Putting the Price in Asset Pricing

Thummim Cho and Christopher Polk*

London School of Economics

First draft: November 2019

This version: December 2020

*Cho: Department of Finance, London School of Economics, London WC2A 2AE, UK. Email: t.cho@lse.ac.uk. Phone: +44 (0)20 7107 5017. Polk: Department of Finance, London School of Economics, London WC2A 2AE, UK and CEPR. Email: c.polk@lse.ac.uk. Phone: +44 (0)20 7849 4917. We thank Tony Berrada, John Campbell, Georgy Chabakauri, John Cochrane, Jack Favilukis, Ben Hebert, Christian Julliard, Leonid Kogan, Lukas Kremens, Dmitry Livdan, Dong Lou, Hanno Lustig, Ian Martin, Stefan Nagel, Cameron Peng, Lukas Schmid, Ran Shi, Andrei Shleifer, Argyris Tsiaras, Dimitri Vayanos, Laura Veldkamp, Tuomo Vuolteenaho, Paul Woolley, and seminar participants at the HEC-McGill Winter Finance Workshop, Red Rock Finance Conference, UBC Winter Finance Conference, Junior Finance Workshop on Valuation and Capital Allocation, IMF, London School of Economics, Stockholm Business School, Swiss Finance Institute - Lugano, and Yale University for useful comments and discussions. Robert Rogers and Amirabas Salarkia provided superb research assistance.

Putting the Price in Asset Pricing

Abstract

We propose a novel way to study asset prices based on price distortions rather than abnormal returns. We derive the correct identity linking current mispricing to subsequent returns, generating a price-level analogue to the fundamental asset pricing equation, $E[MR^e] = 0$, used to study returns. Our empirical test reveals that the CAPM describes the cross-section of prices better than it describes expected short-horizon returns. Despite the improvement, significant mispricing remains. An interaction of book-to-market and quality provides a parsimonious model of CAPM mispricing that both long-term buy-and-hold investors and researchers disciplining models from the price perspective should prioritize.

Keywords: price level, long-horizon returns, mispricing metric, stochastic discount factor, CAPM

JEL classification: G12, G14, G32

We propose a novel way to study asset prices. Our focus is on distortions in *prices*, taking a sharp break from the traditional emphasis on measuring *expected return* distortions, i.e. "abnormal returns."

While abnormal return is certainly an important barometer of capital market efficiency (Fama (1970)), price distortions likely have more important real economic consequences as they may drive the financing and investment decisions of firms.[1] Furthermore, price distortions matter more than short-horizon abnormal returns for investors who commit their capital over a long investment horizon, such as firm managers, policy makers, and other long-term buy-and-hold investors (Cohen, Polk, and Vuolteenaho (2009)).[2]

Price and expected return are synonymous in a model with constant expected returns. In that case, $P_t = \sum_{j=1}^{\infty} E_t\left[D_{t+j}\right]/(1+R)^j$ with D denoting cash flows (dividends), so a distortion in expected one-period return (R) is sufficient to infer a distortion in price (P).[3] However, expected returns are almost certainly time-varying (Campbell and Shiller (1988), Fama and French (1989), Cochrane (1992), Cochrane (2008), and van Binsbergen, Brandt, and Koijen (2012)). Therefore, ex-ante price distortions should be a function of the entire term structure of subsequent abnormal returns, not just the one-period abnormal return.

But is there an exact analytical way to aggregate subsequent abnormal returns correctly over the long run to arrive at ex-ante price distortion? Could such a formula be applied to data to reveal important facts about the cross-section of asset prices?

Our paper makes two contributions. First, we show that the answer to the first question is a surprising *yes*: there is a simple analytical expression relating ex-ante price distortion to subsequent abnormal returns. By correctly aggregating subsequent abnormal returns in order to measure ex-ante mispricing, our exact identity contrasts sharply with ad-hoc ways to aggregate abnormal returns over time, such as the cumulative abnormal return (CAR) measure widely used in em-

[1] Papers studying the link between mispricing and firm activity include Stein (1996), Baker and Wurgler (2002), Baker, Stein, and Wurgler (2003), Shleifer and Vishny (2003), Cohen, Polk, and Vuolteenaho (2009), Polk and Sapienza (2009), van Binsbergen and Opp (2019), and Whited and Zhao (forthcoming).

[2] See also Cochrane (2011): "Focusing on expected returns and betas rather than prices and discounted cashflows . . . makes much less sense in a world with time-varying discount rates" (pp.1063–1064).

[3] That is, up to the asset duration that could differ across assets. Shiller (1984) and Summers (1986) argue that even in this case, persistent price distortion may not generate statistically discernible patterns in expected returns.

pirical studies. Second, we show how to apply our identity to evaluate an asset pricing model based on its ability to explain asset price levels in a manner analogous to the popular time-series regression approach for returns (Black, Jensen, and Scholes (1972)). The close analogy between our approach to prices and the existing approach to returns is attractive, as it allows one to study prices using the apparatus already developed for returns.

When presenting our results on prices, we are careful to recognize that the mispricing we detect could signal either a misspecification of our model of risk or the deviation of price from the intrinsic value of cash flows because of limits of arbitrage, as is always the case when interpreting abnormal returns.[4] Our goal is not to resolve the debate on market efficiency—whether a given "return" anomaly represents mispricing or misspecification of risk—but to carry this debate over to the "price" dimension: Are return anomalies also price anomalies or purely transitory phenomena? Which model of risk explains the cross-section of prices and which characteristics help summarize price deviations from levels implied by the model?

The theoretical part of this paper derives our identity and discusses its implications. We show that "delta" (δ), defined as the percentage deviation of price from value, equals the expected sum of subsequent abnormal returns or "alpha" (α), discounted by the cumulative price-adjusted stochastic discount factor (SDF). In particular,

$$\delta_t = (P_t - V_t)/P_t = -\sum_{j=1}^{\infty} E_t\left[M_{t,t+j}\frac{P_{t+j-1}}{P_t}R^e_{t+j}\right] \quad (1)$$

$$=: -\sum_{j=1}^{\infty} E_t\left[M_{t,t+j-1}\frac{P_{t+j-1}}{P_t}\frac{\alpha_{t+j}}{1+R_{f,t+j}}\right], \quad (2)$$

where $V_t = E_t\left[\sum_{j=1}^{\infty} M_{t,t+j}D_{t+j}\right]$ is the asset's intrinsic value defined as the present value of cash flows, $M_{t,t+j}$ is the cumulative candidate SDF associated with the model of risk (e.g., CAPM), D is dividend, P is price, and $R_{f,t+j}$, R^e_{t+j}, and α_{t+j} are the risk-free rate, excess return (above the risk-free rate), and conditional abnormal return from $t+j-1$ to $t+j$, respectively.[5] The assumption of no explosive bubbles is not restrictive, as it allows for most types of price deviation from value,

[4]This issue is, of course, the famous joint hypothesis problem emphasized in Fama (1970).

[5]$M_{t,t+j}$ is the ratio of marginal utilities in periods t and $t+j$ in consumption-based asset pricing models with time-separable utility. The typical parametrization of the capital asset pricing model (CAPM) implies $M_{t,t+j} = \Pi_{s=1}^{j}\left(b_0 - b_1 R^e_{m,t+s}\right)$ where R^e_m is market return in excess of the risk-free interest rate and (b_0, b_1) are parameters.

including permanent mispricing (e.g., $\delta_{t+j} = \delta \neq 0 \forall j$), which our identity can correctly detect.

The identity is intutive. The economic surplus, relative to the SDF M, from a buy-and-hold strategy on an asset is the net present value of all subsequent abnormal returns. However, since abnormal return at time $t+j+1$ is a rate of return, we can express it in terms of monetary value at time $t+j$ by multiplying abnormal return by P_{t+j} and then rolling the product back one period using the risk-free rate: $V_t - P_t = E_t\left[\sum_{j=0}^{\infty} M_{t,t+j} X_{t+j}^{Abnormal}\right]$, where $X_{t+j}^{Abnormal} = P_{t+j}\frac{\alpha_{t+j+1}}{1+R_{f,t+j}}$ is the abnormal payoff at time $t+j$. Finally, one divides both sides by P_t and changes sign to arrive at the expression for δ_t in equation (2).

Equation (1) implies that not just the magnitude, but also the time and state in which abnormal returns occur determine the extent of ex-ante mispricing. Abnormal returns occurring in a more distant future matter less for mispricing, since these abnormal returns are earned on the portion of the initial asset that excludes the market value of dividend payouts up until that point in the future.[6] Furthermore, abnormal returns occurring after states with either high state price M or high cumulative capital gain at time $t+j-1$ matter more at time t, because in such cases, the abnormal return is earned on a time $t+j-1$ price that matters more for P_t. In contrast, by putting equal weights on abnormal returns occurring in all states and periods, ad-hoc measures such as the CAR can lead to substantial estimation errors.

Our identity implies a price-level analogue of the restriction we test for one-period returns, $0 = E\left[M R^e\right]$. Define $\delta\left(J\right)$ as the unconditional mean of the right-hand quantity in equation (1) replaced with a finite sum over J months after portfolio formation:

$$\delta\left(J\right) = -\sum_{j=1}^{J} E\left[M_{t,t+j}\frac{P_{t+j-1}}{P_t}R_{t+j}^e\right] \tag{3}$$

Choosing $J=1$ and $J=\infty$ imply, respectively,

$$\delta\left(1\right) = -E\left[M_{t+1}R_{t+1}^e\right] \tag{4}$$

$$\delta\left(\infty\right) = -\sum_{j=1}^{\infty} E\left[M_{t,t+j}\frac{P_{t+j-1}}{P_t}R_{t+j}^e\right] \tag{5}$$

[6] In the formula, this conclusion follows from $E_t\left[M_{t,t+j}\frac{P_{t+j}}{P_t}\right]$ converging to zero as $j \to \infty$ under the no-explosive-bubble condition.

Restricting $\delta(1) = 0$ in equation (4) requires the expected one-month return to equal the risk premium, and a deviation from this relation is measured by the abnormal return $\alpha = -\delta(1)/E[M_{t+1}]$. Similarly, restricting $\delta(\infty) = 0$ in equation (5) means that price equals intrinsic value, and a deviation from this relation is mispricing $\delta = \delta(\infty) = E[(P_t - V_t)/P_t]$.

We propose estimating mispricing δ in the same way one estimates abnormal return α based on the time-series approach, as it allows the same test and SDF specification to apply to both the study of prices and returns. In the context of equation (4), one estimates alpha with respect to a factor model specifying M using the sample analogue of $E\left[M_{t+1}R^e_{t+1}\right]$, prescribing M to yield zero abnormal returns on the factor portfolios in sample.[7] Similarly, one estimates mispricing δ using the sample analogue of $-\sum_{j=1}^{J} E\left[M_{t,t+j}\frac{P_{t+j-1}}{P_t}R^e_{t+j}\right]$ for some sufficiently large J, prescribing M to yield zero factor portfolio δs in sample.

To reduce statistical issues associated with overlapping observations, following Fama (1998) and Cohen, Polk, and Vuolteenaho (2009), we estimate δ using an equivalent calendar-time expression, $-\sum_{j=1}^{\infty} E\left[M_{t-j,t}\frac{P_{(t-j),t-1}}{P_{(t-j),t-j}}R^e_{(t-j),t}\right]$, where the $(t-j)$ argument in the subscript indicates that the particular excess returns and cumulative capital gains are earned from buying and holding the particular portfolio formed at time $t-j$. This reformulation of the estimation problem removes the issue of overlapping returns that makes the event-time formulation more challenging to estimate. We find that the post-portfolio-formation horizon of $J = 15$ years (180 months) is sufficient to generate an accurate estimate of δ.[8] We report bootstrap standard errors and p-values that account for the estimator's finite sample behavior, time-series and cross-sectional covariances in the data, and uncertainty in estimating the SDF parameters.

The empirical part of the paper studies the cross-sectional variation in stock prices with respect to the CAPM. We use the CAPM as the candidate SDF not because we advocate the CAPM as the right model of risk, but because (i) its simplicity makes it easier to see how asset pricing with price levels differs from the conventional analysis of returns and (ii) it provides a foundation for multifactor refinements of the model based on price-level analysis. Since mispricing δ depends on the ratio of value to price, sorting stocks on the value-to-book ratio—proxied by a composite

[7] Up to the scaling factor $1/E[M_{t+1}] \approx 1$ needed to interpret the quantity as a rate of return.

[8] Appendix D.2 shows that when computing post-formation returns on a portfolio, no rebalancing—or equivalently, rebalancing based on cumulative capital gains—is the correct approach.

metric dubbed quality (Asness, Frazzini, and Pedersen (2019))—and the book-to-market ratio should lead to a powerful test. Our primary analysis therefore studies quality- and book-to-market-sorted portfolios, first in isolation and then in tandem via a double sort.

The CAPM explains the cross-section of prices fairly well despite its poor ability to explain returns. We first show that, consistent with previous findings, post-formation single-month returns on quality- or book-to-market-sorted quintile portfolios are anomalously negatively related to risk measured by CAPM beta. In contrast, CAPM-implied risk does a particularly good job explaining the prices of quality-sorted portfolios, while extreme book-to-market-sorted portfolios have a larger but also statistically insignificant difference in mispricing δ (Figure 2). The results for the book-to-market are consistent with Cohen, Polk, and Vuolteenaho (2009) while the results for quality at first glance seem to run contrary to Asness, Frazzini, and Pedersen (2019)'s claim that stock prices do not fully reflect variation in quality.

Nevertheless, after double sorting stocks on both characteristics to generate 25 value-and-quality-sorted portfolios, we find large variation in mispricing δ along both dimensions with a spread in CAPM mispricing across the high-quality, inexpensive (high book-to-market) portfolio and the low-quality, expensive (low book-to-market) portfolio of 48 percentage points. This difference is not only economically large but also statistically significant with a t-statistic is 2.31. Hence, the double sort generates the sort of variation a buy-and-hold investor should be interested in exploiting and represents a challenge for future asset pricing models that make understanding price-level risk a priority.

Turning to portfolios sorted on eight other characteristics (quality-to-price, size, momentum, profitability, investment, beta, net issuance, and accruals), we document three main findings. First, a composite measure combining quality and price which we dub quality-to-price effectively captures the large spread in mispricing generated by double-sorting on quality and book-to-market portfolios. Second, of the remaining characteristics we study, only net issuance generates statistically significant spread in δ. Finally, in tests examining portfolios double-sorted on quality-to-price and these other characteristics, we find that the quality-to-price describes the cross-section of CAPM mispricing well, as all of the other anomalies, including net issuance, provide no incremental explanatory value.

In summary, our paper is the first to show how to correctly aggregate abnormal returns into ex-ante price distortion. Empirically, we identify the two key stock characteristics, quality and value, that together are associated with the largest price distortions with respect to the CAPM and hence should be characteristics that buy-and-hold investors who are primarily concerned with market risk exposure should be interested in exploiting. Thus, our approach provides a new challenge to asset pricing models that are not just concerned with capturing variation in average returns over the short run but that also prioritize minimizing model-implied price distortions.

The organization of our paper is as follows. Section 1 reviews related literature. Section 2 presents our framework to price assets in terms of price levels. Section 3 presents data, econometrics, and our primary results related to quality and value. Section 4 extends our analysis to other characteristic sorts. Section 5 concludes.

1 Literature review

There is a long history of accumulating realized abnormal returns in order to proxy for price-level deviations from a benchmark model in the corporate finance literature, namely the well-known cumulative abnormal return (CAR) and buy-and-hold abnormal return (BHAR) methodologies. Work in this area includes Barber and Lyon (1997), Kothari and Warner (1997), Fama (1998), Lyon, Barber, and Tsai (1999), Brav (2000), and Bessembinder, Cooper, and Zhang (2018), among others. Though our method also accumulates future abnormal returns, by correctly discounting the stochastic payoffs associated with mispricing, our delta measure has the exact interpretation as the price deviation from the intrinsic value of cash flows, or equivalently, mispricing from the perspective of long-term shareholders.

Researchers have introduced alternative price-level measures of mispricing. Lee, Myers, and Swaminathan (1999), and others infer the intrinsic value of the firm from its fundamentals. In stark contrast, our formula is an identity linking mispricing to abnormal returns. Black (1986) comments that markets are efficient if prices are within factor of two of intrinsic value but provides no way in which to carefully measure such deviations. Nevertheless, his prior provides a useful benchmark for the economic significance of the price-level misspecifications we measure. Bai, Philippon, and Savov (2016) show that price has become more informative about intrinsic value over time based on the ability of a scaled price ratio to explain subsequent earnings.

The closest mispricing metric to ours is the price-level alpha construct of Cohen, Polk, and Vuolteenaho (2009) (CPV). Unlike that paper, our mispricing identity begins with a clear definition of mispricing that is linked to a specification of the SDF, does not require unobservable quantities such as risk exposures and volatility in the absence of mispricing, and does not rely on the Campbell and Shiller (1988) approximation. The first two differences ensure that δ is a correct price-level measure of model misspecification free of biases that arise from the approximation and the assumption about the unobserved second moments, and the last difference increases our statistical power compared to CPV, as we discuss in more detail in the appendix.

Our exact framework can be used to revisit prior observations about market efficiency. Fama (1970) defines a market as efficient if "prices 'fully reflect' all available information" but goes on to test the efficient market hypothesis using returns, finding that the market is semi-strong form efficient. Shiller (1984) writes, "because real returns are nearly unforecastable, the real price of stocks is close to the intrinsic value . . . is one of the most remarkable errors in the history of economic thought" (pp. 458–459). Summers (1986) provides a numerical example that illustrates this argument, and Campbell (2017) shows how an expected return that follows a persistent AR(1) process may have low volatility but a large effect on the log dividend-price ratio.

Our identity provides a more sophisticated framework in which to understand Shiller's point. It shows that mispricing can be large even if each post-formation alpha is small, as long as alphas are persistent. Furthermore, mispricing can be large even if alphas are small on average if alphas tend to comove strongly with ϕ. The latter channel has been overlooked in the literature but can be quantitatively important: Cho (2020) provides empirical evidence that as arbitrageurs trade away the alphas of equity anomalies such as value and momentum, they can end up exposing these anomalies to the systematic risks they face as arbitrageurs. That is, in the presence of arbitrage with limited capital, initial mispricing in terms of alpha can evolve to become a risk premium associated with an endogenous beta.

Several recent papers tackle different but related topics in the growing literature on asset price distortions and long-term portfolio choice. van Binsbergen and Opp (2019) study a quantitative model of a production economy in which the cost of equity faced by firms may be distorted due to the mispricing and analyze the resulting implications for the real economy. By specifying the exact production technology and frictions in the economy, van Binsbergen and Opp (2019) are able to characterize the impact of expected return distortions on output and perform counterfactuals. In contrast, the novelty of our approach is that it allows the ex-ante price distortion to be estimated without having to specify the process that generates dividends. Hence, our approach is better suited for understanding which firms are mispriced with respect to a candidate SDF as well as providing a specification test of a particular SDF based on price distortions.[9] Of course, the limitation of our approach relative to van Binsbergen and Opp (2019) is that while we provide an easy way to estimate asset price distortions, we cannot draw implications for more direct measures of allocative efficiency such as output.

Chernov, Lochstoer, and Lundeby (2018) propose a new asset-pricing test requiring the linear SDF specification of a factor model to explain both one-period and multi-period factor returns, documenting that popular linear factor models fail to price returns on their own factors accumulated over long horizons. For example, they show that the average four-year gross returns on the market, profitability, and investment factors have annualized misspecification errors that are roughly 7% in absolute magnitude relative to the prediction of the four-year SDF implied by the Fama-French (2015) five-factor model. Our goal of estimating asset price deviation from intrinsic value by correctly aggregating abnormal returns on a buy-and-hold strategy is distinct from their goal of generating high-power asset pricing tests based on long-horizon restrictions on managed portfolios. Importantly, the mispricing identity we obtain for a candidate SDF by iterating the law of motion for mispricing δ forward cannot be obtained in their analysis, which derives a long-horizon condition under the restriction that the short-horizon condition $E\left[MR^e\right]=0$ holds every period following portfolio formation.[10]

Cochrane (2014) shows how mean-variance characterizations can be applied to the stream of long-run payoffs or return opportunities even in a dynamic framework. His analysis shows that optimal dividend payoffs follow a relatively simple analytical form for a mean-variance optimizer

[9] Appendix D.1 provides further details on how our identity and methodology relates to the approach taken by van Binsbergen and Opp (2019).

