This is followed by renegotiation. Finally, the term in square brackets is the upfront CDS premium paid to the protection seller. This consists of two terms because the protection seller has to be compensated for two types of losses: First, in bad states, the firm is liquidated, which triggers a credit event. Second, in intermediate states, the protection seller will inject equity into the firm.

An important remark can be made about the cost of debt financing at this point. It is easy to show that $M < b$, which means the debt is not risk-free. However, the debt value in the active CDS buyer model would be $M = \int_0^{z_R} 0 d\Gamma(z) + \int_{z_R}^{Z} hb d\Gamma(z)$, which is strictly below debt value in the extended model. In other words, for fixed values of k, b, and h, the cost of debt financing is lower in the model in which the dealer can recapitalize the firm. The reason is that in the intermediate states of the world, the protection seller provides financing to the distressed firm and saves it from liquidation. This reduces expected liquidation costs ex ante, which reduces the cost of debt.

Finally, we solve the owner's ex ante investment and financing problem,

$$\begin{aligned} V(k^*, b^*, h(k^*, b^*)) = \max_{k,b} \{ & m(k,b) - k \\ & +(1-\tau) \left[\int_{z_L}^{z_R} (1-\theta)(a(z,k) - b_n) d\Gamma(z) + \int_{z_R}^{Z} (1-\theta)(a(z,k) - b_r) d\Gamma(z) \right] \right\}. \quad (13) \end{aligned}$$

Under the assumption of a uniform distribution for the profitability shock z, one can show that the optimal hedge ratio is $h^* = 1$.[12] We know from the previous steps that $b_n = a$ in the first integral, that $b_r = hb = b$, that $\theta = 0$ in the second integral (from Lemmas 2 and

[12]It is sufficient to show that $\partial M / \partial h$ is positive for all $h \in [0, 1]$, because of our conjecture that $h \leq 1$. For the case $h \leq 1/2$, M is given in the appendix, Equation (20), and the derivative with respect to h is $b(Zk^\alpha - hb)/Zk^\alpha$. This is trivially positive for all $h \in [0, 1/2]$. For the case $h > 1/2$, M is given by Equation (12), and the derivative with respect to h is $b(b - 2hb + Zk^\alpha)/Zk^\alpha$. This is trivially positive at $h = 1/2$. At $h = 1$, it is positive if and only if $b \leq Zk^\alpha$. We will make the assumption $b \leq Zk^\alpha$ at this point and verify that it is true in Equation (15). Under this assumption, the derivative $\partial M / \partial h$ is positive for all $h \in [0, 1]$.

3) because $a = b_n = hb$, and that $z_R = b/k^\alpha$. After making these substitutions, firm value simplifies to

$$V(k,b,h)|_{h=1} = m(k,b) - k + (1-\tau)\int_{z_R}^{Z}(a(z,k)-b)d\Gamma(z),$$

The market value of debt $m(k,b)$ is given in Lemma 4. After substituting, firm value becomes

$$V(k,b,h)|_{h=1} = -k + \int_0^Z zk^\alpha d\Gamma(z) - \tau\int_{z_R}^{Z}(zk^\alpha - b)d\Gamma(z). \tag{14}$$

For an arbitrary level of capital k, the optimal amount of debt can be found by solving the first-order condition with respect to b, which yields

$$b^*(k) = Zk^\alpha. \tag{15}$$

This is strictly higher than the optimal amount of debt in the active CDS buyer model. Using the previous result, one can solve the first-order condition with respect to k to find the optimal level of capital:

$$k^* = \left(\frac{\alpha Z}{2}\right)^{\frac{1}{1-\alpha}}. \tag{16}$$

This level of capital is higher than in the active CDS buyer model. The increase in investment is so large that the firm reaches the first-best level of investment (compare Equations (9) and (16)). In other words, the firm is able to achieve the same capital level as in a world with no frictions.

The emergence of first-best investment is remarkable. It is worthwhile to discuss why this high investment level is possible. The three frictions that constrain investment in this setting are the firm's inability to commit to repaying its debt, bankruptcy costs, and taxes. The lack of commitment is alleviated by the lender's ability to buy CDS protection. With our particular assumption on parameter values and the distribution of z, this problem can be solved completely by increasing the lender's renegotiation payoff to $b_r = hb = b$. As we know from Section 1, however, this creates a new problem, which is a higher likelihood of costly liquidation. This problem is solved by the protection seller's ability to inject equity into the firm. Again, with our parameter assumptions, this problem is fully solved as the probability of bankruptcy drops to zero. Finally, taxes become irrelevant as well, because the CDS market allows the firm to increase leverage so much that it can benefit from the maximum possible tax shield.[13]

Another observation is that even though the probability of liquidation is zero, the debt holder purchases a large amount of CDS contracts. In fact, he is fully hedged, as his hedge ratio is $h^* = 1$. This seems counter-intuitive, but the two outcomes are necessary to sustain the equilibrium of the model. It is precisely *because* the lender purchases a lot of protection that the CDS seller has a strong incentive to save the firm if it is in distress.

A related observation is that the protection seller charges a positive CDS spread upfront, even though a credit event is never triggered in equilibrium. At first sight, this might seem counter-intuitive and unfair. Casual observers might complain that the protection buyer is paying an insurance premium that is too high. But the protection seller is just being fairly compensated for saving the firm from liquidation. He provides a valuable service to society, by avoiding costly liquidation.

To close the comparison of the active CDS buyer model and the extended model, we examine the difference between firm value in the two models. The preceding analysis, particularly the positive effect of a single protection seller on investment, suggests that firm value will increase as well. This is indeed the case, as summarized by the following statement.

