Capital Ratios and Systemic Risk

Regulating non-banks in the syndicated lending market

Click here for the latest version

Louis Bélisle [*†]

December 16, 2020

Abstract

Capital adequacy ratios are a popular way of adding a resiliency buffer to financial firms' balance sheets, but every firm active in the lending market may react differently to the tightening of capital constraints. When applied to financial institutions that are less risk-averse than banks, an increase in capital ratio requirements may increase the level of risk in the lending system, as some institutions prioritise a search-for-yield at the expense of risk management. Assuming a mean-variance preference structure on lenders, I calibrate risk-aversion parameters and funding costs to each lenders in the syndicated loan market and run two counterfactual experiments to assess policy effectiveness at tackling systemic risk. I show, using a structural model of syndicated lending, that regulating non-banks leads them to take on more risk, therefore increasing the overall level of risk in the system, even as the probability of default of the average borrower decreases, the tail risk increases. I also confirm that simply modifying capital ratio on banks leads to a decrease in systemic risk, as expected and as implemented in Basel III. This paper serves as a cautionary tale of pursuing blanket macro-prudential policies when the actors in the market have heterogeneous business models and preferences.

1 Introduction

The organization of the financial system influences the type and level of risk that enters into the market. Different actors with various incentives participate in the lending market, and these incentives often clash. Regulators frequently change policies and requirements to address rising risks and shifting market conditions. However, it is still unclear how regulations changes the strategic responses from the financial institutions to risk-taking. Indeed, more stringent regulation could increase risk-taking behaviour of some institutions and be counterproductive. Similarly, it could be the case that existing regulation still has the capacity to control the system's risk by simply adjusting it at the margin. Given the increasing size of the shadow banking sector, it becomes important to design regulation that addresses the right problems and does not create more.

[*]University of Toronto, 150 St George St, Toronto, ON, M5S 2E9

[†]Email: louis.salmon.belisle@mail.utoronto.ca

It is with this special attention to the variety of financial institutions participating in the lending market that I develop a structural model of syndicated lending where each institution allocates its funding resources according to mean-variance preferences. Each institution endogenously chooses the level of risk it wants to undertake, which in turn influences the level of risk borne by the lending system. I calibrate each individual's risk aversion parameter and cost of funding, which allows me to test the regulatory capital requirement's effect on risk taking and systemic risk. I find that a tightening of regulatory requirement at the intensive margin may reduce systemic risk, while increasing requirements at the extensive margin, i.e., expanding the base of institutions subject to capital requirements, may increase risk-taking and systemic risk.

I adapt a modelling approach from the portfolio optimization literature to the wholesale banking sector. This model allows me to understand how different regulatory environments lead institutions with varying degree of risk aversion to sort into risky or safe loans, and how that affects the lending system's risk. Though each member of the syndicate diversifies their portfolio and minimizes their own exposure to risk, the system as a whole has become riskier.

I conduct the study on the syndicated loan market data from Dealscan, that I pair with balance sheet information from Compustat. I use available linking tables to map the Dealscan borrowers andlLead lenders to their Compustat data. To link the rest of the syndicate members, I create my own matching algorithm using fuzzy matching.

I develop my analysis in three stages. First, I show results of reduced-form regressions summarizing the informational content of the data supporting the theory of search for yield and portfolio convergence. Next, I present a basic static structural model with two actors, a bank and a non-bank, and empirically test its predictions. Then, I extend the basic model to a full structural model with many heterogeneous agents and obtain an equilibrium solution. I empirically estimate the model and based on the results run a counterfactual analysis yielding policy predictions.

1.1 Institutional background

In 2010, the Basel Committee on Banking Supervision agreed to restrict the activities of banks with the aim of reducing financial systemic risk. However, the regulations devised ended up creating incentives for other financial institutions to fill in the gaps, expanding shadow banking activities. The emergent shadow banking sector has since more than doubled in size, particularly in the lending market (Sierra, 2019). To fund the loans they underwrite, banks increasingly rely on loan syndication to attract other financial institutions (Bédard-Pagé, 2019; Federal Reserve Board, 2019). Loan syndication is a process in which a loan underwriter, when faced with a large borrowing request, forms a syndicate of lenders, usually comprised of four or five other banks, that will each pledge parts of the loan amount. The loan is then funded by these four or five institutions, each receiving their share of the returns and bearing the credit risk proportionally.

The syndicated loan market attracted much attention in the aftermath of the financial crisis because of its use of Collateralized Loan Obligations (CLOs), and many saw damning similarities with the infamously collapsed Collateralized Debt Obligations (CDOs). Though CLOs were found to have performed much better under the stresses that doomed the CDOs (Ivashina and Scharfstein, 2010a, 2010b; Benmelech et al., 2012), syndicated loans forming the CLOs have exhibited new forms of risks that current regulations do not fully address (Bruche et al., 2017). There is mounting evidence that the structure of the syndicate itself affects the level of risk (Lim et al., 2014; Cai et al., 2018), but to the best of my knowledge no formal models have been proposed to study the consequences the increased participation of shadow banks has on the risks in the lending market. Related studies focus on the dampening effect of the shadow banking sector on monetary policy transmission (Wang et al., 2018), especially in the mortgage market (Xiao, 2019; Buchak et al., 2020), but none specifically about the adverse effect of regulation on market composition and risk.

Based on empirical research describing the functioning of the syndicated loan market, the two main reasons banks engage in loan syndication are capital constraints and diversification concerns (Simons, 1993). This motivates my model, featuring a mean-variance structure and a regulatory capital constraint in the balance sheet identity. Moreover, with the advent of the standardized syndication contracts, non-bank participants are taking a particularly larger role in the lending business (Armstrong, 2003). The benefits to syndication are mainly due to the higher-yield one can extract from these loans when compared to the bond market, as well as a diversification into industries where an institution would not have loan-origination capabilities. However, instead of diversification, the lending agents could double-down on large loans and end up increasing the system's vulnerability. Moreover, the risk spreading across the different agents creates more complex linkages which are not guaranteed to reduce systemic risk.

Adrian and Ashcraft (2012) discusses evidence of regulatory arbitrage in which less-regulated non-banks would soak up the risk that would normally be taken by banks. This can alleviate moral hazard problem since those smaller agents are less likely to be too big to fail, but can create portfolio convergence and more complex linkages, harder to disentangle in the event of a crisis. I model the banks as "feeding" the non-banks with loans by allowing them to partake in syndicated deals with them, as discussed in Bord and Santos (2012). Based on each agent's capital space and risk appetite, they take a different portion of the loan, and choose loans that fit their optimal portfolio allocation.

While corporate finance literature reveals that capital buffers align incentives of participants by giving them more "skin-in-the-game" (Jensen and Meckling, 1976), similar evidence is mounting within the banking and lending literature (Admati and Hellwig, 2013). It is also theorized that increased capital regulation goes further and makes lenders internalize some systemic risk externalities created by the network nature of banking linkages (Acharya, 2009; Poeschl and Zhang, 2018; Shu, 2020). The model I propose in this paper addresses the different possibilities of increased regulation by differentiating risk-aversion preferences and endogenizing risk-taking behaviours.

The paper is organized as follows. Section 2 motivates the research question by presenting empirical studies pointing to the need for a structural analysis of risks in lending market. Section 2.1 presents stylized facts and reduced-form analyses to motivate the model development. Section 3 comprises of a simplified model used to lay out the basis of the analysis and test a few comparative statics predictions. The main model and contribution of this paper are presented in section 4 where a multi-agent structural model is presented, calibrated, and in section 5 where I conduct counterfactuals and propose policy recommendations.

2 Empirical Foundations

During periods of lower long-term interest rates, non-bank lenders take higher risk, while banks accommodate these choices by originating riskier loans, as uncovered by an investigation of the risk taking behaviour in the US syndicated loan market in recent years in Aramonte et al. (2019). This finding supports the view that, in low-interest rate times, firms are searching for yield.

Much of the risk taking preferences of a lending syndicate is affected by the composition of said syndicate (Lim et al., 2014). Not only do non-banks take more risks, but they also charge a higher premium for taking such risks. This premium cannot be entirely explained by risk compensation, since similar bank-only facilities are priced relatively lower. The price is also much higher when more aggressive hedge funds or private equity funds are members of the syndicate. This suggests that the non-banks hold enough market power to alter the pricing relationship. Underwriting banks are incentivized to syndicate with other banks first, as banks usually require lower spreads and can get capital cheaper. However, when capital is scarce due to tight macroeconomic conditions or regulations or the borrowing firm is in dire need of financing, the underwriter is forced to raise the spread to attract non-bank investors. The implication is that non-bank institutions provide capital when banks cannot, i.e., when capital is restricted or when the borrowing firm is constrained. To avoid unobserved biases, the authors compare pricing within the same loan, i.e., between the different facilities among the same deal package.

Though the previous two studies show that the composition of the syndicate influences loan supply when capital is in higher demand conditions, Ivashina and Scharfstein (2010a, 2010b) add the caveat that loan supply is also affected by negative liquidity shocks and capital shocks to lenders. Syndication allows credit to expand because each bank holds less of a loan on its balance sheet and is able to diversify and extend more credit to more borrowers. Moreover, members of the syndicate can be institutional investors, so more funding opportunities are available to fund the loans, as opposed to only traditional banks activities (such as deposits). However, as a result, over a credit cycle, three shocks can amplify the magnitude of the cycle: shocks to borrowers' collateral, to banks' capital, or to investors' sentiment. These shocks force the Lead banks to hold larger shares of the loan, which then are less willing to expand more loans.

The shadow bank sector can also be thought of as competing for the resources of the investors. A simple structural model of such competition is presented in Xiao (2019), where operating costs and regulations requirements are lower for shadow banks, though they do not have access to the customers as easily as banks due to consumer preference for convenience from the banks' network and ATMs, for example. His findings point towards a shift of market power and deposits from commercial banks to shadow banks, especially during times of monetary tightening. Such shift from the insured to uninsured financial sector leads to amplification of systemic risk.

The model I present in the next sections builds upon the aforementioned empirical findings of search-for-yield and syndication behaviours, while being inspired by the shadow banking models developed by Wang et al. (2018) and Buchak et al. (2020). However, particular attention is given to risk management and reaction to regulations as opposed to monetary policy effects.

2.1 Data

The main dataset used is the Dealscan data, which contains information on the syndicated loan market. Dealscan includes data on the issuance of the loan, namely the date of issuance, the borrower's name, industry (SIC), the amount of the loan, the maturity, the price, as well as details on the lenders like the name of the Lead arranger, of the syndicate participants, the type of institutions each lender is classified as, and in some cases the share of the loan they take on their own balance sheet.

In order to analyse the quality and risks of these loans, I gather balance sheet information on both the borrowers and the lenders from Compustat. I use existing matching table to match the unique company ID from Dealscan to the GVKEY identifier from Compustat. I match the borrowers using linking tables from Chava and Roberts (2008), and I match the Lead arrangers using linking tables from Schwert (2018). However, no tables existed to match the syndicate members, therefore, I developed my own matching table using a fuzzy matching algorithm that I then manually verified to ensure data quality.

2.2 Search for yield

Before describing the model to analyse the risk-taking response to regulations, I present empirical stylized facts that justify the modelling approach taken in section 4. As the main innovation of the model is to expose how financial institutions with heterogeneous risk aversions use syndication to give themselves exposure to different risk-levels, the main stylized fact I present is the type of risk-seeking behaviour that differs between banks and non-banks financial institutions. I show that non-banks exhibit more of a search-for-yield behaviour when their outside options yields are low, indicative of a smaller risk-aversion preference. When scaled up at the market level, this search-for-yield behaviour will be the key driver in increasing systemic risk, as portfolios converge and correlations spike.

After merging the Compustat data with the Dealscan data, I analyse which types of firms borrow on the syndicated loan market. I infer the riskiness of a borrower from a Distance to Default measure based on the Asset to Liability ratio. After computing the borrower-specific ratio for each of the matched sample, I compute the average A/L for borrowers who access their loan through a syndicate only containing banks, and those containing a non-bank. I call the former category Bank Funded, and the latter Market Funded. Figure 1 shows a clear distinction between borrowers based on who lends to them. Non-banks participate in syndicates that tend to have borrowers with lower asset to liability ratios.