[10] In a related work, Favero, Melone, and Tamoni (2020) use the assumption that the prices of assets and factors in a model should be cointegrated to generate an equilibrium correction term that forecasts asset returns. Both Keloharju, Linnainmaa, and Nyberg (2019) and Baba Yara, Boons, and Tamoni (2020) study the extent to which the permanent and temporary components of characteristics differentially describe the cross-section of long-horizon average returns.

that uses simple discounting to weight future utilities, even though the dynamic portfolio strategy that supports these payoffs may be complex. Our δ measure describes the present value of marginal utility gains from adding a particular stream of payoffs and therefore could be useful in describing the optimal portfolio choice for long-term investors.

2 A New Identity: Asset Pricing at Long Horizons

This section defines mispricing as a percentage deviation of price from the intrinsic value of cash flows implied by an asset pricing model: $\delta_t = (P_t - V_t)/P_t$ where $V_t = E_t\left[\sum_{j=1}^{\infty} M_{t,t+j} D_{t+j}\right]$. It then presents this paper's core theoretical result that, under mild assumptions, δ_t equals the sum of subsequent excess returns, discounted by the price-weighted cumulative SDF (Lemma 1):

$$\delta_t = -\sum_{j=1}^{\infty} E_t\left[M_{t,t+j}\frac{P_{t+j-1}}{P_t}R^e_{t+j}\right]. \tag{6}$$

The unconditional statement of the identity, $\delta = -\sum_{j=1}^{\infty} E\left[M_{t-j,t}\frac{P_{t-1}}{P_{t-j}}R^e_t\right]$, is a natural price-level counterpart to the conventional asset pricing equation for abnormal one-period return, $\alpha = E\left[\frac{M_t}{E[M_t]}R^e_t\right]$, and motivates estimating unconditional mispricing δ using the sample analogue

$$\hat{\delta} = -T^{-1}\sum_{t=1}^{T}\sum_{j=1}^{J} M_{t-j,t}\frac{P_{t-1}}{P_{t-j}}R^e_t$$

for $J = 180$ months (15 years) in subsequent sections. Doing so under the restriction that M prices the factor portfolios perfectly in sample makes this estimation approach analogous to the time-series regression for returns in which the factor portfolios are assumed to have zero abnormal returns in sample.

2.1 The environment

Consider an asset with dividends (or coupons for bonds) $\left\{D_{t+j}\right\}_{j=1}^{\infty}$. Our goal is to relate the mispricing (pricing error) of the asset at time t to its subsequent returns. We measure mispricing with respect to the (candidate) stochastic discount factor (SDF) $\left\{M_{t+j}\right\}_{j=1}^{\infty}$, where we use $M_{t,t+j} = \Pi_{s=1}^{j} M_{t+s-1,t+s}$ to denote the cumulative SDF. The SDF we analyze could either be a candidate

SDF to be compared to the one implied by market prices in the sense of Hansen and Jagannathan (1991, 1997), or it could be the SDF of a particular investor in a market in which the the law of one price fails for some assets due to frictions (e.g., Garleanu and Pedersen (2011) and Geanakoplos and Zame (2014)). Note that the derivation below does not rely on M being the true SDF.

2.2 Intrinsic value and mispricing

The intrinsic value, V_t, of the asset is simply the present value of all future dividends:

$$V_t = \sum_{j=1}^{\infty} E_t \left[M_{t,t+j} D_{t+j} \right]. \tag{7}$$

We define mispricing, δ_t, as the deviation of price from value as a percentage of the current price:

$$\delta_t = \frac{P_t - V_t}{P_t}. \tag{8}$$

Hence, $\delta_t > 0$ if the asset is overpriced, and $\delta_t < 0$ if it is underpriced. δ_t can range from $-\infty$ (if $V_t > 0$ and $P_t = 0$) to 1 (if $V_t = 0$ and $P_t > 0$), the opposite of the range for abnormal returns, $[-1, \infty)$.

2.3 Assumptions

To link δ_t to subsequent returns, we make two relatively mild assumptions. The first is the existence of a risk-free asset that satisfies the fundamental asset pricing equation.

Assumption 1. *There is a benchmark asset or a trading strategy such that its time $t+j$ return, denoted $R_{b,t+j}$, satisfies the fundamental asset pricing equation with respect to the candidate SDF M at time $t+j-1$:*

$$E_{t+j-1} \left[M_{t+j} \left(1 + R_{b,t+j} \right) \right] = 1. \tag{9}$$

A natural choice for the benchmark asset b is the risk-free asset proxied by the one-month Treasury bill rate, which the conventional return-based asset pricing literature uses to compute excess returns. The second assumption is a weak form of a no-bubble condition.

Assumption 2. *The present value of the deviation of price and value at the limit $j \to \infty$ is zero:*

$$\lim_{j\to\infty} E_t\left[M_{t,t+j}\left(P_{t+j}-V_{t+j}\right)\right]=0.$$

This assumption is weaker than having two separate no-bubble conditions on price and value, $\lim_{j\to\infty} E_t\left[M_{t,t+j}P_{t+j}\right]=0$ and $\lim_{j\to\infty} E_t\left[M_{t,t+j}V_{t+j}\right]=0$, which imply Assumption (2). The assumption is not particularly restrictive, as it allows for most types of price deviation from value, including permanent mispricing (e.g., $\delta_{t+j}=\delta\neq 0\,\forall j$), which our identity can correctly detect.

2.4 The law of motion for mispricing

Under our definitions and Assumption (1), mispricing follows a simple law of motion. Equation (7) and the law of iterated expectations implies the fundamental asset pricing equation holds for value:

$$1=E_t\left[M_{t+1}\frac{V_{t+1}+D_{t+1}}{V_t}\right]. \tag{10}$$

Next, use equation (8) to substitute the empirically unobserved quantities V_t and V_{t+1} with $V_t=(1-\delta_t)P_t$ and $V_{t+1}=(1-\delta_{t+1})P_{t+1}$ to obtain,

$$\delta_t=1-E_t\left[M_{t+1}\left(1+R_{t+1}\right)\right]+E_t\left[M_{t+1}\frac{P_{t+1}}{P_t}\delta_{t+1}\right]. \tag{11}$$

Finally, following Assumption (1), use $1=E_t\left[M_{t+1}\left(1+R_{b,t+1}\right)\right]$ to express mispricing δ_t at time t in terms of excess return $R^e_{t+1}=R_{t+1}-R_{b,t+1}$ and mispricing δ_{t+1} at time $t+1$:

$$\delta_t=-E_t\left[M_{t+1}R^e_{t+1}\right]+E_t\left[M_{t+1}\frac{P_{t+1}}{P_t}\delta_{t+1}\right]. \tag{12}$$

The law of motion in equation (12) is intuitive. Since $E_t\left[M_{t+1}R^e_{t+1}\right]$ is the conditional abnormal return at time $t+1$ adjusted for the gross risk-free rate ($E_t\left[M_{t+1}R^e_{t+1}\right]=\left(1+R_{f,t+1}\right)^{-1}\alpha_{t+1}$, where α_{t+1} is the abnormal return conditional on time-t information), equation (12) says that underpricing (overpricing) at time t is either "paid out" as a positive (negative) abnormal return or contributes to the remaining mispricing at time $t+1$. The discount factor on δ_{t+1} is the SDF times the capital gain, which is intuitive given that δ_{t+1} is normalized by P_{t+1}. Hence, δ_{t+1} matters more at time t if it arises in a state in which P_{t+1} is high (hence the capital gain term) or has a higher present value (hence the SDF term).

2.5 Relating mispricing to subsequent returns

Iterating the law of motion for mispricing (equation (12)) forward and using Assumption (2) to set $\lim_{j\to\infty} E_t\left[M_{t,t+j}\frac{P_{t+j}}{P_t}\delta_{t+j}\right] = 0$ expresses mispricing as a discounted sum of future excess returns.

Lemma 1. ***(Ex-ante mispricing and subsequent excess returns).*** *Let $V_t = \sum_{j=1}^{\infty} E_t\left[M_{t,t+j}D_{t+j}\right]$ be the intrinsic value of the asset defined as the present value of cash flows with respect to a candidate cumulative SDF $\left\{M_{t,t+j}\right\}$. Under Assumptions (1) and (2), ex-ante mispricing $\delta_t = (P_t - V_t)/P_t$ is the negative of the sum of expected subsequent excess returns discounted by the price-weighted SDF:*

$$\delta_t \equiv \frac{P_t - V_t}{P_t} = -\sum_{j=1}^{\infty} E_t\left[M_{t,t+j}\frac{P_{t+j-1}}{P_t}R^e_{t+j}\right], \tag{13}$$

where P_{t+j-1}/P_t and R^e_{t+j} are the cumulative capital gains and excess returns on the buy-and-hold strategy on the asset j periods following the period in which mispricing is calculated, t.

Note that this formula holds regardless of whether or not M is the correct SDF. For intuition, take a time $t+j-1$ conditional expectation within the expectation and use $E_{t+j-1}\left[M_{t+j}R^e_{t+j}\right] = \left(1+R_{f,t+j}\right)^{-1}\alpha_{t+j}$ to write the identity in terms of abnormal returns.

Corollary 1. ***(Ex-ante mispricing and subsequent abnormal returns).*** *Ex-ante mispricing δ_t can be expressed using subsequent conditional abnormal returns:*

$$\delta_t = -\sum_{j=1}^{\infty} E_t\left[\phi_{t,t+j-1}\frac{\alpha_{t+j}}{1+R_{f,t+j}}\right], \quad \phi_{t,t+j} \equiv M_{t,t+j}\frac{P_{t+j}}{P_t}, \tag{14}$$

where α_{t+1} is the time $t+1$ risk-adjusted abnormal return investor expects to earn conditional on time-t information and $\phi_{t,t+j}$ is a discount factor on abnormal returns.

Several observations about mispricing measured using our novel price-level metric δ follow from equation (14). First, a simple formula relates ex-ante mispricing δ to abnormal returns α.

Remark 1. *Mispricing δ is a stochastically discounted sum of subsequent abnormal returns.*

The asset pricing literature recognizes that some aggregation of abnormal returns over a long horizon can proxy for price-level distortions and has used long-run return measures such as CAR or BHAR as that proxy in numerous applications. Our identity in equation (14) shows that though there is indeed a simple, analytical formula relating initial ex-ante mispricing to subsequent abnormal returns, the formula is clearly distinct from those existing long-run return measures. To the best of our knowledge, we are the first to supply the correct formula measuring price-level distortion.

What follows immediately from the summation formula in equation (14) is that the persistence of abnormal returns, not just the magnitude, matters for mispricing.

Remark 2. *Holding all else fixed, mispricing δ is larger if subsequent abnormal returns are persistent.*

Others have pointed out that the persistence of abnormal returns should matter for price-level distortion (e.g., Cohen, Polk, and Vuolteenaho (2009), Cochrane (2011), van Binsbergen and Opp (2019)), and we confirm this point in our exact relation between mispricing and abnormal returns.

The last set of remarks highlights the importance of the stochastic discounting of abnormal returns. If δ_t is finite, the discount factor on the risk-free-rate adjusted abnormal return must fall over time. More formally, since the single-period component of the discount factor on abnormal returns is $M_{t+j}\frac{P_{t+j}}{P_{t+j-1}}$ and the fundamental asset pricing equation requires $E_t\left[M_{t+j}\left(\frac{P_{t+j}}{P_{t+j-1}}+\frac{D_{t+j}}{P_{t+j-1}}\right)\right]=1$ in the absence of abnormal returns, $M_{t+j}\frac{P_{t+j}}{P_{t+j-1}}$ in general must have an expected value less than 1 once the firm starts paying out dividends. This fact implies that the horizon at which abnormal returns are earned affects the magnitude of the initial mispricing.

Remark 3. *Abnormal returns occurring sooner are in general associated with larger mispricing δ.*

Intuitively, as is the case with any present value formula, the net present value of a buy-and-hold strategy on the asset as a fraction of the initial price—which is what δ represents—depends less on abnormal returns earned far into the future. However, this simple logic is missing in long-run return measures that are widely-used; both CAR and BHAR do not distinguish abnormal returns earned in the near future from those earned far into the future.

The presence of the SDF M in equation (14) implies that the state in which the abnormal return is earned matters.

Remark 4. *Abnormal returns occurring in more valuable states are associated with larger mispricing δ.*

This point is perhaps the most novel insight of our identity, as implies that not just the "expectation," but also the "covariance" matters for how we accumulate subsequent αs into the initial δ. For example, if the market factor is a priced factor, abnormal returns earned following a market crash imply a large deviation of intrinsic value from price, since most asset-pricing models intuitively view dividends or capital gains earned in such a state as more valuable. Thus, this point implies that being able to predict an asset's abnormal returns using past returns on a risk factor would have important implications when quantifying ex-ante mispricing.

Finally, capital gain also matters for the covariance component of mispricing.

Remark 5. *Abnormal returns occurring after relatively large capital gains are associated with larger mispricing δ.*

As a consequence, one cannot simply discount future abnormal returns with the cumulative SDF alone but instead must use a price-weighted cumulative SDF. Capital gain enters into the formula since the abnormal return at time $t+j$ is earned on the $t+j-1$ price. Hence, the abnormal return matters more for mispricing today if it expected to be earned on a high future price. Practically speaking, this component of ϕ means that abnormal returns linked to long-run reversal (De Bondt and Thaler (1985)) count less towards the initial mispricing. This simple intuition has been largely overlooked in the literature's search for a link between abnormal returns and mispricing.[11]

To summarize, our identity in equation (14) highlights that abnormal returns occurring sooner, in more valuable states, or after relatively large capital gains are associated with larger price-level deviations. Specifically, the gross risk-free rate expresses the conditional abnormal return earned at time $t+j$ as a time $t+j-1$ value, the time $t+j-1$ price P_{t+j-1} translates that abnormal return into time $t+j-1$ abnormal cash flow, and the cumulative SDF expresses that abnormal cash flow

[11] Of course, Campbell and Shiller (1988)'s discount parameter implicitly captures the fact that future returns on high dividend-paying assets are worth less in present-value terms.

in today's value. Finally, the formula normalizes the present value of abnormal cash flows with today's price P_t.

2.6 Comparison with one-period tests

The identity allows us to develop asset pricing tests that apply to both prices and returns. Define $\delta(J)$ as the unconditional expectation of the right-hand quantity in equation (13) where the infinite sum is replaced with a finite sum over J periods (months) after portfolio formation:

$$\delta(J) \quad = \quad -\textstyle\sum_{j=1}^{J} E\left[M_{t,t+j}\frac{P_{t+j-1}}{P_t}R^e_{t+j}\right]. \tag{15}$$

This equation is a J-horizon generalization of the asset pricing equation for one-period returns.

Using the covariance identity to decompose the right-hand side, we can also write

$$\sum_{j=1}^{J} E\left[M_{t,t+j}\frac{P_{t+j-1}}{P_t}\right]E\left[R^e_{t+j}\right] = -\delta(J) + \sum_{j=1}^{J} Cov\left(-M_{t,t+j}\frac{P_{t+j-1}}{P_t}, R^e_{t+j}\right), \tag{16}$$

where the quantities $\sum_{j=1}^{J} E\left[M_{t,t+j}\frac{P_{t+j-1}}{P_t}\right]E\left[R^e_{t+j}\right]$ and $\sum_{j=1}^{J} Cov\left(-M_{t,t+j}\frac{P_{t+j-1}}{P_t}, R^e_{t+j}\right)$ can be interpreted as the long-horizon discount rate and long-horizon risk.

To see why equation (15) is a natural long-horizon counterpart to the familiar one-period return pricing equation, take equation (16) for $J = 1$, divide both sides by $E[M_{t+1}]$, and recognize $\alpha = -\delta(1)/E[M_{t+1}]$ as the unconditional abnormal return to write

$$E\left[R^e_{t+1}\right] = \alpha + Cov\left(-\frac{M_{t+1}}{E[M_{t+1}]}, R^e_{t+1}\right), \tag{17}$$

which is the conventional asset pricing equation for expected one-period returns. As $J \to \infty$, on the other hand, equation (15) converges to the identity in Lemma 1 that expresses ex-ante mispricing in terms of subsequent returns over an infinite horizon. For intermediate values of J, the equation expresses the negative of the net present value, per dollar invested, of a strategy that buys the asset today and sell it after J periods. Thus, equation (15) can be used to test the efficiency of asset prices for a multi-period investment horizon.

2.7 Price level and price-level risk

In a time-series return regression, the estimated abnormal return is the difference between realized mean excess return and estimated risk premium. Similarly, it is useful to think of the estimated delta as the error term in the relation between discount rate and risk. Hence, we compute the sample analogues of the long-horizon discount rate and long-horizon risk quantities in equation (16) for $J = 180$ months and call them "price level" and "price-level risk":

$$\underbrace{\sum_{j=1}^{180} E_T \left[M_{t,t+j} \frac{P_{t+j-1}}{P_t} \right] E_{T_j} \left[R^e_{t+j} \right]}_{-1 \times \text{price level}} = - \underbrace{\hat{\delta}(180)}_{\text{estimates } \delta} + \underbrace{\sum_{j=1}^{180} Cov_T \left(-M_{t,t+j} \frac{P_{t+j-1}}{P_t}, R^e_{t+j} \right)}_{\text{price-level risk}}, \tag{18}$$

where the T subscript continues to indicate a sample moment.[12] The price level summarizes all future discount rates on the asset over the next 15 years into a single expression. Ultimately our paper is concerned with understanding variation in prices and the extent to which observed risk explains this variation. Our price level measure, which can be compared across time and across assets and is less contaminated by factors such as expected future profitability than a scaled price ratio such as the market-to-book ratio, allows us to conceptualize and visualize this objective.

When plotting price level and price-level risk in a graph, we divide both by the estimated $\sum_{j=1}^{180} E_T \left[M_{t,t+j} \frac{P_{t+j-1}}{P_t} \right]$ of the market portfolio (and multiply price level by -1), which scales price level and price-level risk to have the same unit as one-month returns. This way, the price-level figures can be compared easily to the expected return figures.

Finally, it is useful to decompose price-level risk into the component arising from contempo-

[12] We prefer to think of the negative of $\sum_{j=1}^{180} E_t \left[M_{t,t+j} \frac{P_{t+j-1}}{P_t} \right] E_T \left[R^e_{t+j} \right]$ as price level so that price level falls as the discount rate rises, but the sign is not important in subsequent analysis.

raneous covariance with the SDF and the remaining intertemporal component:

$$\sum_{j=1}^{180} Cov_T\left(-M_{t,t+j}\frac{P_{t+j-1}}{P_t}, R^e_{t+j}\right) = \underbrace{\sum_{j=1}^{180} E_T\left[\phi_{t,t+j-1}\right] Cov_T\left(-M_{t+j}, R^e_{t+j}\right)}_{\text{Contemporaneous risk}} \\ \underbrace{-\sum_{j=1}^{180} E_T\left[\left(\phi_{t,t+j-1} - E_{T_j}\left[\phi_{t,t+j-1}\right]\right) M_{t+j}\left(R^e_{t+j} - E_{T_j}\left[R^e_{t+j}\right]\right)\right]}_{\text{Intertemporal adjustment}}, \tag{19}$$

The first component is a simple discounted sum of future contemporaneous risk premia, and the second component corrects the first for the fact that future risk premia can covary with past ϕ. That is, for a long-term investor, an asset is risky not only if it has a contemporaneous negative covariance with the SDF, but also if it covaries negatively with the cumulative SDF.