Proposition 1. *(a) Ex ante firm value increases if there is a single protection seller in the CDS market who can intervene in financial distress.*

(b) The debt holder purchases a large amount of CDS contracts even though the probability

[13] Equation (15) shows that the face value of debt is so high that it pushes up the threshold z_R in Equation (14) to $z_R = b/k^\alpha = Z$, so the region $[z_R, Z]$ where the firm would have to pay taxes shrinks to zero.

of liquidation is zero.

(c) The insurance premium in the CDS market is positive, even though a credit event is never triggered in equilibrium.

The first statement can be seen from comparing Equations (7) and (14). These two equations show that for any capital k and debt b, firm value is higher with a single protection seller. Therefore, firm value must be higher at the equilibrium levels of capital and debt as well. The second and third statements have been derived above.

2.1. Testable predictions

By comparing the active CDS buyer model in Section 1 to the extended model in Section 2 we can derive multiple testable predictions.

If there is an exogenous shock, for example, a regulatory change, that prohibits the protection seller from offering financing to the underlying firm in distress, ex ante credit spreads and the probability of liquidation both increase. At the same time, firm value and investment decrease. The major empirical challenge to testing this prediction is that—to the best of our knowledge—a regulatory change of this kind has not yet been passed.

Another prediction is that if there is an exogenous increase in the number of protection sellers for the same underlying firm, then the CDS spread and the probability of liquidation both increase. At the same time, firm value and investment decrease. There are two empirical challenges to testing this prediction. First, as we have mentioned before, the CDS market is very opaque. CDS holdings data for protection buyers and sellers are not publicly available for researchers. This data, to the best of our knowledge, is only accessible to regulatory agencies such as the Federal Reserve Board or the Securities and Exchange Commission. Second, even if we could measure the concentration of protection sellers, we would need an exogenous shock that changes the level of concentration.

2.2. Asymmetric information between protection buyer and seller

We now examine the role of asymmetric information in the CDS market. The subsequent analysis is somewhat speculative since a rigorous extension of the model to allow for asymmetric information would be beyond the scope of this paper. The asymmetric information case is interesting because empirically there is often uncertainty in the market about how many protection sellers there are for a given name. The protection buyer might think that there are many protection sellers, but actually, there could be very few.

An result from the previous two models is that the upfront CDS spread in the active CDS buyer model is higher than the CDS spread in the extended model. To see this, we can write the CDS spread in the active CDS buyer model as

$$C_1 = \int_0^{hb/k^\alpha} hb d\Gamma(z).$$

Also, we can write the CDS spread in the extended model as

$$C_2 = \int_0^{hb/k^\alpha} (b-a) d\Gamma(z),$$

which we know from Equation (19). If the optimal hedge ratio h^* is sufficiently high, it is easy to see that $C_1 > C_2$. In the case of the uniform distribution, this is true, since $h^* = 1$ in both the active CDS buyer model and the extended model. Substituting $h^* = 1$ into the equations above, we see that

$$C_1 = \int_0^{b/k^\alpha} b d\Gamma(z) > \int_0^{b/k^\alpha} (b-a) d\Gamma(z) = C_2. \tag{17}$$

This is important because it suggests that the protection seller has an incentive to use the opacity of the CDS market to hide the fact that he is the only protection seller. If the protection buyer believes that there are many protection sellers, then the protection seller can charge a high premium, C_1. But if he is the sole protection seller in the market, then the true cost of the contract to him is C_2. Since we have just shown that $C_1 > C_2$, he can make a profit this way.

To make the argument in a very rigorous way would require a substantially more complex model. For example, we would need to endogenize the entry decision of prospective protection sellers, to allow for the debt holder to observe the CDS spread and to update his beliefs on how many protection sellers there are in the market, and to solve the equity injection decisions of an arbitrary number of protection sellers.[14]

Instead, we focus on outlining a simple example. The purpose of this example is to illustrate how important the assumption of symmetric information is in the extended model above. This example shows how the welfare implications of CDS investor activism can be very different if a transparent CDS market is replaced with an opaque one.

We assume that the debt holder does not know how many protection sellers are in the market. For simplicity, only one of two extremes can occur: either there is just one protection seller, or there is a continuum of them. The debt holder and the firm observe a quoted CDS spread, which they use as a noisy signal to infer the number of protection sellers. This inference is made difficult by a private shock to the funding costs of the protection sellers. A favorable shock reduces their funding costs, which reduces the CDS spread they quote. Because of this unobservable funding shock, the debt holder and the firm cannot simply figure out the number of protection sellers from the CDS spread. Without the added noise

[14] The extensive form of the game would look something like this: (1) Prospective protection sellers decide whether to enter the CDS market for the underlying firm. The number of protection sellers actually entering the market is denoted by n. (2) Protection sellers quote a CDS spread, which is a function of the number of sellers in the market n. Sellers observe a private shock to their funding costs, which affects the CDS spread they quote. (3) Bondholder and firm observe quoted CDS spread, which serves as a (noisy) signal of the number of protection sellers. (4) Firm chooses capital k and debt b. (5) Bondholder chooses hedge ratio h. (6) Nature chooses profitability shock z. (7) Protection sellers choose how much equity to invest in the firm. (8) Firm repays the debt, renegotiates the debt, or liquidates. Note that stages 4–8 are very similar to our existing model, but with the added complexity in stage 7 that there will be a free-rider problem between the individual protection sellers which needs to be taken into account. Stages 1–3 are completely new, and would add considerable complexity.

from the funding shock, the debt holder would know from the quoted spread whether he is in the active CDS buyer model or in the extended model.