Figure 1 also reveals certain patterns that have been shown elsewhere in the literature and that I explore later in this paper. First, Bank-funded and Market-Funded loans differ considerably from 1995 up to 2007, where they start to move more in lock-step. The Market-funded loans get increasingly "better" quality borrowers while the Bank-funded ones decrease in quality. This could be due to a learning-by-doing pattern from the non-banks, whose market share sharply increased in the early 1990s and stabilized in the mid-2000s, as shown in Figure 2. Second, after an increase in borrower quality from 2007 to 2012, both bank-funded and market-funded lending started to lower the quality of their borrowers, at the onset of the European debt crisis and the low-yield environment it created. This can be

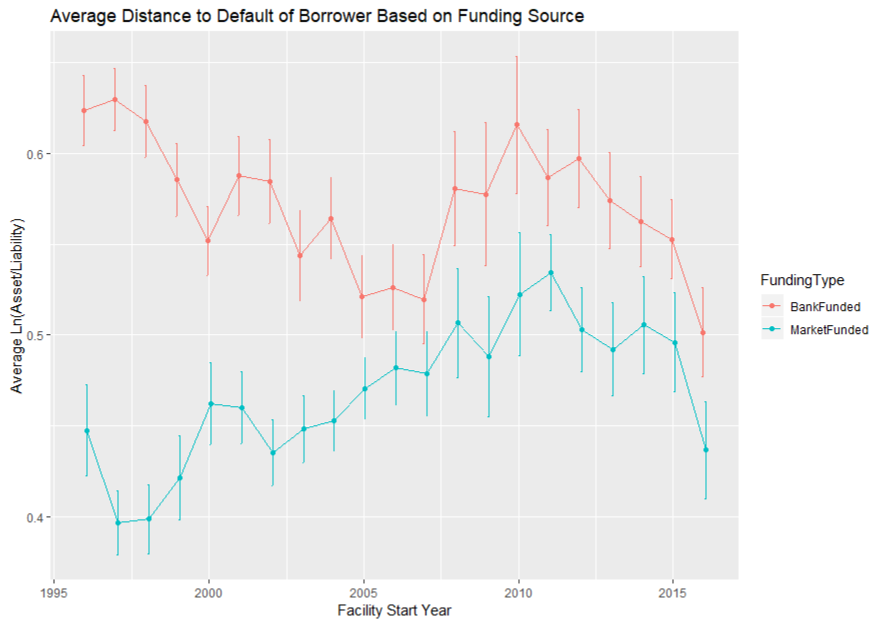

Figure 1: Average Distance to Default by funding source

explained by a search-for-yield behaviour from all parties. These two facts serve as guidance for the development of the model from section 3.

A simple regression can further quantify the qualitative features of Figure 1, corroborating the stylized facts that have been described elsewhere in the literature, e.g., in Aramonte et al. (2019). In my case, Distance to Default serves as a proxy for risk. I estimate the following equation:

$$\ln\left(\frac{A_{t-1}}{L_{t-1}}\right) = \alpha + \beta \text{T10Y} + \gamma \text{Type} + \varepsilon \tag{1}$$

where A and L are the borrower's total assets and liabilities[1], lagged one quarter from the time of the loan, and T10Y is the U.S. Treasury 10 year bond quarterly average yield, and Type is a dummy for the lender's type, indicating a 1 if the syndicate includes a non-bank and 0 otherwise. Lags are included to avoid simultaneity problems due to the timing of loan issuance and decision making. Table 1 shows similar results as reported in previous studies. A decrease in treasury yield leads to a decrease in distance to default, hence an increase in probability of default. Moreover, Market-Funded loans tend to have a higher probability of default than fully bank-funded ones, as inferred by the negative coefficient on "Type: Market Funded". This provides evidence of the search-for-yield stylized fact in my data.

2.3 Portfolio convergence and risk

To test the hypothesis that converging portfolio structure of banks and non-banks stemming from search-to-yield behaviour is in turn associated with increased market risk, I measured portfolio distance, inspired by Cai et al. (2018). Its purpose is to understand how much of the lending by one institution is made to similar borrowers as another

1. Balance sheet data from Compustat is only reported quarterly

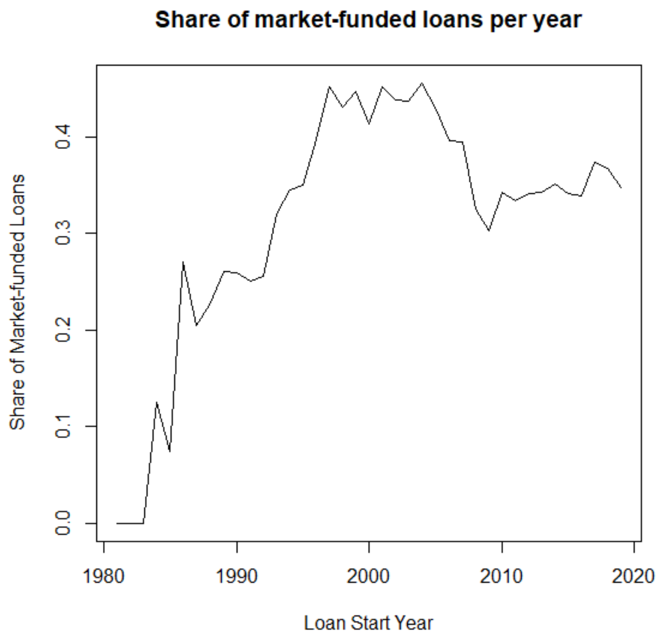

Figure 2: Market Share of Market-Funded Loans

Table 1: Search-for-Yield and Market-funding

	$\ln\left(\frac{A_{t-1}}{L_{t-1}}\right)$
T10Y_{t-2}	0.004^{***}
	(0.001)
Type: Market Funded	-0.100^{***}
	(0.004)
Constant	0.592^{***}
	(0.007)
Observations	57,767
R^2	0.011
Residual Std. Error	0.483 (df = 57764)
F Statistic	315.362^{***} (df = 2; 57764)
Note:	*p<0.1; **p<0.05; ***p<0.01

institution. I first create a monthly profile of each lender's portfolio by its allocation of loans to different industries. Aggregating the previous 12-month period, we can get a relative measure of which industry is being favoured, and since most loans are short-term loans, this 12-months period is a good proxy for the industry-weights of a portfolio as a whole. The distance between each lender is then calculated as the Euclidian distance in the space of industries.

I aggregate a monthly average distance based on funding type and regress this distance on the treasury yield to obtain Table 2. As the treasury yield decreases, the portfolios of banks tend to diverge from that of the syndicate Lead underwriters, indicated by the negative coefficient. This divergence means that the banking sector gets more diversified as each bank reaches for a wider arrays of borrowers. However, the positive coefficient on the treasury yield with respect to market funded syndicates shows that non-banks increase their reliance on syndicates during times of low yields. Their portfolio gets more similar to Lead underwriters' portfolios, causing this decrease in the distance measure. The different reaction observed between banks and non-banks to low-yield environment indicates a different solution to their portfolio optimization problem. I therefore hypothesize that the heterogeneity in preferences indicated by the different reaction to changing financial environment will be exacerbated by prudential regulations.

Table 2: Yield and Portfolio convergence

	Dependent variable:	
	Bank Funded Distance	Mkt Funded Distance
T10Y	-0.002^{***}	0.008^{***}
	(0.001)	(0.002)
Constant	0.141^{***}	0.125^{***}
	(0.004)	(0.007)
Observations	252	252
Note:	*p<0.1; **p<0.05; ***p<0.01	

The two empirical facts discussed above, i.e. search for yield and portfolio convergence, drive the model development of Sections 3 and 4. First, the search for yield behaviour is reflected by financial institutions maximizing their expected returns. Second, the portfolio convergence results indicates that each financial institution is governed by heterogeneous preferences, which is modelled by different risk aversion parameters. The structural model with heterogeneous agents presented in Section 4 allows financial institutions to differ in their risk appetite, while seeking a mean-variance optimized portfolio. This heterogeneity allows a flexible reaction to capital constraints and models how financial institutions react to changes in regulations. In Section 5 I test two main types of regulatory changes, namely at the intensive and extensive margins.

At the intensive margin, I want to verify how banks will react to an increase in the required capital buffer. The working hypothesis is that as they find capital more expensive and since they are more risk-averse, they will increase prudence and the system's risk will decrease. On the other hand, at the extensive margin, I will extend the regulatory requirement of banks onto non-banks and constrain the non-banks' balance sheets to respect the same capital ratios as the banks. This blanket regulation would be justified if banks and non-banks were to react similarly to a higher cost of capital. However, as we have seen in this section, non-banks tend to increase their search for yield and might be more risk takers than banks. This is what the calibration exercise of section 4.2 will reveal, and the results on systemic risk will show that blanket, uniform regulation is maladapted to reduce risk in the presence of non-banks in the lending system.

3 Basic Structural Model

The model presented in this section is intended to highlight the distinction that syndication makes to lending and portfolio allocation decisions when financial institutions differ in their risk tolerance. This basic static model comprises 2 actors, one Lead underwriter L, and one syndicate member M, coming together to lend to two potential borrowers to compose their portfolio. Each lender has mean-variance preferences over their allocation, that only differ in their risk aversion parameter γ_j, as shown in equation (2).

The Lead underwriter L is seeking to maximize expected returns and minimize the variance of its portfolio by controlling two variables, namely the price of the loan r_i and the weights it keep on its balance sheet $w_{i,L}$. However, in doing so, it must respect two conditions. First, each loan must be fully financed, so the sum of weights across syndicate members $j \in \{L, M\}$ must add up to 1. Second, the amount of loans the Lead owns on its balance sheet must not exceed regulatory capital constraints. These capital constraints are measured as a fraction $\underline{\mathrm{e}}$ of the institution's equity E. Therefore, the sum of loans kept on balance sheet across all borrowers $i \in \{1, 2\}$ must not exceed $\frac{E_l}{\underline{\mathrm{e}}}$, as described in equation (3). The syndicate member M is solving a similar problem, albeit without control over r_i and without the constraint (4) of ensuring full funding of the loan sizes.

$$\max_{r_i, w_{i,L}} \mathbb{E}\left[\sum_i w_{i,L} B_i \widetilde{r}_i\right] - \frac{1}{2}\gamma_j \operatorname{Var}\left[\sum_i w_{i,L} B_i \widetilde{r}_i\right] \tag{2}$$

$$\text{s.t.} \sum_i w_{i,L} B_i \leq \frac{E_L}{\underline{\mathrm{e}}} \tag{3}$$

$$\sum_j w_{i,j} = 1, \forall i \tag{4}$$

The return on the loan $\widetilde{r}$ is a function of the risk of default of the borrower λ_i, which is itself a parameter of a Bernoulli random variable. I define $\widetilde{r}_i = (1 - \theta\widetilde{\lambda}_i)r_i$, where r_i is the rate chosen by the Lead underwriter. The parameter θ is the recovery rate of the defaulted loan, and $\widetilde{\lambda}_i$ is the Bernoulli(λ_i) random variable representing default, with mean λ_i and variance $\lambda_i(1 - \lambda_i)$.

The Lead underwriter can also choose the price it charges and therefore has an expanded maximization problem that I solve in two parts: first, the Lead underwriter chooses the price, then chooses the weights to allocate within its portfolio. The Lead underwriter must also worry about fulfilling the entirety of the loan size B_i, something that does not enter the syndicate member's problem. The syndicate member, labeled by subscript M, therefore solves a mostly identical problem as the Lead underwriter, except it does not include the choice of price r and the constraint of fulfilling the entire loan demand B_i.

Assuming a simple environment of only two lenders, two borrowers problem, I can present more explicitly the problem as follows. The Lead underwriter is maximising over mean and variance by choosing a price r_i and portfolio weights $w_{i,L}$

$$
\begin{aligned}
\max_{r_i, w_{i,M}} \ & \mathbb{E}[w_{1,L}B_1(1-\theta\widetilde{\lambda}_1)r_1 + w_{2,L}B_2(1-\theta\widetilde{\lambda}_2)r_2] \\
& \quad - \frac{1}{2}\gamma_M \operatorname{Var}[w_{1,L}B_1(1-\theta\widetilde{\lambda}_1)r_1 + w_{2,L}B_2(1-\theta\widetilde{\lambda}_2)r_2] && (5) \\
\text{s.t.} \ & w_{1,L}B_1 + w_{2,L}B_2 \leq \frac{E_L}{\underline{\mathrm{e}}} && (6) \\
& w_{1,L} + w_{1,M} = 1, \ \& \ w_{2,L} + w_{2,M} = 1 && (7) \\
& w_{i,j} \geq 0 \ \ \forall i, j && (8)
\end{aligned}
$$

The other lending financial institution can then choose to participate in the syndicate as a member, and solve the following maximisation problem

$$
\begin{aligned}
\max_{w_{i,M}} \ & \mathbb{E}[w_{1,M}B_1(1-\theta\widetilde{\lambda}_1)r_1 + w_{2,M}B_2(1-\theta\widetilde{\lambda}_2)r_2] \\
& \quad - \frac{1}{2}\gamma_M \operatorname{Var}[w_{1,M}B_1(1-\theta\widetilde{\lambda}_1)r_1 + w_{2,M}B_2(1-\theta\widetilde{\lambda}_2)r_2] && (9) \\
\text{s.t.} \ & w_{1,M}B_1 + w_{2,M}B_2 \leq \frac{E_M}{\underline{\mathrm{e}}}, \quad w_{i,M} \geq 0 \ \ \forall i && (10)
\end{aligned}
$$

3.1 Solution Concepts

The previous optimization problems can be solved analytically to yield the financial institutions' decision functions. As the Lead's decisions will depend on the reaction from the member, I start by presenting the solution for the member's problem, then the Lead's portfolio allocation problem, and finally the Lead's pricing choice.

