The Empirical Relevance of the Okishio Theorem: An Autoregressive Distributed Lag Approach

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Abstract

The empirical relevance of the theoretically robust Okishio Theorem has not been verified since Shaikh's famous critique of its criterion for the choice of technique, which motivates this paper. Our investigation of the US nonfinancial corporate economy (1947-2018) using an autoregressive distributed lag approach finds that the steadily increasing organic composition of capital has a significantly positive long-run effect on the average rate of profit when controlling for the real wage rate. This finding thus provides empirical support for the Okishio Theorem.
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JEL Classifications: B5, O3

1 Introduction

The Okishio Theorem (Okishio, 1961) states that, as long as the real wage rate is fixed, *any* technical change (in one or multiple industries) that is profitable under the prevailing prices will raise the economy-wide long-run equilibrium rate of profit. This is in stark contrast with Marx's argument that the ever-increasing organic composition of capital tends to decrease the general rate of profit, leading many to examine the robustness of the Okishio Theorem as well as to reflect on Marx's hypothesis.[1]

It turns out that the Okishio Theorem is theoretically robust considering the presence of fixed capital (Okishio and Nakatani, 1975; Roemer, 1979), joint production (with some extra restrictions) (Bidard, 1988), product innovation (Nakatani and Hagiwara, 1997; Fujimori, 1998), or historical cost accounting (Laibman, 2001; Tazoe,

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[1] It is beyond the scope of this paper to examine Marx's hypothesis in greater depth.

2011). However, Shaikh's critique (1978) on the relevance of Okishio's criterion for the choice of technique, i.e., profitability of a technical change, has only been commented on theoretical grounds (see, e.g., Roemer, 1979; Nakatani, 1980; Steedman, 1980) while the critique is empirical in nature. The primary goal of this paper is to econometrically examine whether the Okishio Theorem holds in reality. To the best of our knowledge, this is the first paper to do so.

Based on data of the post-WWII U.S. nonfinancial corporate economy, our econometric exercise using the autoregressive distributed approach provides support for the Okishio Theorem. Specifically, using the organic composition of capital (OCC) as an indicator of production technique and holding the real wage rate constant, the ever-increasing OCC has a long-run positive effect on the average rate of profit at the 1 percent significance level. We further argue that the Okishio Theorem is simply a formal confirmation of the economic intuition that lower production cost leads to higher economic efficiency, which is also explicitly noted by Marx (see the concluding section), while the actual division of the fruits from increased efficiency is subject to class struggle. Therefore, we propose to go beyond the Okishio Theorem and focus more on how class struggle interacts with the process of technical change.

This paper proceeds as follows: Section 2 reviews the empirical literature on the Okishio Theorem. Section 3 through 6 in turn present the econometric model, data issues, estimation strategy, and results. The last section suggests a future line of research and concludes.

2 The Okishio Theorem

To facilitate our derivation of an empirically testable hypothesis, it is necessary to recapitulate the theoretical Okishio Theorem in a multisector setting. Assume constant returns to scale, no joint production, and the turnover period is uniform across sectors. Let R be the equilibrium rate of profit; let M be the $n \times n$ constant capital input (flow, including depreciation) matrix whose generic element M_{ij} denotes the material amount of good i needed to produce 1 unit of good j; let K be the matrix of the capital advanced (stock) with the same dimensions. Let l be the $1 \times n$ labor input vector (per unit of output), p the $1 \times n$ price vector, x the $n \times 1$ gross output, and b the $n \times 1$ real wage bundle consumed by 1 unit of labor power. The equilibrium price system is as follows:

$$p = RpK + pM + pbL, \tag{1}$$

which is derived from the definition of the rate of profit when it is equalized across sectors:

$$\vec{R} = [p - (pM + pbL)] \oslash (pK). \tag{2}$$

From (1) and (2) we can see that the equilibrium rate of profit coincides with the average rate of profit in the long-run equilibrium in the following way. Let x be the equilibrium output that is associated with the price equilibrium. Multiply (1) through by x and rearrange, we have $px = RpKx + pMx + pbLx$, or:

$$R = \frac{px - (pMx + pbLx)}{pKx}. \tag{3}$$

Within a short horizon where p and b are fixed, Okishio's criterion for the choice of technique is that capitalists will adopt a new production technique $\{K', M', L'\}$ as long as it is profitable, i.e.:

$$p > RpK' + pM' + pbL', \tag{4}$$

or equivalently $p = \tilde{R}pK' + pM' + pbL'$ where $\tilde{R} > R$. If this is true, then in the long run after all kinds of adjustment, the new equilibrium emerges as:

$$p' = R'p'K' + p'M' + p'bL'. \tag{5}$$

With some standard assumptions, it must be the case that $R' > R$. This is the Okishio Theorem.[2]

