Product Concentration and Asset Prices

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Abstract

We document adecreasing product concentration within firms since the mid-2000s. Firms consolidating market power have been actively expanding product scope and diversifying production. Lower product concentration within firms is associated with higher valuation ratios, but lower productivity, expected returns, and idiosyncratic volatility. We explain these relations in a general equilibrium model of multiproduct firms with endogenous firm and industry boundaries. Parameters governing the flexible production technology are identified using the GIV approach of Gabaix and Koijen (2020) by exploiting fat tails in the distribution of product mix. The estimated model can explain the recent productivity slowdown and declining trend in idiosyncratic volatility.

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1 Introduction

Firm sboundaries have been rapidly changing over the past few decades through the creation and destruction of products. The growing ubiquity of multiproduct firms in increasingly concentrated industries has changed the scope and intensity of product market competition. We document a declining pattern in average product concentration within firms – measured at the barcode level – since the early 2000s, and that most of the decreasing trend arises from large firms expanding their product range outside of their core competence. The decline in firm product concentration over time coincides with an increasing trend in average industry concentration, both of which are depicted in Figure 1. We also find a strong negative relation between product concentration and market power across firms, suggesting that industry leaders are consolidating market power by diversifying production. This paper uncovers economic forces shaping the diverging trends in product and industry concentration through the lens of a general equilibrium model of multiproduct firms.

The expanding scope of multiproduct firms has important aggregate consequences especially given their prominent role in the economy. Bernard, Redding, and Schott (2010) document that multiproduct firms account for over 91% of output despite only representing less than half of the total number of firms. These firms are also very active in varying their product mix. Broda and Weinstein (2010) show that the majority of product creation and destruction occurs within the boundaries of the firm, highlighting the importance of an intra-firm extensive margin. Our paper links the increasing trend in product diversification to the recent productivity slowdown and declining pattern in idiosyncratic volatility.[1] We provide novel panel evidence that measures of product diversification at the barcode level are negatively associated with both productivity and idiosyncratic volatility within the firm, supporting the aggregate trend relations.

We explain the negative relation between product and industry concentration in a general

[1]Syverson (2017) and Byrne, Fernald, and Reinsdorf (2016) are examples highlighting a productivity slowdown and Herskovic, Kelly, Lustig, and Van Nieuwerburgh (2016) document recent trends in idiosyncratic volatility.

equilibrium model of multiproduct firms competing in oligopolistic markets that features endogenously evolving firm and industry boundaries. Firms use a flexible manufacturing technology that uses capital with an endogenously chosen product scope (e.g., Eckel and Neary (2010)), subject to aggregate productivity shocks. They can invest resources to expand the variety of differentiated products (*intra-firm extensive margin*), where marginal costs increase as the firm increases variety further away from core competence products to capture increasing adaptation costs. Firms can also make intensive margin adjustments by accumulating capital in conjunction with the capital allocation decision across products. Incumbent firms finance both extensive and intensive margin investments using internal and external funds. External financing is subject to convex costs and aggregate cost shocks (e.g., Gomes (2001), Eisfeldt and Muir (2016), and Belo, Lin, and Yang (2014)).

At the industry level, differentiated firms compete in a Cournot game by strategically choosing product scope and intensity with an captured through the net entry of firms (*inter-firm extensive margin*), determined by a free entry condition. A competitive fringe of startup firms without internal funds require external financing to cover a fixed cost of entry, subject to random cost shocks (e.g., capturing changes in antitrust regulation). The new firms are assumed to be in the startup phase for one period before starting production.

Our model is calibrated to explain the first and second moments of product and industry concentration, investment rates in capital and product scope, entry dynamics, idiosyncratic volatility, and valuation ratios. The model also reproduces the negative relation between product diversification and productivity within firms observed in the data. As a firm expands further away from the core range of products, production efficiency declines, consistent with our empirical findings. More broadly, we provide evidence at a more granular level for supporting the negative dynamic relation between corporate diversification and productivity within firms documented at the plant level in Schoar (2002). Each product is assumed to be subjected to project-specific risk in the model. Therefore, firm idiosyncratic risk can be reduced by increasing product variety, which allows the model to explain the negative link between product diversification and idiosyncratic volatility within firms.

The three aggregate shocks in our model, productivity, external financing, and entry costs, affect the dynamics industry and firm boundaries in significantly different ways. We show through impulse response functions that the financing and entry costs are central for generating the negative relation between product and industry concentration observed over the past decade, while productivity shocks generate positive comovement. A good productivity shock increases the marginal product of capital, inducing firms to invest more in the intensive and extensive margins. Extensive margin investment expands product scope product concentration within firms. Higher aggregate productivity also attracts the entry of new firms lowering industry concentration.

The shocks to external financing and entry costs both generate negative comovement between product and industry concentration. A positive external financing cost shock more severely affects startup firms than incumbent firms given that new firms rely exclusively on external financing to fund the fixed cost of entry while incumbent firms use a combination of internal and external financing to finance operations. Higher external financing costs reduces firm entry rates, leading to a rise in industry concentration. Incumbent firms exploit their increased market power by raising markups across their products. At the margin, the increase in profits from higher market power dominates the adverse effects from reduced external financing capacity, such that incumbent firms increase extensive and intensive margin investments, reducing product concentration. A positive entry cost shocks also reduce entry rates, increasing industry concentration. Incumbents consolidate market power by charging higher markups and expanding their product range.

In a quantitative model experiment, estimated shocks are fed into our calibrated model to explain the diverging trends in industry and product concentration for the period, 2004 - 2015, for which we have product data. We then relate these concentration patterns to the aggregate productivity slowdown and decline in the idiosyncratic volatility of fundamentals. The process for external financing costs is estimated to fit the average cost series from Eisfeldt and Muir (2016) and the entry cost process is treated as a residual for matching product concentration dynamics. The entry and external financing cost shocks are key for reproducing the negative relation between the concentration measures. We find that the increasing trend in external financing costs explains 40% of the decrease in product concentration and 10% of the increase in industry concentration. Our results therefore suggest that external financing costs are therefore a quantitatively important entry barrier for startup firms. The entry cost shock explains the remaining 60% of product concentration and 17.5% of the trend in industry concentration. The financing and entry cost shocks jointly can explain 33% of the observed productivity slowdown and 15% of the decline in the trend component of idiosyncratic volatility.

Our paper relates to the literature studying product creation in dynamic general equilibrium models (e.g., Jaimovich and Floetotto (2008) and Bilbiie, Ghironi, and Melitz (2012)) and those featuring multiproduct firms (e.g., Jovanovic (1993), Broda and Weinstein (2010), Bernard, Redding, and Schott (2010), and Eckel and Neary (2010)). Our paper also relates to dynamic models studying corporate diversification (e.g., Gomes and Livdan (2004)). We complement this literature, but differ along the following dimensions. First, we show that financing frictions can help explain the observed trends in product concentration within the boundaries of the firm that are documented in our paper. Second, show we how rising external financing costs in the early 2000s contributed to an increase in product diversification among incumbent firms, helping to explain the productivity slowdown and downward trend in idiosyncratic volatility.

The firm problem builds on dynamic firm models featuring financial frictions (e.g., Gomes (2001), Jermann and Quadrini (2012), Hennessy and Whited (2005), Eisfeldt and Muir (2016), Gourio (2013), Belo, Lin, and Yang (2014), Gilchrist, Schoenle, Sim, and Zakrajšek (2017), Begenau and Salomao (2018), Bianchi, Kung, and Morales (2019)). The industry framework relates to equilibrium models connecting competition to asset prices (e.g., Loualiche (2016), Bustamante and Donangelo (2017), Corhay (2015), Corhay, Kung, and

Schmid (2019a), Dou, Ji, and Wu (2019)) and paradigms linking asset prices to innovation (e.g., Lin (2012), Lin, Palazzo, and Yang (2017), Kung and Schmid (2015), Kung (2015), Garleanu, Panageas, and Yu (2012), and Kogan and Papanikolaou (2010)). Our mechanism for generating time-varying idiosyncratic risks relates to Herskovic, Kelly, Lustig, and Van Nieuwerburgh (2016), Decker, D'Erasmo, and Moscoso Boedo (2016), and Chen, Strebulaev, Xing, and Zhang (2018). We differ from these contributions by distinguishing between product creation within and across the boundaries of the firm. We also provide empirical and theoretical evidence that accounting for the intra-firm extensive margin is important for explaining the recent productivity slowdown and declining trend in idiosyncratic volatility.

Examining product concentration within firms relates to papers documenting increasing patterns in product concentration *within households* (e.g., Neiman and Vavra (2018), Kaplan, Menzio, Rudanko, and Trachter (2019), Jaravel (2018)) despite greater heterogeneity in consumption bundles across households. We document a declining trend in product concentration *within firms*, driven by industry leaders, providing a supply-side view of how the largest firms are acquiring greater market power by creating new products to meet the growing demand for variety across households. Argente, Lee, and Moreira (2018) studies product reallocation within the boundaries of the firm. Our framework also relates to models studying the implications of the recent acceleration in market power, especially among industry leaders, on real decisions (e.g., Gutiérrez and Philippon (2016), Autor, Dorn, Katz, Patterson, and Van Reenen (2017), Barkai (2016), De Loecker and Eeckhout (2017), Hartman-Glaser, Lustig, and Zhang (2017), and Corhay, Kung, and Schmid (2019b)). We complement this literature by showing how the industry leaders have been consolidating market power by reducing product concentration and diversifying production outside of the core.

The paper is organized as follows. Section 2 presents the dynamic general equilibrium model. Section 3 describes the key model mechanisms. The first part of Section 4 describes the data and outlines the stylized facts. The second half of Section 4 compares model simulations with the data. Section 5 concludes.

2 Model

This section presents the benchmark general equilibrium model used to examine the key determinants of product concentration within the boundaries of the firm. The economy is populated by a large number of differentiated multiproduct firms that operate in various industries. Each firm produces differentiated products where the stock of products (i.e., the firm boundary) is endogenously determined through *extensive margin* investments. They also accumulate physical capital through *intensive margin* investments, allocated across the different products. Production is subject to both aggregate technology shocks and project-specific disturbances. Investment is financed through internal and external financing, where external financing is subject to convex costs and aggregate disturbances. Industry boundaries are endogenously determined by the entry and exit of firms.

A competitive fringe of startup firms can enter by paying a fixed stochastic startup cost common across firms that can only be financed with external funds. The entry cost shock captures aggregate fluctuations in the barriers to entry. The household side is characterized by a representative agent with recursive preferences following Epstein and Zin (1989) and Bansal and Yaron (2004). These preferences are quantitatively important for replicating the relation between the idioynscratic volatility of cash flows and of stock returns.

2.1 Household

We assume a representative household with recursive preferences defined over aggregate consumption, $\mathcal{C}_t$:

$$\mathcal{U}_t = \left(\mathcal{C}_t^{1-1/\psi} + \beta \left(E_t \left[\mathcal{U}_{t+1}^{1-\theta}\right]\right)^{\frac{1}{1-\theta}}\right)^{\frac{1}{1-1/\psi}}, \tag{1}$$

where $\theta \equiv 1 - \frac{1-\gamma}{1-1/\psi}$, γ captures the degree of relative risk aversion, ψ is the elasticity of intertemporal substitution, and β is the subjective discount rate.

The household inelastically provides labor services to firms in a competitive labor market, and receives the wage rate $\mathcal{W}_t$ for each hour worked. Without loss of generality, we normalize the total labor endowment to 1. The household owns all firms in the economy and receives the aggregate dividend payout from the production sector, $\mathcal{D}_t$. Solving the household problem yields a set of Euler equations that price all securities in the economy. The equilibrium one-period pricing kernel is

$$\mathcal{M}_{t,t+1} = \beta \left(\frac{\mathcal{C}_{t+1}}{\mathcal{C}_t}\right)^{-1/\psi} \left(\frac{\mathcal{U}_{t+1}}{E_t[\mathcal{U}_{t+1}^{1-\theta}]^{\frac{1}{1-\theta}}}\right)^{-\theta} \tag{2}$$

2.2 Product demand

The final consumption good, $\mathcal{Y}_t$, is produced by a competitive final goods producer using a continuum of industry goods, normalized to the interval $[0, 1]$:

$$\ln(\mathcal{Y}_t) = \int_0^1 \ln(Y_{jt}) dj, \tag{3}$$

where Y_{jt} is the quantity of industry good j demanded. Solving the expenditures minimization implies that the demand for the industry j good is:

$$Y_{jt} = \mathcal{Y}_t P_{jt}^{-1}, \tag{4}$$

where P_{jt} is the price of industry good j.

Industries are themselves composed of a continuum of firms producing differentiated products.[2] In particular, there is a mass N_t of firms, and each firm produces a mass Ω_{it} of products. Industry goods are produced using a CES production technology over all products produced in the industry:

$$Y_{jt} = \left[\int_0^{N_t} \int_0^{\Omega_{it}} y_{it}(\omega)^{\frac{\nu-1}{\nu}} \, d\omega \, di\right]^{\frac{\nu}{\nu-1}}, \tag{5}$$

[2]Because industries are symmetric in the benchmark model, we omit the j subscript in the following unless it is necessary to avoid confusion.

where ν is the elasticity of substitution across products in an industry. Subscript i identifies a firm and ω identifies a particular product produced by firm i.

The firm profit maximization implies that the demand for a variety of product ω produced by a firm i is:

$$y_{it}(\omega) \quad = \quad p_{it}(\omega)^{-\nu} P_{jt}^{\nu-1} \mathcal{Y}_t, \tag{6}$$

where $P_{jt} = \left(\int_0^{N_t} \int_0^{\Omega_{it}} p_{it}(\tau)^{1-\nu} d\tau di \right)^{\frac{1}{1-\nu}}$ is the industry j price index.

2.3 Product technology

The firm produces differentiated products using labor and capital. Labor is obtained from competitive labor markets, while the stock of capital is accumulated through investment, i_{it}, and evolves as

$$k_{it+1} = (1 - \delta_k) k_{it} + \Gamma(i_{it}/k_{it}) k_{it}, \tag{7}$$

where δ_k is the depreciation rate of capital and $\Gamma(.)$ is a function capturing convex adjustment costs in the accumulation of capital.

Each period, the firm allocates a portion of its labor and capital to the production of various products over the interval $[0, \Omega_{it}]$. The production function for a product ω is given by

$$y_{it}(\omega) = \left(e^{a_t + g_t} \, l_{it}(\omega) \right)^{1-\alpha} k_{it}(\omega)^{\alpha}, \tag{8}$$

where $k_{it}(\omega)$ and $l_{it}(\omega)$ are the amount of capital and labor allocated to the production of product ω.

