Bubble-driven Financial Cycles and Macroeconomic Policies*

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Abstract

This paper analyzes the efficacy of several policy instruments for counteracting financial bubbles generated in the banking sector. We augment a New Keynesian macroeconomic framework by endogenizing boundedly-rational expectations on asset values of loan portfolios. We show how a credit bubble can develop from a financial innovation, among other shocks. Our model quantitatively captures the negative skewness of historical US credit market cycles characterized by occasional credit market run-ups and reversals. In policy experiments we find that an endogenous capital requirement significantly reduces the impact of a credit bubble while "leaning against the wind" central bank intervention proves to be less effective.

JEL: E44, E52; **Keywords:** Credit bubble, Kalman gain, Countercyclical capital buffers, CCyB, Basel III, Credit-to-GDP gap

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1 Introduction

The running-up of financial bubbles followed by financial crashes is a frequent phenomenon in modern economies. The build-up of the bubble and the crash usually set off a number of amplification mechanisms linked to the presence of financial frictions and spillover effects in the real economy. Indeed, "procyclicality" is a relevant feature of financial cycles for macroeconomists and policymaking: hence the focus on business cycle fluctuations and financial crises (Borio et al. (2001), Danielsson et al. (2004), Kashyap and Stein (2004), Brunnermeier et al. (2009), Adrian and Shin (2010)).

In this paper we formalize the processes of bubble creation and asset price inflation to provide a setting for analyzing monetary policy and efficacy of regulatory instruments. In particular, we consider a real or rational trigger of the credit bubble in the form of a financial innovation and an irrational or behavioral extrapolation of past loan growth into the future asset price.

To define policy instruments and measure their efficacy in counteracting financial bubbles, we augment a standard New-Keynesian macroeconomic model with loan management technology and endogenous equity holdings for banks. We consider several policy options (*i*) a conventional monetary policy reaction to changes in overall loans ("leaning against the wind"); (*ii*) a macroprudential measure that exogenously increases the target level of capital (*iii*) an endogenous capital requirement that reacts to the credit-to-GDP gap.

To investigate the role of stabilization policies we look through the lens of a general equilibrium model with supply-side financial frictions which incorporates a bubble generating process in the credit market. We consider a model with bank monitoring and inside money creation features (see Goodfriend and McCallum (2007)) with interbank transactions in the credit market. We contribute to the literature in several ways. First, we model the process of bubble creation in the loan market following Branch and Evans (2005) in assuming that agents act as econometricians when forecasting and use this bounded rationality on banks when evaluating the change in loan value. Specifically, we assume that learning by agents (determined by the signal-to-noise ratio of their available information) in the model gives rise to endogenously driven waves of optimism and pessimism, which lead to sustained credit booms and bursts whenever agents observe increasing or decreasing fundamental values of loans. Under the assumption that balance sheets are continuously marked-to-market, asset price changes show up immediately in changes in net worth, eliciting responses from financial intermediaries making net worth strongly procyclical (Adrian and Shin, 2008 and 2010) as the loan bubble feeds into banks' equity values featuring a banking sector transmission through endogenous bank capital (see also Gerali et al., 2011). We show that our model is able to quantitatively capture the historical US credit market cycles especially the negative skewness and the large support of the empirical distribution which reflect the presence of occasional credit market run-ups and reversals, i.e., periods in which prices grow considerably faster (or slower) than the growth of the economy. Second, in the proposed set-up we analyze the stabilizing effects of several policy options. We find that a "leaning against the wind" policy is less effective in reducing the size of financial bubbles than an endogenous rule for the capital requirement reacting to the credit-to-GDP-gap. Raising the overall capital levels increases volatility but improves welfare through lower spreads. A welfare analysis, which accounts for expectation formation, gives highest rank a policy reaction that makes use of an endogenous capital requirement. We are, to our knowledge, the first to assess these measures in one single framework.

1.1 Motivation

Financial cycles, characterized by the build-up of an asset price bubble are less frequent than average business cycles. However, when the bubble bursts, its economic consequences remain for a long period of time (see Reinhart and Rogoff (2011), and Brunnermeier and Oehmke (2013)). As a consequence of the emergence of a credit bubble, López-Salido et al. (2017) show that loans can be mispriced so any reversal in credit market conditions will lead to an inward shift in the credit supply and a contraction of economic activity.

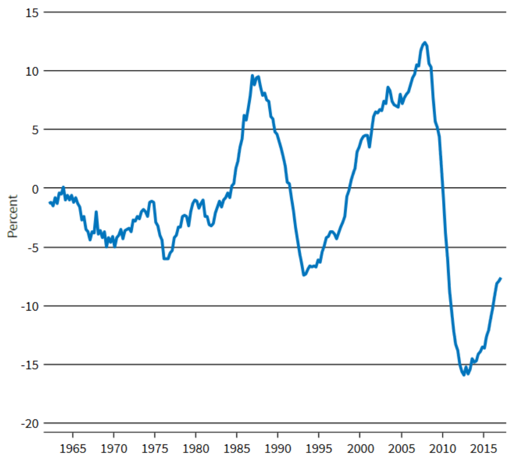
We define an asset price and credit bubble as having three phases: 1) the creation of the bubble (potentially triggered by financial innovation); 2) a period of inflation; and 3) a sudden bubble burst (or implosion). During the boom the price of an asset deviates from its intrinsic value, i.e. it is not just determined by supply and demand forces. Such a deviation features a positive feedback mechanism. In a burst asset prices suddenly fall inducing a negative feedback mechanism, sometimes even below the intrinsic value. These interactions can amplify economic fluctuations and possibly lead to serious financial distress and prolonged economic disruption.

To measure the financial cycle we employ the credit-to-GDP gap published by the Bank of International Settlements (BIS). The credit-to-GDP gap (according to the definition by the BIS) is the difference between the credit-to-GDP ratio and its long-run trend.[1] The credit-to-GDP gap reflects, in reduced form, the build-up of excessive credit, i.e. of a credit bubble. Focusing on a credit measure, we can abstract from specific asset classes affected by the bubble such as housing, or stock markets. Figure 1 shows the development of the credit-to-GDP gap in the United States, Japan and several European countries. For all countries plotted, we see sizeable swings in this measure.

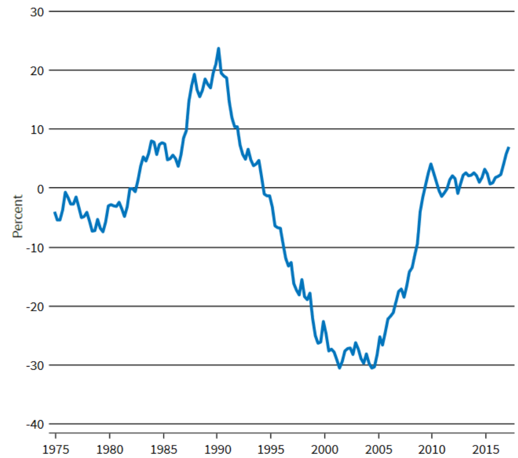
Panel (a) shows swings of a magnitude of 25% from peak to trough for the United States in the last financial cycle. The coupling of low interest rates and financial innovation in the form of mortgage securitization fueled a housing prices bubble. When it burst in 2007 this led to one of the longest and deepest economic downturns in US history (for a summary, see

[1]In the BIS database the credit-to-GDP ratio is total credit to the private non-financial sector and captures total borrowing from all domestic and foreign sources as input data (https://www.bis.org/statistics/c_gaps.htm).

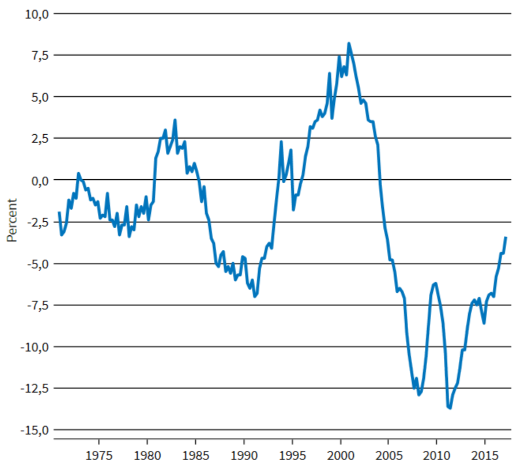
Figure 1: Credit-to-GDP Gap, Quarterly.



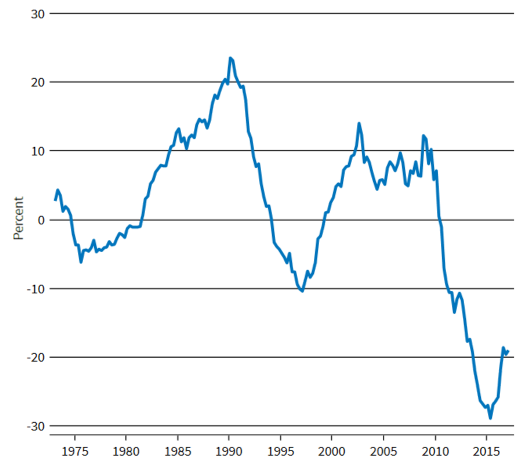
(a) United States



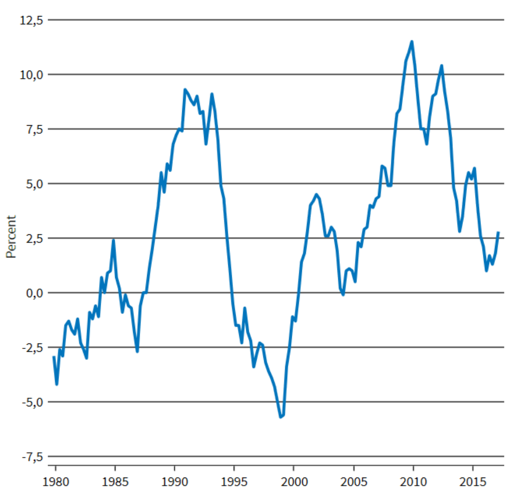
(b) Japan



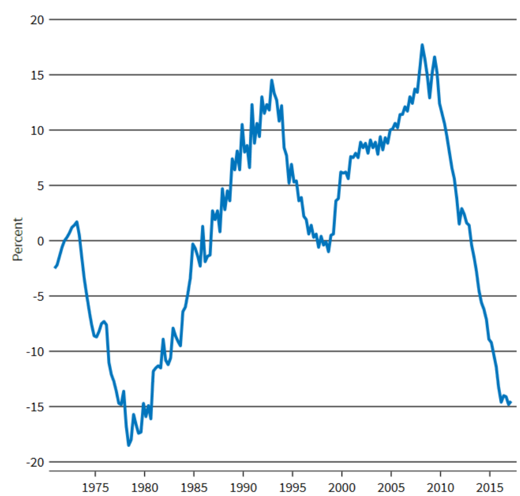
(c) Germany



(d) United Kingdom



(e) France



(f) Italy

Source: BIS. Based on total credit to the private non-financial sector, as percentage of GDP. The credit-to-GDP gap is defined as the difference between the credit-to-GDP ratio and its long-run trend in percentage points. The long-run trend is calculated using a one-sided Hodrick-Prescott filter with a lambda of 400,000.

Brunnermeier et al. (2009) and Brunnermeier and Oehmke (2013)). Panel (b) shows that the Japanese economy experienced a long-lasting financial cycle with a swing of 50% from peak to trough in the credit-to-GDP measure. Japan faced a very deep crisis after the end of the real estate and stock markets boom in the early 1990s leading to the so-called "lost decade" of the 1990s and enduring low growth during the 2000s. Several European countries have recently been dealing with major financial cycles that are documented by swings in the credit-to-GDP gap for the United Kingdom in panel (d) (40% swing peak-to-trough) and Italy in panel (f) (30% swing peak-to-trough). Germany and France (panel (c) and (e) respectively) experienced swings of a lesser extent and did not have major domestically-driven financial crises.

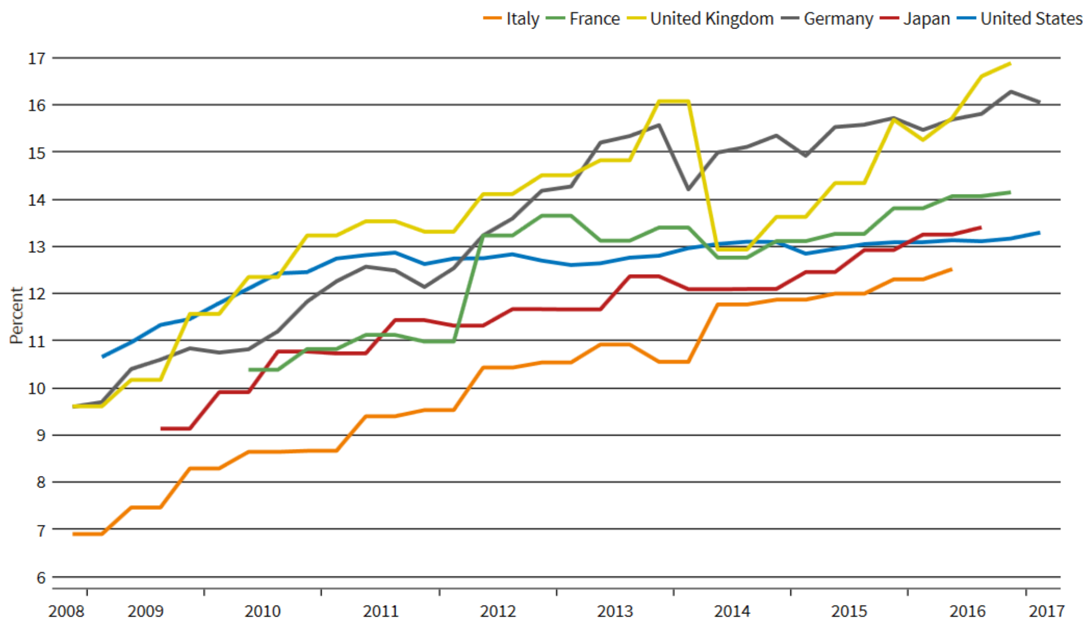
Several policy options can be considered for reducing the probability and severity of future financial crises. First, central banks can use monetary policy to "lean against the wind" of asset prices and credit booms by adopting a higher policy interest rate than would be implied by the inflation target and the stabilization of the output gap (BIS (2014) and (2015)). Second, in the aftermath of the global financial crisis, new capital rules were adopted increasing the risk-weighted capital requirements of banks. Policy switches to stricter capital requirements can be observed in the US, Japan and Europe. Figure 2 shows the ratio between Tier 1 capital (the sum of common stocks plus retained bank profits) over risk-weighted assets where risk-weighted assets assign the highest weight (50%) to loans. The ratio has been steadily increasing in the aftermath of the crisis. Other measures such as a stricter loan-to-value (LTV) ratio and leverage limits have similar macroeconomic effects; so that higher capital requirements serve as a proxy for several regulatory measures which stay fixed over the financial cycle.

Third, countercyclical capital buffers (CCyB) are an integral part of the Basel III capital standards. The credit-to-GDP gap is a "common reference point under Basel III to guide the build-up of countercyclical capital buffers". All major economies have adopted regulations regarding the CCyB, and its activation has received some opposition in the US, Japan, and major European economies. In this paper we build a single framework in which the CCyB together with other policy alternatives can be evaluated.