Shaikh (1978, 1980) contends that, rather than to increase the short-run rate of profit as suggested by the Okishio Theorem, capitalists instead choose new production techniques in order to cut down the production unit cost $pM + pbL$, or equivalently, to increase the profit margin $p - pM - pbL$ to create room for price cutting and more market shares. If Shaikh's criterion is true, it is possible that a new technique that raises the profit margin might lower the short-run rate of profit at the prevailing prices if a large investment in the capital stock is adopted at the same time. Despite the theoretical critiques of Shaikh's criterion, (Roemer, 1979; Nakatani, 1980; Steedman, 1980), Shaikh's own critique is no trivial, and it can only be effectively answered with empirical research now that the debate involves judging the empirical relevance of the two competing criteria. This topic seems to be once in Marx's research agenda, although he never does it. Right after Marx (Marx and Engels, 1996, pp. 623) has elaborated how competition, accumulation (concentration of capital) and credit (centralization of capital) lead to the increase in the organic composition of capital in Capital Volume I, Engles, in the third German edition, immediately puts a footnote:

> In Marx's copy there is here the marginal note: "Here note for working out later; if the extension is only quantitative, then for a greater and a smaller capital in the same branch of business the profits are as the magnitudes of the capitals advanced. If the quantitative extension induces qualitative change, then the rate of profit on the larger capital rises simultaneously.

Based on the context of that chapter, "quantitative extension" here means the accumulation of capital without the change in the organic composition of capital (constant return to scales in production) while "qualitative extension" means the accumulation

[2]The proof is available on request. It can also be proved that this result holds even when we consider differential turnover periods, different schedules of wage payment, and unproductive labor.

of capital ("the larger capital") with an increase in the organic composition of capital. The last sentence in his note indicates that Marx is actually hypothesizing Okishio's criterion, i.e., the increasing organic composition of capital will lead to an increase in the short-run rate of profit for the larger capital that adopts the technical change. But he is cautious enough to cast it as a hypothesis and set it for future validation.[3]

Which criterion is true? It is hard to find an answer at the micro level from the real world data. On the one hand, the material input matrices M and K are not officially recorded in most capitalist countries.[4] On the other hand, prices in the real world change very frequently and it is hard to evaluate whether a technical change is profitable or profit-margin-increasing based on changing prices. Even if we are able to find some proxy data, the measurement errors might outweigh the fact that these two criteria might be highly overlapped, rendering the comparison unreliable. As a result, we choose not to empirically compare the two criteria, but instead directly estimate whether a predominant type of technical change in an economy has a long-run positive effect on the average rate of profit by controlling for the real wage rate. If it does, it provides support for the Okishio Theorem and implies that the dominant technical change is profitable according to the reverse of the Okishio Theorem (Dietzenbacher, 1989).

So far in the literature, there are only a few empirical studies on the Okishio Theorem. Morimoto (2013) argues, by using descriptive data, that the discovery of the Okishio Theorem is mainly motivated by the rise in the rate of profit in the 1950s in Japan. Park (2005) investigates how well Okishio's criterion predicts the evolution of actual capitalist economies by simply counting the ups and downs of the expected rate of profit. Tazoe (2013) use the Japanese input-output tables to impute the equilibrium rate of profit and finds a rising trend in the imputed series, which does not directly investigate the relationship between the actual rate of profit and technical change. By also using the input-output tables, Hashimoto (2018) finds that for 124 out of 1400 Japanese industries where technical change takes place, the cost criterion is observed, while the relationship between technical change and rate of profit is also left untreated. These studies are all descriptive in nature and might not have revealed the true causal link that the Okishio Theorem suggests. Our exercise below aims to fill in this lacuna.

3 The measurement of main variables

Moving from the theoretical setting to an empirical one poses the difficulty of data availability. Since a comprehensive material input-output relation is seldom recorded at the industry or economy level, and most national accounts only systematically record economy- or industry-level sales, we have to transform the material input-

[3]Groll and Orzech (1987) read this note as a textual evidence that Marx's has abandoned his own law of falling rate of profit. We argue against this reading for the reason given in the concluding section.

[4]The Soviet Union and China in some past periods did produce such national accounts in material terms.

output relations according to the available data. We therefore focus on the aggregate economy for which data series are much longer than industry-level data,[5] and empirically examine the Okishio Theorem in a long enough period. Therefore, our measurement of the rate of profit is according to equation (3), and the real wage rate is measured as:

$$W = \bar{p}b, \tag{6}$$

where $\bar{p}$ is the base-year price vector.

To measure technical change, we use Marx's concept organic composition of capital and follow Shaikh's (1990) interpretation and formulation as the constant-capital/labor ratio in real terms, or what is called "capital-deepening" in the mainstream literature. Using our notations above, OCC at the aggregate level is defined as:

$$C = \frac{\bar{p}Fx}{\bar{W}Lx} \tag{7}$$

where F is the fixed-capital matrix (a component of K), and $\bar{W}$ the base-year wage rate. Output x serves as the aggregating vector. Alternatively, one can use depreciation as a proxy for constant capital. In this way, the change in OCC only reflects the change in the technical composition of capital (represented by F and L) as well as in the relative weights of industries in the aggregating process (x). The so-measured OCC confirms Marx's observation that OCC tends to rise in capitalist countries since the upward trend turns out to be very pronounced and steady as shown in figure 1 as well as by Jones (2017) and Gourio and Klier (2015).