The log aggregate productivity process, $a_t + g_t$, is common across all firms and features both a stationary (a_t) and a non-stationary (g_t) component, which evolve as:

$$a_t = (1-\rho_a)\bar{a} + \rho_a a_{t-1} + \sigma_a \epsilon_{at} \tag{9}$$

$$g_{t+1} - g_t = \bar{x}_g + x_{gt} \tag{10}$$

$$x_{gt} = \rho_{xg} x_{gt-1} + \sigma_{xg} \epsilon_{xgt} \tag{11}$$

where ϵ_a and ϵ_{xgt} are standard normal random variables, $\bar{a}$ is a scale parameter that pins down the average level of productivity, and $\bar{x}_g$ is the mean growth rate of productivity. The presence of persistent processes in productivity growth x_t generates low-frequency components that are used to generate long-run risk as in Bansal and Yaron (2004).

2.4 Product scope within firms

The product scope of the firm is determined by production costs in a flexible manufacturing technology framework (e.g., Eckel and Neary (2010)). Flexible manufacturing allows firms to possess a "core competence" in the production of a specific product. However, production becomes less efficient as the firm deviates from its core competence and expands its product variety.

We capture this inefficency by assuming that capital gets relatively less productive when allocated away from its core. In particular, the total stock of capital available for production at time t satisfies:

$$k_{it} = \int_0^{\Omega_{it}} k_{it}(\omega)\omega^{\chi} d\omega, \tag{12}$$

where ω^{χ} is a function that captures the inefficiencies coming from deviating from the core and $\nu^{-1} > \chi > 0$ captures the sensitivity of the firm's productivity to deviating from the core.[3] We assume that the firm's core competency is zero, which allows us to simplify the

[3] This parameter restriction ensures that the marginal product of an additional product variety is always positive.

derivations without affecting our main results.

Similarly for labor, the cost of a unit of hour increases as we deviate from the core. This additional cost can be interepreted as a reduced form way to capture the diseconomies of scales stemmimg from product diversification. Denoting the competitive wage by $\mathcal{W}_t$, the marginal cost of a unit of labor allocated to product ω is

$$W_t(\omega) = \omega^{\chi} \mathcal{W}_t. \tag{13}$$

The firm can adjusts its product scope by creating new products. Denoting the firm product development expenditures by s_{it}, new products are created using the following technology:

$$\varphi_{it} = \bar{\varphi}\, \bar{\Omega}_t^{1-\eta} s_{it}^{\eta}, \tag{14}$$

where $\bar{\varphi}$ is a scale parameter, and $\eta \in [0, 1]$ is the elasticity of new product with respect to product development expenditures and guarantees decreasing return to scale in the product creation rates, φ_{it}/Ω_t.[4]

Every period, a randomly selected proportion of products δ_Ω becomes obsolete and disappears from the firm's product line. Thus the firm's product scope evolves as:

$$\Omega_{it+1} = \varphi_{it} + (1 - \delta_\Omega)\Omega_{it}. \tag{15}$$

Operating a product line also entails fixed operating costs. In particular, to operate in a given product market, the firm pays a fixed cost of operation of f for each variety produced. The level of f affects the average profitability of a product line and is key to pin down the average number of products sold by the firm.

[4] The assumption that the firm takes the stock of products as given is not key for our results, but makes the optimal choice for scope more simple.

2.5 Dividends and financing

The firm derives revenues by selling its products in various markets. The profit made by the firm on a specific product line ω is,

$$\pi_{it}(\omega) = p_{it}(\omega)y_{it}(\omega) - \omega^{\chi}\mathcal{W}_t l_{it}(\omega) - f. \tag{16}$$

The firm profit is also subject to a series of $i.i.d.$ product-specific shocks $z_i(\omega)$, which are assumed to be normally distributed, $z_i(\omega) \sim N(0, \sigma_z)$, and observed after all production decisions are made. Although the shocks are $i.i.d.$ at the product-level, we allow these shocks to be contemporaneously correlated at the firm-level, that is

$$corr(z_i(\omega), z_i(\tau)) = \rho_z, \ \forall \omega, \tau \in \Omega_{it}, \text{and } \omega \neq \tau. \tag{17}$$

The total amount of idiosyncratic risk faced by a firm, $\tilde{z}_i$, is defined as the average product-specific risk over the entire product portofolio, i.e.,

$$\tilde{z}_i \equiv \frac{1}{\Omega_{it}} \int_0^{\Omega_{it}} z_i(\omega) d\omega \tag{18}$$

This specification for the idiosyncratic risk has three main advantages. First, it captures, in a reduced form, the *diversification benefits* of increasing the firm's scope that we observe in the data. Second, it generates heterogeneity across firms within an industry in a tractable way. Third, this specification nests the case where there is no diversification stemming from product creation (i.e., $\rho_z = 1$), in which case, $\tilde{z}_i$ is simply a firm-specific iid shock.

Using the properties of the normal distribution, it can be shown that $\tilde{z}_i$ is normally distributed, $\tilde{z}_i \sim N(0, \sigma_{it})$, where the total amount of idiosyncratic risk at the firm-level is given by[5]

$$\sigma_{it}^2 = \bar{\sigma}_z^2 \left((1-\rho_z) \frac{1}{\Omega_{it}} + \rho_z \right). \tag{19}$$

Eq. 19 highlights the important link that exists between a firm's product space Ω_{it} and idiosyncratic risk. Firms that are more diversified across products have lower idiosyncratic volatility (i.e. $\partial \sigma_{it}^2 / \partial \Omega_{it} \leqslant 0$). Importantly, the magnitude of the diversification effect of product scope is related to the magnitude of the correlation across product shocks, ρ_z.

When firms are hit by a sufficiently negative $\tilde{z}_i$ shock, they need to obtain external financing from financial markets. External financing, (i.e., negative dividends), are subject to equity dilution costs. More specifically, we assume that obtaining financing through financial markets involves a dilution cost of ϱ_t that is proportional to the amount of equity issued.

Accordingly, the real dividend is

$$d_{it} = \int_0^{\Omega_{it}} \pi_{it}(\omega)\, d\omega - i_{it} - s_{it} - z_i \sigma_{it} + \varrho_t \min\left\{ d_{it}, 0 \right\}, \tag{20}$$

where $z_i = \tilde{z}_i / \sigma_{it} \sim N(0,1)$. In the following, we denote by $H(\cdot)$ and $h(\cdot)$ the cumulative distribution and probability density functions of the standard normal distribution.

Note that the presence of dilution costs create a wedge between the internal and external financing of the firm. When the firm issues equity, i.e. $d_{it} < 0$, the firm only receives $(1-\varrho_t)d_{it}$, while financing through internal cash flow is costless.

2.6 Industry equilibrium

Firms generate revenues in product markets in which they compete with other firms. Industries are characterized by an oligopolistic market structure where firms play a static Cournot-Nash game in each period within their respective industry. More specifically, firms

[5]See technical appendix for details.

choose the production for each product, taking the production decisions of rivals as fixed. Because firms are relatively large in their industry, but are small relative to the broader economy, individual firms impact the industry price, but do not affect aggregate factor prices such as the wage rate, $\mathcal{W}_t$.

The optimization problem of the firm is to choose which products to produce and in what quantity, in order to maximize the value of the firm, subject to the demand for the firm's products, the capital and product accumulation constraint, the production decisions of the other firms in the industry, and the external equity financing constraint, that is,

$$
\begin{aligned}
v_{i,t} \quad &= \max_{\substack{s_{it},\Omega_{it+1},i_{it},k_{it+1}, \\ \{k_{it}(\omega),l_{it}(\omega),p_{it}(\omega)\}_{\omega\in[0,\Omega_{it}]}}} \quad d_{it} + (1-\delta_n)\ E_t[\mathcal{M}_{t,t+1} v_{i,t+1}] \qquad (21) \\
\text{s.t.} \quad & d_{it} = \int_0^{\Omega_{it}} \pi_{it}(\omega)\ d\omega - i_{it} - s_{it} - z_i\sigma_{it} + \varrho_t \min\{d_{it}, 0\} \\
& k_{it+1} = (1-\delta_k)k_{it} + \Gamma(i_{it}/k_{it})k_{it} \\
& \Omega_{it+1} = \bar{\varphi}\ \bar{\Omega}_t^{1-\eta} s_{it}^{\eta} + (1-\delta_\Omega)\Omega_{it} \\
& k_{it} = \int_0^{\Omega_{it}} k_{it}(\omega)\omega^{\chi} d\omega \\
& y_{it}(\omega) = p_{it}(\omega)^{-\nu} P_{jt}^{\nu-1} \mathcal{Y}_t \\
& \sigma_{it}^2 = \bar{\sigma}_z^2 \left((1-\rho_z)\frac{1}{\Omega_{it}} + \rho_z \right),
\end{aligned}
$$

where $\mathcal{M}_{t,t+1}$ is the one period stochastic discount factor and $(1-\delta_n)$ is the probability of survival of the firm (which we describe in the next section).

2.7 Entry

We assume the existence of a competitive fringe of potential startup firms, which are identical prior to entry. When a firm is established, an entry cost f_t^E is paid to create the firm. Because newly created firms have no internal source of financing, the entirety of the entry cost is financed through external financing. In other words, the total amount of resource needed to enter is $f_t^E/(1-\varrho_t)$. Once the firm has successfully entered the industry, it starts production in the next period.

In equilibrium, firms enter as long as the expected value of entry is greater than the total entry cost, which leads to the following free-entry condition:

$$\frac{f_t^E}{1-\varrho_t} = v_{E,t}, \tag{22}$$

where $v_{E,t} = (1-\delta_n)\beta E_t[v_{t+1}]$ is the expected value of entry and is equal to the ex-dividend market value of equity.

Following Loualiche (2016), we assume the following specification for the entry cost:

$$f_t^E = f^E \left(\frac{N_{E,t}}{N_t}\right)^{\zeta_E^{-1}} \mathcal{Y}_t, \tag{23}$$

where we scale the entry cost by aggregate output $\mathcal{Y}_t$ to keep entry costs quantitatively relevant along the balanced growth path. ζ_E measures the elasticity of the entry rate to the market value of a new entrant. Effectively, ζ_E^{-1} acts as an adjustment cost that ensures that entry is smooth over the business cycle. The fixed entry case is the particular case where entry is perfectly elastic, i.e., $\zeta_E^{-1} = 0$.

Each period, a randomly selected number of firms δ_n defaults and leaves the industry. Thus, the mass of firms in an industry evolves as:

$$N_{t+1} = (1-\delta_n)N_t + N_{E,t}. \tag{24}$$

2.8 Equilibrium and aggregation

As discussed before, industries are symmetric, so the j index can be dropped. Within industries, the only difference across firms results from the product-specific shock, which can be aggregated at the firm level as an idiosyncratic shock, z_i. Because the firm-specific shocks are observed after all production decisions are made, none of the policies of the firm depend on particular realizations of z_i, except for the equity financing decision. *Ex-ante*, z_i

only affects firm policies because it impacts the probability that the firm will need equity financing in the future. Therefore the i-subscript can also be dropped and the only relevant variable for the cross-section of firms in the industry is the proportion of firms that requires external financing, $(1 - H(z_t^\star))$, which is obtained by applying the law of large numbers.

Using the symmetric nature of the economy and imposing the free entry condition, we can derive the aggregate resource constraint of the economy:

$$\mathcal{Y}_t = \mathcal{C}_t + N_{Et} f_{Et} + N_t i_t + N_t s_t + N_t \Omega_t f + R_t \tag{25}$$

where $R_t = \frac{\varrho_t}{1-\varrho_t}\left(\sigma_t \int_{z_t^\star}^{\bar{z}} (z - z_t^\star)\, dH(z) + N_{Et} f_{Et}\right)$ is the amount of resource lost due to costly external financing.

3 Optimal firm policies

In this section, we discuss the firm's optimal policies, which are obtained by solving the firm's problem defined in (21).[6] Firms choose their optimal product scope in an environment characterized by imperfect competition and financing frictions. It is therefore expected that all these forces interact in equilibrium. Indeed, we show that these key model ingredients have important implications for the firm's optimal boundaries and for aggregate quantities. We then assess the quantitative relevance of these implications in subsequent sections.

3.1 Financing decision

Financing frictions affect incumbents and new entrants in a very different way. New entrants do not generate internal financing and must finance the entirety of the entry cost using external financing. As a result, the shadow value of an additional dollar of financing for an entrant is $\theta_t^E = \frac{1}{1-\varrho_t}$.

[6] Refer to Appendix A for the derivation details

In contrast, incumbent firms have access to both external and internal funding. External financing is used only when internal funds are insufficient, which happens when the realization of the firm-specific shock is higher than the no-financing threshold $z_t^\star$. Accordingly, incumbents use external financing with a probability $(1 - H(z_t^\star))$ and use internal financing with probability $H(z_t^\star)$. While obtaining one dollar externally costs $\frac{1}{1-\varrho_t}$, internal financing is costless. Therefore the shadow value of an additional dollar of funding for an incumbent firm is

$$\begin{aligned} \theta_t &= H(z_t^\star) + (1 - H(z_t^\star))\frac{1}{1-\varrho_t} \\ &= 1 + (1 - H(z_t^\star))\frac{\varrho_t}{1-\varrho_t}. \end{aligned} \tag{26}$$

The presence of financing frictions has two opposite effects on incumbent firms. First, external financing costs affect the shadow value of funding, θ_t, and impact the firm policies. For example, if financing becomes more costly, the firm might choose to cut on investment or product creation to reduce the need for external funds. This effect is consistent with the existing literature (e.g., Eisfeldt and Muir (2016)).

Second and unique to our model, an increase of financing cost also deters entry. This happens because startups are financially constrained and require external financing to pay the entry cost.[7] The resulting increase in barrier to entry helps incumbents consolidate market power and grow, resulting in higher investment and increased product creation. Thus, the free-entry effect alleviates the negative impact of financing costs and could even benefit incumbents – if strong enough. Ultimately, which of the two effects ultimately dominate is a quantitative question, which we explore in our calibrated model.

[7] In fact, the marginal cost of financing is always higher for startups than incumbents because $\theta_t^E > \theta_t$.

3.2 Firm scope

In a flexible manufacturing environment with product-specific shocks, the firm scope is an important determinant of both the volatility and the level of a firm's profit.

Product diversification and productivity. The firm is most efficient at producing products close to its core competence. Therefore, when a firm deviates further away from its core, the overall productivity is reduced. The effect of product scope on productivity can be visualized by aggregating the total output of a firm over its range of products. In particular, total firm output is characterized by a standard Cobb-Douglas production function where total factor productivity, $\tilde{Z}_t$, is endogenous and depends on the variety of products:

$$y_t = \tilde{Z}_t \, k_t^{\alpha} \, l_t^{1-\alpha} \tag{27}$$

$$\tilde{Z}_t = \bar{\kappa}\Omega_t^{-\chi} \, e^{(1-\alpha)(g_t+a_t)}, \tag{28}$$

where $\bar{\kappa} = \frac{1-\chi\phi\nu}{1-\chi\nu}$.

It is straightforward to see that $\partial\tilde{Z}_t/\partial\Omega_t < 0$ when $\chi > 0$, that is, firms with more product diversity are less productive, all else being equal.