For brevity, we only consider the case where the true number of protection sellers is one, but the debt holder (and the firm) believe that there is a continuum of them. In the model, the reason for this is that the debt holder is unsure whether a particular CDS quote is caused by a certain number of protection sellers or by certain funding costs. As we have seen in Equation (17), a continuum of protection sellers will charge a higher CDS spread, other things being equal. Observing such a high quote, the debt holder cannot determine whether it is caused by a continuum of sellers with low funding costs or by a single seller with high funding costs. A single seller will use this opportunity to hide his type from the debt holder. The argument is similar to the model in Maug (1998), among others.

Absent the funding costs, the CDS spread that a continuum of protection sellers would choose is given by C_1 in Equation (17). The CDS spread chosen by a single protection seller, absent the effect of funding costs, would be C_2, also in Equation (17). After taking into account funding costs, but without assuming a particular value for them, the CDS spread for a continuum of sellers would be below C_1, and for a single seller, it would be above C_2. Even without solving the equilibrium explicitly, we know that the resulting equilibrium CDS spread will be a convex combination of C_1 and C_2, or

$$C = xC_1 + (1 - x)C_2, \tag{18}$$

where $x \in (0, 1)$. The quoted CDS spread C is what the debt holder would use to base his hedging decision on. Without knowing x explicitly, we cannot fully solve the bondholder's hedging decision. However, we can still say something about the qualitative nature of the resulting equilibrium. Given that the optimal hedge ratio in both the active CDS buyer model and the extended model is $h^* = 1$, it is reasonable to assume that the bondholder will choose the same hedge ratio in the model with asymmetric information.

Since the true number of protection sellers is one, the equity injection and default resolution stage of the game will be as in the extended model. Liquidation will never occur in equilibrium, debt will be repaid in the region $a < b$, and will be renegotiated in the region $a \geq b$. The protection seller never suffers a credit event, and he only needs to recapitalize the firm in the amount of $b - a$ in the states $a < b$. As a result, his expected costs are summarized by C_2 in Equation (17). However, as argued before, his upfront cash inflow is C. We have established that $C > C_2$, and the difference $C - C_2$ is the protection seller's profit.

This outcome is very different from the outcome of either the active CDS buyer model or the extended model. In both of those models, the protection seller makes zero profit. What allows him to make a positive profit now? It is the fact that he can hide his identity in the opacity of the CDS market. The bondholder believes that there might be multiple protection sellers and that there might be liquidation in the future. Therefore, the bondholder is willing to pay a higher CDS spread. However, the true probability of liquidation is zero, which reduces the expected cash outflow of the protection seller.

3. Discussion and policy implications

3.1. Coase Theorem

Some of our results might seem to follow directly from the Coase Theorem. After allowing the protection seller to the bargaining table, the efficient outcome of no liquidation is reached. Since there are no transaction costs associated with bargaining between the owner and the protection seller, this outcome seems to be an implication of the Coase Theorem. However, our results go considerably beyond the Coase Theorem.

The Coase Theorem—as used in most applications—is about how ex post bargaining can lead to efficient outcomes. However, some of our most interesting results are not the efficient outcome ex post, but what happens ex ante. And even what happens ex post in our model is different from the simple Coasian prediction.

Ex ante, we show that the lender buys more CDS insurance upfront because he wants to give a strong incentive to the protection seller to intervene. In equilibrium, the ex ante probability of needing the insurance is zero, and yet the lender buys a lot of insurance. Finally, the beneficial effects of intervention ex post allow the firm to borrow more and invest more at time zero.

Ex post, our model is different from Coase as well. In the context of our model, we to refer to ex post as everything that happens after the profit shock z realizes (see the timeline in Figure 3). If the lender is hedged with a CDS contract, and if the realization of z is sufficiently low, then the lender's outside option is very high relative to the firm's asset value. This makes bargaining infeasible and so the firm is liquidated. The lender's tough stance in debt renegotiation effectively pushes the firm into bankruptcy. To avoid a negative cash flow shock, the protection seller makes a payment to the owner—not to the lender—to avoid liquidation. This is different from the Coasian setup because it is the lender who effectively causes the deadweight cost of liquidation, but the protection seller makes a payment to a different party—the owner—to avoid the inefficiency.

To make this distinction even clearer, note that our results go through even if the equity injection by the dealer does not trigger a payment from the firm to the lender. All that we need is that the dealer makes a cash injection into the firm. Even if the cash is held on the balance sheet and not paid out to the lender, the net debt of the firm is reduced, default is avoided, and the results are the same as in Proposition 1.[15]

[15] To see this, one can re-interpret our model as follows: a can be seen as the physical assets of the firm, i.e., without cash holdings, b can be interpreted as net debt, and b_n as the new net debt after the cash injection. If the firm makes default decisions based on net debt, then our equations and results remain the same.

Instead of the Coase Theorem, our results are more reminiscent of the theoretical literature on how carefully chosen ex ante contracts can solve ex post hold-up problems or conflicts of interest. For example, Aghion and Bolton (1992) show how a standard debt contract can solve a financing problem between an entrepreneur and an investor. Assigning control rights in a non-contingent way to the entrepreneur would prohibit her from raising sufficient financing. But giving state-contingent control rights to the investor solves that problem. In another example, Noldeke and Schmidt (1995) show that the famous hold-up problem of Hart and Moore (1988) can be solved by an option contract between the two parties, assuming that actions are verifiable.

Our model implies that adding a CDS contract to the lender's portfolio and allowing the protection seller to intervene solves two problems: (i) the problem that the owner cannot commit not to renegotiate the debt contract in good states of the world, and (ii) the excessive liquidation resulting from the fact that the lender becomes reluctant to renegotiate the debt.