1.2 Related literature

Modern economies experience financial cycles with episodes of large movements in asset prices that cannot be explained by changes in economic fundamentals. Such evidence has triggered a growth of literature on financial bubbles in macroeconomic models. Brunnermeier and Oehmke (2013) analyze the amplification mechanisms of rational bubbles in asset prices. In models of rational bubbles, agents hold a bubble asset because its price is expected to rise in the future. The implied explosive nature of the price path in this class of models is consistent with the observed run-up phases to many financial crises. Bubbles in asset prices can emerge in an overlapping generation setting (Galí (2014), (2017)). Furthermore, and in contrast with the

Figure 2: Tier1 Capital to Risk Weighted Assets



Source: IMF

earlier literature on rational bubbles, the introduction of nominal rigidities allows the central bank to affect the real interest rate and thereby affect the magnitude of the bubble. Bubbles in asset prices can arise and survive because of several classes of frictions in the underlying economic model. A more recent strand of literature deals with these counterfactual implications by incorporating borrowing constraints. For example, in the model by Martin and Ventura (2012), entrepreneurs face financing constraints meaning they can only borrow a fraction of the future value of their firm. When such financing constraints are present, bubbles can have a crowding-in effect, and thus allow the productive set of entrepreneurs to increase investments. Lending is not intergenerational, but in our view can be more adequately explained through decisions made in the banking sector. In addition, in this class of models, the process of new cohorts entering the model with an intrinsically worthless asset is constructed exogenously; instead, we model how a loan asset is valued differently according to expectation formation and driven by (standard) fluctuations in the economy.

Bubbles in asset prices can also originate in heterogeneous beliefs (Scheinkman and Xiong (2003), Xiong (2013)). In a market in which agents disagree about an asset's fundamental value and short sales are constrained, an asset owner is willing to pay a price higher than their own expectation of the asset's fundamental because they expect to resell the asset to a future optimist at an even higher price. Such speculative behavior leads to a bubble component in asset prices. The bubble component builds on the fluctuations of investors' heterogeneous beliefs. It is possible to analyze welfare implications of belief distortions based on models with heterogeneous beliefs (Brunnermeier, Simsek and Xiong (2014)). These findings question the efficient markets notion that rational speculators always stabilize prices. These findings are consistent with models in which rational investors may prefer to ride bubbles because of predictable investor sentiment, heterogeneous beliefs and limits to arbitrage (Brunnermeier and Nagel, 2004). Boswijk, Hommes and Manzan (2007) estimate a model with fundamentalist and chartist traders whose relative shares evolve according to an evolutionary performance criterion. The authors show that their model can generate a run-up in asset prices and subsequent mean-reversion to fundamental values.

Unlike much of the current literature on the role of financial frictions in macroeconomics (Bernanke et al (1999); Kiyotaki and Moore (1997)), López-Salido et al. (2017) suggest that investor sentiment in credit markets can be an important driver of economic fluctuations. More recent behavioral theories of the credit cycle (Bordalo, Gennaioli, and Shleifer, 2017; Greenwood, Hanson, and Jin, 2016) also try to elucidate sentiment-related aspects of the credit cycle to explain (i) why agents become overoptimistic, generating credit booms thereby compressing credit spreads, and (ii) the associated macroeconomic dynamics. Bordalo et al. (2017) and Adam et al. (2017) assume that credit cycles arise from a particular psychological model of belief formation, which is inherently extrapolative. Specifically, expectations about credit are overly influenced by the current state of the economy, so that when there is good news about fundamentals, investors become too optimistic and real activity picks up. In our case it is the banking sector which forms expectations based on the positive news on fundamentals (both on growth and loan value) and becomes over-optimistic, generating endogenously driven waves of optimism and pessimism which lead to sustained credit booms and bursts whenever agents observe increasing or decreasing fundamentals. Greenwood et al. (2016) present a model for extrapolating default rates that also delivers many credit cycle facts. Bubbles can also originate from near-rational behavior of agents. Lansing (2010) shows how near-rational bubbles can arise under learning dynamics when agents forecast a composite variable depending on the future asset price. Branch and Evans (2011) present a model where agents learn about risk and return and show how it gives rise to bubbles. DeLong et al. (1990) show how the pricing effects of positive feedback trading can both originate and amplify bubbles in asset prices. In our setup, we incorporate the notion of bounded-rationality through a Kalman filter recursion.

The control of bubble-burst episodes in the financial sector has led to a series of proposals. Benes and Kumhof (2012) analyze, in a Dynamic Stochastic General Equilibrium (DSGE) setting, the implication of the Chicago Plan. The Chicago Plan enables softenening of credit cycles by preventing banks from creating excessive inside money during credit booms and then dismantling it during economic downturns. More recent macroprudential policies try to influence the supply of credit taking a system-wide approach. In the absence of macroprudential policy the monetary authority reacts to an adverse change in financial conditions by using the policy rate to affect the refinancing conditions of financial intermediaries (Blinder et al. (2008), Carlstrom and Fuerst (1997)). Woodford (2012) finds a complementary role for macroprudential policy alongside interest rate policy. Svensson (2012) argues in favor of a clear assignment to financial stability and price stability. Most of the macroprudential tools discussed in the literature are targeted at the bank's regulatory capital to address potential vulnerabilities on the demand side of credit.[2] Studies which raise the importance of supply side features identify short-term debt refinancing of banks as a major source of vulnerability and financial innovation in the form of new financial instruments used in the interbank market.[3] Endogenous capital requirements (Borio (2012)) have been proposed as a part of macroprudential policies. A key element of these proposals is to address the procyclicality of the financial sector by building up buffers in good times, when financial vulnerabilities emerge, so as to be able to drain them in bad times, when financial strain materializes. If effective macroprudential frameworks were in place, capital and liquidity buffers could be drained to control the building up of the bubble. By setting up a comprehensive banking sector within our macroeconomic model, we can evaluate the efficacy of interest rate policy, fixed capital requirements and countercyclical requirements in one single framework.

Sections 2 and 3 of this paper describe the model and its equilibrium. Section 4 gives the results of quantitative experiments. Section 5 shows the welfare analysis, before section 6 concludes.

2 Model

In this section, we first describe a financial sector comprising the banking system including the central bank and then the real side with households and firms.

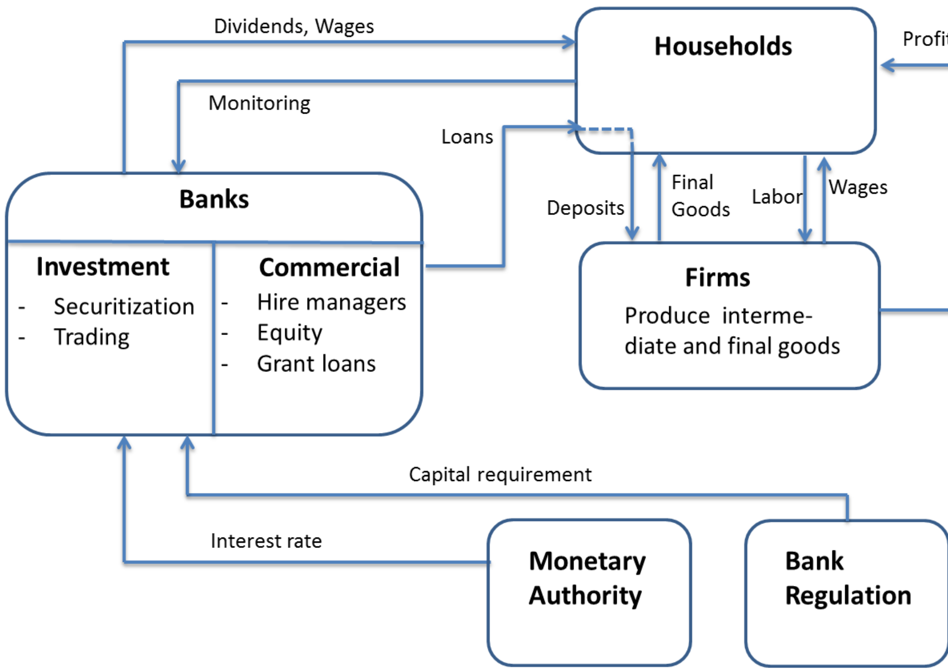
The agents in this economy, and their interconnections, are summarized in a flow chart in Figure 3. The real part of the model comprises households in which one part consumes by spending money, and the other part works for firms and in the banking sector. Households can acquire goods by taking up a loan from the banking sector, which is repaid at the end of the period. By means of the bank loan, the households receive deposits which they use to pay firms in exchange for goods. Households can invest in bank shares, own firms and save in bonds. There are monopolistically-competitive intermediate firms and a continuum of final

[2]Examples of models with limited borrowing capacity of households are Bernanke, Gertler and Gilchrist (1999) and Kiyotaki and Moore (1997). Corrado and Schuler (2017), among others, analyze the effects of several macroprudential policy measures in a model with cash-in-advance households in which banks trade excess funds in the interbank lending market. They conclude that stricter liquidity measures along with a moderate capital requirement, directly limit inside money creation, thereby reducing the severity of a breakdown in interbank lending.

[3]Justiniano et al. (2015), Gertler and Karadi (2011) and Gertler and Kiyotaki (2010) focus on the role of endogenous leverage constraints for banks to trigger credit supply disruptions. De Fiore and Uhlig (2015) and Klug et al. (2018) analyze how financial friction shocks affect the business cycle.

good producing firms which together form the production sector.

Figure 3: Model Overview



We define the two-layered banking sector as being comprised of commercial and investment banks and a central monetary authority. The financial sector comprises commercial banks which are active in the traditional banking business, i.e. in the provision of credit to households, and investment banks which are able to trade loan portfolios with each other. The commercial bank employs managers, manages equity and provides loans to households with which they are able to buy consumption goods, while the investment bank securitizes bank loans, which are sold via interbank trading. Finally, the model includes a monetary authority, which sets the riskless interest rate, and a bank regulator, which sets the capital requirement.

2.1 Banks

Banks feature a loan management stage where they lend to households. We allow for interbank transactions in the form of trades of securitized loan portfolios which are performed at the bank headquarter level. We model productivity, i.e. efficiency in loan production, and lending constraints in the form of equity capital in the financial sector. Figure 4 provides the timeline of decisions taken by the commercial and investment banks. We illustrate in the following each of these decisions.

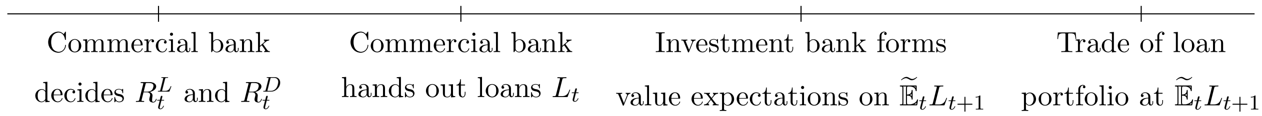


Figure 4: Timeline of events in each period t

There is a mass 1 of banks with a lending and trading stage. The commercial bank decides on the interest rate spread given the optimal amount of loans and the capital structure. The bank maximizes its profits, $\omega_{B,t}$, i.e.

$$max\mathbb{E}_t \sum_{t=0}^{\infty} \left[R_t^L \frac{L_t}{P_t} - R_t^D \frac{D_t}{P_t} - \frac{\kappa_e}{2} \left(\frac{e_t}{L_t} - \tau \right)^2 - w_t m_t \right] + \widetilde{\mathbb{E}}_t \mathbf{V_t}, \tag{1}$$

where L_t is the overall loan portfolio, D_t is the household bank deposits and e_t is bank equity, while m_t refers to monitoring work[4] and w_t is the real wage. Banks face a quadratic cost related to a deviation from the optimal ratio of bank equity versus the loan portfolio. The capital requirement ratio τ. $\widetilde{\mathbb{E}}_t \mathbf{V_t}$ represents the expected value function of the bank for the loan trading stage.

Loan management. The individual bank balance sheet constraint has to hold, i.e.

$$L_t = D_t + e_t \tag{2}$$

Banks decide on the amount of monitoring work performed by workers whose labor is supplied to the banking sector by households and remunerated by the real wage. The size of the loan portfolio is determined by the following loan management technology

$$\frac{L_t}{P_t} = A2_t m_t^{1-\alpha} \tag{3}$$

with $A2_t$ being the efficiency in loan production, which is subject to shocks following a AR(1) process

$$A2_t = \rho_2 A2_{t-1} + \epsilon_t^2 \tag{4}$$

where ϵ_t^2 is an i.i.d. shock. We call productivity shocks to the efficiency of the bank loan production function, $A2_t$, "financial innovation shocks".

[4] We introduce bank loan monitoring as an essential feature of the banking sector as in Goodfriend and McCallum (2007) "Finally, the analytical heart of the banking sector specification is a model of loan production or, more accurately, loan management. Monitoring plays a role as banks need to perform loan management in order to provide loans." It could be interpreted as a reduced form of solving an asymmetric information problem (moral hazard) in the banking sector.

Optimal loan provision gives the external finance premium, i.e.

$$R_t^L - R_t^D = \frac{v w_t m_t}{(1-\alpha)c_t} - \kappa_e \left(\frac{e_t}{L_t}\right)^2 \left(\frac{e_t}{L_t} - \tau\right). \tag{5}$$

Deposit demand for transaction purposes by households triggers demand for loans. Total loan demand, L_t, and total deposit demand, D_t, are derived from the household money in advance condition.

Loan trading. Bank headquarters can securitize loans into tradable loan portfolios, and exchange them with each other.[5] The seller of a securitized loan can obtain additional profits, while the buyer of the securitized loan gets an additional loan asset with an expected (higher) return. The value function for the trading stage at the bank headquarter reads as follows

$$\mathbf{V_t} = \frac{1}{2}[V_t^s + V_t^b], \tag{6}$$

with V_t^s being the profit of a seller and V_t^b the profit of a buyer. This mechanism allows for marking-to-market of price changes in loan portfolios.

Profits, dividends and retained earnings. Bank profits, $\omega_{B,t}$, consist of profits from the commercial bank and from loan trading, i.e.

$$\omega_{B,t} = \omega_{R,t} + \omega_{T,t} \tag{7}$$

Commercial bank profits, $\omega_{R,t}$, are given by the in-period return over equity and monitoring costs, i.e.

$$\omega_{R,t} = R_t^L \frac{L_t}{P_t} - R_t^D \frac{D_t}{P_t} - \frac{\kappa_e}{2}\left(\frac{e_t}{L_t} - \tau\right)^2 - w_t m_t. \tag{8}$$

The value of trading profits of the headquarter, $\omega_{T,t}$, are given as

$$\omega_{T,t} = \widetilde{\mathbb{E}}_t \mathbf{V_t}. \tag{9}$$

The share of profits, ϕ_Ψ, which is paid out as dividends is given by

$$\Pi_t^\Psi = \phi_\Psi \omega_{B,t}. \tag{10}$$

The remaining share, $(1-\phi_\Psi)$, is booked as a profit to the bank's equity capital e_t. The law of

[5]See Brunnermeier (2009) for a detailed treatment on loan securitization according to the "originate and distribute model" characterizing bank behavior before the financial crisis of 2007-08 in the US and Goswani etal. (2009) for an analysis of macro-financial linkages of securitization.

motion for bank capital, e_t, which is our proxy for Tier 1 capital, is then

$$e_t = (1-\delta_e)e_{t-1} + (1-\phi_\Psi)\omega_{B,t-1}, \tag{11}$$

where the dying rate of bank capital, δ_e, captures the sunk cost of bank capital management.

