# Intangibles, Concentration, and the Labor Share[*]

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## Abstract

Over the past three decades, the U.S. business sector has been characterized by increasing concentration and decreasing measured labor share. Over the same period, investment in BEA-measured intangible capital, mainly software and R&D, has grown rapidly as a share of total business income. This paper develops a quantitative general equilibrium model of firm dynamics and shows that intangibles play a key role in understanding the trends in measured labor share and concentration jointly, taking both the intangible-related technical change and measurement issue of labor share into consideration. The model is consistent with important aspects of firm behavior at the micro level. An intangible-investment-specific technical change (IISTC) shifts the distribution of firms toward large, intangible-intensive firms with low labor shares. When the IISTC is calibrated to match the observed decline in the relative price of intangible investment goods, the model can account for a significant part of the observed rise in concentration and the observed decline in the measured labor share jointly.

**Keywords**: Intangible Investment, Concentration, Labor Share, Technical Change

**JEL Classification**: D21, D25, D33, E13, E22, E23, E25, L25, O31, O33, O34

# 1 Introduction

Over the past three decades, the U.S. business sector has been characterized by increased industrial concentration in terms of the employment share and market share of very large firms at the national level as well as declined measured labor share. Over the same period, BEA-measured investment in intangibles, mainly software and R&D, has risen relative to total business income. These three trends may be linked to each other. Autor, Dorn, Katz, Patterson, and Van-Reenen (2019) find that the rise in industrial concentration and the fall in labor share are positively associated. However, the potential force that drives this relationship is still an open question.

Intangibles may play a central role in understanding the trends in measured labor share and concentration jointly. First, the measurement of labor share depends on the measurement of intangibles. Second, declining labor share can be driven by some intangible-related technology advances, such as a rapid fall in the relative price of intangibles, particularly software, due to some potential substitutability between labor and intangible capital. Moreover, intangible capital, compared to traditional physical capital, has some distinct economic features such as non-rivalry and scalability. These features promote economies of scale, making firms with high productivity in producing intangibles become larger, which may have enabled the rise in industrial concentration.

This paper quantitatively explores the role of intangibles in explaining the secular change in concentration and measured labor share over the recent three decades. Compared to the existing literature that jointly studies the evolution of labor share and concentration, particularly, Aghion, Bergeaud, Boppart, Klenow, and Li (2019); Akcigit and Ates (2019); Autor, Dorn, Katz, Patterson, and Van-Reenen (2019); DeLoecker and Eeckhout (2017), the novelty of this paper is twofold. First, it shows that intangibles-related technological advances contribute to a significant part of the decline in measured labor share while simultaneously taking the measurement concern of labor share addressed in Koh, Santaeulalia-Llopis, and Zheng (2016); McGrattan and Prescott (2010b) into consideration - whether capitalizing intangible expenditures or not will affect the measurement of labor share. More specifically, this paper shows that no matter whether intangibles are treated as final output, when the intangible-related technical change is targeted to match the decline in the relative price of intangibles from the data, the model accounts for around half of the decline in labor share

of both measures. Second, this paper exploits the distinct economic property of intangible capital, i.e. the usage of intangible capital is non-rival in producing different types of goods within each individual firm, to generate that firms with higher productivity in producing intangibles are more intangible-capital-intensive and larger, which is consistent with the micro evidence. Intangible-capital-intensive firms benefit more and become even larger compared to the rest firms as producing intangibles becomes cheaper, which leads to increased industrial concentration.[1]

My approach is to develop a general equilibrium model of firm dynamics where firms produce both consumption/physical investment goods bundles and intangible investment goods, and the former serves as the numeraire in this economy. Firms have heterogeneous productivities in producing numeraire goods and intangible investment goods. An intangible-investment-specific technical change (IISTC) is modeled as a permanent increase in the aggregate productivity of producing intangible investment goods relative to that of numeraire goods. As a production input, intangible capital differs from physical capital in two aspects. First, intangible capital is firm-specific in the sense that each individual firm accumulates its own intangible capital within firm. Second, the usage of intangible capital is non-rival in the sense that it enters the production function for both numeraire goods and intangible investment goods rather than being split between them. Due to these two properties, the firm-specific heterogeneity results in varied factor shares of income across firms in the long run. Firms with high productivity in producing intangibles are large and intangible-capital intensive, and more intangible-capital intensive firms are also less labor intensive. These firms benefit disproportionately from the IISTC due to a general equilibrium effect: The technical change pushes up the equilibrium wage. Consequently, output and labor are reallocated from small firms with high labor shares to large firms with low labor shares. This leads to the decline in the aggregate labor share and the rise of concentration together.

I also enrich my model with two other features. First, I allow for endogenous entry and exit of firms. This helps generate distributions of firm size and firm age closer to their empirical counterparts as well as amplifies the impact of the intangible-investment-specific technical change (IISTC) on concentration. Second, I introduce financial frictions: Incumbent firms

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[1]The novelty of this paper also differentiates it from the existing literature (e.g. Eden and Gaggl (2018); Lashkari, Bauer, and Boussard (2019)) that study the implications of the decline in the price of ICT capital on labor share or concentration since ICT capital also contains physical capital, thus unable to utilize the economic property of intangible capital.

cannot issue equity, and they face borrowing constraints that restrict leverage to a multiple of collateralizable assets, as in Evans and Jovanovic (1989). Adding such a financial friction contributes to a realistic firm life cycle. Both features matter for the quantitative results on firm dynamics and concentration driven by the IISTC.

I calibrate the model to the U.S. business sector under the assumption that it was at the steady state in the early 1980s to match a rich set of macro and micro moments. In particular, I discipline production function to target BEA-measured income shares. I also discipline firm-specific production technology including firms' productivity processes on producing both numeraire and intangible investment goods to capture two key empirical facts: (i) more intangible-intensive firms are larger, and (ii) firm size distribution is skewed, in the sense that a small proportion of highly productive firms account for a very large share of total employment and total final output.

I then show that the calibrated model can reproduce firm-level cross-sectional predictions that are not targeted directly but are consistent with the micro evidence. The most important prediction is the negative correlation between firm size and firm-level labor share documented by Autor, Dorn, Katz, Patterson, and Van-Reenen (2019) using U.S. Census data.

Finally, I use the calibrated model to quantify the aggregate long-term impact of the intangible-investment specific technical change, which is disciplined by the observed decline in the relative price of intangible-investment goods over the period 1980-2016 from the data. In particular, I compare two steady states: One is the initial steady state calibrated to the early 1980s and the other is the new steady state after the technical change has occurred.

I find that when the IISTC is calibrated to match the observed decline in the relative price of intangible investment goods, the model can explain both the observed decline in the current BEA-measured labor share by around 60% and the decline in measured labor share of the pre-1999 revision—when intangibles are not treated as final output—by around 40%. Moreover, it accounts for around 90% of the observed increase in the employment share of large firms (with more than 500 employees) and of the increase in the share of total sales going to the top 10% firms. In addition, the model also predicts more than two-thirds of the observed reduction in annual firm entry rate.

To investigate the mechanisms that lead to the above results, I consider a number of extensions to the baseline model by altering its key features one by one to identify the most

essential elements of the baseline model and the deep parameters that drive the key relationships that generate the main results of the paper. Enabling the model to generate a realistic skewed size distribution of firms and sufficiently large intangible-intensive firms matters for the quantitative results. Otherwise, the negative correlation between firm size and firm-level labor share would not be strong enough, and thus, the results on the declined labor share and increased concentration driven by the IISTC would be weakened.

My paper also has implications on the aggregate elasticity of substitution of factors in production. By deriving the aggregate elasticity of substitution of factors in production numerically, I find that the aggregates of production factors—physical and intangible capital, as well as labor and intangible capital—are more substitutable than those implied by an aggregate Cobb-Douglas production function. That is, the aggregation of heterogeneous firms' production mimics the behavior of an aggregate CES production function with elasticity of substitution among factors greater than one.

**Related Literature**   This paper relates to several strands of literature. First, it contributes to the voluminous literature on the evolution of the labor share in general. Using aggregate-level data, Elsby, Hobijn, and Sahin (2013) document the decline in labor compensation as a share of national income for the U.S., and Karabarbounis and Neiman (2014) show that it is also a global phenomenon. Later studies, such as Autor, Dorn, Katz, Patterson, and Van-Reenen (2019) and Kehrig and Vincent (2018), confirm this trend using micro data from the U.S. Census. The exact magnitude of the decline in measured labor share is still being debated, due to measurement issues such as how to treat the labor portion of the proprietor's income (Elsby, Hobijn, and Sahin, 2013; Gollin, 2002) and intangible capital (Koh, Santaeulalia-Llopis, and Zheng, 2016). However, there is consensus that the labor share has indeed been falling.[2] While Karabarbounis and Neiman (2014) emphasize that the decline in labor compensation share in both the U.S. and abroad primarily represents a within-industry rather than a between-industry phenomenon, Autor, Dorn, Katz, Patterson, and Van-Reenen (2019) and Kehrig and Vincent (2018), by exploring firm heterogeneity, find that the fall in

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[2]Koh, Santaeulalia-Llopis, and Zheng (2016) argues that the labor share would have declined little if investments into intangible capital were treated as expenditures rather than investments. However, the accounting treatment of intangibles cannot mechanically explain a decline in the payroll-to-sales ratio, or the rising concentration of sales which Autor et al. (2019) find to be correlated with declining labor shares at the industry level using Census data.

the labor share is driven largely by between-firm reallocation. The decline in the measured labor share has spurred a growing literature that seeks to account for it using a wide set of economic mechanisms: increased openness to international trade and outsourcing (Elsby, Hobijn, and Sahin, 2013; Sun, 2020); an increase in concentration that causes increasing profit rates (Autor et al., 2019; Barkai, 2017), and increasing markups (DeLoecker and Eeckhout, 2017); automation (Acemoglu and Restrepo, 2018); and the decline in the relative price of investment goods (or IT investment) that increases the substitution of capital (or IT capital) for labor (Eden and Gaggl, 2018; Karabarbounis and Neiman, 2014; Lashkari, Bauer, and Boussard, 2019).

I contribute to the literature by highlighting the role of intangibles in explaining the evolution of labor share and firm concentration jointly. More specifically, I take both the measurement issue of labor share as well as the technological change originating from intangible capital into consideration. The measurement of intangible capital and intangible investment has important macroeconomic implications. McGrattan and Prescott (2010b) and Koh, Santaeulalia-Llopis, and Zheng (2016) argue that the macro measures, such as GDP and income shares, depend on how intangibles are measured and how intangible capital rents are allocated between labor income and capital income. Guided by these papers, I explicitly incorporate intangible capital and intangible investment into the model so it features a direct mapping between the model itself and the BEA data, but what makes my paper different from theirs is that my framework can also speak to firm-level patterns and links micro-level heterogeneity to macro outcomes, not only on measured income shares but also on concentration and firm dynamics.

Second, my work complements a set of recent papers concerning increased concentration in the U.S., which focus on either the increased employment share, or sales share of large firms, or both. Some papers (e.g., Aghion et al. (2019); Barkai (2017); DeLoecker and Eeckhout (2017); Gutierrez and Philippon (2017); Liu, Mian, and Sufi (2019)) argue that rising concentration is due to rising mark-ups and declining competition between firms, while others believe that higher market concentration may not necessarily imply higher market power of firms, consistent with Syverson (2004a,b). Autor et al. (2019); Bessen (2016); and Hopenhayn, Neira, and Singhania (2019) argue that rising concentration is a result of the expansion of more productive firms, although the expansion is driven by different forces. For Hopen-

hayn, Neira, and Singhania (2019), the driving force is the declining growth rate of labor force participation. Bessen (2016) documents that sales concentration is strongly correlated with the use of information and communication technologies at the industry level. In a similar vein, Crouzet and Eberly (2019) observe that a firm's market share in its industry is higher when its intangible capital intensity is higher, and this relationship holds between firms of the same industry and within firms over time. Understanding the sources of rising concentration can be very important since different sources have vastly different welfare and policy implications. If rising concentration is the outcome of rising market power and reducing competition, which may limit innovation, lead to resource misallocation, and hurt the economy in the long-run, then the government may seek to break up overly large and dominant firms into smaller ones. If, on the other hand, it is led by technological innovation, which favors some highly successful and productive firms such as Apple, Amazon, and Google, then policymakers may not need to worry too much. My work proposes a new driving force for increasing concentration by utilizing the distinct economic property of intangible capital and is in line with the latter strand of literature: Rising market concentration is the natural consequence of a technical change that favors highly productive, intangible-intensive firms.

Three recent papers study labor share and concentration jointly: Akcigit and Ates (2019); Autor et al. (2019); and Aghion et al. (2019), but all different from my paper in substantive ways. Autor et al. (2019) mainly focus on the empirical part while this paper proposes a quantitative general equilibrium framework to quantify a possible channel to explain the trends in labor share and concentration. Akcigit and Ates (2019) explores quantitatively which force plays a dominant role among multiple candidates (i.e. lower effective corporate tax, higher R&D subsidy, higher entry cost, and lower knowledge diffusion from frontier firms and the lagged ones) in explaining the decline in labor share and the slowdown of the overall dynamism (e.g. increased concentration) of the U.S. economy. Aghion et al. (2019) emphasize that reducing the firm-level costs of spanning multiple markets—possibly as a result of IT advances—is the driving force behind both the decreased labor share and increased concentration.

Compared to these papers, I attempt to reconcile two important empirical papers — one uses macro data and other other uses micro data — in a quantitative general equilibrium framework. Koh, Santaeulalia-Llopis, and Zheng (2016) use national-account data (BEA) and show that whether capitalizing intangible expenditures affects measurement of labor share.

The magnitude of the decline in measured labor share is smaller without treating intangibles as final output. This papers shows that the IISTC accounts for half of decline in measured labor share regardless of treatment of intangibles. Autor et al. (2019) use firm-level census data and document several important empirical patterns that potentially reveal the channels through which labor share declines and concentration rises. They find that the decline in labor share is not due to a within-firm effect but due to a between-firm effect as well as a negative correlation between firm size and firm-level labor share which is defined as compensation divided by sales of a firm. Based on these empirical findings, they argue that any aggregate shocks that favor more productive, large firms would lead to increasing concentration and declining labor share on the aggregate level jointly. My paper builds up a general equilibrium model that well replicates these key empirical findings at firm-level in their paper and identify the IISTC is such an aggregate shock.