3.2. Alternative contracts

One of the main results in Section 2 is that the probability of liquidation is zero in equilibrium. The reduction in expected bankruptcy costs is one of the main reasons for the increase in firm value going from the model in Section 1 to the model in Section 2. However, this raises the question of whether the same outcome could be reached through an alternative contractual arrangement. In particular, why do the owner and the lender not come to an agreement that avoids costly liquidation?

Our model actually allows for such negotiations between the two parties. If there were no CDS market, then the owner and the lender would both agree that avoiding liquidation is in both parties' interest, and they would agree on debt renegotiation. This model is solved formally in Internet Appendix C. As a result, the probability of liquidation without a CDS market would be zero.[16]

However, the introduction of a CDS market and allowing CDS trading and intervention by the lender destroys that outcome. The lender wants to improve his ex post bargaining position, so he purchases CDS contracts ex ante. As a result, he demands so much in renegotiation that in certain states of the world debt renegotiation becomes infeasible, which leads to liquidation. Therefore, the owner and the lender cannot reach the efficient outcome of no liquidation through a bilateral arrangement.

3.3. Policy implications

Our analysis has several important policy implications. First, we show that having a protection seller who interferes with the debt restructuring of a financially distressed firm is not necessarily reducing firm value. This is not a trivial insight, as some market commentators argue that the recent cases of CDS investor intervention in Table 1 are evidence that the CDS market is absurd and dysfunctional.[17] Also, speculative protection buyers who would benefit from a credit event consider an unexpected intervention of a protection seller as unfair to them.

Our results suggest that the ability of a protection seller to interfere with debt restructuring actually improves firm value. Firm investment increases, the probability of liquidation goes down, and credit spreads narrow. Under certain assumptions on the distribution of future profitability and certain parameter values, firm investment can even achieve the first-best level, and the probability of liquidation can drop to zero.

[16]Even though liquidation never occurs in the no-CDS model, the outcome is far from efficient. The reason is that the probability of strategic default—or debt renegotiation in good states of the world—is much higher, which depresses firm value ex ante.

[17]E.g., Financial Times, "Time to wipe out the absurd credit default swap market", May 11, 2018.

The resulting equilibrium may seem unfair because the probability of liquidation is zero, but the protection buyer still pays a positive CDS spread. We argue that this is, in fact, necessary: The positive up-front CDS spread compensates the protection seller for his ex post cash injections in the distressed firm.

Our results also imply that a smaller number of protection sellers is better because it increases the incentives of the seller to save the distressed firm. This is in contrast to the intuitive notion that a large number of market participants is always better. Our result is analogous to theories on the disadvantages of dispersed bondholders (e.g., Gertner and Scharfstein, 1991). The result is policy-relevant today, as there are concerns that the inter-dealer CDS market is too concentrated.[18] To the extent that a smaller number of dealers is correlated with a smaller number of protection sellers, our results suggest that a concentrated inter-dealer market can have its benefits.

All these results are based on the assumption of symmetric information between protection buyers, protection sellers, and the underlying firm. In particular, we assume that everyone knows how many protection sellers there are and how much protection they have sold. We argue that this is an important assumption: If we relax it, it is not necessarily true that the firm value increasing equilibrium still prevails. We show that a protection seller always has an incentive to pretend ex ante that he cannot rescue the firm, to sell protection at a high CDS spread, and to rescue the firm ex post.

Empirically, it is not unlikely that this can happen. The CDS market is very opaque, and no regular investor knows how many protection sellers there are, how much protection they have sold, and whether they have deep pockets to inject cash into the underlying firm. Therefore, we argue that it is possible that regulation that improves the transparency of the CDS market can increase firm value. Other authors have proposed disclosure requirements in the CDS market as well (e.g., Bolton and Oehmke, 2011), although for different reasons.

[18]The Wall Street Journal, "Big banks agree to settle swaps lawsuit", September 12, 2015.

For example, one could introduce reporting requirements for protection sellers. If someone has a large position in the CDS market, he would need to make that position public. This is similar to reporting requirements in the stock market, where investors who hold more than 5% of a firm's stock need to report their holdings to the SEC.

We do not take a stance on what it means precisely to have a large position. Future research will be needed to answer that question. For example, a large position could be defined as a notional value of a CDS position that exceeds 5% of the face value of debt of the underlying firm. Alternatively, the top 5 protection sellers by dollar amount for each reference entity would have to be reported each trading day.

4. Conclusion

We document a recent empirical trend of increased intervention by CDS investors in the restructuring of financially distressed firms. Using a simple model, we analyze the effects of CDS intervention on firm value. We start with an active CDS buyer model, without intervention by the protection seller, which highlights the costs and benefits created by a CDS market. We then extend the model to allow for intervention by the CDS seller.

Our main result is that firm value is not necessarily reduced when we allow for CDS intervention. This is true in spite of the seemingly unfair outcome that the protection buyer has to pay a CDS spread upfront that is high relative to the low probability of liquidation. We show that the lender, who is the typical protection buyer in our model, is not worse off by paying the CDS spread, because the seller will bail out the firm in the future, which is good for the lender. We also show that the lower probability of bankruptcy causes ex ante firm borrowing costs to go down, which in turn leads to higher investment and firm value.

Our results depend on symmetric information between all related parties. In particular, we assume that the protection buyer and the firm know how many protection sellers there are.

This number determines the CDS seller's incentive to intervene ex post. Empirically, the CDS market is very opaque. Therefore, we argue that any policy measure to reduce asymmetric information about the incentives of protection sellers has the potential to increase firm value.