Expectation formation mechanism for future loan valuation. We deviate from the fully-informed agent paradigm in the expectation mechanism for loan valuation.[6] Several contributions to the literature support a deviation from the rational expectations assumption. As in Branch and Evans (2005) we assume that banks act as econometricians when forecasting and implement boundedly-rational beliefs, $\mathcal{P}$, when they evaluate the change in the value of loan portfolios.[7] Specifically, we assume that learning from past observations tends to generate sustained credit booms, to the extent that agents become more optimistic or pessimistic about the credit market conditions whenever they are positively or negatively surprised by increasing fundamental values. The estimation follows a Kalman filter recursion where the banking sector monitors past growth in value of loans, i.e.

$$L_t = L_{t-1} + g_t + v_t \tag{12}$$

with v_t being a short run fluctuation around g_t, the economy's growth rate, which is unobserved and affects the fundamental value of loans. We assume that v_t is i.i.d. with variance σ_v. The short-run fluctuation, v_t, differs conceptually from shocks to $A2_t$, the efficiency of loan management. In their expectation formation process bank agents monitor past changes in loan value and the growth rate of the economy, g_t. They do not consider the contribution of (aggregate) bank productivity, $A2_t$, when they form expectations about the growth in future value of loans which is linked, via the deposit-in-advance constraint, to the growth rate of the economy. The expectation operator of the not-fully informed agent based on equation (14), reads as:

$$\widetilde{\mathbb{E}}_t L_{t+1} = L_t + \widetilde{\mathbb{E}}_t(g_{t+1} + v_{t+1}) \tag{13}$$

where $\widetilde{\mathbb{E}}_t$ is the expectation operator with an adaptive expectation formation.

[6]Experimental evidence as described in Mauersberger and Nagel (2018) illustrates causes and effects of bubbles in a setup with identical subjects endowed with the same number of shares, in which money pays no interest. Information about fundamental values is contained in the dividend payments of shares per period. There is a finite horizon after which shares are worthless. Trading is done via a call market with orders cleared at a single price. Everyone can choose to be a buyer or seller in this market.After the start of trading, the contract prices rise above the fundamental value. As the fundamental value falls toward the end of the trading periods, the contract prices settle below the fundamental value.

[7]Branch and Evans (2005) compare the effectiveness of several prediction models for economic growth. The Constant Gain Method and specifically the Simple Method provide the best results with regards to the expectations formation of economic agents that adapt to continuous structural changes in the economy.

We assume that the fundamental growth rate, g_t, is an AR(1) process around an expected mean, g, i.e.:

$$g_t = g + \delta_g g_{t-1} + \eta_t \tag{14}$$

where η_t is an i.i.d. shock with variance σ_η. The specification implies that fundamental innovations are unpredictable, but that the expected fundamental varies over time in a persistent way. By replacing in equation (13), the expectation of g_t from (14) the expression for the loan portfolio becomes:[8]

$$\widetilde{\mathbb{E}}_t L_{t+1} = L_t + g + \delta_g \widetilde{\mathbb{E}}_t g_t \tag{15}$$

We can also see that the forecasting model implies:

$$\widetilde{\mathbb{E}}_t L_{t+n} = L_t + \sum_{j=0}^{n} \left(g + \delta_g^{n-j} \widetilde{\mathbb{E}}_t g_t \right) \tag{16}$$

so the medium forecast of boundedly-rational agents is determined by the current stat, L_t, and by drifting beliefs on the fundamental of the value of loans, $\widetilde{\mathbb{E}}_t g_t$. Investors rationally maximize infinite horizon utility but hold subjective priors about the asset process that we allow to differ from the rational expectations prior which can be solved separately. Bayesian updating of beliefs then gives rise to self-reinforcing credit market optimism that results in a credit boom.

Subjective Beliefs. For the expectation of the fundamental value of loans, $\widetilde{\mathbb{E}}_t g_t$, banks use a linear filter combining information from last period's forecast for growth, $\widetilde{\mathbb{E}}_{t-1} g_t$, and an updating part which depends on innovations in the observer equation on the value of loans:

$$\widetilde{\mathbb{E}}_t g_t = \widetilde{\mathbb{E}}_{t-1} g_t + \kappa (L_t - L_{t-1} - \widetilde{\mathbb{E}}_{t-1} g_t) \tag{17}$$

which is

$$\widetilde{\mathbb{E}}_t g_t = (1-\kappa) \widetilde{\mathbb{E}}_{t-1} g_t + \kappa (L_t - L_{t-1}). \tag{18}$$

Theorem 1 (Optimal value of κ) *The parameter κ is the optimal Kalman Gain which is (approximately) equal to the signal-to-noise ratio $\kappa = \frac{\sigma_\eta^2}{\sigma_\eta + \sigma_v^2}$. If there is no noise in the fundamental value, $\sigma_\eta^2 = 0$, then $\kappa = 0$ so the fundamental value in each period is only affected by last period growth trend, g_{t-1}. This is a case of boundedly-rational expectations where there is*

[8] Milani (2005) tests whether agents have rational expectations with regards to economic growth or show learning behavior. The latter presumes that agents do not fully know the underlying economic model with all its parameters, so they forecast the future based on their observed values from previous periods. The adaptive learning method improves the fit of DSGE models. Similarly, Eusepi and Preston (2011) introduce a model in which agents do not have full knowledge of the economic processes, but predict future realizations by extrapolation from historical patterns in observed data. This results in a higher volatility and a higher persistence of macroeconomic variables which corresponds with the observed data.

no credit bubble.[9] *If there is a high degree of noise in the fundamental value then* $lim_{\sigma_\eta^2 \to \infty} \kappa = 1$ *so now the short-run credit market conditions will affect agents' beliefs. Appendix D derives the optimal Kalman Gain.*

The learning model predicts optimism on the fundamental value of loans to co-move positively with the growth in the value of loans: during a credit boom investors will optimistically update their beliefs on the fundamental value and this feeds back into the credit market.

By replacing the subjective belief on the fundamental value, $\widetilde{\mathbb{E}}_t g_t$, into the loan aggregate dynamics in (15), multiplying out and collecting terms we derive the expectation of the future loan value:

$$\widetilde{\mathbb{E}}_t(L_{t+1}) = L_t + g + \delta_g(1-\kappa)\widetilde{\mathbb{E}}_{t-1}g_t + \delta_g\kappa(L_t - L_{t-1}) \tag{19}$$

The size of the bubble is determined by extrapolating the pace of loan growth which makes it backward looking.This results in the law of motion for future loan values:

$$\widetilde{\mathbb{E}}_t L_{t+1} = L_t + g + \delta_g b_t \tag{20}$$

with

$$b_t = (1-\kappa)\widetilde{\mathbb{E}}_{t-1}g_t + \kappa(L_t - L_{t-1}). \tag{21}$$

Thus the presence of a bubble, b_t, creates an amplification effect in the value of the loan portfolio.

Rational Expectations. The model with boundedly-rational agents nests the Rational Expectation equilibrium when $g_t = g_{t-1} = 0$, $\sigma_\eta = 0$, and therefore $\kappa = 0$, that is the prior belief of the variance of the drift term of the unobserved fundamental value is zero:

$$\lim_{\sigma_\eta \to 0} \widetilde{\mathbb{E}}_t L_{t+1} = \mathbb{E} L_{t+1} = L_t \tag{22}$$

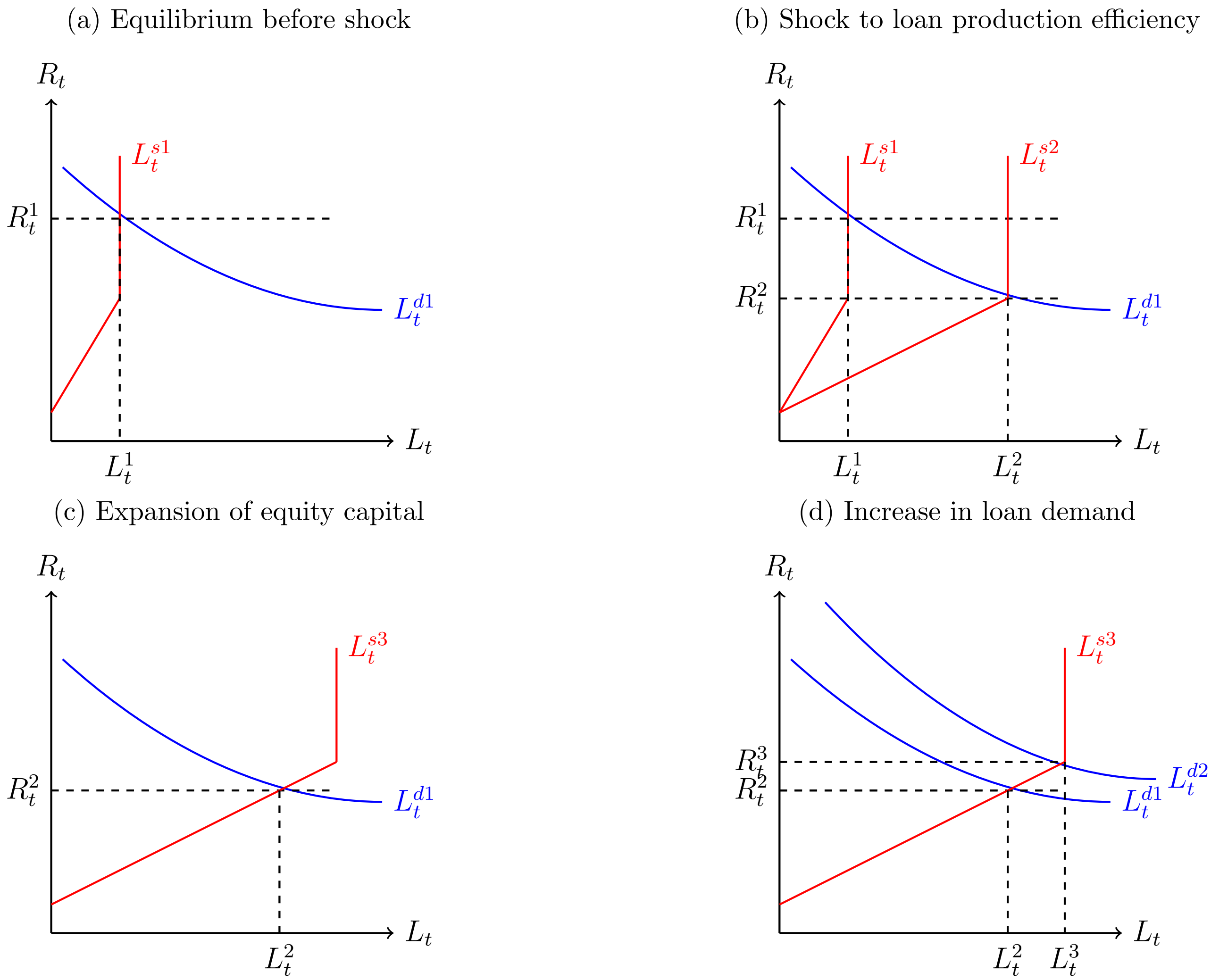
Mechanism for financial innovation shock. We examine a financial innovation shock in isolation to study the interaction of this shock with the expectation formation for the loan portfolio value in Figure 5. Panel (a) shows an initial equilibrium in the loan market. The financial innovation shock to the efficiency in loan production, $A2_t$, allows for an outward shift in the loan supply curve, L^s, as illustrated in panel (b). Given the expectation formation in loan valuation, the trading stage results in a higher level of bank equity, e_t (see panel (c)). As the amount of outstanding loans is linked to optimal bank capital, rising equity capital through trading activities allows the bank headquarter to expand further on the total amount of loans, L_t. On the demand side of credit, higher dividend payments create wealth effects for the household sector – leading to rising output and employment– which shifts credit demand

[9] When $\kappa = 0$ we have a case of pure confirmatory bias where only the most recent fundamental value, $\widetilde{\mathbb{E}}_{t-1}g_{t-1}$, enters the belief formation mechanism.

curve, L^d, outwards, by the size of output growth, g_t, as shown in panel (d), and thereby absorbing the higher lending capacity of banks.

The dynamics stemming from the expectation formation process is inherently extrapolative and positive changes of the observed fundamentals, L_{t-1} and g_{t-1}, lead to a narrowing of loan spreads, here labelled as R_t,[10] an increase in the quantity of loans, L_t and an acceleration of output growth, g_t.

Figure 5: Mechanism for loan expansion



The mechanism leads to endogenous reversals of bank lending as after several periods with narrowed credit spreads, growth of the fundamentals will be below high expectations. This leads to a widening of the credit spread. This comes through extrapolative expectation formation with regard to loans, L_t, but also from a change in the growth rate of the economy, g_t.[11]

[10] In the quantitative part labelled as external finance premium (EFP), with $EFP = R_t^L - R_t^D$.

[11] See also López-Salido et al. (2017) who show that a reversal of credit market conditions lead to inward shift in credit supply resulting in a contraction of economic activity.

It should be noted that the extrapolation mechanism in the loan valuation also applies in case of productivity or monetary shocks, as both lead to a change in loans. Here, we study a pure financial shock in isolation to illustrate its effect on the banking system and the macro economy.

2.2 Households

There is a mass one of infinitely-lived households with the utility described by

$$\max \ E_0 \sum_{t=0}^{\infty} \beta^t \left[log(c_t) + \phi_l log(1 - l_t^s - m_t^s) \right] \tag{23}$$

where c_t is consumption, l_t^s labor provided to the production sector and m_t^s labor provided to the banking sector. ϕ_l reflects the weight of leisure. Households can only acquire consumption goods by spending bank deposits D_t which means that they face a money in advance constraint given by

$$P_t c_t \leq v D_t \tag{24}$$

Their budget is described by the following inequality which involves the interest payments on loans, L_t, and deposits, D_t:

$$\begin{aligned} c_t + \frac{B_{t+1}}{P_t} + \frac{D_{t+1}}{P_t} - \frac{L_{t+1}}{P_t} + \frac{Q_t^{\Psi}}{P_t}\Psi_t \quad \leq \quad & w_t(l_t^s + m_t^s) + (1 + R_t^B)\frac{B_t}{P_t} ... \\ & ... + (1 + R_t^D)\frac{D_t}{P_t} - (1 + R_t^L)\frac{L_t}{P_t} + \frac{Q_t^{\Psi} + \Pi_t^{\Psi}}{P_t}\Psi_{t-1} + \Pi_t^F \end{aligned} \tag{25}$$

Here B_t is savings in government bonds,[12] w_t the real wage for production or banking labor, P_t the price level, and R_t^D, R_t^L, and R_t^B interest rates on the respective assets and liabilities. Q_t^{Ψ} represents the equity price and Ψ_t the equity investments. Π_t^{Ψ} relates to dividend payments for bank equity.

Optimal household decisions regarding work provided to the production sector and banking sector imply the following relationship

$$\lambda_t w_t = \frac{\phi_l}{(1 - l_t^s - m_t^s)}. \tag{26}$$

[12]Households can save part of their income in (riskless) bonds at the end of the period. Bonds are held in net zero supply as the households in aggregate cannot sell the bonds. However, consumption-savings decisions affect the bond rate.