As a cautious remark, the results of my paper do not mean—and are far from implying— that the intangible-investment specific technical change is the only driver of the observed trends for measured intangible investment, measured labor share, and concentration. Compared with papers that have a horse race structure,[3] this paper attempts to jointly explain several empirical facts using one driving force. Hence, I abstract my model from many other factors.[4]

**Layout**  The paper is organized as follows. Section 2 describes the model set-up and defines a recursive competitive equilibrium. Section 3 discusses calibration and cross-sectional implications respectively. In Section 4, I quantify the aggregate long-run effect of the intangible-investment-specific technical change (IISTC) and inspect the mechanisms that lead to the main results. Section 5 concludes the paper.

## 2  Model

In this section, I build up a quantitative framework. I incorporate the production technology a la McGrattan and Prescott (2010b) with decreasing returns to scale into an equilibrium model of firm dynamics, that dates back to the classic competitive settings (Jovanovic (1982);

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[3]See, for example, Akcigit and Ates (2019).

[4]Potential candidates that may drive one or more trends among labor share, concentration, and intangible investment include decline in real interest rate, decline in effective corporate tax rate, higher R&D subsidies, higher entry cost, lower knowledge diffusion, rising markups, change in labor force participation, globalization and trade, change in legal forms, among others.
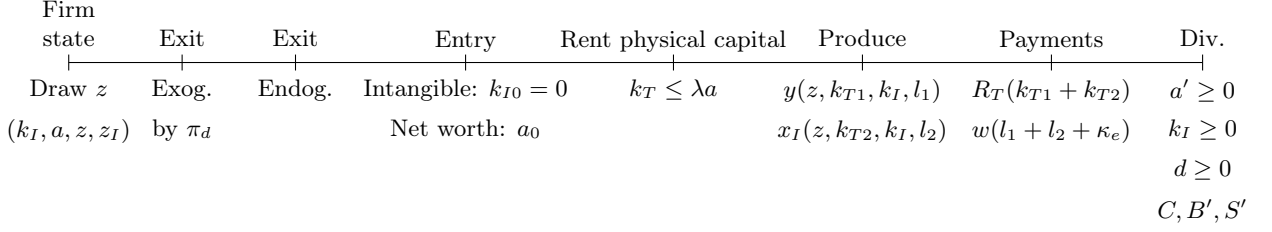
Firm state | Exit | Exit | Entry | Rent physical capital | Produce | Payments | Div.

Draw $z$ | Exog. | Endog. | Intangible: $k_{I0}=0$ | $k_T \le \lambda a$ | $y(z,k_{T1},k_I,l_1)$ | $R_T(k_{T1}+k_{T2})$ | $a' \ge 0$

$(k_I,a,z,z_I)$ | by $\pi_d$ | | Net worth: $a_0$ | | $x_I(z,k_{T2},k_I,l_2)$ | $w(l_1+l_2+\kappa_e)$ | $k_I \ge 0$

$d \ge 0$

$C,B',S'$

Figure 1: Timeline of the Model

Hopenhayn (1992)).

Time is discrete and infinite. There is a continuum of firms that are perfectly competitive in the final goods market and produce both a consumption/physical investment goods bundle and intangible investment goods. Consumption/physical investment goods bundle serves as the numeraire in this economy. Firms own tangible assets, accumulate intangible capital through producing intangible investment goods and rent physical capital subject to a borrowing collateral constraint. Firms are also subject to persistent shocks to individual productivity, which, together with endogenous entry and exit, yield heterogeneity in production. Households are identical and infinitely-lived.

I abstract the model from industry-level heterogeneity as if there is only one sector in this economy in order to focus on the impact of firm heterogeneity on the macroeconomy. I consider a stationary equilibrium without aggregate uncertainty.

Before describing firms' problem in detail, I outline the precise timing of the model, summarized in Figure 1. Within a period, the events unfold as follows: (i) realization of the productivity shocks for incumbent firms; (ii) endogenous and exogenous exit of incumbents; (iii) realization of initial productivity and entry decision of potential entrants; (iv) production and revenues from sales; (v) payment of wage bill, operation expenses, and physical capital rental costs; (vii) firm's decisions on dividend payment, intangible capital and tangible assets for next period, and household consumption/saving decisions.

In the following sub-sections, I present my model economy and examine the optimization problems of firms, followed by the household problem, and the definition of stationary competitive equilibrium and aggregation.

## 2.1 Production Technology

The production technology extends the one used in McGrattan and Prescott (2010b) to a heterogenous-firm set-up. More specifically, each individual firm produces a consumption/physical investment goods bundle $y$ (numeraire) and intangible investment goods $x_I$ using two types of capital - physical and intangible capital, and labor according to the following technologies (to differentiate firm-specific variables from variables common to all firms, I index each individual firm by $i$):

$$y(i) = Az(i) \left[ \left( k_{T1}(i)^{(1-\mu)} k_I(i)^{\mu} \right)^{1-\alpha} (l_1(i))^{\alpha} \right]^{\eta(i)} \tag{1}$$

and

$$x_I(i) = A_I z_I(i) \left[ \left( k_{T2}(i)^{(1-\mu)} k_I(i)^{\mu} \right)^{1-\alpha} (l_2(i))^{\alpha} \right]^{\eta(i)} \tag{2}$$

Among the parameters, $z(i)$ is a firm-specific productivity shock on the production of numeraire goods, following a Markov process. $z_I(i)$ is a firm-specific productivity shock on the production of the intangible investment goods, which is time-invariant and unevenly distributed across firms.[5] $\eta(i)$ is a firm-specific permanent shock on decreasing-returns-to-scale (DRS) technology with $0 < \eta < 1$.[6] The value of $A_I$ represents the aggregate productivity of producing intangible investment goods. I call an increase in $A_I$ relative to the aggregate productivity of producing numeraire goods, $A$, an intangible-investment-specific technical change (IISTC).

There are two features of the production technology that are key to the main results of the paper. First, intangible capital is firm-specific in the sense that each individual firm accumulates its own intangible capital by producing intangible investment goods in house

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[5]A possible source of persistent differences in productivity on producing intangibles is their business processes. Consider Amazon versus Barnes & Noble. While Barnes & Noble relies on its chains of physical stores, Amazon developed an online platform to sell books. The different business processes employed by the two companies determine the different amount of resources they devote to investing on intangibles and their efficiency. Moreover, firms like Amazon have established successful business models and logistics that are evidently hard to copy and reverse engineer. As a robust check, making $z_I$ a persistent shock, following a Markov process as well, does not alter my results. I target the persistence of this Markov process to match the persistence of intangible-investment to total assets ratio at the firm-level from the data, which turns out to be a number close to 1.

[6]One could interpret this as the consequence of "span of control" with some limited entrepreneur's management skill (see Lucas (1978)). As a result, there will be a non-trivial firm heterogeneity in the stationary equilibrium and the most productive firms will not take all of the market.

at each period. This means that there is not a common market for intangible assets to be traded. Without this assumption, firms with highest productivity in producing intangibles would produce all the intangible investment goods in the economy. Consequently, there would be always constant income shares across firms regardless of the firm-level heterogeneity if no other elements such as overhead are introduced to the model. However, since the model works through the reallocation of labor among firms with different labor shares, this is a key assumption. Moreover, this assumption is also reasonable in the sense that in the data, a large fraction BEA-measured intangible investment is indeed done in house such as own-account R&D and own-account software.

Second, intangible capital $k_I(i)$ has a distinct characteristic compared to other factors of production. As an input to both types of goods, intangible capital $k_I(i)$ is not split between producing the two types of goods, $y(i)$ and $x_I(i)$, as is the case for physical capital $k_{T1}(i), k_{T2}(i)$ and labor $l_1(i), l_2(i)$. An existing software is used both to produce consumption goods and to develop new softwares. This non-rivalry property, together with the assumption that intangible capital is firm-specific, break the usual constant factor share result with a standard Cobb-Douglas technology. For example, the profit share of each firm in this economy is not simply $(1-\eta)$ because a firm with high productivity in producing intangible investment goods $z_I$ can accumulate intangible capital more efficiently, thus reaping more intangible capital rents which means larger profit relative to its value-added (defined as $y(i)+p(i)x_I(i)$ to be consistent with the national account's measure).

Although there is no explicit market price of intangible assets, there is a shadow price of intangible investment goods $x_I(i)$ in terms of the numeraire goods $y(i)$ for each individual firm $i$, which is determined by:

$$p(i) = \frac{1}{A_I z_I(i)} \Big/ \frac{1}{A z(i)} \tag{3}$$

Later on I will use these shadow prices when I map my model to the data on the aggregate relative price of intangible investment goods.

## 2.2 Financial Frictions

Firms face two financial constraints. First, the capital decision involves borrowing physical capital from financial intermediaries (banks) in intra-period loans. Because of imperfect contractual enforcement frictions, firms can appropriate a fraction $1/\lambda$ of the capital received by

banks, with $\lambda > 1$. To preempt this behavior, a firm renting $k_T$ units of physical capital is required to deposit $k_T/\lambda$ units of the collateral with the bank. This guarantees that, ex post, the firm does not have an incentive to abscond with the capital. I assume that only tangible assets can serve as collateral, and likewise, intangible capital cannot be liquidated if the firm exits since there is no market for intangible assets to be traded.[7] Thus, I assume firms face collateral constraints of the form, $k_T \le \lambda a$.

Second, I assume that firms may only issue equity upon entry: an incumbent must keep nonnegative dividends payments. The model requires both constraints, otherwise, the collateral borrowing constraint can be easily circumvented. The non-negative dividend constraint captures two key facts about external equity documented in the corporate finance literature. First, firms face significant costs of issuing new equity, both direct flotation costs (see, for example, Smith (1977)) and indirect costs (see, for example, Asquith and Mullins (1986)). Second, firms issue external equity very infrequently (DeAngelo, DeAngelo, and Stulz (2010)). The specific form of the non-negativity constraint is widely used in the macro literature because it allows for efficient computation of the model in general equilibrium (See Khan and Thomas (2013), Khan, Senga, and Thomas (2017), Ottonello and Winberry (2018), among others). An alternative set-up is to introduce costly equity issuance, which would play a similar role.[8]

Introducing financial frictions contributes to a more realistic firm life cycle since it hinders the births of start-ups and slows the expansions of young firms. Otherwise, firms jump to their optimal scale almost immediately after their births. Since a main goal of this paper is to study the impact of a technical change that drives intangible investment on firm dynamics, having a model that features realistic firm distribution and life cycles is key.[9]

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[7]The main reason, which is consistent with my model set-up, is that there are limited, and sometimes no markets on which intangible assets can be readily sold to other potential users. Intangible assets are either too firm-specific (e.g. human capital) or not easily separated from the firms and transferred to other users (e.g. proprietary databases or software, or in-process R&D). The consequence is that business lending against intangible assets is very difficult. Based on the results from Falato et al. (2018) that uses a large sample of syndicated loans to US corporations for which a detailed breakdown of types of collateral used is available, only a very few of them (patents and brands) can be used as collateral and only an extremely small minority of secured syndicated loans (about 3% of total loan value) do use them as collateral.

[8]Costly equity issuance can be introduced in the form of proportional costs of equity issues (e.g., Begenau and Salomao (2019); Cooley and Quadrini (2001); Gomes (2001); Hennessy and Whited (2005)) and quadratic costs (e.g., Hennessy and Whited (2007)). As a robust check, I allow part of the firms (corresponds to public firms) to issue equity at a cost and find there is almost no change in the quantitative results. See Table 7 in subsection 4.2.

[9]From this perspective, the assumption that only tangible assets can be used as collateral is not essential. It is made to be consistent with the empirical evidence. Making intangible capital as collateralizable as tangible

## 2.3 Entry and Exit

I model firm entry and exit based on the standard approaches in the literature.[10] In each period, incumbent firms may exit the economy either by an exogenous probability or by their endogenous decisions. Due to financial frictions at the firm-level, individual states of productivity, intangible capital, and tangible assets jointly affect incumbents' endogenous exit decisions. Together with endogenous entry decisions by potential entrants, my model is able to capture key moments of firm dynamics.

### Entry

At each period, there is a fixed mass of potential entrants $M_0$. Potential entrants draw a productivity of consumption/physical investment goods $z_0$ from the ergodic distribution $\Gamma_0$. Similarly, they draw a productivity on intangible investment goods $z_I$ as well as the DRS technology $\eta$ from distributions $\Gamma_{z_I}$ and $\Gamma_\eta$ respectively. They start with zero intangible capital and the same positive amount of tangible assets $a_0$, which is financed by an equity injection from households. This is the only time when firms are able to issue equity.

Firms, after observing their draws, decide whether to enter the market by paying the fixed entry cost $\kappa_e$, denominated in labor units, which can be interpreted as labor utilized for entry such as entrepreneurs in start-ups. Since firms start with zero intangible capital, they need to invest on intangible capital in the first period they enter the market given their initial states, $(z_0, z_I, \eta)$, and start producing both types of goods in the following period. Let $o(z_0, z_I, \eta) \in \{0, 1\}$ denote the entry decision rule.

The value of entry is:

$$v^e(z_0, z_I, \eta) = \max_{k_I'} - k_I' + \beta \int_Z \frac{U'\left(C'\right)}{U'(C)} v^0\left(k_I', (1+r) a_0, z', z_I, \eta\right) dH\left(z'|z_0\right) \qquad (4)$$

where $v^0(\cdot)$ is the value of of an operating firm at the beginning of each period, a function

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assets will not alter the results on concentration and firm dynamics. I will show this result in Section 4. There are other set-ups that may also be able to achieve this goal. For example, we can make physical capital accumulated rather than rented and introduce physical capital adjustment cost instead of the collateral constraint.

[10]Hopenhayn (1992) is the seminal work in this literature with industry dynamics driven by firms' endogenous entry and exit. Clementi and Palazzo (2016) modify the timing of entry in the Hopenhayn model to investigate the business cycle implications of firm dynamics. I follow an approach similar to them, while introducing the exogenous exit as in Khan and Thomas (2013).

of $(k_I, a, z, z_I, \eta)$. Since any individual firm is owned by a representative household in the economy, the household's stochastic discounting factor $\frac{U'(C')}{U'(C)}$ will show up in the firm's optimization problem. Note that the initial investment on intangibles for start-ups is different from the intangible investment made by incumbents. Incumbents accumulate intangible capital through producing intangible investment goods based on the existing intangible capital stock, while entrants, starting with zero intangible capital, make a one-time investment on intangibles against their future value, based on their initial states $(z_0, z_I, \eta)$.