Product diversification and idiosyncratic volatility. Firm face product-specific shocks that are not perfectly correlated. Therefore, by adding additional products, the firm can reduce its exposure to idiosyncratic risk via diversification. This can be seen from the equation characterizing the firm-specific volatility, σ_{it}:

$$\sigma_{it}^2 = \bar{\sigma}_z^2 \left((1-\rho_z)\frac{1}{\Omega_{it}} + \rho_z \right). \tag{29}$$

When the product-specific shocks are not perfectly correlated (i.e., $\rho_z < 1$), it is straightforward to show that $\partial\sigma_{t+1}/\partial\Omega_{t+1} < 0$.

Optimal scope. We now turn to the optimal firm product variety policy. In equilibrium, the firm chooses its range of products or scope, Ω_t, so that the marginal cost of creating a new product is equal to its marginal benefit. The optimal decision is given by the following Euler equation:

$$Q_t^{\Omega}\theta_t = (1-\delta_n)E_t\left[\mathcal{M}_{t+1}\left(\theta_{t+1}\left(\pi(\Omega_t)+(1-\delta_\Omega)Q_{t+1}^{\Omega}\right)+\underbrace{\frac{\partial\sigma_{t+1}}{\partial\Omega_{t+1}}\left(\theta_{t+1}-h(z_{t+1}^{\star})\frac{\varrho_{t+1}}{1-\varrho_{t+1}}\right)}_{\text{diversification benefits}}\right)\right], \quad (30)$$

where $Q_t^{\Omega}=\frac{1}{\bar{\varphi}\eta}\frac{s_t^{1-\eta}}{\bar{\Omega}_t^{1-\eta}}$ is the shadow value of an additional unit of product for an unconstrained firm (similar in spirit to the marginal q for capital). Note that because external financing is costly, all costs and benefits are multiplied by the shadow value of funding in the period, θ_t .

The cost of expanding the scope (left-hand-side in Eq. 30) is given by the shadow value of creating an additional unit of product. The marginal benefits of increasing the firm's core (right-hand-side in Eq. 30) consists of the extra profit earned on the marginal product variety, $\pi(\Omega_t)$, plus the residual value of the product next period and the diversification benefits of extending the scope.

Product concentration and firm scope. In the empirical section of the paper, we measure product diversification at the firm-level using a product concentration measure (PCM) defined as:

$$PCM_{it} = \int_0^{\Omega_{it}}\left(\frac{p_{ijt}(\omega)y_{ijt}(\omega)}{\int_0^{\Omega_{it}}p_{ijt}(\omega)y_{ijt}(\omega)\,d\omega}\right)^2 d\omega. \quad (31)$$

This measure has the advantage of accounting for the relative size of each product in the product portfolio of the firm. In the context of the model, one can show that the PCM is equal to:

$$PCM_{it} = \Upsilon\Omega_{it}^{-1}, \quad (32)$$

where $\Upsilon = (1 - \nu\phi\chi)^2/(1 - 2\nu\phi\chi)$. Therefore, there exists an inverse relation between PCM and the firm scope.

3.3 Product pricing

The optimal pricing policy for each product variety yields the following markup rule over the marginal cost of production, $mc_{it}(\omega)$:

$$p_{it}(\omega) = \mu_{it}\, mc_{it}(\omega), \tag{33}$$

where μ_{it} is the equilibrium price markup and equal to:

$$\mu_{it} = \frac{\nu}{\nu - 1} (1 - ms_{it})^{-1}, \tag{34}$$

where $ms_{it} \equiv \frac{p_{it} y_{it}}{\int_0^{N_t} p_{kt} y_{kt} dk}$ is the market share of firm i in the industry.

Note two things about the firm pricing policy. First, because of the decreasing return to scope, the marginal cost of production increases as the firm expands its product variety. Therefore, the firm charges a higher price for products that are outside of the core. Second, the equilibrium markup is time-varying and depends on the firm's relative market share in the industry. In the symmetric equilibrium, the market share of each firm is equal to the Herfindhal-Hirschman industry concentration index $HHI_t = N_t^{-1}$ and the price markup simplifies to:

$$\mu_t = \frac{\nu}{\nu - 1} (1 - HHI_t)^{-1}. \tag{35}$$

Therefore all else being equal, firms operating in more concentrated industries will be able to charge a higher price markup on all their products.

4 Quantitative results

This section presents a quantitative analysis of the model. We start by describing the data and stylized facts related to product concentration within firms. All the macroeconomic data series are obtained from standard sources and are available on the FRED website. We then estimate two new key parameters model and describe the calibration. Using the calibrated model, we present a quantitative evaluation of the key channels linking product diversity, financing frictions, productivity, and idiosyncratic volatility and generate new predictions that we then test in the data.

4.1 Data description

The product data comes from the Kilts-Nielsen Consumer Panel Dataset which is a longitudinal panel of approximately 40,000-60,000 U.S. households who continually provide information to Nielsen about what products they buy, in addition to where and when they make purchases. Panelists use a Nielsen-provided device to enter details about each of their shopping trips, including the date and store where the purchases were made. They also scan the barcode of each purchased good and enter the number of units purchased. Nielsen samples all states and major markets. The panelists are geographically dispersed and the panel composition is designed to be representative of the total United States population. Each panelist is assigned a projection factor, which corrects for sampling error and therefore can be used to project nationwide sales. The panel data ranges between 2004-2015 and contains 69.4 million unique observations at the barcode $\times$ household $\times$ date level. Overall the database includes 3.2 million unique barcodes that Nielsen estimates to cover approximately 30% of consumer spending.

Nielsen stores barcodes in the standard Universal Product Code (UPC) format. The UPCs are 12-digit codes which uniquely identify a product. For each product, Nielsen keeps track of three levels of aggregation: brand, product module, and product group. For example, the barcode "*004900000463*" represents a six-pack of Coca-Cola which belongs to the brand "*Coca-Cola*", product module "*Soft Drinks*" and the product group "*Carbonated Beverages*". The difficulty is that neither of these attributes of bar codes uniquely identifies the company that produces the product. In order to map UPCs to firms we make use of the GS1 US database.[8] GS1 is a not-for-profit organisation that develops and maintains global standards for barcodes. They are responsible for issuing and managing barcodes of more than 1.5 million companies worldwide. The first 6-10 digits of an UPC are GS1 company prefixes. Each company prefix uniquely identifies the company that produces the product. It is not possible to know the prefix length of a given UPC without searching it on the GS1 US database. Therefore, we feed each unique barcode in the Nielsen Database into the GS1 US database and get a company prefix, name, and address for each good in our sample. We assigned 44,760 company names to 2.25 million products. We then match company names from the GS1 US database to Compustat database. We proceed in two steps. First, we fuzzy match GS1 company names with Compustat company names using bigram string comparators and manually checked the matched results. This procedure allowed us to match 209 company names. Second, we manually reviewed companies that sell more than 200 UPC and matched them with Compustat.[9] Our final sample has 186 public companies that sell around 180 thousand different products.

Our sample contains on average per year 170 to 180 thousand different products that belong to an average of 1026 product modules and 114 product groups. The number of different products have been steadily increasing.[10] On average firms sell around 519 products but there is large heterogeneity among firms. The median firm sells only around 35 products. In addition, the firms in our sample are large – 85% of the market share is condensed in

[8] This is an approach similar to the one used in Argente, Lee, and Moreira (2018).

[9] We have also extensively used Google searches to match GS1 company names to Compustat. For example a product sold under Fritolay Co. is actually owned by Pepsi Co. or products sold by The Holmes Group are now a part of Newell Rubbermaid (due to past acquisitions).

[10] Broda and Weinstein (2010) defined firm level as the first 6 digits of UPC. This approach is not fully correct as often the it is the first 7 to 10 digits of an UPC that are assigned to a firm. Their assumption however has minor impact on the results as most firms indeed have a 6 digit UPC.

firms that sell more than \$2bn. Only the smallest firms in the sample sell few products and have them condensed in a small number of brands, modules and product groups. Over 95% of sales in 2015 come from firms that sell more than 1,000 products belonging to several different brands and product groups. This highlights the nature of multiproduct firms in the retail market.

4.1.1 Measures of product diversification

We build two measures of product diversification: product concentration and product creation/destruction.

Product Concentration. We define our empirical measure of product concentration by adapting the product concentration measure of the model (see, Eq. 31). In particular, for each firm/UPC i/j in our sample, we calculate total sales and the associated sale share in year t:

$$\text{Share}_{i,j,t} = \left(\frac{Sales_{i,j,t}}{\sum_{j=1}^{J} Sales_{i,j,t}} \right) \tag{36}$$

Our main product concentration measure (PCM) at the firm level is therefore the sum of the squared expenditure shares:

$$PCM_{i,t} = \sum_{j=1}^{J} (\text{Share}_{i,j,t})^2 \tag{37}$$

This is a measure of how concentrated are firm sales in a single product (the concept is akin to the Herfindahl-Hirschman which has been widely used to study industry concentration). A firm that sells only one product would have a PCM index of 1, and a firm that sells two products with equal weight on its total revenue would have a product concentration measure of 0.5. Of course different firms have different sizes and therefore to aggregate these firm-level PCMs we take an weighted average of individual firm level PCMs:

$$\text{PCM}_t^{agg} = \sum_{i=1}^{N} \alpha_{i,t} \text{PCM}_{i,t} \tag{38}$$

where $\alpha_{i,t}$ is the share of revenue of firm i at time t in total revenue in our sample.

Product scope dynamics. Within firm product concentration is mainly driven by how much firms innovate and expand their product scope. We define measures of product creation and destruction in the spirit of Broda and Weinstein (2010). Define entry rate as the number of new goods in period t relative to period s as a share of the total number of products in period t and exit rate as defined as the number of products that existed in period t but not in period s divided by the number of total products in period s. Creation and destruction are sales-weighted analogues of the two measures above:

$$\text{Creation } (t, s) = \frac{\text{Value of New UPCs } (t, s)}{\text{Total Value } (t)} \tag{39}$$

$$\text{Destruction } (t, s) = \frac{\text{Value of Disappearing UPCs } (t, s)}{\text{Total Value } (s)} \tag{40}$$

In the model, firms face different idiosyncratic shocks that affect their creation and destruction of products, thus go one step further than Broda and Weinstein (2010) aggregate measures of creation and destruction, and compute them at the firm level i. These measures are the empirical analogues of equation 14 in the model which describes how firms create and destroy new products.

The first column of table 1 reports the aggregate measures defined above for the overall sample period (11-year period). It reveals that 79% of products that existed in 2015 did not exist in 2004. Similarly, 77% of products that were available for purchase in 2004 were no longer available in 2015. The fact that creation is lower than entry rate and destruction is lower than exist rate, implies that new introduced and destroyed products have lower share of sales than existing products. This makes sense as new products take time to gain market share and most of the products destroyed have lower sales vis-à-vis continuing products. The last column of the table reports 1-year medians. It reveals that per year the median entry rate of products is around one-fourth of existing products and exit rate is around 23%.

As documented by Broda and Weinstein (2010) at the aggregate level, net-creation is procyclical, i.e. higher in expansions and lower in recessions (as after the crisis).

4.2 Parametrization

This section describes the calibration of the quantitative model. We estimate two new parameters that are unique to our model using the GIV method and calibrate the remaining parameters using steady state targets and values from the existing literature.

4.2.1 Estimated parameters using Granular Instrumental Variables

Our model has two parameters that are novel in the literature. The first parameter is χ which governs the sensitivity of the firm's productivity to deviating from the core i.e., the further away the firm is from its core competency the lower its productivity. The second parameter is the correlation across product shocks ρ_z, that drives the diversification benefits stemming from the firm scope. We estimate both of these parameters using the Granular Instrumental Variables (GIV) method (Gabaix and Koijen (2020)), relying on the fat-tail of the distriction of firm-level product shares. Figure 2 plots the histogram of product shares, pooled across all firms and years. The distribution of portfolio shares exhibits a fat right tail. Following Galaasen, Jamilov, Juelsrud and Rey (2020) we fit a Pareto I density to the data and estimate a Pareto rate of 1.8.

The key equation that describes the decreasing return to scope is equation (27). This equation cannot however be estimated by standard OLS methods because $\tilde{Z}_t$ is an endogenous variable that depends on the variety of products, which in turn depends on all other state variables of the model. This can be seen through equation (30) that states that the marginal cost of creating a new product or expanding production of an existing product depends on idiosyncratic volatility, external financing costs, the stock of capital from the firm and the stochastic discount factor. Therefore, use GIV to estimate equation (27).

To take the model to the data we start by using the fact that, in the model, there is a one-to-one mapping between product scope and product concetration (see equation 32). This is useful, as it is relatively easier to measure product concentration in the data.

Let $Sh_{j,i,t}$ denote the share of of product j, sold by firm i at time t such that:

$$PCM_{S,i,t} \equiv \Upsilon\Omega_{it}^{-1} = \sum_{j=1}^{N_i} Sh_{j,i,t}Sh_{j,i,t} \tag{41}$$

Thus as before $PCM_{S,i,t}$ is the value-weighted sum of shares of each product sold by firm i. Write equation (27) from the model in logs and replace product scope with the equivalent product concentration measure:

$$\log(PCM_{S,i,t}) = c + \frac{1}{\chi} z_{i,t} + \epsilon_t \tag{42}$$

where c is a constant, ϵ_t is an aggregate productivity shock and $z_{i,t}$ is the log idiosyncratic productivity of firm i at time t. We are interested in estimating the parameter χ from the above equation.

To do so we combine the above equation with the Euler equation for product scope. This equation does not have a closed form solution, but in general, the share sold of a product, can be approximated using a log-linear relationship at the product level:

$$Sh_{j,i,t} = \phi^1 z_{i,t} + \Phi^\star C_{i,t} + \eta_t + u_{j,i,t} \tag{43}$$

where $Sh_{j,i,t}$ is the share of product j sold by firm i at time t, $C_{i,t}$ is a vector of firm level-controls, η_t is an aggregate shock (possibly due to changes in the SDF), and $u_{j,i,t}$ are product specific demand shocks.

It is difficult to estimate χ due to the fact that ϵ_t and η_t can be correlated. To do so, we follow an instrumental variable approach and rely on granular instruments (Gabaix and

Koijen (2020)). This identification strategy exploits differences in the size-weighted and unweighted average idiosyncratic shocks experienced by firms at the product level. The GIV is formally constructed the following way:

$$g_{i,t} := PCM_{S,i,t} - PCM_{E,i,t} \equiv u_{S,i,t} - u_{E,i,t} \tag{44}$$

where $PCM_{E,i,t} = \frac{1}{N_i}\sum_{j=1}^{N_i} Sh_{j,i,t}$ is the average share of each product in firm's i portfolio.

For the GIV to work we need that $g_{i,t}$ is uncorrelated with ϵ_t (exclusion restriction) and correlated with $z_{i,t}$ (relevance condition). The first restriction cannot be empirically tested but it makes economic sense as it is unlikely that a shock that affects a demand for a single product would be correlated with ϵ_t (an aggregate productivity shock). The relevance of the instrument stems from the assumption a demand shock to a product, makes the firm to have to readjust their production lines and product scope, and due to time-to-build this must affect the firm's idosyncratic productivity and idiosyncratic volatility. This assumption is verified in the first stage regression (Table 2 reports the first stage results).