The Euler equation with respect to bonds reads as

$$\begin{aligned}\mathbb{E}_t\Lambda_{t,t+1}(1+R^B_{t+1}) &= 1 \\ \Lambda_{t,t+1} &\equiv \mathbb{E}_t\beta\left\{\frac{\lambda_t P_{t+1}}{\lambda_{t+1}P_t}\right\}\end{aligned} \tag{27}$$

The Euler equation for the pricing of equity, Ψ_t, assuming no direct utility from equity holdings,[13] gives

$$1=\beta\mathbb{E}_t\left\{\frac{\lambda_{t+1}}{\lambda_t}\frac{Q^\Psi_{t+1}+\Pi^\Psi_{t+1}}{Q^\Psi_t}\frac{P_t}{P_{t+1}}\right\} \tag{28}$$

For each household i the cash in advance constraint

$$P_t c_t(i) = vD_t(i) \tag{29}$$

generates an individual loan demand

$$L_t(i) = D_t(i) \tag{30}$$

Household i obtains individual deposits $D_t(i)$ through loans $L_t(i)$ from each bank j, i.e.

$$L_t(i) = \int_0^1 L_t(i,j)dj \tag{31}$$

and

$$D_t(i) = \int_0^1 D_t(i,j)dj, \tag{32}$$

where the individual demands are determined by the interest rate ratio of the bank j vs. the aggregate interest rate level. Finally, optimal holdings of bank deposits, D_t, are determined by

$$\frac{1}{\lambda_t} = c_t + \frac{L_t}{P_t}\left[R^L_t - R^D_t\right], \tag{33}$$

which relates them to consumption and the marginal cost of holding loans, i.e. the aggregate interest spread.[14]

2.3 Firms

Production of consumer goods involves two stages with intermediate inputs. The final goods firm produces a composite good, y_t, by combining intermediate goods, $y_t(i)$, through a constant

[13] We abstract from the possibility that agents could draw prestige related to social status from owning banks and other wealth items, see Kumhof et al. (2015) for the alternative approach.

[14] Subsequently, we leave out the respective subscript as each household is identical.

elasticity of substitution (CES) aggregator, i.e.

$$y_t = \left(\int_0^1 y_t(i)^{1-\epsilon} di \right)^{\frac{1}{1-\epsilon}} \tag{34}$$

The profit function of intermediate firm i is given by

$$\Pi_t^F(i) = y_t(i) - w_t l_t(i) \tag{35}$$

Intermediate goods are produced by employing labor, l_t, according to the following technology

$$y_t(i) = A1_t l_t(i)^{1-\eta} \tag{36}$$

In (36), $A1_t$ is a shock to productivity in goods production, similar to a standard TFP shock in the real-business-cycle literature, whose mean increases over time at the trend growth rate of g.

There is a probability of θ that firms are not able to change the price in a given period. Thus firms setting the price have to solve the following multi-period problem (Calvo (1983) pricing), i.e.

$$\sum_{k=0}^{\infty} \theta^k \mathbb{E}_t \{ R_{t,t+k}^B y(i)_{t+k|t} (P_t^*(i) - \mathcal{M}MC_{t+k}) \} = 0 \tag{37}$$

with P_t^* being the optimal price set in period t.

2.4 Monetary authority

The policy rate follows a Taylor (1993) rule which reacts to inflation, π_t, and fluctuations in output, y_t, i.e.

$$R_t^p = \left(R_{t-1}^p\right)^{\rho} \left(\frac{\pi_t}{\pi^*}\right)^{(1-\rho)\phi_\pi} \left(\frac{y_t}{y_{t-1}}\right)^{(1-\rho)\phi_y} A3_t \tag{38}$$

with $A3_t$ following the following AR(1) process

$$A3_t = \rho_3 A3_{t-1} + \epsilon_t^3. \tag{39}$$

We modify the Taylor rule for further experimentation in the quantitative section. We do not consider the inclusion of the output gap since, as also stressed by Galí (2008), this implicitly assumes observability of the latter variable. That assumption is unrealistic because determination of the output gap and its movements requires an exact knowledge of (i) the economy 'true model', (ii) the values taken by all its parameters including κ, and (iii) the realized value (observed in real time) of all the real and financial shocks impinging on the economy (see Corrado and Schuler, 2017).

3 The Equilibrium

This section presents a closed form solution for the equilibrium. We consider equilibria with rational expectations at the side of households. Banks optimize intra-period with regard to their commercial lending and inter-period with regard to trading of loan portfolio governed by their price expectations described in section 3.2. We describe expectations for the special case where all banks hold the same subjective beliefs, $\mathcal{P}$, and where these beliefs imply no uncertainty about future loan portfolio prices. Assuming no uncertainty allows key insights to be derived from how the equilibrium price of the loan portfolio depends on banks' beliefs and their evolution over time. By imposing market clearing in the labor, goods, credit, stock and bond markets, the model can be solved for the equilibrium solution.

Theorem 2 (Equilibrium) *Households maximize their utility by choosing optimal sequences* $\{c_t, l_t^s, m_t^s, B_t, L_t, \Psi_t\}$. *The intermediate firm* i *chooses optimal prices* $P_t^*(i)$, *given its cost function with labor input* $\{l_t(i)\}$. *The final goods producing firm provides* $\{y_t\}$ *through a cost-minimal combination of intermediate goods* $y_t(i)$. *Commercial banks maximize profits by lending to households* $\{L_t\}$, *by receiving funds in the form of deposits* $\{D_t\}$, *and from trade in loan portfolios* $\{L_{t+1}\}$ *according to the law of motion below. Commercial banks provide loans* L_t *to households. Markets clear in each period* t, *i.e. for output* $y_t{=}c_t$, *bond holdings of the households clear as well as total stock holdings* $\Psi = \int_0^1 \Psi_t(i)$. *Labor markets clear, i.e.* $l_t^s{=}l_t$ *and* $m_t^s{=}m_t$.

We solve the model for its equilibrium, calculate non-stochastic steady states and linearize the model around the steady state. Upon log-linearizing and combining the relevant equilibrium conditions, we obtain a system of equations which characterize the dynamics of the economy in the neighborhood of the non-stochastic steady state. There are three forcing variables: productivity shocks $a1_t$, financial innovation shocks $a2_t$, and monetary shocks $a3_t$. We list the main equilibrium conditions in Appendix A.

Equilibrium Conditions. Investors rationally maximize infinite horizon utility but hold subjective beliefs, $\mathcal{P}$, about the asset process that we allow to differ from the prior rational expectations which can be solved separately. For the case with subjective beliefs the law of motion for future loan values is

$$\widetilde{\mathbb{E}}_t L_{t+1} = L_t + g + \delta_g b_t \tag{40}$$

with

$$b_t = (1-\kappa)\widetilde{\mathbb{E}}_{t-1} g_t + \kappa(L_t - L_{t-1}) = \tag{41}$$

$$f\left(\widetilde{\mathbb{E}}_{t-1} g_t, (L_t - L_{t-1})\right). \tag{42}$$

Bayesian updating of beliefs then gives rise to self-reinforcing credit market optimism that results in a credit boom. Thus the presence of a bubble, b_t, creates an amplification effect in the value of the loan portfolio.

From the household's equilibrium conditions, given the market clearing and bank's balance sheet equilibrium defined in Theorem 2, a credit bubble drives up consumption and therefore output and inflation. We have a financial accelerator that translates belief-driven credit fluctuation into the real sector since for households the cash in advance constraint

$$P_t c_t = v D_t \tag{43}$$

generates an individual loan demand

$$L_t = D_t \tag{44}$$

The accelerator is such that it gives rise to positive co-movement between credit, consumption and inflation. While credit supplied by the banking sector under rational expectations only depends on the optimal loan monitoring work provision, it can deviate from its rational expectation value under subjective beliefs and generate over or under credit provisions linked to the endogenous bubble formation mechanism.

Solution Dynamics. Under fully rational expectations the set-up is standard as households' decisions depend only on the history of the fundamental shocks $Z_t = (a1_t, a2_t, a3_t)$.

Under subjective beliefs $\mathcal{P}$ the state $S_t = \left(Z_t, F_t^b\right)$ describing the aggregate economy at time t also includes the lagged and predetermined values of the fundamentals $F_t^b = \left(\widetilde{\mathbb{E}}_{t-1} g_t, L_{t-1}\right)$ which are required to describe the evolution of beliefs described in (41).

The equilibrium dynamics can be described by a log-linear state transition function V that maps current states and future shocks into future states

$$S_{t+1} = V\left(S_t, Z_{t+1}\right) \tag{45}$$

The outcome function $G(.)$ maps current states into economic outcomes for the remaining variables

$$\left(c_t, l_t^s, m_t^s, B_t, L_t, \Psi_t\right) = G\left(S_t\right) \tag{46}$$

4 Quantitative results and policy experiments

In the following section, we first describe the estimation of κ and the benchmark calibration for the simulation of the model. Then we show impulse responses for a financial bubble shock. Finally, we employ the simulated model for several policy experiments.

4.1 Estimation of signal-to-noise ratio κ

We estimate equations (105) and (106) of the Kalman filter mechanism and use the values for σ_η^2 and σ_v^2 to calculate the Kalman gain, κ. The Kalman gain κ takes the form

$$\kappa \simeq \frac{\sigma_\eta^2}{\sigma_\eta^2 + \sigma_\nu^2}$$

and results approximately in

$$\kappa \simeq 0.359$$

Refer to Appendix D.2 for the estimation results.

4.2 Benchmark calibration

The model is calibrated to quarterly frequencies matching endogenous aggregates and interest rates to observable data. We assume zero average inflation. The household discount factor is set to 0.99 implying an annual real rate of interest of 4% for the riskless bond rate R^B.

Table 1: Parameters

β	discount factor	0.997
η	concavity in production	0.34
α	concavity in loan management	0.38
ϕ_l	weight of leisure in utility	0.7
ϵ	Dixit-Stiglitz parameter	6
τ	equity target level	0.11
v	velocity of money	0.31
θ	share of firms without price reset	0.77
$\mathcal{M}$	price markup	1.2
ϕ_π	weight of inflation in policy function	1.5
ϕ_y	weight of output in policy function	0.5
ρ	smoothing in policy function	0.25
ω_Ψ	share of dividends in bank profits	0.68
δ_e	equity depreciation	0.025
κ_e	leverage deviation cost	4
ϵ_L	elasticity loan demand	3.5

The share of intermediate firms which cannot reset their price in a given period is $\theta = 0.77$. The Dixit-Stiglitz parameter, ϵ, is set to 6 generating a mark-up of 20%. The velocity of money v is set to 0.31 on the basis of average GDP to M3 after the US subprime crisis. The capital requirement ratio τ is set to 11%. For further experimentation we change τ to 15%.

We assume a coefficient η equal to 0.34 for the concavity of labor in the production function of the intermediate product; for loan management we choose a coefficient α equal to 0.38. We set

Table 2: Implied Steady-States

R^P	policy rate	0.0084
R^B	bond rate	0.0101
R^D	deposit rate	0.0067
R^L	loan rate	0.0169
c	consumption	0.6244
$\frac{l}{c}$	production work	0.4900
$\frac{D}{c}$	deposits	2.0145
$\frac{L}{c}$	loans	2.0145
w	wage	0.9588
$\frac{m}{c}$	monitoring work	0.0246
ω_B	bank's profits	0.0239
ϕ_Ψ	share of bank's profits paid as dividends	0.7690
e	equity	0.2215
Π^Ψ	bank's dividends	0.0184
Q^Ψ	equity price	1.8258

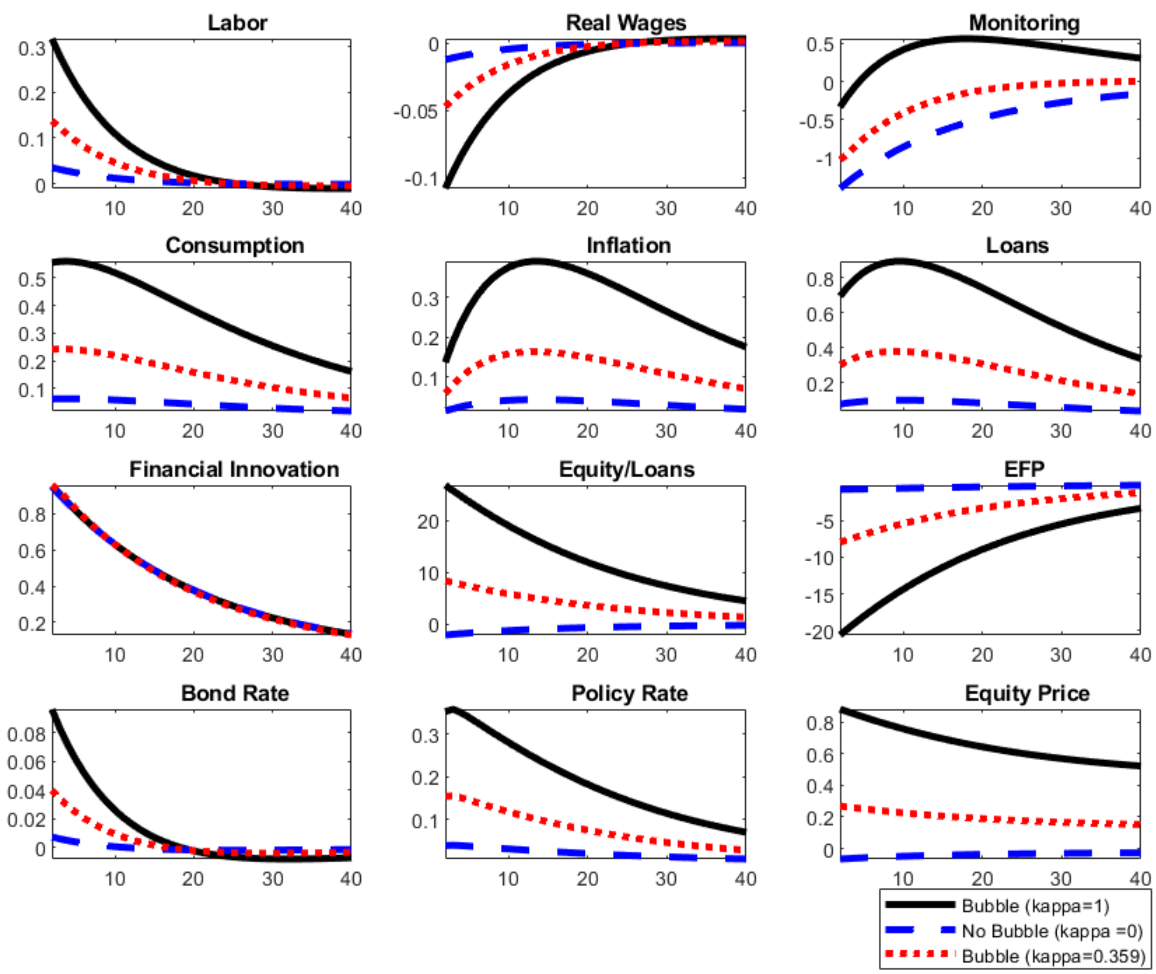
total labor supplied in steady state to 1/2 hours, following Goodfriend and McCallum (2007). The share of working time devoted to banking services is 2%. This implies that a share of 49% of total time is in the production sector and 1% in the banking sector. Following Gerali et al. (2011) we calibrate the banking parameters to replicate data averages for commercial bank interest rates and spreads. We calibrate the steady states to $R^B = 4\%$ p.a. and $R^P = 3.36\%$ p.a. This implies an annualized return for $R^D = 2.6\%$ p.a. and a loan rate $R^L = 6.7\%$ p.a. From the derivation of the implied steady states of the model we have it that 76% of profits are paid as dividends assuming that equity depreciates at 10% p.a ($\delta_e = 0.025$).