The firm chooses to enter the market if and only if the value of entry exceeds start-up costs $\kappa_e$, denominated in labor units, plus household equity injection $a_0$,

$$v^e (z_0, z_I, \eta) - w\kappa_e \geq a_0$$

Entrants start with relative low net worth compared to mature firms. Due to the fixed start-up cost denominated in labor units, an increase in wage rate (resulting from technological advances) may suppress firm entry.[11]

**Exit**

At the beginning of each period, firms are informed of their respective status of exit which takes place before production. First, there is a fixed probability of exit, $\pi_d$, which is common across firms. The remaining firms that survive from this exogenous death shock need to pay $\kappa_o$ units of labor in order to continue operation in the next period. This fixed cost of operation denominated in labor can be interpreted as overhead labor such as expenses on hiring HR and accountants, which creates a binary exit decision. If a firm does not pay this cost, it has to exit the economy permanently with liquidation value equal to its net worth in terms of tangible assets $a$. Thus, only firms continuing to the next period make inter-temporal decisions on investment and saving after paying $\kappa_o$. Due to the fixed operation cost, denominated in labor units, an increase in the wage rate will have a larger impact on small firms. This is relevant for quantitative results, as I show in Section 4.

Both endogenous and exogenous exit are necessary elements to this model. With a fixed

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[11]This is in line with Jo and Senga (2019). They show that in a general equilibrium set-up similar to my framework where heterogeneous firms face credit constraints, increased factor prices, due to credit subsidies, reduce the number of firms in production.

probability of exogenous exit, all firms have equal chances to exit, regardless of firm size. This assumption helps the model reproduce the empirical distribution of firm size and firm age by allowing turnovers of large-mature firms (e.g. job destruction in large and mature firms). The endogenous exit margin of firm dynamics enables relatively less-profitable firms to endogenously choose to exit. Financially constrained and unproductive firms are more likely to exit and have higher job destruction rates. This is consistent with the empirical evidence that small and young firms have higher exit and job destruction rates in general. To validate this set-up, I check the exit rate of firms with different sizes in terms of employment as a non-targeted moment.

Let $v^0(\cdot)$ be the value of an operating firm at the beginning of the current period, before its survival from exogenous exit is known. Accordingly, define $v^1(\cdot)$ as a surviving firm's value, before making its decision to pay the operation cost $\kappa_o$ in terms of labor. Finally, if a firm decides to continue to the next period, its value is given by $v(\cdot)$. Once a firm exits, its liquidation value equals its net worth in terms of tangible assets, $a$, since firm-specific intangible capital has no value outside the firm. Let $e(k_I, a, z, z_I, \eta) \in \{0, 1\}$ denote the endogenous exit decision rule. Then firms' value can be written as:

$$v^0(k_I, a, z, z_I, \eta) = \pi_d \cdot a + (1 - \pi_d) v^1(k_I, a, z, z_I, \eta) \tag{5}$$

$$v^1(k_I, a, z, z_I, \eta) = \max_e \{(e \cdot a, (1 - e) \cdot v(k_I, a, z, z_I, \eta)\} \tag{6}$$

**Incumbent firm**

The recursive form of the problem of incumbent firms is given by

$$v(k_I, a, z, z_I, \eta) = \max_{k_I', a', l_1, l_2, k_{T1}, k_{T2}, d} \left\{ d + \beta \mathbb{E} \left[ \frac{U'\left(C'\right)}{U'(C)} v^0\left(k_I', a', z', z_I, \eta\right) \mid z \right] \right\} \tag{7}$$

s.t.

$$\underbrace{d}_{dividend} + \underbrace{px_I}_{intangible\ invest.} + a' = \underbrace{y + px_I}_{NIPA\ income} - \underbrace{wl}_{wage} - \underbrace{w\kappa_o}_{overhead\ labor} - \underbrace{(r + \delta_T)k_T}_{rental\ cost} + (1 + r)a$$

$$y = Az \left[ \left( k_{T1}^{(1-\mu)} k_I^\mu \right)^{1-\alpha} (l_1)^\alpha \right]^\eta$$

$$x_I = A_I z_I \left[ \left( k_{T2}^{(1-\mu)} k_I^\mu \right)^{1-\alpha} (l_2)^\alpha \right]^\eta$$

$$k_I' = (1 - \delta_I) k_I + x_I, x_I \geq 0$$

$$k_T = k_{T1} + k_{T2}, l = l_1 + l_2$$

$$k_T \leq \lambda a, d \geq 0$$

Note that there is irreversibility in intangible investment since the production of $x_I$ is always non-negative. This is consistent with that intangible assets are firm-specific and there is no market for them to be traded.

To help understand the budget constraint and preface how I take the model to the data, define firm debt by the identity $b := k_{T1} + k_{T2} - a$, with the understanding that $b < 0$ denotes savings. Making this substitution reveals an alternative formulation of the model in which the firm owns its physical capital rather than rent it and faces a constraint on leverage: $b \leq \theta k_T$ where $\lambda = 1/(1 - \theta)$. With state vector $(k_I, k_{T1}, k_{T2}, b, z, z_I)$, the firm faces the following budget constraint:

$$\underbrace{d}_{dividend} + \underbrace{x_{T1} + x_{T2}}_{physical\,investment} + \underbrace{px_I}_{intangible\,investment} = \underbrace{y + px_I}_{NIPA\,income} - \underbrace{wl}_{wage} - \underbrace{w\kappa_o}_{overhead\,labor} - r_t b_t + \underbrace{b_{t+1} - b_t}_{\triangle Borrowing}$$

## 2.4 Representative Households

Assume that there is a unit measure of identical households in the economy. In each period, households consume, supply labor inelastically, and invest in one-period risk-free bonds and firms' shares:

$$W(B, S) = max_{B', S', C > 0} U(C) + \beta W \left( B', S' \right) \tag{8}$$

subject to

$$C + B' + QS' = w\bar{N} + (D + Q) S + (1 + r)B$$

where $B$ are one period risk-free bonds, $S$ are shares of the mutual fund composed of all firms in the economy, and $D$ are aggregate dividends per share. The household takes as given the return of risk-free bonds $(1+r)$, the share price $Q$, and the price of the consumption, which is the numeraire, so normalized to 1. In steady states, from the first-order conditions for deposits and share holdings, I obtain $1/(1+r) = \beta$ and $Q = \beta(Q + D)$, which implies a time-invariant rate of return of $r = \beta^{-1} - 1$ on both bonds and shares. The household is therefore indifferent over portfolios. For simplicity, I assume $U(C) = C$. Because of risk neutrality, households are indifferent over the timing of consumption as well.

## 2.5   Stationary Equilibrium

The state space for an incumbent firm is $S = K_I \times A \times Z \times Z_I \times H$ where $(k_I, a, z, z_I, \eta) \in S$. To simplify the exposition of the equilibrium, it is convenient to use $s \equiv (k_I, a, z, z_I, \eta)$ and $s_0 \equiv (z_0, z_I, \eta)$ as the argument for incumbents' and entrants' decision rules. Also, denote with $\varphi$ the stationary measure of incumbent firms at the beginning of the period, following the draw of firm-level persistent productivity, before the exogenous exit shock. Accordingly, denote $\varphi^e$ as the mass of actual entrants. Denote $\varphi^p$ as the distribution of producing firms that survives from exogenous shocks and decide to continue, and denote $\varphi^{ex}$ as the distribution of exiting firms (including both exogenous and endogenous).

**Definition**   A **stationary recursive competitive equilibrium** is a collection of firms' decision rules $\left\{ k_I'(s), a'(s), d(s), k_{T1}(s), k_{T2}(s), l_1(s), l_2(s), e(s), o(s_0) \right\}$, value functions $\left\{ v^0, v^1, v, v^e \right\}$, a measure of entrants $\varphi^e$, a distribution of firms $\varphi$, wage $w$, policy functions for households $\left\{ C, B', S' \right\}$ with the associated value function $W$ that solve the optimization problems and clear markets in the following conditions.

1. The decision rules $\left\{ k_I'(s), a'(s), d(s), k_{T1}(s), k_{T2}(s), l_1(s), l_2(s), e(s), o(s_0) \right\}$ solve the firm's problems (4), (5), (6), and (7), $\left\{ v^0, v^1, v, v^e \right\}$ are the associated value functions, and $\varphi^e$ is the mass of entrants implied by

$$\varphi_e = M_0 \int_Z \int_{Z_I} \int_H o(s_0) \, d\Gamma_\eta d\Gamma_{z_I} d\Gamma_0 \tag{9}$$

2. $W$ solves (8) , and $\left\{ C, B', S' \right\}$ are the associated policy functions.

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3. Labor markets clear:

$$\bar{N} = \int_S \left(l_1(s) + l_2(s) + \kappa_o\right) d\varphi^p + \int_S \kappa_e d\varphi^e \tag{10}$$

4. Goods markets clear (resource constraints hold):

$$C_h + K_T - (1 - \delta_T) K_T + M_0 \int_Z \int_{Z_I} \int_H a_0 \cdot o(s_0) \, d\Gamma_\eta d\Gamma_{z_I} d\Gamma_0 - \int_S a(s) d\varphi^{ex} = Y \tag{11}$$

where $Y = \int_S y(s) d\varphi^p$ and $K_T = \int_S k_T(s) d\varphi^p$

5. Shares markets clear (by Walras' Law) at $S = 1$ with share price

$$Q = \int_S v(s) \, d\varphi + M_0 \int_Z \int_{Z_I} \int_H o(s_0) v^e(s_0) \, d\Gamma_\eta d\Gamma_{z_I} d\Gamma_0$$

and aggregate dividends

$$D = \pi_d \int_S a(s) d\varphi + (1 - \pi_d) \int_S \{[1 - e(s)] \, d(s) + e(s) \, a(s)\} \, d\varphi - M_0 \int_Z \int_{Z_I} \int_H a_0 \cdot o(s_0) \, d\Gamma_\eta d\Gamma_{z_I} d\Gamma_0$$

6. The distribution of firms, $\varphi$, is a fixed point where its transition is consistent with the policy functions and the law of motion for $\varphi$, which is given by

$$\varphi\left(\mathcal{K}_I \times \mathcal{A} \times \mathcal{Z} \times \mathfrak{Z}_\mathfrak{J} \times \mathcal{H}\right) = (1 - \pi_d) \int_S [1 - e(s)] \, \mathbf{1}_{\left\{k_I'(s) \in \mathcal{K}_I\right\}} \mathbf{1}_{\left\{a'(s) \in \mathcal{A}\right\}} \Gamma\left(\mathcal{Z}, z\right) d\varphi$$
$$+ M_0 \int_Z \int_{Z_I} \int_H i(s_0) \, \mathbf{1}_{\left\{k_I'(s_0) \in \mathcal{K}_I\right\}} \mathbf{1}_{\left\{a'(s_0) \in \mathcal{A}\right\}} \Gamma\left(\mathcal{Z}, z\right) d\Gamma_\eta d\Gamma_{z_I} d\Gamma_0 \tag{12}$$

# 3    Calibration and Cross-sectional Implications

I numerically solve the model by using non-linear methods, and find a stationary equilibrium where individual decisions are consistent with market clearing prices.[12] In subsection 3.1, I discuss how I map from the model to the data. In subsection 3.2, I calibrate the model to be consistent with observed data for the U.S. business sector. Once the model is calibrated, in subsection 3.3, I explore the main cross-sectional implications of the calibrated model, which are going to inform the aggregate results in Section 4.

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[12]See Appendix C for computational details.

### 3.1 Variables of interest

Key moments for parameterizing the model are the implied aggregate intangible investment (in terms of numeraire goods), income shares and the distribution of firms.

**Price** First, intangibles considered in this paper are BEA-measured including software, R&D, and artistic originals. There are potentially other types of intangibles such as organizational capital and human capital as emphasized in Corrado, Hulten, and Sichel (2005),[13] but due to the measurement issue, they are not considered in this paper.

The aggregate intangible investment in terms of numeraire goods in the model is defined by aggregating all the individual firms' production on intangible investment goods $x_I$ evaluated at the firm-specific shadow price $p$. That is, $PX_I = \int_S p(s)x_I(s)d\varphi^p$ where $P$ is the aggregate relative price of intangible investment goods. Here, I do not derive $P$ explicitly but take it together with the aggregate quantity of intangible investment $X_I$. Note that intangibles in the model has no price since intangible capital is accumulated in house and thus, no market for it to be traded. In the data, a large part of intangible investment is indeed produced in-house and not sold in the market. Because own-account software and R&D is not sold in the market, the BEA estimates the own-account production of software and R&D as the sum of costs of production (i.e., wages, nonwages, and intermediates) plus a markup (Crawford et al., 2014; Moylan, 2001).[14] The best way to infer a "price" from the model and map it to BEA data is working through each individual firm's shadow price $p$ which is determined by the inverse of the relative productivity in producing intangible investment goods of that firm.

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[13] Corrado, Hulten, and Sichel (2005) propose a framework to estimate for the values of intangible investments, a substantial part of which are unmeasured by national accounts. They consider three broad categories: computerized information including software and customized database; R&D and intellectual property; and economic competencies including brand, firm-specific human capital (e.g. costs of developing workforce skills such as the on-the-job training and tuition payments for job-related education) and organizational structures. National accounts like BEA, only measure software, artistic originals, and, most recently, R&D as intangible investment. Corrado et al. (2016) estimate the intangible investment as a share of gross value-added to be 0.15 for business sector in 2016, while for BEA-NIPA, this number is only 0.055.

[14]There are three types of software: (i)-(ii) prepackaged and custom software, which are estimated from the benchmark I-O accounts that, in turn, are based on receipts for software products; and (iii) own-account software, which is calculated by multiplying the number of programmers and systems analysts times a an estimate for the share of time they spend doing tasks associated with non-embedded software development, times a national median wage rate for programmers and systems analysts, times various factors that cover nonwage compensation costs and intermediate inputs, based on BLS employment-by-industry data. For R&D, there are two types: (i) purchased R&D, which is funded by one entity, but produced by another entity; and (ii) own-account R&D, which is produced for an entity's own use. Similar to how BEA estimates software, market prices are used to value purchased R&D, while the value of own-account R&D is estimated as the sum of production costs.

**Measured Labor Share** I compute two different measures of the labor income share, depending on whether BEA treats intangibles as expenditures or investments. To be consistent with BEA's definition post-2013-revision, own-account intangibles (including software, R&D, and artistic originals) are considered as final output That is, the final output for BEA's definition post-2013-revision should be represented by $Y + PX_I = \int_S y(s)d\varphi^p + \int_S p(s)x_I(s)d\varphi^p$ in the model. Hence, labor income share (post-2013-revision) in the model is defined as labor compensation divided by the the gross value-added of the domestic business sector, taking into intangibles as final output:

$$S_N = \frac{w\bar{N}}{Y + PX_I} = \frac{w\left[\int_S (l_1(s) + l_2(s) + \kappa_o)\, d\varphi^p + \int_S \kappa_e d\varphi^e\right]}{\int y(s)d\varphi^p + \int p(s)x_I(s)d\varphi^p} \tag{13}$$

where $P$ is the aggregate price of intangible investment goods in terms of consumption/physical investment.