We emphasize that our analysis is limited to two types of CDS intervention: A hedged lender who is a tough negotiator in an out-of-court debt restructuring and a protection seller who can avoid a credit event by infusing capital into the firm. Other kinds of CDS interventions, such as so-called "manufactured defaults" and "orphaned CDSs", are outside of the scope of our paper and are left for future research. Also, our analysis is limited by several simplifying assumptions that make the model tractable. For example, we only have a single lender, we do not endogenize the number of protection sellers, and we ignore the role of liquidity in the CDS market and the bond market (see Oehmke and Zawadowski (2015) for the effects on the bond market).

Keeping these caveats in mind, our results have important policy implications. First, any regulation that prohibits intervention by protection sellers might reduce firm value instead of increasing it. Second, improved reporting requirements in the CDS market that resemble the SEC rules for the stock market can increase the likelihood that the CDS market increases firm value.

Figure 1: Timeline of events in the active CDS buyer model

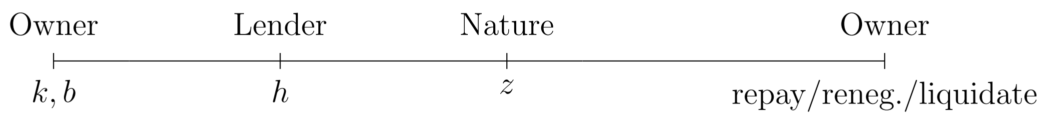


Figure 2: Optimal default decision in the active CDS buyer model

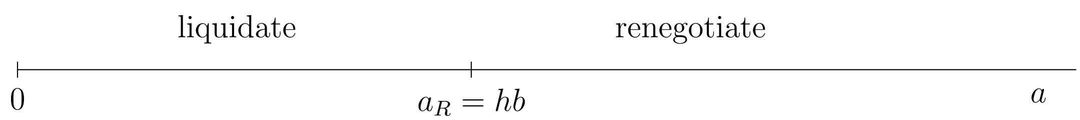


Figure 3: Timeline of events in the extended model

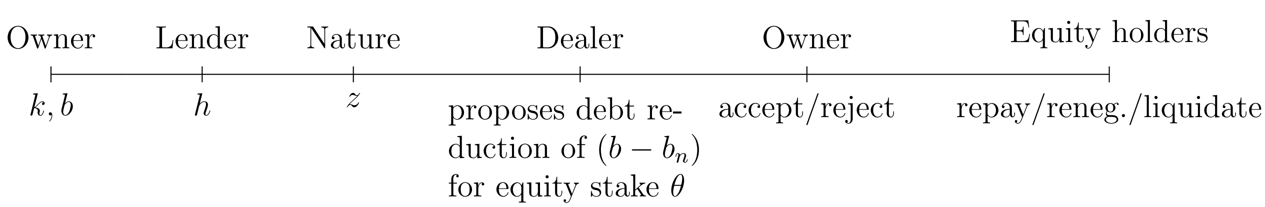
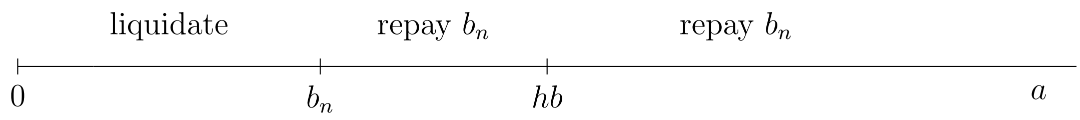


Figure 4: Optimal default decision – extended model

If $b_n \leq hb$:



If $b_n > hb$:

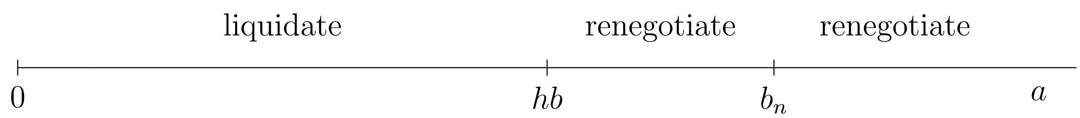
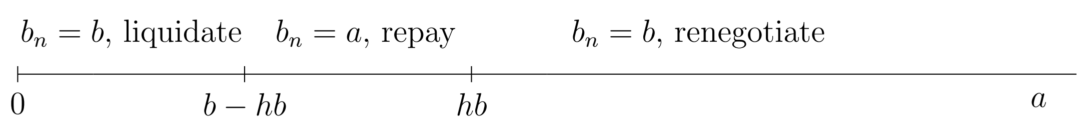
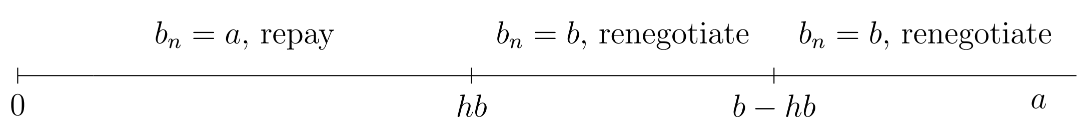


Figure 5: Optimal intervention and optimal default decision – extended model

If $b - hb < hb$ (i.e., $h > 1/2$):



If $b - hb \geq hb$ (i.e., $h \leq 1/2$):