Table 3: Calibration of exogenous shocks

Persistence		
ρ_1	productivity	0.95
ρ_2	financial innovation	0.9
ρ_3	monetary policy	0.9
Volatility		
σ_1	productivity	0.72%
σ_2	financial innovation	0.82%
σ_3	monetary policy	1%

The technology shocks are assumed to be quite persistent, with a standard deviation equal to 0.72% and an autoregressive parameter 0.95. The shock to the policy rate has a standard deviation equal to 0.82%, and an autoregressive parameter of 0.9, and for the financial innovation shock we assume a higher standard deviation of 1% and an autoregressive parameter equal to 0.9. Shocks to the Total Factor Productivity (TFP) have a relatively prolonged effect on macroeconomic variables. The bubble shock is modelled as being somewhat persistent due to its effects on loan creation. Monetary policy coefficients on inflation and the output are 1.5 and 0.5. The rest of the parameters, implied steady states and interest rates used in the calibration are given in Tables 1-3.

Figure 6: Impulse responses to a financial innovation with different values for Kalman gain



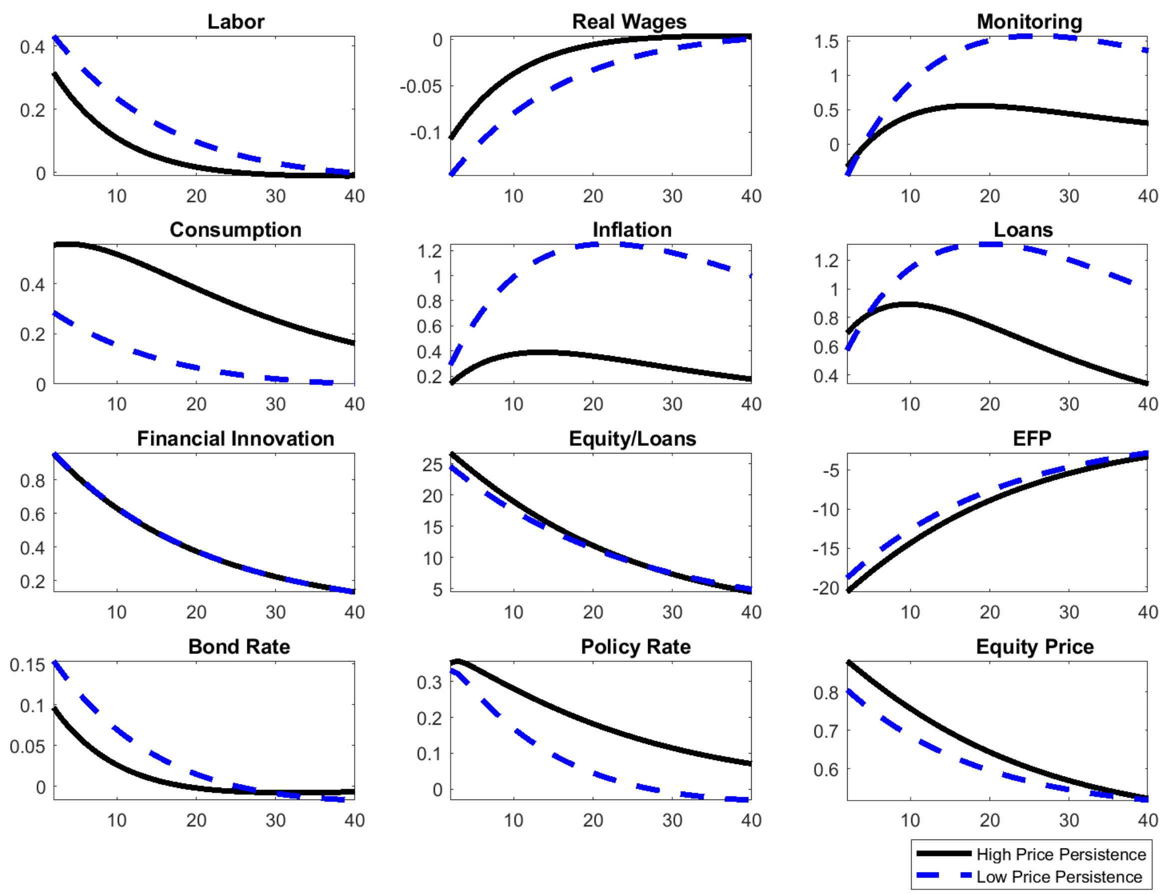
Note: All interest rates are shown as absolute deviations from the steady state, expressed in percentage points. All other variables are percentage points deviations from the implied steady state value.

4.3 Impulse response to a financial innovation shock

We conduct experiments with a shock to bank monitoring to illustrate the model mechanisms.

The impulse response function to a financial innovation shock is shown in Figure 6. In the financial bubble cases, i.e. $\kappa = 1$ and $\kappa = 0.359$ as estimated, the financial innovation leads to a reduction in monitoring needs to service given transaction money demand. Simultaneously, the bank spread (external finance premium, or EFP) is lowered. The amount of loans handed out by banks and the equity price rise on impact. They then further increase due to the positive feedback mechanism of the financial bubble. The spread is further compressed down by the expansion in the supply of loans while equity prices increase further. Hence, the presence of a bubble generates an amplification effect in the financial sector (via loans and equity) and in the real sector (via consumption funded by transaction money demand). The financial bubble has a direct impact on inflation and due to staggered pricing, it also has real effects on consumption in addition to the initial financial innovation.

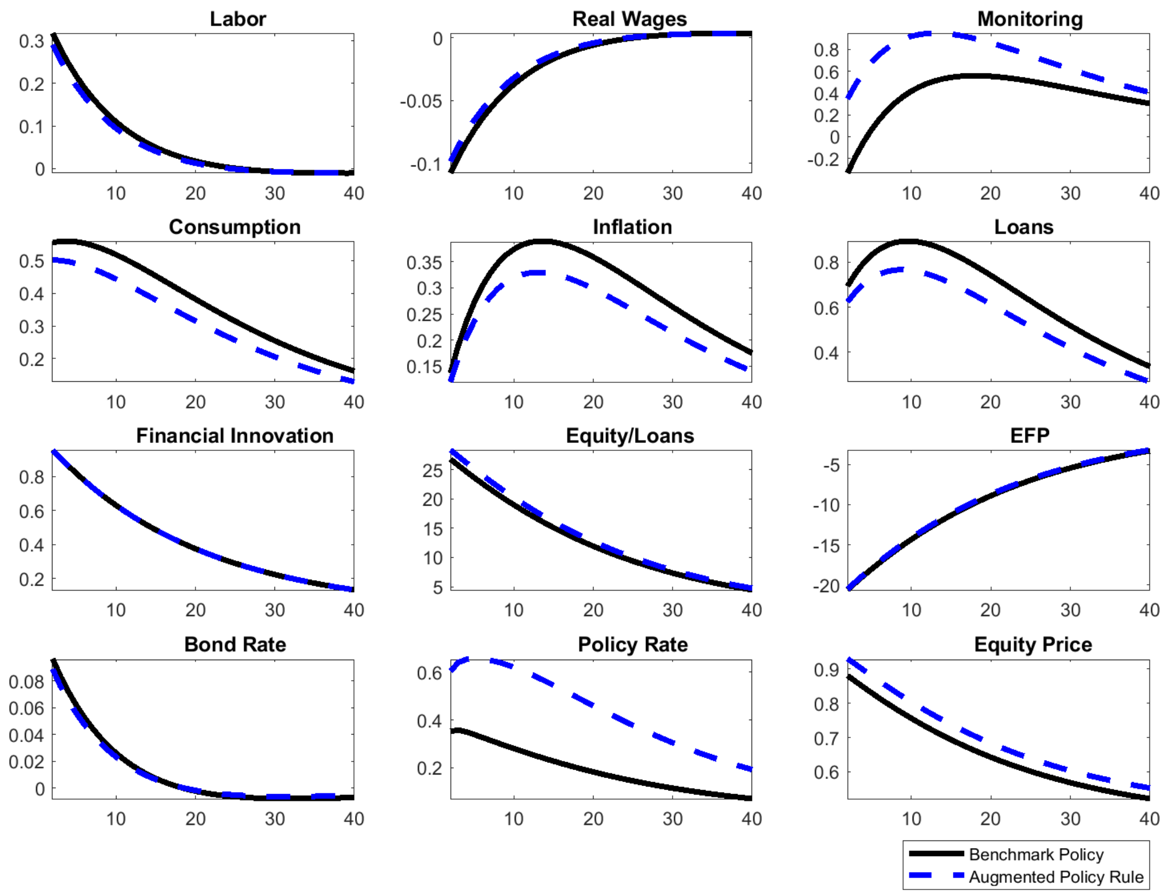
Figure 7: Impulse responses to a financial innovation with high and low price inertia



Note: All interest rates are shown as absolute deviations from the steady state, expressed in percentage points. All other variables are percentage point deviations from the implied steady state value.

We illustrate the effect of the staggered pricing mechanism (as in Calvo (1983)) under a

Figure 8: Impulse response of financial innovation with different monetary policy reaction



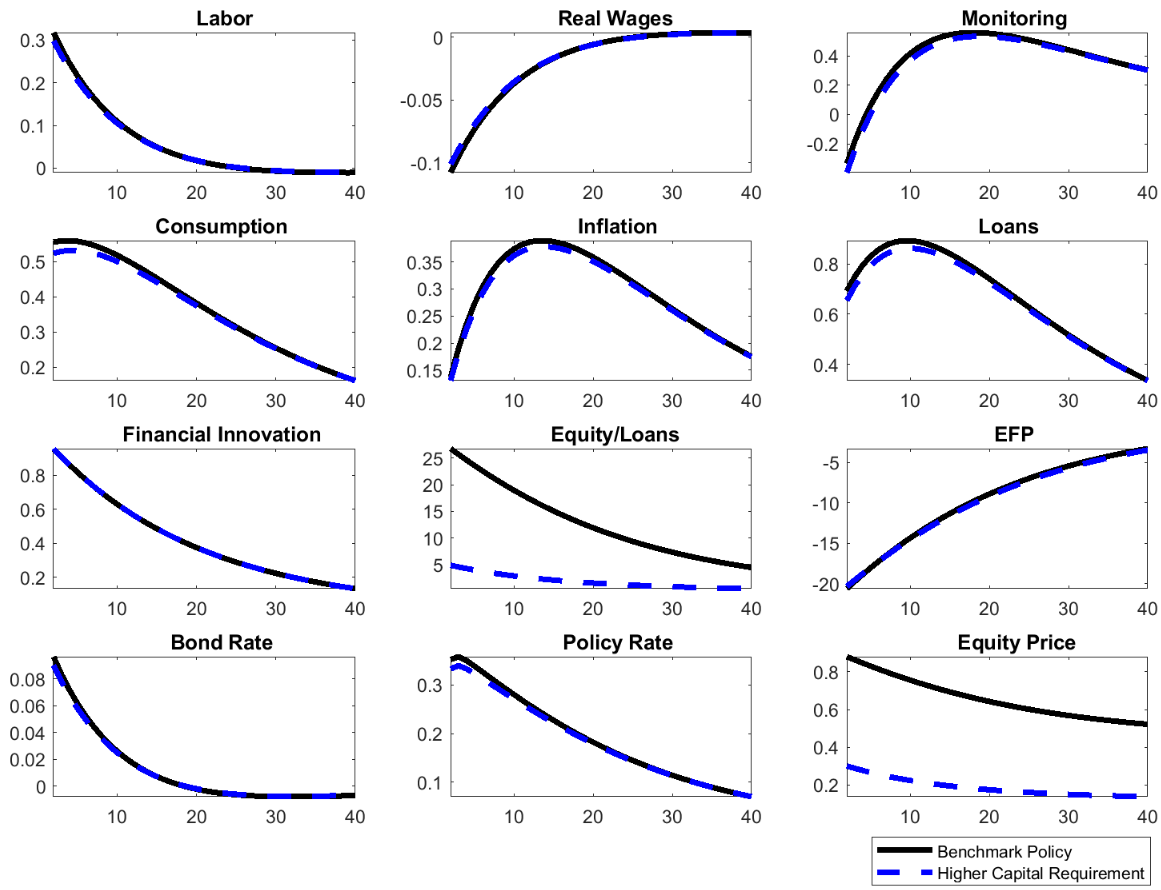
Note: All interest rates are shown as absolute deviations from the steady state, expressed in percentage points. All other variables are percentage points deviations from the implied steady state value.

financial innovation shock (Figure 7) and $\kappa = 1$. We find that with higher price persistence less adjustment is channeled through inflation and real effects are higher. Hence, the effects of a financial bubble differ for economies depending on the degree of price flexibility. In particular, the assumption of sticky prices makes monetary policy non-neutral, allowing it to influence the size of the bubble; on the other hand, price stickiness makes it possible for aggregate loan fluctuations to influence aggregate demand and, hence, output and employment.