3.1.1 Member's Solution Concept

When the member's capital ratio constraint is binding, i.e., when the member is subject to regulation, its optimization problem yields the following weight:

$$
w_{1,M} = \frac{(1-\theta\lambda_1)r_1 - (1-\theta\lambda_2)r_2 + \frac{E_M}{\underline{\mathrm{e}}}\gamma_M\lambda_2(1-\lambda_2)\theta^2 r_2^2}{B_1\gamma_M[\lambda_1(1-\lambda_1)\theta^2 r_1^2 + \lambda_2(1-\lambda_2)\theta^2 r_2^2]} \tag{11}
$$

$$
w_{2,M} = \frac{(1-\theta\lambda_2)r_2 - (1-\theta\lambda_1)r_1 + \frac{E_M}{\underline{\mathrm{e}}}\gamma_M\lambda_1(1-\lambda_1)\theta^2 r_1^2}{B_2\gamma_M[\lambda_1(1-\lambda_1)\theta^2 r_1^2 + \lambda_2(1-\lambda_2)\theta^2 r_2^2]}. \tag{12}
$$

The size of funds allocated to one borrower can therefore be interpreted as containing two parts. The first one is the relative gain, scaled by the risk aversion parameter, in Sharpe ratio by allocating more balance sheet space to one borrower than to the other. The second component is the effect of the relative variance of the alternative investment opportunity, scaled by the potential balance sheet space.

$$w_{1,M} = \frac{1}{B_1}\left(\frac{1}{\gamma_M}\underbrace{\frac{\mathbb{E}[\widetilde{r}_1 - \widetilde{r}_2]}{\mathrm{Var}(\widetilde{r}_1 + \widetilde{r}_2)}}_{\text{Rel. Sharpe Ratio}} + \frac{E_M}{\underline{\mathrm{e}}}\underbrace{\frac{\mathrm{Var}(\widetilde{r}_2)}{\mathrm{Var}(\widetilde{r}_1 + \widetilde{r}_2)}}_{\text{Relative Variance}}\right) \tag{13}$$

The solution for $w_{2,M}$ is symmetric. Therefore, the syndicate member solves a relatively standard investment problem, only with the added balance sheet's capital ratio constraint.

In case the member is not regulated as tightly as banks, or has extra capital space, the constraint is non-binding, i.e. $\nu = 0$, and the weight on each asset simply follows that asset's Sharpe ratio, scaled by the lender's risk aversion.

$$w_{1,M} = \frac{1}{B_1}\left(\frac{1}{\gamma_M}\underbrace{\frac{\mathbb{E}[\widetilde{r}_1 - \widetilde{r}_2]}{\mathrm{Var}(\widetilde{r}_1 + \widetilde{r}_2)}}_{\text{Rel. Sharpe Ratio}}\right) \tag{14}$$

The solution for $w_{2,M}$ is symmetric.

3.1.2 Lead's Solution Concept

Now, once the Lead underwriter solves for the allocation strategy for the syndicate's member, it can solve its own maximisation problem. In order to make the solution analytically tractable, I assume that the Lead first chooses the price which would clear the market, then finds its optimal allocation.

Solving the problem backwards, as the Lead's choice of weights will also solve the asset allocation problem, with the added constraint of ensuring that the loan is fully funded. The Lagrangian for this problem is expressed as

$$\begin{aligned}\max_{w_{1,L},w_{2,L}} \; & \mathbb{E}[w_{1,L}B_1(1-\theta\widetilde{\lambda}_1)r_1 + w_{2,L}B_2(1-\theta\widetilde{\lambda}_2)r_2] - \frac{1}{2}\gamma_L \mathrm{Var}[w_{1,L}B_1(1-\theta\widetilde{\lambda}_1)r_1 + w_{2,L}B_2(1-\theta\widetilde{\lambda}_2)r_2] \\ & - \nu\left(w_{1,L}B_1 + w_{2,L}B_2 - \frac{E_L}{\underline{\mathrm{e}}}\right) - \eta_1(w_{1,L} + w_{1,M} - 1) - \eta_2(w_{2,L} + w_{2,M} - 1)\end{aligned} \tag{15}$$

In practice, the Lead would never issue a loan in which it cannot fully fulfill the loan amount, and as long as the loans required by the borrowers do not exceed the total capital space available by both lenders, i.e., $B_1 + B_2 \leq \frac{E_L}{\underline{\mathrm{e}}} + \frac{E_M}{\underline{\mathrm{e}}}$, the two constraints on η_1 and η_2 are always satisfied, and never as a hard limit. Therefore, the Lead's allocation can be interpreted in the same way as the Member's decision in equation (13) as consisting of a relative Sharpe ratio and a relative variance. However, the main difference is that the Lead now must take into account the member's choices and the difference between the potential investments' variances.

Finally, the Lead chooses the price of the loan that will Lead the member to participate with the appropriate amount. Since the Lead's problem in this two lenders problem is solvable with respect to the member's choice, one could derive the FOCs with respect to the price using the weights as functions. However, for the sake of presentation, it is useful to find the closed form solution for the price conditional on the weights. This will be the closest solution concept to the multi-agent, large-scale problem of section 4.

The solution of the optimization problem (15) is thus

$$r_i = \frac{1}{\gamma_L} \frac{\mathbb{E}\left[1-\theta\widetilde{\lambda}\right]}{w_{i,L} B_i \theta^2 \mathrm{Var}\left(\widetilde{\lambda}\right)} \tag{16}$$

Recall that $w_{i,L}B_i$ is the quantity of funds provided by lender L to borrower i, so the Lead actually chooses the cashflow of income $r_i w_{i,L} B_i$ that balances the expected recovery rate of the loan with the variance of a potential default, scaled by the lender's aversion to that variance.

3.2 Model's intuition

The main observation to make coming from this simple model is the link between the Lead and the Member's decision. While the member takes into account its own portfolio, the Lead must balance the fulfillment of the loans with the desire of the member to allocate funds to this loan. I include in appendix B the Lead's allocation weights when incorporating the Member's.

A key fact to gather from equations (13) and (14) is that members with lower risk aversion will allocate more funds into loans yielding a larger return, regardless of the variance difference with the alternative asset. Moreover, when a lender reaches its capital ratio constraint, which can be cause by a tightening of the ratio or a broadening of the coverage of regulated institutions, the allocated weight now takes into account the limited capital space. We can see that due to this limited space and desire for diversification, a lender could go from a null weight in an asset, due to a negative relative Sharpe ratio, to a positive weight, due to the relative variance and capital space consideration.

This allocation's constraint will drive the share of risky borrowers that will get loans when regulatory oversight expands to all lenders. Now that capital is more expensive, those with lower risk aversion will then increase their participation in risky assets in order to compensate the loss of revenue and diversification they were able to get when they did not have such restriction. This increase in cost of capital, though encouraging diversification at the individual lender's level, will increase the amount of risk borne by the system as a whole. This effect will be even more exacerbated in the full model as I introduce different funding costs for banks and non-banks, which will eat much of that revenue generation and encourage non-banks to take even more aggressive stances when constrained.

4 Multi-agent Model

One of the main feature of syndication is the ability of the Lead bank to attract multiple other financial institutions to participate in the loan. It is therefore an important component to add to the previously presented model more than one potential syndicate member available. Assume that the Lead bank, upon receiving a loan request from a borrower, may now call on a smaller bank, or a non-bank financial institution to participate in the loan syndication process. The following model is inspired by Buchak et al. (2020), Xiao (2019), and Wang et al. (2018).

Syndication offers benefits to each of the participating actors, be it the Lead bank, the member bank, or the borrower itself. The main drivers of syndication for lenders is diversification and capital constraints (Simons, 1993).

Syndication allows the Lead bank to collect extra fees, compete with bond issuance market, and maintain a full service for a large client, especially when such client is considering taking a large debt and faces the loan or the bond route (Armstrong, 2003). For borrowers, having a bank syndicate its loan allows to keep a relationship with a known underwriter and keep administration costs low due to monitoring. A syndicated loan allows to take on a large debt without going through the lengthy process of a bond issuance. The member banks that would join in a syndication would benefit from reduced origination costs, since much of the legwork is done by the Lead bank, but most importantly, syndication may allow member banks to participate in a market segment in which they have a thin presence, either due to their different geographic footprint or industry penetration than from the Lead bank (Bruche et al., 2017). Non-bank financial institutions also benefit from joining a lending syndicate as syndication opens up a different type of asset class that has a different correlation with the rest of the market while offering relatively high yields. It allows institutional investors to further diversify their portfolio, and many have argued that corporate syndicated loans exhibit a lower volatility and a better hedge against interest rate risk, while offering higher recovery rate than bonds in the event of a default since loans are usually senior to bonds (Bédard-Pagé, 2019). Finally, for all lenders involved, syndication is beneficial due to the increased information taken into account by credit rating agencies to provide credit risk assessments. These agencies usually take into consideration the probability of default (PD) as well as the loss given default (LGD) when assessing syndicated credit products, whereas most bonds are rated by simply considering the probability of default and omitting the loss given default (Król and Williams, 2019).

Therefore, when developing the objective functions of the different participants, I will be modelling separately the Borrowers, the Lead Agents, and the Syndicate Members. The borrower approaches the Lead of its choice, based on price, size possible, and relationship. The borrower can chose between taking no loan at no price but no size, issuing a bond at a high price and high size, or among all the potential Lead banks, each with a differentiated loan product.

Once the Lead Agent is approached to make a loan, it gets fees and the rate on the loan, but must expend some origination costs for the setup. The Lead can then choose how to make the loan. It can either refuse to give the loan, lend in a bilateral agreement where it is the sole lender, or syndicate the loan and find members to participate in the loan. It then chooses how much of the loan will be syndicated and which members to approach, based on the required rate of return that the members would need to be incentivized to participate, as well as a relationship component. This relationship preference at the syndicate level is justified by the history of the syndication lending business, where most loans used to be done as Club Deals, where a select number of large members would syndicate as lending clubs based heavily on relationship. Though nowadays most syndication follows a transactional system, there are still anecdotes of traders and structurers contacting a preferred list of potential members first based on past deals and expected future deals first, then expanding the calls to others if not enough participants have been secured to fund the loan (B. Nordin, personal communication, August 23, 2019). Though the profession is increasingly streamlined, finance remains a personal business where contacts matter and relationships are groomed with care.

Finally, the members will be approached by the Lead agents to participate in a syndication. They then make a decision based on a diversification optimization problem where they can benefit from the diversification that the syndicated loan might offer to their portfolio. They also consider the expected return of the loan, as well as the PD and LGD. They therefore choose between a loan and an outside option in High yield bonds with a lower recovery rate. They benefit from a reduced origination cost, but must still perform due diligence so face a small origination cost.

4.1 Model

4.1.1 Borrowers

Borrowers of different risk levels make loan requests of size B_i to banks the entirety of which must be financed for the loan to be issued. They choose the lender based on the rate offered, but also an unobserved convenience yield parameter and an idiosyncratic utility shock.

$$u_{ij} = \underbrace{-\alpha_i B_i r_j(\lambda_i)}_{\text{loan rate}} + \underbrace{\beta_i x_{ij}}_{\text{Relationship}} + \underbrace{\xi_j}_{\text{Convenience}} + \underbrace{\varepsilon_{ij}}_{\text{Idiosyncratic}} \tag{17}$$

where i is borrower specific, j is Lead bank specific. The parameter β_i captures the relationship effect on borrower i of loans with a Lead j indicated by x_{ij}. The error term ε_{ij} is a mean-zero pair-specific idiosyncratic utility shock following an extreme value distribution of type one with cumulative distribution $F(\varepsilon) = \exp\{\exp(-\varepsilon)\}$.