4 The econometric model and hypothesis

We adopt a linear characterization of the relationship among the natural-log-transformed variables partly because some of our data are unit-free indexes which only preserve the growth rates of their actual levels, and we can estimate the more meaningful elasticities among variables from the following log-log model:

$$\ln R_t = \alpha + \psi \cdot \ln C_t + \omega \cdot \ln W_t + \sum_i \chi_i \cdot \ln Z_{it} + u_t, \tag{8}$$

where ψ and ω are the elasticities of R with respect to C and W, and Z_{it} are other control variables including the rate of investment (I, accumulation of capital), number of mergers and acquisitions to number of corporations (A, centralization of capital), and the relative price of fixed capital with respect to labor (P, wage pressure), which are the three factors that Marx thinks are driving the change in the OCC (Marx, 1996). Since these other factors might also affect R, it is important to control for them to rule out endogeneity bias. We also control for the terms of trade (T) since the US is an open economy such that trade might affect R and C at the same time.

[5] For example, the annual US input-output tables only begin at 1997 while the national account series dates back to 1947 or even earlier.

Furthermore, because there might be short-run adjustment processes and costs due to the adoption of new production techniques (Lucas, 1967; Foley and Sidrauski, 1970; Romer, 2012), we are also interested in differentiating between the short-run and long-run effects of technical change, and adopt the following ARDL(r, q) model (Enders, 2015, 267–76):

$$\begin{aligned}\ln R_t =& \theta + \sum_{j=1}^{r} \beta_j \cdot \ln R_{t-j} + \sum_{j=0}^{q} \psi_j \cdot \ln C_{t-j} + \\ & \sum_{j=0}^{q} \omega \cdot \ln W_{t-j} + \sum_{i} \sum_{j=0}^{q} \chi_{ij} \cdot \ln Z_{i,t-j} + v_t\end{aligned} \tag{9}$$

The theoretical advantage of the ARDL model is threefold. First, it allows the effects of C and W on R to play out in several time periods: The increase in C entails reorganization and adjustment (for example, installing new machines, laying off workers, and coordinating the new labor process) of the production process. As a result, the effect of increasing C might be different between the first period when capitalists have to pay adjustment costs such as dismissal compensation, and the latter periods. Second, it can bridge the gap between the long-run equilibrium examined in the Okishio Theorem and the fact that the actual economy is not always in equilibrium, but rather oscillates around a center of gravity (Duménil and Lévy, 2002). The ARDL model can trace the effect of a technical change in a sufficiently long time span, which averages out the short-run fluctuations of the economy. Third, by including sufficient lags of all variables, the problem of endogeneity can be technically alleviated (Pesaran et al., 1999) .

It is expected by the Okishio Theorem that, in the long run, the rising OCC will raise the rate of profit if the real wage rate is constant as long as Okishio's criterion for the choice of technique is in play; conversely, if we observe a rising general rate of profit under a constant real wage rate, the underlying technical changes in the economy must be predominantly profitable when the new technique is initially adopted under the old set of prices, i.e., Okishio's criterion is in play (Dietzenbacher, 1989). Therefore, the parameters of interest are ψ_0 which is the contemporaneous (impact) effect of C on R, and the β_j's that serve to transfer the effect of C into future periods. The interpretation of ψ_j for $j > 0$ is not straightforward, the partial effect ρ in year $t + j$ of one unit permanent increase in C in year t will be:

$$\rho_{t+j} = \sum_{i=1}^{min\{p,j\}} \beta_i \cdot \rho_{t+j-i} + \psi_j \tag{10}$$

where $\psi_j = 0$ if $j > q$. The interim effect η of period $t + s$ will simply be the cumulative sum of all partial effects up to this period:

$$\eta_{t+s} = \sum_{j=0}^{s} \rho_{t+j}. \tag{11}$$

The long-run effect when time goes to infinity, assuming the system is stable ($|\sum_j \beta_j| < 1$), is calculated by:

$$\lambda = \sum_{j=0}^{\infty} \rho_{t+j} = \frac{\sum_{j=0}^{q^c} \psi_j}{1 - \sum_{j=1}^{p} \beta_j} \tag{12}$$

and our ultimate interest is to test the hypothesis:

$$\lambda > 0. \tag{13}$$

5 Data

We use the time series of the US aggregate nonfinancial corporate economy for the period 1947-2018. Most original data are all from official sources: the National Income and Product Accounts (NIPA) and the Fixed Assets Accounts (FAA) published by Bureau of Economic Analysis (BEA), as well as from the Bureau of Labor Statistics (BLS) and the Federal Reserve Board (FRB). The measurement and source of data for each variable is summarized in Table xxx. The nonfinancial corporate economy accounts for some 70% of the business sector (Duménil and Lévy, 2002). Treating the nonfinancial corporate economy enables us to examine the purest form of capitalist production by excluding the non-incorporated business whose profit motives might not be as strong as the corporate business, and by excluding the financial sector which is subject to huge fluctuations. Data for merger and acquisitions are from Institute of Merger and Acquisition.