Correspondingly, labor income share (pre-1999-revision) when own-account intangibles are not treated as final output in the model is defined as:

$$S_{N_{pre}} = \frac{w\bar{N}}{Y} \tag{14}$$

For aggregate measured income shares, I focus on the corporate sector for two reasons: first, clearer measurement of labor income and profit.[15] Second, the way I model firms is more consistent with (non-financial) corporations.

**Distribution of Firms** For the distribution of firms, I focus on all the employer firms in the U.S. business sector of which the corporate firms are just a subset.[16] The way I deal with this discrepancy is as follows. Since corporations are usually very large firms,[17] in the model, I filter corporate firms based on firm size in terms of final output and choose a cutoff for $y + px_I$,

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[15]For the corporate sector, labor income is unambiguously defined as the compensation of employees, but for the non-corporate sector such as partnership or sole proprietorship, since owners also use their own time to contribute to the production, part of the proprietor's income should be allocated to labor income. The exact magnitude of the decline in labor income income is affected by the treatment of the labor portion of proprietor's income (See Gollin (2002), Elsby, Hobijn, and Sahin (2013)).

[16]The reason why I focus on the distribution of all employer firms is that the data on firm distribution is only available for all the employer firms rather than for firms in corporate sector or for all the firms in the private sector.

[17]For example, based on Dyrda and Pugsley (2019) that uses U.S. census data, 20% of all firms are corporations, accounting for 90% of all sales in the early 1980s.

| Parameter | Description | Value | Source/Target |
|---|---|---|---|
| $\beta$ | Discount factor | 0.990 | Annual risk-free rate $= 4\%$ |
| $M_0$ | Mass of potential entrants | 0.002 | Measure of incumbents $= 1$ |
| $\delta_T$ | Physical capital depreciation rate | 0.050 | NIPA fixed asset tables |
| $\delta_I$ | Intangible capital depreciation rate | 0.215 | NIPA fixed asset tables |

Table 1: Parameter values set externally

call it $\bar{v}$, such that the total income generated by all the firms with $y + px_I \geq \bar{v}$ divided by the total income generated by all the firms in the economy (i.e. $Y + PX_I$ in equation (13)) equals the corporate income as a share of the domestic business income from BEA-NIPA.[18] Any moments generated from my model that are used to calibrate parameters to match their empirical counterpart from Compustat also use this filtering criterion. In addition, I compare the national account generated from the calibrated model with the BEA-NIPA. The two are close to each other (See Table 8).

To ensure better estimates and consistency for intangible investment, the starting point of the time-series considered in the paper is 1975 because it is the first year that the Federal Accounting Standards Board (FASB) required firms to report R&D.

## 3.2 Calibration

I begin with the subset of parameters calibrated externally, and then consider those estimated within the model. Data moments are averages over 1980 - 1985 unless otherwise specified.[19] For empirical moments related to concentration (e.g. employment share and market share of large firms), what I report is the weighted average by industry since the model in this paper abstracts from industry-level heterogeneity.

### 3.2.1 Externally Calibrated

The model period is one quarter. I set $\beta = 0.99$ to replicate an annualized risk-free rate of 4 percent. Since the measure of potential entrants $M_0$ scales the distribution of entrants $\varphi_e$ (see equation 12), I choose $M_0$ to normalize the total measure of incumbent firms to 1. I use BEA fixed asset tables which include both flows and stocks to compute the depreciation rate of physical capital $\delta_T = 0.05$ as well as the depreciation rate of intangible capital $\delta_I = 0.215$ (see Appendix A for more details). Table 1 summarizes these parameter values.

### 3.2.2 Internally Calibrated

Table 2 lists the remaining 17 parameters of the model that are set by minimizing the distance between an equal number of empirical moments and their equilibrium counterparts in the model.[20] It also lists the targeted moments, their empirical values, and their simulated values from the model. Even though every targeted moment is determined simultaneously by all parameters, in what follows I discuss each of them in relation to the parameter for which, intuitively, that moment yields the most identification power.

There are two key sets of parameters to be calibrated within the model: those that characterize the production technology and those that characterize the heterogeneity across firms in the corresponding state variables of productivity and DRS technology. I start with disciplining the parameters in production technology for both numeraire goods $y$ and intangible investment goods $x_I$. Recall that

$$y = Az \left[ \left( k_{T1}^{(1-\mu)} k_I^{\mu} \right)^{1-\alpha} (l_1)^{\alpha} \right]^{\eta}$$

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[18] Admittedly, there also exists discrepancy between the coverage of BEA-business sector, which covers both employer and non-employer firms, and the coverage of BDS/LBD, which only covers employer firms. However, since non-employer firms do not contribute to the total employment and only takes a very small portion in terms of sales (which is 2.48% based on SBO 2007), I assume they cover the same firms. See Table 11 for a comparison among the datasets for the coverage of firms.

[19] Ideally, I would only use data on firms, since financial constraints apply at the firm level. However, since some moments are only available at the establishment-level and my model indeed does not differentiate between firms and establishments, I use firm data whenever I have a choice, and establishment data only when firm-level data is not available.

[20] Specifically, the vector of parameters $\Psi$ is chosen to minimize the minimum-distance-estimator criterion function $f(\Psi) = (m_{data} - m_{model}(\Psi))' W (m_{data} - m_{model}(\Psi))$, where $m_{data}, m_{model}$ are the vectors of moments in the data and model, and $W = diag(1/m_{data}^2)$ is a diagonal weighting matrix.

| Parameter | | Value | Moment | Data | Model |
|---|---|---|---|---|---|
| **Production Technology - consumption/physical investment goods & intangible investment goods** | | | | | |
| Labor share | $\alpha$ | 0.670 | Labor compensation/value-added | 0.640 | 0.640 |
| Intangible capital share | $\mu$ | 0.225 | Intangible investment/value-added | 0.032 | 0.032 |
| **Permanent Productivity on intangible invest. goods** $z_{\mathrm{I}} \in \left\{ z_{\mathrm{I}}^{L}, z_{\mathrm{I}}^{H} \right\}$ | | | | | |
| High - low gap | $z_{\mathrm{I}}^{H}/z_{\mathrm{I}}^{L}$ | 3.990 | Share of sales going to top 10% | 0.519 | 0.493 |
| Mass: $z_{\mathrm{I}}^{H}$ firms | $\mu_{z_{\mathrm{I}}^{H}}$ | 0.050 | Intangible-intensive firms | | |
| **Firm Size Distribution** | | | | | |
| Low DRS in prod. | $\eta_L$ | 0.750 | | | |
| Mid DRS in prod. | $\eta_M$ | 0.800 | | | |
| High DRS in prod. | $\eta_H$ | 0.905 | Firm size distribution (BDS) | see Table 3 | |
| Mass: $\eta_L$ firms | $\mu_L$ | 0.690 | | | |
| Mass: $\eta_H$ firms | $\mu_H$ | 0.065 | | | |
| **Persistent Productivity on Consumption/Physical Invest. Goods** $z$ **(AR1)** | | | | | |
| Persistence | $\rho_z$ | 0.795 | Autocorrelation coefficient for log revenues | 0.170 | 0.170 |
| Standard deviation | $\sigma_z$ | 0.296 | Std. of log revenues | 0.540 | 0.579 |
| **Entrants** | | | | | |
| Initial tangible asset | $a_0$ | 1.210 | Start-up debt/value-added rel. to aggregate debt/value-added | 1.739 | 1.645 |
| Initial productivity (mean) | $\hat{z}_0$ | 0.391 | Average start-up size rel. to average incumbent size | 0.296 | 0.295 |
| **Financial Friction** | | | | | |
| Collateral parameter-low value | $\lambda$ | 8.500 | Aggregate debt/value-added | 0.608 | 0.605 |
| **Entry and Exit** | | | | | |
| Exog. exit prob. | $\pi_d$ | 0.075 | 5-year survival rate | 0.485 | 0.547 |
| Entry cost | $\xi_e$ | 6.000 | Annual entry rate | 0.125 | 0.114 |
| Operating cost | $\xi_o$ | 0.035 | Average firm size | 20.47 | 20.66 |

Table 2: Parameter calibrated internally

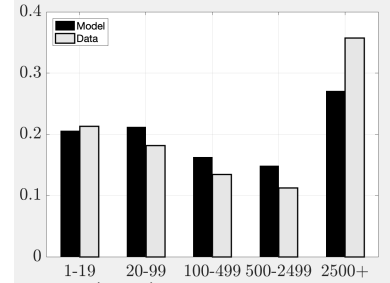| | Population Share (%) | | Employment Share (%) | | |
|---|---|---|---|---|---|
| Employees | **Data** | **Model** | **Data** | **Model** | |
| 1 to 19 | 88.97 | 88.97 | 21.35 | 22.61 | |
| 20 to 99 | 9.30 | 9.30 | 18.19 | 22.06 | |
| 100 to 499 | 1.41 | 1.41 | 13.45 | 18.22 | |
| 500 to 2499 | 0.24 | 0.24 | 11.27 | 11.17 | |
| 2500+ | 0.08 | 0.08 | 35.74 | 25.94 | |



Table 3: Firm Size Distribution: Model v.s. Data (p.p.)

and

$$x_I = A_I z_I \left[ \left( k_{T2}^{(1-\mu)} k_I^{\mu} \right)^{1-\alpha} (l_2)^{\alpha} \right]^{\eta}$$

I assume both technologies take a Cobb-Douglas structure and are symmetric in the sense that they have exactly the same parameters of factor shares, i.e. $(\alpha, \mu, \eta)$, because I do not have additional information on whether, and the degree to which, these inputs are substitutes or complements at the firm-level, following the literature involving intangible capital as a production input.[21] The labor share $\alpha$ and the intangible capital share $\mu$ are chosen so that the model predictions for labor compensation as a share of gross value-added and the intangible investment as a share of gross value-added are consistent with the BEA data.[22]

The other key set of parameters is calibrated to capture two empirical facts: (i) more intangible-intensive firms are large; and (ii) firm size distribution is highly skewed, because of two main reasons. First, a major goal of this paper is to study the impact of a technological change on concentration, so having firm distribution matched to the data at the initial steady state is important. More importantly, how well this model is able to match the firm distribution determines how well this model is able to reproduce a key relationship (i.e. a negative correlation between firm size and firm-level labor share documented by Autor, Dorn, Katz, Patterson, and Van-Reenen (2019)) that will affect the aggregate results on concentration and labor share, as I show later.

In particular, I do two things to match firm distribution. First, I calibrate the distribution of productivity in producing intangible investment goods $z_I$ to capture the fact that intangible-intensive firms are large on average. In the baseline case, I consider $z_I$ follows a two-point distribution with support $\left\{ z_I^L, z_I^H \right\}$ to target the moment that top 10% firms in terms of intangible-investment-to-total-assets ratio account for 51.9% of total sales in the

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[21]For example, in McGrattan and Prescott (2010a), McGrattan and Prescott (2010b), and Bhandari and McGrattan (2019), they all have intangible capital as a production input and they assume that firms produce at Cobb-Douglas technology because there is not sufficient information from the data to estimate accurately the elasticity of substitution among the factors physical capital, intangible capital, and labor in production.

[22]In my model, intangible capital share and intangible investment as a share of gross value-added give the same information following the assumption from Karabarbounis and Neiman (2014) that the ratio of the nominal value of the capital stock to nominal investment is constant and that the required rate of return on capital is constant (See Section IV.B of Karabarbounis and Neiman (2014)), together with the depreciation rate of intangible capital computed using BEA data consistently.

early 1980s.[23] This is in line with Crouzet and Eberly (2019).[24] In subsection 4.2, I allow $z_I$ to take more values, and compare the results with the baseline case. Second, I let the permanent heterogeneity in the scale parameter $\eta$ match the skewed firm size distribution in terms of employment from the Business Dynamic Statistics (BDS). I consider a three-point distribution with support $\{\eta_L, \eta_M, \eta_H\}$, leaving five unknown parameters: (i-iii) the values of $\eta_L, \eta_M, \eta_H$; and (iv)–(v) the fractions of low and high DRS firms $\mu_L, \mu_H$. This heterogeneity dramatically improves the results on matching the skewed firm size distribution,[25] see Table 3 for a comparison between the model and the data.[26]

The persistent firm-specific productivity of producing a consumption/physical investment goods bundle, $z$, follows an $AR(1)$ process in logs: $log z' = log Z + \rho_z log z + \varepsilon'$, with $\varepsilon' \sim N(-\sigma_z^2/2, \sigma_z)$ and $\varepsilon'$ uncorrelated with $z_I$. I choose the persistence of the productivity process $\rho_z = 0.795$ so that the model generates an autocorrelation coefficient for log revenues equal to 0.82.[27] Similarly, I choose the standard deviation of productivity shocks $\sigma_z = 0.296$ so that the model generates a standard deviation of log revenues equal to 1.56, which is the average standard deviation in my data across industries and time (for the periods 1980 - 1985).

The initial productivity distribution on producing numeraire goods for entrants $\Gamma_0$ is assumed to be exponential in order to better match the empirical skewed firm size distribution. The mean of the initial productivity $\hat{z}_0$ is chosen to match the average size of start-

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[23]This is a moment I document from Compustat. As I have mentioned before, I am trying to target the whole demographics of employer firms in the U.S. economy-not only the very large public firms. To fix this discrepancy, I filter Compustat firms based on the same criterion as the corporate firms. That is, after I do the simulation, I choose a cutoff for $y + px_I$, call it $\bar{v}$, such that the total income generated by all the firms with $y + px_I \geq \bar{v}$ divided by the total income generated by all the firms in the economy equals the corporate income as a share of the domestic business income from BEA-NIPA, and then I compute the relevant moments only for firms with $y + px_I \geq \bar{v}$. For measurement of intangible capital at the firm-level, see Appendix A.2

[24]They find that a firm's market share in its industry is higher when its intangible capital to total assets is higher, and this relationship holds between firms of the same industry, within firms over time, and controlling for year effects. I show my model is able to reproduce their results in Table 10 of Appendix A.2.

[25]This method follows Gavazza, Mongey, and Violante (2018). Permanent heterogeneity in productivity might also be used to match these facts (see Elsby and Michaels (2013); Kaas and Kircher (2015)), but heterogeneity in $\eta$ also generates small old firms alongside young large firms, thus decoupling age and size, which tend to be too strongly correlated in standard firm dynamics models with mean reverting productivity as Gavazza, Mongey, and Violante (2018) suggest. An alternative way to match the firm-size distribution is to employ a non-Gaussian process for the persistent firm-specific productivity shocks (see Jo and Senga (2019)).