We follow the procedure described in Gabaix and Koijen (2019) to get an estimate of χ. We define the variable $\tilde{Sh}_{j,i,t} = Sh_{j,i,t} - PCM_{E,i,t}$ and use principal components analysis on $\tilde{Sh}_{j,i,t}$ to get an estimate of η_t that we denote as η_t^e.

We then estimate $\chi^{inv} \equiv 1/\chi$ via the regression:

$$PCM_{i,t} \equiv \omega_{S,t} = k + \chi^{inv}\hat{z}_{i,t} + \lambda'\eta_t^e + e_t \tag{45}$$

where $\hat{z}_{i,t}$ is the instrumented $z_{i,t}$.

We follow a similar procedure to estimate the parameter ρ_z from the model. The model equation we are looking to estimate is (29) which relates volatility and product scope. This equation is highly non-linear so we make a taylor-expansion around its steady state and estimate:

$$PCM_{S,i,t} = \beta\sigma^2_{i,t} + \epsilon^z_t \qquad (46)$$

where $\beta \equiv \frac{(1-\rho_z)/P\bar{C}M+\rho_z}{(1-\rho_z)/P\bar{C}M}$ and $P\bar{C}M$ is the log steady state value of the product concentration measure. From the estimated beta we can backout the value of ρ_z.

4.2.2 Calibrated parameters

We now describe how the remaining parameters are calibrated. For convenience, we have summarized all the parameter values in Table 3. The capital share α is set to 0.33 which is a standard value in the macroeconomics literature. The depreciation rate of capital δ_k is equal to 1.26% to match the mean investment rate in our sample. We model the capital adjustment cost along the lines of Jermann (1998), with a convexity parameter ζ_k of 8.[11] This value allows us to generate a relative investment volatility consistent with the data.

χ captures the inefficiencies associated with producing away from the firm's core. We estimate this parameter using the granular instrumental variables as described on the previous section. We estimate χ to be 0.105. We set the product obsolescence rate δ_Ω to 4.92% to match the mean product creation rate from the data and set the firm death rate δ_n to 2% as in Bilbiie, Ghironi, and Melitz (2012). The elasticity of new product patents with respect to R&D, η, is 0.65, which allows the model to be consistent with the relative volatility of product creation in the data sample. We choose $\sigma_a = 1.05\%$ and $\sigma_{xg} = 0.1\sigma_a$ and set the persistence of the productivity process to $\rho_a = \rho_{xg} = 0.95$. These values are close to those used in Croce (2014) and allows the model to generate an output volatility of about 2.20% per annum. We also assume that corr(ϵ_a,ϵ_{xg})=1. This assumption is consistent with the productivity process arising in a model where growth is endogenously determined (e.g., see Kung (2015)) and allows to reduce the number of exogenous shocks in the model. The

[11] In particular, we assume that capital accumulates as follows: $k_{t+1} = (1-\delta_k)k_t + \Phi(i_t/k_t)k_t$, where $\Phi(i_t/k_t) = \left(\frac{\alpha_1}{1-\frac{1}{\zeta_k}}(i_t/k_t)^{1-\frac{1}{\zeta_k}} + \alpha_2\right)$, and α_1 and α_2 are chosen so that there is no adjustment costs in the steady state.

average level of productivity $\bar{a}$ is set to generate an book-to-market ratio of approximately 0.5 as in the data and productivity grows at a annual rate of 2% in the steady state.

The entry cost parameter f_E determines the average number of firms in the industry while the price elasticity across products ν determines the degree of market power of each firm over its products. We choose these two parameters to jointly generate an industry price markup of 40% and an industry-level concentration index in the steady state close to 78.24 as in the data. The elasticity of entry rate to the value of entry, ζ_E is set 5 (e.g. Loualiche (2016)). The fixed cost of operation for each product f determines the number of products sold by firms. We choose $f = 0.15$ such that the steady state product concentration measure is 60.79 as in the data.

We calibrate the steady state external financing cost, $\bar{\varrho}$, to 0.3 as in Cooley and Quadrini (2001). The idiosyncratic shock z is the only source of cross-sectional heterogeneity within an industry and is the key driver of external equity financing needs. The volatility of the idiosyncratic shock σ_z is chosen to generate a probability of having to use external financing of 2.5% (e.g., see Eisfeldt and Muir (2016)). The correlation across product shocks ρ_z drives the diversification benefits stemming from the firm scope. We estimate ρ_z to be 0.15 (using GIV as described in the previous section).

On the preference side, the subjective discount factor β is chosen to generate a low level of the risk-free rate, consistent with the data. The preference parameters controlling risk tastes, ψ (intertemporal elasticity of substitution) and γ (coefficient of relative risk aversion), are calibrated to 2 and 10, respectively, and are in the standard range of values in the long-run risks literature (e.g., Bansal and Yaron (2004). This preference configuration implies that the representative agent prefers an early resolution of uncertainty (i.e., $\psi > 1/\gamma$), which implies that the price of risk for low-frequency consumption growth uncertainty is positive.

4.3 Model Results

This section presents a quantitative analysis of our model. The model is solved at the quarterly frequency using third order perturbation methods around the deterministic steady state after detrending all non-stationary variables by e^{n_t}.

4.3.1 Model fit

Table 4 reports several key moments obtained from the model and the data. The simulated moments targeted in the calibration match their empirical counterparts relatively well. For instance, the model replicates the first and second moments of key macroeconomic variables such as consumption and output growth. The mean and volatility of the capital investment and the product creation rate also line up very well with the data. Importantly, the concentration moments – both for products and industries – are close to their empirical targets.

The model also fares well on asset pricing moments. The average book-to-market ratio is equal to 0.44, close to the data average. Notably, the model generates and a low and stable risk-free rate and a sizeable risk premium. Both moments are within the confidence bounds of their empirical counterparts. The quantitative success of the model for asset prices is due to the combination of long-run risk in productivity with a representative agent that has a preference for early resolution of uncertaintly.

In short, Table 4 suggests that our calibrated model provides a fair representation of US multi-product firms and the economy. We now explore new implications of the model that we test in the data.

4.3.2 Dynamics

We start by exploring the model dynamics using impulse response functions. Figure 4 plots the impulse response function to an increase in a. An increase in productivity makes producing an additional product more profitable and firms increase investment in product creation. The resulting increase in firm scope leads to better product diversification and the product concentration measure, PCM, drops. As a result, idiosyncratic volatility falls and the model endogenously generates countercyclical idiosyncratic volatility.

As firms become more valuable, more entrants find it optimal to enter and aggregate entry increases leading to a drop in industry concentration, HHI. Therefore, product and industry concentration move in the same direction in response to a productivity shock.

There are two forces that affect equilibrium productivity and that work in opposite directions. On the one hand, productivity increases following the exogenous shock a. On the other, the higher product diversification hurts firm overall productivity because of the decreasing return to scope. We find that the former effect dominates so that both product creation and productivity are procyclical as in the data.

Figure 5 looks at the responses to an increase in the cost of external financing ϱ. As discussed earlier, the increase in financing cost has two opposite effects on incumbent firms. On the one hand, it increases the shadow value of funding, θ_t, and drive firms to cut on investment or product creation to reduce the need for external funds. On the other hand, when external financing is more costly, entry becomes relatively more expensive and fewer firms enter, which leads to an increase in industry concentration. The drop in competition increases product demand for incumbent firms which respond by expanding their operations (both in terms of physical capital and product scope) in order to take advantage of these new profit opportunities. We find that on average, the second effect dominates so that an increase in financing costs leads to an increase in industry concentration but a decrease in firm's product concentration. Note that these effects are in sharp contrast to those stemming from productivity shocks where PCM and HHI are positively correlated.

Because the firm scope increases following the financing shock, firms are better diversified and idiosyncratic risk drops. At the same time, the wider firm scope creates inefficiencies in production and leads to a fall in total factor productivity. Interestingly, time-varying competition creates an amplification mechanism that propagates the effects of financing cost shocks. This happens because firms become larger as a result of the financing shock, which creates an endogenous barrier to entry for new firms, that is, setting up a new firm is now much more expensive. This effect persists long after the initial financing shock.

Figure 6 investigates the impulse response function to an entry cost shock. To obtain these plots, we extend the benchmark model with an exogenous process that affects the magnitude of the entry cost. In particular, we replace f_{Et} for $f^{\star}_{Et}$, where $f^{\star}_{Et}$ is given by

$$f^{\star}_{Et} = f_{Et}\, e^{x_{Et}} \tag{47}$$

$$x_{Et} = \rho_E\, x_{Et} + \sigma_E\, \epsilon_{Et} \tag{48}$$

Overall, the responses to an entry cost shock are qualitatively similar to that of a financing shock. This happens because an increase in financing cost makes entry more costly, which effectively acts as a barrier to entry shock x_{Et}. Note however that because the pure entry cost shock does not have a direct impact on incumbents (e.g., through the cost of financing), the resulting responses will generally be stronger than for the financing shock.

Given how similar the impulse-response functions are, it is challenging to empirically identify the source of the shocks. In our quantitative experiment, we identify financing shocks by relying on the fact that financing costs shocks affect the average cost of external financing of incumbents while the entry cost shock does not.

5 Quantitative experiment

The shocks to external financing and entry costs both generate a negative comovement between product and industry concentration. In this section, we document an interesting empirical pattern, industry concentration has gone up while product concentration has gone down. Using our calibrated model, we ask how much of the aggregate trends in industry and product concentration is explained by the financing and entry cost shocks individually. We also explore the related implications for the aggregate trends in productivity and idiosyncratic volatility.

We start by describing these trends in the data. Average industry concentration has accelerated over the past two decades (e.g., Grullon, Larkin, and Michaely (2018)). Panel A from Figure 1 plots the trend in average industry concentration, measured by the Herfindahl-Hirschman Index (HHI), for public US firms between 1985-2015. To construct the HHI index, every year we sum up the squared total sales of each firm in a given NAICS 3-digit industry divided by the aggregate number of firms in the industry. We then average the resulting HHIs and use a Hodrick-Prescott filter to get the trend in industry concentration. Overall industry concentration has increased around 40% between 1985 and 2015.

In contrast, product concentration within firms (Eq. 38) has been decreasing on average. Panel B from the same figure shows that sharp contrast with industry concentration, product concentration has decreased around 10% between 2004 and 2015. Firms have been diversifying their portfolio of products and have started to have less concentrated sales. Our model predicts that a decrease in firm-level product scope (and concentration) implies a decrease in idiosyncratic productivity and idiosyncratic volatility (as we documented panel-wise in Table 4). Given the decrease in aggregate product concentration seen in the data we would expect a similar aggregate decrease in idiosyncratic productivity and volatility. Panel A from Figure 3 plots the trend in average firm-level idiosyncratic volatility and Panel B plots the trend in average idiosyncratic productivity across firms. Both have been decreasing over the past decade. We now ask how much of these trends can our model explain with the aggregate financing and entry cost shocks.

The cost of external financing is driven by the exogenous process ϱ_t, which captures the additional cost for each dollar raised externally. We obtain a measure of these costs from Eisfeldt and Muir (2016) who estimate the average cost of external finance per dollar raised for the 1980-2014 period at the annual frequency. The average cost per dollar raised X_t is linked to ϱ_t by the following relation:[12]

$$X_t = \frac{1}{1 - \varrho_t}. \tag{49}$$

The resulting series is then fed into our calibrated model. Figure 7 presents a graphical representation of the results. The increase in external financing cost starting in 2000 led to a reduction in firm entry rates and to a rise in industry concentration (upper two panels of Figure 7). At the same time, incumbent firms exploit their increased market power by raising markups across their products and expand their product variety. Product concentration falls, while industry concentration rises. Comparing the model implied trend (dotted blue line) to the data (solid red line), we notice similar patterns. Table 6 reports the relative change in the two concentration measures between 2005 and 2014 for the data (first column) and for the financing shock model experiment (second column). In the data, product concentration has dropped 17%, while industry concentration has increased 26%. Our calibrated model suggests that financing costs account for about 43% (31%) of the change in product (industry) concentration over the past decade. These results confirm the importance of financing frictions in explaining product creation decisions within and across the boundaries of the firm.

Financing cost shocks, by affecting product diversity, are also an important determinant of both productivity and idiosyncratic volatility. We then ask how much of the aggregate trend in productivity and volatility can be explained by the financing shock. The lower two panels in Figure 7 plots the trend, both for the model and the data, for the aggregate sales volatility and the total factor productivity. Here again, the model captures the empirical trends very well. In terms of economic magnitude, the financing shock explains around 4% of the drop in idiosyncratic volatility and 37% of the drop in aggregate productivity.

The entry cost shock creates similar dynamics as the financing shock. To estimate the

[12] Our model is calibrated at the quarterly frequency, so we linearly interpolate the empirical series $\hat{X}_t$ and normalize the shocks so that the first autocorrelation and standard deviation of the resulting quarterly series matches the initial annual series for $\hat{X}_t$.

additional contribution of the entry cost shock to the observed trends, we simulate a model counterfactual with both financing and entry cost shocks. The entry cost process is obtained as a residual for matching product concentration dynamics.[13] The third column in Table 6 presents the simulation results from the model. The entry cost shock explains the remaining 53% of product concentration and 43% of the trend in industry concentration. Importantly, the financing and entry cost shocks jointly can explain 86% of the observed productivity slowdown and 10% of the decline in the trend component of idiosyncratic volatility, suggesting that product diversification within the boundaries of the firm is an important driver of the trends in market power, idiosyncratic volatility, and productivity.

6 Cross-sectional evidence

The benchmark model abstracts from cross-sectional heterogeneity by focusing on symmetric industries. This section outlines an extension of our baseline model that allows for ex-post cross-sectional heterogeneity in product concentration across firms. We use the extended model to derive additional predictions that we then test in the data.

We model heterogeneity across industries by assuming that there are two industry types, 1 and 2, equally distributed across the unit interval. The two sectors are initially the same, but face industry-specific shocks to the product operating cost. More specifically, the product operating cost is industry f becomes:

$$f_{jt} = f + x_{fjt}, \tag{50}$$

where $x_{fjt} = \rho_{xf} x_{fjt-1} + \sigma_{xf} \epsilon_{fjt}$ and $\epsilon_{fjt} \sim N(0, 1)$. f_{jt} is a natural candidate to generate cross-sectional difference in product concentration because it directly affects the average

[13]Note that the estimated entry cost process will be a conservative estimate of the rising entry cost documented in the literature, e.g., Corhay, Kung, and Schmid (2019b). An alternative would be to obtain the entry cost process as a residual for matching industry concentration dynamics, which would result in a larger estimate of the entry cost.

profitability of a product line. It is therefore a key determinant of the average number of products sold by the firm.

