

Population Dynamics and Optimal Family Policies

Anson Zhou
University of Wisconsin-Madison
anson.zhou@wisc.edu



Introduction

Since 1980s, an increasing number of countries have started adopting formal family policies to address demographic pressures stemming from low fertility. Aggregate fertility is also crucial for long-run economic growth (Jones 2020).

Family policies include, but are not limited to, baby bonus, universal childcare, and child tax credits. These expenditures are quite sizable. In 2015, expenditures on family policies amount to 3% of GDP on average among OECD countries.

A number of empirical papers have studied the effectiveness of financial incentives in encouraging fertility (c.f. Stone 2020). Yet several important **policy design questions** remains unexplored:

- 1 What are the impacts of family policies beyond current generation?
- 2 With multiple policy instruments to raise fertility, which one(s) should we use?
- 3 In a study of population dynamics, Chu and Koo (1990) argues against policies that raise fertility among the low-income population as it hurts the distribution of human capital. Is this a complete and sensible policy recommendation?

In this paper, we provide answers to these questions using a tractable and micro-founded model.

Simple Model

We first discuss the **planner's problem** to highlight normative principles in the optimal distribution of fertility. Three key ingredients for optimal policy:

Intergenerational transmission Fertility allocation changes the human capital distribution in the long-run due to intergenerational linkages

Costs of childbearing Fertility allocation changes aggregate output and consumption as it distributes the costs of raising children to agents with different market productivity

Distributive justice of fertility Fertility allocation changes utility of households. Assumptions on how social welfare function evaluates this change needs to be specified

Besides fertility profile, the planner's problem also offers interesting insights into education policies:

- Education expenditures make people better workers. It can also make them better parents
- Education policies could be used to complement or compensate changes in intergenerational transmission forces induced by family policies

Quantitative Model

The decentralized economy extends the model in de La Croix and Doepke (2004) with family policies. It is an **overlapping-generations model** with **quality-quantity tradeoff in fertility choice** and **Ramsey taxation** by government

- Households are heterogeneous in human capital h . They choose consumption (c), fertility (n), leisure (l), and child investments (e) to solve:

$$u(h) = \max_{c,n,l,e} \log(c) + \nu \log(n \cdot \mathbb{E}h') + \zeta \log(l) - \mathcal{C}(N)$$

$$\text{subject to } c + \frac{ne}{\text{total investments}} = \frac{y}{\text{labor income}} - \frac{\mathcal{T}(y,n)}{\text{net taxes}}$$

$$y = wh(1 - \frac{n^p \chi}{\text{child costs}} - l) + \frac{\mathcal{F}(h,n)}{\text{family benefits}}$$

where $\mathcal{C}(N)$ denotes congestion externalities. Child human capital h' production function:

$$h' = \underbrace{\zeta}_{\text{scalar}} \cdot \underbrace{\epsilon}_{\text{i.i.d. shock}} \cdot \underbrace{h^\theta}_{\text{IGE}} (\underbrace{E}_{\text{public edu.}} + \underbrace{e}_{\text{parent inv.}})^\gamma$$

where the intergenerational transmission h^θ captures genes, interaction within family, and also progressivity in the education system

- Representative firm takes labor as the sole input:

$$Y = \frac{\exp(\mathcal{A}(N))}{\text{production externalities}} \cdot \frac{H}{\text{aggregate labor supply}}$$

- Government balanced budget with revenues from progressive taxes $\mathcal{T}(y,n)$ depending on gross income and # of children. Expenditures include:
 - (1) Family benefits consisting of:

$$\mathcal{F}(h,n) = \underbrace{\alpha_1 \cdot n}_{\text{baby bonus}} + \underbrace{\alpha_2 \cdot h \cdot n^p}_{\text{universal childcare}}$$

- (2) Public education expenditure per child E
 - (3) Other exogenous expenditures X per capita
- Endogenous equilibrium human capital distribution $F_{\mathcal{P}}(h)$** depends on household choices and government policies. We prove its existence and uniqueness in the steady-state