The choice set for the borrowers is $\{0, 1, \ldots, J, J+1\}$, where 0 represents cash, and $J+1$ is bond financing, and the other are differentiated lenders. The parameter α represent the sensitivity to the lending rate for borrower of type i. Each borrower arrives in the market with an exogenously determined probability of default λ_i. This borrower's problem was not considered in the basic model of Section 3, where there was only one Lead and one Member, with the Lead choosing over two borrowers. In the multi-agents context, the borrowers' optimization will define the set of lenders that become Lead agents, described in the next part of the model below. It's optimization takes the following form:

$$u_i^* = \max_j u_{ij}(B_i, r_j, x_{i,j}, \xi_i) \tag{18}$$

4.1.2 Lead agents

I assume Lead lenders are risk-averse portfolio optimizers that must choose assets among the borrowers that approach them, or allocate their capital in either a risk-free bond or a high-yield bond. The lenders must always adhere to the balance sheet identity of equating their assets with their liabilities plus deposits, and certain lenders will face a capital ratio regulatory requirement. This capital ratio requirement, denoted $\underline{\mathrm{e}}_j$, is either 0 in the case of non-banks, or the ratio determined by the Basel Accords, currently set at 8% (Basel Comittee on Banking Supervision, 2019). The problem follows a classic Markowitz (1952) portfolio optimization problem where the Lead's universe of assets that it has access to is the borrowing requests that come its way from the borrowers plus a risk-free bond and a high-yield bond. The main difference from Markowitz's Modern Portfolio Theory comes from the constraints put on the portfolio optimization, namely the balance sheet identity, the capital ratio requirements, and financing costs.

Indeed, financing the loans is costly. Financing can be done by either using the bank's balance sheet or by syndicating the loan. Syndicating the loan "costs" the lender the potential returns it would have earned on a larger-sized loan, but it doesn't use balance sheet capital space. Funding the loans using the institution's own balance sheet is limited by the regulatory limit ratio $\underline{\mathrm{e}}$ of equity E_j, therefore implying an opportunity cost to each dollar funded by the lender's own balance sheet.

In order to fund using syndication, the loan must be attractive enough for outside financial institutions to participate in the syndicate as members. To make the loan syndicate more attractive, the Lead must offer higher return rates, which might hamper its demand share in the underwriting phase. This interplay between attractiveness of the loan on both sides of the market (corporate borrowers and syndicate members) therefore also implies a shadow price of syndication.

To use its balance sheet, a lender can either use equity funding or liability funding. The two types of liability funding allowed are either through market liabilities, obtained by non-reservable borrowing, or through deposit liabilities, obtained by un-modelled demand deposits. Banks that have access to deposits can use deposits to expand the asset-side of their balance sheet at no cost.

On the other hand, funding the loans using market liabilities in the form of non-reservable borrowing is costly. The costs consist of the price to pay on the borrowing, as well as market risks and credit risks. Following Kashyap and Stein (1995), the market funding cost is modeled as an increasing function of the amount raised above r_0 the risk-free rate, which can be proxied by the Federal Funds rate. I incorporate the opportunity costs of capital space mentioned above into a financing cost function $g(\cdot)$ defined below as part of the Lead lender's optimization problem.

The lenders receive benefits from the loans from their expected returns and some client relationship benefit. The total costs they incur consist of origination costs, which will differ between Lead and member agents, and of financing costs, which will differ between banks and non-banks.

There are I borrowers, and J Lead lenders. Let $\boldsymbol{B}$ be the vector of borrower's loan requests dollar amount. Let the vector $\boldsymbol{w}_j = (w_{0,j}, w_{1,j}, \ldots, w_{I+1,j})'$ the shares of each asset i that the Lead bank j keeps on its balance sheet. Asset $i = 0$ represents the risk-free bond, and asset $i = I + 1$ represents the high-yield bond. Define the dollar amount kept by Lead j on its balance sheet as $\boldsymbol{L}_j = \boldsymbol{w}_j \cdot \boldsymbol{B}$, the dot product of the vector of weights with the vector of borrowers' loan requests and other two available assets. Let $\mathcal{A}_j$ be the set of $i \in \{1, \ldots, I\}$ that lender j lends to. This set will be determined by the borrowers' choice of section 4.1.1. Let $\widetilde{\boldsymbol{R}}_j = \boldsymbol{R}_j \cdot (\mathbf{1} - \boldsymbol{\theta}\boldsymbol{\Lambda})$ be the vector of realized return, which is composed of lending rates $\boldsymbol{R}_j$ set by firm j, a random shock indicating default, $\boldsymbol{\Lambda}$, and the loss given default of the loans $\boldsymbol{\theta}$. The balance sheet's asset side is composed of the loans and bonds weighted by the lender, and the liability side is composed of deposits D_j and non-reservable funds N_j, as well as equity E_j.

The utility of lender j is represented by the following equation with constraints:

$$\pi_j = \underbrace{\mathbb{E}[\boldsymbol{L}_j'\widetilde{\boldsymbol{R}}_j]}_{\text{Expected Returns}} \underbrace{-\gamma_j \operatorname{Var}[\boldsymbol{L}_j'\widetilde{\boldsymbol{R}}_j]}_{\text{Risk Aversion}} \underbrace{-g(N_j, D_j, E_j, \boldsymbol{w}_j, \boldsymbol{B}, r_0)}_{\text{Financing Costs}} \underbrace{-\zeta_j \boldsymbol{B}'\vec{\mathbb{I}}\{i \in \mathcal{A}_j\}}_{\text{Origination Costs}} \underbrace{+\boldsymbol{\beta}_j'\vec{\mathbb{I}}\{i \in \mathcal{A}_j\}}_{\text{Relationship Benefits}} \tag{19}$$

$$\text{s.t. } \boldsymbol{L}_j'\mathbf{1} = N_j + D_j + E_j \leftarrow \text{ Balance sheet identity} \tag{20}$$

$$E_j \geq \underline{\mathrm{e}}_j(\boldsymbol{L}_j'\mathbf{1} - L_0) \leftarrow \text{ Capital ratio requirement} \tag{21}$$

$$\sum_j w_{i,j} = 1, \, \forall i \in \mathcal{A}_j \leftarrow \text{Full Loan Funding} \tag{22}$$

$$w_{i,j} \geq 0, \;\; \forall i, j \leftarrow \text{No Loan Short-Sell} \tag{23}$$

The returns are non-deterministic because even though the Lead sets the lending rate, there is a probability of default associated with each borrower. So Lead j sets the rates $\boldsymbol{R}_j$ but the realized return $\widetilde{\boldsymbol{R}}_j$ depends on the realization of the vector of Bernoulli random variables $\boldsymbol{\Lambda}$ representing the borrowers' default realization.

I specify a functional form for the cost function $g(\cdot)$, inspired by the forms from Buchak et al. (2020) and Wang et al. (2018) as follows:

$$g(N_j, D_j, E_j, \boldsymbol{w}_j, \boldsymbol{B}, r_0) = r_0 + \left[\frac{E_j}{\boldsymbol{L}_j'\mathbf{1} - D_j} - \underline{e}\right]^{-\phi} \times N_j^2 \tag{24}$$

where N_j is non-reservable borrowing, and D_j is the level of deposits, taken here as exogenous to the model.

Note that the basic model presented in section 3 is a special case of the above model, with $I = 2$, $J = 2$, and where financing and origination costs, as well as potential relationship benefits, are set to zero. Not that the basic model is much closer to Markowitz (1952), but still include the significant difference of considering the syndication fulfillment problem, as well as the capital adequacy ratio.

The Lead lender then solves the above system of equations by maximizing over loan prices $\boldsymbol{R}_j$ and asset allocation $\boldsymbol{w}_j$.

$$\pi_j^* = \max_{\boldsymbol{R}_j, \boldsymbol{w}_j} \pi_j(\boldsymbol{R}_j, \boldsymbol{w}_j, N_j, D_j, E_j, \boldsymbol{B}) \tag{25}$$

4.1.3 Members

The members have a similar optimization problem, but are not able to choose the return rate, and do not have access to the benefits of leading the syndicate. Their origination costs will however be reduced and can be proportional to the amount invested, since they might want to perform more due diligence if they invest large amounts, but smaller ones if they have marginal involvement in the loan. They choose the amount of funds they want to allocate to each loan based on the composite vector of offered return rates $\boldsymbol{R}^+ = \sum_j \boldsymbol{R}_j$. Since only one Lead bank would be offering a positive rate per borrower, this vector $\boldsymbol{R}^+$ should consist of the risk-free rate, and all the offered rates by all the leads to all the borrowers receiving a loan, and the rate of the high-yield bond.

The syndication member's problem is stated below. There are K Member lenders active on the market. Note the notation for the on-balance sheet share of a loan as $\boldsymbol{S}_k = \boldsymbol{w_k}\boldsymbol{B}$ to clearly differentiate between assets taken as a Lead j and those taken as a Member k of a syndicate.

$$\pi_k = \underbrace{\mathbb{E}[\boldsymbol{S}_k' \widetilde{\boldsymbol{R}}^+]}_{\text{Expected Returns}} \underbrace{-\gamma_k \operatorname{Var}[\boldsymbol{S}_k' \widetilde{\boldsymbol{R}}^+]}_{\text{Risk Aversion}} \underbrace{-g(E_k, \boldsymbol{S}_k, \boldsymbol{B}, r_0)}_{\text{Financing}} \underbrace{-\zeta_k \boldsymbol{S}_k' \mathbf{1}}_{\text{Origination}} \tag{26}$$

$$\text{s.t. } \boldsymbol{S}_k' \mathbf{1} = N_k + E_k + D_k \leftarrow \text{ Balance sheet identity} \tag{27}$$

$$\text{and } E_k \geq \underline{\mathrm{e}}(\boldsymbol{S}_k' \mathbf{1} - S_0) \leftarrow \text{ Capital ratio requirement} \tag{28}$$

$$w_{i,k} \geq 0, \ \ \forall i, k \leftarrow \text{No Loan Short-Sell} \tag{29}$$

Note that both leads and members can be banks or non-banks, the only difference will be in their level of capital requirement and in their access to deposits on their balance sheet or not.

The Member lender then solves the above system of equations by maximizing over asset allocation $\boldsymbol{w}_k$.

$$\pi_k^* = \max_{\boldsymbol{w}_k} \pi_k(\boldsymbol{w}_k, N_k, D_k, E_k, \widetilde{\boldsymbol{R}}^+, \boldsymbol{B}) \tag{30}$$

4.1.4 Equilibrium

In the first step, borrowers maximize utility by choosing the best Lead j for their loan L_i.

$$\text{Borrower problem: } \max_j \ -\alpha_i B_i r_j + \beta_i x_{ij} + \xi_j + \varepsilon_{ij} \tag{31}$$

and Lead optimize their portfolio by setting the interest rates for their potential customers, taking costs as given.

$$\text{Lead problem 1: } \max_{\boldsymbol{R}_j} \mathbb{E}[\boldsymbol{L}_j' \widetilde{\boldsymbol{R}}_j] - \gamma_j \operatorname{Var}[\boldsymbol{L}_j' \widetilde{\boldsymbol{R}}_j] - g(E_j, \boldsymbol{L}_j, \boldsymbol{B}, r_0) - \zeta_j \boldsymbol{B}' \vec{\mathbb{1}}\{i \in \mathcal{A}_j\} + \boldsymbol{\beta}_j' \vec{\mathbb{1}}\{i \in \mathcal{A}_j\} \tag{32}$$

In the second step, leads optimize their portfolio weights to account for the financing costs, taking the interest rates offered to their customers as given

$$\text{Lead problem 2: } \max_{\boldsymbol{L}_j} \mathbb{E}[\boldsymbol{L}_j' \widetilde{\boldsymbol{R}}_j] - \gamma_j \operatorname{Var}[\boldsymbol{L}_j' \widetilde{\boldsymbol{R}}_j] - g(E_j, \boldsymbol{L}_j, \boldsymbol{B}, r_0) - \zeta_j \boldsymbol{B}' \vec{\mathbb{1}}\{i \in \mathcal{A}_j\} + \boldsymbol{\beta}_j' \vec{\mathbb{1}}\{i \in \mathcal{A}_j\} \tag{33}$$

and the members optimize their portfolio based on the rate offered by each loan and the leftover loan to be financed. Whatever is not picked up by members must be financed externally, increasing the costs to the Lead bank.