[26]Since employment is not a state variable in my model, there is no way to know the number of employees of each firm directly. I divided firms into five groups based on the population share from the data (this is why the model can perfectly match the data in terms of the population share) and then I can get the employment cutoff for each group of firms. Finally, I can compute the employment share of each group of firms from my model and compare the results with the data.

[27]The value of the persistence in this paper falls into the range of the persistence estimates (0.757, 0.966) in Foster, Haltiwanger, and Syverson (2008).

ups relative to that of incumbent firms. For the initial tangible assets, $\bar{a}_0$, I calibrate it to match the start-up debt-to-output ratio relative to the aggregate debt-to-output ratio, which is $1.4/0.8 = 1.75$.[28] Calibrating the initial conditions of entrants to match the data is important since it matters for firm distribution and the results on concentration and firm dynamics. Having start-ups too large compared to the data results in increased firm entry and declined concentration when the same technology shock that drives intangible investment is fed into the model.

For financial frictions, I calibrate the collateral parameter $\lambda$ which is economy-wide to match the aggregate debt-to-value-added ratio for the private business sector. For the remaining three parameters regarding firm dynamics: exogenous exit probability $\pi_d$, entry cost $\kappa_e$, and the operating cost $\kappa_o$, I calibrate them to match three moments from the data: 5-year survival rate, annual entry rate, and the average firm size in terms of employment.

## 3.3 Cross-Sectional Implications

I now explore the main cross-sectional implications of the model at its steady-state equilibrium, calibrated to the early 1980s. Table 4 reports some empirical moments not targeted in the calibration and their model-generated counterparts. The model can replicate fairly well the distribution of employment by firm age, which is not explicitly targeted. The average exit rate of firms with less than 20 employees relative to that of firms with more than 500 employees generated from the model is not far from its empirical counterpart. This speaks to the importance of introducing exogenous death shock, without which the average exit rate of small firms relative to large will be a very large number.[29]

As a key non-targeted moment, I show that my model can reproduce the negative relationship between firm size and firm-level labor share, documented by Autor, Dorn, Katz, Patterson, and Van-Reenen (2019). More specifically, they regress the payroll-to-sales ratio on each firm's sales as a fraction of total sales using LBD Census data for six main sectors. They

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[28]The reason why for $\bar{a}_0$, I target the relative ratio rather than the direct ratio is because for start-up debt to output data, I only have it for the year 2004, which is from the Kauffman Survey (See Robb and Robinson (2014)). Assuming the relative relationship between start-up debt to output and aggregate debt to output is stable overtime. By using the data for aggregate debt to output in 2004 and the average of 1980-1985, I can infer the start-up debt to output averaged for 1980-1985.

[29]There are also technical reasons for introducing the exogenous death shock. First, it is a simple way to avoid the situation where financial frictions are irrelevant when firms survive long enough (see Khan and Thomas (2013)). Second, without the exogenous death shock or if its value is very small, it is much harder to find the stationary distribution of firms.

| Moment | Data | Model | Source |
|---|---|---|---|
| Employment share: $Age \leq 1$ | 0.036 | 0.042 | |
| Employment share: $Age \in (1,10]$ | 0.297 | 0.201 | BDS |
| Employment share: $Age \geq 11$ | 0.667 | 0.757 | |
| Exit rate $< 20$/exit rate ($> 500$) | 32.11 | 22.67 | |
| **Regression coefficient of payroll-to-sales on firm size (sales)** | **-0.837** | **-1.102** | ADKPV |
| Top 10% concentration (sales) | 0.675 | 0.714 | |
| Persistence of intangible capital/total assets | 0.780 | 0.751 | Compustat |

ADKPV: Autor, Dorn, Katz, Patterson, and Van-Reenen (2019), LBD census data -Various sectors; regression coefficient within the range $[-2.37, -0.35]$, with value-added weighted average -0.837

Table 4: Non-Targeted Moments

get a range of the estimates on the regression coefficients: $[-2.37, -0.35]$.[30] A value-added-weighted average of these coefficients is -0.837. Using the simulated data from the calibrated model,[31] I find a regression coefficient of $-1.102$, which is within the range and fairly close to the weighted average of the data -0.837. The share of sales going to the largest 10% of firms is also a non-targeted moment but is well captured by the model. There are two key elements that drive the negative correlation between firm size and firm-level labor share: (1) heterogeneity in $z_I$; and (2) overhead labor.

In addition, that the persistence of the intangible capital to total assets in the model is close to its empirical counterpart validates the introduction of the persistent idiosyncratic productivity on intangible investment goods.[32] If shocks on $z_I$ are less persistent, the value of persistence generated from the model will be even smaller.

In Figure 2, I plot the average firm size in terms of employment over the life cycle generated from the baseline economy as well as a counterfactual economy without financial frictions (when $\lambda \to \infty$). Then I compare them with the data. The baseline model features a realistic life cycle dynamics due to two elements: financial frictions and costly accumulation of intangible capital. As we can see, the first element plays a dominant role. Without financial

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[30]The six sectors are wholesale trade, finance, manufacturing, retail trade, utilities + transportation, and services, where wholesale trade has the most negative correlation, i.e. -2.37, and service has the least negative correlation, i.e. -0.35. The coefficient of manufacturing sector is -0.90.

[31]Here is how I map from the model to the data: since the model abstracts from intermediate goods, there is no difference between sales and value-added. Moreover, since the firm-level census data still treats intangibles such as software and R&D as expenses rather than investments, I let $y$ to be sales in my model. That is, I regress $w(l_1 + l_2 + \kappa_o)/y$ on $y/Y$. I also check what will happen if I regress $w(l_1 + l_2 + \kappa_o)/(y + px_I)$ on $(y + px_I)/(Y + PX_I)$ instead.

[32]See subsection 4.2 where I allow $z_I$ following AR(1) process and the quantitative results on concentration and labor share driven by the technical change do not change.
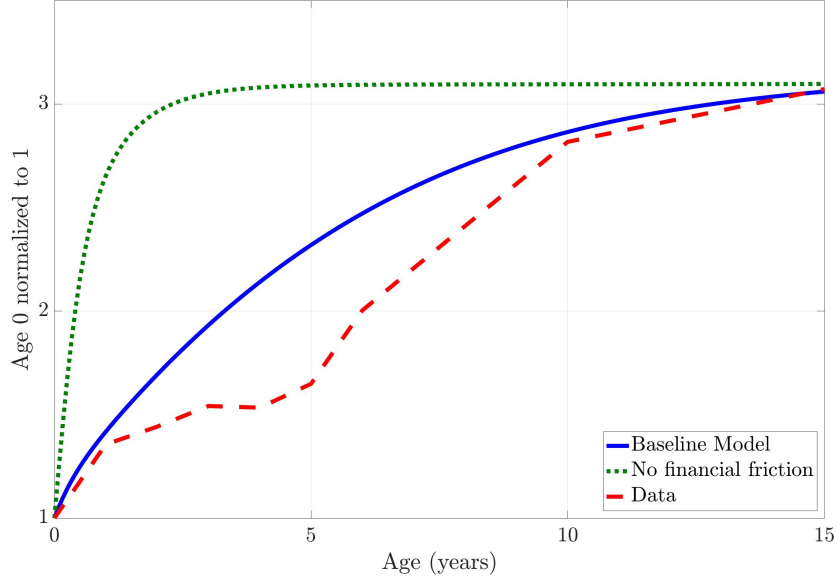
Figure 2: Average Life Cycle of Firms: Model v.s. Data

frictions, firms jump to their optimal level almost immediately which affects the aggregate results on concentration non-trivially in response to a technology shock (see subsection 4.2 for more details).

In Figure 3, I plot the average firm size in terms of employment and value-added, growth rates of firms in terms of tangible assets, and intangible capital-to-physical capital ratio for firms with different productivity on producing intangible investment goods from birth to maturity. Panel A, B, and C show that firms with high $z_I$, those with (permanent) high productivity in producing intangibles, account for the upper tail in the size and growth rate distributions. These firms take more advantage of an intangible-investment-specific technical change. Together with Panel D, these figures imply that firms with higher $z_I$ are more intangible-capital intensive and larger (in terms of employment and value-added) as well. The growth patterns for firms with different intangible capital intensity are also in line with Crouzet and Eberly (2019) where they find that firms with higher intangible capital intensity are larger and grow faster.
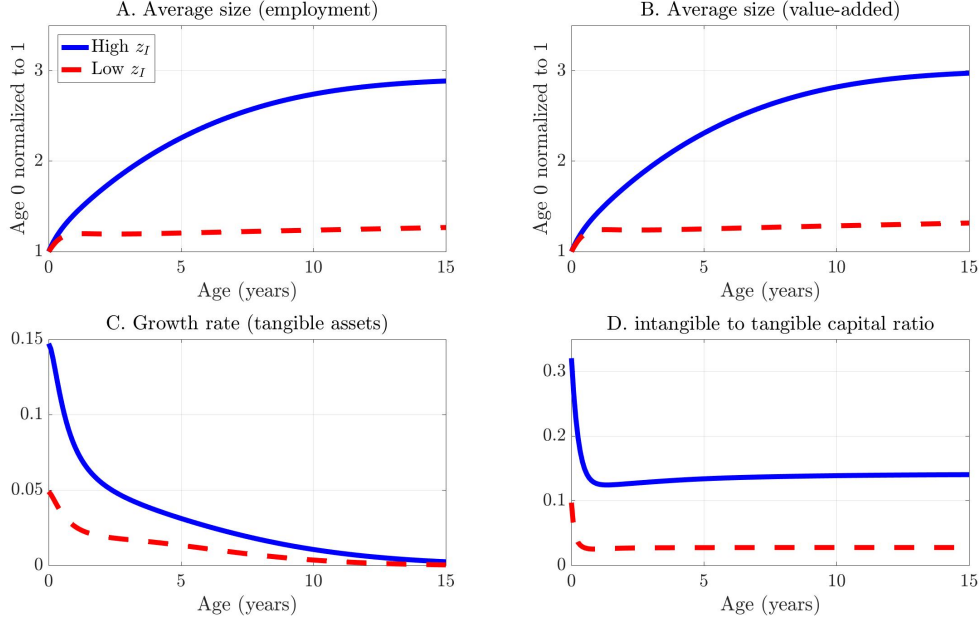
Figure 3: Average Life Cycle of Firms: high $z_I$ firms v.s. low $z_I$ firms

## 3.4   Discussion

Before moving forward, I summarize key elements of the model, their calibration strategy, and the role they would play in generate the main results in next section. Firms' heterogeneity in my framework is introduced in three dimensions: (i) Firms differ in their productivity in producing the numeraire goods, $z$, which is a persistent shock, following a Markov process; (ii) they also differ in permanent productivity in producing intangible investment goods, $z_I$; and (iii) they operate different DRS technology, which is permanent as well. Given $z$ and $\eta$, firms with higher $z_I$ are larger and more intangible-capital intensive as well. This structure implies that the effects of the intangible-investment-specific technical change (IISTC), manifested as a permanent increase in the aggregate productivity in producing intangibles, $A_I$, on firms are heterogeneous across the population: firms with high $z_I$ benefit disproportionately from this technical change. Distribution on $z_I$ is disciplined to capture an empirical fact that more intangible-intensive firms are large, and heterogeneity in $z_I$ plays a key role in generating the results on both the decline in the aggregate labor share and the rise of concentration. On the other hand, introducing the heterogeneity in DRS technology $\eta$ may work in the opposite direction in generating the declined labor share because if the IISTC shifts the distribution

of firms toward larger firms with higher $\eta$ which are the ones with the lower share of income accruing to the fixed factor and thus, with higher labor share, then the aggregate labor share will increase. Heterogeneity in $\eta$ helps to match the skewed firm size distribution, which improves the quantitative results on the increased concentration significantly but only weakens the results on the labor share decline slightly due to the non-rivalry property of intangible capital, as I will show in subsection 4.2. Since the goal of this paper is to account for the rise of concentration and the decline in measured labor share simultaneously, I need both elements and they need to balance somewhat to match different aspects of the data.

In addition, overhead labor is another element of my model that matters for the quantitative results. First, it contributes to generating a negative correlation between firm size and firm-level labor share. Second, it may enable IISTC—the improved aggregate productivity in producing intangibles—to prevent start-ups from entering the market when the cost of this technological advance, which is the increased equilibrium wage, dominates the benefit of it.[33] I verify that, by calibrating the initial conditions of start-ups to match the data, this is indeed the case in Section 4.

## 4  Main Results

This section uses the calibrated model from Section 3 to study the aggregate implications of an intangible-investment-specific technical change (IISTC). The main results are summarized in subsection 4.1 where I quantify the long-term impact of the IISTC on measured labor share and concentration. In subsection 4.2, I investigate the key mechanism that drives the quantitative results.

### 4.1  Aggregate Implications

The intangible-investment-specific technical change (IISTC) is modeled as a permanent increase in the aggregate productivity in producing intangible investment goods $A_I$ to match the decline in the relative price of intangible investment goods from the BEA data, which is constructed as the ratio of the price of investment in intangible capital including software,

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[33]This is in line with literature that studies the sources of the decline in entrepreneurship in the U.S. over the past three decades, arguing that the decline comes from that the outside option of being an entrepreneur becomes better such as the higher skilled wage premium, as in Salgado (2018).

| | Start of sample | | End of sample | | Change (pp.) | |
|---|---|---|---|---|---|---|
| | Model | Data | Model | Data | Model | Data |
| Labor share $\frac{wL}{Y+PX_I}$ (post-2013 revision) | 0.640 | 0.640 | 0.610 | 0.594 | -3.0 | -4.6 |
| Labor share $\frac{wL}{Y}$ (pre-1999 revision) | 0.671 | 0.665 | 0.659 | 0.641 | -1.2 | -2.4 |

Start of the sample: 1980-1985 average (1975-2016 linear trend);
End of the sample: 2011-2016 average (1975-2016 linear trend)

Table 5: Aggregate Implications: Measured Labor Share

R&D, and artistic originals to the price of the bundle of consumption and investment in traditional physical capital including structures and equipment. For 1980 - 2016, the relative price of intangible investment goods declines by 44%.[34]

To be consistent with how BEA constructs the price indices, the aggregate relative price of intangible investment goods in new steady state relative to its value in initial steady state is given by the Fisher formula:

$$P_{new,initial} = \sqrt{\frac{\int p_{initial} x_{\mathrm{I},initial} d\varphi^p_{initial}}{\int p_{new} x_{\mathrm{I},initial} d\varphi^p_{initial}} \times \frac{\int p_{initial} x_{\mathrm{I},new} d\varphi^p_{new}}{\int p_{new} x_{\mathrm{I},new} d\varphi^p_{new}}}$$

where $p$ is the shadow price of the intangible investment goods relative to the numeraire, as in equation 3. Then the percentage change in the aggregate relative price of intangible investment goods from the initial steady state to the new steady state is $\%\Delta = (P_{new,initial} - 1) \times 100$. In the model, I increase the aggregate productivity in producing intangible investment goods $A_I$ by 79% in order to match the 44% of the observed decline in the relative price from the data.