2.8 Long-horizon asset pricing equations in calendar time

Our identity facilitates a price-level analogue of the well-known asset pricing equation for returns. Define δ as the unconditional expectation of the right-hand quantity in equation (13):

$$\delta = E\left[\delta_{t_0}\right] = -\sum_{j=1}^{\infty} E\left[M_{t_0,t_0+j}\frac{P_{t_0+j-1}}{P_{t_0}} R^e_{t_0+j}\right], \tag{20}$$

where we use t_0 to denote "event time," the time at which ex-ante mispricing is measured. Defining $t \equiv t_0 + j$ to denote "calendar time," the time at which excess returns on a asset or a trading strategy are realized, we can rewrite equation (20) as

$$\delta = -\sum_{j=1}^{\infty} E\left[M_{t-j,t}\frac{P_{t-1}}{P_{t-j}} R^e_t\right]. \tag{21}$$

The calendar-time approach for measuring (unconditional) mispricing in (21) is especially convenient when measuring the mispricing of a buy-and-hold trading strategy that trades a type of asset. Indeed, Fama (1998) emphasizes the usefulness of similar calendar-time techniques in his discussion of the literature on post-event, long-horizon abnormal returns, and Cohen, Polk, and Vuolteenaho (2009) exploit a calendar-time approach in their CAPM price-level tests, providing a direct precedent.

In particular, consider a portfolio formed every t_0 based on the specific buy-and-hold trading strategy in question (e.g. buy the bottom quintile of value stocks) and denote its buy-and-hold excess returns and cumulative capital gains by $\left\{R^e_{(t_0),t_0+j}\right\}_{j=1}^{\infty}$ and $\left\{P_{(t_0),t_0+j}/P_{(t_0),t_0}\right\}_{j=1}^{\infty}$, where (t_0) in the subscript indicates that the particular excess returns and cumulative capital gains are earned from buying and holding the particular portfolio formed at time t_0. The event-time expression for δ is thus

$$\delta = -\sum_{j=1}^{\infty} E\left[M_{t_0,t_0+j}\frac{P_{(t_0),t_0+j-1}}{P_{(t_0),t_0}}R^e_{(t_0),t_0+j}\right] \tag{22}$$

In words, unconditional δ is the expected value of the discounted sum of price-weighted excess returns realized at times t_0+1, t_0+2, ... from a portfolio formed at t_0. This definition makes it clear that δ_{t_0} is almost certainly quite correlated across events as observations of δ_{t_0} that are nearby in calendar time necessarily reference returns that overlap in calendar time. For example, the 15-year realized delta for a value portfolio formed in 1990 and a value portfolio formed in 1991 will have 14 years of overlapping returns on quite similar portfolios.[13] That sort of induced correlation significantly complicates statistical inference.

Now consider the calendar-time expression,

$$\delta = -\sum_{j=1}^{\infty} E\left[M_{t-j,t}\frac{P_{(t-j),t-1}}{P_{(t-j),t-j}}R^e_{(t-j),t}\right]. \tag{23}$$

In words, unconditional δ is now the expected value of the discounted sum of price-weighted excess returns realized at time t from portfolios formed at $t-1$, $t-2$, etc. By redefining δ_t in this way, the excess returns at a particular point in calendar time that show up in relevant δ_{t_0} observations are recombined into a single observation. Thus, each δ_t observation measures the mispricing on a specific composite calendar-time portfolio that contains all of the time-t alpha information that is relevant for δ. As a consequence, all cross-sectional correlation in the component portfolios formed at $t-1$, $t-2$, etc. is captured in the resulting composite portfolio's return realization and thus the associated δ_t realization. Therefore, δ_t, unlike δ_{t_0}, is not hard-wired to be correlated through time. Indeed, since short-term (e.g., monthly or yearly) returns are difficult to predict,

[13] Indeed, even if the trading strategy being evaluated is based on a transitory signal, time-t returns on portfolios formed at $t-1$, $t-2$, etc. based on this transitory signal will still likely load on market, industry, and other systematic sources of return variation, so that the overlapping nature of δ_{t_0} remains problematic.

there should be only minimal amounts of correlation in δ_t which standard techniques can easily take into account.[14]

2.9 The method of moments estimator, standard errors, and test statistics

We estimate mispricing δ with the 180-month sample analogue $\hat{\delta}(180)$ which provides a good approximation of the infinite sum, since both the discount factor $M_{t-j,t}\frac{P_{t-j,-1}}{P_{t-j}}$ and the conditional abnormal return $E_{t-j}[M_t R_t^e]$ are small after j =15 years for any portfolio formed at time $t-j$ (Appendix D.3).[15] For comparison, we also estimate $\hat{\delta}(1)$ for 1-month returns, which measures the extent to which expected one-month returns are abnormal as well as value of J between 1 and 180 in some of our tests.[16] The GMM estimator of a test portfolio's $\delta(J)$ is

$$\hat{\delta}(J) = -\sum_{j=1}^{J} E_T\left[M_{t-j,t}(\boldsymbol{b})\frac{P_{(t-j),t-1}}{P_{(t-j),t-j}}R^e_{(t-j),t}\right], \tag{24}$$

where $\boldsymbol{b}$ denotes the loadings that candidate risk factors have on the SDF.

To be consistent with the implicit assumption in conventional asset pricing regressions with returns, we model the SDF as having constant coefficients on linear factors $\boldsymbol{f}$: $M_t = b_0 - \boldsymbol{b}_1'\boldsymbol{f}_t$. We estimate the SDF parameters $\boldsymbol{b}$ by requiring the SDF to price the factors perfectly in sample, which is analogous to a time-series asset pricing regression using returns where the factor portfolios have zero abnormal returns by definition.[17]

Rather than use the risk-free rate as the benchmark asset, we exploit the fact that we require the model SDF to imply a zero δ for the market portfolio in sample. Therefore, we measure returns in excess of the market return. This choice has at least two benefits. First, by benchmarking

[14] Figure B1 plots the autocorrelations of extreme portfolios double sorted on quality and book-to-market to show that the autocorrelations are small.

[15] Appendix Figure B4 shows that the finite sum $-\sum_{j=1}^{J} E_T\left[M_{t-j,t}\frac{P_{(t-j),t-1}}{P_{(t-j),t-j}}R^e_{(t-j),t}\right]$ indeed plateaus around $J = 15$ years in the data, implying that the remaining terms in the infinite sum are likely to make little difference to the infinite sum.

[16] $\hat{\delta}(1)$ estimates $\delta(1)$, a scaled multiple of the estimated abnormal return α in a conventional return regression: $\delta(1) = E[M_{t+1}]\alpha$.

[17] In particular, we require the model SDF to explain perfectly the one-period market returns and to price the J-period delta of the market portfolio perfectly in sample. When $J = 1$, the two conditions coincide, so we add the condition that the model SDF explains one-month Treasury bill rates perefctly in sample. Appendix Table B2 reports the SDF parameter estimates and associated confidence intervals for each of the six horizons we study. The resulting estimates are relatively stable across horizons.

test assets against the market, we remove common variation in returns associated with the market factor, improving the precision of our estimates. Second, recent papers have argued that the T-bill return is affected by the special liquidity of short-term US government debt, which makes it near-money (Krishnamurthy and Vissing-Jorgensen (2012); nagel2016liquidity). By using the market as a reference asset, we ensure that that sort of pricing distortion does not directly affect our estimates of δ.[18]

To account for the possible non-normality of our estimator $\hat{\delta}$ in a finite sample, rather than relying on the generalized method of moments (GMM) asymptotics, we compute bootstrap standard errors and p-values. We use a block bootstrap procedure (Kunsch (1989)) that accounts for cross-sectional covariances across our test portfolios and factor portfolios as well as possible time-series covariances. (Recall that our calendar-time approach leads to small time-series covariances.) Specifically, we construct each bootstrap sample by parsing together T/B randomly selected (with replacement) blocks of length B, where T is the number of portfolio formation months in the original sample ($T = 859$ months from June 1948 to December 2019) and the block length B denotes the number of consecutive portfolio formation months. When doing so, we use the same time blocks for both all test portfolios and factor portfolios, thereby retaining the cross-sectional covariance structure within a period and accounting for the uncertainty in estimating $\boldsymbol{b}$. We use a block length of 12 and generate 10,000 simulations for each bootstrap.

The primary test statistic we look at is the estimated difference in δs between the two extreme quintile portfolios ("Hi - Lo"), which we use to reject or not reject an asset pricing model for each characteristic sort. We also compute the Wald statistic that tests the hypothesis that deltas on all of a set of characteristic-sorted portfolios are equal to 0.

3 Asset Pricing Tests Using Price Levels

We examine how well the simple unconditional CAPM explains the prices of stock portfolios sorted on two natural characteristics for price-level tests: the book-to-market ratio (B/M) and quality. The former directly measures book-to-price while the latter is a proxy for the (intrinsic)

[18]We grant that the T-bill rate does affect our estimation of the parameter b_0 in the SDF and thus may indirectly influence our estimates. However, in results not shown, we have confirmed that our conclusions are broadly robust to a reasonable normalization of b_0 that avoids using the T-bill.

value-to-book ratio (V/B). Compared to how poorly it explains the cross-section of returns of portfolios based on a univariate sort on either ratio, the CAPM does a surprisingly better job explaining the corresponding cross-section of prices. We use this result to illustrate how asset pricing with price levels works and why it can generate drastically different results from asset pricing with returns. We then show that a double sort on B/M and our proxy for V/B nonetheless leads to larger price-level errors.

3.1 Data

We combine monthly stock price data from the Center for Research in Security Prices (CRSP) and annual accounting data from CRSP/Compustat Merged (CCM) to create our basic merged dataset. We use one-month Treasury bill rates from Kenneth French's data library (originally from Ibbotson Associates) as the risk-free rate and the market excess return from the same data library as the market factor.

To estimate price-level errors of characteristic-sorted portfolios, we need post-formation returns over a long horizon of 180 months (15 years) after the initial portfolio formation. That is, we form value-weight quintile portfolios each month at t based on the NYSE quintile cutoffs and compute the post-formation returns on these portfolios over $t+1, \ldots, t+180$. Post-formation returns at $t+j$ for the portfolio formed at t are buy-and-hold returns that do not reinvest dividends into the same or different stocks.[19] In summary, our data is three dimensional, as we illustrate in Table B1: we have post-formation returns for 10 different portfolios (or 25 for two-way sorted portfolios), for T different portfolio formation periods, and for J different post-formation periods. Our baseline data use post-formation returns over 1948m6–2019m12. Our portfolio sorts begin in 1933m6. However, some characteristics data begin later in 1957m6, which is when we can first compute accounting-based characteristics based on the annual Compustat dataset

Table 1 provides descriptive statistics for the portfolios formed from a univariate sort on each of the ten characteristics we consider in the rest of the paper: book-to-market ratio, quality, quality-

[19]These $t+j$ returns are equivalent to returns earned by forming a new portfolio every month at $t+j-1$ based on initial time-t weights adjusted by the cumulative capital gain from time t to time $t+j-1$. The rebalancing based on cumulative capital gain is the correct approach for our purpose of inferring the initial price level of the portfolio based on subsequent returns, since it mirrors how the returns earned by investing the dividend payments for an individual asset do not enter into our formula. See the exact argument in Appendix C.

to-price, size, momentum, profitability, investment, beta, net issuance, and accruals. Appendix B provides further details on the data construction.

3.2 Basic risk adjustment with the market factor

We use the CAPM to model the SDF, as it provides the basic risk adjustment upon which multi-factor models are built:[20]

$$M_t = b_0 - b_1 R^e_{m,t}. \tag{25}$$

We choose b_0 and b_1 to make the price-level error of the market portfolio as well as the one-month Treasury bill zero in sample. This makes our approach analogous to the time-series approach to estimating unconditional abnormal return αs, which makes the in-sample abnormal return on the factor portfolios zero and also implicitly assumes the one-month Treasury bill to have a zero abnormal return by taking an excess return with respect to it.

3.3 Quality and B/M as the primary sorting characteristics

A powerful test on price levels requires test assets that a priori are likely to exhibit large variation in mispricing δ. To do this, recall that δ measures the percentage deviation of value from price, which can be rewritten in terms of intrinsic value over book equity V/B and book equity over market price B/M (where V, M, and B are measured per share so that $M = P$):

$$\delta_t = 1 - \frac{V_t}{P_t} = 1 - \frac{V_t}{B_t} \times \frac{B_t}{M_t}. \tag{26}$$

Hence, holding the other ratio fixed, a variation in either V/B or B/M implies variation in δ. This motivates us to sort stocks based on these two ratios.

Since the value-to-book ratio V/B is unobserved, we follow Asness, Frazzini, and Pedersen (2019) to use a composite z-score measure called *quality* as a proxy for V/B. Rewriting Gordon's

[20] Our analysis in subsequent sections will provide a direction for future research on multifactor models of price levels. For example, it would be natural to consider the intertemporal CAPM specification of Campbell, Giglio, Polk, and Turley (2018), which incorporates stochastic volatility into the ICAPM framework of Campbell and Vuolteenaho (2004) to significantly reduce the pricing errors relative to the CAPM in standard SDF return tests.

growth model in the absence of mispricing as

$$\frac{V}{B} = \frac{profitability \times payout\,ratio}{required\,returns - growth}, \tag{27}$$

they use *quality* measured by a z-score that rewards profitable, fast-growing, safe, and high-payout stocks to proxy for the value-to-book ratio: [21]

$$quality \equiv z\left(z_{\text{profitability}} + z_{\text{growth}} + z_{\text{safety}} + z_{\text{payout ratio}}\right) \propto \frac{V}{B} + noise \tag{28}$$

The book-to-market ratio B/M may contain different information from the value-to-book ratio V/B, since a distortion in the discount rate due to abnormal returns affects B/M but not V/B:

$$\frac{B}{M} = \left(\frac{profitability \times payout\,ratio}{abnormal\,returns + required\,returns - growth}\right)^{-1}. \tag{29}$$

Since the B/M ratio accounts for variation in long-horizon discount rates due to abnormal returns whereas the V/B ratio does not, double sorting on *quality* and B/M should lead to larger δ variation and hence an even more powerful test. In particular, as we illustrate graphically in Figure 1, low-quality stocks that are expensive (low *quality* and B/M) are likely to be overpriced and have a positive δ, whereas high-quality stocks that are inexpensive (high *quality* and B/M) are likely to be underpriced with a negative δ.

We note in passing that, in the context of Gordon's model, B/M depends not only on discount rates, but also on factors such as growth and profitability. Indeed, Cohen, Polk, and Vuolteenaho (2003) show that roughly 80% of the cross-sectional variation in book-to-market ratios reflects predictable differences in expected future profitability. This fact makes B/M, or its reciprocal M/B, a less appealing measure of price level than our identity-motivated measure of price level or mispricing.

[21]Theoretically, the effect of the payout ratio is ambiguous in a fully dynamic model of the value-to-book ratio. However, empirically, higher payout ratio appears to be associated with a higher firm value.

3.4 Explaining the prices of portfolios sorted by quality or B/M

To what extent does the CAPM explain the cross-section of prices of portfolios formed by a univariate sort on either quality or the book-to-market ratio? Specifically, does the CAPM explain price levels better than it explains returns, and if so, why?

To provide a reference point, we begin with conventional asset pricing tests using returns. The left two panels in Figure 2 show that the CAPM does a very poor job explaining the cross-section of returns on quintile portfolios sorted on quality or book-to-market ratio. High quality stocks earn higher returns than low quality stocks despite having a lower market beta (Asness, Frazzini, and Pedersen (2019)). Similarly, High B/M or "value" stocks earn higher returns than low B/M or "growth" stocks despite having a lower market beta (e.g., Fama and French (1992)). Hence, both $quality$ and B/M sorts lead to a cross-sectional relation between risk and returns that is opposite to what the CAPM predicts.

Asset pricing tests with price levels generate meaningfully different results. The right two panels in Figure 2 plot the cross-sectional relation between price level and price-level risk but with a scaling factor that divides both quantities by the value of $\sum_{j=1}^{180} E\left[M_{t,t+j}P_{t+j-1}/P_t\right]$ for the market and multiplying the price level by -1 for an easier comparison with the left two panels. The long-horizon discount rate and long-horizon risk that scaled price level and scaled price-level risk respectively capture display a positive cross-sectional relation with a fitted line reasonably close to the 45 degree line, suggesting that the CAPM does a decent job describing these portfolios' price-level risk that buy-and-hold investors would care about.

To provide a formal analysis, Tables 2 and 3 report the estimated δs and our key Hi-Lo test statistic along with standard errors (in parentheses) and p-values (in square brackets) for J: $\{1\text{mo}, 1\text{yr}, 3\text{yrs}, 5\text{yrs}, 10\text{yrs}, 15\text{yrs}\}$ with $J = 1$ month generating conventional time series return regression results and $J = 15$ years (180 months) proxying price-level results given by $J \to \infty$. The intermediate values of J allow us to see how the performance of the asset pricing model changes as the return horizon increases gradually from 1 month to 15 years.

Table 2 shows that high-quality stocks are undervalued and low-quality stocks are overvalued from the perspective of CAPM investors with a short investment horizon of $J = 1$ month or 1 year. However, for $J = 5$ or more years, the estimated δs are statistically indistinguishable from zero for all quality-sorted portfolios and imply that the market price correctly accounts for the quality difference. For example, for $J = 15$, we find that high-quality stocks are relatively cheaper than low-quality stocks by only 13 percentage points with an associated t-statistic of 0.67. Our conclusion, based on an identity that gives an exact expression for ex-ante mispricing, is contrary to the conclusion drawn by Asness, Frazzini, and Pedersen (2019), whose analysis instead studies either cumulative five-year returns or a cross-sectional regression of the M/B ratio on quality.

Table 3 reports the result for B/M-sorted portfolios. Compared to quality-sorted portfolios, B/M portfolios show larger price-level errors with respect to the CAPM. Growth stocks are estimated to be 23 percentage points more overpriced than value stocks but this difference has a t-statistic less than one.

The mechanism: Why does asset pricing with price levels look different?

Even if both returns and betas of extreme quintile portfolios converge after portfolio formation (Keloharju, Linnainmaa, and Nyberg (2019)), we should continue to see a negative cross-sectional relation between the discounted sums of risks and returns that both $quality$- and B/M-sorted portfolios exhibit in the return regression (left two panels in Figure 2). That is, a mere convergence in returns and risks after portfolio formation cannot explain our results. In order for the sign of the negative cross-sectional relation to flip from negative to positive as we move from the return to the price-level perspective, at least one of the following should occur for the extreme decile portfolios. Either the return spread crosses (flips sign) at some point after portfolio formation; the beta spread crosses; or the intertemporal adjustment component of price-level risk in equation (19) has an opposite cross-sectional pattern to the contemporaneous risk component, thereby undoing the puzzling pattern that the high-return quintile 5 has a lower market beta than the low-return quintile 1 immediately following portfolio formation.

The primary reason why price levels of $quality$-sorted portfolios line up well with their price-level risks is because of the long-run behavior of their post-formation realized returns. Figure 3 shows that the realized returns are initially higher on high-quality stocks than on low-quality stocks for the few months following the portfolio formation, after which the low-quality stocks often have higher realized returns than high-quality stocks. This makes the long-horizon discount rate (price level) higher for low-quality stocks than high-quality stocks, as we expect from their long-horizon risk (price-level risk). Another important contribution comes from the intertemporal adjustment. Figure B2 shows that the intertemporal component of price-level risk is larger in high-quality stocks than in low-quality stocks which gives them relatively more price-level risk. Thus, high-quality stocks are riskier for investors with a long investment horizon, since their returns tend to be low after the stock market experiences a series of negative shocks; this would be the case, for instance, if high quality stocks' market beta tends to come from the exposure of their returns to market-level cash flow news. For example, Campbell, Polk, and Vuolteenaho (2010) shows that accounting variables that are often associated with quality forecast variation in this component of beta.

In contrast, price-level risks of B/M-sorted portfolios tend to line up with price levels reasonably well, as the beta spread between high- and low-B/M portfolios crosses. Figure 3 shows that the market beta of the high-B/M portfolio is low at the time of the portfolio formation but rises above that of the low-B/M portfolio, which steadily declines over the post-formation periods. This fact, first documented in Cohen, Polk, and Vuolteenaho (2009), makes value stocks riskier for a long-term investor.[22]

Quality and B/M double sort

Double sorting stocks based on *quality* and B/M to generate 25 portfolios resurrects the ability of these characteristics to explain larger variation in δs (Table 4). Furthermore, the variation in δ across the two dimensions of the table is consistent with our conjecture in Figure 1.