Further remarks on the measurement of the rate of profit is necessary since there are many ways to do this in the literature (Basu and Vasudevan, 2013). Our main measure (R) of the rate of profit is the ratio between aggregate (after-tax) profits (with inventory valuation adjustment and capital consumption adjustment) and the net worth of the economy, rather than following the convention of using fixed assets as the denominator. This is because one of the central concerns in the Okishio Theorem is the criterion for the choice of technique: the for-profit capitalists would presumably choose a new technique that maximizes the rate of profit that is of utmost importance to them. Because capital advanced (tied up) for production not only includes fixed assets, but also inventories of circulating constant capital and the wage funds (Brody, 1970). Furthermore, because of the modern complex financial relations, we think net worth is the most accurate measure of capital advanced in modern capitalist production, and the return on net worth (equity) is a key indicator for investment decision in the real world corporate finance practice. However, as robustness check, we also report results based on the alternative measure ($R.f$) using non-residential fixed assets as the denominator. As an alternative measure (D) of the OCC, we also use depreciation of non-residential fixed assets in place of fixed assets. Another note is that we do not differentiate between productive and unproductive economic activities since this is not a central concern in the debate on the Okishio Theorem, and the literature still lacks a clear-cut and analytic criterion for this differentiation (Laibman, 1992). Therefore, this issue will be reserved for future research. Table

1 reports the summary statistics and figure 1 the time series of the main variables. Between 1947 and 2018, the rate of profit is subject to huge fluctuations. There is a slightly downward trend in the rate of profit before the 1990s, and a slightly upward one since the 1990s. OCC (C or D) and W steadily increase for the whole period.

Table 1: Summary statistics, US nonfinancial corporate economy (1947–2018)

Variable	Unit	Obs.	Min.	Mean	Max.	Sd.
R	Percent	72	3.14	5.15	7.90	1.31
R.f	Percent	72	3.47	5.87	9.86	1.41
C	1947 = 1	72	1	1.81	2.90	0.56
D	1947 = 1	72	1	3.66	8.41	2.28
W	1947 = 1	72	1	1.95	2.62	0.44
A	Percent	72	0.02	0.16	0.37	0.08
I	Permille	72	0.08	0.11	0.14	0.01
P	1947 = 1	72	0.44	0.64	1	0.16
T	1947 = 1	72	0.64	0.79	1.01	0.12

Notes:

6 Econometric procedure and results

Our estimation method is Ordinary Least Square. Although the potential problem of omitted variable can be dealt with by including relevant controls and also by the ARDL approach, the potential problem of reverse causality remains. To partly rule out this problem, we also perform the Granger causality test in a vector autoregression setting. The econometric procedure is summarized as follows.

Step 1: Unit root test. We start by testing for the potential stochastic trends (unit roots) in the variables, which, if present, will require further special treatment to avoid spurious regression. Philips (2018) offers a comprehensive recipe as regards what to do in different circumstances within the ARDL approach. We heed the advice there.

Table 2 reports the result of a battery of unit root tests performed on each variable in log-levels. We rely on the plots of the tested variable as well as their autocorrelation and partial autocorrelation functions (figure 2) to choose the lags and types for the respective tests. Given the low power of any unit root test in a small-sample setting, we conclude that all variables except P are stationary, and P is I(1).[6]

Step 2: Model selection. We adopt the "from general to specific" method for model selection, which starts with a sufficiently large number of lag lengths, and then pair down the model step by step using F-test to remove the lags that are not significant in explaining the dependent variable. To ensure the model is well-specified,

[6] First difference of all the log-levels are stationary. Results are available on request.