Finally, the members solve the optimization problem by choosing how much they want to allocate to each asset in the universe available. This problem is akin to a Markowitz problem since they only choose how to split a predefined budget.

$$\text{Member problem: } \max_{\boldsymbol{S}_k} \mathbb{E}[\boldsymbol{S}_k' \widetilde{\boldsymbol{R}}^+] - \gamma_k \operatorname{Var}[\boldsymbol{S}_k' \widetilde{\boldsymbol{R}}^+] - g(E_k, \boldsymbol{S}_k, \boldsymbol{B}, r_0) - \zeta_k \boldsymbol{S}_k' \mathbf{1} \tag{34}$$

$$\text{s.t. } \boldsymbol{S}_k' \mathbf{1} = E_k + N_k + D_k, \quad \text{and } \boldsymbol{S}_k' \mathbf{1} \leq \frac{E_k}{\underline{e}} \tag{35}$$

4.1.5 Solution concept using a quadratic cost function

I assume that the cost of funding increases quadratically with the amount of non-reservable borrowing made, as in Wang et al. (2018), $g(\cdot) = \frac{1}{2}\phi_N N^2$. In the following subsection, I translate the matrix notation in the summation notation for ease of derivation.

To solve the model presented above, I start with the Members' decision strategy, since their strategy will be taken into consideration by the Lead when it makes its allocation and pricing decision. I then follow with the Lead's allocation decision, taken the prices as given, since this decision comes after the price is accepted by the borrower and thus the pricing function considers the allocation strategy already. The pricing function incorporates the previous two decision rules. Finally, the borrowers' solutions are solved last, once I know how each other decision will be impacted by their choice.

Members

$$\max_{S_{i,k}} \sum_i S_{i,k} m_i - \gamma_k \sum_h \sum_i S_{h,k} S_{i,k} \sigma_h^r \sigma_i^r \rho_{h,i} - \frac{1}{2}\phi_N (\sum_i S_{i,k} - E_k - D_k)^2 - \zeta_k \sum_i S_{i,k} - \nu(\sum_i S_{i,k} - \frac{E_k}{\underline{e}}) \tag{36}$$

and the Karush-Kuhn-Tucker conditions become

$$m_i - \gamma_k S_{i,k} \sigma_i^{r\,2} - \gamma_k \sum_{h \neq i} S_{h,k} \sigma_h^r \sigma_i^r \rho_{h,i} - \phi_N (\sum_h S_{h,k} - E_k - D_k) - \zeta_k - \nu = 0, \ \forall i, k \tag{37}$$

$$\nu(\sum_i S_{i,k} - \frac{E_k}{\underline{e}}) = 0, \ \forall k \tag{38}$$

Now if the condition is slack, this means $\nu = 0$ and the solution is

$$S_{i,k} = \frac{1}{2\sigma_i^{r\,2}\gamma_k + \phi_N} \left[m_i^r + \phi_N (E_k + D_k - \sum_{h \neq k} S_{h,k}) - \gamma_k \sum_{h \neq i} S_{h,k} \sigma_h^r \sigma_i^r \rho_{h,i} - \zeta_k \right] \tag{39}$$

but if the condition is binding, we get

$$S_{i,k} = \frac{1}{2\sigma_i^{r\,2}\gamma_k} \left[m_i^r - \gamma_k \sum_{h \neq i} S_{h,k} \sigma_h^r \sigma_i^r \rho_{h,i} - m_0 \right] - \frac{1}{2\sigma_i^{r\,2}} \left[S_{0,k}\sigma_0^2 + \sum_h S_{h,k} \sigma_h^r \sigma_0 \rho_{h,0} \right] \tag{40}$$

where $i = 0$ is the risk-free asset. We obtain this form by solving the FOC for ν and equating the solution for any $i \in \{1, \dots, N\}$ to the risk-free security.

Furthermore, if we assume that the risk free asset is cash and has no returns nor variance, we can get the simplified form

$$S_{i,k} = \frac{1}{2\sigma_i^{r\,2}\gamma_k} \left[m_i^r - \gamma_k \sum_{h \neq i} S_{h,k} \sigma_h^r \sigma_i^r \rho_{h,i} \right] \tag{41}$$

Lead Share The Lead share problem will be very similar to the member problem except for the costs associated with the origination and the relationship benefits. Since the Lead has to bear the costs for the entire underwriting, no matter what portion of the loan it keeps on its books, the origination cost ζ_j is not impacted by the size of the share L_j. Similarly, since the Lead bank gets the relationship benefit not matter how much of the loan it keeps, β_j is not affected by L_j. Therefore, the maximization problem looks similar to the members'. The Lead obtains the same form of share choice as the member shown above.

If the condition is slack, this means $\nu = 0$ and the solution is

$$L_{i,j} = \frac{1}{2\sigma_i^{r\,2}\gamma_j + \phi_N} \left[m_i^r + \phi_N (E_j + D_j - \sum_{h \neq} L_{h,j}) - \gamma_j \sum_{h \neq i} L_{h,j} \sigma_h^r \sigma_i^r \rho_{h,i} - \zeta_j \right] \tag{42}$$

but if the condition is binding, we get

$$L_{i,j} = \frac{1}{2{\sigma_i^r}^2\gamma_j}\left[m_i^r - \gamma_j \sum_{h\neq i} L_{h,j}\sigma_h^r\sigma_i^r\rho_{h,i} - m_0\right] - \frac{1}{2{\sigma_i^r}^2}\left[L_{0,j}\sigma_0^2 + \sum_h L_{h,j}\sigma_h^r\sigma_0\rho_{h,0}\right] \tag{43}$$

where $i = 0$ is the risk-free asset.

Lead Pricing When choosing a price, the Lead will solve the following maximization problem

$$\begin{aligned}\max_{r_i} \sum_i L_{i,j} r_i \mathbb{E}(1-\theta\lambda_i) - \gamma_j \sum_h \sum_i L_{h,j}L_{i,j}r_h r_i \theta^2 \sigma_h\sigma_i\rho_{h,i} - \frac{1}{2}\phi_N(\sum_i L_{i,j} - E_j - D_j)^2 \\ - (\zeta+\beta) - \nu(\sum_i L_{i,j} - \frac{E_j}{\underline{e}})\end{aligned} \tag{44}$$

with the First-Order-Conditions yielding

$$r_i = \frac{\mathbb{E}(1-\theta\lambda_i) - 2\gamma_j\theta\sum_h L_{h,j}r_h\sigma_h\sigma_i\rho_{h,i}}{2\gamma_j\sigma_i^2\theta^2 L_{i,j}} \tag{45}$$

Borrowers The borrowers are solving a discrete choice problem where they observe all the prices that the leads can offer them and will then choose which Lead to approach in order to maximize their utility. They make decision based on the price, the size, and a relationship and convenience coefficient that captures unobserved heterogeneity between lenders' offers. For the sake of the model's tractability, I assume that borrower's loan defaults are independent of each other.

4.2 Estimation

The solution concepts described in subsection 4.1.5 can then be summarized in three equations, each with a single parameter to be estimated. First, the Lead's pricing problem becomes a series of pricing equations with respect to the borrowers probability of default λ_i, its default variance σ_i^2, the Lead lender's retained share of the loan L_{ij}, and the Lead lender's risk aversion parameter γ_j.

$$r_i = \frac{1}{\gamma_j}\frac{\mathbb{E}[1-\theta\lambda_i]}{2\sigma_i^2 L_{ij}} \tag{46}$$

Recall that I assume that the default occurrence is a Bernoulli random variable, therefore $\sigma_i^2 = \theta^2\lambda_i(1-\lambda_i)$.

Once I obtain each Lead lender's risk aversion parameter γ_j, I can then use the Lead's Share Problem that will identify the cost of funding ϕ_N. As I assume all lenders face the same borrowing cost function on the market, I use the subset of unconstrained Lead lenders to estimate ϕ_N

$$\phi_N = \frac{2r_i^2\sigma_i^2\gamma_j L_{i,j} - r_i\mathbb{E}[1-\theta\lambda_i]}{E_j + D_j - L_{i,j}} \tag{47}$$

With the cost of borrowing on the market, I can now estimate the risk aversion parameters of all the constrained leads and members by MLE on the respective solutions exposed in section 4.1.5.

4.2.1 Probability of Default Estimation

A large portion of the analysis rests upon estimating the probability of default of the borrowers and assessing how the lenders react to it. I have chosen for the most common approach in the literature, namely estimating the PD from a Distance to Default measure implied by the Merton model (Black and Scholes, 1973; Merton, 1974; Vasicek, 1997; Crosbie and Bohn, 2002; Vassalou and Xing, 2004; Duffie et al., 2007). I implemet the measure of distance to default from Duffie et al. (2007) as it is particularly adapted to Compustat data, and define A_i as the firm's balance sheet assets, B_i as its liabilities, r_0 as the risk-free rate, $\sigma_{i,A}$ as the assets historical volatility, and T as the time to maturity of the loan.

$$\text{Distance to Default: } DtD_i = \frac{\ln \frac{A_i}{B_i} + \left(r_0 - \frac{\sigma_{i,A}^2}{2}\right) T}{\sigma_{i,A}\sqrt{T}} \tag{48}$$

$$\text{Probability of Default: } \lambda_i = \Phi(-DtD_i) \tag{49}$$

where Φ is the standard Normal cumulative distribution function. I use as recovery rate $\theta = 0.4$ at is commonly the case for debt products (Bush et al., 2011).

4.2.2 Risk aversion of different types of financial institutions

By estimating equation (46), we can obtain estimates of each Lead lender's risk aversion parameter. The estimation methodology yields a different risk aversion parameter for each bank participating in a syndicate. Though each equation could potentially yield a different parameter for each loan, I aggregate by bank and find the best fit for the parameter using maximum likelihood estimation. The following regression verifies that banks are more risk-averse than non-banks.

$$\gamma_j = \alpha + \mathbb{1}_{j,\text{Pension}} + \mathbb{1}_{j,\text{Insurance}} + \mathbb{1}_{j,\text{Hedge Fund}} + \epsilon_j \tag{50}$$

Table 3 shows the resulting difference in average risk aversions of non-bank financial institutions when compared to banks, as classified in the DealScan dataset. As expected, banks have the highest risk-aversion parameter, pension plans are not significantly different from them, and hedge funds have a lower risk aversion than banks. Insurance companies seem to have a lower risk aversion than hedge funds, but it should be noted that the category of Hedge Funds is quite diverse and vast, so different risk profiles are to be expected and it is possible that, on average, they appear more risk-averse.

5 Counterfactuals

To measure the effect of the syndicate's structure on the lending system's risk, I compare the potential loss distribution of the syndicated loan market under the current regulatory environment to various counterfactual scenarios with changing regulations. The potential loss distribution is measured by factoring the probability of default (PD) of each asset with the loss given default (LGD) of the asset, and the firm's exposure at default (EAD) to the asset.

Table 3: Risk Aversion by Institution's Type

	Dependent variable:
	Risk Aversion
Constant	0.003***
	(0.0001)
Pension	−0.002
	(0.002)
Insurance	−0.002***
	(0.0005)
HedgeFund	−0.001***
	(0.0001)
Observations	1,877
R^2	0.064
Note:	*p<0.1; **p<0.05; ***p<0.01

The EAD is assumed to be the weighted size of the loan $w_{i,j}B_i$ that each firm allocates to its portfolio, as I do not observe secondary markets or hedging strategies. Moreover, based on previous studies, I assume that the LGD is 60%, since credit instruments often have a recovery mechanism that allows the lender to recover a portion of the defaulted loan through restructuring. Finally, the PD of each loan is estimated as (49). Thus, the Loss distribution used to compare counterfactual scenarios takes the following form:

$$PL = \sum_{i,j} PD_i \times LGD_i \times EAD_{i,j} \tag{51}$$

$$= \sum_{i,j} \lambda_i \times \theta \times w_{i,j} B_i \tag{52}$$

When estimating counterfactual scenarios, the main driver of changes in Potential Loss between scenarios is $w_{i,j}$, the allocated weights of each lender to each borrower. Changes in regulation will affect the cost of capital of lenders and change their portfolio allocation. This weight variable is also affected by the price impact that different regulations might have on Lead lender's choice of r_i, since Members seek to maximize expected returns. Hence, the two choice variables of the structural model's Lead lender have direct impact on the lending market's Potential Loss distribution. Both of these choice variables are dependent on the model's key innovation, which is to model each financial institution as having a separate risk-aversion parameter γ_j, estimated through the model calibration of section 4.2.