Each quarter, we sort the two industries based on the level of product concentration measure and denote by High the industry with a high PCM and by Low the industry with a low PCM. Table 5 reports a series of moments depending on the level of product concentration. Firms with a higher cost of producing a variety invests less in product creation and display a higher level of product concentration. Because they are more specialized, these firms have a higher level productivity. But specialization comes at the expense of lower diversification, which translates into a higher idiosyncratic profit volatility.

Firms with high PCM also have a higher exposure to systematic risk. This happens because firms face decreasing return to scope and scope is procyclical (e.g., see Figure 4). Therefore endogenous product scope dynamics provide a hedge against aggregate productivity shocks – the scope hedge. Because high PCM firms have a smaller scope, they have a smaller hedge against productivity risk and are thus more risky. This increased risk stemming from flexible manufacturing adds to the increased profit volatility, which increases the likelihood of high PCM firms needing costly external finance. Indeed more concentrated firms require external financing twice as much as less concentrated firms. Because the need for external financing increases in bad times, when profits are low, more concentrated firms will be riskier for this reason as well.

Panel B of Table 5 reports the excess returns and CAPM beta for two investment strategies – one that invests in the High PCM portfolio and one that invests in the Low PCM porfolio. We find that PCM is a significant driver of the cross-section of asset prices and affects the exposure of the firm to systematic risk. This exposure is mainly driven by the scope hedge effect documented above. Indeed, in untabulated results, we simulated the model without decreasing return to scope (i.e., $\chi = 1$) and found the following. At the aggregate level, removing the scope hedge effect significantly increases risk, leading to an increase in the equity risk premium and a drop in the risk-free rate. In the cross-section, we find that the High minus Low portfolio return drops from 1.40% to 0.17% because muting the scope hedge effect removes a key driver of the cross-section of firm risk.

We then test these unique predictions in the data. Each year we sort firms into two portfolios: the first composed of firms with high product concentration and the second composed of firms with low product concentration. Firms with a product concentration above (below) the median are characterized as having high (low) concentration. For each portfolio of firms we then compute its value-weighted net creation (creation - destruction), investment rate, idiosyncratic total factor productivity, idioynscratic volatility and book-to-market. The three rightmost columns of Panel A from Table 5 report these results: the first of these columns reports the moments for the high concentration portfolio, and second the moments for the low concentration portfolio and the last column the difference between the two portfolios and its statistical significance. To test the statistical significance of the difference we run a panel regression where the independent variable is the the variable under analysis (net creation, investment rate, etc) and the dependent variable is a dummy that takes a value of 1 if the firm belongs to the high product concentration portfolio and zero otherwise. In line with the model, firms with higher product concentration have lower net-creation and investment and higher idiosyncratic productivity, volatility and book-to-market.

Finally, the three leftmost columns of Panel B from the same table report the asset pricing moments. The first row of Panel B reports the log-return on each portfolio and the second row their CAPM betas. In line with the model, firms with high product concentration have higher returns and higher CAPM betas.

7 Conclusion

This paper documents a recent trend of how industry leaders have been reducing product concentration and adding new products outside of their core. The rapid expansion of firm boundaries coincides with the acceleration in market power among the largest firms, suggesting that industry leaders are consolidating market share by expanding horizontally. We explain the diverging patterns in firm product concentration and industry concentration in a general equilibrium model of multiproduct firms with endogenously evolving firm and industry boundaries. We show that external financing and entry costs shocks are important for explaining the negative relation between product and industry concentration unfolding over time, while productivity shocks generate a positive relation.

The increasing trend towards more diverse production among firms with increasing market power has important consequences for aggregate fluctuations. Using estimated shocks in a counterfactual analysis, we first show that a rising trend in external financing and entry costs can quantitatively explain the diverging trends in product and industry concentration. We then show that our model can account for a large fraction of the observed productivity slowdown and declining pattern in idiosyncratic volatility due to the increasing product diversification within incumbent firms. A firm expanding product scope further away from their core competence reduces production efficiency, lowering firm productivity. Greater product diversification also reduces firm-specific risk. Panel evidence supports the model mechanisms, as measures of product diversification are negatively related to both productivity and uncertainty within firms. Overall, our paper highlights the growing importance of product diversification within the boundaries of the firm for reconciling trends in market power and productivity.

A Optimal conditions

A.1 Demand for industry goods

The final goods firm problem consists of choosing the optimal bundle of industry goods Y_{jt} for $j \in [0, 1]$. The firm's production technology is:

$$\mathcal{Y}_t \quad = \quad e^{\int_0^1 \ln(Y_{jt}) dj}. \tag{51}$$

We derive the optimal demand for industry goods by minimizing total expenditures for a given level of production $Y_t^\star$. The Lagrangian of the cost minimization problem writes

$$\mathcal{L}_t = \min_{\{Y_{jt}\}_{i \in [0,1]}} \int_0^1 P_{jt} Y_{jt} \, dj + \Lambda_{\xi,t} \left(\mathcal{Y}_t^\star - e^{\int_0^1 \ln(Y_{jt}) dj} \right), \tag{52}$$

where P_{jt} is the price of industry good j.

The first-order condition (FOC) with respect to Y_{jt} is

$$\Lambda_{\xi,t} \frac{\mathcal{Y}_t}{Y_{jt}} = P_{jt} \tag{53}$$

Using the expression above for any two industries $j, k \in [0, 1]$:

$$Y_{jt} = Y_{k,t} \left(\frac{P_{it}}{P_{k,t}} \right)^{-1} \tag{54}$$

Now, taking logs on both sides of the equation, integrating over j and taking exponential on both sides, we get

$$\mathcal{Y}_t = Y_{k,t} P_{k,t} e^{-\int_0^1 \ln(P_{jt}) dj} \tag{55}$$

The price index $\mathcal{P}_t$ is such that:

$$\mathcal{P}_t \mathcal{Y}_t = \int_0^1 P_{jt} Y_{jt} \, dj \tag{56}$$

Therefore the aggregate price index is:

$$\mathcal{P}_t = e^{-\int_0^1 \ln(P_{jt}) dj}, \tag{57}$$

which we take as our numéraire, that is $\mathcal{P}_t = 1$. Therefore the demand for industry good Y_{jt} is:

$$Y_{jt} \;\; = \;\; \mathcal{Y}_t P_{jt}^{-1}. \tag{58}$$

A.2 Demand for products

The optimization problem of industry j consists in choosing the optimal bundle of products $y_{it}(\omega)$ for $i \in [0, N_t]$, and $\omega \in [0, \Omega_{it}]$ in order to minimize production costs, for any given production level $Y^{\star}_{j,t}$. The Lagrangian of the problem is:

$$\mathcal{L}_{\xi,j,t} = \min_{\{y_{it}(\omega)\}_{\omega \in \Omega_{it}}} \int_0^{N_t} \int_0^{\Omega_{it}} p_{it}(\omega) y_{it}(\omega) \; d\omega \; di + \Lambda_{\xi,j,t} \left(Y^{\star}_{jt} - \left[\int_0^{N_t} \int_0^{\Omega_{it}} y_{it}(\omega)^{\frac{\nu-1}{\nu}} \; d\omega \; di \right]^{\frac{\nu}{\nu-1}} \right) \tag{59}$$

where $\Lambda_{\xi,j,t}$ is the associated Lagrange multiplier.

The first order necessary conditions are:

$$\Lambda_{\xi,j,t} \left[\int_0^{N_t} \int_0^{\Omega_{it}} y_{it}(\omega)^{\frac{\nu-1}{\nu}} \; d\omega \; di \right]^{\frac{\nu}{\nu-1}-1} y_{it}(\omega)^{\frac{\nu-1}{\nu}-1} = p_{it}(\omega), \qquad \text{for } \omega \in \Omega_{it}, i \in [0, N_t] \tag{60}$$

Using the expression above, for any two products ω and τ,

$$y_{it}(\tau) = y_{it}(\omega) \left(\frac{p_{it}(\tau)}{p_{it}(\omega)} \right)^{-\nu} \tag{61}$$

Now, raising both sides of the equation to the power of $\frac{\nu-1}{\nu}$, integrating over τ and i, then raising both sides to the power of $\frac{\nu}{\nu-1}$, we get

$$Y_{jt} = y_{it}(\omega)\frac{\left(\int_0^{N_t}\int_0^{\Omega_{it}} p_{it}(\tau)^{1-\nu}d\tau di\right)^{\frac{\nu}{\nu-1}}}{p_{it}(\omega)^{-\nu}} \tag{62}$$

rearranging terms,

$$Y_{jt}\frac{p_{it}(\omega)^{-\nu}}{y_{it}(\omega)} = \left(\int_0^{N_t}\int_0^{\Omega_{it}} p_{it}(\tau)^{1-\nu}d\tau di\right)^{\frac{-\nu}{1-\nu}} \tag{63}$$

The industry j price index is defined as the price P_{jt} such that the total expenditure is equal to $P_{jt}Y_{jt}$. Therefore, substituting Eq. 63, we get

$$P_{jt}Y_{jt} = \int_0^{N_t}\int_0^{\Omega_{it}} p_{it}(\omega)y_{it}(\omega)d\omega \tag{64}$$

$$= \int_0^{N_t}\int_0^{\Omega_{it}} p_{it}(\omega)\frac{Y_{jt}p_{it}(\omega)^{-\nu}}{\left(\int_0^{N_t}\int_0^{\Omega_{it}} p_{it}(\tau)^{1-\nu}d\tau di\right)^{\frac{-\nu}{1-\nu}}}d\omega, \tag{65}$$

which yields the following expression for industry j price index:

$$P_{jt} = \left(\int_0^{N_t}\int_0^{\Omega_{it}} p_{it}(\tau)^{1-\nu}d\tau di\right)^{\frac{1}{1-\nu}}. \tag{66}$$

Therefore the demand for a product variety $y_{it}(\omega)$ is:

$$y_{it}(\omega) = p_{it}(\omega)^{-\nu}P_{jt}^{\nu}Y_{jt}. \tag{67}$$

Therefore, one can express the inverse demand function for a product as:

$$p_{it}(\omega) = y_{it}(\omega)^{-\frac{1}{\nu}}\mathcal{Y}_t Y_{jt}^{-\frac{\nu-1}{\nu}}. \tag{68}$$

A.3 Firm's problem

The idiosyncratic shocks z_i's are assumed to be observed after all production decisions are made. Therefore the first order necessary conditions are all taken after integrating for the joint distributions of the z_i's shocks. The realizations of the idiosyncratic shocks only matters for the equity issue decision, which consists of a threshold rule, i.e., issue equity when $z_i > z_t^\star$, and no equity issue otherwise. The equity issue threshold is the value for z that makes the dividend equal to zero:

$$z_i^\star = \frac{\int_0^{\Omega_{it}} \left(y_{it}(\omega)^{1-\frac{1}{\nu}} \mathcal{Y}_t^{\frac{1}{1}} Y_{jt}^{-\frac{\nu-1}{\nu}} - \omega^\chi \mathcal{W}_t l_{it}(\omega) - f \right) \, d\omega - i_{it} - s_{it}}{\sigma_{it}}. \tag{69}$$

Substituting for $p_{it}(\omega)$ using the product demand schedule, the firm's optimization problem in Lagrangian form writes:

$$\begin{aligned}
\mathcal{L}_i = E_0 \sum_{t=0}^{\infty} M_{0,t}(1-\delta_n)^t \Bigg\{ \; & \sigma_{it} \int_{\underline{z}}^{z_{it}^\star} (z_i^\star - z) \, dH(z) + \frac{1}{1-\varrho_t} \sigma_{it} \int_{z_{it}^\star}^{\bar{z}} (z_i^\star - z) \, dH(z) \\
& + Q_{it}^{k\mathcal{L}} \left((1-\delta_k) k_{it} + i_{it} - k_{it+1} \right) \\
& + Q_{it}^{\Omega\mathcal{L}} \left((1-\delta_\Omega)\Omega_{it} + \bar{\varphi} \, \bar{\Omega}_t^{1-\eta} s_{it}^\eta - \Omega_{it+1} \right) \\
& + \Lambda_{it}^{k\mathcal{L}} \left(k_{it} - \int_0^{\Omega_{it}} k_{it}(\omega) \omega^\chi d\omega \right) \Bigg\}
\end{aligned}$$

where

$$\sigma_{zit}^2 = \bar{\sigma}_z^2 \left((1-\rho) \frac{1}{\Omega_{it}} + \rho_z \right),$$

$$z_i^\star \sigma_{it} = \int_0^{\Omega_{it}} \left(y_{it}(\omega)^{1-\frac{1}{\nu}} \mathcal{Y}_t Y_{jt}^{-\frac{\nu-1}{\nu}} - \omega^\chi \mathcal{W}_t l_{it}(\omega) - f \right) \, d\omega - i_{it} - s_{it}, \text{ and}$$

$z \sim N(0,1)$.

The associated first order necessary conditions are:

$$
\begin{aligned}
[i_{it}]:&\ \theta_{it}\frac{\partial z^{\star}_{it}\sigma_{it}}{\partial i_{it}} + Q^{k\mathcal{L}}_{it} = 0\\
[s_{it}]:&\ \theta_{it}\frac{\partial z^{\star}_{it}\sigma_{it}}{\partial s_{it}} + Q^{\Omega\mathcal{L}}_{it}\bar{\varphi}\eta\bar{\Omega}_t^{1-\eta}s_{it}^{\eta-1} = 0\\
[k_{it+1}]:&\ -Q^{k\mathcal{L}}_{it} + (1-\delta_n)E_t\left(\mathcal{M}_{t,t+1}\left[\Lambda^{k\mathcal{L}}_{it+1} + Q^{k\mathcal{L}}_{it+1}(1-\delta_k)\right]\right) = 0\\
[\Omega_{it+1}]:&\ -Q^{\Omega\mathcal{L}}_{it} + (1-\delta_n)E_t\mathcal{M}_{t,t+1}\left[\theta_{it+1}\frac{\partial z^{\star}_{it+1}\sigma_{it+1}}{\partial\Omega_{it+1}} + \frac{\partial\sigma_{it+1}}{\partial\Omega_{it+1}}\left(\int_{\underline{z}}^{z^{\star}_{it+1}}(z^{\star}_{it+1}-z)dH(z) + \frac{1}{1-\varrho_{t+1}}\int_{z^{\star}_{it+1}}^{\bar{z}}(z^{\star}_{i}\right.\right.\\
&\ +(1-\delta_n)E_t\mathcal{M}_{t,t+1}\left[Q^{\Omega\mathcal{L}}_{it+1}(1-\delta_\Omega) - \Lambda^{k\mathcal{L}}_{it+1}k_{it+1}(\Omega_{it+1})\Omega^{\chi}_{i,t+1}\right] = 0\\
[k_{it}(\omega)]:&\ \theta_{it}\frac{\partial z^{\star}_{it}\sigma_{it}}{\partial k_{it}(\omega)} - \Lambda^{k\mathcal{L}}_{it}\omega^{\chi} = 0\\
[l_{it}(\omega)]:&\ \theta_{it}\frac{\partial z^{\star}_{it}\sigma_{it}}{\partial l_{it}(\omega)} = 0
\end{aligned}
$$

where $\theta_{it} = 1 + (1 - H(z^{\star}_t))\frac{\varrho_t}{1-\varrho_t}$ is the shadow value of a dollar of internal financing.