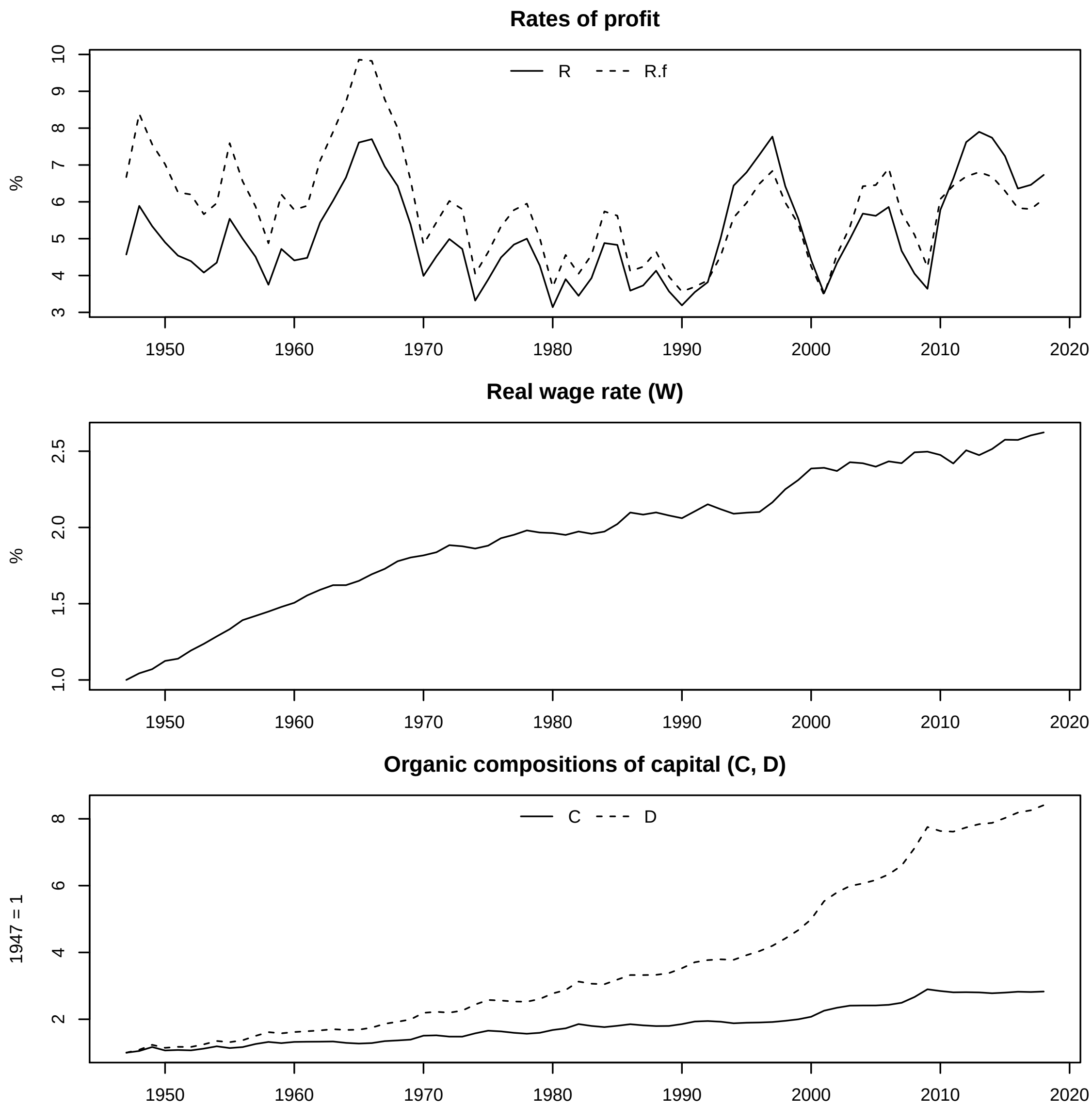

Figure 1: Time series of main variables

Notes: See notes in Table 1.

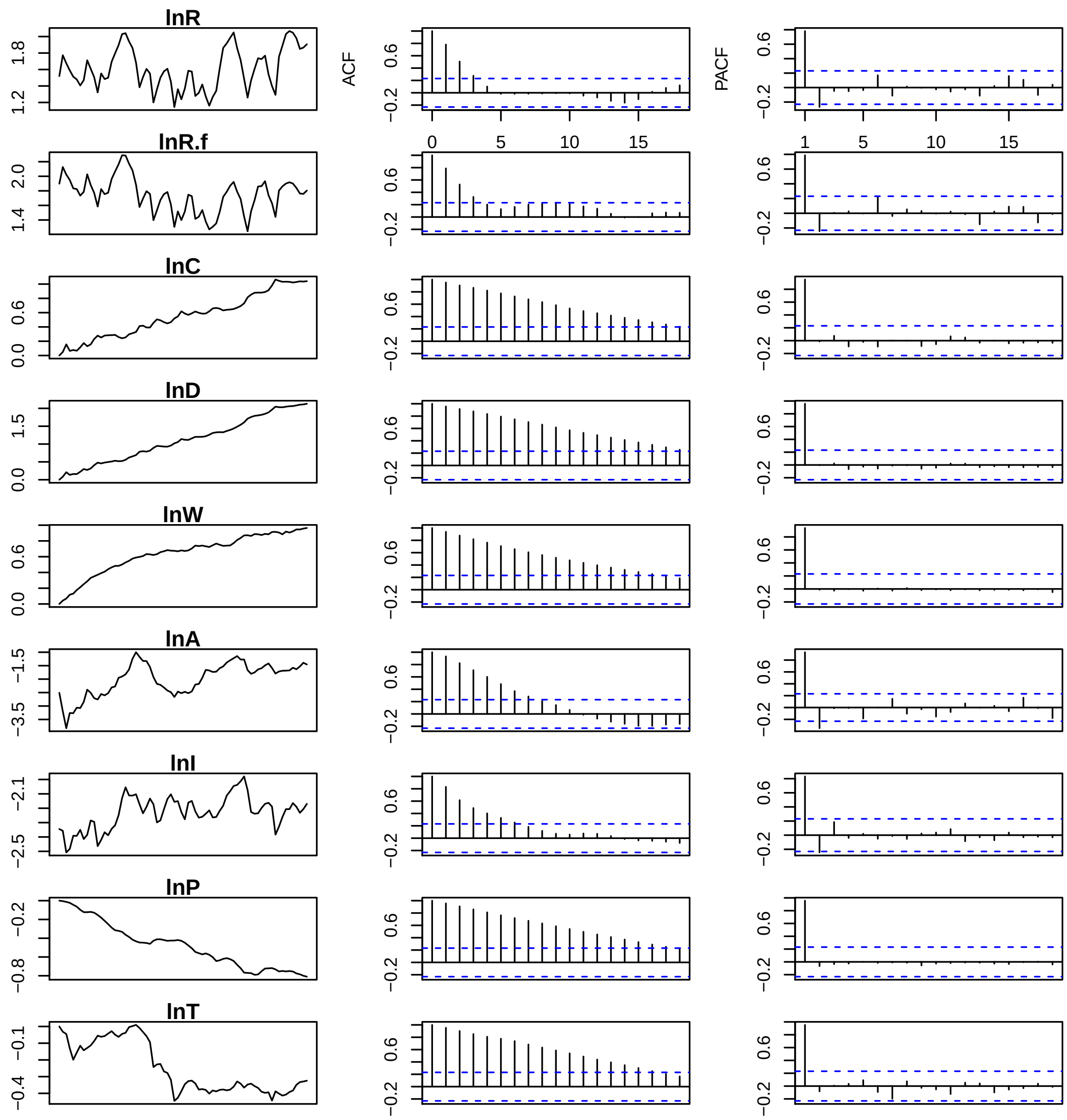

Figure 2: Plots of log-levels, their ACFs, and PACFs

Notes: Dashed lines represent 95% confidence interval.