To assess systematic and systemic risk, I use an array of common risk metrics. The first is the Value-at-Risk (VaR) of the industry's loan portfolio, consisting of the aggregate positions and distribution of probability of default of the loans. The VaR is typically reported at the 95% level, which is simply the dollar value of the ninety-fifth percentile of the potential loss distribution. VaR is a common risk measure but is not always sufficient, especially when interested in systemic risk. A complementary measure is expected shortfall (ES), the expected value of the tail end of the loss distribution. Its estimation follows the same procedure as the VaR, but goes one step further in that after having found the ninety-fifth percentile of the loss distribution, I then estimate the mean of the values between the ninety-fifth and one hundredth percentile. Finally, from the expected shortfall, one can estimate the marginal expected shortfall (mES) as the derivative with respect to each agent's individual contribution to the loss distribution to assess each financial institution's contribution to aggregate risk, considered an indicator of the institution's systemic importance (Rösch and Scheule, 2008).

Using these metrics, I am able to compare the risk distribution across different policy-relevant scenarios. Below, I present counterfactual results from changing the regulatory capital requirement, $\underline{e}$. First I impose the same regulatory constraint on non-banks as I do to banks. This allows me to gauge how much risk is taken on by the non-bank sector in corporate lending. Second, I estimate the system's risk under a range of level of capital requirement, to understand how constraining Basel III's requirement actually are. Both of these counterfactual experiments help understand how the lending market might be affected by its structure, and how fine-tuning regulations can alleviate potential problems.

5.1 Baseline estimation

Using the methodology of section 4.2, we can estimate the potential loss distribution of our system of banks and non-banks under the current set of regulations. Figure 3 shows the density function of the potential losses, as well as the Value-at-Risk and the Expected Shortfall. As expected, this distribution is a long-tailed one, with most of the losses being of smaller caliber.

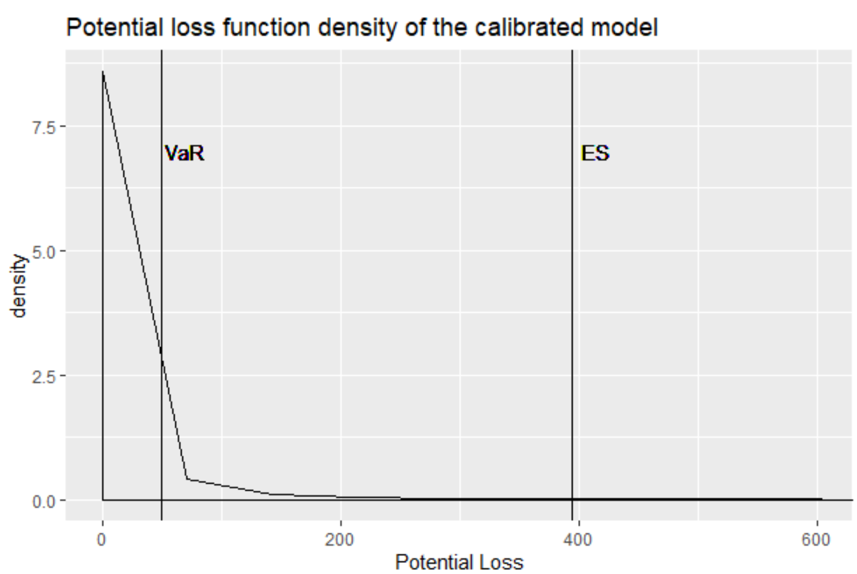

Figure 3: Potential Loss using calibrated parameters

5.2 Regulatory requirements

5.2.1 Extensive margin: Impose capital constraints on all lenders

I run a counterfactual scenario in which all financial institutions, both banks and non-banks, in order to participate in the lending market, must comply with the same required level of capital to fund their loans. Under this scenario, I estimate the potential loss function presented in figure 4. There are three notable results we can observe from the figure. First, the Value-at-Risk is slightly lower than in the baseline case, but by a marginal amount.

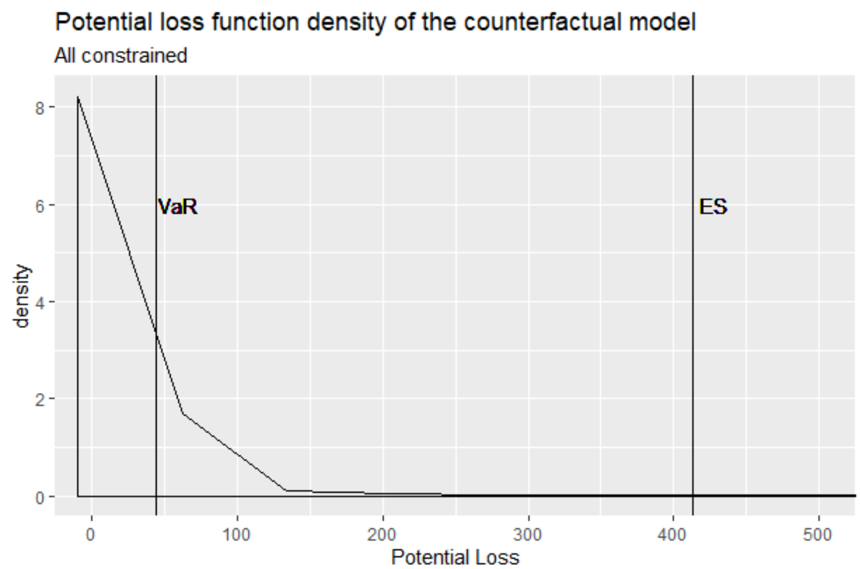

Figure 4: Potential Loss when all FIs are constrained

Second, and more importantly, is that the Expected Shortfall is much higher. This suggests that, though 95% of potential losses might be lower than under the current regulatory requirements, in the 5% of cases where we experience a severe shock, we can expect the losses to be much higher. This increase in Expected Shortfall is further evidence of the fat-tailed nature of the loss distribution. So tightening regulation on all financial institutions might push some actors into taking even greater risks, pushing the ES further out in the tail.

The third observation to make is that the mass to the left of the Value-at-Risk is considerably more concentrated towards zero. This is because of a credit-rationing effect of increasing the regulatory requirements. The institutions who would have participated more heavily in a lending syndicate are now constrained by their capital requirement and can only lend out a smaller portion of the loan, thus reducing the availability of larger sized loans to worthy borrowers. When plotting this change in potential loss, it becomes clear that the mass moves leftwards, while the tail moves rightward, as shown on Figure 5.

Figure 6 overlaps the two densities, with the solid lines representing the density and risk measures of the calibrated baseline model and the dashed lines representing the density and the risk measures of the counterfactual scenario where all financial institutions are bound by the capital requirement ratio. It is clear from this graph that the density shifts and the risk increases when the requirement start binding non-banks with greater risk appetite.

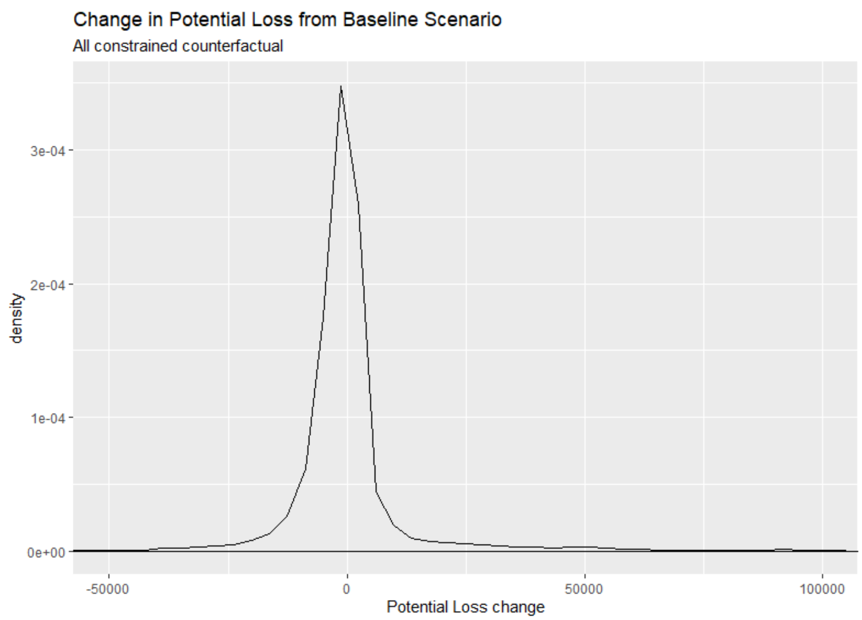

Figure 5: Change in Potential Loss when all FIs are constrained

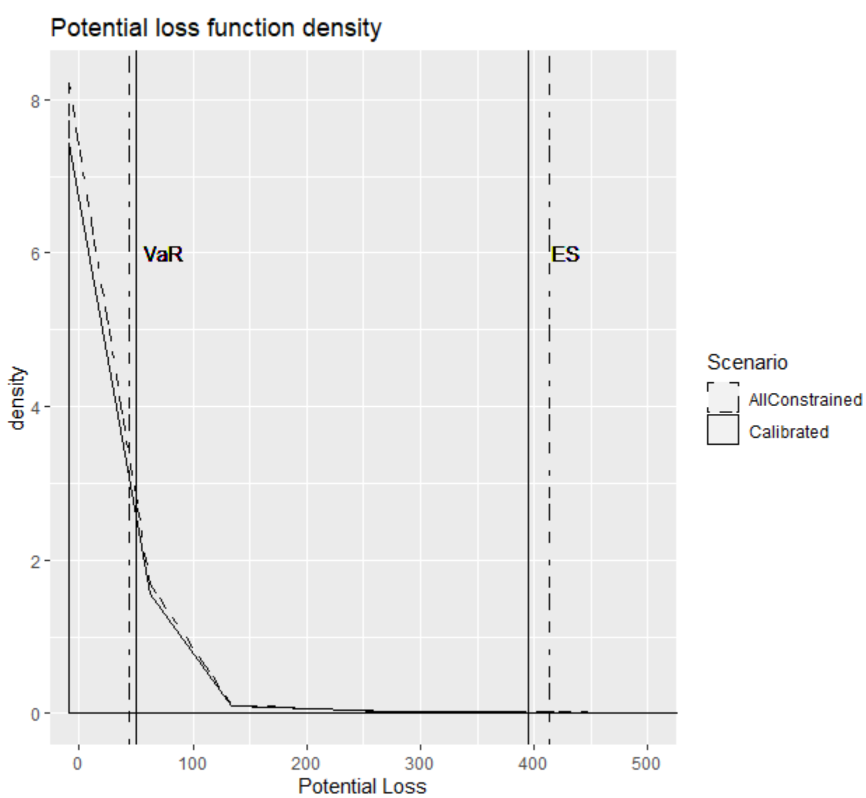

Figure 6: Comparison when imposing capital constraints on all lenders

When decomposing the variation of risk between banks and non-banks syndicates, using Montgomery-decomposition (De Boer, 2008), I observe both types of syndicate increasing the exposure to risk, though non-bank syndicates increase their potential loss exposure by 2.28 times the increase observed in bank-only syndicates. The change in marginal Expected Shortfall (mES) can also be indicative that the increase in risk comes from the pull from the non-bank syndicates. It has been used to measure financial institutions' contribution to systemic risk (Acharya et al., 2017) and is useful in my context to understand which parts of the lending market contributes most to risk-taking. Marginal Expected shortfall is calculated as the partial derivative of the expected shortfall with respect to a specific individual or type's share S_i, i.e.,

$$mES_i = \frac{\partial ES_\alpha(PL)}{\partial S_i} = \mathbb{E}[PL_i | PL \geq VaR_\alpha(PL)] \tag{53}$$

When computing the marginal expected shortfall under the baseline scenario, banks are 6.7% under the overall ES while non-banks are 0.89% above. However, under the counterfactual scenario, banks are now only 5.25% under the aggregated ES, while non-banks are 2.82% above. Therefore I conclude that even though the risk decreases marginally under a blanket regulation, the non-banks syndicate members significantly increase their participation in risky investments, contributing to a large extent to the increase in the system's expected shortfall.