Also, note that under the normality assumption,

$$
\begin{aligned}
\int_{\underline{z}}^{z^{\star}_{it}} z dH(z) + \frac{1}{1-\varrho_t}\int_{z^{\star}_{it}}^{\bar{z}} z dH(z) &= -h(z^{\star}_{it}) + \frac{1}{1-\varrho_t}h(z^{\star}_{it})\\
&= h(z^{\star}_{it})\frac{\varrho_t}{1-\varrho_t},
\end{aligned}
$$

so that,

$$
\begin{aligned}
\int_{\underline{z}}^{z^{\star}_{it}}(z^{\star}_{it}-z)dH(z) + \frac{1}{1-\varrho_t}\int_{z^{\star}_{it}}^{\bar{z}}(z^{\star}_{it}-z)dH(z) &= G(z^{\star}_{it}) + \frac{1-G(z^{\star}_{it})}{1-\varrho_t} - h(z^{\star}_{it})\frac{\varrho_t}{1-\varrho_t}\\
&= \theta_{it} - h(z^{\star}_{it})\frac{\varrho_t}{1-\varrho_t}.
\end{aligned}
$$

Firms are oligopolistic in their industry and compete in a Cournot-Nash setting, so they take into account their impact on the total industry demand when choosing the optimal quantity of product:

$$\begin{aligned}
\frac{\partial \int_0^{\Omega_{it}} \pi_{it}(\tau)\, d\tau}{\partial k_{it(\omega)}} = & \left(1-\frac{1}{\nu}\right) y_{it}(\omega)^{-\frac{1}{\nu}} \mathcal{Y}_t Y_{jt}^{-\frac{\nu-1}{\nu}} \frac{\partial y_{it}(\omega)}{\partial k_{it}(\omega)} \\
& + \int_0^{\Omega_{it}} y_{it}(\tau)^{1-\frac{1}{\nu}} \mathcal{Y}_t Y_{jt}^{-\frac{\nu-1}{\nu}-1} \left(-\frac{\nu-1}{\nu}\right) \frac{\partial Y_{jt}}{\partial k_{it}(\omega)}\, d\tau \\
= & \left(1-\frac{1}{\nu}\right) \left(\frac{y_{it}(\omega)}{Y_{jt}}\right)^{-\frac{1}{\nu}} \frac{\partial y_{it}(\omega)}{\partial k_{it}(\omega)} \\
& + \left(-\frac{\nu-1}{\nu}\right) \int_0^{\Omega_{it}} \left(\frac{y_{it}(\tau)}{Y_{jt}}\right)^{1-\frac{1}{\nu}} d\tau \left(\frac{y_{it}(\omega)}{Y_{jt}}\right)^{-\frac{1}{\nu}} \frac{\partial y_{it}(\omega)}{\partial k_{it}(\omega)} \\
= & \left(\frac{\nu-1}{\nu}\right) \left(1-\frac{1}{N_{jt}}\right) p_{it}(\omega) \frac{\partial y_{it}(\omega)}{\partial k_{it}(\omega)}
\end{aligned}$$

where we used the symmetry condition across industries and firms, i.e., $\mathcal{Y}_t = Y_{jt}$ and $y_t = y_{it}$, as well as the fact that:

$$\frac{\partial Y_{jt}}{\partial k_{it}(\omega)} = \frac{\partial Y_{jt}}{\partial y_{it}(\omega)} \frac{\partial y_{it}(\omega)}{\partial k_{it}(\omega)} = \left(\frac{y_{it}(\omega)}{Y_{jt}}\right)^{-\frac{1}{\nu}} \frac{\partial y_{it}(\omega)}{\partial k_{it}(\omega)}$$

Similarly for labor:

$$\frac{\partial \int_0^{\Omega_{it}} \pi_{it}(\tau)\, d\tau}{\partial l_{it(\omega)}} = \left(\frac{\nu-1}{\nu}\right) \left(1-\frac{1}{N_{jt}}\right) p_{it}(\omega) \frac{\partial y_{it}(\omega)}{\partial l_{it}(\omega)} - \omega^{\chi} W_t,$$

and the firm's scope:

$$\begin{aligned}
\frac{\partial \int_0^{\Omega_{it}} \pi_{it}(\tau)\, d\tau}{\partial \Omega_{it}} = & \; y_{it}(\Omega_{it})^{1-\frac{1}{\nu}} \mathcal{Y}_t Y_{jt}^{-\frac{\nu-1}{\nu}} - \mathcal{W}_t l_{it}(\Omega_{it}) \Omega_{it}^{\chi} - f \\
& + \int_0^{\Omega_{it}} y_{it}(\tau)^{1-\frac{1}{\nu}} \mathcal{Y}_t Y_{jt}^{-\frac{\nu-1}{\nu}-1} \left(-\frac{\nu-1}{\nu}\right) \frac{\partial Y_{jt}}{\partial \Omega_{it}}\, d\tau \\
= & \; p_{it}(\Omega_{it}) y_{it}(\Omega_{it}) \left(1-\frac{1}{N_t}\right) - f - \mathcal{W}_t l_{it}(\Omega_{it}) \Omega_{it}^{\chi} \\
= & \; p_{it}(\Omega_{it}) y_{it}(\Omega_{it}) \left(\frac{\nu}{\nu-1} - (1-\alpha)\right) \left(\frac{\nu-1}{\nu}\right) \left(1-\frac{1}{N_t}\right) - f,
\end{aligned}$$

where I used the fact that

$$\frac{\partial Y_{jt}}{\partial \Omega_{ijt}} = \frac{\nu}{\nu-1} Y_{jt}^{\frac{1}{\nu}} y_{it}(\Omega_{it})^{1-\frac{1}{\nu}}.$$

Since we focus on the symmetric equilibirum where all firms within the same industry make identical decisions, so the i-subscript can be dropped and $P_{jt} = 1$. In the following, we denote $\phi \equiv \left(\frac{\nu-1}{\nu}\right)$. Replacing for the partial derivatives calculated above and imposing the symmetry conditions across firms within an industry, the set of first order condition becomes:

$$\begin{aligned}
&Q_t^k = 1 \\
&Q_t^\Omega = \frac{1}{\bar{\varphi}\eta} \frac{s_t^{1-\eta}}{\bar{\Omega}_t^{1-\eta}} \\
&Q_t^k = (1-\delta_n) E_t \mathcal{M}_{t,t+1} \left[\Lambda_{t+1}^k + Q_{t+1}^k (1-\delta_k)\right] \\
&Q_{it}^\Omega \theta_{it} = (1-\delta_n) E_t \mathcal{M}_{t,t+1} \left[\theta_{it+1} \left(p_{t+1}(\Omega_{t+1}) y_{t+1}(\Omega_{t+1}) \mu_{t+1}^{-1} \left(\phi^{-1} - 1\right)\right) - f\right] \\
&\qquad + (1-\delta_n) E_t \mathcal{M}_{t,t+1} \left[Q_{it+1}^\Omega \theta_{it+1} (1-\delta_\Omega) + \frac{\partial \sigma_{it+1}}{\partial \Omega_{it+1}} \left(\theta_{it+1} - g(z_{it+1}^\star) \frac{\varrho_{t+1}}{1-\varrho_{t+1}} \right) \right] \\
&\omega^\chi \Lambda_t^k \mu_t = p_{it}(\omega) \alpha \frac{y_{it}(\omega)}{k_{it}(\omega)} \\
&\omega^\chi W_t \mu_t = p_{it}(\omega)(1-\alpha) \frac{y_{it}(\omega)}{l_{it}(\omega)} \\
&\frac{\partial \sigma_{zt}}{\partial \Omega_t} = -\frac{(1-\rho)}{2\Omega_t^2} \frac{\bar{\sigma}_z^2}{\sigma_{zt}}
\end{aligned}$$

where $Q_t^k = Q_t^{k\mathcal{L}}/\theta_t$, $Q_t^\Omega = Q_t^{\Omega\mathcal{L}}/\theta_t$, and $\Lambda_t^k = \Lambda_t^{k\mathcal{L}}/\theta_t$.

Note that we defined the price markup, μ_t, as the product price over the real marginal cost of production, we have:

$$\mu_t = \frac{p_{it}(\omega)}{\frac{\Lambda_{ij,t}^k}{\partial y_{it}(\omega)/\partial k_{it}(\omega)}} = \frac{\nu}{\nu-1} \left(1 - \frac{1}{N_t}\right)^{-1}$$

A.4 Firm-level aggregation

The firm makes decision on a continuum of products $\omega \in [0, \Omega_t]$. In this section, we provide details on the aggregation of decisions at the firm level as well as market clearing in product markets.

First we aggregate the FOC with respect to $k_t(\omega)$:

$$\begin{aligned}\int_0^{\Omega_t} y_t(\omega)^\phi \, d\omega &= \frac{\mu_t}{\alpha} \int_0^{\Omega_t} \frac{\Lambda_t^k k_t(\omega)\omega^\chi}{Y_t^{\frac{1}{\nu}}} \, d\omega \\ &= \frac{\mu_t}{\alpha} \frac{\Lambda_t^k}{Y_t^{\frac{1}{\nu}}} k_t\end{aligned}$$

where we define $k_t = \int_0^{\Omega_t} k_t(\omega)\omega^\chi \, d\omega$. Replacing the left-hand-side using the definition of Y_t, we get

$$\begin{aligned}Y_t &= \mu_t \frac{\Lambda_t^k}{\alpha} N_t k_t \\ Y_t &= \mu_t \frac{W_t^B}{1-\alpha} N_t\end{aligned}$$

where W_t^B is the total wage bill.

To find the firm's total stock of capital and output, we first express the optimal amount of capital allocated to the production of ω as a function of aggregate state variables using the FOC w.r.t. $k_{it}(\omega)$:

$$\begin{aligned} l_{it}(\omega) &= \frac{1-\alpha}{\alpha} \frac{\Lambda_t^k}{W_t} k_{it}(\omega) \\ \Rightarrow \qquad y_t(\omega) &= k_t(\omega)^\alpha (e^{a_t+g_t} l_t(\omega)^{1-\alpha} \\ &= \left(e^{a_t+g_t} \frac{1-\alpha}{\alpha} \frac{\Lambda_t^k}{W_t} \right)^{1-\alpha} k_t(\omega)\end{aligned}$$

Using the FOC with respect to capital, we obtain an expression for the pricing decision of product ω:

$$p_t(\omega) = \omega^{\chi}\mu_t\left(\frac{\Lambda_t^k}{\alpha}\right)^{\alpha}\left(\frac{W_t/e^{a_t+g_t}}{1-\alpha}\right)^{1-\alpha}.$$

Using the inverse demand function and replacing for $p_t(\omega)$ and $y_t(\omega)$, we obtain:

$$k_t(\omega) = \left[\frac{\Lambda_t^k\omega^{\chi}}{\alpha}\mu_t\right]^{-\nu}\left(\left(e^{a_t+g_t}\frac{1-\alpha}{\alpha}\frac{\Lambda_t^k}{W_t}\right)^{1-\alpha}\right)^{\nu\phi} Y_t$$

The firm's total capital stock is obtained by multiplying both side by ω^{χ} and then integrating over ω:

$$k_t = Y_t\left[\frac{\mu_t\Lambda_t^k}{\alpha}\right]^{-\nu}\left(\left(e^{a_t+g_t}\frac{1-\alpha}{\alpha}\frac{\Lambda_t^k}{W_t}\right)^{1-\alpha}\right)^{\nu\phi}\frac{\Omega_t^{1-\chi\phi\nu}}{1-\chi\phi\nu}$$

In a similar way, the firm's total output is obtained by aggregating over ω:

$$\begin{aligned} y_t &= \int_0^{\Omega_t} y_t(\omega)d\omega \\ &= k_t^{\alpha}(e^{a_t+g_t}l_t)^{1-\alpha}\Omega_t^{-\chi}\frac{1-\chi\phi\nu}{1-\chi\nu} \end{aligned}$$

Finally, we impose the market clearing condition in product markets and use the definition of Y_t :

$$\begin{aligned} Y_t^{\phi} &= N_t\int_0^{\Omega_t} y_t(\omega)^{\phi}\, d\omega \\ &= N_t y_t^{\phi}\frac{(1-\chi\nu)^{\phi}}{1-\chi\phi\nu}\Omega_t^{1-\phi} \end{aligned}$$

The revenues on the marginal product is:

$$p_t(\Omega_t)y_t(\Omega_t) = y_t(\Omega_t)^{\phi}Y_t^{\frac{1}{\nu}} = \left(\left(e^{a_t+g_t}\frac{1-\alpha}{\alpha}\frac{\Lambda_t^k}{W_t}\right)^{1-\alpha} k_t(\Omega_t)\right)^{\phi} Y_t^{\frac{1}{\nu}}$$

The total labor market clears, that is:

$$\bar{L} = N_t l_t = \frac{1-\alpha}{\alpha} \frac{\Lambda_t^k}{W_t} N_t k_t$$

A.5 Household problem

We assume a representative household with recursive preference. The household inelastically supplies labor to firms. The total labor endowment is normalized to one, i.e., $\bar{L} = 1$. In addition, the household chooses how much stocks of firm ij, Z_{ijt}, to buy. The resource constraint is:

$$\mathcal{C}_t + \int_0^1 \int_0^{N_t + N_{Et}} (V_{ijt} - D_{ijt}) Z_{ijt+1} didj = \mathcal{W}_t^B + \int_0^1 \int_0^{N_t} V_{ijt} Z_{ijt} didj,$$

where the total labor income is equal to the combined labor income collected from supplying labor to the production of individual products in all firms and all industries, i.e., $\mathcal{W}_t^B = \int_0^1 \int_0^{N_{jt}} \int_0^{\Omega_{it}} \omega^\chi W_t l_{ijt}(\omega)\, d\omega didj$.

Solving for the optimal consumption decision yields a pricing kernel that prices all financial securities in the economy:

$$\mathcal{M}_{t+1} = \beta \left(\frac{U_{t+1}}{E_t(U_{t+1}^{1-\theta})^{\frac{1}{1-\theta}}} \right)^{-\theta} \left(\frac{\mathcal{C}_{t+1}}{\mathcal{C}_t} \right)^{-\frac{1}{\psi}}.$$

A.6 Aggregate resource constraint

Industries and firms are symmetric, so we can drop the ij subscript. Also, imposing the market clearing condition in financial markets, i.e., $Z_{ijt} = 1$, and applying the law of large number for the mass of surviving firms, we obtain:

$$\mathcal{C}_t + N_{Et}(V_t - D_t) = \mathcal{W}_t^B + N_t D_t$$

The representative firm's dividend when the idiosyncratic shock z is null is:

$$D_t^{z=0} = \sigma_t z_t^\star = \frac{Y_t}{N_t} - W_t^B - \Omega_t f - i_t - s_t.$$

Given that some firms require costly external financing when $z > z_t^\star$ and using the law of large numbers, the average dividend paid by a firms is:

$$D_t \;=\; D_t^{z=0} + D_t^{z=0}\frac{\varrho_t}{1-\varrho_t}\int_{z_t^\star}^{\bar{z}}\left(1-\frac{z}{z_t^\star}\right)dH(z)$$

Puting it all together, we obtain the aggregate resource constraint after imposing the free entry condition:

$$\mathcal{C}_t + N_{Et}f_{Et} \;=\; Y_t - N_t\Omega_t f - N_t i_t - N_t s_t - R_t$$

where $R_t = \frac{\varrho_t}{1-\varrho_t}\left(\sigma_t\int_{z_t^\star}^{\bar{z}}(z-z_t^\star)\,dH(z) + N_{Et}f_{Et}\right) = \frac{\varrho_t}{1-\varrho_t}\left(g(z_t^\star) - (1-G_t) + N_{Et}f_{Et}\right)$ is the amount of resources lost due to costly external financing.