4.4 Policy experiments

We test how effective several monetary and macroprudential policies are in this set-up when we are in a bubble economy, i.e. $\kappa = 1$. In Figure 8 we study the effect of monetary policy reacting to changes in overall loans, L_t, setting the weight $\phi_L = 1.5$. This would modify the

Figure 9: Impulse response of financial innovation with higher capital requirement



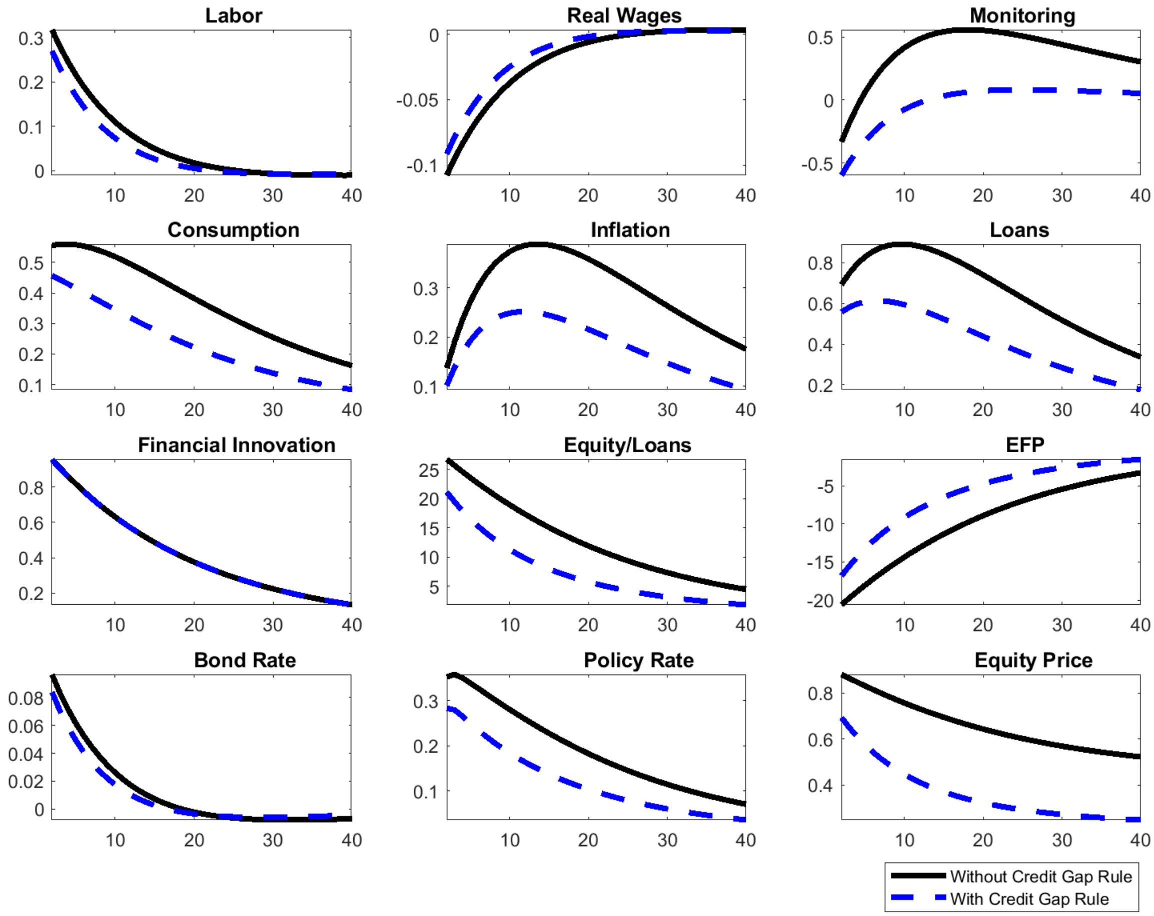
Note: All interest rates are shown as absolute deviations from the steady state, expressed in percentage points. All other variables are percentage points deviations from the implied steady state value.

Taylor rule of the monetary authority as follows:

$$R_t^p = \left(R_{t-1}^p\right)^\rho \left(\frac{L_t}{L_{t-1}}\right)^{(1-\rho)\phi_L} \left(\frac{\pi_t}{\pi^*}\right)^{(1-\rho)\phi_\pi} \left(\frac{y_t}{y_{t-1}}\right)^{(1-\rho)\phi_y} A3_t. \tag{47}$$

We see in the impulse response that the modified Taylor rule has some effect on inflation and consumption as the policy reaction doubles. A reaction to overall loan growth barely affects bank leverage, the credit margin (external finance premium, EFP) or the equity price. We conclude that a "leaning against the wind" policy has some effect in reducing the size of the financial bubble. The next experiment uses a macroprudential measure by increasing the target level of the capital requirement ratio, τ, from a level of 11% towards 15%, which affects the

Figure 10: Impulse response of financial innovation with endogenous capital requirement



Note: All interest rates are shown as absolute deviations from the steady state, expressed in percentage points. All other variables are percentage point deviations from the implied steady state value.

profits at the headquarter level:

$$\Pi_t^B = \sum_{t=0}^{\infty} \left[R_t^L \frac{L_t}{P_t} - R_t^D \frac{D_t}{P_t} - \frac{\kappa_e}{2} \left(\frac{e_t}{L_t} - \tau \right)^2 - w_t m_t \right] \tag{48}$$

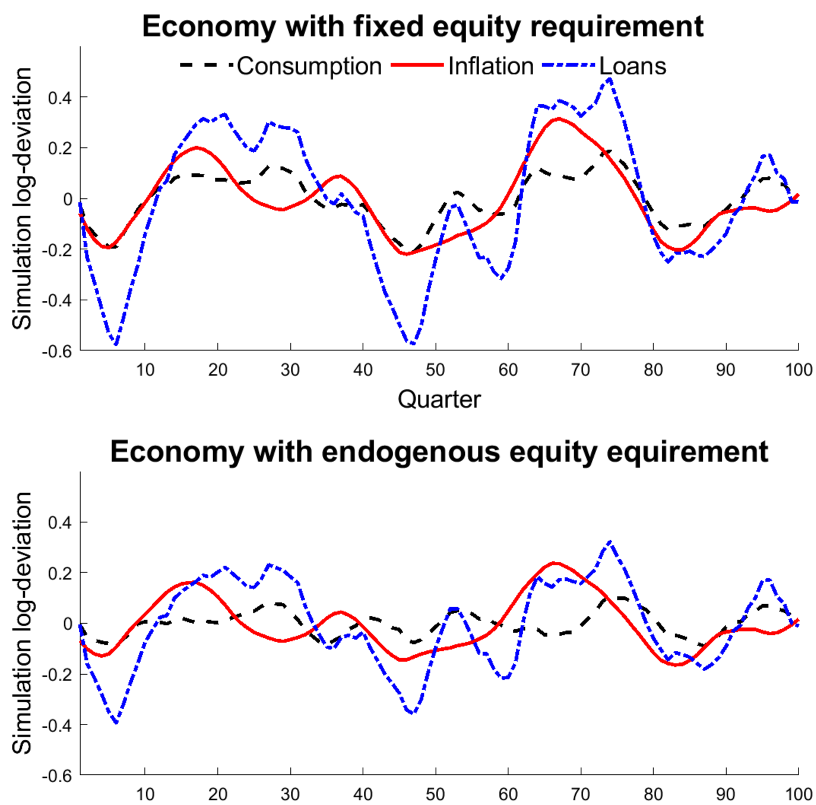
The increased capital requirement ratio leads to a slight reduction of the impact of the shock as demonstrated in Figure 9. The response of inflation and consumption is dampened. This works through the equity price which increases by less during a financial shock. On the other hand, the negative impact on the spread between the loan and deposit rate is reduced, meaning that the financial sector can better absorb the bubble shock.

Finally, in Figure 10 we introduce an endogenous capital requirement, τ_t, set by a regulator along the following rule reacting to the credit-to-GDP gap:

$$\tau_t = \overline{\tau} + \kappa_\tau \left(\frac{L_t}{Y_t} - \frac{L}{Y} \right) \tag{49}$$

Under the rule reacting to the credit-to-GDP gap we study the behavior of the financial bubble compared to the base case. While the equity price falls more than in the base case (but less than under the exogenous increase in target capital), the reaction of the spread (EFP) is dampened. The stabilization comes from the combination of a lower equity price and a limited response of monitoring. The endogenous increase in the necessary bank equity holdings counteracts the initial cost reduction from the financial innovation shock. The side effects of the financial bubble are reduced and inflation and consumption react significantly less. The visible impact on the target variables, inflation and consumption, and the limited reaction of other variables let us conclude that an endogenous requirement is effective in precisely working in the required way without adversely affecting other macroeconomic variables.

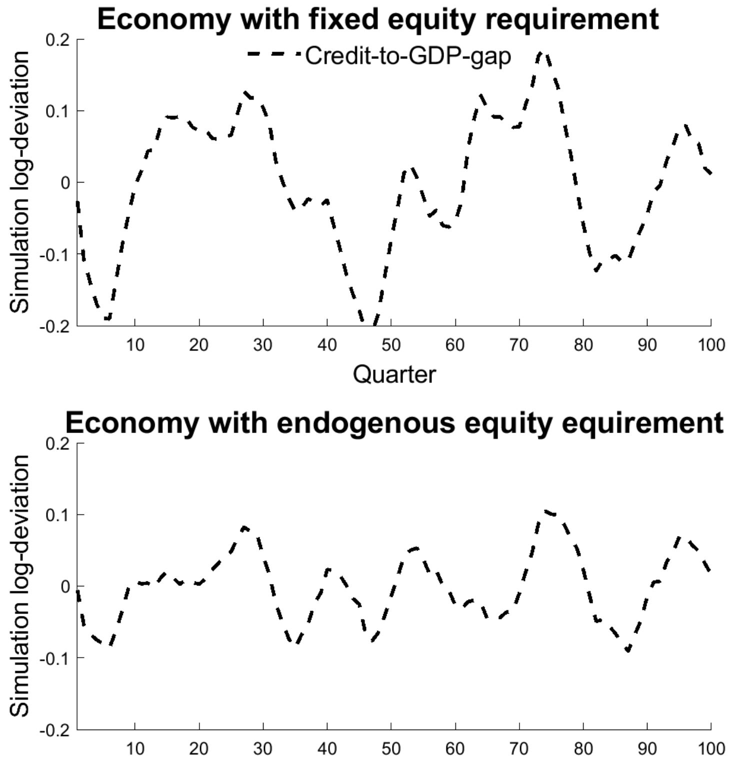
Figure 11: Simulation of the economy with fixed and endogenous capital requirement



Note: Two-year moving average of deviations in consumption, inflation and loans.

Supporting this reasoning, Table 4 shows that under an endogenous macroprudential rule

Figure 12: Simulation of credit-to-GDP gaps with fixed and endogenous capital requirement



Note: Two-year moving average of deviations in total credit-to-GDP.

the volatility of consumption and loans is significantly attenuated while the volatility of equity and equity prices increases. The volatility of consumption is particularly attenuated in a bubble economy with $\kappa = 1$ but also with moderate values of $\kappa = 0.359$ an endogenous capital requirement has stabilizing effects in the real economy.

This is illustrated in Figure 11 which gives a simulation of the main variables consumption, inflation and loans in a bubble economy ($\kappa = 1$). We see that the amplitude of the cycle is much smaller in the case of the endogenous equity requirement. In particular, credit booms and busts are attenuated. Furthermore, the simulation shows a stabilizing effect of the endogenous rule on the real economy and on inflation.

In the simulation we use all the array of shocks, i.e. productivity shocks, financial friction shocks, monetary shocks. The bubble mechanism endogenously reacts to these shocks.

On the motivation of this paper in section 1.1. we show, in Figure 12, how an endogenous requirement (countercyclical capital buffer) helps to dampen financial cycles. The amplitude of the credit-to-GDP gap is reduced from more than 30% peak-to-trough to 20% peak-to-trough. Financial cycles also become shorter as the countercyclical buffer is a self-correcting mechanism

Table 4: Model Moments

	Benchmark		Credit Gap Rule	
	S.D.	Corr.	S.D.	Corr.
$\kappa = 1$				
Consumption	0.16497	1	0.11331	1
Equity	4.42854	0.63471	7.34583	0.43092
Equity Price	0.13782	0.59156	0.23655	0.38796
Loans	0.44273	0.94411	0.32735	0.69627
$\kappa = 0.359$				
Consumption	0.15734	1	0.10294	1
Equity	1.99801	0.41801	6.77659	0.17558
Equity Price	0.06152	0.30354	0.21952	0.12874
Loans	0.44066	0.96513	0.32536	0.72256
$\kappa = 0$				
Consumption	0.14815	1	0.11315	1
Equity	2.26443	0.58141	6.76117	0.23365
Equity Price	0.06928	0.50493	0.21734	0.18775
Loans	0.43408	0.97765	0.33474	0.74822

Note : *Corr.* is given relative to consumption.

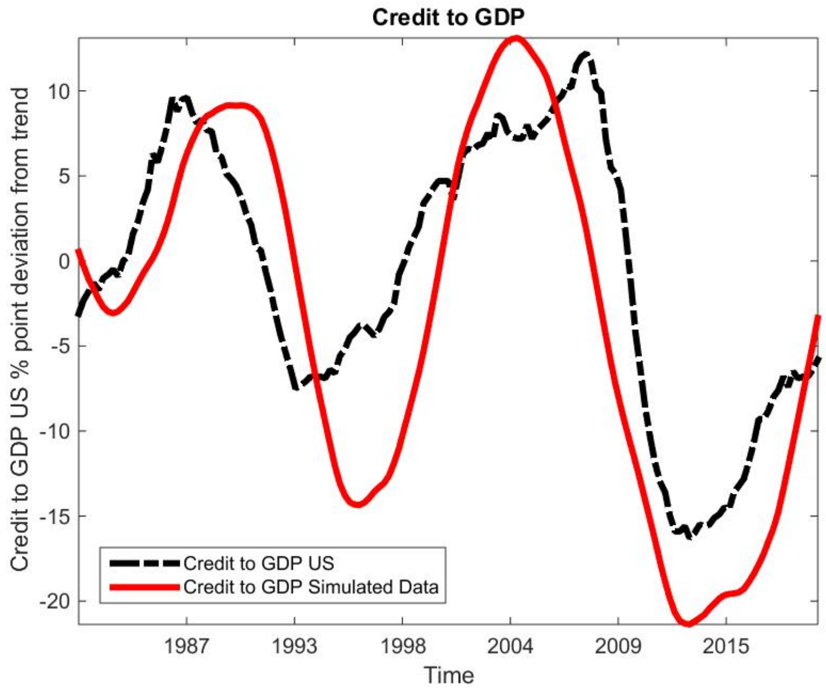
of excessive deviations in the credit-to-GDP measure.

4.5 Empirical validation

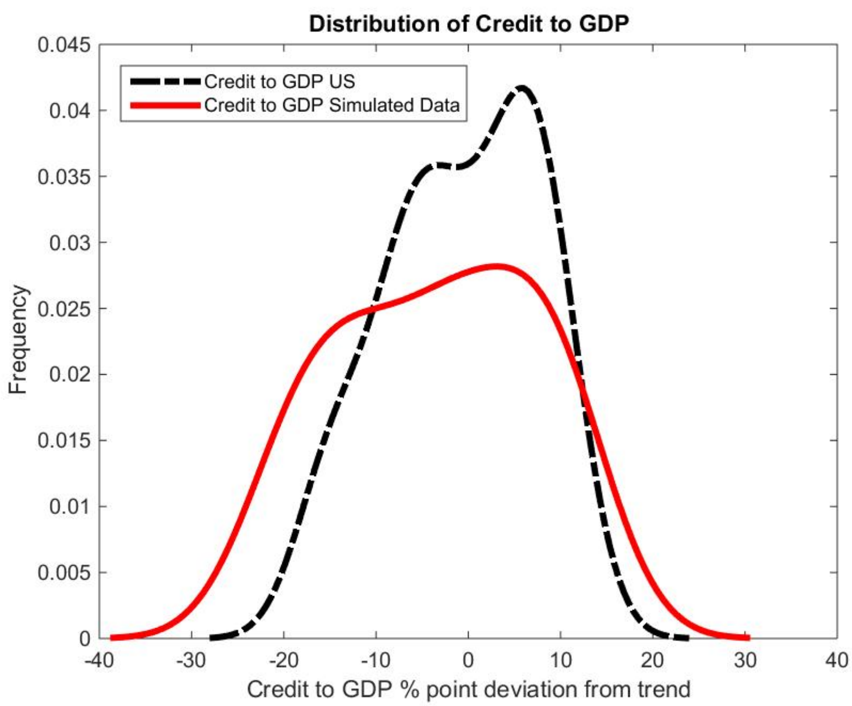
We present an adaptive expectation approach for quantitatively capturing US credit market cycles. The negative skewness and the large support of the empirical distribution capture the presence of occasional credit market run-ups and reversals, i.e., periods in which prices grow considerably faster (or slower) than the growth of the economy.