Funding Type	PD	Facility Size	PL	mES
Bank funded	5.23%	385.83	8.329	513.34
Market Funded	7.96%	721.44	30.087	769.38

Table 4: Calibrated Model: Average risk summary

Funding Type	PD	Facility Size	PL	mES
Bank funded	5.23%	183.98	8.371	235.48
Market Funded	7.96%	746.08	31.901	448.78

Table 5: Counterfactual all constrained: Average risk summary

5.2.2 Intensive margin: Changing regulatory capital requirements on banks

The other main policy of interest is to test whether the capital requirement level is at an appropriate level. I test this policy by re-estimating the model under 100 different scenarios of increasing level of capital requirement. This counterfactual takes into consideration the implicit cost of balance sheet funding from equation (24). I start from a requirement of 0.1%, which basically amounts to no requirement since this level is almost guaranteed to not be binding, and I increase it up to a level of 35%. The result is shown in Figure 7. Note that I only change the level of capital requirement on banks and do not impose such constraint on the non-banks.

In Figure 7, the risk levels are shown to be a non-monotonic function of the capital requirement. The risk decreases up to a ratio of about 40%, and then slowly starts to increase again. It is also interesting to not that the skewness of the distribution decreases, as the Expected Shortfall shrinks faster than the Value-at-Risk. In the end, the current capital requirement ratio imposed by Basel III of 8% is definitely contributing to the soundness of the lending market.

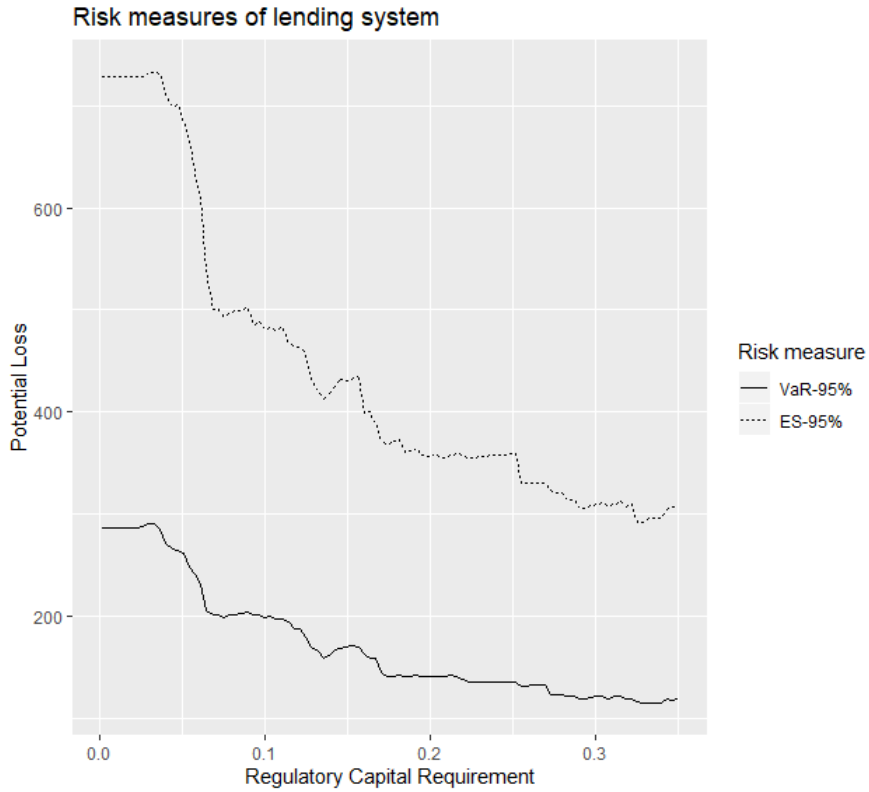

Figure 7: Testing risk levels under different regulatory scenarios

6 Conclusion

I develop a structural model with mean-variance preferences over portfolio allocation. Using this model, I estimate the impact of capital requirements on systemic risk and risk distribution in the corporate syndicated lending market. Consistent with empirical literature, I find that banks are more risk averse than hedge funds, and that hedge funds provide a inflow of funds to expand the lending market. I analyse whether this inflow of unregulated funds increases risk taking behaviour in the lending market.

Through counterfactual analysis, I estimate that constraining all lenders to the same set of regulatory requirement would reduce the availability of funds by credit rationing, while increasing significantly the expected shortfall of the system and having minimal impact on reducing the Value-at-Risk. The main result of the paper suggests that extending regulatory requirement to non-banks would make the system more prone to extreme shocks while reducing the quantity of credit in the economy. In the current system, banks use the increase in participation of non-banks in order to extend credits to more borrowers of adequate risk level while the non-banks absorb much of the risk induced by the expansion of credit supply.

This analysis provides important insight into the issue of whether the shadow banking system should not be regulated merely as an extension of banking, but rather as an affiliate but distinct market that complements many of the functionalities of the traditional banking system by providing funds and reallocating risks.

References

Acharya, V. 2009. "A theory of Systemic Risk and Design of Prudential Bank Regulation." *Journal of Financial Stability.*

Acharya, V.V., L.H. Pedersen, T. Philippon, and M. Richardson. 2017. "Measuring Systemic risk." *Review of Financial Studies* 30.

Admati, A., and M. Hellwig. 2013. *The Bankers' New Clothes.* Princeton, NJ: Princeton University Press.

Adrian, T., and A.B. Ashcraft. 2012. "Shadow Bank Regulation." *Annual Review of Financial Economics* 4.

Aramonte, S., S.J. Lee, and V. Stebunovs. 2019. "Risk taking and low longer-term interest rates: Evidence from the U.S. syndicated term loan market." *Journal of Banking & Finance.* doi:https://doi.org/10.1016/j.jbankfin.2019.02.010.

Armstrong, J. 2003. *The Syndicated Loan Market: Developments in the North American Context.* Technical report. Financial System Review. Bank of Canada.

Basel Comittee on Banking Supervision. 2019. *Basel III: Minimum capital requirements for market risk.* Technical report. Retrieved on December 14 2020 from https://www.bis.org/bcbs/publ/d457.pdf. Bank for International Settlements.

Bédard-Pagé, G. 2019. *Non-Bank Financial Intermediation in Canada: An Update.* Staff Discussion Paper 2. Available at https://www.bankofcanada.ca/2019/03/non-bank-financial-intermediation-in-canada-an-update/. Bank of Canada, March.

Benmelech, E., J. Dlugosz, and V. Ivashina. 2012. "Securitization without adverse selection: The case of CLOs." *Journal of Financial Economics.*

Black, F., and M. Scholes. 1973. "The Pricing of Options and Corporate Liabilities." *The Journal of Political Economy* 81:637–654.

Bord, V., and J.A.C. Santos. 2012. "The rise of originate to distribute model and the role of banks in financial intermediation." *Economic Policy Review.*

Bruche, M., F. Malherbe, and R. Meisenzahl. 2017. "Pipeline Risk in Leveraged Loan Syndication." Available at https://ssrn.com/abstract=2958675, *FEDS Working Paper No. 2017-048* (April).

Buchak, G., G. Matvos, T. Piskorski, and A. Seru. 2020. *Beyond the Balance Sheet Model of Banking: Implications for Bank Regulation and Monetary Policy.* Working Paper 25149. Available at https://www.nber.org/papers/w25149. National Bureau of Economic Research.

Bush, N., B.M. Hambly, H. Haworth, L. Jin, and C. Reisinger. 2011. "Stochastic evolution equations in portfolio credit modelling." *SIAM Journal on Financial Mathematics.*

Cai, J., F. Eidam, A. Saunders, and S. Steffen. 2018. "Syndication, interconnectedness, and systemic risk." *Journal of Financial Stability.*

Chava, S., and M. Roberts. 2008. "How does financing impact Investment? The role of debt covenants." *Journal of Finance.*

Crosbie, P.J., and J.R. Bohn. 2002. *Modeling default risk.* Technical report. KMV, LLC.

De Boer, P. 2008. "Additive Structural Decomposition Analysis and Index Number Theory: An Empirical Application of the Montgomery Decomposition." *Economic Systems Research* 20.

Duffie, D., L. Saita, and K. Wang. 2007. "Multi-period corporate default prediction with stochastic covariates." *Journal of Financial Economics.*

Federal Reserve Board. 2019. *Financial Stability Report.* Technical report. Available at https://www.federalreserve.gov/publications/files/financial-stability-report-201905.pdf.

Ivashina, V., and D. Scharfstein. 2010a. "Bank lending during the financial crisis of 2008." *Journal of Financial Economics* 97 (July).

———. 2010b. "Loan Syndication and Credit Cycles." *American Economic Review: Papers and Proceedings* 100 (May).

Jensen, M. C., and W.H. Meckling. 1976. "Theory of the firm: Managerial behavior, agency costs and ownership structure." *Journal of Financial Economics* 3.

Kashyap, A. K., and J. C. Stein. 1995. "The impact of monetary policy on bank balance sheets." In *Carnegie-Rochester Conference Series on Public Policy,* vol. 42. Elsevier.

Król, J., and N. Williams. 2019. *Non-bank lending in the European Union.* Technical report. Alternative Credit Council.

Lim, J., B. Minton, and M. Weisbach. 2014. "Syndicated Loan spreads and the composition of the syndicate." *Journal of Financial Economics.*

Markowitz, H.M. 1952. "Portfolio Selection." *The Journal of Finance* 7 (March): 77–91. doi:10.2307/2975974.

Merton, R. 1974. "On the pricing of corporate debt: The risk structure of interest rates." *Journal of Finance* 29:449–470.

Poeschl, J., and X. Zhang. 2018. "Bank Capital Regulation and Endogenous Shadow Banking Crises." *MPRA Working Paper.*

Rösch, D., and H. Scheule, eds. 2008. *Stress Testing for financial institutions: Applications, Regulations and Techniques.* RiskBooks.

Schwert, M. 2018. "Bank Capital and Lending Relationship." *Journal of Finance* 73, no. 2 (April).

Shu, C. 2020. "Endogenous Risk-Exposure and Systemic Instability." Avaialble at https://ssrn.com/abstract=3076076, *SSRN.*

Sierra, M. 2019. "Leveraged loan market size doubles in ten years, private credit explodes." Accessed April 30, 2020. https://www.reuters.com/article/levloan-decade/leveraged-loan-market-size-doubles-in-ten-years-private-credit-explodes-idUSL1N28U0QQ.

Simons, K. 1993. "Why do banks syndicate loans?" *New England Economic Review.*

Vasicek, O.A. 1997. "An equilibrium characterization of the term structure." *Journal of Financial Economics 5.*

Vassalou, M., and Y. Xing. 2004. "Default Risk in Equity Returns." *Journal of Finance* 59.

Wang, Y., T. Whited, Y. Wu, and K. Xiao. 2018. "Bank Market Power and Monetary Policy Transmission: Evidence from a Structural Estimation." Available at https://ssrn.com/abstract=3049665, *Working Paper.*

Xiao, K. 2019. "Monetary transmission through Shadow Banks." Available at https://ssrn.com/abstract=3348424.