I report the main results in Table 5 and 6. The start of the sample I am targeting is the early 1980s and the end of the sample is the 2010s. For measured labor share, the data I am targeting is the year 1980 (initial steady state) versus the year 2016 (new steady state after the technical change has occurred) in the linear trend of the BEA-measured labor share for 1975-2016.[35] As shown in Table 5, the intangible-investment-specific technical change (IISTC),

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[34]For more details about the construction of the relative price of intangible investment, see Appendix A.

[35]Choosing 1975 as the starting year is to ensure better estimates and consistency for intangible investment because 1975 is the first year that the Federal Accounting Standards Board (FASB) required firms to report R&D, and hence, the measured labor share.

| | Start of sample | | End of sample | | Change (pp.) | |
|---|---|---|---|---|---|---|
| | Model | Data | Model | Data | Model (pp.) | Data (pp.) |
| Annual Firm Entry rate | 11.0 | 12.5 | 7.8 | 8.0 | -3.2 | -4.5 |
| Employment Share: Large Firms (500+) | 37.1 | 47.0 | 41.6 | 51.7 | +4.5 | +4.7 |
| Employment Share: Mature Firms (11 years +) | 75.7 | 66.7 | 83.5 | 80.5 | +7.8 | +13.7 |
| Top 10% concentration (Sales) | 71.4 | 67.5 | 76.3 | 72.8 | +4.9 | +5.3 |

Start of the sample: 1980-1985 average; End of the sample: 2011-2016 average

Table 6: Aggregate Implications: Concentration

or fallen relative prices of intangible investment goods is able to explain both the decline in BEA-measured labor share of post-2013 revision and of pre-1999 revision when investments into intangible capital were treated as expenditures rather than investments, by around 50%. This is consistent with the increased intangible investment as a share of gross value-added, which implies that the drop in the aggregate labor income share would be smaller if intangibles are not treated as final output.

For statistics related to concentration, the data I am focusing on is the average of 1980-1985 (initial steady state) versus the average of 2011-2016 (new steady state with the technical change). For concentration, I look at three measures: (1) the employment share of large firms with more than 500 employees; (2) the employment share of old firms with more than 11 years of operation; and (3) the share of sales going to the largest 10% of firms.[36] As shown in Table 6, the IISTC, targeted to match the decline in the relative price of intangible investment goods, accounts for around 2/3 of the decline in annual firm entry rate, 95% of the increase in the employment share of large firms, more than half of the increase in the employment share of mature firms, and around 92% of the rise of the market concentration.

## 4.2 Inspecting the Mechanisms

**Alternative Set-ups** To identify the most essential elements of the baseline model and the deep parameters that drive the key relationships that generate the results on concentration and labor share led by the intangible-investment-specific technical change (IISTC), I consider

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[36]The data on the first two measures comes from BDS. The data on concentration in terms of sales comes from Autor et al. (2019) for various sectors weighted by industry sales shares, and the empirical moment is the average of the years 1987-1992.

a number of alternative set-ups. The cases I am considering are: (1) no heterogeneity in $\eta$; (2) no heterogeneity in $z_I$; (3) persistent $z_I$ shocks (AR1); (4) no overhead labor; (5) no financial friction ($\lambda \to \infty$); (6) making intangible capital $k_I$ as collateralizable as tangible assets; and (7) allowing equity issuance at a cost. Moreover, in order to highlight the importance of having two types of technologies to produce numeraire goods and intangible investment goods with firm-specific productivity in producing intangibles, $z_I$, I consider a model where firms only operate one type of technology and accumulate intangible capital at a price that is common across firms rather than producing it at firm-specific productivity, as in the baseline case. I recalibrate parameters in all of these experiments to reproduce the same set of moments in the data as I have done in the baseline economy except for the first two cases without which the skewed firm size distribution cannot be matched well.

I list the results in Table 7. In this table, I first report three cross-sectional moments in the first column which inform the macro moments in the second column. I check how each element of my baseline model helps to generate the three cross-sectional moments, thus contributing to the macro results on concentration and the labor share, driven by the IISTC.

Heterogeneity in DRS technology $\eta$ is the first key element in the baseline model. It plays an essential role in matching the skewed firm size distribution in terms of employment, thus contributing significantly to the results on the increased concentration. On the other hand, introducing heterogeneity in $\eta$ dampens the results on the declined labor share. Without this element, the aggregate labor share declines even more compared to the baseline model. The reasons are that due to the technical change, firm distribution shifts towards large firms with high $\eta$, but high $\eta$ also means high labor share on the firm-level because firms with high $\eta$ are the ones with the lower share of income accruing to the fixed factor. This has a positive impact on the aggregate labor share. However, as we can see from the third line in Table 7, with the introduction of the heterogeneity in $\eta$, the magnitude of the decline in labor share is only reduced slightly. The reasons are as follows. Firms with higher $\eta$ requires more $k_I$ for producing $y$. Since the same $k_I$ is also used to produce $x_I$, the non-rivalry property of intangible capital helps firms escape the DRS technology. That is, labor share at firm-level in the baseline model is not monotone with $\eta$, as it is in a standard decreasing-returns-to-scale Cobb-Douglas production function.

Heterogeneity in $z_I$ is another key element in the baseline model. It plays an essential role

in matching the fact that more intangible-intensive firms (in terms of intangible-investment-to-total-assets) are larger (in terms of market share), thus contributing significantly to the results on declined labor share. Introducing permanent shocks to $z_I$ is also important to match the skewed firm size distribution, thus contributing to the the results on the rise of concentration.

| | Cross-sectional Moment | | | Macro Moment | | | |
|---|---|---|---|---|---|---|---|
| Cases | Emp. share large firms (500+) | Market share top 10% firms (intan. invest.) | Regression coefficient labor share on firm size | △ Concentration emp. | sales | entry | △ Labor share |
| Data | 47.0 | 51.9 | -0.84 | 4.7 | 5.3 | -4.5 | -4.6 |
| Baseline | 37.1 | 49.3 | -1.10 | 4.5 | 4.9 | -3.2 | -3.0 |
| No heterog. $\eta$ | 12.1 | 45.3 | -1.21 | 0.5 | 0.4 | -0.0 | -3.1 |
| No heterog. $z_I$ | 23.8 | 25.6 | -0.78 | 1.3 | 1.4 | -0.2 | -1.2 |
| $z_I \sim AR(1)$ | 37.2 | 50.1 | -1.11 | 4.6 | 5.0 | -3.2 | -3.0 |
| No overhead | 36.2 | 49.1 | -1.03 | 4.2 | 4.3 | +0.0 | -2.1 |
| $\lambda \to \infty$ | 50.2 | 73.6 | -1.85 | 27.7 | 23.5 | +11.4 | -3.3 |
| $k_I$ collateral | 37.4 | 49.7 | -1.10 | 4.5 | 4.9 | -3.2 | -3.0 |
| Equity issu. | 36.9 | 49.1 | -1.10 | 4.5 | 4.9 | -3.2 | -3.0 |
| *One type of technology* | | | | | | | |
| Full model | 37.2 | 25.3 | -0.69 | -0.2 | 0.4 | -1.1 | +1.0 |
| No heterog. $\eta$ | 11.7 | 23.2 | -0.78 | 0.1 | 0.3 | +0.6 | -0.7 |
| No overhead | 37.2 | 25.4 | 2.59 | -1.3 | 0.4 | +0.1 | +3.9 |
| Neither hetero. $\eta$ nor overhead | 11.9 | 25.2 | >100 | 0.1 | 0.3 | +0.7 | 0 |

Table 7: Alternative Set-ups

To illustrate the importance of the set-up where firms operate two types of technologies to produce both numeraire goods $y$ and intangible investment goods $x_I$ with different productivity in producing $x_I$, I consider an alternative model in which the recursive problem of incumbent firms is as follows:

$$v\left(k_I, a, z, \eta\right) = \max_{k_I', a', l, k_T, d} \left\{ d + \beta \mathbb{E} \left[ \frac{U'\left(C'\right)}{U'(C)} v^0 \left(k_I', a', z', \eta\right) \mid z \right] \right\} \tag{15}$$

s.t.

$$\underbrace{d}_{dividend} + \underbrace{P_I \left( k_I' - (1 - \delta_I) k_I \right)}_{intangible\ invest.} + a' = \underbrace{y}_{NIPA\ income} - \underbrace{wl}_{wage} - \underbrace{(r + \delta_T) k_T}_{rental\ cost} + (1 + r) a$$

$$y = Az \left[ \left( k_T^{(1-\mu)} k_I^{\mu} \right)^{1-\alpha} l^{\alpha} \right]^{\eta}$$

$$k_T \leq \lambda a, d \geq 0$$

where $P_I$ denotes the price of intangible investment relative to numeraire. Note that the differences between this model set-up, where I call it the "one type of technology" case, and the baseline model are as follows. In "one type of technology" case, (i) there is only one type of technology to produce one type of goods $y$ that can be used for investment in both physical capital and intangible capital as well as for consumption; and (ii) there is no heterogeneity in productivity in producing intangibles, so the price of intangible investment is no longer firm-specific, and instead, all the firms face the same relative price of intangible investment, $P_I$.

Correspondingly, the aggregate labor share (post-2013 revision) when intangibles are treated as final output is define as

$$S_{N,alter.} = \frac{w\bar{N}}{Y} = \frac{w \left[ \int_S (l + \kappa_o) \, d\varphi^p + \int_S \kappa_e d\varphi^e \right]}{\int y d\varphi^p}$$

and the aggregate labor share (pre-1999 revision) is defined as

$$S_{N,pre,alter.} = \frac{w\bar{N}}{Y - P_I X_I} = \frac{w \left[ \int_S (l + \kappa_o) \, d\varphi^p + \int_S \kappa_e d\varphi^e \right]}{\int y d\varphi^p - P_I \int x_I d\varphi^p}$$

At the initial steady state, $P_I = 1$. I then decrease $P_I$ to match the decline in the relative price of intangible investment goods from the data.

I report the results under "one type of technology" and label the model defined in equation (15) as the "full model", which includes both the elements of heterogeneity in DRS technology $\eta$ and the overhead labor, as the baseline model does. In this case, the negative correlation between firm size and firm-level labor share generated from the model purely relies on the introduction of the overhead labor. That is, given $\eta$, a larger firm (due to higher $z$) has a lower labor share because of the fixed cost $\kappa_o$, denominated in labor units. Even though the model is still able to generate the negative correlation between firm size and firm-level labor share, the aggregate labor share increases. The reasons are that a decrease in $P_I$ shifts the distribution of firms towards firms with high $\eta$ which have a high labor share, and this effect dominates the effect that a larger firm (i.e. more productive firms) has a lower labor share. This can be seen by comparing the case "Full model" with the case "No overhead" under "one type of technology": when there is no overhead labor, the correlation between firm size and firm-level labor share becomes positive, and the aggregate labor share increases even more.

The discussions above highlight the importance of having permanent productivity shocks to the production of intangible investment goods, $z_I$, which contributes both to the decline in aggregate labor share and the rise of concentration. While introducing heterogeneity in DRS technology $\eta$ does help match the skewed firm size distribution, thus contributing to the results on increased concentration, it only works in a set-up where firms operate two types of technologies to produce both numeraire goods $y$ and intangible investment goods $x_I$, at different productivity in producing $x_I$.

In the baseline economy, productivity on producing intangible investment goods $z_I$ is a permanent shock. Modeling it as a persistent shock following an AR(1) process and introducing more grids do improve the results but to a very limited extent.[37] Overhead labor contributes to better results as well, to a limited extent on concentration in terms of sales and employment. However, it improves the results on firm entry significantly because the cost of the technological advances (i.e. the increased equilibrium wage) dominates the benefit of them for start-up firms when calibrating their initial conditions (i.e. wealth and productivity)

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[37]I increase the number of grids of $z_I$ from two (in the baseline) to five, and calibrate the persistence of the productivity process $\rho_{z_I}$ as well as the standard deviation of productivity shocks $\sigma_{z_I}$ to target the persistence of the intangible investment to total assets ratio and the standard deviation of the intangible investment to total assets ratio respectively.

to match the data.

The next three cases are about financial frictions. In the first one, I let the collateral parameter $\lambda$ go to infinity so that there is no financial friction. In that case, the moments on employment share of large firms and market share of the top 10% in terms of intangible-investment-to-total-assets ratio overshoot the data. As we can see from Figure 2, this set-up does not feature a realistic life-cycle of firms in the sense that firms jump to their optimal level almost immediately after their birth compared to the baseline. Consequently, the moments on concentration are very sensitive to the value of technology parameters. An improved aggregate technology on producing intangibles disciplined by the decline in the relative price of intangible investment goods increases the concentration in terms of sales and employment drastically. The increase in the model, in percentage point, is almost six times as large as in the data. Moreover, it makes the annual entry rate increase by 10 percentage points, while its empirical counterpart declines by 4.5 percentage points. These results imply that having a model with realistic life cycle of firms is important to study concentration and firm dynamics.[38] In the second case, intangible capital $k_I$ is made to be equally collateralizable as tangible assets so that the collateral constraint becomes $k_T \leq \lambda a + (\lambda - 1) p k_I$.[39] Such financial friction also contributes to a realistic life cycle of firms so there is almost no change on the results of concentration compared to the baseline case. The implies the non-collateralizability assumption of intangible capital is not crucial to explain the change in firm concentration (or labor share).[40] Since in my model, intangible capital is accumulated within firm and cannot be traded, assuming that intangible capital has no collateral value seems more natural. I choose my baseline set-up also to be consistent with the empirical evidence as well as the existing literature that studies implications of a lack of collateral value for intangible capital (see Ai et al. (2019); Caggese and Perez-Orive (2018); Chen (2014); Falato et al. (2018); Garcia-Macia

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[38]In Aghion et al. (2019), the concentration in terms of sales share of top 10% firms increases by 35.1 percentage points (while the data is 5.3 p.p.) driven by fallen firm-level costs of spanning multiple markets matched to the change in between component of labor share from the data

[39]This is derived from: $b' \leq \theta \left( k_T' + p k_I' \right)$ where $b := k_T - a$, plus the timing assumption following Moll (2014) and Midrigan and Xu (2014).