Table 2: Unit root tests

Variable	Type	Lag	ADF	PP	DF-GLS	ZA	Break
lnR	constant	2	-0.2	-3.02**	-3.15***	-4.8***	1972
lnR.f	constant	2	-0.51	-3.03**	-2.83***	-4.96***	1972
lnC	constant, drift	1	-0.64	-3.2*	-3.49**	-4.54	2007
lnD	constant, drift	1	-0.4	-2.81	-3.04**	-4.78	2000
lnW	constant, drift	1	-4.08***	-3.84**	-0.71	-4.12	1951
lnA	constant, drift	2	-3.25**	-2.34	-3.13**	-3.53	1972
lnI	constant	2	-0.84	-2.53	-2.01**	-4.12	1963
lnP	constant, drift	1	-1.67	-1.79	-2.27	-3.71	1973
lnT	constant, drift	1	-1.55	-1.62	-1.8	-5.12***	1973

Notes: The null hypothesis of all tests is that the tested series has a unit root. The last column reports the break point identified by the Zivot-Andrews (ZA) test that allows for one structural break. *p<0.1; **p<0.05; ***p<0.01.

we carry out the Dickey-Fuller unit root test to make sure the residuals are stationary and the Ljung-Box test to rule out serial autocorrelation. If the residuals turn out to be non-stationary, our interpretation of the unit root test results in the first step might be inappropriate and need further adjustment. Ruling out serial correlation in the residual is necessary to have consistent estimates.

We use the 10 percent significance level as our screening criterion. We starts with 3 lags. None of the 3rd lag of all variables is individually significant, nor are they jointly significant with an F test. The same is true 2 lags are used. When we use 1 lag, the significant patterns of different variables are mixed. We use 1 lag for as final models. The regression results are reported in Table 3. The resulting three alternative models all pass the Dickey-Fuller unit root test in the residuals at the 1% significance level as well as the Ljung-Box serial autocorrelation test almost at 10%. Model (1) is our main model with R (measured with net worth) and C (measured with real non-residential fixed assets) being the dependent variable and the proxy for a production technique. Model (2) switches from using C to using D (measured with real depreciation of non-residential fixed assets). Model (3) uses $R.f$ (measured with non-residential fixed assets) instead of R. Note the coefficients of OCC are individually significant at 1% (except $\ln C$ in model 3) and $\ln W$ has the expected negative sign, though insignificantly different from zero. However, the interpretations of these coefficients in the ARDL framework is not straightforward.

Step 3: With the well-specified models, we extract the relevant OLS estimates and calculate the impact (time = 0), partial, interim, and long-run effects of OCC on the rate of profit according to equation (10), (11), and (12). Table 4 records these effects within a 10-year horizon. At time 0 where the new technique is adopted, the impact effect of the technical change on the rate of profit is negative across the three models, which confirms the intuition that the rate of profit takes a dip immediately after the new production technique is adopted due to adjustment costs. The positive effect of the new technique is to be revealed after 1 year when the partial effect is

Table 3: ARDL regression results

	Dependent variable:		
	R	R	R.f
	(1)	(2)	(3)
lnR.L1	0.58*** (0.10)	0.63*** (0.11)	
lnR.f.L1			0.50*** (0.10)
lnC	−1.81*** (0.67)		−1.31* (0.67)
lnC.L1	2.54*** (0.69)		2.01*** (0.64)
lnD		−1.92*** (0.71)	
lnD.L1		2.21*** (0.71)	
lnW	−1.10 (1.20)	−1.04 (1.24)	−0.59 (1.13)
lnW.L1	0.34 (1.09)	0.49 (1.12)	−0.04 (1.03)
lnA	−0.02 (0.10)	−0.03 (0.10)	0.02 (0.10)
lnA.L1	0.04 (0.11)	0.04 (0.11)	−0.08 (0.11)
lnI	0.92** (0.40)	0.80** (0.38)	1.02*** (0.38)
lnI.L1	−0.58* (0.31)	−0.64* (0.33)	−0.59* (0.30)
lnP.D1	0.91 (1.16)	0.87 (1.19)	1.30 (1.10)
lnP.D1.L1	0.03 (1.00)	0.01 (1.03)	0.70 (0.95)
lnT	1.97*** (0.45)	1.96*** (0.47)	1.95*** (0.42)
lnT.L1	−1.51*** (0.43)	−1.55*** (0.44)	−1.10** (0.41)
Constant	1.73 (1.09)	1.22 (0.99)	2.01** (0.95)
DF-stat	-5.27	-5.32	-4.92
LB-p	0.09	0.1	0.09
Observations	70	70	70
Adjusted R^2	0.81	0.80	0.81

Notes: The DF test critical value at 1% is -2.6. The null hypothesis of the Dickey-Fuller test (1 lag is used with a constant term) is the tested series has a unit root. The null hypothesis of the Ljung-Box test (1 lag is used) is that the series is not autocorrelated. *p<0.1; **p<0.05; ***p<0.01.

also highest. The interim (cumulative) effect only turns positive after 1 year in model (3), 2 years in model (1), and 3 years in model (2). As time lapses, the partial effect approach zero and the interim effect converges. The second column of Table 5 reports the cumulative effect as time goes to infinity, which is the long-run effect we define in equation (12). Next, we carry out inference to see if these positive long-run effects are statistically significantly different from zero.