Firm	Year	Summary	Source
Codere	2013	Protection buyer (Blackstone GSO) offers financing to Codere in return for technical default. The losing party is the protection seller (unknown).	Bloomberg
Caesars Ent.	2014	Protection buyer (Elliott) pushes for early bankruptcy. Protection seller (Blackstone GSO) is against bankruptcy. Elliott is also a major bondholder. Blackstone GSO is a major loan holder. Eventually, Caesars files for bankruptcy before CDS maturity, Elliott profits.	WSJ
Forest Oil	2014	Financially distressed Forest Oil wants to merge with Sabine Oil & Gas, a competitor, to avoid default. Protection buyers (unknown) purchase stocks in order to vote against the deal.	WSJ
RadioShack	2014	BlueCrest Capital Management, DW Investment Management and Saba Capital Management sell CDS protection on RadioShack. When RadioShack becomes financially distressed, the protection sellers offer new loans to avoid default.	WSJ
Norske Skog	2016	Protection seller (Blackstone GSO) keeps financially distressed firm alive and collects CDS spread.	Reuters
iHeartMedia	2016	iHeartMedia misses a payment on a bond owed to its subsidiary, which triggers a CDS credit event. This eliminates the CDS contracts of bondholders, which might make a future debt restructuring easier for the firm.	WSJ
Matalan	2017	Protection sellers (unknown group of hedge funds) offer financing to keep financially distressed firm alive. They request that the new debt is issued by a new legal entity.	FT
Hovnanian	2018	Protection buyer (Blackstone GSO) offers financing in return for default of a subsidiary of Hovnanian.	WSJ
McClatchy	2018	Protection seller (Chatham) provides financing to McClatchy, asks to move debt to subsidiary. Protection seller collects CDS spread on parent company with zero default risk. Deal is cancelled a few months later.	Bloomberg

(*continued*)

Table 1: **Summary of recent cases of CDS investor involvement**.

Firm	Year	Summary	Source
Supervalu	2018	United Natural Foods wants to acquire Supervalu and needs to issue new debt to finance the deal. Protection buyers (unknown) for Supervalu would lose money in the deal, because their CDS protection would become worthless. They convince debt underwriter (Goldman Sachs) to make Supervalu a co-borrower of the new debt.	Bloomberg
Windstream	2019	Protection buyer (Aurelius Capital Management) sues Windstream for violating bond covenant. Windstream files for bankruptcy.	WSJ
Neiman Marcus	2019	Protection buyer (Aurelius Capital Management) pushes Neiman Marcus to link more of its debt to CDS contracts.	WSJ
Thomas Cook	2019	Bondholders hedged with CDSs threaten to block a debt restructuring because they want a credit event to be triggered. Additionally, a protection seller offers financing to the firm.	FT

Table 1: **Continued**

	Owner	Dealer	Lender
Debt repayment	$(1-\theta)(a-b_n)$	$\theta(a-b_n)$	b_n
Debt renegotiation	$(1-\theta)(a-b_r)$	$\theta(a-b_r)$	b_r
Liquidation	0	$-hb$	hb

Table 2: **Extended model – terminal payoffs for each player.** The symbol a denotes the asset value of the firm; b is the face value of debt; b_n is the new face value of debt after a possible equity injection; b_r is the face value of debt that is agreed on in a debt renegotiation; h is the lender's CDS hedge ratio; θ is the fraction of the firm owned by the CDS dealer after a possible equity injection.

References

Aghion, Philippe, and Patrick Bolton, 1992, An Incomplete Contracts Approach to Financial Contracting, *Review of Economic Studies* 59, 473–494.

Bartram, Söhnke M, Jennifer S Conrad, Jongsub Lee, and Marti G Subrahmanyam, 2018, Credit Default Swaps Around the World: Investment and Financing Effects, *SSRN Electronic Journal*.

Batta, George E, and Fan Yu, 2018, Credit Derivatives and Firm Investment, *SSRN Electronic Journal*.

Bolton, Patrick, and Martin Oehmke, 2011, Credit Default Swaps and the Empty Creditor Problem, *Review of Financial Studies* 24, 2617–2655.

Campello, Murillo, and Rafael Matta, 2016, Investment Risk, CDS Insurance and Firm Financing, *SSRN Electronic Journal*.

Chang, Xin Simba, Yangyang Chen, Sarah Qian Wang, Kuo Zhang, and Wenrui Zhang, 2019, Credit Default Swaps and Corporate Innovation, *Journal of Financial Economics* 134, 474–500.

Colonnello, Stefano, Matthias Efing, and Francesca Zucchi, 2018, Empty Creditors and Strong Shareholders: The Real Effects of Credit Risk Trading, *Journal of Financial Economics* forthcoming.

Danis, Andras, and Andrea Gamba, 2018, The Real Effects of Credit Default Swaps, *Journal of Financial Economics* 127, 51–76.

Gertner, Robert, and David Scharfstein, 1991, A Theory of Workouts and the Effects of Reorganization Law, 46, 1189–1222.

Guest, Paul M, Nikolaos Karampatsas, Dimitris Petmezas, and Nickolaos G Travlos, 2017, Credit Default Swaps, Financial Distress and Corporate Acquisitions, *SSRN Electronic Journal*.

Hart, Oliver, and John Moore, 1988, Incomplete Contracts and Renegotiation, *Econometrica* 56, 755–785.

Kim, Gi H, 2016, Credit derivatives as a commitment device: Evidence from the cost of corporate debt, *Journal of Banking and Finance* 73, 67–83.

Maug, Ernst, 1998, Large Shareholders as Monitors: Is There a Trade-Off between Liquidity and Control?, *The Journal of Finance* 53, 65–98.

Narayanan, Rajesh, and Cihan Uzmanoglu, 2018, Credit Default Swaps and Firm Value, *Journal of Financial and Quantitative Analysis* 53, 1227–1259.

Noldeke, Georg, and Klaus M Schmidt, 1995, Option Contracts and Renegotiation: A Solution to the Hold-up Problem, *The RAND Journal of Economics* 26, 163.