Mispricing δ declines as we move from top to bottom, which amounts to holding B/M fixed while increasing *quality*, and quality appears to be an especially strong predictor of CAPM mispricing among growth (low B/M) stocks. Similarly, δ declines as we move from left to right, which amounts to holding *quality* fixed while increasing B/M, and this variation leads to statistically significant differences in mispricing among low-quality stocks. Moving diagonally from the

[22]We discuss in Appendix D.1 how our mispricing metric δ improves on the mispricing metric of Cohen, Polk, and Vuolteenaho (2009).

top left to the bottom right generates the largest variation in δs. We estimate low-quality, low-B/M stocks to be 48 percentage points more overpriced than high-quality, high-B/M stocks with a t-statistic of 2.31. Indeed, the price-level variation of Table 2 is exactly the sort of variation a buy-and-hold investor should be interested in exploiting and represents a challenge for future asset pricing models that make understanding price-level risk a priority.[23]

4 Are Return Anomalies Price-level Anomalies?

We next study the extent to which the CAPM explains price-level variation associated with eight additional characteristics known to be associated with cross-sectional variation in average returns: size, momentum, profitability, investment, beta, net issuance, accruals, and a characteristic that combines quality and book-to-market: the "quality-to-price" ratio. Specifically, we follow first convert quality and book-to-market to z scores and then add the results together.[24] The first row of Table 5 documents that our quality-to-price variable generates economically large (41 percentage points) and statistically significant (t-statistic of 2.47) spread in our δ measure.

We next turn to individual anomalies. The first four represent a set of prominent return anomalies that (in conjunction with value and the market factor) make up the widely-used Fama-French-Carhart six-factor model. The next three are chosen for their potential conceptual link to price-level distortions. As noted in Section 3, the CAPM tends to explain price-level variation better than it explains short-horizon return variation. Among the seven characteristics studied, only quality-to-price and net issuance are price-level anomalies with respect to the CAPM.

4.1 Prominent return anomalies

We begin with the four prominent return anomalies. Fama and French (2015) argue that profitability, investment, and size are characteristics that are important in summarizing the cross-section of returns, and price momentum has been a prominent return anomaly since Jegadeesh and Titman (1993). To what extent are these prominent return characteristics associated with variation in price levels unrelated to CAPM price-level risk?

[23] Appendix Tables B3, B4, and B5 confirm that our conclusions are robust to only using the modern subsample to form quality-sorted, book-to-market-sorted, or quality-and-book-market-sorted portfolios.

[24] The acknowledgement for first using this characteristic goes to Asness, Frazzini, and Pedersen (2019), who call this characteristic "quality at a reasonable price."

Size and momentum

Size and momentum are interesting to study from the price-level perspective, given that momentum strongly predicts the cross-section of average returns but is a rather transitory firm characteristic while size weakly predicts the cross-section of average returns but is a rather persistent firm characteristic. In particular, Cohen, Polk, and Vuolteenaho (2009) highlight that signal persistence is an important consideration when moving from the conventional return perspective to the price-level perspective, a point that Cochrane (2011) subsequently emphasizes.

> "For example, since momentum amounts to a very small time-series correlation and lasts less than a year, I suspect it has little effect on long-run expected returns and hence the level of stock prices. Long-lasting characteristics are likely to be more important. Conversely, small transient price errors can have a large impact on return measures" (p.1064).

Consistent with Cochrane's conjecture, Table 5 shows that momentum is not a statistically significant predictor of CAPM mispricing. The difference in δ between the high momentum and low momentum portfolios is 14 percentage points with a t-statistic of 1.70. Also consistent with Cochrane's view, the persistence of the size characteristic does generate larger price-level variation than one might expect from its relatively weak one-period alpha. However, the difference in price-level errors of the two extreme size portfolios is not statistically significant.

Profitability and investment

Table 5 documents that profitability-sorted portfolios are associated with price-level errors that are statistically insignificant. Moreover, the point estimates are consistent with high profitability companies being overpriced and low profitability companies being underpriced. As a consequence, the behavioral interpretation of abnormal returns earned on profitability-based trades is more consistent with investor overreaction to news about profitability rather than fundamental undervaluation (overvaluation) of profitable (unprofitable) companies (Novy-Marx (2013) and Asness, Frazzini, and Pedersen (2019)). The result is also consistent with the observation that cross-sectional variation in the marginal product of capital, which our profitability measure could proxy for, does not necessarily imply misallocation of capital (David, Schmid, and Zeke (2019)). Price-level variation is also not anomalous for investment-sorted portfolios; price levels of portfolios sorted by asset growth are explained by variation in price-level risk.

4.2 Characteristics conceptually related to mispricing

Other stock characteristics are interesting to analyze using our mispricing measure either due to their mechanical link to price-level risk (beta) or their conceptual association with mispricing (net issuance and accruals) vis-ï¿œ-vis the endogenous choices of managers. We explain the conceptual link that each characteristic has to mispricing δ and study the extent to which the characteristic is associated with price-level variation that cannot be explained by exposure to market risk that the CAPM captures.

Net share issuance

A series of papers argue that share repurchase (issuance) indicates undervaluation (overvaluation) as perceived by firm managers (Loughran and Ritter (1995); Ikenberry, Lakonishok, and Vermaelen (1995)). Thus, to the extent that firm managers are long-term investors in the firm, net issuance could be a useful proxy for mispricing with respect to the CAPM. The spread in mispricing δs is large: Table 5 shows that high issuance firms are estimated to be modestly underpriced by nearly 5%, but low (negative) issuance firms are estimated to be underpriced by 34%. That is, share repurchases appear to be strong signals of CAPM underpricing while share issuances are not indicative of CAPM overpricing. The difference in δ's across the quintiles of 29 percentage points has an associated t-statistic of 2.37.

Beta

Equation (19) shows that price-level risk can be decomposed into two terms, i) a discounted sum of contemporaneous covariances between the SDF M and returns and ii) the sum of intertemporal covariances between the price-adjusted SDF ϕ and returns. The persistence of market beta implies that market beta sorts have the potential to generate large variation in the first of these two terms, the contemporaneous risk component of CAPM price-level risk. In particualar, if the resulting variation in price-level risk is not compensated with corresponding variation in price levels, large δs could arise.

Table 5 shows an estimated difference in δs of 45 percentage points across the high- and low-beta portfolios, suggesting that the large variation in risk generated by the beta sort does not lead to a correspondingly large variation in price levels. However, this estimate is only marginally statistically significant with a t-statistic of 2.02.

Accruals

Earnings management proxied by accruals (Sloan (1996)) is an interesting phenomenon to analyze with our price-level identity, as it is typically motivated as being the result of companies with adverse operating results managing earnings to inflate the value of their firm as perceived by outsiders. Thus, if the firms are successful in managing earnings, high accruals may proxy for the gap between the market price set by investors and the intrinsic value perceived by firm managers. Table 5 shows results inconsistent with this interpretation of accruals as the measured deltas of the extreme quintile portfolios are economically and statistically insignificant.

4.3 Double sorts on characteristics and the quality-to-price ratio

Table 6 synthesizes our analysis by examining double sorts of the quality-to-price ratio and each of the above seven characteristics. Specifically, we sort stocks into three by three portfolios based on independent NYSE breakpoints. The left-hand side of the table reports the δ and associated t-statistic for each of the nine portfolios. The right-hand-side of the table reports the δ's associated with the combination of the nine portfolios that results in either a quality-to-price neutral portfolio that bets on the second characteristic or a characteristic-neutral portfolio that bets on the quality-to-price ratio.

There are two important takeaways from the table. First, after controlling for the quality-to-price characteristic, there is no incremental information in the characteristics we study, with the exception of momentum whose p-value is 0.047. This finding is true even for net issuance, which showed significant δ variation in a univariate sort. Second, across all of the rows in Table 6, the quality-to-price ratio always generates economically and statistically significant variation in CAPM δ. Indeed, the t-statistic on our characteristic-neutral quality-to-price hedge portfolios are higher than the corresponding single-sort value in row one of Table 5 (2.47) for all but two of the rows in Table 6.

5 Conclusion

Our novel model misspecification measure, delta, precisely links future alpha to current price-level deviations. In stark contrast to existing measures, our approach correctly recognizes that abnormal returns occurring sooner, in more valuable states, or after relatively large capital gains should be associated with larger price-level distortions. Our primary tests reveal that the cross-section of average price levels of either book-to-market- or quality-sorted portfolios do not strongly reject the CAPM; however, portfolios formed from double sorts on these two variables generate large variation in mispricing. We show that among all other prominent return anomalies, only net equity issuance sorts produce significant price-level distortions relative to the CAPM, and that variation is subsumed by the quality-to-price ratio, a composite signal that incorporates the information in CAPM mispricing contained in book-to-market and quality.

As a consequence, our novel mispricing measure and the associated results provide better identification of the stocks that a buy-and-hold mean-variance investor should find attractive/unattractive. Moreover, our approach highlights where new models that aim to explain both short- and long-run patterns in markets should focus. Indeed, by providing an exact metric of the extent to which a candidate asset-pricing model explains variation in prices, we aim to advance future research in both asset pricing and corporate finance. For the former, our economically-important price-level metric could provide a useful lens through which to distinguish among risk-based, behavioral-based, and institutional-friction-based explanations for well-known empirical patterns in markets. For the latter, our measure of mispricing with respect to a risk model may refine the results of a large literature (e.g. Baker and Wurgler (2002) and Shleifer and Vishny (2003)) that aims to link a firm's investment and financing decisions to mispricing but instead often simply assumes that using the book-to-market equity ratio suffices in that regard.

References

Asness, Clifford S., Andrea Frazzini, and Lasse Heje Pedersen. 2019. "Quality minus junk." *Review of Accounting Studies* 24 (1):34–112.

Baba Yara, Fahiz, Martijn Boons, and Andrea Tamoni. 2020. "New and Old Sorts: Implications for Asset Pricing."

Bai, Jennie, Thomas Philippon, and Alexi Savov. 2016. "Have financial markets become more informative?" *Journal of Financial Economics* 122 (3):625–654.

Baker, Malcolm, Jeremy C Stein, and Jeffrey Wurgler. 2003. "When does the market matter? Stock prices and the investment of equity-dependent firms." *The Quarterly Journal of Economics* 118 (3):969–1005.

Baker, Malcolm and Jeffrey Wurgler. 2002. "Market timing and capital structure." *Journal of Finance* 57 (1):1–32.

Barber, Brad M. and John D. Lyon. 1997. "Detecting long-run abnormal stock returns: The empirical power and specification of test statistics." *Journal of Financial Economics* 43 (3):341–372.

Bessembinder, Hendrik, Michael J. Cooper, and Feng Zhang. 2018. "Characteristic-based benchmark returns and corporate events." *Review of Financial Studies* 32 (1):75–125.

Black, Fischer. 1986. "Noise." *Journal of Finance* 41 (3):528–543.

Black, Fischer, Michael C. Jensen, and Myron Scholes. 1972. "The capital asset pricing model: Some empirical tests." *Studies in the theory of capital markets* 81 (3):79–121.

Brav, Alon. 2000. "Inference in long-horizon event studies: a Bayesian approach with application to initial public offerings." *Journal of Finance* 55 (5):1979–2016.

Campbell, John. 2017. *Financial decisions and markets: A course in asset pricing*. Princeton University Press.

Campbell, John Y., Stefano Giglio, Christopher Polk, and Robert Turley. 2018. "An intertemporal CAPM with stochastic volatility." *Journal of Financial Economics* 128 (2):207–233.

Campbell, John Y., Christopher Polk, and Tuomo Vuolteenaho. 2010. "Growth or glamour? Fundamentals and systematic risk in stock returns." *Review of Financial Studies* 23 (1):305–344.

Campbell, John Y. and Robert J. Shiller. 1988. "The dividend-price ratio and expectations of future dividends and discount factors." *Review of Financial Studies* 1 (3):195–228.

Campbell, John Y. and Tuomo Vuolteenaho. 2004. "Bad beta, good beta." *American Economic Review* 94 (5):1249–1275.

Chernov, Mikhail, Lars A. Lochstoer, and Stig R.H. Lundeby. 2018. "Conditional dynamics and the multi-horizon risk-return trade-off." NBER working paper series.

Cho, Thummim. 2020. "Turning alphas into betas: Arbitrage and endogenous risk." *Journal of Financial Economics* 137 (2):550–570.

Cochrane, John H. 1992. "Explaining the Variance of Price-Dividend Ratios." *Review of Financial Studies* 5 (2):243–280.

———. 2008. "The Dog That Did Not Bark: A Defense of Return Predictability." *Review of Financial Studies* 21 (4):1533–1575.

———. 2011. "Presidential address: Discount rates." *Journal of Finance* 66 (4):1047–1108.

———. 2014. "A Mean-Variance Benchmark for Intertemporal Portfolio Theory." *Journal of Finance* 69 (1):1–49.

Cohen, Randolph B., Christopher Polk, and Tuomo Vuolteenaho. 2003. "The value spread." *Journal of Finance* 58 (2):609–641.

———. 2009. "The price is (almost) right." *Journal of Finance* 64 (6):2739–2782.

David, Joel M., Lukas Schmid, and David Zeke. 2019. "Risk-Adjusted Capital Allocation and Misallocation."

Davis, James L., Eugene F. Fama, and Kenneth R. French. 2000. "Characteristics, Covariances, and Average Returns: 1929 to 1997." *Journal of Finance* 55 (1):389–406.

De Bondt, Werner F. M. and Richard Thaler. 1985. "Does the stock market overreact?" *Journal of Finance* 40 (3):793–805.

Fama, Eugene F. 1970. "Efficient Capital Markets: A Review of Theory and Empirical Work." *Journal of Finance* 25 (2):383–417.

———. 1998. "Market efficiency, long-term returns, and behavioral finance." *Journal of Financial Economics* 49 (3):283–306.

Fama, Eugene F. and Kenneth R. French. 1989. "Business conditions and expected returns on stocks and bonds." *Journal of Financial Economics* 25 (1):23 – 49.

———. 1992. "The cross-section of expected stock returns." *Journal of Finance* 47 (2):427–465.

———. 2015. "A five-factor asset pricing model." *Journal of Financial Economics* 116 (1):1–22.

Favero, Carlo A., Alessandro Melone, and Andrea Tamoni. 2020. "Factor Models with Drifting Prices."

Gabaix, Xavier. 2007. "Linearity-generating processes: A modelling tool yielding closed forms for asset prices."

Garleanu, Nicolae and Lasse Heje Pedersen. 2011. "Margin-based asset pricing and deviations from the law of one price." *Review of Financial Studies* 24 (6):1980–2022.

Geanakoplos, John and William R. Zame. 2014. "Collateral equilibrium, I: A basic framework." *Economic Theory* 56 (3):443–492.

Hansen, Lars Peter and Ravi Jagannathan. 1991. "Implications of security market data for models of dynamic economies." *Journal of Political Economy* 99 (2):225–262.

———. 1997. “Assessing specification errors in stochastic discount factor models.” *Journal of Finance* 52 (2):557–590.

Ikenberry, David, Josef Lakonishok, and Theo Vermaelen. 1995. “Market underreaction to open market repurchases.” *Journal of Financial Economics* 39 (2–3):181–2018.

Jegadeesh, Narasimhan and Sheridan Titman. 1993. “Returns to buying winners and selling losers: Implications for stock market efficiency.” *Journal of Finance* 48:65–91.

Keloharju, Matti, Juhani Linnainmaa, and Peter Nyberg. 2019. “Long-term discount rates do not vary across firms.”

Kothari, S.P. and Jerold B. Warner. 1997. “Measuring long-horizon security price performance.” *Journal of Financial Economics* 43 (3):301–339.

Krishnamurthy, Arvind and Annette Vissing-Jorgensen. 2012. “The aggregate demand for treasury debt.” *Journal of Political Economy* 120 (2):233–267.

Kunsch, Hans R. 1989. “The Jackknife and the Bootstrap for General Stationary Observations.” *Annals of Statistics* 17 (3):1217–1241.

Lakonishok, Josef, Andrei Shleifer, and Robert W. Vishny. 1994. “Contrarian investment, extrapolation, and risk.” *Journal of Finance* 49 (5):1541–1578.

Lee, Charles M.C., James Myers, and Bhaskaran Swaminathan. 1999. “What is the Intrinsic Value of the Dow?” *Journal of Finance* 54 (5):1693–1741.

Loughran, Tim and Jay R. Ritter. 1995. “The new issues puzzle.” *Journal of Finance* 50 (1):23–51.

Lyon, John D., Brad M. Barber, and Chih-Ling Tsai. 1999. “Improved methods for tests of long-run abnormal stock returns.” *Journal of Finance* 54 (1):165–201.

Novy-Marx, Robert. 2013. “The other side of value: The gross profitability premium.” *Journal of Financial Economics* 108 (1):1–28.

Polk, Christopher and Paola Sapienza. 2009. “The stock market and corporate investment: A test of catering theory.” *Review of Financial Studies* 22 (1):187–217.

Rosenberg, Barr, Kenneth Reid, and Ronald Lanstein. 1985. “Persuasive evidence of market inefficiency.” *Journal of Portfolio Management* 11 (3):9–16.

Shiller, Robert J. 1984. “Stock prices and social dynamics.” *Brookings papers on economic activity* 2:457–510.

Shleifer, Andrei and Robert W. Vishny. 2003. “Stock market driven acquisitions.” *Journal of Financial Economics* 70 (3):295–311.

Shumway, Tyler. 1997. “The delisting bias in CRSP data.” *Journal of Finance* 52 (1):327–340.

Shumway, Tyler and Vincent A. Warther. 1999. “The delisting bias in CRSP’s Nasdaq data and its implications for the size effect.” *Journal of Finance* 54 (6):2361–2379.

Sloan, Richard G. 1996. “Do stock prices fully reflect information in accruals and cash flows about future earnings?” *Accounting Review* 71 (3):289–315.

Stein, Jeremy. 1996. “Rational Capital Budgeting in an Irrational World.” *The Journal of Business* 69 (4):429–55.

Summers, Lawrence H. 1986. “Does the stock market rationally reflect fundamental values?” *Journal of Finance* 41 (3):591–601.

van Binsbergen, Jules, Michael Brandt, and Ralph Koijen. 2012. “On the Timing and Pricing of Dividends.” *American Economic Review* 102 (4):1596–1618.

van Binsbergen, Jules H. and Christian C. Opp. 2019. “Real anomalies.” *Journal of Finance* 74 (4):1659–1706.

Vuolteenaho, Tuomo. 2002. “What drives firm-level stock returns?” *Journal of Finance* 57 (1):233–264.

Whited, Toni M. and Jack Zhao. forthcoming. “The Misallocation of Finance.” *Journal of Finance* .

Table 1: **Descriptive Statistics**

The table describes the ten characteristics we use to study CAPM mispricing. Column 2 reports the sample period over which post-formation returns for $j = 1$ through 180 months are available. Columns 3–5 report the CAPM alphas of the lowest and highest portfolio quintiles as well as the difference in the alphas between the two portfolios. We report Newey-West standard errors with six lags in parentheses. Column 6 reports Persistence, the value-weighted probability that the characteristic decile of a stock in the portfolio changes after a year. The remaining columns report the time-series average of the pairwise cross-sectional correlations among the characreristics. We use quintile numbers to compute these correlations to ensure that the correlations are not driven by outliers. See Appendix B for a detailed description of the way we construct these ten characteristics.