Figure 13 shows that the simulated credit-to-GDP from our adaptive model specification and real data for the United States are matching over the business cycle. Panel (a) displays data for the credit-to-GDP gap, i.e. its deviation from trend in the period from 1981 to 2018. It shows the cyclical swing in the 1980s reaching a peak in 1987 followed by a trough in mid-1990s. The ensuing phase resulted in an even larger upward movement in the measure driven by the dotcom and housing boom. In the global financial crisis the credit-to-GDP measure fell dramatically from above 10% to -15%. Our simulation with $\kappa = 1$ is able to match the data for the credit-to-GDP gap, especially with regard to the amplitude of the cycles. As the amplitude increases compared to the period before the 1990s, this indicates that the US economy became more bubble-driven over time. Panel (b) shows the kernel distribution of the US credit-to-GDP gap compared to the simulated data. Particularly, the estimated negative skewness of real and simulated data are similar.

Figure 13: Credit to GDP Data and Simulation.



(a) Simulation of historical data



(b) Kernel distribution

Source: BIS. This data is based on total credit to the private non-financial sector, as a percentage of GDP. The credit-to-GDP gap is defined as the difference between the credit-to-GDP ratio and its long-term trend in percentage points. The long-term trend is calculated using a one-sided Hodrick-Prescott filter with a lambda of 400,000. Credit to GDP US skewness=-0.3215; Credit to GDP Simulated Data skewness=-0.3582

5 Welfare

We have approximated welfare by employing a second-order Taylor expansion to utility and derived the loss function using the labor demand function, the marginal cost function and the money in advance constraint.[15]

Table 5: Welfare

	Output volatility	Inflation volatility	Welfare loss	Δ Loss[1]
$\kappa = 1$				
Benchmark	0.16497	0.05174	0.05996	-
Monetary Policy Reaction	0.14301	0.05109	0.05229	-12.8%
Higher Capital Requirement[2]	0.17271	0.05216	0.05263	-12.2%
Credit-Gap Rule	0.11331	0.04168	0.04149	-27.7%
$\kappa = 0.359$				
Benchmark	0.15734	0.05321	0.05738	-
Monetary Policy Reaction	0.15013	0.05291	0.05486	-4.4 %
Higher Capital Requirement[2]	0.15684	0.05435	0.04722	-17.7%
Credit-Gap Rule	0.10294	0.04194	0.03790	-34.0%
$\kappa = 0$				
Benchmark	0.14815	0.05223	0.05414	-
Monetary Policy Reaction	0.15920	0.05236	0.05799	1.1%
Higher Capital Requirement[2]	0.16190	0.05390	0.04896	-14.7%
Credit-Gap Rule	0.11315	0.04150	0.04143	-27.8%

[1] Relative to respective Benchmark.
[2] Calculated on the basis of different steady-states implied by the higher capital requirement.

The loss function reads as

$$\mathfrak{L}_t = \varphi\sigma_{\tilde{y}}^2 + \varpi\sigma_{\pi}^2, \tag{50}$$

with φ and ϖ resulting from model parameters. We use the approximation to quantify the welfare rankings which result from the monetary and macroprudential rules. Table 5 shows the welfare losses for the different regimes. A higher capital requirement is slightly procyclical, but incorporating changes to the steady state improves the welfare result. By means of a higher equity requirement the spread and monitoring reaches minimal levels placing an upper bound for possible welfare gains in well-capitalized economies.[16] When we are facing a bubble economy ($\kappa = 1$) the endogenous capital requirement via a credit-gap rule reduces the welfare loss by

[15]The full derivation can be found in section C of the Appendix.

[16]We take into account lower credit spreads (EFP) in steady state in higher capitalized economies (see Gambacorta and Shin (2016)) through a loan production function which incorporates different equity levels through the loan production efficiency $A2_t$. We arrive at a welfare gain of up to 0.01 in consumption units. The welfare gain is calculated by incorporating equity implicitly into the Cobb-Douglas function for loan management.

27.7%, while the monetary policy reaction reduces welfare losses by just 12.8%. The credit-gap rule performs better in terms of welfare than the policy reaction to loan growth as it precisely targets bank leverage. It is also worth noting that welfare for credit rule is best in combination with $\kappa = 0.359$ as it reduces welfare losses by more than 34% while the monetary policy reaction reduces welfare losses by only 4.4%. In addition, there are higher losses with a monetary policy reaction in case of no bubble ($\kappa = 0$) as in this case the authorities are over-responsive to lending conditions.

In all cases for the Kalman gain, κ, the credit-gap rule performs better than the augmented Taylor rule. The endogenous increase in the necessary bank equity holdings counteract the extrapolative effects from the expectation formation of banks. As bank equity and the credit spread are the main channels for the development of credit bubbles and reversals, an endogenous capital requirement is effective in precisely working in the required way without adversely affecting other macroeconomic variables, while the Taylor rule instead does not target bank spreads.

6 Conclusions

In this paper, we set up a framework for the causes and effects of a financial bubble. We use this model to shed light on recent policy debates on the use of monetary and macroprudential instruments. We augment a standard New-Keynesian macroeconomic model with a loan management technology and endogenous equity holdings for banks in order to define policy instruments and measure their efficacy in counteracting financial bubbles.

The financial bubble features the deviation of the value of an asset from its equilibrium value, as well as a positive feedback mechanism for the value deviation. The analytical framework shows how a financial bubble can develop from the credit supply side with banks following behavioral functions where learning by agents in the model gives rise to endogenous credit booms whenever agents observe increasing fundamental values. The extrapolation comes from a linear filter on the available information for which we provide a micro-foundation for the belief structure by deriving and estimating the (endogenous) signal-to-noise ratio.

Our adaptive expectation approach quantitatively captures the occasional market run-ups and reversals of the US credit market cycles. We show that the simulated credit-to-GDP and real data are matching over the business cycle, i.e. the amplitude of the cycles in credit-to-GDP are similar and also the estimated skewness.

We test several measures to determine whether they can effectively reduce the impact of a financial bubble. We find that a macroprudential rule that reacts to the credit-to-GDP gap proves to be the most effective measure to prevent a bubble from growing. This is because of its inherent tendency to increase costs which counteracts the reduced need for monitoring following a financial innovation. A central bank intervention against the financial bubble ("leaning against the wind") is less effective. Our welfare analysis shows that volatility increases, but overall welfare improves when introducing a higher fixed capital requirement. In a bubble economy an endogenous requirement reduces welfare losses by more than double compared to a monetary policy reaction. The credit-gap rule performs better in terms of welfare than the policy reaction to loan growth as it precisely targets bank leverage. It is also worth noting that welfare for credit rule is best in combination with moderate bubbles as it reduces welfare losses by more than seven times compared to a monetary policy reaction. We therefore provide a comprehensive rationale for the use of countercyclical capital buffers.

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A Linearised model

Let $\widehat{x}$ denote the deviation of a variable x from its steady state. The model can then be reduced to the following linearized system of equations:

1) Supply of production and monitoring labor

$$\widehat{\lambda}_t + \widehat{w}_t = \frac{l}{1-l-m}\widehat{l}_t + \frac{m}{1-l-m}\widehat{m}_t \tag{51}$$

2) Demand for production labor

$$\widehat{w}_t = -\eta\widehat{l}_t + a1_t \tag{52}$$

3) Monitoring demand

$$\frac{1}{\lambda}\widehat{\lambda}_t + c\widehat{c}_t + LR^L\left(\widehat{L}_t + \widehat{R}_t^L\right) + LR^D\left(\widehat{L}_t - \widehat{R}_t^D\right) = 0 \tag{53}$$

4) Production

$$\widehat{c}_t = (1-\eta)\,\widehat{l}_t + a1_t \tag{54}$$

5) Loan provision:

$$\widehat{L}_{t+1} = (1-\alpha)\widehat{m}_t + \delta_g\widehat{b}_t + a2_t \tag{55}$$

6) Money in advance constraint

$$\widehat{c}_t + \widehat{P}_t = \widehat{D}_t \tag{56}$$

7) Inflation

$$\widehat{\pi}_t = \widehat{P}_t - \widehat{P}_{t-1} \tag{57}$$

8) Calvo (1983) pricing

$$\widehat{\pi}_t = \beta\mathbb{E}_t\widehat{\pi}_{t+1} + \vartheta\widehat{mc}_t \tag{58}$$

with $\vartheta = \frac{(1-\theta)(1-\beta\theta)}{\theta}\frac{1-\eta}{1-\eta+\eta\epsilon}$

9) Marginal cost

$$\widehat{mc}_t = \widehat{w}_t - \frac{1}{1-\eta}\left(\eta\widehat{c}_t\right) + (1-\eta)\,\widehat{l}_t \tag{59}$$

10) Bond holding

$$\widehat{B}_t = 0 \tag{60}$$

11) Stock holding

$$\hat{\Psi} = 0 \tag{61}$$

12) Loans

$$\widehat{L}_t = \widehat{D}_t \tag{62}$$

13) Equity

$$e(1-\delta_e)\widehat{e}_{t-1} - e\widehat{e}_t\widehat{\omega}_{B,t-1} = 0 \tag{63}$$

14) Bond rate

$$\widehat{R}_t^B = \widehat{\pi}_t + \widehat{\lambda}_t - \mathbb{E}_t\widehat{\lambda}_{t+1} \tag{64}$$

15) Equity price

$$\mathbb{E}_t\left(\lambda\widehat{\lambda}_{t+1} + Q^\Psi\widehat{Q}_{t+1}^\Psi + \Pi^\Psi\widehat{\Pi}_{t+1}^\Psi - \widehat{\pi}_{t+1}\right) - \left(\lambda\widehat{\lambda}_t + Q^\Psi\widehat{Q}_t^\Psi\right) = 0 \tag{65}$$

16) Loan spread

$$R^L R_t^L = \frac{\epsilon_L}{\epsilon_L - 1}\begin{bmatrix} R^D R_t^D + \frac{vw_t m_t}{(1-\alpha)c}\left(\hat{w}_t + \hat{m}_t - \hat{c}_t\right) \\ -\frac{\kappa_e e}{L^2}\left(2\frac{e}{L} - \tau\right)\widehat{e}_t - \frac{\kappa_e e}{L^2}\left(3\frac{e}{L} - 2\tau\right)\widehat{L}_t \end{bmatrix} \tag{66}$$

17) Deposit rate

$$\widehat{R}_t^D = \widehat{R}_t^P \tag{67}$$

18) Policy feedback rule

$$\widehat{R}_t^P = (1-\rho)\left(\phi_\pi\widehat{\pi}_t + \phi_c\widehat{c}_t\right) + \rho\widehat{R}_{t-1}^P + a3_t \tag{68}$$

19) Bank Profit

$$\begin{aligned} \omega\widehat{\omega}_t \quad = \quad & R^L L\left(\widehat{R}_t^L + \widehat{L}_t\right) - R^D L\left(\widehat{R}_t^D + \widehat{D}_t\right) - \\ & \frac{\kappa_e e}{L}\left(\frac{e}{L} - \tau\right)\left(\widehat{e}_t - \widehat{L}_t\right) - wm\left(\widehat{w}_t + \widehat{m}_t\right) + \delta_g\widehat{b}_t \end{aligned} \tag{69}$$

20) Dividends:

$$\widehat{\Pi}_t^\Psi = \widehat{\omega}_B \tag{70}$$

21) EFP:

$$efp\ efp_t = R^L \widehat{R}_t^L - R^D \widehat{R}_t^D \tag{71}$$

22) Bubble

$$\widehat{b_t} = \delta_g (1-\kappa)(\widehat{c}_t - \widehat{c}_{t-1}) + \delta_g \kappa (\widehat{L}_t - \widehat{L}_{t-1}) \tag{72}$$

There are 22 equations and 22 variables.

B Calculating Steady States

There is no technological progress, i.e. $A1_t = A1 = 1$ and $A2_t = A2 = 1$, and no price change i.e. $P_t = P = 1$.

$$1 + R^B = \frac{1}{\beta} \tag{73}$$

R^D

$$R^D = R^P \tag{74}$$

R^L

$$R^L = \chi_{\epsilon_L} \left[R^D + \frac{vwm}{(1-\alpha)c} \right] \tag{75}$$

c

$$c = l^{1-\eta} \tag{76}$$

D

$$D = \frac{c}{v} \tag{77}$$

w

$$w = (1-\eta)\, l^{-\eta} \tag{78}$$

L

$$L = D \tag{79}$$

m

$$m = \left(\frac{L}{Q} \right)^{\frac{1}{1-\alpha}} \tag{80}$$

λ

$$\lambda = \frac{\phi_l}{w(1-l-m)} \tag{81}$$

ω_B

$$\omega_B = \left(R^L - R^D\right) L - wm \tag{82}$$

e

$$e = \frac{(1-\phi_\Psi)}{\delta_e}\omega_B \tag{83}$$

ϕ_Ψ

$$\begin{aligned} \tau &= \frac{e}{L} = \frac{(1-\phi_\Psi)}{\delta_e}\frac{\omega_B}{L} \\ \phi_\Psi &= 1 - \frac{\tau\delta_e L}{\omega_B} \end{aligned}$$

Π^Ψ

$$\Pi^\Psi = \phi_\Psi \omega_B \tag{84}$$

Q^Ψ

$$\begin{aligned} 1 &= \beta E_t \left\{ \frac{Q^\Psi + \Pi^\Psi}{Q^\Psi} \right\} \qquad (85) \\ Q^\Psi &= \frac{\beta \Pi^\Psi}{(1-\beta)} \end{aligned}$$

$R^L - R^D$

$$R^L - R^D = \frac{R^D}{\epsilon_L - 1} + \frac{\epsilon_L}{\epsilon_L - 1}\left[\frac{v w_t m_t}{(1-\alpha)c}\right] \tag{86}$$

C Welfare

Defining $\hat{x}_t$ as the log deviation from the steady state ($\hat{x}_t = x_t - x$), each variable can be restated as a second order approximation of its relative deviation from the variable's steady state, which reads as:

$$\frac{X_t - X}{X} \simeq \hat{x}_t + \frac{1}{2}\hat{x}_t^2$$

From the problem, above household utility is described by additive functions of consumption and leisure

$$U_t = log(c_t) + \phi_l log(1 - l_t^s - m_t^s) \tag{87}$$

Taking the deviation from the steady state we get

$$U_t - U = \frac{1}{c}(c_t - c) - \frac{\phi_l}{1 - l^s - m^s}(l_t^s - l^s) - \frac{\phi}{1 - l^s - m^s}(m_t^s - m^s) \tag{88}$$

$$-\frac{1}{2}\frac{1}{c^2}(c_t - c)^2 + \frac{1}{2}\frac{\phi_l}{(1 - l^s - m^s)^2}(l_t^s - l^s)^2 + \frac{1}{2}\frac{\phi_l}{(1 - l^s - m^s)^2}(m_t^s - m^s)^2 \tag{89}$$

Simplifying further

$$= \hat{c}_t - \frac{1}{2}\hat{c}_t^2 - \frac{\phi_l}{1 - l^s - m^s}(l^s \hat{l}_t^s + m^s \hat{m}_t^s) + \frac{1}{2}\frac{\phi_l}{(1 - l^s - m^s)^2}c(l^{s2}\hat{l}_t^{s2} + m^{s2}\hat{m}_t^{s2}) \tag{90}$$