Oehmke, Martin, and Adam Zawadowski, 2015, Synthetic or Real? The Equilibrium Effects of Credit Default Swaps on Bond Markets, *Review of Financial Studies* 28, 3303–3337.

Subrahmanyam, Marti G, Dragon Yongjun Tang, and Sarah Qian Wang, 2014, Does the Tail Wag the Dog?: The Effect of Credit Default Swaps on Credit Risk, *Review of Financial Studies* 27, 2927–2960.

Wong, Tak-Yuen, and Jin Yu, 2018, Credit Default Swaps and Debt Overhang, *SSRN Electronic Journal*.

Appendices

A. Proof that the optimal hedge ratio is $h \leq 1$ in the extended model in Section 2

We solve the model by backwards induction under the assumption $h > 1$, and show that the resulting debt value cannot be optimal from the debt holder's perspective.

From Figure 4, because for $h > 1$ we have necessarily $b < hb$, considering the case $b_n \leq b < hb$, the firm's default decision is

In order to determine the outside option of the initial owner in case she rejects a proposal from the protection seller, we also derive the optimal default decision of the owner without an equity injection (this figure is analogous to Figure IA.1 in the internet appendix).

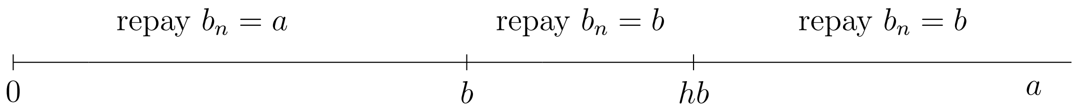
Using the same logic as in the proof of Lemma 2, we can derive the optimal θ in the case $h > 1$:

$$\theta = \begin{cases} 1 - \frac{a-b}{a-b_n} & \text{if } a \geq b, \\ 1 & \text{if } a < b. \end{cases}$$

Analogously to the proof of Lemma 3, one can derive the optimal b_n, which becomes:

$$b_n = \begin{cases} b & \text{if } a \geq b, \\ a & \text{if } a < b. \end{cases}$$

Both cases, $a \geq b$ and $a < b$, are followed by repayment of debt. Having the optimal θ and b_n, we can summarize the solution graphically as follows:



Similarly to Equation (19), we can now derive the market value of debt,

$$M = \int_0^{b/k^\alpha} [\underbrace{(b-a)}_{\text{early repaym.}} + \underbrace{a}_{\text{remaining repaym.}}] d\Gamma(z) + \int_{b/k^\alpha}^{Z} \underbrace{b}_{\text{full repayment}} d\Gamma(z) - \underbrace{\int_0^{b/k^\alpha} (b-a) d\Gamma(z)}_{\text{upfront CDS premium}},$$

which can be simplified to

$$M = \int_0^{b/k^\alpha} a d\Gamma(z) + \int_{b/k^\alpha}^{Z} b d\Gamma(z).$$

Because this expression does not depend on h, the debt holder cannot do better (nor worse) by increasing his hedge ratio beyond 1.

B. Proof of Lemma 1

From Equation (11), debt renegotiation is only feasible if $a \geq hb$. Also, we know that if renegotiation is feasible, the equity holders prefer repayment to renegotiation if $b_n \leq hb$. If renegotiation is infeasible, the equity holders prefer repayment to liquidation if $a \geq b_n$. Combining these three inequalities produces the stated result.

C. Proof of Lemma 2

We consider two possible cases: $b_n \leq hb$ and $b_n > hb$. As for the first case,

- if $a \geq hb$, the owner's payoff if the offer is accepted is $(1-\theta)(a-b_n)$, corresponding to debt repayment, and if the offer is rejected it is $a - b_r(b) = a - hb$, corresponding to renegotiation.[19] Equating the two payoffs we have

$$\theta = 1 - \frac{a - hb}{a - b_n}.$$

[19]Notice that the $b_r(b)$ is the b_r under rejection, i.e., without a debt reduction. Under our parameter assumptions, the two expressions for b_r are the same, but in general they are not. For this reason, we use the notation $b_r(b)$, not b_r.

Because $a \geq hb \geq b_n$, then $\theta \leq 1$, with strict inequality if either of the two inequalities $a \geq hb$ and $hb \geq b_n$ is strict.

- if $b_n \leq a < hb$, the payoff if the offer is accepted is $(1-\theta)(a-b_n)$ from debt repayment, and 0 from liquidation if the offer is rejected. From the equality of the two payoffs we find $\theta = 1$;
- if $a < b_n$, the payoff is 0 from liquidation regardless if the offer is accepted or rejected. The dealer can choose any $\theta \in [0, 1]$. We assume that in equilibrium, $\theta = 1$.

As for the case $b_n > hb$,

- if $a \geq b_n$, the owner's payoff if the offer is accepted is $(1-\theta)(a-b_r(b_n)) = (1-\theta)(a-hb)$, deriving from renegotiation of the debt b_n, and if the offer is rejected it is $a - b_r(b) = a - hb$, corresponding to renegotiation of the debt b. Equating the two payoffs we have $\theta = 0$;
- if $hb \leq a < b_n$, the payoff if the offer is accepted is $(1-\theta)(a-b_r(b_n))$, from renegotiation of b_n, and if the offer is rejected it is $a - b_r(b)$, from renegotiation of b. Hence, $\theta = 0$;
- if $a < hb$, the payoff is 0 from liquidation regardless if the offer is accepted or rejected. As before, we assume that in equilibrium, $\theta = 1$.