		CAPM alpha				Correlation									
	Sample	Lo	Hi	Hi - Lo	Persistence	Qlty	B/M	QMP	Size	Mom	NI	Beta	Prof	Inv	Acc
Quality	Jun48-Dec19	-0.25	0.19	0.45	0.41	1.00									
		(-3.25)	(4.40)	(4.00)											
Book-to-market	Jun48-Dec19	-0.04	0.25	0.29	0.48	-0.36	1.00								
		(-0.77)	(2.63)	(2.13)											
Quality-to-price	Jun48-Dec19	-0.26	0.38	0.64	0.38	0.55	0.47	1.00							
		(-4.39)	(4.59)	(6.38)											
Size	Jun48-Dec19	0.07	0.01	-0.06	0.78	0.19	-0.26	-0.07	1.00						
		(0.67)	(0.23)	(-0.51)											
Momentum	Jun48-Dec19	-0.51	0.32	0.83	0.14	0.07	0.01	0.07	0.05	1.00					
		(-4.61)	(4.36)	(5.14)											
Net issuance	Jun48-Dec19	0.24	-0.25	-0.50	0.34	-0.37	-0.10	-0.43	-0.00	-0.01	1.00				
		(4.62)	(-4.27)	(-5.06)											
Beta	Jun48-Dec19	0.26	-0.41	-0.67	0.46	-0.23	-0.09	-0.27	0.04	-0.04	0.13	1.00			
		(3.36)	(-4.66)	(-4.62)											
Profitability	Jun72-Dec19	-0.26	0.10	0.36	0.46	0.48	-0.44	0.03	0.22	0.05	-0.10	-0.02	1.00		
		(-2.83)	(1.88)	(2.66)											
Investment	Jun72-Dec19	0.24	-0.18	-0.43	0.22	0.00	-0.26	-0.24	0.05	-0.03	0.21	0.08	0.17	1.00	
		(3.04)	(-2.62)	(-3.24)											
Accruals	Jun72-Dec19	0.05	-0.22	-0.27	0.25	-0.11	-0.03	-0.14	-0.04	-0.03	0.05	-0.02	-0.02	0.25	1.00
		(0.74)	(-3.37)	(-2.26)											

Table 2: **Pricing Quality-sorted Portfolios: Returns vs. Prices**

The sample period is 1948m6–2019m12. The table shows that the CAPM does a good job describing the cross-section of prices of portfolios sorted on quality (the last row) but a poor job describing the cross-section of one-month returns (the first row). Quality is the composite metric introduced by Asness, Frazzini, and Pedersen (2019) to proxy for the ratio of intrinsic value-to-book ratio (V/B). We form five value-weighted portfolios by sorting stocks based on quality. We form these portfolios and track post-formation returns for 15 years. In the first "return" row, δ measures -1 times the average one-month abnormal return (over the gross one-month risk-free rate):

$$\delta(1) \quad = \quad -E\left[M_t R_t^e\right].$$

Positive (negative) δ here means negative (positive) one-month abnormal return. The reported δs in the last row are estimated values of mispricing defined as

$$\delta = E\left[\frac{P_t - V_t}{P_t}\right] \approx \delta(180) \quad = \quad -E\left[\sum_{j=1}^{180} M_{t-j,t} \frac{P_{(t-j),t-1}}{P_{(t-j),t-j}} R^e_{(t-j),t}\right],$$

where $(t-j)$ denotes the portfolio formation month and t denotes the month in which returns are realized. In the remaining rows, the reported δ estimates illustrate how the estimated $\delta(J) = -E\left[\sum_{j=1}^{J} M_{t-j,t} \frac{P_{(t-j),t-1}}{P_{(t-j),t-j}} R^e_{(t-j),t}\right]$ changes as J takes values less than 180. We use the SDF implied by the unconditional CAPM, $M_{t-j,t} = \Pi_{s=1}^{j}\left(b_0 - b_1 R^e_{m,t-s}\right)$, where b_0 and b_1 are chosen such that the market portfolio have in-sample α and $\delta(J)$ being zero. This estimate automatically implies δ estimates for the quintile portfolios as well as the difference in δs between the two extreme quintiles. We report bootstrap standard errors that account for time-series and cross-sectional covariances in the data in parentheses and p-values in brackets.

	$100 \times \delta$ (t-statistic)						
J	Lo	2	3	4	Hi	Hi - Lo	p(Hi - Lo)
1mo	0.25	0.05	0.06	-0.04	-0.19	-0.45	0.000
("return")	(3.27)	(0.77)	(1.36)	(-0.95)	(-4.26)	(-3.96)	
1yr	2.20	0.47	0.49	-0.80	-2.24	-4.44	0.001
	(2.48)	(0.55)	(0.86)	(-1.72)	(-3.50)	(-3.19)	
3yrs	3.23	-0.16	0.98	-2.12	-5.33	-8.57	0.042
	(1.33)	(-0.07)	(0.69)	(-1.47)	(-2.57)	(-2.04)	
5yrs	2.78	-1.34	0.43	-3.44	-6.81	-9.58	0.162
	(0.73)	(-0.36)	(0.17)	(-1.57)	(-1.90)	(-1.38)	
10yrs	3.22	-1.76	-0.64	-7.04	-10.28	-13.50	0.311
	(0.42)	(-0.26)	(-0.10)	(-1.81)	(-1.35)	(-0.93)	
15yrs	1.18	-0.48	-2.72	-10.75	-12.06	-13.24	0.449
("price")	(0.12)	(-0.05)	(-0.30)	(-1.67)	(-1.09)	(-0.67)	

Table 3: **Pricing B/M-sorted Portfolios: Returns vs. Prices**

The sample period is 1948m6–2019m12. The table shows that the CAPM does an adequate job describing the cross-section of prices of portfolios sorted on the book-to-market equity ratio (the last row) but a poor job describing the cross-section of one-month returns (the first row). We form five value-weighted portfolios by sorting stocks based on the book-to-market ratio. We form these portfolios and track post-formation returns for 15 years. In the first "return" row, δ measures -1 times the average one-month abnormal return (over the gross one-month risk-free rate):

$$\delta(1) \quad = \quad -E\left[M_t R^e_t\right].$$

Positive (negative) δ here means negative (positive) one-month abnormal return. The reported δs in the last row are estimated values of mispricing defined as

$$\delta = E\left[\frac{P_t - V_t}{P_t}\right] \approx \delta(180) \quad = \quad -E\left[\sum_{j=1}^{180} M_{t-j,t} \frac{P_{(t-j),t-1}}{P_{(t-j),t-j}} R^e_{(t-j),t}\right],$$

where $(t-j)$ denotes the portfolio formation month and t denotes the month in which returns are realized. In the remaining rows, the reported δ estimates illustrate how the estimated $\delta(J) = -E\left[\sum_{j=1}^{J} M_{t-j,t} \frac{P_{(t-j),t-1}}{P_{(t-j),t-j}} R^e_{(t-j),t}\right]$ changes as J takes values less than 180. We use the SDF implied by the unconditional CAPM, $M_{t-j,t} = \Pi_{s=1}^{j}\left(b_0 - b_1 R^e_{m,t-s}\right)$, where b_0 and b_1 are chosen such that the market portfolio have in-sample α and $\delta(J)$ being zero. This estimate automatically implies δ estimates for the quintile portfolios as well as the difference in δs between the two extreme quintiles. We report bootstrap standard errors that account for time-series and cross-sectional covariances in the data in parentheses and p-values in brackets.

	$100 \times \delta$ (t-statistic)						
J	Lo	2	3	4	Hi	Hi - Lo	p(Hi - Lo)
1mo	0.04	0.00	-0.13	-0.13	-0.25	-0.29	0.035
("return")	(0.78)	(0.09)	(-2.10)	(-1.54)	(-2.56)	(-2.10)	
1yr	0.38	-0.22	-1.51	-1.87	-3.05	-3.43	0.048
	(0.54)	(-0.40)	(-1.76)	(-1.71)	(-2.58)	(-1.97)	
3yrs	-0.14	-0.84	-3.52	-6.02	-8.36	-8.22	0.119
	(-0.06)	(-0.56)	(-1.53)	(-1.98)	(-2.50)	(-1.54)	
5yrs	-0.28	-0.99	-4.80	-9.48	-12.64	-12.35	0.169
	(-0.07)	(-0.41)	(-1.44)	(-2.10)	(-2.29)	(-1.34)	
10yrs	0.45	-2.69	-7.14	-15.79	-18.91	-19.37	0.229
	(0.05)	(-0.58)	(-0.98)	(-1.77)	(-1.92)	(-1.12)	
15yrs	1.22	-4.54	-11.78	-20.41	-22.01	-23.24	0.277
("price")	(0.09)	(-0.72)	(-1.11)	(-1.49)	(-1.65)	(-0.95)	

Table 4: **Pricing Quality-and-B/M-sorted Portfolios**

The sample period is 1948m6–2019m12. The table shows that mispricing relative to the CAPM is large for portfolios double-sorted on quality and the book-to-market equity ratio. We form 25 value-weighted portfolios by sorting stocks based on quality within each book-to-market equity quintile. We form portfolios and track post-formation returns for 15 years.. The reported δs are estimated values of mispricing defined as $\delta = E\left[\frac{P_t - V_t}{P_t}\right] \approx \delta(180), = -E\left[\sum_{j=1}^{180} M_{t-j,t} \frac{P_{(t-j),t-1}}{P_{(t-j),t-j}} R^e_{(t-j),t}\right]$, where $(t-j)$ denotes the portfolio formation month and t denotes the month in which returns are realized. We use the SDF implied by the unconditional CAPM, $M_t = b_0 - b_1 R^e_{m,t}$, where b_0 and b_1 are chosen such that the market portfolio have in-sample α and $\delta(J)$ being zero. We report bootstrap standard errors that account for time-series and cross-sectional covariances in the data in parentheses and p-values in brackets.

	$100 \times \delta$ (t-statistic) [p-value]					
	Quality					
Book-to-market	Lo	2	3	4	Hi	Hi - Lo
Lo	14.8 (1.97)	11.0 (1.17)	2.2 (0.19)	-4.0 (-0.26)	-7.4 (-0.36)	-22.1 (-1.15), [0.210]
2	10.4 (1.27)	6.1 (0.72)	-0.2 (-0.02)	-14.5 (-1.74)	-22.5 (-2.04)	-32.9 (-2.07), [0.038]
3	3.9 (0.32)	-1.3 (-0.12)	-9.0 (-0.73)	-19.6 (-1.63)	-28.7 (-2.14)	-32.6 (-2.12), [0.039]
4	-1.7 (-0.15)	-16.3 (-1.40)	-20.9 (-1.51)	-25.6 (-1.68)	-34.6 (-1.58)	-32.9 (-1.68), [0.074]
Hi	-9.6 (-0.70)	-17.3 (-1.39)	-21.5 (-1.43)	-21.3 (-1.55)	-33.5 (-2.02)	-23.9 (-1.63), [0.083]
Hi - Lo	-24.3 (-1.40), [0.116]	-28.3 (-1.56), [0.095]	-23.7 (-0.99), [0.269]	-17.4 (-0.69), [0.438]	-26.1 (-0.79), [0.352]	

Cheap quality − expensive junk (HH-LL)	
$100 \times (\delta_{HH} - \delta_{LL})$	-48.2
(t-statistic)	(-2.31)
[p-value]	[0.033]

Table 5: **Pricing Anomaly-sorted Portfolios**

The sample period is 1948m6–2019m12 except for profitability, investment, and accruals, which have a sample period of 1972m6–2019m12. The table reports estimated mispricings with respect to the CAPM for portfolios sorted on prominent return anomaly characteristics or characteristics conceptually linked to mispricing. For each characteristic, we form five value-weighted portfolios and track post-formation returns for 15 years. Quintile Hi (Lo) denotes stocks with the highest (lowest) value of the characteristic. The reported δs are estimated values of mispricing defined as $\delta = E\left[\frac{P_t - V_t}{P_t}\right] \approx \delta(180), = -E\left[\sum_{j=1}^{180} M_{t-j,t} \frac{P_{(t-j),t-1}}{P_{(t-j),t-j}} R^e_{(t-j),t}\right]$, where $(t-j)$ denotes the portfolio formation month and t denotes the month in which returns are realized. We use the SDF implied by the unconditional CAPM, $M_t = b_0 - b_1 R^e_{m,t}$, where b_0 and b_1 are chosen such that the market portfolio has an in-sample α and $\delta(J)$ being zero. We report bootstrap standard errors that account for time-series and cross-sectional covariances in the data in parentheses and p-values in brackets.

	$100 \times \delta$ (t-statistic)						
Sort	Lo	2	3	4	Hi	Hi - Lo	p(Hi - Lo)
Quality-to-price	11.13	0.44	-6.58	-22.07	-29.79	-40.92	0.026
	(1.82)	(0.07)	(-0.73)	(-2.30)	(-1.97)	(-2.47)	
Size	-17.77	-22.00	-18.64	-13.15	-3.29	14.47	0.394
	(-1.22)	(-1.65)	(-1.60)	(-1.95)	(-0.59)	(0.75)	
Momentum	-14.29	-12.08	-9.67	-5.39	-0.67	13.62	0.067
	(-2.07)	(-2.48)	(-2.02)	(-1.18)	(-0.14)	(1.70)	
Profitability	-22.53	-14.94	-14.93	-3.10	4.33	26.86	0.256
	(-1.32)	(-1.44)	(-1.65)	(-0.35)	(0.28)	(0.88)	
Investment	-20.19	-21.02	-11.00	0.38	6.44	26.63	0.069
	(-2.27)	(-2.34)	(-1.64)	(0.05)	(0.68)	(1.71)	
Net issuance	-34.05	-20.76	-9.57	0.21	-4.81	29.25	0.023
	(-2.39)	(-1.82)	(-1.40)	(0.03)	(-0.82)	(2.37)	
Beta	-30.64	-23.45	-11.52	3.04	14.01	44.65	0.050
	(-2.23)	(-2.46)	(-1.43)	(0.32)	(1.40)	(2.02)	
Accruals	-3.73	-19.34	-7.56	-3.04	9.48	13.22	0.330
	(-0.42)	(-2.03)	(-0.79)	(-0.37)	(1.07)	(0.92)	

Table 6: **Quality-to-Price Ratio as the Prime Predictor of Mispricing**

The sample period is 1948m6–2019m12 except for rows three, four, five, and seven which are limited to 1972m6–2019m12 because of data availability. The table reports estimates of CAPM mispricing for portfolios formed by sorting stocks into three by three portfolios based on independent NYSE breakpoints for the quality-to-price ratio and the particular characteristic listed in column one. The quality-to-price ratio combines quality and book-to-market by taking the sum of their z scores. The left-hand side of the table reports the δ and associated t-statistic for each of the nine portfolios. The right-hand-side of the table reports the δs associated with the combination of the nine portfolios that results in either a quality-to-price-neutral portfolio that bets on the second characteristic or a characteristic-neutral portfolio that bets on the quality-to-price ratio. We report p-values in brackets.

	Quality-to-price ratio sort									Q/P neutral	Second sort neutral
	1 (Low)			2			3 (High)			$\frac{1}{3}*((13+23+33)$	$\frac{1}{3}*((31+32+33)$
Second sort →	1	2	3	1	2	3	1	2	3	$-(11+21+31))$	$-(11+12+13))$
Second characteristic											
Size	-8.05	-5.40	10.40	-22.71	-21.83	-5.64	-35.62	-27.86	-28.67	14.16	-29.70
	(-0.44)	(-0.57)	(1.72)	(-1.74)	(-2.44)	(-0.64)	(-2.39)	(-2.39)	(-1.87)	(0.83), [0.355]	(-3.15), [0.011]
Momentum	2.02	6.93	10.45	-19.04	-9.33	-2.84	-32.60	-29.79	-23.91	11.11	-35.24
	(0.37)	(1.05)	(2.05)	(-3.04)	(-1.32)	(-0.34)	(-2.32)	(-2.28)	(-1.90)	(1.94), [0.047]	(-2.68), [0.020]
Profitability	-3.98	9.26	15.28	-25.40	-13.81	3.54	-38.43	-36.70	-34.88	17.25	-43.53
	(-0.34)	(0.82)	(1.35)	(-1.73)	(-1.82)	(0.18)	(-1.73)	(-2.39)	(-2.11)	(0.95), [0.226]	(-2.76), [0.019]
Investment	-1.09	6.56	13.08	-19.16	-3.95	-0.25	-37.42	-39.80	-31.67	12.95	-42.48
	(-0.14)	(0.77)	(1.65)	(-2.05)	(-0.31)	(-0.02)	(-2.26)	(-2.24)	(-1.85)	(1.39), [0.121]	(-2.70), [0.020]
Net issuance	12.49	-12.99	8.77	-17.75	-11.83	-4.16	-42.48	-25.63	-33.57	6.26	-36.65
	(0.65)	(-0.59)	(1.82)	(-1.48)	(-0.72)	(-0.55)	(-2.10)	(-1.58)	(-2.18)	(0.64), [0.450]	(-2.00), [0.043]
Beta	-13.70	0.67	20.08	-26.71	-7.01	3.24	-45.20	-34.18	-23.13	28.60	-36.52
	(-1.15)	(0.09)	(2.23)	(-2.35)	(-0.55)	(0.28)	(-2.88)	(-1.91)	(-0.90)	(1.68), [0.081]	(-2.30), [0.030]
Accruals	2.53	8.04	18.51	0.02	-10.79	-0.85	-37.06	-37.44	-32.92	6.42	-45.50
	(0.27)	(1.17)	(1.60)	(0.00)	(-0.74)	(-0.07)	(-1.86)	(-2.34)	(-2.31)	(0.51), [0.586]	(-2.90), [0.018]

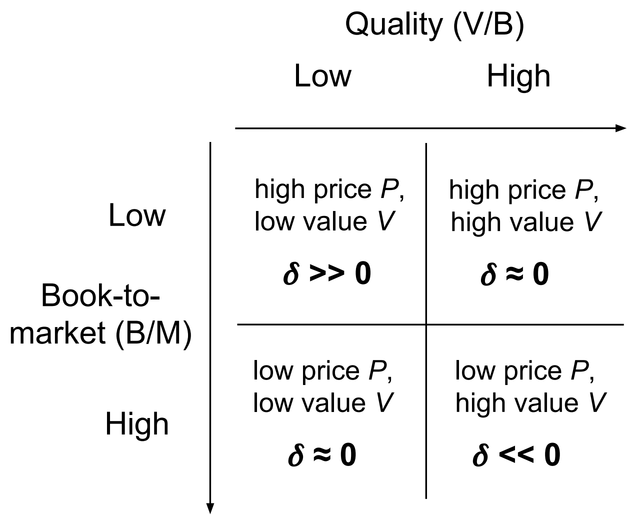

Figure 1: **Double Sorting on Quality and B/M: Illustration**

This diagram illustrates how a double sort on quality and the book-to-market equity ratio should generate a large cross-sectional variation in mispricing δ if quality proxies for the intrinsic value-to-book equity ratio. Quality is a composite measure that proxies for the ratio of intrinsic value-to-book ratio.

Figure 2: **Explaining Returns vs. Prices: Quality and Book-to-Market**

The sample period is 1948m6–2019m12. The left panels report the cross-sectional relation between realized mean excess return and mean excess return predicted by the CAPM for 5 quality- or B/M-sorted sorted portfolios. The right panels report the cross-sectional relation between estimated price levels and price-level risk with respect to the CAPM. The right panels are scaled to have comparable units to one-month returns: Scaled price level (scaled price-level risk) is $-1\times$price level (price-level risk) divided by the estimated $\sum_{j=1}^{180} E_T\left[M_{t-j,t}\frac{P_{(t-j),t-1}}{P_{(t-j),t-j}}\right]$ of the market portfolio, where price level and price-level risk correspond to the portfolio's $-\sum_{j=1}^{180} E_T\left[M_{t-j,t}\frac{P_{(t-j),t-1}}{P_{(t-j),t-j}}\right]E_T\left[R^e_{(t-j),t}\right]$ and $\sum_{j=1}^{180} Cov_T\left(-M_{t-j,t}\frac{P_{(t-j),t-1}}{P_{(t-j),t-j}}, R^e_{(t-j),t}\right)$ such that the estimated mispricing is the sum of the two expressions. The 45-degree dash line indicates the perfect cross-sectional relation between risk and discount rates.