B Technical note on iid shocks

We assume that $z_i(\omega) \sim N(0,\sigma_z)$, and that $z_i(\omega)$'s are uncorrelated across time. We allow for contemporaneous correlation across $z_i(\omega)$'s within a time period, for a given firm. In particular, we assume that that $corr(z_i(\omega), z_i(\tau)) = \rho_z$, $\forall\omega,\tau \in \Omega_{it}$, and $\omega \neq \tau$. Defining

$$\tilde{z}_i = \frac{1}{\Omega_t}\int_0^{\Omega_{it}} z_i(\omega)d\omega$$

From the properties of the normal distribution, we know that $\tilde{z}_i$ is normally distributed with first and second moment derived below.

$$E[\tilde{z}_i] = \frac{1}{\Omega_t} \int_0^{\Omega_{it}} E[z_i(\omega)] d\omega = 0,$$

and

$$\begin{aligned}
Var[\tilde{z}_i] &= Var\left[\frac{1}{\Omega_t} \int_0^{\Omega_{it}} z_i(\omega) d\omega\right] \\
&= \frac{1}{\Omega_t^2} \int_0^{\Omega_{it}} \int_0^{\Omega_{it}} E\left[z_i(\omega) z_i(\tau)\right] d\tau d\omega \\
&= \bar{\sigma}_z^2 \left((1-\rho_z)\frac{1}{\Omega_{it}} + \rho_z\right)
\end{aligned}$$

Note that this specification nests the specification without the "diversification effect". Just set $\rho_z = 1$. As soon as $\rho_z < 1$, there is some diversification effects from increasing the firm's scope. To see this,

$$\frac{\partial \sigma[\tilde{z}_i]}{\partial \Omega_{it}} = -\frac{\sigma_z}{2\Omega_{it}^2} \frac{1-\rho_z}{\sqrt{(1-\rho_z)\frac{1}{\Omega_{it}} + \rho_z}}.$$

C HHIs

The firm-level sales HHI is defined as:

$$\begin{aligned}
HHI_i &= \int_0^{\Omega_{it}} \left(\frac{p_{ijt}(\omega) y_{ijt}(\omega)}{\int_0^{\Omega_{it}} p_{ijt}(\omega) y_{ijt}(\omega)\ d\omega}\right)^2 d\omega \\
&= \frac{\int_0^{\Omega_{it}} y_{ijt}(\omega)^{2\phi}\ d\omega}{\left(\int_0^{\Omega_{it}} y_{ijt}(\omega)^{\phi}\ d\omega\right)^2} = \frac{\int_0^{\Omega_{it}} k_{ijt}(\omega)^{2\phi}\ d\omega}{\left(\int_0^{\Omega_{it}} k_{ijt}(\omega)^{\phi}\ d\omega\right)^2} \\
&= \frac{\int_0^{\Omega_{it}} \omega^{-2\nu\phi\chi}\ d\omega}{\left(\int_0^{\Omega_{it}} \omega^{-\nu\phi\chi}\ d\omega\right)^2} = \frac{\Omega_{it}^{1-2\nu\phi\chi}}{\Omega_{it}^{2-2\nu\phi\chi}} \frac{(1-\nu\phi\chi)^2}{(1-2\nu\phi\chi)} \\
&= \frac{1}{\Omega_{it}} \frac{(1-\nu\phi\chi)^2}{(1-2\nu\phi\chi)}
\end{aligned}$$

The industry-level sales HHI is defined as:

$$\begin{aligned} HHI &= \int_0^{N_{jt}} \left(\frac{p_{ijt} y_{ijt}}{\int_0^{N_{jt}} p_{ijt} y_{ijt} \, di} \right)^2 di \\ &= \int_0^{N_{jt}} \left(\frac{P_{jt} Y_{jt} / N_{jt}}{\int_0^{N_{jt}} P_{jt} Y_{jt} / N_{jt} \, di} \right)^2 di \\ &= \frac{1}{N_{jt}} \end{aligned}$$

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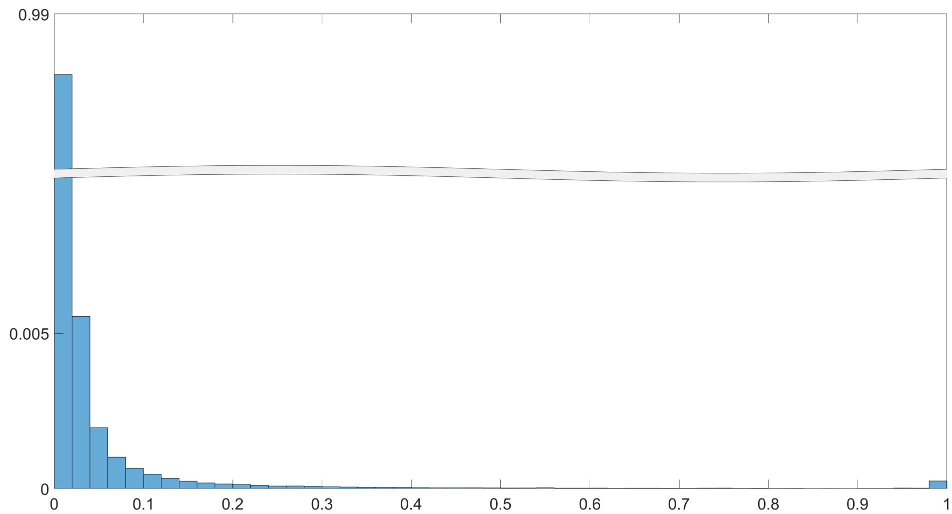
Figure 1: Trends in industry and product concentration

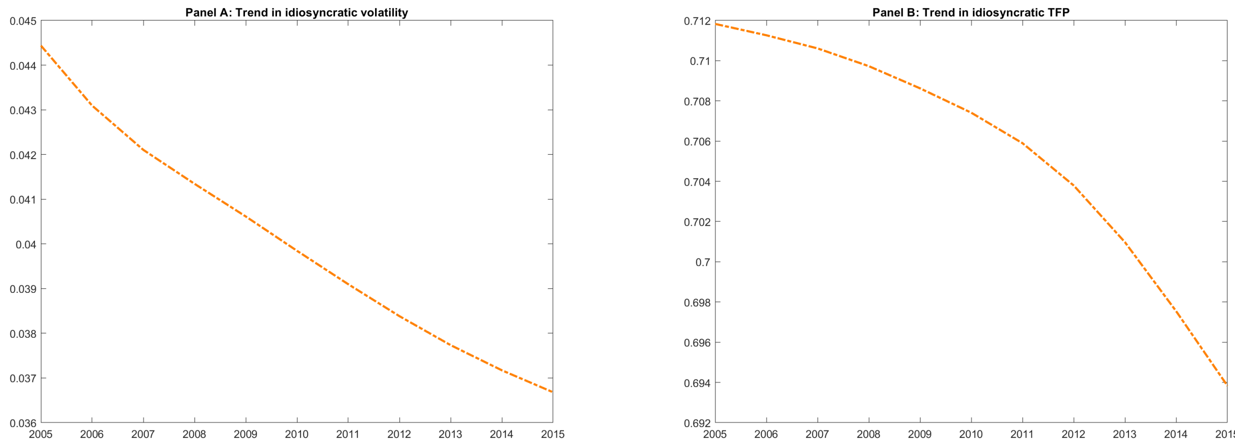
This figure plots the average trend industry Herfindahl-Hirschman Index (HHI), for public US firms (panel A) and the average trend in firm-level product concentration (panel B). We used Hodrick-Prescot filters to extract the trend from the raw time-series.

Figure 2: Product portfolio shares



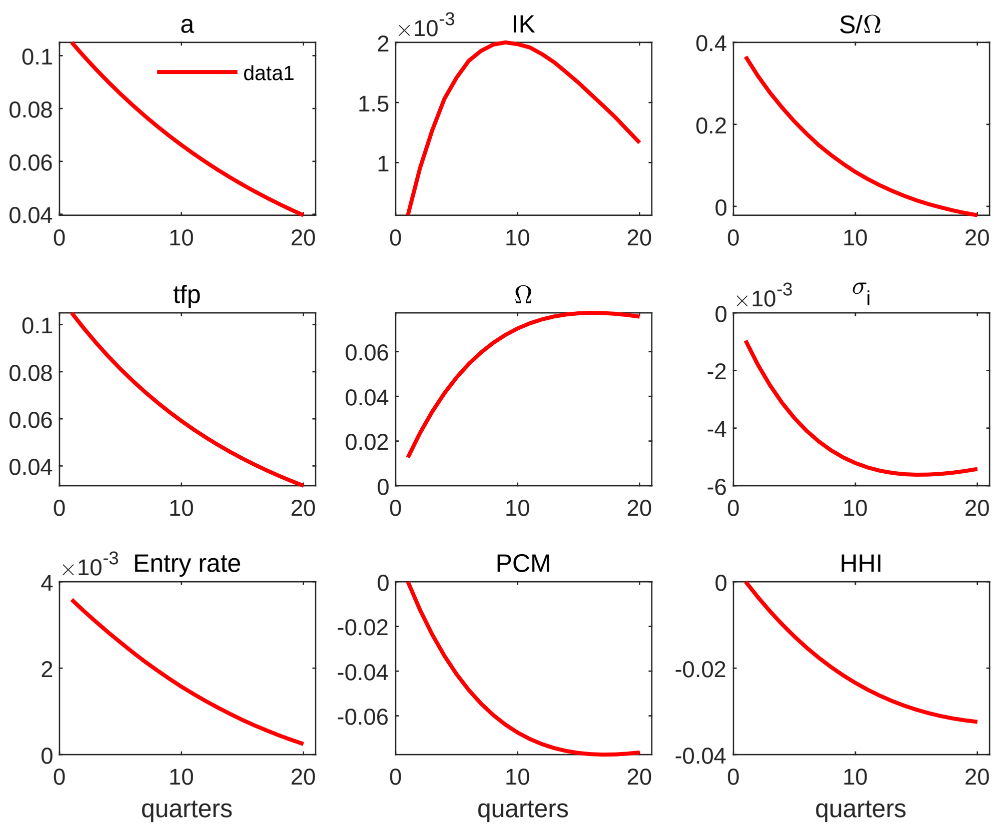
This figure plots the distribution of product shares in a the firm product portfolio. For each year and firm in our sample we compute the share of a given product in the firm portoflio. This figure plots all such shares, pooled across all firms and years. We break the y-axis of the figure to highlight the tail of the distribution.

Figure 3: Idiosyncratic Volatility and Idiosyncratic TFP



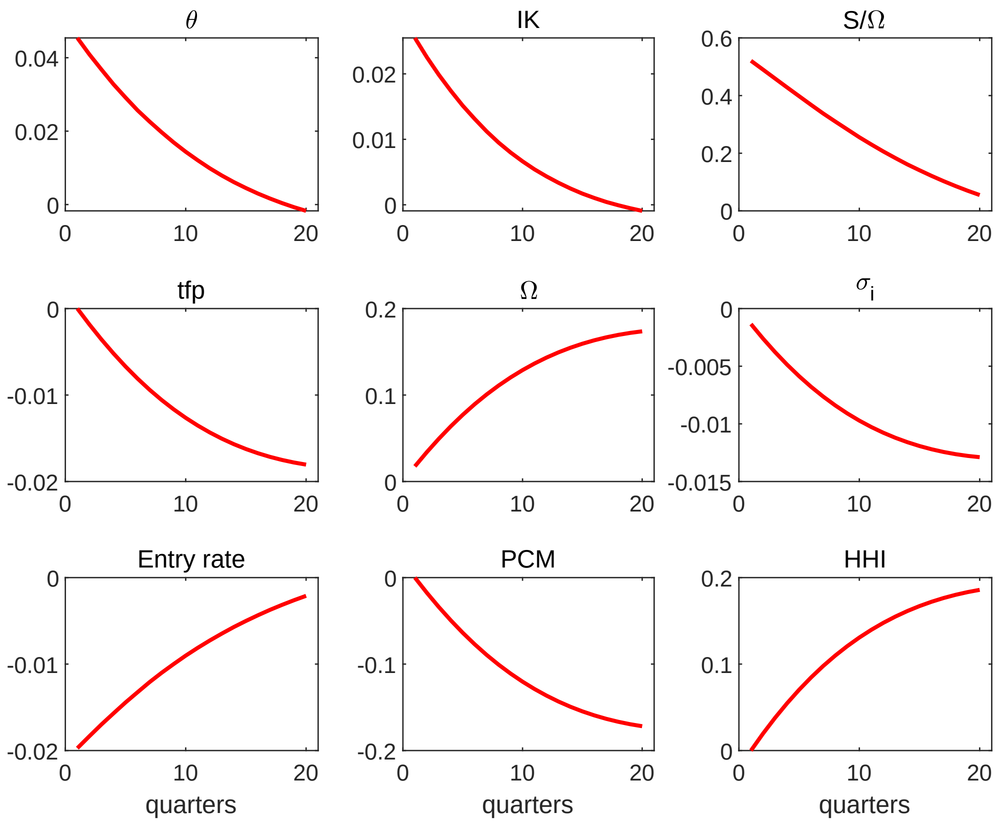
This figure plots the average firm-level idiosyncratic volatility (Panel A) and idiosyncratic total factor productivity (Panel B). We follow İmrohoroğlu and Tüzel (2014) methodology to compute idiosyncratic volatility and Herskovic, Kelly, Lustig, and Van Nieuwerburgh (2016) to compute idiosyncratic total factor productivity. We used Hodrick-Prescot filters to extract the trend from the raw time-series.