Table 4: Partial and interim effects of OCC on the rate of profit

	(1)		(2)		(3)	
Time	Partial	Interim	Partial	Interim	Partial	Interim
0	−1.81	−1.81	−1.92	−1.92	−1.31	−1.31
1	1.49	−0.31	1	−0.93	1.35	0.04
2	0.87	0.55	0.63	−0.30	0.68	0.72
3	0.50	1.06	0.39	0.09	0.34	1.06
4	0.29	1.35	0.25	0.34	0.17	1.22
5	0.17	1.52	0.16	0.50	0.08	1.31
6	0.10	1.62	0.10	0.60	0.04	1.35
7	0.06	1.68	0.06	0.66	0.02	1.37
8	0.03	1.71	0.04	0.70	0.01	1.38
9	0.02	1.73	0.02	0.72	0.01	1.39
10	0.01	1.74	0.02	0.74	0	1.39

Table 5: Long run OCC elasticity of rate of profit

Model	Estimate	Standard error	First percentile
(1)	1.75	0.49	0.28
(2)	0.77	0.22	0.25
(3)	1.39	0.42	0.24

Step 4: We use the bootstrap method to generate the sampling distribution for our long-run effect estimate, and test our main hypothesis. The basic idea is to use our observable sample and carry out random re-sampling and estimation of the same model for many times (1000 in our case). Then we use the distribution of these 1000 estimates as our sampling distribution. To see the significance or the long-run estimate, we simply look at the probability of the long-run effect estimate being positive according to this sampling distribution.[7]

Figure 3 plots the histograms of the bootstrap sampling distributions of the three models. They look like a normal distribution, with most simulated estimates lying

[7] For inference, because our estimator λ is nonlinear in the model estimates, we do not use the delta method which is appropriate for linear or quasi-linear estimator or in large sample, but can lead to unforeseeable bias for nonlinear estimator, especially we have a small sample.

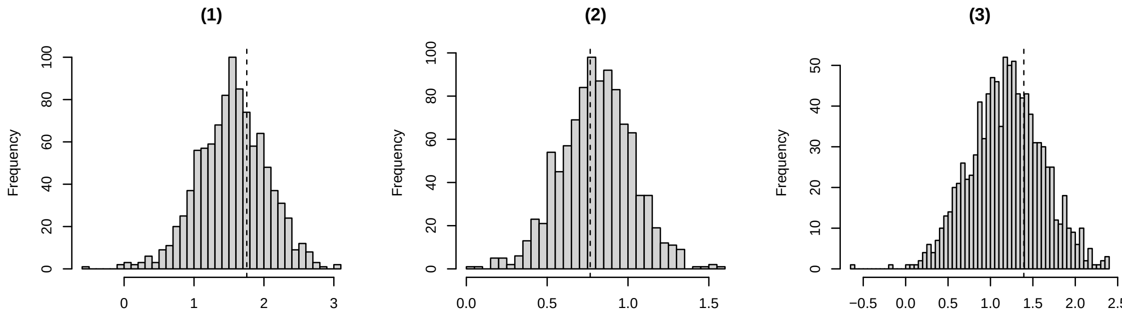

Figure 3: Bootstrap sampling (1000 runs) distribution of long-run effect λ
Notes: Dashed lines represent the actual estimates.

above zero. The standard errors, first percentiles are reported in table 5 along with the actual estimates. Inference based on a normal distribution indicates the long-run effect are significantly different from zero at 1% significance level since the lower bound (about 2.5 standard errors below the estimate) of the confidence interval is above zero. Or alternatively, the first percentile of the bootstrap distribution is above zero, which means at least 99% of the simulated estimates are positive. Therefore, we conclude that the long-run effect of OCC on the rate of profit is significantly positive.

Step 5: In the end, we perform the Granger causality test in a vector autoregressive framework to make sure that the causality we observe actually runs from OCC to the rate of profit, not the other way around. The Granger causality test bast on the trivariate VAR(1) that includes the three main variables confirms this, as is reported in table 6. For example, in Model (1), the null hypothesis that the past realizations of $\ln C$ does not explain the current realization of $\ln R$ if rejected at 1% significance level, while the null hypothesis that the past realizations of $\ln R$ does not explain the current realization of $\ln C$ cannot be rejected at 10% significance level. The same conclusion holds similarly in the other two models. Thus we tentatively conclude that our ARDL analysis does not suffer from the reverse causality problem.[8]

7 Discussion

In this paper we have found strong empirical evidence supporting the Okishio Theorem using data for the US non-financial economy, though its empirical robustness at more general level is left for future research. To conclude this paper, let us return to a fundamental question: What is Okishio's Theorem really about? Or put differently: How could a profitable technical change not raise the equilibrium rate of profit given the real wage rate is fixed? To us, the Okishio Theorem is nothing more than a formal confirmation of the economic intuition that if the cost of production is reduced, the economic efficiency of the system will be raised. This rise in economic efficiency, in

[8]Note that the Granger causality test does not deal with contemporaneous causal relationship whose direction is hard to test in principle.