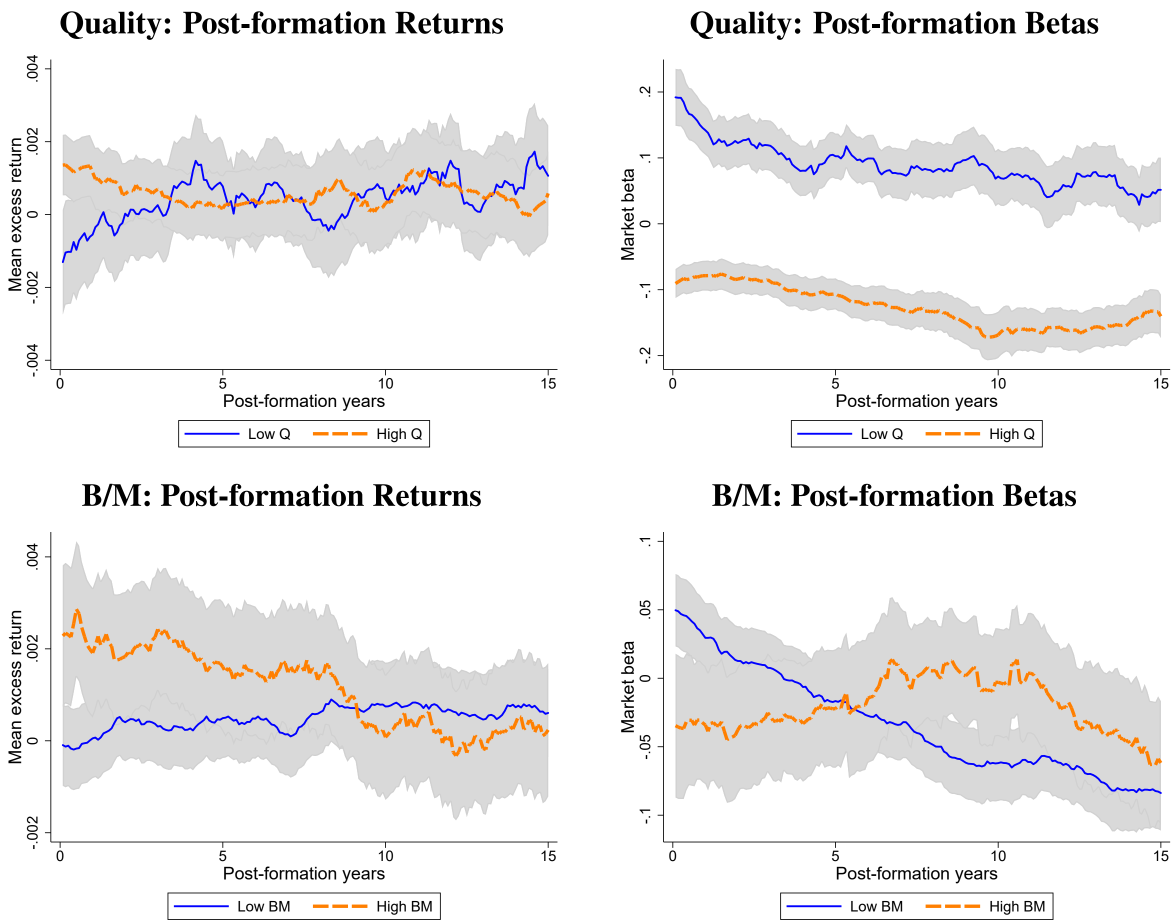

Figure 3: **Post-formation Behavior of Return and Risk: Quality and B/M**

The sample period is 1948m6–2019m12. The left panel reports the post-formation mean excess returns of the two extreme quintile portfolios, and the right panel reports their post-formation market betas. Excess return is taken over the market return. The extreme quintile portfolios are formed by sorting stocks based on quality or the book-to-market equity ratio, and we track post-formation returns over the subsequent 15 years.

Online Appendix to “Putting the Price in Asset Pricing”

A Additional Tables and Figures

Table B1: **Post-formation Returns: Illustration**

The table describes our three-dimensional data structure. Our data consist of overlapping samples of returns over 180 post-formation months on portfolios formed in 1038 different months (1933m6–2019m11) for 5 different characteristic quintiles. However, our estimation takes the calendar-time approach that leads to minimal serial correlations.

Portfolio	Number of post-formation months j	Formation month t	Calendar month $t+j$	Return	Capital gain
BM1	1	1933m6	1933m7	...	...
BM1	1	1933m7	1933m8	...	...
$\vdots$	$\vdots$	$\vdots$	$\vdots$		
BM1	1	2019m11	2019m12	...	...
BM1	2	1933m7	1933m8	...	...
$\vdots$	$\vdots$	$\vdots$	$\vdots$		
BM1	180	2004m12	2019m12	...	...
BM2	1	1933m6	1933m7	...	...
$\vdots$	$\vdots$	$\vdots$	$\vdots$		
BM5	180	2004m12	2019m12	...	...

Table B2: **SDF Parameters**

The table reports the estimated parameters of the model SDF, $b_0 - b_1 R^e_{m,t}$, we use for basic risk adjustment. We estimate the parameters by requiring it to explain perfectly the one-period market returns and requiring the market to have $\delta(J) = 0$ in sample. In the bracket is the 95% confidence interval based on bootstrap.

J	b_0	b_1
1mo	1.020	3.598
	[1.005,1.046]	[2.019,5.493]
1yr	1.016	3.104
	[1.003,1.037]	[1.850,4.706]
3yrs	1.015	3.032
	[1.003,1.037]	[1.829,4.801]
5yrs	1.015	3.057
	[1.003,1.039]	[1.838,4.958]
10yrs	1.014	2.830
	[1.003,1.036]	[1.729,4.788]
15yrs	1.013	2.776
	[1.002,1.035]	[1.662,4.748]

Table B3: **Pricing Quality-sorted Portfolios: Returns vs. Prices (Modern Subsample)**

The sample period is 1972m6–2019m12. The table shows that the CAPM does a good job describing the cross-section of prices of portfolios sorted on quality (the last row) but a poor job describing the cross-section of one-month returns (the first row). Quality is the composite metric introduced by Asness, Frazzini, and Pedersen (2019) to proxy for the ratio of intrinsic value-to-book ratio (V/B). We form five value-weighted portfolios by sorting stocks based on quality. We form these portfolios and track post-formation returns for 15 years. In the first "return" row, δ measures -1 times the average one-month abnormal return (over the gross one-month risk-free rate):

$$\delta(1) \quad = \quad -E\left[M_t R_t^e\right].$$

Positive (negative) δ here means negative (positive) one-month abnormal return. The reported δs in the last row are estimated values of mispricing defined as

$$\delta = E\left[\frac{P_t - V_t}{P_t}\right] \approx \delta(180) \quad = \quad -E\left[\sum_{j=1}^{180} M_{t-j,t} \frac{P_{(t-j),t-1}}{P_{(t-j),t-j}} R^e_{(t-j),t}\right],$$

where $(t-j)$ denotes the portfolio formation month and t denotes the month in which returns are realized. In the remaining rows, the reported δ estimates illustrate how the estimated $\delta(J) = -E\left[\sum_{j=1}^{J} M_{t-j,t} \frac{P_{(t-j),t-1}}{P_{(t-j),t-j}} R^e_{(t-j),t}\right]$ changes as J takes values less than 180. We use the SDF implied by the unconditional CAPM, $M_{t-j,t} = \Pi_{s=1}^{j}\left(b_0 - b_1 R^e_{m,t-s}\right)$, where b_0 and b_1 are chosen such that the market portfolio have in-sample α and $\delta(J)$ being zero. This estimate automatically implies δ estimates for the quintile portfolios as well as the difference in δs between the two extreme quintiles. We report bootstrap standard errors that account for time-series and cross-sectional covariances in the data in parentheses and p-values in brackets.

	$100 \times \delta$ (t-statistic)						
J	Lo	2	3	4	Hi	Hi - Lo	p(Hi - Lo)
1mo	0.23	-0.01	0.02	-0.05	-0.16	-0.38	0.009
("return")	(2.26)	(-0.19)	(0.45)	(-0.96)	(-2.73)	(-2.66)	
1yr	2.02	-0.40	-0.09	-1.07	-1.69	-3.71	0.035
	(1.87)	(-0.36)	(-0.14)	(-1.82)	(-2.12)	(-2.15)	
3yrs	3.10	-1.59	-0.29	-3.11	-4.24	-7.34	0.154
	(1.04)	(-0.53)	(-0.17)	(-1.86)	(-1.67)	(-1.42)	
5yrs	2.20	-3.22	-1.78	-4.68	-5.40	-7.59	0.359
	(0.48)	(-0.68)	(-0.61)	(-1.88)	(-1.22)	(-0.90)	
10yrs	0.67	-4.79	-4.69	-8.63	-8.02	-8.69	0.571
	(0.08)	(-0.59)	(-0.62)	(-2.00)	(-0.85)	(-0.51)	
15yrs	-2.05	-3.98	-7.64	-12.87	-9.50	-7.45	0.718
("price")	(-0.17)	(-0.34)	(-0.68)	(-1.77)	(-0.66)	(-0.30)	

Table B4: **Pricing B/M-sorted Portfolios: Returns vs. Prices (Modern Subsample)**

The sample period is 1972m6–2019m12. The table shows that the CAPM does an adequate job describing the cross-section of prices of portfolios sorted on the book-to-market equity ratio (the last row) but a poor job describing the cross-section of one-month returns (the first row). We form ten value-weighted portfolios by sorting stocks based on the book-to-market ratio. We form these portfolios and track post-formation returns for 15 years. In the first "return" row, δ measures -1 times the average one-month abnormal return (over the gross one-month risk-free rate):

$$\delta(1) \quad = \quad -E\left[M_t R_t^e\right].$$

Positive (negative) δ here means negative (positive) one-month abnormal return. The reported δs in the last row are estimated values of mispricing defined as

$$\delta = E\left[\frac{P_t - V_t}{P_t}\right] \approx \delta(180) \quad = \quad -E\left[\sum_{j=1}^{180} M_{t-j,t} \frac{P_{(t-j),t-1}}{P_{(t-j),t-j}} R^e_{(t-j),t}\right],$$

where $(t-j)$ denotes the portfolio formation month and t denotes the month in which returns are realized. In the remaining rows, the reported δ estimates illustrate how the estimated $\delta(J) = -E\left[\sum_{j=1}^{J} M_{t-j,t} \frac{P_{(t-j),t-1}}{P_{(t-j),t-j}} R^e_{(t-j),t}\right]$ changes as J takes values less than 180. We use the SDF implied by the unconditional CAPM, $M_{t-j,t} = \Pi_{s=1}^{j}\left(b_0 - b_1 R^e_{m,t-s}\right)$, where b_0 and b_1 are chosen such that the market portfolio have in-sample α and $\delta(J)$ being zero. This estimate automatically implies δ estimates for the quintile portfolios as well as the difference in δs between the two extreme quintiles. We report bootstrap standard errors that account for time-series and cross-sectional covariances in the data in parentheses and p-values in brackets.

	$100 \times \delta$ (t-statistic)						
J	Lo	2	3	4	Hi	Hi - Lo	p(Hi - Lo)
1mo	0.08	-0.07	-0.13	-0.16	-0.27	-0.35	0.054
("return")	(1.10)	(-1.17)	(-1.57)	(-1.41)	(-2.21)	(-1.95)	
1yr	0.80	-1.03	-1.54	-2.28	-3.73	-4.53	0.037
	(0.93)	(-1.49)	(-1.36)	(-1.65)	(-2.63)	(-2.12)	
3yrs	0.97	-2.57	-3.81	-7.38	-10.26	-11.23	0.070
	(0.34)	(-1.41)	(-1.30)	(-2.02)	(-2.76)	(-1.81)	
5yrs	2.16	-4.05	-6.47	-11.53	-15.70	-17.86	0.094
	(0.43)	(-1.45)	(-1.53)	(-2.14)	(-2.54)	(-1.66)	
10yrs	4.91	-7.43	-10.86	-20.46	-23.76	-28.66	0.132
	(0.46)	(-1.40)	(-1.22)	(-1.95)	(-2.09)	(-1.39)	
15yrs	6.66	-10.08	-17.22	-26.33	-27.90	-34.55	0.176
("price")	(0.39)	(-1.34)	(-1.28)	(-1.59)	(-1.76)	(-1.13)	

Table B5: **Pricing Quality-and-B/M-sorted Portfolios (Modern Subsample)**

The sample period is 1972m6–2019m12. The table shows that mispricing relative to the CAPM is large for portfolios double-sorted on quality and the book-to-market equity ratio. We form 25 value-weighted portfolios by sorting stocks based on quality within each book-to-market equity quintile. We form portfolios and track post-formation returns for 15 years. The reported δs are estimated values of mispricing defined as $\delta = E\left[\frac{P_t - V_t}{P_t}\right] \approx \delta(180), = -E\left[\sum_{j=1}^{180} M_{t-j,t} \frac{P_{(t-j),t-1}}{P_{(t-j),t-j}} R^e_{(t-j),t}\right]$, where $(t-j)$ denotes the portfolio formation month and t denotes the month in which returns are realized. We use the SDF implied by the unconditional CAPM, $M_t = b_0 - b_1 R^e_{m,t}$, where b_0 and b_1 are chosen such that the market portfolio have in-sample α and $\delta(J)$ being zero. We report bootstrap standard errors that account for time-series and cross-sectional covariances in the data in parentheses and p-values in brackets.

	$100 \times \delta$ (t-statistic) [p-value]					
	Quality					
Book-to-market	Lo	2	3	4	Hi	Hi - Lo
Lo	16.5 (1.79)	13.9 (1.12)	5.7 (0.38)	0.4 (0.02)	2.5 (0.09)	-14.0 (-0.56), [0.509]
2	8.3 (0.84)	1.3 (0.12)	-5.3 (-0.51)	-23.0 (-2.33)	-30.3 (-2.28)	-38.7 (-2.10), [0.045]
3	0.4 (0.03)	-4.1 (-0.29)	-15.8 (-0.99)	-26.0 (-1.81)	-37.1 (-2.31)	-37.5 (-2.05), [0.040]
4	-5.0 (-0.39)	-23.2 (-1.59)	-28.6 (-1.59)	-32.3 (-1.80)	-39.0 (-1.52)	-34.0 (-1.53), [0.105]
Hi	-13.7 (-0.85)	-24.3 (-1.68)	-30.8 (-1.68)	-26.1 (-1.53)	-38.5 (-2.02)	-24.8 (-1.47), [0.116]
Hi - Lo	-30.2 (-1.40), [0.115]	-38.2 (-1.69), [0.076]	-36.5 (-1.22), [0.154]	-26.6 (-0.84), [0.322]	-41.0 (-1.00), [0.221]	

	Cheap quality − expensive junk (HH-LL)	
	$100 \times (\delta_{HH} - \delta_{LL})$ (t-statistic) [p-value]	-55.0 (-2.29) [0.032]

Table B6: **Quantitative Analysis: Paramter Selection**

The table reports the parameter values used in the quantitative analysis of delta and its approximations. $\hat{g}_*$, $\hat{\sigma}_g$, and $\hat{\rho}_g$ in the data are calculated by first computing g_t as the 12-month moving average of $0.5 \times ROE_t$, where 0.5 is an approximate steady-state plowback ratio, and taking the mean, standard deviation, and serial correlation of g_t. All values are monthly.

Panel A. Common parameters			
R_f	0.004		
g_*	0.005	π_*	0.003
σ_g	0.003	σ_π	0.002
σ_x	0.003	σ_ω	$\sigma_x/100$
σ_e	0.495		
ρ_g	0.9	ρ_π	0.9
ρ_x	0.96	ρ_ω	0.9
$\widetilde{g}_0$	0	$\widetilde{\pi}_0$	0
$\widetilde{\omega}_0$	0		
Panel B. Parameters that differ across analyses			
	No permanent mispricing	Permanent return distortion	Permanent risk distortion
x_*	0	$[-0.6, 0.1]$	$[-0.25, 0.7]$
$\tilde{x}_0$	$[-14, 29]$	0	0
ω_*	0	0	$x_*/20$

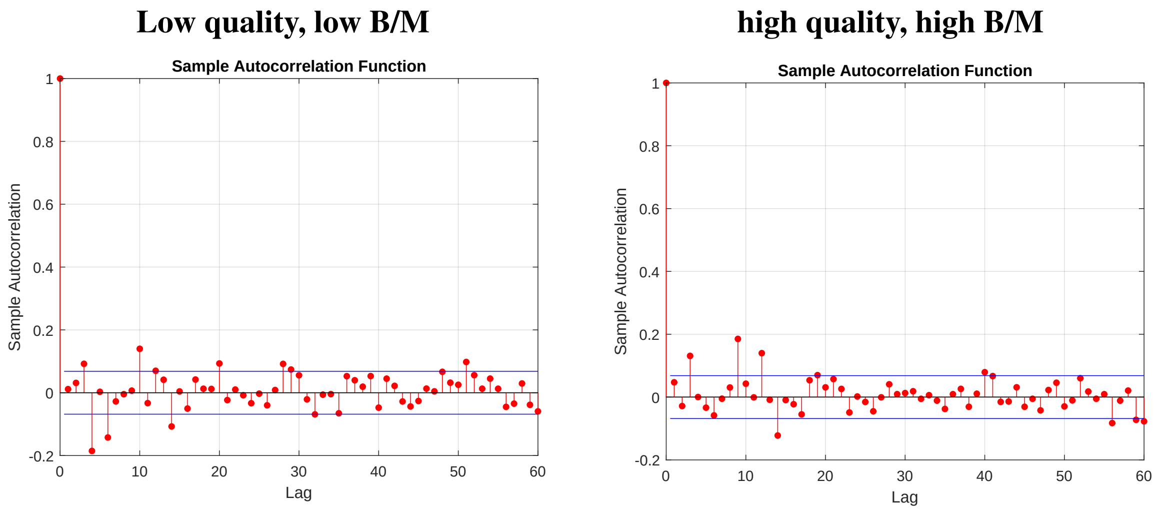

Figure B1: **Calendar-time Delta: Autocorrelation**

The figure plots the autocorrelation function of the two extreme quality and B/M double sorted portfolios. They show that the autocorrelations are small, especially after 12 lags. The blue lines denote the 2 standard error bounds.

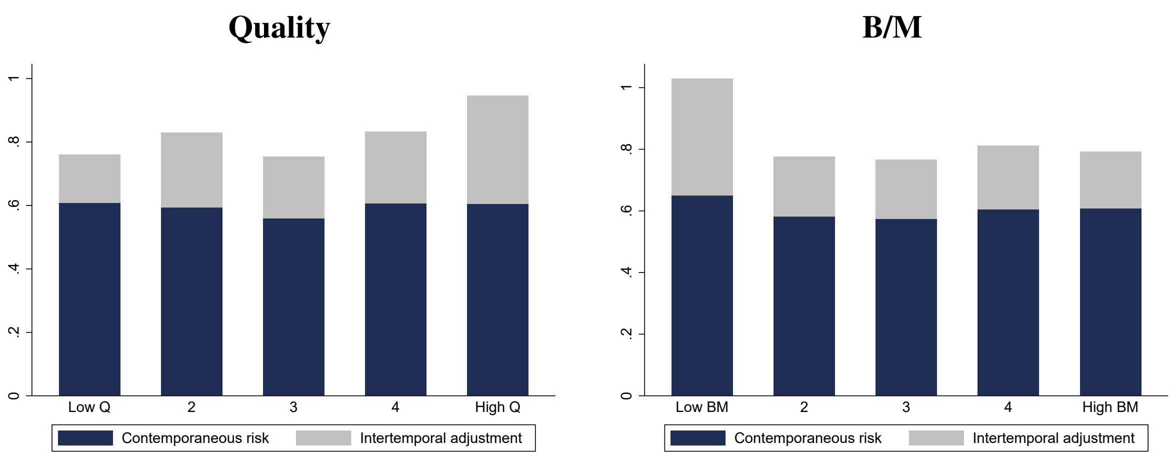

Figure B2: **Decomposing Price-level Risk: Quality and B/M**

These figures decomposes price-level risk into its contemporaneous and intertemporal components:

$$\sum_{j=1}^{180} Cov_T\left(-M_{t-j,t}\frac{P_{(t-j),t-1}}{P_{(t-j),t-j}}, R^e_{(t-j),t}\right) = \underbrace{\sum_{j=1}^{180} E_T\left[\phi_{t-j,t-1}\right] Cov_T\left(-M_t, R^e_{(t-j),t}\right)}_{\text{Contemporaneous risk}}$$

$$\underbrace{-\sum_{j=1}^{180} E_T\left[\left(\phi_{t-j,t-1} - E_{t-j}\left[\phi_{t-j,t-1}\right]\right) M_t\left(R^e_{(t-j),t} - E_{t-j}\left[R^e_{(t-j),t}\right]\right)\right]}_{\text{Intertemporal adjustment}},$$

where $(t-j)$ denotes the portfolio formation period and $\phi_{t-j,t-1} = M_{t-j,t}P_{(t-j),t-1}/P_{(t-j),t-j}$. Although contemporaneous risk is a large fraction of the overall price-level risk, both the intertemporal component appears more important for the cross-sectional variation in price-level risk.