A Notation

Borrower i's loan request size: B_i

Available assets: $\boldsymbol{B} = \begin{bmatrix} B_0 \\ B_1 \\ \vdots \\ B_I \\ B_{I+1} \end{bmatrix}$

Lead j's deals: $\vec{\mathbb{1}}\{i \in \mathcal{A}_j\}$

Lead j's share of loan kept on balance sheet: $\boldsymbol{w}_j = \begin{bmatrix} w_{0j} \\ w_{1j} \\ \vdots \\ w_{Ij} \\ w_{I+1j} \end{bmatrix}$

Lead j's amount size: $\boldsymbol{L}_j = \boldsymbol{w}_j \cdot \boldsymbol{B} = \begin{bmatrix} w_{0j}B_0 \\ w_{1j}B_1 \\ \vdots \\ w_{Ij}B_I \\ w_{I+1,j}B_{I+1} \end{bmatrix}$

Lead j's offered interest rate: $\boldsymbol{R}_j = \begin{bmatrix} r_0 \\ r_{1j} \\ \vdots \\ r_{Ij} \\ r_{I+1} \end{bmatrix}$

Syndication Share: $\boldsymbol{V} = \sum_k \boldsymbol{v}_k = \sum_k \begin{bmatrix} v_{0,k} \\ v_{1,k} \\ \vdots \\ v_{I,k} \\ v_{I+1,k} \end{bmatrix}$ s.t. $\sum_k \boldsymbol{v}_k + \sum_j \boldsymbol{w_j} \leq 1$

Syndication Size: $\boldsymbol{S} = \boldsymbol{V} \cdot \boldsymbol{B}$

Syndication Size of member k: $\boldsymbol{S}_k = \boldsymbol{v}_k \cdot \boldsymbol{B}$

Lead j's required financing: $\boldsymbol{B}'\vec{\mathbb{1}}\{i \in \mathcal{A}_j\}$

Syndication size of Lead j's deals$\boldsymbol{S}'\vec{\mathbb{1}}\{i \in \mathcal{A}_j\}$

Lead j's market financing: $N_j = (\underbrace{\boldsymbol{B}}_{\text{Required}} - \underbrace{\boldsymbol{L}_j'}_{\text{Balance Sheet}} - \underbrace{\boldsymbol{S}}_{\text{Syndication}})\underbrace{\vec{\mathbb{1}}\{i \in \mathcal{A}_j\}}_{\text{Lead } j\text{'s deals}}$

B Toy Model: Lead's allocation solution

To match the member's contribution to the syndicate so that each loan is fully funded, i.e., $w_{i,L} = 1 - w_{i,M}$, the Lead's optimal allocation will yield the following weights

$$w_{1,L} = \frac{(1-\theta\lambda_1)r_1 - (1-\theta\lambda_2)r_2 + \gamma_M B_1 \lambda_1(1-\lambda_1)\theta^2 r_1^2 - \left(\frac{E_M}{\underline{\mathrm{e}} B_2} - \frac{B_1}{B_2}\right)\gamma_M \lambda_2(1-\lambda_2)\theta^2 r_2^2}{B_1 \gamma_M [\lambda_1(1-\lambda_1)\theta^2 r_1^2 + \lambda_2(1-\lambda_2)\theta^2 r_2^2]} \tag{54}$$

$$w_{2,L} = \frac{(1-\theta\lambda_2)r_2 - (1-\theta\lambda_1)r_1 + \gamma_M B_2 \lambda_2(1-\lambda_2)\theta^2 r_2^2 - \left(\frac{E_M}{\underline{\mathrm{e}} B_1} - \frac{B_2}{B_1}\right)\gamma_M \lambda_1(1-\lambda_1)\theta^2 r_1^2}{B_2 \gamma_M [\lambda_1(1-\lambda_1)\theta^2 r_1^2 + \lambda_2(1-\lambda_2)\theta^2 r_2^2]} \tag{55}$$

We can then reduce the Lead underwriter's problem to one of maximisation with respect to the price of the loans, which needs to balance the member's incentives with the condition of fulfilling the entirety of the loan, while also managing its own balance sheet regulatory requirements. Assuming independence between the two loans, the maximization problem of the Lead underwriter becomes:

$$\begin{aligned} &\max_{r_1,r_2} w_{1,L}(r_1,r_2) r_1 B_1(1-\theta\lambda_1) + w_{2,L}(r_1,r_2) r_2 B_2 (1-\theta\lambda_2) \\ &\qquad - \frac{1}{2}\gamma_L \left[\left(w_{1,L}(r_1,r_2) B_1 \theta r_1\right)^2 \lambda_1(1-\lambda_1) + \left(w_{2,L}(r_1,r_2) B_2 \theta r_2\right)^2 \lambda_2(1-\lambda_2)\right] \qquad (56) \\ &\text{s.t. } w_{1,L} B_1 + w_{2,L} B_2 \le \frac{E_L}{\underline{\mathrm{e}}} \qquad (57) \end{aligned}$$

with the weights $w_{1,L}$ and $w_{2,L}$ being functions defined in (54) and (55).

Deriving the first order conditions of the above model yields the following solution:

$$\begin{aligned} \text{w.r.t. } r_1: \;& \left(w_{1,L}'^{(r_1)} r_1 + w_{1,L}\right) B_1(1-\theta\lambda_1) + w_{2,L}'^{(r_1)} r_2 B_2 (1-\theta\lambda_2) \\ & - \frac{1}{2}\gamma_L \left[2(w_{1,L} r_1 B_1 \theta)\left(w_{1,L}'^{(r_1)} r_1 + w_{1,L}\right) B_1\theta\lambda_1(1-\lambda_1) + 2(w_{2,L} r_2 B_2 \theta)\left(w_{2,L}'^{(r_1)} r_2\right) B_2 \theta \lambda_2(1-\lambda_2)\right] \\ & - \nu\left(w_{1,L}'^{(r_1)} B_1 + w_{2,L}'^{(r_1)} B_2\right) = 0 \qquad (58) \\ \text{w.r.t. } r_2: \;& \left(w_{2,L}'^{(r_2)} r_1\right) B_1(1-\theta\lambda_1) r_1 + \left(w_{2,L}'^{(r_2)} r_2 + w_{2,L}\right) B_2 (1-\theta\lambda_2) \\ & - \frac{1}{2}\gamma_L \left[2(w_{1,L} r_1 B_1 \theta)\left(w_{1,L}'^{(r_2)} r_1\right) B_1\theta\lambda_1(1-\lambda_1) + 2(w_{2,L} r_2 B_2 \theta)\left(w_{2,L}'^{(r_2)} r_2 + w_{2,L}\right) B_2 \theta \lambda_2(1-\lambda_2)\right] \\ & - \nu\left(w_{1,L}'^{(r_2)} B_1 + w_{2,L}'^{(r_2)} B_2\right) = 0 \qquad (59) \\ \text{w.r.t. } \nu: \;& w_{1,L} B_1 + w_{2,L} B_2 - \frac{E_L}{\underline{\mathrm{e}}} = 0 \qquad (60) \end{aligned}$$

These FOCs rely on the derivatives of the Lead's portfolio weights described in (54) and (55). The conditions (58) and (59) can be combined to equate $\nu = \nu$ to yield

$$\left\{ \left(w_{1,L}^{\prime(r_1)} r_1 + w_{1,L}\right) B_1(1-\theta\lambda_1) + \left(w_{2,L}^{\prime(r_1)} r_2\right) B_2(1-\theta\lambda_2) \right.$$
$$-\frac{1}{2}\gamma_L \left[2(w_{1,L} r_1 B_1 \theta)\left(w_{1,L}^{\prime(r_1)} r_1 + w_{1,L}\right) B_1\theta\lambda_1(1-\lambda_1) + \right.$$
$$\left.\left. 2(w_{2,L} r_2 B_2 \theta)\left(w_{2,L}^{\prime(r_1)} r_2\right) B_2\theta\lambda_2(1-\lambda_2)\right]\right\}$$
$$\times \left(w_{1,L}^{\prime(r_2)} B_1 + w_{2,L}^{\prime(r_2)} B_2\right) = \left\{ \left(w_{2,L}^{\prime(r_2)} r_1\right) B_1(1-\theta\lambda_1) r_1 + \left(w_{2,L}^{\prime(r_2)} r_2 + w_{2,L}\right) B_2(1-\theta\lambda_2) \right.$$
$$-\frac{1}{2}\gamma_L \left[2(w_{1,L} r_1 B_1 \theta)\left(w_{1,L}^{\prime(r_2)} r_1\right) B_1\theta\lambda_1(1-\lambda_1) \right.$$
$$\left.\left. +2(w_{2,L} r_2 B_2 \theta)\left(w_{2,L}^{\prime(r_2)} r_2 + w_{2,L}\right) B_2\theta\lambda_2(1-\lambda_2)\right]\right\}$$
$$\times \left(w_{1,L}^{\prime(r_1)} B_1 + w_{2,L}^{\prime(r_1)} B_2\right) \tag{61}$$

The derivatives of the portfolio weights (54) and (55) are expressed as:

$$\frac{\partial w_{1,L}}{\partial r_1} = \frac{[(1-\theta\lambda_1) + 2\gamma_M B_1\lambda_1(1-\lambda_1)\theta^2 r_1] \times B_1\gamma_M[\lambda_1(1-\lambda_1)\theta^2 r_1^2 + \lambda_2(1-\lambda_2)\theta^2 r_2^2]}{\gamma_M^2 B_1^2[\lambda_1(1-\lambda_1)\theta^2 r_1^2 + \lambda_2(1-\lambda_2)\theta^2 r_2^2]^2}$$
$$- \frac{\left[(1-\theta\lambda_1) r_1 - (1-\theta\lambda_2) r_2 + \gamma_M B_1\lambda_1(1-\lambda_1)\theta^2 r_1^2 - \left(\frac{E_M}{\underline{e}B_2} - \frac{B_1}{B_2}\right)\gamma_M\lambda_2(1-\lambda_2)\theta^2 r_2^2\right] \times 2\gamma_M B_1\lambda_1(1-\lambda_1)\theta_1^2 r_1}{\gamma_M^2 B_1^2[\lambda_1(1-\lambda_1)\theta^2 r_1^2 + \lambda_2(1-\lambda_2)\theta^2 r_2^2]^2} \tag{62}$$

$$\frac{\partial w_{2,L}}{\partial r_1} = \frac{\left[-(1-\theta\lambda_1) - 2\gamma_M\left(\frac{E_M}{\underline{e}B_1} - \frac{B_2}{B_1}\right)\lambda_1(1-\lambda_1)\theta^2 r_1\right] \times B_2\gamma_M[\lambda_1(1-\lambda_1)\theta^2 r_1^2 + \lambda_2(1-\lambda_2)\theta^2 r_2^2]}{\gamma_M^2 B_2^2[\lambda_1(1-\lambda_1)\theta^2 r_1^2 + \lambda_2(1-\lambda_2)\theta^2 r_2^2]^2}$$
$$- \frac{\left[(1-\theta\lambda_2) r_2 - (1-\theta\lambda_1) r_1 + \gamma_M B_2\lambda_2(1-\lambda_2)\theta^2 r_2^2 - \left(\frac{E_M}{\underline{e}B_1} - \frac{B_2}{B_1}\right)\gamma_M\lambda_1(1-\lambda_1)\theta^2 r_1^2\right] \times 2\gamma_M B_2\lambda_1(1-\lambda_1)\theta^2 r_1}{\gamma_M^2 B_2^2[\lambda_1(1-\lambda_1)\theta^2 r_1^2 + \lambda_2(1-\lambda_2)\theta^2 r_2^2]^2} \tag{63}$$

$$\frac{\partial w_{1,L}}{\partial r_2} = \frac{\left[-(1-\theta\lambda_2) - 2\gamma_M\left(\frac{E_M}{\underline{e}B_2} - \frac{B_1}{B_2}\right)\lambda_2(1-\lambda_2)\theta^2 r_2\right] \times B_1\gamma_M[\lambda_1(1-\lambda_1)\theta^2 r_1^2 + \lambda_2(1-\lambda_2)\theta^2 r_2^2]}{\gamma_M^2 B_1^2[\lambda_1(1-\lambda_1)\theta^2 r_1^2 + \lambda_2(1-\lambda_2)\theta^2 r_2^2]^2}$$
$$- \frac{\left[(1-\theta\lambda_1) r_1 - (1-\theta\lambda_2) r_2 + \gamma_M B_1\lambda_1(1-\lambda_1)\theta^2 r_1^2 - \left(\frac{E_M}{\underline{e}B_2} - \frac{B_1}{B_2}\right)\gamma_M\lambda_2(1-\lambda_2)\theta^2 r_2^2\right] \times 2\gamma_M B_1\lambda_2(1-\lambda_2)\theta^2 r_2}{\gamma_M^2 B_1^2[\lambda_1(1-\lambda_1)\theta^2 r_1^2 + \lambda_2(1-\lambda_2)\theta^2 r_2^2]^2} \tag{64}$$

$$\frac{\partial w_{2,L}}{\partial r_2} = \frac{[(1-\theta\lambda_2) + 2\gamma_M B_2\lambda_2(1-\lambda_2)\theta^2 r_2] \times B_2\gamma_M[\lambda_1(1-\lambda_1)\theta^2 r_1^2 + \lambda_2(1-\lambda_2)\theta^2 r_2^2]}{\gamma_M^2 B_2^2[\lambda_1(1-\lambda_1)\theta^2 r_1^2 + \lambda_2(1-\lambda_2)\theta^2 r_2^2]^2}$$
$$- \frac{\left[(1-\theta\lambda_2) r_2 - (1-\theta\lambda_1) r_1 + \gamma_M B_2\lambda_2(1-\lambda_2)\theta^2 r_2^2 - \left(\frac{E_M}{\underline{e}B_1} - \frac{B_2}{B_1}\right)\gamma_M\lambda_1(1-\lambda_1)\theta^2 r_1^2\right] \times 2\gamma_M B_2\lambda_2(1-\lambda_2)\theta^2 r_2}{\gamma_M^2 B_2^2[\lambda_1(1-\lambda_1)\theta^2 r_1^2 + \lambda_2(1-\lambda_2)\theta^2 r_2^2]^2} \tag{65}$$