Now we rewrite m^s and l^s in terms of output. Production labor demand l_t is given by

$$l_t = \left(\frac{Y_t}{A1_t}\right)^{\frac{1}{1-\eta}} \int_0^1 \left(\frac{p_t(i)}{P_t}\right)^{-\frac{\epsilon}{1-\eta}} di \tag{91}$$

and according to the lemmas in Galì (2008)

$$\int_0^1 \left(\frac{p_t(i)}{P_t}\right)^{\frac{-\epsilon}{1-\eta}} di \simeq 1 + \frac{1}{2}\left(\frac{\epsilon}{1-\eta}\right)\frac{1}{\Theta} var_i\{p_t(1)\} \tag{92}$$

Log-linearizing the above condition

$$(1-\eta)\hat{l}_t = \hat{y}_t - a1_t + (1-\eta) log \int_0^1 \left(\frac{p_t(i)}{P_t}\right)^{-\frac{\epsilon}{1-\eta}} di \tag{93}$$

Loan management demand, m_t, is given by $\frac{L_t}{P_t} = A2_t m_t^{1-\alpha}$, together with the money in advance constraint $c_t \leq \frac{vL_t}{P_t}$ we derive the log-linearized expression for m_t

$$\hat{m}_t = \frac{1}{1-\alpha}\hat{c}_t - a2_t \tag{94}$$

Substitution by $\hat{c}_t = \hat{y}_t$

$$U_t - U = \tag{95}$$

$$\hat{y}_t - \frac{1}{2}\hat{y}_t^2 + \frac{1}{1 - l^s - m^s}\left[\frac{\phi_l}{1-\eta}(l^s[\hat{y}_t - a1_t + (1-\eta) log \int_0^1 \left(\frac{p_t(i)}{P_t}\right)^{-\frac{\epsilon}{1-\eta}} di) + \phi_l m^s \left(\frac{1}{(1-\alpha)}\hat{y}_t\right)\right] \tag{96}$$

$$= \hat{y}_t - \frac{1}{2}\hat{y}_t^2 - \phi_l[\nu(\hat{y}_t + \frac{\epsilon}{2\Theta} var_i\{p_t(i)\}) - \frac{1}{2}\nu^2(\hat{y}_t - a1_t)^2] - \phi_l[\mu \hat{y}_t - \frac{1}{2}\mu^2(\hat{y}_t - a2_t)^2] + t.i.p. \tag{97}$$

where $\nu = \frac{l^s}{(1-l^s-m^s)(1-\eta)}$ and $\mu = \frac{m^s}{(1-l^s-m^s)(1-\alpha)}$, $\Theta \equiv \frac{1-\eta}{1-\eta+\eta\epsilon}$ and t.i.p. are terms which are not affected by monetary policy. Using Woodford's (2003) result

$$\sum_{t=0}^{\infty} \beta^t var_i\{p_t(i)\} = \lambda \sum_{t=0}^{\infty} \beta^t \pi_t^2 \tag{98}$$

Finally, we collect all terms on the rhs:

$$U_t - U = (1 - \phi_l(\nu + \mu) - \frac{1}{2}\phi_l(\nu^2 + \mu^2))\hat{y}_t - \frac{1}{2}(1 - \phi_l(\nu^2 + \mu^2))\hat{y}_t^2 - \frac{1}{2}\frac{\nu\phi_l\epsilon}{\Theta\lambda}\pi^2 + t.i.p. \tag{99}$$

Under $\phi_l = 0.65$ to yield roughly 1/2 of available time working in either goods production or banking, similar to Goodfriend and McCallum (2007), $\nu + \mu$ cancels out from the first expression.

The welfare measure is therefore approximated by:

$$\mathcal{W} = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t (U_t - U) = -\frac{1}{2}\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[(1 - \phi_l(\nu^2 + \mu^2))\tilde{y}_t^2 + \frac{\nu\phi_l\epsilon}{\Theta\lambda}\pi_t^2\right] + t.i.p. \tag{100}$$

Restating gives

$$\mathcal{W} = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t (U_t - U) = -\frac{1}{2}\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\varphi\tilde{y}_t^2 + \varpi\pi_t^2\right] + t.i.p., \tag{101}$$

with

$$\varphi = 1 - \phi_l(\nu^2 + \mu^2) \tag{102}$$

and

$$\varpi = \frac{\nu\phi_l\epsilon}{\Theta\lambda}. \tag{103}$$

The welfare function can be expressed in terms of a quadratic loss function

$$\mathfrak{L}_t = \varphi\sigma_{\tilde{y}}^2 + \varpi\sigma_{\pi}^2. \tag{104}$$

D Kalman Filter

We first derive an expression for κ and then solve for its value based on regression results.

D.1 Derivation of the Kalman gain κ

Suppose we have a state, g_t, which evolves according to:[17]

$$g_t = \delta_g g_{t-1} + \eta_t, \tag{105}$$

where η_t is a shock. The state g_t is not necessarily observed. We observe a variable $L_t - L_{t-1}$ which is a linear combination of the state.

$$L_t - L_{t-1} = g_t + \nu_t \tag{106}$$

with ν_t a potential measurement error.

We use a linear filter to forecast the state. In particular, our forecast of the current state is equal to our forecast from the previous period of today's state plus an "updating" part that depends on innovations in the observer equation:

$$\widetilde{\mathbb{E}}_t g_t = \widetilde{\mathbb{E}}_{t-1} g_t + \kappa(L_t - L_{t-1} - \widetilde{\mathbb{E}}_{t-1} g_t) \tag{107}$$

Our objective is to choose κ to minimize the variance of our errors. Let $\epsilon_t = g_t - \widetilde{\mathbb{E}}_t g_t$. We choose κ to minimize the variance of this. The variance of this error can be written:

$$var(g_t - \widetilde{\mathbb{E}}_t g_t) = var(g_t - (\widetilde{\mathbb{E}}_{t-1} g_t + \kappa(L_t - L_{t-1} - \widetilde{\mathbb{E}}_{t-1} g_t)) \tag{108}$$

Now plug in for $L_t - L_{t-1}$

$$var(g_t - \widetilde{\mathbb{E}}_t g_t) = var(g_t - (\widetilde{\mathbb{E}}_{t-1} g_t + \kappa(g_t + \nu_t - \widetilde{\mathbb{E}}_{t-1} g_t)) \tag{109}$$

Simplify,

$$var(g_t - \widetilde{\mathbb{E}}_t g_t) = var(g_t - \widetilde{\mathbb{E}}_{t-1} g_t - \kappa(g_t - \widetilde{\mathbb{E}}_{t-1} g_t) - \kappa\nu_t) \tag{110}$$

$$var(g_t - \widetilde{\mathbb{E}}_t g_t) = var((1-\kappa)(g_t - \widetilde{\mathbb{E}}_{t-1} g_t) - \kappa\nu_t) \tag{111}$$

Now, because ν_t is independent, we can move the variance operator through

$$var(g_t - \widetilde{\mathbb{E}}_t g_t) = (1-\kappa)^2 var(g_t - \widetilde{\mathbb{E}}_{t-1} g_t) + \kappa^2 var(\nu_t) \tag{112}$$

To minimize, we want to take the derivative with respect to κ and set it equal to zero. We rewrite $var(g_t - \widetilde{\mathbb{E}}_t g_t)$ as $P_{t|t}$ and $var(g_t - \widetilde{\mathbb{E}}_{t-1} g_t)$ as $P_{t|t-1}$ and $var(\kappa\nu_t)$ as σ_ν^2 such that

$$P_{t|t} = (1-\kappa)^2(P_{t|t-1}) + \kappa^2\sigma_\nu^2 \tag{113}$$

[17] see Sims (2010) https://www3.nd.edu/∼esims1/kalman_filter.pdf

To minimize, we take the derivative with respect to κ and set it equal to zero.

$$0 = -2(1-\kappa)(P_{t|t-1}) + 2\kappa\sigma_\nu^2 \tag{114}$$

which then yields

$$\kappa = \frac{P_{t|t-1}}{P_{t|t-1} + \sigma_\nu^2} \tag{115}$$

We need to come up with a measure of $P_{t|t-1}$ to make this operational. The definition of $P_{t|t-1}$ is:

$$P_{t|t-1} = var(g_t - \widetilde{\mathbb{E}}_{t-1}g_t) \tag{116}$$

In words, this is the ex-ante variance of the forecasts, whereas $P_{t|t}$ is the ex-post variance. Our forecast of the state in period t conditional on information available in $t-1$ will simply result from plugging in our forecast of the $t-1$ state into the equation governing the evolution of the actual state:

$$\widetilde{\mathbb{E}}_{t-1}g_t = \delta_g \widetilde{\mathbb{E}}_{t-1}g_{t-1} \tag{117}$$

This is the forecast because the shock term is zero in expectation. We know that the actual state obeys:

$$g_t = \delta_g g_{t-1} + \eta_t \tag{118}$$

Plug this in:

$$P_{t|t-1} = var(\delta_g g_{t-1} + \eta_t - \delta_g \widetilde{\mathbb{E}}_{t-1}g_{t-1}) \tag{119}$$

Using the independence of η_t, we can move the expectations operator through to get:

$$P_{t|t-1} = \delta_g^2 var(g_{t-1} - \widetilde{\mathbb{E}}_{t-1}g_{t-1}) + var(\eta_t) \tag{120}$$

But $var(g_{t-1} - \widetilde{\mathbb{E}}_{t-1}g_{t-1}) = P_{t-1|t-1}$. Because this is unconditional variance (in the sense of not conditioning on particular realization), we have $P_{t-1|t-1} = P_{t|t}$. We have from above:

$$P_{t|t} = (1-\kappa)^2(P_{t|t-1}) + \kappa^2\sigma_\nu^2 \tag{121}$$

Plug this in:

$$P_{t|t-1} = \delta_g^2 \left(1-\kappa\right)^2 P_{t|t-1} + \delta_g^2\kappa^2\sigma_\nu^2 + var(\eta_t) \tag{122}$$

$$P_{t|t-1} = \delta_g^2(1-\kappa)^2(P_{t|t-1}) + \delta_g^2\kappa^2\sigma_\nu^2 + \sigma_\eta^2 \tag{123}$$

Because of κ is a function of $P_{t|t-1}$, this is one expression in one unknown, and can in principle be solved for $P_{t|t-1}$. We can in fact simplify this further:

$$P_{t|t-1} = \delta_g^2(P_{t|t-1} - 2\kappa P_{t|t-1} + \kappa^2(P_{t|t-1} + \sigma_\nu^2)) + \sigma_\eta^2 \tag{124}$$

Now plug in the derived value for κ in one point for now (something is going to drop out):

$$P_{t|t-1} = \delta_g^2(P_{t|t-1} - 2\kappa P_{t|t-1} + \kappa^2(P_{t|t-1} + \sigma_\nu^2)) + \sigma_\eta^2 \tag{125}$$

Remember

$$\kappa = \frac{P_{t|t-1}}{P_{t|t-1} + \sigma_\nu^2} \tag{126}$$

$$\tag{127}$$

Now we are left with

$$P_{t|t-1} = \delta_g^2(P_{t|t-1} - 2\kappa P_{t|t-1} + \kappa P_{t|t-1}) + \sigma_\eta^2 \tag{128}$$

Now additional terms cancel out

$$P_{t|t-1} = \delta_g^2(P_{t|t-1} - \kappa P_{t|t-1}) + \sigma_\eta^2 \tag{129}$$

or

$$P_{t|t-1} = \delta_g^2((1-\kappa)P_{t|t-1}) + \sigma_\eta^2 \tag{130}$$

Now plug in for the derived expression of κ again

$$P_{t|t-1} = \delta_g^2((1 - \frac{P_{t|t-1}}{P_{t|t-1} + \sigma_\nu^2})P_{t|t-1}) + \sigma_\eta^2 \tag{131}$$

So

$$P_{t|t-1}(1 - \delta_g^2(1-\kappa)) = \sigma_\eta^2 \tag{132}$$

$$P_{t|t-1}\left(1 - \delta_g^2\left(1 - \frac{P_{t|t-1}}{P_{t|t-1} + \sigma_\nu^2}\right)\right) = \sigma_\eta^2 \tag{133}$$

$$P_{t|t-1}\left(1 - \delta_g^2\frac{\sigma_\nu^2}{P_{t|t-1} + \sigma_\nu^2}\right) = \sigma_\eta^2 \tag{134}$$

This is now one expression in one unknown. This has a cubic solution. To make things operational instead we guess and verify that

$$P_{t|t-1} = \sigma_\eta^2 \tag{135}$$

This gives us

$$\sigma_\eta^2 \left(1 - \delta_g^2 \frac{\sigma_\nu^2}{\sigma_\eta^2 + \sigma_\nu^2}\right) = \sigma_\eta^2 \tag{136}$$

which is

$$1 - \delta_g^2 \frac{\sigma_\nu^2}{\sigma_\eta^2 + \sigma_\nu^2} = 1 \tag{137}$$

which is approximately true for a low value for δ_g and a low signal-to-noise ratio .

Finally, we arrive at an approximate value for κ.

$$\kappa \simeq \frac{\sigma_\eta^2}{\sigma_\eta^2 + \sigma_\nu^2} \tag{138}$$

D.2 Estimation of the Kalman gain κ

We estimate the equations (105) and (106). By incorporating constants we disentangle the cyclical fluctuation from long-run patterns.

Table 6: Dependent Variable: Change in GDP and Loans

Variable	$dlog(GDP_t)$	**Variable**	$dlog(loans_t)$
$dlog(GDP_{t-1})$	0.496658	$dlog(GDP_t)$	0.502729
	(0.051254)		(0.075688)
$cons$	0.007786	$cons$	0.012553
	(0.000969)		(0.001430)
N	289	N	290
R^2	0.247	R^2	0.133
S.E. of regression	0.009467	S.E. of regression	0.013987

The Kalman gain takes the form

$$\kappa \simeq \frac{\sigma_\eta^2}{\sigma_\eta^2 + \sigma_\nu^2}$$

Resulting in

$$\kappa \simeq 0.358582415 \tag{139}$$

We verify the approximate solution with a numerical solution for κ from the estimation results of Table 6. Specifically, we replace the values for δ_g=0.496658, $\sigma_v^2 = (0.013987)^2 = 0.000196$ and $\sigma_\eta^2 = (0.009467)^2 = 0.000089$ in equation (133):

$$P_{t|t-1}\left(1-(0.496658)^2\left(\frac{(0.000196)}{P_{t|t-1}+(0.000196)}\right)\right)=(0.000089)$$

Finally, by replacing the solution for $P_{t|t-1}$=$1.059\,7\times10^{-4}$ in equation (115):

$$\kappa=\frac{P_{t|t-1}}{P_{t|t-1}+\sigma_v^2}$$

$$\kappa=\frac{1.059\,7\times10^{-4}}{1.059\,7\times10^{-4}+(0.000196)}$$

We obtain a value for κ=0.35093.

E Data Documentation

- We draw on the Bureau of Economic Analysis, that provides: The Gross Domestic Product (SAAR, Bil.$) are extracted from Haver.
- The Loans & Leases in Bank Credit: All Commercial Banks (SA, Bil.$) is provided by the Federal Reserve Board.