Table 6: Trivariate Granger causalisty test with VAR(1)

Model (1)	F	df	df_r	p
ln$R$				
ln$C$	9.48	1.00	65.00	0.00
ln$W$	5.57	1.00	65.00	0.02
ALL	5.78	2.00	65.00	0.00
ln$C$				
ln$R$	2.80	1.00	65.00	0.10
ln$W$	8.62	1.00	65.00	0.00
ALL	5.52	2.00	65.00	0.01
ln$W$				
ln$R$	21.23	1.00	65.00	0.00
ln$C$	4.56	1.00	65.00	0.04
ALL	13.79	2.00	65.00	0.00

Model (2)	F	df	df_r	p
ln$R$				
ln$D$	4.23	1.00	67.00	0.04
ln$W$	3.29	1.00	67.00	0.07
ALL	2.20	2.00	67.00	0.12
ln$D$				
ln$R$	0.25	1.00	67.00	0.62
ln$W$	0.14	1.00	67.00	0.70
ALL	0.17	2.00	67.00	0.84
ln$W$				
ln$R$	4.89	1.00	67.00	0.03
ln$D$	2.20	1.00	67.00	0.14
ALL	4.42	2.00	67.00	0.02

Model (3)	F	df	df_r	p
ln$R.f$				
ln$C$	5.55	1.00	67.00	0.02
ln$W$	6.65	1.00	67.00	0.01
ALL	3.33	2.00	67.00	0.04
ln$C$				
ln$R.f$	0.01	1.00	67.00	0.94
ln$W$	1.30	1.00	67.00	0.26
ALL	0.69	2.00	67.00	0.50
ln$W$				
ln$R.f$	4.41	1.00	67.00	0.04
ln$C$	1.66	1.00	67.00	0.20
ALL	3.15	2.00	67.00	0.05

the new equilibrium, is either expressed in rising profitability (if the real wage rate is fixed), or in rising real wage rate (if the rate of profit is fixed), or in both. The actual distribution of the fruits from the technical change fundamentally depends on class struggle, rather than any ex ante principles.

Perhaps a more important question to most Marxists is: Does the Okishio Theorem invalidate Marx's law? It depends on what one sees as the assumption of Marx's law, given its ambiguity in (Marx, 1962, Chapter 13) which is only in a raw state and published posthumously thanks to Engels's editing efforts. Most of the time Marx assumes a constant rate of exploitation, but sometimes states the law also holds even if the rate of exploitation is rising. Roemer (1981, 102–3) proves that in the context of capital-using and labor-saving (CU-LS, a special type of rising organic composition of capital), a fixed real wage rate entails a rising rate of exploitation. If a constant rate of exploitation is instead assumed, the rate of profit could rise or fall due to a profitable CU-LS technical change (Laibman, 1982; Bidard, 2004; Foley, 2009); in the case of rising rate of exploitation, a falling rate of profit might still be compatible with a profitable CU-LS technical change as long as the real wage rate rise to some degree; in the case of fixed real wage rate, there is no room for the rate of profit to fall. Only when one interprets Marx's law as that the rate of profit must fall due to a profitable CU-LS technical change no matter how high the rate of exploitation becomes, is Marx's law nullified by the Okishio Theorem that serves as a counterexample. This might be too extreme an interpretation that Marx himself might not endorse.

If Marx were alive, he would not be surprised by the Okishio Theorem at all. When he is writing about the effect of increasing labor productivity in the sector that produces means of production on the profitability of other sectors, he comments:

> The characteristic feature of this kind of saving of constant capital arising from the progressive development of industry is that the rise in the rate of profit in one line of industry depends on the development of the productive power of labour in another. Whatever falls to the capitalist's advantage in this case is once more a gain produced by social labour, if not a product of the labourers he himself exploits. Such a development of productive power is again traceable in the final analysis to the social nature of the labour engaged in production; to the division of labour in society; and to the development of intellectual labour, especially in the natural sciences. What the capitalist thus utilises are the advantages of the entire system of the social division of labour. It is the development of the productive power of labour in its exterior department, in that department which supplies it with means of production, whereby the value of the constant capital employed by the capitalist is relatively lowered and consequently the rate of profit is raised. (Marx and Engels, 1981, pp. 85).

As Marx has rightly predicted, if we assume the rise in labor productivity was a result of a cost-reducing technical change, the relative price of the products in that "exterior department" is proved to decline necessarily (Dietzenbacher, 1989). The consequent rise in the rate of profit in any other department that use the cheapened means of production can be interpreted as a rise of the general rate of profit, either because of the ubiquitous use of the cheapened means of production, or because of the interlocking input-output relationships. Therefore, not only has Shibata (1934, 1939) predicted the Okishio Theorem, as is acknowledged by Okishio (1961), but also has Marx.

Moving forward, we think a more fruitful research agenda would be to transcend the fixed real wage assumption and focus on how technical change interact with class struggle, through which the distribution is determined from a Marxian perspective, or what (Roemer, 1981, pp. 145) calles the "social consequences" of technical change.

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