D. Proof of Lemma 3

Motivated by Lemma 2, and the expression of the optimal θ, we consider two possible cases: $a > hb$ and $a < hb$. As for the first case:

- If the dealer chooses a sufficiently low b_n such that $b_n \leq hb$, because this would be a case of repayment of b_n, his payoff is

$$\max_{b_n} \left\{-(b - b_n) + \theta(a - b_n)\right\}.$$

  After substituting for $\theta = 1 - \frac{a-hb}{a-b_n}$ from Lemma 2, this simplifies to $\max_{b_n}(-b + hb)$, which is independent of b_n and negative.

- If the dealer chooses a sufficiently high b_n such that $b_n > hb$, because this in this case the debt would be renegotiated, his payoff is

$$\max_{b_n} \left\{ -(b - b_n) + \theta(a - b_r(b_n)) \right\}.$$

Based on Lemma 2, the optimal θ is 0 in this case. Hence, the payoff simplifies to $\max_{b_n}(-(b - b_n))$, which is maximized at $b_n = b$. This implies that no recapitalization takes place, and the dealer's payoff is zero, which is higher than in the case considered above.

Therefore, if $a > hb$, it is optimal for the dealer not to reduce the debt.

As for the second case, $a < hb$:

- suppose the dealer chooses a sufficiently low b_n such that $b_n \leq hb$.
  - If he chooses a sufficiently low b_n such that $b_n \leq a < hb$, because this would be a case of repayment, he gets

    $$\max_{b_n} \left\{ -(b - b_n) + \theta(a - b_n) \right\}.$$

    After substituting for $\theta = 1$ from Lemma 2, this simplifies to $\max_{b_n}(a - b)$, which is independent of b_n and negative. Because $a < hb$, then $a < b$, and therefore the payoff is negative.
  - Otherwise, if he chooses a value of b_n such that $b_n > a$, then he gets

    $$\max_{b_n} \left\{ -(b - b_n) - hb \right\},$$

    because this would be a case of liquidation.

  We now show that the dealer is better off by choosing b_n such that he is in the case $b_n \leq a < hb$ rather than in the case $b_n > a$. This is the case because, for the dealer's payoff in the two cases, it is true that $a - b > \max_{b_n} \left\{ -(b - b_n) - hb \right\}$, and this inequality is equivalent to $a > \max_{b_n} \left\{ b_n - hb \right\}$. In the last inequality, the left-hand side is always positive, and the right-hand side is negative or zero, since $b_n \leq hb$ by assumption. We conclude that $b_n \leq a$.

- If the dealer chooses a sufficiently high b_n such that $b_n > hb$, then this would be a case of liquidation with payoff

$$\max_{b_n} \{-(b - b_n) - hb\}.$$

The maximum payoff is $-hb$, which is attained at $b_n = b$.

Comparing the optimal payoffs in the cases $b_n \leq hb$ and $b_n > hb$ reveals that there is no globally optimal b_n. If $-hb > a - b$, it is optimal to choose $b_n = b$, which is followed by liquidation. Otherwise, the optimal new debt is $b_n = a$, which is followed by repayment.

We conclude the case $a < hb$ as follows: If $-hb > a - b$, or $a < b - hb$, it is optimal to choose $b_n = b$, which is followed by liquidation. Otherwise, the optimal new debt is $b_n = a$, which is followed by repayment.

To describe the optimal b_n across all possible cases, we have to take into account two thresholds, hb and $b - hb$. It is easy to show that $hb < b - hb$ is equivalent to $h < 1/2$. Hence, we have Figure 5.

E. Proof of Lemma 4

From the proof of Lemma 3, if $a \geq hb$, there is renegotiation of the debt, no intervention by the dealer, and no payment from the CDS contract. On the other hand, if $a < hb$ the equilibrium strategy depends on whether h is higher or lower than $1/2$, see Figure 5. If $h \leq 1/2$, with $b_n = a$ there is early repayment in the amount of $b-a$, plus final repayment of the remaining debt b_n, and no payment from the CDS contract. If $h > 1/2$, there is the same policy if $b - hb \leq a < b$, and if $a < b - hb$ there is no intervention followed by liquidation, with the compensation from the CDS contract.

Hence, if $b - hb > hb$, the expected total payoff to the debt is

$$M = \int_0^{z_R} [\underbrace{(b-a)}_{\text{recapitalization}} + \underbrace{a}_{\text{debt paym.}}] d\Gamma(z) + \int_{z_R}^{Z} \underbrace{hb}_{\text{renegotiation}} d\Gamma(z) - \underbrace{\int_0^{z_R} (b-a) d\Gamma(z)}_{\text{CDS premium}}, \quad (19)$$

where $z_R = hb/k^\alpha$. The last term is the expectation of the payment by the dealer, which corresponds to the recapitalization of the firm by $b - a$. This expression simplifies to

$$M(k, b, h) = \int_0^{z_R} a d\Gamma(z) + \int_{z_R}^{Z} hb d\Gamma(z). \quad (20)$$

If $b - hb < hb$, then the total expected payoff to the debt is

$$M = \int_0^{z_L} (\underbrace{0}_{\text{bond payoff}} + \underbrace{hb}_{\text{CDS payoff}})d\Gamma(z) + \int_{z_L}^{z_R} [\underbrace{(b-a)}_{\text{recapitalization}} + \underbrace{a}_{\text{debt paym.}}]d\Gamma(z)$$
$$+ \int_{z_R}^{Z} \underbrace{hb}_{\text{renegotiation}} d\Gamma(z) - \underbrace{\left[\int_0^{z_L} hbd\Gamma(z) + \int_{z_L}^{z_R} (b-a)d\Gamma(z)\right]}_{\text{CDS premium}},$$

where $z_L = (b - hb)/k^\alpha$ is the liquidation threshold for this case.