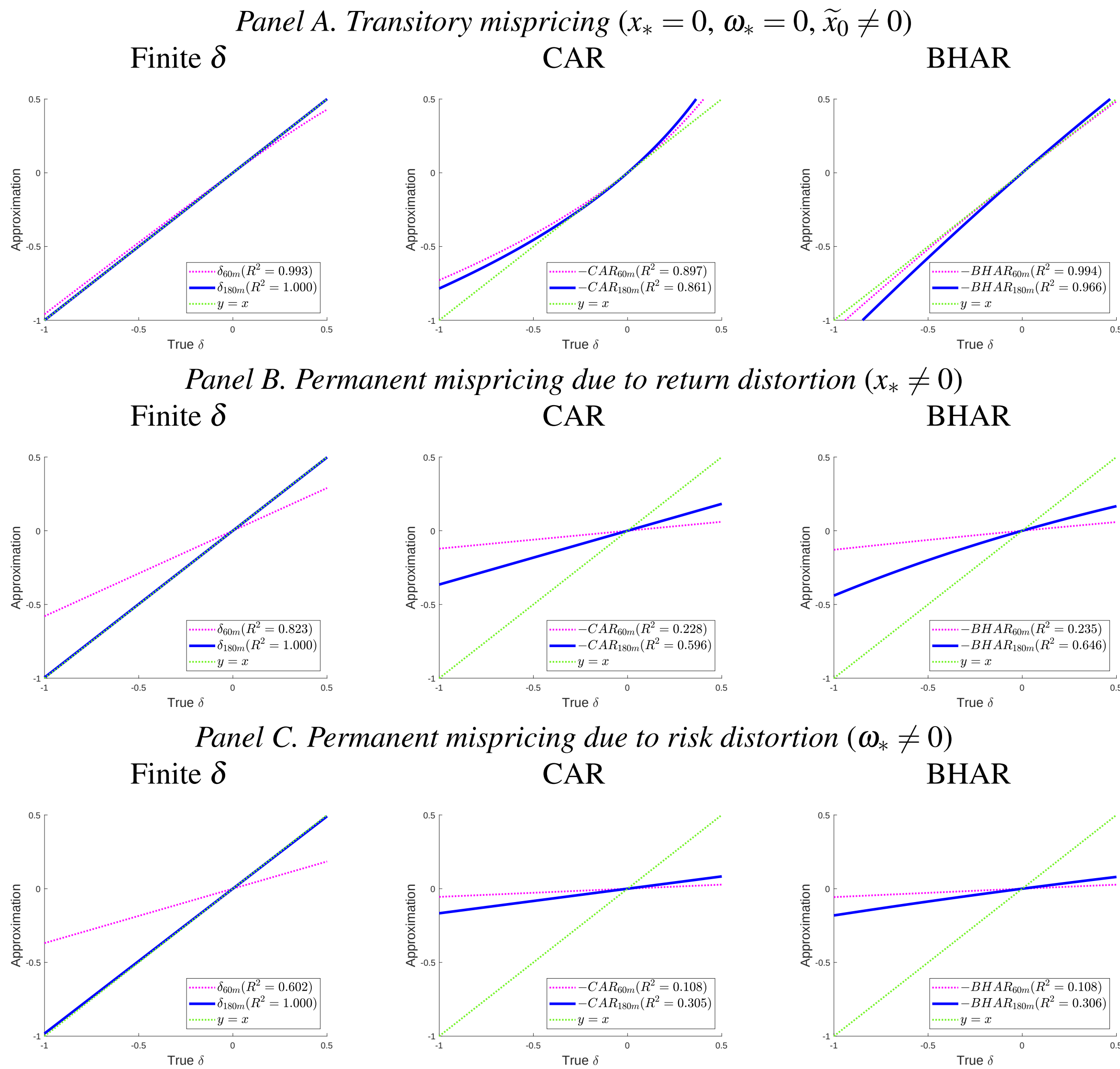

Figure B3: **Quantitative Comparison of Mispricing Metrics**

These figures compare the ability of a finite δ, CAR, and BHAR to proxy for the true δ using the quantitative model specified in Section D.3.1 and parameter values in Table B6. Finite δ based on 180 post-formation months, defined as $\delta(180) = -\sum_{j=1}^{180} E\left[M_{t,t+j}\frac{P_{t+j-1}}{P_t}R^e_{t+j}\right]$ is a near-perfect proxy for the actual δ defined as $\delta = E\left[\frac{P_t - V_t}{P_t}\right] = -\sum_{j=1}^{\infty} E\left[M_{t,t+j}\frac{P_{t+j-1}}{P_t}R^e_{t+j}\right]$. This is true even when δ does not converge to zero but to a nonzero steady-state value due to expected return distortion (Panel B) or risk distortion (Panel C). When there is no permanent mispricing, the negative of CAR and BHAR based on J post-formation months, defined respectively as $-CAR(J) = -\sum_{j=1}^{J} E[\alpha_{t+J}]$ and $-BHAR(J) = -E\left[\Pi_{j=1}^{J}(1+\alpha_{t+j}) - 1\right]$, can sometimes be a decent proxy for the true δ. However, they deviate substantially from the true δ in the presence of even a small amount of permanent mispricing.

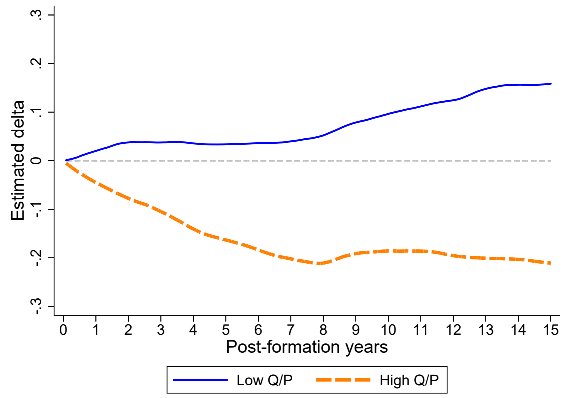

Figure B4: **Estimated δ by the Choice of Finite Post-formation Periods J**

The figure plots the values of δ estimated based on J post-formation years for the extreme quintile quality-to-price portfolios, where J is the horizontal axis. Sample period is 1948m6–2019m12.

B Data Appendix

B.1 Basic data adjustments

We use domestic common stocks (CRSP share code 10 or 11) listed on the three major exchanges (CRSP exchange code 1, 2, or 3). We replace missing prices with the average bid-ask price when available and drop observations with missing share or price information in the previous month. We code missing returns as zero returns and add delisting returns to returns. If delisting returns are missing but the CRSP delisting code is 500 or between 520 and 584, we use -35% (-55%) as the delisting returns for NYSE and AMEX stocks (for NASDAQ stocks) (Shumway (1997) and Shumway and Warther (1999)). To compute capital gains, we use the CRSP split-adjustment factor (CFACPR) to ensure that capital gains are not affected by split events. The market factor is downloaded from Kenneth French's website.

We use NYSE breakpoints when sorting stocks throughout the analysis and always study value-weight portfolios. Still, to prevent market microstructure issues from affecting our results, we exclude microcaps defined as market equity below the bottom 10% NYSE size cutoff when forming stock portfolios. However, to avoid a look-ahead bias, if after portfolio formation a stock's size falls below the 10 percentile NYSE cutoff, we still keep it in the portfolio. This also applies to the size portfolios; we form 5 portfolios sorted by size using stocks that exclude microcaps.

B.2 Characteristics and portfolios

An important stock characteristic with which we form portfolios is the quality measure of Asness, Frazzini, and Pedersen (2019) defined as a z-score based on four characteristics—profitability, growth, safety, and payout ratio—that determine the market-to-book ratio in a Gordon growth model and in the absence of mispricing: quality $= z\left(z_{\text{profitability}} + z_{\text{growth}} + z_{\text{safety}} + z_{\text{payout ratio}}\right)$. The four characteristic z scores are in turn obtained as an equal weighted average of z scores based different measures of each characteristic. When some of the underlying measures are missing, the z score is taken over all available measures. In the pre-Compustat period, we use the book equity numbers that Davis, Fama, and French (2000) collected from the Moody's Industrial, Public Utility, Transportation, and Bank and Finance Manuals to calculate measures that require book equity data. Quality is computed once a year at the end of June. See Asness, Frazzini, and Pedersen (2019) for further details.

Another important stock characteristic is the book-to-market-equity (B/M) ratio computed each year in June. B/M ratio is the stock's book value of equity in the previous fiscal year divided by its market value of equity in December of the previous calendar year. Book value of equity is defined as stockholders' equity SEQ (Compustat item 144) plus balance sheet deferred taxes and investment tax credit $TXDITC$ (item 35) minus book value of preferred stock ($BE = SEQ + TXDITC - BPSTK$). Book value of preferred stock $BPSTK$ equals the preferred stock redemption value $PSTKRV$ (item 56), preferred stock liquidating value $PSTKL$ (item 10), preferred stock $PSTK$ (item 130), or zero depending on data availability. If SEQ is unavailable, we set it equal to total assets AT (item 6) minus total liabilities LT (item 181). If $TXDITC$ is unavailable, it is assumed to be zero. In the pre-Compustat period, we use the book equity data from Davis, Fama, and French (2000). We treat zero or negative book values as missing.

We interact quality and the book-to-market ratio to compute the *quality-to-price* ratio, which Asness, Frazzini, and Pedersen (2019) call "quality at a reasonable price." The idea is that if quality proxies for the value-to-book ratio (V/B), the product of quality and book-to-market (B/M) should give us a proxy for the ratio of intrinsic value to price, which is what $\delta = 1 - V/P$ seeks to estimate. We compute quality-to-price as the z score of the sum of quality and the z score for the log book-to-market ratio.

We also examine portfolios sorted by seven additional characteristics: size, momentum, net issuance, beta, profitability, investment, and accruals. The first four characteristics can be computed in the pre-Compustat period, whereas the last three characteristics are available only in the post-Compustat period. Size is market equity calculated at the end of each month. Momentum is calculated is the cumulative gross return over the previous 12 months excluding the month before the portfolio formation and is also computed at the end of each month. Net issuance is calculated annually at the end of each June and is the split-adjusted growth in shares outstanding over the previous 12 months. Beta is the trailing 3-year market beta (minimum of 2 years) calculated each month based on overlapping 3-day returns.

We define profitability and investment as in Fama and French (2015). Profitability is computed each year in June. Operating profitability ("profitability") in calendar year y is operating profits in fiscal year $y-1$ over book value of equity in fiscal year $y-1$, where operating profits equals sales *SALE* (Compustat item 12) minus cost of goods sold *COGS* (item 41), interest and related expenses *XINT* (item 134) (if available), and selling, general, and administrative expenses *XSGA* (item 132) (if available). Asset growth ("investment") is also computed each year in June, and investment in calendar year y is total assets in fiscal year $y-1$ divided by total assets in fiscal year $y-2$. Accruals measures the degree to which earnings come from non-cash sources and is defined according to Sloan (1996).

C Empirical Appendix

C.1 Empirical support for the 15-year finite δ

Does a finite δ using post-formation returns over 15 years perform well in practice as it does in simulations of the population of data? Recall that to estimate δ using 15-year post-formation returns, we replace the infinite sum in equation (13) with a finite sum over $J = 180$ months (15 years). However, since both the discount factor $M_{t,t+j-1}P_{t+j-1}/P_t$ and the conditional abnormal return α_{t+j} that appear in the expression are likely to decay over post-formation years, the finite sum over 15 post-formation years provides a good approximation of the infinite sum. In particular, Figure B4 shows that the estimated δ based on J post-formation periods plateaus around J =13 years.

D Theory Appendix

D.1 Comparing δ to Other Metrics of Price-level Error

Some readers may be interested in how our price-level measure of mispricing, $\delta_t = \frac{P_t - V_t}{P_t}$, compares to existing measures of mispricing or long-term return.

Market-to-book ratio

The market-to-book-equity ratio, closely related to long-run reversal, is one popular measure of mispricing (De Bondt and Thaler (1985), Rosenberg, Reid, and Lanstein (1985), and Lakonishok, Shleifer, and Vishny (1994)). However, the market-to-book-equity ratio is a highly imperfect measure of mispricing, since factors other than mispricing can be influence the ratio. The decomposition of Vuolteenaho (2002) and Cohen, Polk, and Vuolteenaho (2003) shows that the log market-to-book-equity ratio mb_t is approximately,

$$mb_t = \sum_{j=1}^{\infty} \rho^{j-1} E_t \left[roe_{t+j} \right] - \sum_{j=1}^{\infty} \rho^{j-1} E_t \left[\alpha_{t+j} + \lambda_{M,t+j} \beta_{M,t+j} \right] + \frac{1}{2} \sum_{j=1}^{\infty} \rho^{j-1} Var_t \left(r_{t+j} \right), \quad (30)$$

where roe_{t+j} is the log return on equity, α_{t+j} is abnormal return, $\lambda_{M,t+j}\beta_{M,t+j}$ is the risk premium implied by the SDF M, and r_{t+j} is log return. Hence, besides the distortion in the discount rate due to α_{t+j}, other factors such as earnings growth, risk, and volatility can affect cross-sectional and time-series variation in the market-to-book-equity ratio.

Price-level alpha

Cohen, Polk, and Vuolteenaho (2009) (CPV) were the first to propose an identity for measuring price-level distortions. They define the fundamental value of a stock as the present value of future dividends discounted with the discount factors that would have prevailed in the absence of mispricing:

$$V_t = \sum_{j=1}^{\infty} E_t \left[\frac{D_{t+j}}{\Pi_{s=1}^{j} \left(1 + R_{v,t+s}\right)} \right], \quad (31)$$

where $R_{v,t+s}$ is the return on V_t. They then show that price-level alpha α_t^{price}, defined as the log deviation of price from value, approximately equals

$$\begin{aligned} \alpha_t^{price} &= -\textstyle\sum_{j=1}^{\infty} \rho^{j-1} E_t\left[r_{t+j} - r_{v,t+j}\right] \\ &= -\textstyle\sum_{j=1}^{\infty} \rho^{j-1} E_t\left[R_{t+j} - \lambda_{M,t+j}\beta_{M,v,t+j}\right] + \frac{1}{2}\sum_{j=1}^{\infty} \rho^{j-1}\left(Var_t\left(r_{t+j}\right) - Var_t\left(r_{v,t+j}\right)\right), \end{aligned} \tag{32}$$

where r denotes log return, R denotes simple return, and $\beta_{M,v,t+j}$ is quantity of risk in the absence of mispricing. Hence, if the distortion in the volatility of log return due to mispricing is small,

$$\underbrace{-\sum_{j=1}^{\infty} \rho^{j-1} E_t\left[R_{t+j}\right]}_{\text{CPV price level}} \approx \alpha_t^{price} - \underbrace{\sum_{j=1}^{\infty} \rho^{j-1} E_t\left[\lambda_{M,t+j}\beta_{M,v,t+j}\right]}_{\text{CPV price-level risk}}. \tag{33}$$

CPV use two different methodologies to test the CAPM using a price-level perspective. In their first methodology, CPV use the above relation to run a cross-sectional regression that explains the price level based on price-level risk with respect to the CAPM, using cash-flow betas measured by the exposure of a portfolio's return on equity to the market's return on equity to estimate market betas in the absence of mispricing. If the convexity adjustment is done correctly, this approach could be useful in rejecting the null of no mispricing, since under that null the volatility distortion should be zero, but using their identity to draw a conclusion beyond the rejection of the null is difficult due to the potentially large volatility distortion along with measurement error introduced from estimating $\beta_{M,v,t+j}$ with their cash-flow beta proxy. Our identity addresses this problem by expressing mispricing in terms of observable quantities (other than the SDF loadings).

In their second methodology, CPV instead use a calendar-time approach that studies the monthly returns on trading strategies designed to approximate a buy-and-hold investor's experience in price-level units. This method allows them not only to examine long-horizon effects using CAPM betas rather than cash-flow betas but also to perform their tests at a higher frequency, with more reliable statistical tools. In particular, CPV study the monthly returns on what they call N-year price-to-book deciles. These deciles are portfolio strategies that invest in N-year decile composite portfolios. In particular, each of these composite portfolio deciles are the aggregation of the corresponding decile sorted on year t–1 price-to-book ratios, the corresponding decile sorted on year $t-2$ price-to-book ratios, ... , and the corresponding decile sorted on year t–N price-to-book ratios. To combine the time-t returns on these year t–n sorted component portfolios so that the composite portfolio reflects a buy-and-hold experience, CPV weight each of these different component portfolios with a negative, exponentially-declining weight motivated by Campbell and Shiller. Namely, CPV use weights that are the Campbell-Shiller discount parameter ρ to the power of years from the sort minus one. Like CPV, our calendar-time approach combines the contemporaneous returns on portfolios formed at different look-back dates to avoid the statistical issues created by overlapping returns. In contrast to CPV, we combine those portfolios using weights that depend on the past price-weighted cumulative SDF realization as prescribed by our identity.

The misspecification metric in van Binsbergen and Opp (2019)

van Binsbergen and Opp (2019) introduce a production economy in which they generate several insights about real distortions arising from abnormal returns. An important quantity in their analysis is log abnormal return defined as $\widetilde{\alpha}_t \equiv P_t - E_t\left[M_{t+1}\left(D_{t+1}+P_{t+1}\right)\right]$, where M_{t+1} is a candidate stochastic discount factor. Rewriting this expression, iterating it forward, and imposing a transversality condition implies,

$$P_t = \sum_{j=0}^{\infty} E_t\left[M_{t,t+j+1} e^{-\sum_{s=0}^{j}\widetilde{\alpha}_{t+s}} D_{t+j+1}\right]. \tag{34}$$

This expression, although not the main point of their paper, is useful in our context as follows. If P_t differs from intrinsic value defined as $V_t = \sum_{j=0}^{\infty} E_t\left[M_{t,t+j+1} D_{t+j+1}\right]$ because of frictions, then the above expression implies that the price deviation from value is an aggregation of abnormal returns:

$$P_t - V_t = \sum_{j=0}^{\infty} E_t\left[M_{t,t+j+1} D_{t+j+1}\left(e^{-\sum_{s=0}^{j}\widetilde{\alpha}_{t+s}} - 1\right)\right]. \tag{35}$$

This expression also allows van Binsbergen and Opp (2019) to anticipate that price distortion depends not just on the magnitude of the alphas, but also on their persistence. However, the expression involves the dividend process and is therefore difficult to take to data without putting more structure on how dividends evolve.

Estimating the intrinsic value directly

Another approach to estimating mispricing could be to estimate the intrinsic value directly using dividend data:

$$V_t = \sum_{j=1}^{\infty} E_t \left[M_{t,t+j} D_{t+j} \right]. \tag{36}$$

Estimating intrinsic value in this way is problematic because the truncation of the infinite sum at some finite J would leave out a large fraction of the intrinsic value.[25]

Cumulative abnormal return (CAR)

One popular measure of long-term return is the cumulative abnormal return defined as the simple sum of abnormal returns over a period of time:

$$CAR_t = -\sum_{j=1}^{\infty} E_t \left[\alpha_{t+j} \right] \tag{37}$$

(written with a sign flip so that, like our δ_t, positive abnormal returns means a negative CAR_t). How well can CAR proxy for ex-ante mispricing of the portfolio?