- The Ramsey problem of the government maximizes **steady-state social welfare of those who are actually born**:

$$\text{SWF}_{\mathcal{P}} = \underline{u} + \left[\int (u(h) - \underline{u})^{\frac{\psi-1}{\psi}} dF_{\mathcal{P}}(h) \right]^{\frac{\psi}{\psi-1}}$$

where $1/\psi$ governs inequality-aversion

- A key difficulty lies in the uncertain magnitude of population externalities $\mathcal{A}(N)$ and $\mathcal{C}(N)$. We make further progress by **decomposing the Ramsey problem into two parts**:

(1) Maximize $\text{SWF}_{\mathcal{P}}$ subject to an additional constraint on aggregate fertility with given \bar{N} :

$$\int n^*(h) dF_{\mathcal{P}}(h) = \bar{N}$$

assuming population externalities \mathcal{C}, \mathcal{A} are zero

(2) Trace out **Reproduction Possibility Frontier (RPF)** by varying "fertility target" \bar{N}

Calibration and Counterfactuals

The model is calibrated to match the tax system in the U.S. and moments including intergenerational mobility, fertility-income profile, and childraising costs. Total fertility rate (TFR) is 1.92 children per women. Three policy counterfactuals are enacted as validation exercises. In each counterfactual, the government spend 0.13% of GDP on family or education policies. This amount translates to a \$5,000 direct cash payment to newborns, a 1% reduction in fixed costs of children, or a 4% increase in public education expenditures. Results in Table 1 suggests:

- The model generates untargeted fertility elasticities that are **quantitatively consistent with empirical estimates** using past policies
- Baby bonus is more cost-effective in the short-run, but its long-run costs in terms of output per capita is larger than the short-run due to deteriorations in human capital distribution
- Each "naive" policy tool has its strengths and weaknesses. Maximizing social welfare requires using family policies (affecting fertility) and education policies (affecting output) jointly

	short run - initial steady-state			long run - new steady-state		
	baby bonus	childcare	education	baby bonus	childcare	education
aggregate fertility	+0.04	+0.025	unchanged	+0.04	+0.025	-0.001
output per capita	-0.49%	-0.47%	unchanged	-0.72%	-0.49%	+0.5%

Table 1: Fertility and Output Responses to Family and Education Policies

Normative Analysis

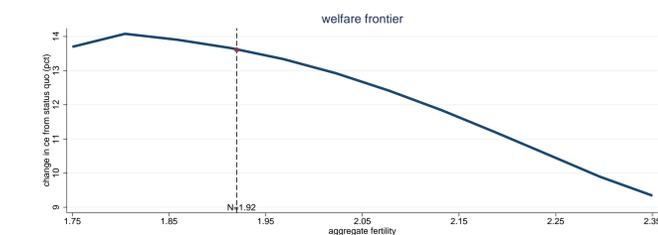
We solve for the optimal policy with:

- $\bar{N} = 2.1$ - replacement fertility in steady-state
- $\psi = 0.1$ - welfare principle close to maxmin
- $\alpha_1, \alpha_2 \geq 0$ - no explicit taxes on childbearing
- Majority support for policy reform

The solution to **optimal policy** highlights:

- 1 Subsidized childcare - reduce child costs by 6%
- 2 Expanding public education by 15.4%
- 3 Ex ante consumption equivalence increase by 2.2% despite output per capita reduces by 2%
- 4 Fertility increase is similar across income levels
- 5 Transition is mostly completed in two generations

Note that baby bonus, the more cost-effective family policy in the counterfactual analysis is left unused. This is because when both education and family policies are allowed to change, optimal policy chooses subsidized childcare as it has more synergy with public education due to complementarities in the human capital production function. Baby bonus is used only when intergenerational transmission channel is weaker, e.g. when education system is more progressive. Lastly, we vary \bar{N} to explore whether replacement level fertility is desirable in the first place and trace out the reproduction possibility frontier, i.e. what is the highest social welfare obtainable under each \bar{N} .



The figure indicates that under given assumptions, RPF has a local maximum at $\bar{N} \approx 1.8$. Further research on measuring population externalities are needed to determine "optimal aggregate fertility".