Figure 4: IRF productivity shock



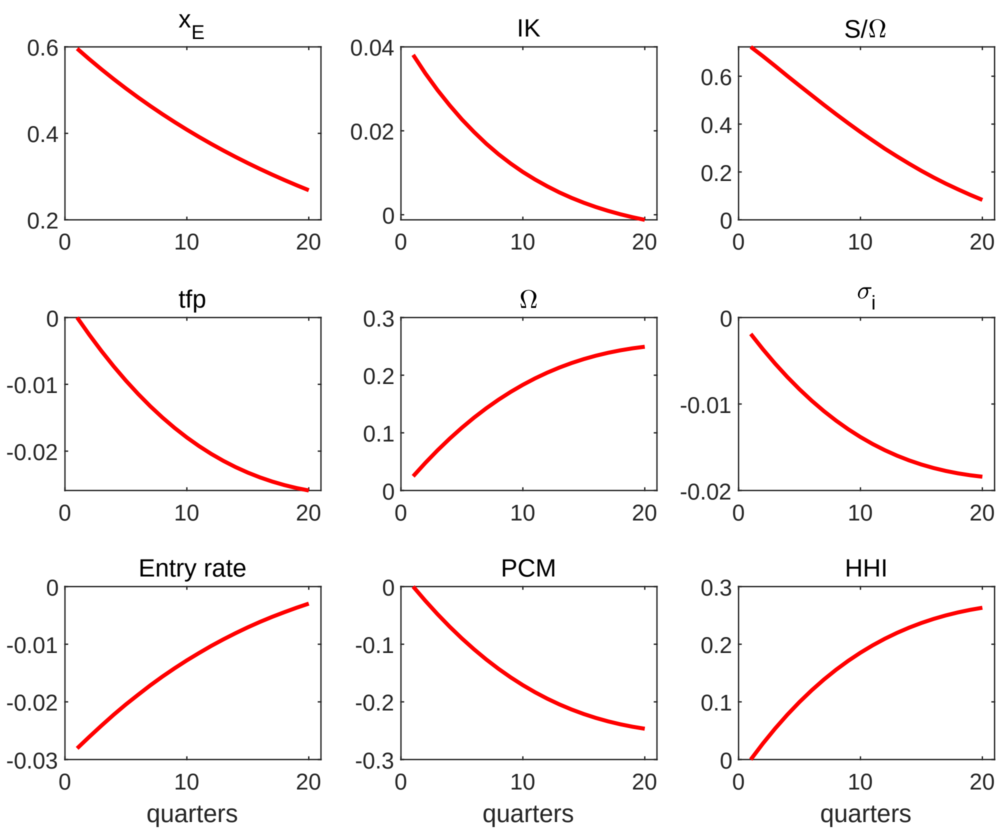
This figure plots the impulse response functions to a positive productivity shock in the benchmark model for the exogenous technology process a, the investment-to-capital ratio IK, product creation rate S/Ω, total factor productivity tfp, the product scope Ω, idiosyncratic volatility σ_i, the indsutry entry rate, and the firm and industry concentration. All values on the y-axis are percentage deviation from the steady state.

Figure 5: IRF EQUITY FINANCING SHOCK



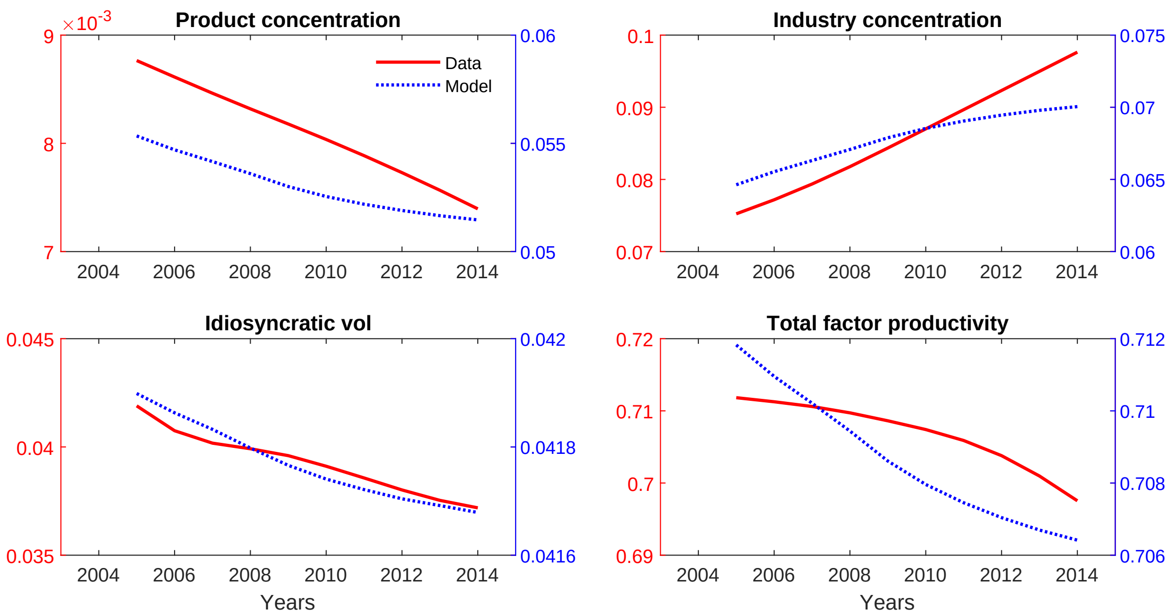
This figure plots the impulse response functions to an increase in external financing costs x_t in the benchmark model for the shadow value of funding θ, the investment-to-capital ratio IK, product creation rate S/Ω, total factor productivity tfp, the product scope Ω, idiosyncratic volatility σ_i, the indsutry entry rate, and the firm and industry concentration. All values on the y-axis are percentage deviation from the steady state.

Figure 6: IRF ENTRY COST SHOCK



This figure plots the impulse response functions to an increase in entry costs x_{Et} in the benchmark model. To obtain these IRF, we augment the model with time-varying entry cost shocks as follows $f_t^E = f^E e^{x_{Et}} (N_{E,t}/N_t)^{\zeta_E^{-1}}$, where $x_{Et} = \rho_E x_{Et} + \sigma_E \epsilon_{Et}$. We report responses for the following variables: the entry cost shock x_{Et}, the investment-to-capital ratio IK, product creation rate S/Ω, total factor productivity tfp, the product scope Ω, idiosyncratic volatility σ_i, the indsutry entry rate, and the firm and industry concentration. All values on the y-axis are percentage deviation from the steady state.

Figure 7: QUANTITATIVE EXPERIMENT



This figure plots the time series for the average product concentration measure, the industry concentration, the average sales idiosyncratic volatility, and the total factor productivity in the data (solid red line) and for a model experiment where we fed the external financing cost process from Eisfeldt and Muir (2016). The reported trends are obtained after applying a *hp*-filter to the data.

Table 1: Product Entry and Exit in the United States

	11-year	5-year	1-year
Period	2004-2015	2010-2015	median
Entry Rate	0.79	0.62	0.24
Creation	0.63	0.42	0.08
Entrant Relative to size	0.46	0.46	0.28
Exit Rate	0.77	0.61	0.23
Destruction	0.47	0.24	0.02
Exit relative to size	0.26	0.20	0.08

This table shows the averages and medians at the firm-level of product entry rate and exit rate over the Nielsen 2004-2015 sample (first column) a post-crisis sample (second column) and overall 1-year medians (last column). Product entry rate (exit rate) is defined as the ratio of the number of new (disappearing) UPCs to total number of UPCs. Creation (destruction) is defined as the value of new (disappearing) UPCs divided by the total value of products sold.

Table 2: Granular Instrumental Variables estimation

Panel A: Estimation of χ

	First stage	Second stage	First stage	Second stage	First stage	Second stage	First stage	Second stage
TFP	-0.0212**	9.511*	-0.0205**	12.41**	-0.0528***	5.178***	-0.0463***	7.936***
	(0.00994)	(4.891)	(0.00961)	(6.177)	(0.00973)	(1.258)	(0.00949)	(1.907)
$PC_{1,t}$	0.0975*	-1.009			0.326***	-2.014***		
	(0.0551)	(0.682)			(0.0559)	(0.539)		
$PC_{2,t}$	0.0303	-0.372			0.0455	-0.319		
	(0.0294)	(0.335)			(0.0315)	(0.220)		
$PC_{3,t}$	0.0606**	-0.500			0.125***	-0.603**		
	(0.0277)	(0.422)			(0.0293)	(0.268)		
Observations	1,649	1,649	1,649	1,649	1,649	1,649	1,649	1,649
Year FE		Yes		Yes		Yes		Yes
Industry FE		Yes (30)		Yes (30)		Yes (12)		Yes (12)

Panel B: Estimation of ρ_z

	First stage	Second stage	First stage	Second stage	First stage	Second stage	First stage	Second stage
Idiosyncratic volatility	-0.0276***	7.298***	-0.0221***	11.52***	-0.0378***	7.223***	-0.0345***	10.64***
	(0.00314)	(1.471)	(0.00312)	(2.122)	(0.00291)	(1.048)	(0.00294)	(1.291)
$PC_{1,t}$	0.168***	-1.312***			0.216***	-1.887***		
	(0.0174)	(0.305)			(0.0167)	(0.310)		
$PC_{2,t}$	-0.0236**	0.0883			-0.0236**	0.0871		
	(0.00929)	(0.127)			(0.00943)	(0.131)		
$PC_{3,t}$	-0.0231***	0.245**			-0.00980	0.115		
	(0.00876)	(0.117)			(0.00878)	(0.119)		
	1,649	1,649	1,649	1,649	1,649	1,649		1,649
		Yes		Yes		Yes		Yes
		Yes (30)		Yes (30)		Yes (12)		Yes (12)

This table presents the estimates of our IV strategy. The first stage regressions reported in Columns (1), (3), (5) and (7). Columns (2), (4), (6), and (8) report the second stage regresison. Panel A from the table reports the estimation of $\chi^{inv} \equiv 1/\chi$ and Panel B from the table reports the estimation of $\beta \equiv \frac{(1-\rho_z)/P\bar{C}M+\rho_z}{(1-\rho_z)/P\bar{C}M}$. $PC_{i,t}$ is the ith principal component that we extracted based on $\tilde{Sh}_{j,i,t}$.

Table 3: Parameter Values

Parameter	Description	Model
A. Deep parameters		
β	Subjective discount factor	0.981
ψ	Elasticity of intertemporal substitution	2
γ	Risk aversion	10
α	Capital share	0.33
χ	Degree of decreasing return to scope	0.105
ν	Price elasticity of products	4.85
f_E	Entry cost	766.34
f	Fixed cost of operations in product markets	0.15
$\bar{a}$	Average level of productivity	2.71
$4\bar{x}_g$	Average growth rate of productivity	2.00%
δ_k	Depreciation rate of capital stock	1.26%
δ_Ω	Product obsolescence rate	4.92%
δ_n	Firm obsolescence rate	2.00%
ζ_k	Capital adjustment cost parameter	8.00
ζ_E	Elasticity of entry to the value of entry	5.00
η	Elasticity of new products to R&D	0.65
ρ_z	Correlation across product-specific shocks	0.15
σ_z	Standard deviation of product-specific shocks	36.19
$\bar{\varrho}$	Equity dilution cost parameter	0.30
B. shocks		
ρ_a	Persistence of a_t	0.95
σ_a	Conditional volatility of a_t	1.05%
ρ_{xg}	Persistence of x_{gt}	0.95
σ_{xg}	Conditional volatility of x_{gt}	0.11%
$\rho_{x\varrho}$	Persistence of x_t	0.96
$\sigma_{x\varrho}$	Conditional volatility of x_t	0.60%

This table summarizes the parameter values used in the benchmark calibration of the model.

Table 4: Summary statistics

	Data	Model
A. Firm and macroeconomic quantities		
$\mathrm{E}[\Delta y]$	2.00%	2.00%
$\mathrm{E}[I/K]$	5.06%	5.05%
$\mathrm{E}[S/\Omega]$	12.78%	12.79%
$\mathrm{E}[ivol]$	33.15%	10.87%
$\mathrm{E}[BM]$	0.51	0.44
$\mathrm{E}[PCM]$	60.79	50.35
$\mathrm{E}[HHI_{\text{industry}}]$	78.24	70.69
$\xi_{\sigma\Omega}$	-0.10	-0.06
$\sigma[\Delta y]$	2.20%	2.14%
$\sigma[\Delta c]$	1.40%	0.84%
$\sigma[\Delta i]/\sigma[\Delta y]$	2.65	2.41
$\sigma[S/\Omega]/\sigma[I/K]$	2.70	2.85
$\sigma[BM]$	0.21	0.02
$\sigma[PCM]$	21.13	1.91
$\sigma[HHI_{\text{industry}}]$	11.97	1.75
B. Asset pricing moments		
$\mathrm{E}[r_f]$	1.00%	1.02%
$\sigma[r_f]$	1.17%	0.38%
$\mathrm{E}[r_d - r_f]$	5.84%	2.28%
$\sigma[r_d - r_f]$	17.87%	5.59%

This table presents the means and standard deviations for key firm variables from the data and the benchmark model. I/K is the capital investment rate, S/Ω is the product investment rate, $ivol$ is the idiosyncratic return volatility, BM is the book-to-market ratio, $\xi_{\sigma\Omega}$ is the elasticity of a firm idiosyncratic volatility with respect to the number of products, Δy, Δc, and Δi are the growth rate of output, consumption, and investment respectively, r_f is the risk-free rate and $r_d - r_f$ is the equity risk premium. All variables are annualized when applicable. HHI is multiplied by 1000.

Table 5: Summary statistics – cross-section

	High	Low	HML	High	Low	HML
	A. Firm quantities - model			B. Asset pricing moments - data		
PCM	76.33	25.43	50.90	105.82	6.32	99.50***
E$[S/\Omega]$	-9.90%	13.89%	-23.79%	0.31%	5.80%	-5.49***
E$[I/K]$	4.80%	5.67%	-0.87%	3.75%	5.05%	-1.29%***
E$[TFP]$	2.56	2.42	0.13	0.13	-0.40	0.616***
E$[\sigma]$	2.74	2.67	0.07	1.44	1.12	0.295***
E$[BM]$	0.46	0.40	0.06	0.38	0.35	0.0279**
	B. Asset pricing moments - model			B. Asset pricing moments - data		
E$[r_d - r_f]$	2.94	1.56	1.39	11.27%	9.35%	1.90%
E$[\beta_{CAPM}]$	1.21	0.82	0.38	1.06	0.79	0.252***

This table presents a series of moments conditional on the product concentration measure, PCM. All variables are annualized when applicable. HHI is multiplied by 1000.

Table 6: Quantitative experiment

	Data	Financing	Financing+Entry
PCM	-16.99%	-7.27%	-16.69%
$HHI_{industry}$	26.07%	8.05%	19.22%
$ivol$	-11.93%	-0.53%	-1.11%
tfp	-2.03%	-0.76%	-1.75%

This table reports the percentage change between 2004 and 2014 for the product concentration measure (PCM), the industry concentration (HHI), the sales idiosyncratic volatility ($ivol$), and the total factor productivity (tfp). The first column reports moment from the data. The second column reports model moments obtained after feeding in the model the external financing cost process from Eisfeldt and Muir (2016). The last column reports model moments obtained after feeding in the model both the external financing cost process and the entry cost process. Percentage changes are obtained after isolating trends using a *hp*-filter.