To see how CAR relates to price-level mispricing δ, rewrite equation (14) as

$$\delta_t = -\sum_{j=1}^{\infty} E_t \left[w_{t,t+j} \right] E_t \left[\alpha_{t+j} \right] - \sum_{j=1}^{\infty} Cov_t \left(w_{t,t+j}, \alpha_{t+j} \right), \tag{38}$$

where

$$w_{t,t+j} = M_{t,t+j-1} \frac{P_{t+j-1}}{P_t} \frac{1}{1+R_{f,t+j}}. \tag{39}$$

Hence, CAR is an exact measure of mispricing when $E_t \left[w_{t,t+j} \right] = 1$ and $Cov_t \left(w_{t,t+j}, \alpha_{t+j} \right) = 0$. These conditions would approximately hold, for example, if abnormal return tends to be stable (i.e., conditional abnormal return equals unconditional abnormal return), the gross monthly risk-free rate is close to 1, and the portfolio has a very high duration such that cumulative capital gain approximately equals cumulative return (in which case $E_t \left[M_{t+s} \frac{P_{t+s}}{P_{t+s-1}} \right] \approx E_t \left[M_{t+s} \left(1 + R_{t+s} \right) \right] \approx 1 \Longrightarrow E_t \left[\Pi_{s=1}^{j-1} \left(M_{t+s} \frac{P_{t+s}}{P_{t+s-1}} \right) \right] \approx 1$ when abnormal returns are small and both the SDF and returns exhibit little serial covariance). Nevertheless, Cho (2020) shows that returns on anomaly trad-

[25] For example, suppose V_t follows a Gordon growth model with expected dividend growth rate g and constant discount rate R. $\sum_{j=J+1}^{\infty} E_t \left[D_t \left(1+g \right)^j / \left(1+R \right)^j \right] = \left(\frac{1+g}{1+R} \right)^J E_t \left[D_t \frac{1+g}{R-g} \right] = \left(\frac{1+g}{1+R} \right)^J V_t$. This means, if $g = 5\%$ and $R = 10\%$, about 40% (10%) of V_t comes from dividends occurring 20 (50) years after t.

ing strategies depend importantly on shocks to the capital of arbitrageurs proxied by aggregate funding liquidity and the aggregate arbitrageur's portfolio, suggesting that these conditions are violated.

"Discounted" CAR (DCAR)

Under slightly less strong assumptions than the ones specified for CAR, mispricing δ_t is approximately a discounted sum of subsequent abnormal returns with a simple geometric discount factor. To see this, start with Equation (38) and continue to assume $Cov_t\left(w_{t,t+j},\alpha_{t+j}\right)=0$. Next, note that if monthly risk-free rates are small and both the SDF and returns exhibit little serial covariance,

$$E_t\left[w_{t,t+j}\right]\approx\Pi_{s=1}^{j-1}E_t\left[M_{t+s}\frac{P_{t+s}}{P_{t+s-1}}\right]=\Pi_{s=1}^{j-1}E_t\left[M_{t+s}\left(1+R_{t+s}\right)\frac{1}{1+D_{t+s}/P_{t+s}}\right]. \tag{40}$$

Then, replace $\frac{1}{1+D_{t+s}/P_{t+s}}$ with the Campbell and Shiller (1988) discount factor $\rho=\frac{1}{1+\overline{D/P}}$ (where $\overline{D/P}$ is the long-run average of the dividend-price ratio) and assume that abnormal returns are small ($E_t\left[M_{t+s}\left(1+R_{t+s}\right)\right]\approx 1$) to obtain

$$E_t\left[w_{t,t+j}\right]\approx\rho^{j-1}, \tag{41}$$

where Campbell (2017) suggests using ρ of 0.95–0.96. Hence, under these strong assumptions, we can write mispricing as a sum of subsequent abnormal returns discounted at a constant rate:

$$\delta_t\approx-\sum_{j=1}^{\infty}\rho^{j-1}E_t\left[\alpha_{t+j}\right]. \tag{42}$$

We call this discounted CAR and analyze this potential metric together with simple CAR below.

An ex-post identity for mispricing

Our identity uses the SDF to discount future cash flows, so some readers would wonder whether defining ex-post realized returns as the discount factor yields a similar identity. Such an approach yields an identity that holds both ex-ante and ex-post, but it involves the unobserved return in the absence of mispricing:

$$\delta_t=-\sum_{j=1}^{\infty}\frac{1}{\Pi_{s=1}^{j}\left(1+R_{v,t+s}\right)}\left(R_{t+j}-R_{v,t+j}\right), \tag{43}$$

where R_v is the return on fundamental value V.

D.2 Portfolio δ

In practice, one would typically estimate the δ of a portfolio of stocks, which requires expressing the portfolio δ as a function of post-formation capital gains and returns on the portfolio. These capital gains and returns should be those based on a buy-and-hold strategy that does not rebalance the portfolio (or equivalently, use the original weight times the stock's cumulative capital gain to rebalance the portfolio every month). If $w_{i,t}$ is the portfolio weight on security i at the time of portfolio formation t,

$$\begin{aligned}
\delta_t &= \sum_{i=1}^{N} w_{i,t}\delta_{i,t} \\
&= \sum_{i=1}^{N} w_{i,t}\left(-\sum_{j=1}^{\infty} E_t\left[M_{t,t+j}\frac{P_{i,t+j-1}}{P_{i,t}}R^e_{i,t+j}\right]\right) \\
&= -\sum_{j=1}^{\infty} E_t\left[M_{t,t+j}\sum_{i=1}^{N}\left(w_{i,t}\frac{P_{i,t+j-1}}{P_{i,t}}R^e_{i,t+j}\right)\right] \\
&= -\sum_{j=1}^{\infty} E_t\left[M_{t,t+j}\sum_{i\in N_{t+j}}\left(w_{i,t}\frac{P_{i,t+j-1}}{P_{i,t}}R^e_{i,t+j}\right)\right],
\end{aligned} \tag{44}$$

where N_{t+j} denotes the set of firms surviving (not delisted) at the end of $t+j-1$ and therefore have return data for $t+j$. Hence,

$$\begin{aligned}
\delta_t &= -\sum_{j=1}^{\infty} E_t\left[M_{t,t+j}\left(\sum_{i\in N_{t+j}} w_{i,t}\frac{P_{i,t+j-1}}{P_{i,t}}\right)\left(\sum_{i\in N_{t+j}}\frac{w_{i,t}\frac{P_{i,t+j-1}}{P_{i,t}}R^e_{i,t+j}}{\sum_{i\in N_{t+j}} w_{i,t}\frac{P_{i,t+j-1}}{P_{i,t}}}\right)\right] \\
&= -\sum_{j=1}^{\infty} E_t\left[M_{t,t+j}\frac{P_{t+j-1}}{P_t}R^e_{t+j}\right],
\end{aligned} \tag{45}$$

where

1. we normalize the time t portfolio price P_t to be 1.
2. the buy-and-hold time $t+j-1$ portfolio price is $P_{t+j-1} = \sum_{i\in N_{t+j}} w_{i,t}\frac{P_{i,t+j-1}}{P_{i,t}}$.
3. the buy-and-hold portfolio weight on asset i between $t+j-1$ and $t+j$ is
$$w_{i,t+j} = \frac{w_{i,t}\frac{P_{i,t+j-1}}{P_{i,t}}}{\sum_{i\in N_{t+j}} w_{i,t}\frac{P_{i,t+j-1}}{P_{i,t}}}$$
4. the buy-and-hold portfolio excess return is then given by $R^e_{t+j} = \sum_{i\in N_{t+j}} w_{i,t+j}R^e_{i,t+j}$.

D.3 Quantitative Analysis[26]

This section provides a quantitative model of the intrinsic value and price processes. The intrinsic value process follows a linearity-generating process of Gabaix (2007), and the price process also follows a similar form that yields linearity in both the price-dividend and the value-dividend ratios.

D.3.1 A linearity-generating model of price and intrinsic value

Basic structure. The stochastic discount factor (SDF) follows

$$M_{t+1} = \frac{1+\varepsilon_{M,t+1}}{1+R_f}, \tag{46}$$

The asset has a growth rate of cash flows (dividends)

$$\frac{D_{t+1}}{D_t} = 1 + g_t + \varepsilon_{D,t+1}, \tag{47}$$

where

$$\pi_t = -Cov_t\left(\varepsilon_{M,t+1}, \varepsilon_{D,t+1}\right) \tag{48}$$

is the time-varying risk premium associated with the cash flows. The state variables (g_t, π_t) follow an AR(1) process

$$\begin{aligned} \widetilde{g}_{t+1} &= g_{t+1} - g_* = \xi_t \rho_g \widetilde{g}_t + \varepsilon_{g,t+1} \\ \widetilde{\pi}_{t+1} &= \pi_{t+1} - \pi_* = \xi_t \rho_\pi \widetilde{\pi}_t + \varepsilon_{\pi,t+1}, \end{aligned} \tag{49}$$

where ξ_t is the linearity generating "twist" that preserves linearity of the price-dividend process as a function of the state variables:

$$\xi_t = \frac{1+g_*-\pi_*}{1+g_t-\pi_t}. \tag{50}$$

[26] For the results in this section, we are indebted to Robert Rogers and Ran Shi for the extraordinary skill with which they provided a framework that formed the basis of our quantitative model. Robert Rogers also provided a groundwork for our simulation results by conducting a thorough research on suitable parameter values and generating the first batch of simulation results.

We assume that the shocks $\varepsilon_{\pi,t+1}$ and $\varepsilon_{g,t+1}$ have a zero mean and are uncorrelated with

$$M_{t+1}\frac{D_{t+1}}{D_t}. \tag{51}$$

Mispricing shocks. We consider a particular form of deviation from the correct price. Suppose that the arbitrageur's cost of capital varies over time, generating a wedge in the pricing equation denoted by x_t.

$$1-\frac{D_t}{P_t}\frac{1}{1+R_f}x_t = E_t\left[M_{t+1}\frac{P_{t+1}+D_{t+1}}{P_t}\right] \tag{52}$$

That is, as in Cho (2020), arbitrageurs do not have unlimited capital and their time-varying capital generates variation in x_t. Here, a positive x_t means overpricing and a negative x_t means underpricing. x_t follows

$$\widetilde{x}_{t+1} = \xi_t\rho_x\widetilde{x}_t + \varepsilon_{x,t+1} \tag{53}$$

and its exposure to risk is measured by ω_t:

$$Cov_t\left(M_{t+1}\frac{D_{t+1}}{D_t}, \varepsilon_{x,t+1}\right) = \frac{\omega_t}{1+R_f}, \tag{54}$$

where

$$\widetilde{\omega}_{t+1} = \xi_t\rho_\omega\widetilde{\omega}_t + \varepsilon_{\omega,t+1}. \tag{55}$$

Assume that $\varepsilon_{\omega,t+1}$ is uncorrelated with $M_{t+1}D_{t+1}/D_t$.

The price process. The setting above gives a price-to-dividend process and a value-to-dividend process that are linear in the state variables. To see this, conjecture the following price-to-dividend process:

$$\frac{P_t}{D_t} = \beta_{p,0} + \beta_{p,g}\widetilde{g}_t + \beta_{p,\pi}\widetilde{\pi}_t + \beta_{p,x}\widetilde{x}_t + \beta_{p,\omega}\widetilde{\omega}_t. \tag{56}$$

Plugging this into both sides of the pricing equation with limited arbitrage (equation (52)) implies

$$\begin{aligned}\left(1+R_f\right)\left(\beta_{p,0} + \beta_{p,g}\widetilde{g}_t + \beta_{p,\pi}\widetilde{\pi}_t + \beta_{p,x}\widetilde{x}_t + \beta_{p,\omega}\widetilde{\omega}_t\right) - \left(\widetilde{x}_t + x_*\right) \\ = \left(\kappa_* + \widetilde{g}_t - \widetilde{\pi}_t\right)\left(1+\beta_{p,0}\right) \\ +\kappa_*\left(\beta_{p,g}\rho_g\widetilde{g}_t + \beta_{p,\pi}\rho_\pi\widetilde{\pi}_t + \beta_{p,x}\rho_x\widetilde{x}_t + \beta_{p,\omega}\rho_\omega\widetilde{\omega}_t\right) + \beta_{p,x}\left(\widetilde{\omega}_t + \omega_*\right),\end{aligned} \tag{57}$$

where

$$\kappa_* = 1 + g_* - \pi_* \tag{58}$$

Matching coefficients,

$$
\begin{aligned}
\left(1+R_f\right)\beta_{p,0}-x_* &= \kappa_*\left(1+\beta_{p,0}\right)+\beta_{p,x}\omega_* \\
\left(1+R_f\right)\beta_{p,g} &= 1+\beta_{p,0}+\kappa_*\beta_{p,g}\rho_g \\
\left(1+R_f\right)\beta_{p,\pi} &= -\left(1+\beta_{p,0}\right)+\kappa_*\beta_{p,\pi}\rho_\pi \\
\left(1+R_f\right)\beta_{p,x}-1 &= \kappa_*\beta_{p,x}\rho_x \\
\left(1+R_f\right)\beta_{p,\omega} &= \kappa_*\beta_{p,\omega}\rho_\omega+\beta_{p,x}
\end{aligned}
\tag{59}
$$

Solving these equations,

$$
\begin{aligned}
\beta_{p,0} &= \tfrac{\kappa_*+x_*+\beta_{p,x}\omega_*}{1+R_f-\kappa_*} \\
\beta_{p,g} &= \tfrac{1+R_f+x_*+\beta_{p,x}\omega_*}{\left(1+R_f-\kappa_*\right)\left(1+R_f-\kappa_*\rho_g\right)} \\
\beta_{p,\pi} &= -\tfrac{1+R_f+x_*+\beta_{p,x}\omega_*}{\left(1+R_f-\kappa_*\right)\left(1+R_f-\kappa_*\rho_\pi\right)} \\
\beta_{p,x} &= \tfrac{1}{1+R_f-\kappa_*\rho_x} \\
\beta_{p,\omega} &= \tfrac{\beta_{p,x}}{1+R_f-\kappa_*\rho_\omega}
\end{aligned}
\tag{60}
$$

The intrinsic value process. Under the null of a correct pricing model, $\widetilde{x}_t = x_* = \widetilde{\omega}_t = \omega_* = 0$. In this case,

$$
\frac{V_t}{D_t} = \beta_{v,0}+\beta_{v,g}\widetilde{g}_t+\beta_{v,\pi}\widetilde{\pi}_t, \tag{61}
$$

where

$$
\begin{aligned}
\beta_{v,0} &= \tfrac{\kappa_*}{1+R_f-\kappa_*} \\
\beta_{v,g} &= \tfrac{1+R_f}{\left(1+R_f-\kappa_*\right)\left(1+R_f-\kappa_*\rho_g\right)} \\
\beta_{v,\pi} &= -\tfrac{1+R_f}{\left(1+R_f-\kappa_*\right)\left(1+R_f-\kappa_*\rho_\pi\right)}.
\end{aligned}
\tag{62}
$$

The process for $\Delta_t = P_t - V_t$. It is useful to consider the process for the price deviation from intrinsic value. This is given as the difference between equation (56) and equation (61):

$$
\frac{\Delta_t}{D_t} = \beta_{\Delta,0}+\beta_{\Delta,g}\widetilde{g}_t+\beta_{\Delta,\pi}\widetilde{\pi}_t+\beta_{\Delta,x}\widetilde{x}_t+\beta_{\Delta,\omega}\widetilde{\omega}_t, \tag{63}
$$

where

$$\begin{aligned}
\beta_{\Delta,0} &= \tfrac{x_*+\beta_{p,x}\omega_*}{1+R_f-\kappa_*} \\
\beta_{\Delta,g} &= \tfrac{x_*+\beta_{p,x}\omega_*}{\left(1+R_f-\kappa_*\right)\left(1+R_f-\kappa_*\rho_g\right)} \\
\beta_{\Delta,\pi} &= -\tfrac{x_*+\beta_{p,x}\omega_*}{\left(1+R_f-\kappa_*\right)\left(1+R_f-\kappa_*\rho_\pi\right)} \\
\beta_{\Delta,x} &= \tfrac{1}{1+R_f-\kappa_*\rho_x} \\
\beta_{\Delta,\omega} &= \tfrac{\beta_{p,x}}{1+R_f-\kappa_*\rho_\omega}
\end{aligned} \tag{64}$$

Delta and alphas. Hence, we also have analytical expressions for the delta and the alphas:

$$\delta_t = \frac{P_t/D_t - V_t/D_t}{P_t/D_t}. \tag{65}$$

To compute conditional alphas, note that by the definition of x_t, alpha is simply

$$\alpha_{t+1} = -\frac{x_t}{P_t/D_t}. \tag{66}$$

Use δ denote the unconditional mean value of δ_t.

Finite delta. In practice, we cannot use the exact identity in equation (2) to estimate δ and instead use a finite sum over J periods to proxy for the infinite sum. To compute finite delta over J post-formation periods (denoted $\delta(J)$) in this model, note that

$$\delta(J) = \delta - E\left[M_{t,t+J}\frac{P_{t+J}}{P_t}\delta_{t+J}\right] = \delta - E\left[M_{t,t+J}\frac{D_{t+J}}{D_t}\frac{P_{t+J}/D_{t+J} - V_{t+J}/D_{t+J}}{P_t/D_t}\right]. \tag{67}$$

Note

$$M_{t+j}\frac{D_{t+j}}{D_{t+j-1}} = \frac{1}{1+R_f}\left(1+g_{t+j-1}-\pi_{t+j-1}+e_{t+j}\right), \tag{68}$$

where

$$e_{t+j} = \frac{1}{1+R_f}\left[\left(1+g_{t+j-1}\right)\varepsilon_{M,t+j}+\varepsilon_{D,t+j}+\varepsilon_{M,t+j}\varepsilon_{D,t+j}+\pi_t\right] \tag{69}$$

is a mean-zero error:

$$E_{t+j-1}\left[e_{t+j}\right] = 0. \tag{70}$$

We choose a constant σ_e for our simulations. Assuming that $Cov_{t+j-1}\left(\varepsilon^2_{M,t+j},\sigma^2_{D,t+j}\right)$, $Cov_{t+j-1}\left(\varepsilon_{M,t+j},\varepsilon_{M,t+j}\varepsilon_{D,t+j}\right)$, and $Cov_{t+j-1}\left(\varepsilon_{D,t+j},\varepsilon_{M,t+j}\varepsilon_{D,t+j}\right)$ are all close to zero,

$$Var_{t+j-1}\left(e_{t+j}\right) \approx \frac{1}{\left(1+R_f\right)^2}\left[\left(1+g_{t+j-1}\right)\sigma^2_M+\sigma^2_D-2\left(1+g_{t+j-1}\right)\pi_{t+j-1}+\sigma^2_M\sigma^2_D+\pi^2_{t+j-1}\right] \tag{71}$$

Plugging in the parameter values and replacing g_{t+j-1} and π_{t+j-1} with their steady-state values g_* and π_* allow us to pick a value for σ_e that is consistent with the rest of the model.

CAR and BHAR.

We compute CAR as the sum of expected alphas over J periods after portfolio formation at time t:

$$CAR(J) = \sum_{j=1}^{J} E\left[\alpha_{t+j}\right] \tag{72}$$

Similarly, we compute BHAR as the expected cumulative alphas over J periods:

$$BHAR(J) = E\left[\Pi_{j=1}^{J}\left(1+\alpha_{t+j}\right)-1\right] \tag{73}$$

D.3.2 How well do finite δ, CAR, and BHAR proxy for the true δ?

The linearity-generating model above can be used to generate different kinds of insights on mispricing and abnormal returns. Here, we focus on what it implies about the relative performance of a finite δ, CAR, and BHAR and use the parameter values specified in Table B6.

Figure B3 shows that a finite δ based on 180 post-formation months is a near-perfect proxy for the actual δ. This is true even when δ does not converge to zero but to a nonzero steady-state value due to permanent mispricing associated with either an expected return distortion (Panel B) or a distortion in risk (Panel C). When there is no permanent mispricing, the negative of CAR and BHAR can sometimes be a decent proxy for the true δ_t. However, they deviate substantially from the true δ_t in the presence of even a small amount of permanent mispricing.