[40]However, if the focus of the paper is to explain the debt-financing patters of firms with different intangible intensity, firms' cash holdings, or the cyclicality of equity returns, this assumption will be the key to the results. See, for example, Wang (2017), Falato et al. (2018), and Ai et al. (2019). Consistent with these papers, the baseline model featuring the assumption that intangible capital cannot be used as collateral contributes to explaining the rise of corporate saving flows driven by the intangible-investment specific technical change since early 1990s, but this is not going to be the focus on of my paper. Rise of corporate saving is a byproduct of the decline in labor share. See Chen, Karabarbounis, and Neiman (2017).

(2017); Wang (2017)). In the third case, I allow part of the firms (corresponding to public firms) to issue equity at a cost: $d + \boldsymbol{H}\left(d\right)\mathbf{1}_{\{\mathbf{d}\leq\mathbf{0}\}}$ where $\boldsymbol{H}\left(d\right) = -\iota|d|$ and $\iota$ is chosen to match the equity-to-asset ratio in Compustat, while the rest of the firms are still facing non-negative payment conditions. There is almost no change in the quantitative results.

To sum up, by considering alternative set-ups, I verify three key features of the baseline model that contribute to the results in Section 4: First, introducing heterogeneity in $\eta$ is important, but only works when firms operate two types of technologies to produce both numeraire goods $y$ and intangible investment goods $x_I$ with different productivity in producing $x_I$; second, whether $z_I$ is a permanent shock or a persistent shock is not crucial but having heterogeneity in $z_I$ is important; third, introducing financial frictions are necessary but whether enabling intangible capital to be collateralizable or not is not important. I choose it to be non-collateralizable in order to be consistent with my model set-up and the empirical evidence from the existing literature.

**Aggregate Elasticity of Substitution**   Finally, I derive the aggregate elasticity of substitution numerically. Given that the distribution of firms shifts towards more productive and more intangible-capital intensive firms with lower labor shares, I want to explicitly see if the aggregation of individual firms mimics an aggregate CES production technology with elasticity of substitution among factors greater than one, in response to an IISTC. [41]

Since there is no specific form of the aggregate production function in my model, I compute the aggregate elasticity of substitution between factors in production as follows. First, I compute the aggregates of factor inputs - physical capital stock $K_T$, intangible capital stock $K_I$, and labor $N$ as well as the factor prices $R_T, R_I, w$ for the three types of inputs respectively at both initial steady state and new steady state with the IISTC. Then the aggregate elasticity

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[41]Consider a simple example where there are only two perfectly competitive firms: one labor-intensive firm with production technology $Y_1 = L$ and the other is a capital-intensive firm with production technology $Y_2 = AK$. There are constant labor shares within each firm. Suppose there is capital-specific technical change such that $A$ increases. Under certain conditions, as capital becomes relatively cheaper, it is substituted for labor on the aggregate-level. For more details, see Karabarbounis (2018).

of substitution between $K_T$ and $K_I$ is computed using a mid-point rule as follows:

$$\sigma_{K_I K_T} = \frac{dln\left(\frac{K_I}{K_T}\right)}{dln\left(\frac{R_T}{R_I}\right)} \underset{mid\,point\,rule}{\cong} \frac{\frac{\left(\frac{K_I}{K_T}\right)_{final} - \left(\frac{K_I}{K_T}\right)_{initial}}{\frac{1}{2}\left[\left(\frac{K_I}{K_T}\right)_{final} + \left(\frac{K_I}{K_T}\right)_{initial}\right]}}{\frac{\left(\frac{R_T}{R_I}\right)_{final} - \left(\frac{R_T}{R_I}\right)_{initial}}{\frac{1}{2}\left[\left(\frac{R_T}{R_I}\right)_{final} + \left(\frac{R_T}{R_I}\right)_{initial}\right]}}$$

The aggregate elasticity of substitution between intangible capital $K_I$, and labor $N$ , $\sigma_{K_I,N}$, and between physical capital $K_T$, and labor $N$ , $\sigma_{K_T,N}$ are computed in the same manner.

I aggregate intangible capital as $K_I = \int k_I d\varphi^p$ which implicitly assumes that $k_I$ produced by each individual firms are perfectly substitutes. This is consistent with how I aggregate intangible investment goods $x_I$ for each individual firm in subsection 4.1. Physical capital used to produce consumption/physical investment goods $y$ is aggregated as $K_{T1} = \int k_{T1} d\varphi^p$. Physical capital used to produce intangible investment goods $x_I$ is aggregated as $K_{T2} = \int k_{T2} d\varphi^p$. Then aggregate physical capital is $K_T = \int (k_{T1} + k_{T2}) d\varphi^p$. I assume the physical capital rental rate is $R_T = \int (r + \delta_T + \zeta) d\varphi^p$ where $\zeta$ is the Lagrange multiplier of the collateral constraint faced by each individual firm, which is a non-negative number.[42] Finally, the model's equilibrium aggregate cost of intangible capital is defined by:

$$R_I = \frac{\int y d\varphi^p + \int p x_I d\varphi^p - WN - R_T K_T}{K_I}$$

Admittedly, defining $R_I$ in this way makes $R_I K_I$ not only include intangible capital rents, i.e. profit generated from $K_I$, but also profit due to decreasing returns to scale technology, because the current framework cannot separate the two. A monopolistic competition set-up may make it possible to separate profit due to owning intangible capital from profit due to downward-sloping demand which has the same effect as the DRS technology. However, I find that the results on aggregate elasticity of substitution is very robust to how I define $R_I$. I provide more details on an alternative monopolistic competition set-up when I discuss the

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[42]Note that physical capital $k_{T1}$ used to produce $y$ for each individual firm is generated by solving a *static* profit maximization problem, so the aggregate cost of physical capital $k_{T1}$ used to produce $y$ is $R_{T1} = \int (r + \delta_T + \zeta) d\varphi^p$. Physical capital $k_{T2}$ used to produce $x_I$ for each individual firm is generated after the choice of intangible capital $k_I'$ for next period is made. More specifically, given $k_I'$ (see Appendix C for algorithm), I can solve for $x_I$, which is equal to $k_I' - (1 - \delta_I) k_I$. Based on production technology for $x_I$ (equation (2)), I have $k_T^{(1-\mu)(1-\alpha)\eta} l_2^{\alpha\eta} = \frac{k_I' - (1-\delta_I)k_I}{A_I z_I k_I^{\mu(1-\alpha)\eta}}$. Then I get $k_T, l_2$ separately by solving a maximization problem given factor prices $R_T = \int (r + \delta_T + \zeta) d\varphi^p$ and $w$.

relationship of this paper with Autor et al. (2019) in the next section.

The results are

$$\sigma_{K_I,K_T} = 1.432, \sigma_{K_I,N} = 1.534, \sigma_{K_T,N} = 0.333$$

We can see that the aggregate elasticity of substitution between intangible capital $K_I$ and physical capital $K_T$ as well as between intangible capital $K_I$ and labor $N$ in production both exceed one, with $\sigma_{K_I,K_N}$ slightly larger. This is because $R_I$ drops driven by the decline in the relative price of intangible investment goods, while $R_T$ slightly increases since more firms' collateral constraints are binding after the technical change, making the effective rental rates of physical capital become larger, and the increase in $w$ is larger than the increase in $R_T$. The aggregate elasticity of substitution between physical capital $K_T$ and labor $N$ is much smaller and less than one. Therefore, the aggregation of heterogenous firmsâ production mimics the behavior of an aggregate CES production function with elasticity of substitution between capital and labor greater than one, as in Karabarbounis and Neiman (2014), and identifies that it is intangible capital that drives this relationship.

**Relationship with Autor et al. (2019)** Autor et al. (2019) presents a simple illustrative model of superstar firms where there is an industry characterized by heterogeneous firms, imperfect competition in the product market, and fixed costs of overhead labor. They show that an increase in product market competition as indexed by consumers' sensitivity to price generates a reallocation of the output and labor from less productive firms to more productive ones ("superstar firms"). This is in contrast to the "rising mark-ups" story (DeLoecker and Eeckhout (2017); Gutierrez and Philippon (2017)) where a significant part of the increase in industry concentration and the decline in the aggregate labor share can be explained by rising mark-ups.

I modify the benchmark model to incorporate imperfect competition. Producers of the final good on consumption/physical investment goods bundle are perfectly competitive and produce aggregate output $Y_t$ by combining intermediate goods $y_{it}$ according to a CES function $Y_t = \left( \int_i y_{it}^{\frac{\varepsilon-1}{\varepsilon}} di \right)^{\frac{\varepsilon}{\varepsilon-1}}$, where $\varepsilon > 1$ is the elasticity of substitution between varieties. Denoting by $p_{it}$ the price of variety $i$ and by $P_t$ the price of output, the profit maximization problem of the final goods producer yields the demand functions $y_{it} = (p_{it}/P_t)^{-\varepsilon} Y_t$. Intermediate goods are monopolistically competitive and, therefore, there are economic profits in equilibrium equal

to a fraction $s_\pi = 1/\varepsilon$ of total income generated from $Y_t$ . $Y_t$ is used for consumption and physical investment. I normalize the price of $Y_t$ to one which implies $P_t = \left( \int_i p_{it}^{1-\varepsilon} di \right)^{\frac{1}{1-\varepsilon}} = 1$.

Heterogeneous firms as in the benchmark model now produce intermediate goods that are used to produce consumption/physical investment goods bundle $Y_t$. Note that $\eta < 1$ previously due to DRS technology in the benchmark model now is due to the downward-sloping demand function, and $\eta = 1 - 1/\varepsilon$, so I modify the production technologies to constant-returns-to-scale as follows:

$$y(i) = Az(i) \left( k_{T1}(i)^{(1-\mu)} k_I(i)^\mu \right)^{1-\alpha} (l_1(i))^\alpha$$

and

$$x_I(i) = A_I z_I(i) \left( k_{T2}(i)^{(1-\mu)} k_I(i)^\mu \right)^{1-\alpha} (l_2(i))^\alpha$$

The budget constraint of individual firms' problem becomes

$$\underbrace{d_{it}}_{dividend} + \underbrace{p_{Iit} x_{It}}_{intangible\ invest.} + a_{it}' = \underbrace{p_{it} y_{it} + p_{Iit} x_{Iit}}_{firm's\ revenue} - \underbrace{w_t l_{it}}_{wage} - \underbrace{w_t \kappa_o}_{overhead\ labor} - \underbrace{(r_t + \delta_T) k_{Tit}}_{rental\ cost} + (1+r) a_{it}$$

where $p_{Iit}$ is the firm-specific shadow price of intangible investment goods produced by each individual firm and $p_{Ii} = \frac{1}{A_I z_{Ii}} / \frac{1}{A z_i p_i}$.

The simple illustrative model of Autor et al. (2019) shows that an increase in product market competition (as measured by an increase in $\varepsilon$) increases concentration in terms of market shares, reallocates output to the low labor share firms, and will decrease the aggregate labor share, as long as the between-firm effect dominates the within-firm effect. The intuition is that an increase in competition, i.e. an increase in $\varepsilon$, favors firms with high productivity, which, due to the overhead labor, have low labor shares. As $\varepsilon$ increases, high productivity firms capture a larger share of the market. They also show that an increase in $\varepsilon$ raises the productivity threshold for firms' production which reallocates labor from less productive firms with high labor share to more productive firms with low labor share. Simultaneously, however, there is an offsetting within-firm effect because an individual firm's markup of price over marginal cost is declining in $\varepsilon$ since $markup = \frac{\varepsilon}{\varepsilon - 1}$. Since the labor share is declining in the markup, and so increasing in $\varepsilon$, within firms the labor share will tend to rise with

higher competition, i.e. higher $\varepsilon$. Hence the overall effect of competition on the labor share is ambiguous and depends on the balance of the within and between-firm effects.

A version of this mechanism is present in the modified version of my model as well. An increase in $\varepsilon$ has a positive impact on concentration in terms of employment share and market share and a negative impact on aggregate measured labor share in my model. This implies that my model nests the case illustrated in Autor et al. (2019). The benchmark model is also consistent with this result in the sense that the IISTC shock shifts firm distribution towards firms with larger $\eta$ (i.e. DRS technology), which means a larger $\varepsilon$ if you consider $\eta = 1 - 1/\varepsilon < 1$ due to downward-sloping demand function.

Autor et al. (2019) concludes that any exogenous shock that favors more productive firms will allocate more market share and labor to superstar firms with low labor share in equilibrium, thus reducing the aggregate labor share. In my paper, I identify the decline in the relative price of intangibles ("intangible-investment-specific technical change (IISTC)") as such a shock.

## 5    Conclusion

In this paper, I propose a general equilibrium framework of firm dynamics highlighting the role of intangibles that can potentially account for two important macroeconomic trends of the U.S business sector over the past three decades: (i) declined measured labor income share; and (ii) increased concentration in large firms in terms of employment and sales at the national level. I show that a significant part of these phenomena can be explained by a rapid fall in the relative price of intangible investment goods, particularly software, manifested as an increase in the aggregate productivity on producing intangible investment goods relative to that on consumption/physical investment goods, where I call it an "intangible-investment-specific technical change (IISTC)." In my theory, individual firms operate at a non-standard Cobb-Douglas technology to produce both types of goods with heterogeneous productivity, and they aggregate as if the elasticity of substitution among factors in production is greater than one, triggered by the IISTC. Cross-sectionally, my model can well replicate the negative correlation between firm-level labor share and firm size from Autor, Dorn, Katz, Patterson, and Van-Reenen (2019). Firms with higher productivity on producing intangible-investment goods are more intangible-capital intensive and larger, and they benefit disproportionately

from the IISTC. Consequently, the IISTC shifts firm distribution towards large firms with low share, which leads to rising concentration and declining labor share simultaneously.

By reconciling the empirical facts regarding labor share and concentration using both aggregate and census data, my work emphasizes the importance of a comprehensive approach that links changes in micro-level heterogeneity to macro-level outcomes when analyzing the drivers of prominent empirical trends in the U.S. business sector.

Taken together, my results have several broad implications. First, they highlight the role intangible investment played in accounting for the trends of measured labor share. The declined measured labor share can reflect both technological change and improved measurement. Because intangible investment is largely unobservable, its measurement is challenging (Bhandari and McGrattan (2019); Corrado et al. (2016); Corrado, Hulten, and Sichel (2005); Karabarbounis and Neiman (2018); McGrattan and Prescott (2010b)). However, since an essential aspect of the U.S. macroeconomic model is the factor distribution of income which relies explicitly on the measurement of intangible investment and has important implications on other macro trends such as measured TFP,[43] future research efforts should be devoted to reasonably defining the boundaries of intangibles, accurately measuring intangibles at both firm-level and aggregate-level as well as figuring out the factor distribution of intangible capital rents.

Second, since the focus of this paper is to study the driving forces for the evolution of the labor share of the U.S. business sector rather than its consequences, my model concentrates on firms' side and abstracts from household heterogeneity. However, over the past five decades, the U.S. households have experienced rising inequality and uneven growth (Heathcote, Perri, and Violante (2010, 2020); Lippi and Perri (2019); Moll, Rachel, and Restrepo (2019)). Moreover, income percentiles at the lower half of the distribution of the U.S. households have stagnated since the early 1980s, and although incomes at the 95th percentile increased significantly from 1980 to 2000, it has become stagnant too since 2000. Understanding the potential implications of the secular change in the factor distribution of income on rising inequality as well as the patterns of uneven, "disappearing" growth that have experienced by the U.S. households can be another important direction for future research.

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[43]For example, in Guvenen et al. (2018), they show that increasingly common profit-shifting practices related to intangible capital, in which an multinational enterprise effectively underprices intangible capital when "sold" from one of its entities in a high-tax jurisdiction to another of its entities in a low-tax jurisdiction, have non-trivial impacts on national statistics especially the aggregate productivity growth rates.

Third, my results also indicate that a large fraction of the slowdown of the business dynamism (e.g. the decline in firm entry) and rising concentration is the natural consequences of technological advances such as falling software prices that favor the large, intangible-intensive, and highly productive firms which are more adaptable to transitioning towards a more intangible-intensive economy, thereby increasing their efficiency and advancing their market share. In other words, when the decline of the relative price of intangibles is matched to the data, my model is able to account for a significant part of the increase in concentration without involving mark-ups. Therefore, viewed through the lens of my model, policymakers may not need to worry too much about the increased concentration as well as declined firm creation and the overall dynamism of the U.S. economy.

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# Appendix A: Data and Measurement

In this appendix, I describe the data used in constructing the empirical moments to discipline the model and the intangible-investment specific technical change (IISTC). Subsection A.1 provides the details of the sources and construction of the aggregate data series. In particular, I discuss how I construct the relative price of intangible investment goods in terms of consumption/physical investment goods in Subsection A.1.2. I also show that my model can reproduce the national accounts table in Subsection A.1.3. Subsection A.2 is about the firm-level data I use. In particular, I discuss how I measure intangible capital at the firm-level to be consistent with BEA in subsection A.2.1.

## A.1 The Construction of Aggregate Data Series

All the aggregate series are retrieved for the period 1975-2016.[44] There are three sources of data that I use:

**National Income and Product Accounts (NIPA-BEA)**   NIPA 1.7.5, NIPA 1.12, NIPA 1.13, NIPA 1.14, NIPA 2.3.3, NIPA 2.3.5, NIPA 5.3.4, NIPA 5.3.5

**Fixed Assets Accounts (FAT-BEA)**   FAT 1.1, FAT 1.3, FAT 2.1, FAT 2.4, FAT 4.7

**Flow of Funds**   Table L.102: Aggregate balance sheet data for the U.S

### A.1.1 Depreciate Rate by Type of Capital

I construct the net stock of capital and depreciation of capital for traditional physical capital and for intangible capital (corresponding to IPP capital in BEA). Since I focus on the private sector only, the net stock of traditional physical capital is the private sector nonresidential structures, equipment, and residential capital. The net stock of IPP capital is only in nonresidential.

The net stock of capital by type of capital (BEA-FAT 1.1, 2.1):

Private IPP: $K^{IPP}$

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[44]Choosing 1975 as the starting year is to ensure better estimates and consistency for intangible investment (because 1975 is the first year that the Federal Accounting Standards Board (FASB) requires firms to report R&D), and hence, the measured factor shares of income

Private physical: $K^T = K^{P,ST,NRes} + K^{P,EQ,NRes} + K^{P,Res}$

The depreciation by type of capital (BEA-FAT 1.3, 2.4):

Private IPP: $DEP^{IPP}$

Private physical: $DEP^T = DEP^{P,ST,NRes} + DEP^{P,EQ,NRes} + DEP^{P,Res}$

The capital depreciation rate by type of capital is then:

$$\delta_I = \frac{DEP^{IPP}}{K^{IPP}}$$

and

$$\delta_T = \frac{DEP^T}{K^T}$$

### A.1.2 Relative Price of Intangible Investment

I construct the relative price of investment in traditional physical capital and in intangible capital (corresponding to IPP capital in BEA). The price of the bundle of consumption/physical investment good is the numeraire.

I first construct the price index for consumption $P_t^C$. Let $P_t^C$ be the price index for nondurable goods and service good $i$ in year $t$, computed as the ratio between nominal consumption of good $i$, $C_{it}$, and the quantity index of good $i$, $QC_{it}$, i.e. $P_t^C = \frac{C_{it}}{QC_{it}}$, for $i \in \{ND, SV\}$. Let $s_t^{C_i} = \frac{C_{it}}{C_{NDt}+C_{SVt}}$ be the corresponding nominal share of good $i$ in period $t$. All the variables are from NIPA 2.3.3 and 2.3.5. Denote the growth rate of a variable $x_t$ to be $\lambda(x_t) = \frac{x_t}{x_{t-1}} - 1 \approx ln\left(\frac{x_t}{x_{t-1}}\right)$. Then, the growth rate of the *Törnqvist* price index for consumption is

$$\lambda\left(P_t^C\right) = \sum_i \frac{s_t^{C_i} + s_{t-1}^{C_i}}{2} \lambda\left(P_t^{C_i}\right)$$

The level of the consumption price index is recovered recursively:

$$P_t^C = P_{t-1}^C \left[1 + \lambda\left(P_t^C\right)\right]$$

where $P_0^C$ is normalized to 1 at the initial period.

Second, I construct the price of investment in traditional physical capital including structures and equipment. For price of investment in structures $P_t^{ST}$, I use price index for consumption $P_t^C$ constructed in step 1 as a proxy. In computing the price index of equipment invest-

ment, I use a *Törnqvist* price index for private residential equipment investment, $P_t^{EQ,Res}$, and private non-residential equipment investment, $P_t^{EQ,NRes}$, from NIPA Table 5.3.4. Let $s_t^{EQ,Res}$ and $s_t^{EQ,NRes}$ be the share of private residential and non-residential equipment investment of total equipment investment using data from NIPA Table 5.3.5. The the growth rate of the price index of equipment is

$$\lambda\left(P_t^{EQ}\right) = \left(\frac{s_t^{EQ,Res} + s_{t-1}^{EQ,Res}}{2}\right)\lambda\left(P_t^{EQ,Res}\right) + \left(\frac{s_t^{EQ,NRes} + s_{t-1}^{EQ,NRes}}{2}\right)\lambda\left(P_t^{EQ,NRes}\right)$$

Then in computing the price index of traditional physical investment, I use a *Törnqvist* price index again for structures and equipment constructed above. The growth rate of the price index of the traditional physical investment is given by:

$$\lambda\left(P_t^{T}\right) = \left(\frac{s_t^{EQ} + s_{t-1}^{EQ}}{2}\right)\lambda\left(P_t^{EQ}\right) + \left(\frac{s_t^{ST} + s_{t-1}^{ST}}{2}\right)\lambda\left(P_t^{ST}\right)$$

where $s_t^{EQ}, s_t^{ST}$ are the share of equipment and structures investment of total traditional physical investment using data from NIPA Table 5.3.5.

Third, I construct the price of consumption/physical investment bundle using results in first two steps. Let $s_t^C = \frac{C_{NDt}+C_{SVt}}{C_{NDt}+C_{SVt}+invest_{EQ}+invest_{ST}}$ be the corresponding nominal share of consumption goods in the sum of consumption and traditional physical investment in period $t$ where $invest_{EQ}, invest_{ST}$ are nominal investment in equipment capital and structures capital respectively from NIPA Table 5.3.5. Similarly, Let $s_t^T = \frac{invest_{EQ}+invest_{ST}}{C_{NDt}+C_{SVt}+invest_{EQ}+invest_{ST}}$ be the corresponding nominal share of traditional physical investment in the sum of consumption and traditional physical investment in period $t$. Using a *Törnqvist* price index again for consumption and traditional physical investment constructed above. The growth rate of the price index of the consumption/physical investment bundle is given by:

$$\lambda\left(P_t^{C,T}\right) = \left(\frac{s_t^C + s_{t-1}^C}{2}\right)\lambda\left(P_t^C\right) + \left(\frac{s_t^T + s_{t-1}^T}{2}\right)\lambda\left(P_t^T\right)$$

Then the level of the price indices of consumption/physical investment bundle is recovered recursively as

$$P_t^{C,T} = P_{t-1}^{C,T}\left[1 + \lambda\left(P_t^{C,T}\right)\right]$$

Fourth, I construct the price of investment in IPP, which is only available for non-residential investment. I use the price index for IPP Investment $P_t^I$ available in NIPA Table 5.3.4. As in constructing $P_t^C$, normalize the price index of the IPP investment to 1 at the initial period as well.

Finally, the relative price of investment (using the consumption/physical investment as numeraire) is defined as

$$p_t = \frac{P_t^I}{P_t^{C,T}}$$

### A.1.3 National Accounts

The national accounts for the model can be expressed mathematically in terms of shares of income and product, where total income is $Y_b + \bar{Y}_{nb}$ where $Y_b = P_Y Y + P_I X_I = \int p_y y d\varphi^p + \int p_I x_I d\varphi^p$ represents business income and $\bar{Y}_{nb}$ denotes nonbusiness income. $\bar{Y}_{nb}$ and government expenditure $\bar{G}$ are included as exogenous source of income.

Gross Value-added (GVA) of corporate sector is defined by:

$$GVA = \underbrace{(1-\tau)Q_C}_{After\,tax\,Gross\,Output} - \underbrace{P_{Mt}M_t}_{Intermediates} + \underbrace{\tau Q_C}_{Net\,taxes\,on\,production} = \underbrace{\int_{y+px_I \geq \bar{v}} y d\varphi^p + \int_{y+px_I \geq \bar{v}} x_I d\varphi^p}_{Final\,output} + \tau Q$$

Labor income of corporate sector is defined by:

$$WL_C = w \int (l_1 + l_2 + \kappa_o) d\varphi^p + \tilde{w} \int_S \kappa_e d\varphi^e$$

Net Operating Surplus (NOS) of corporate sector:

$$NOS = \int_{y+px_I \geq \bar{v}} [y + px_I - w(l_1 + l_2 + \kappa_o) - (r + \delta_T) k_T] d\varphi^p$$

Identity:

$$\underbrace{Y_C + PX_{IC} + \tau Q_C}_{GVA} = \underbrace{WL}_{labor\,income} + NOS + \underbrace{\overbrace{\int \delta_T k_T d\varphi^p}^{depreciation} + \overbrace{\tau Q_C}^{net\,prod.\,taxes}}_{Capital\,income}$$

|  |  | Data | Model |
|---|---|---|---|
| **Income Shares** |  |  |  |
| Business income | $\left( \int y d\varphi^p + \int px_I d\varphi^p + \tilde{Q} \right)/Y_a$ | 0.791 | 0.791 |
| Corporate business income | $\left( \int_{y+px_I \geq \bar{v}} y d\varphi^p + \int_{y+px_I \geq \bar{v}} px_I d\varphi^p + \tau Q_C \right)/Y_a$ | 0.608 | 0.608 |
| Corporate labor income | $wL_C/Y_a$ | 0.385 | 0.385 |
| Net operating surplus | $NOS/Y_a$ | 0.100 | 0.124 |
| Consumption of fixed capital | $\left( \int_{y+px_I \geq \bar{v}} \delta_T k_T d\varphi^p \right)/Y_a$ | 0.075 | 0.051 |
| Production taxes | $\tau Q_C/total\ income$ | 0.048 | 0.048 |
| Noncorporate business income | $\left( \int_{y+px_I < \bar{v}} y d\varphi^p + \int_{y+px_I < \bar{v}} px_I d\varphi^p + \tau Q_{NC} \right)/Y_a$ | 0.183 | 0.183 |
| Nonbusiness income | $\bar{Y}_{nb}/total\ income$ | 0.209 | 0.209 |
| **Product Shares** |  |  |  |
| Private Consumption | $C_h/Y_a$ | 0.612 | 0.632 |
| Private Investment | $(K_T - (1-\delta_T)K_T + PX_I)/Y_a$ | 0.182 | 0.144 |
| Gov't Investment + consumption | $\bar{G}/total\ income$ | 0.206 | 0.219 |

Table 8: National Account Shares, Data and Benchmark Model

Then corporate business income/total business income is given by:

$$\frac{Y_C + PX_{IC} + \tau Q_C}{\underbrace{\left( Y_b + \bar{Y}_{nb} \right)}_{Y_a}}$$

The national accounts of this economy can be summarized in Table 8.

## A.2 Firm-level Data

For measurement of intangible capital at the firm-level, I use Compustat North America-Capital IQ, which provides annual accounting data for publicly listed U.S. firms. This data set fits our purpose well because firm-level R&D investment data are available and because it is well-suited to study U.S. firmsâ financial aspects due to its rich firm characteristics and industry information. I exclude foreign firms, government-sponsored firms, public utilities and financial firms, as is commonly done in the investment literature. I also exclude mergers, acquisitions, and observations with extreme values. To be consistent with the aggregate data, I focus on the period 1975 - 2016. After following the standard data cleaning procedures (see, for example, Imrohoroglu and Tuzel (2014); Ottonello and Winberry (2018)), I end up with 14734 firms in total and 153,505 firm-year observations. The representativeness of the dataset is fairly good: total assets Compustat/Flow of Funds (non-financial corporate sector) ranges from 50 to 75%. Again, since I focus on all the employer firms in my model, so I filter a subset

of firms in my model based on firm size with the criterion specified in the main text to target moments constructed using Compustat data.

### A.2.1 Measurement of Intangible Capital at the Firm-level

Existing literature attempts to measure intangible capital either directly or indirectly. The indirect approach is to construct a proxy using aggregate stock market or national accounting data (e.g. Karabarbounis and Neiman (2018); McGrattan and Prescott (2010b)). These approaches measure intangibles as unexplained (by physical capital) residuals of stock market value or firm productivity. The other approach is to construct aggregate measures of the different components of intangible capital directly (Corrado, Hulten, and Sichel (2005)) using a wide range of aggregate datasets including NIPA, the Services Annual Survey (SAS), surveys of employer-provided training from BLS. The aggregate data on intangibles considered in this paper is measured following the method developed by Corrado, Hulten, and Sichel (2005), henceforth, CHS. The biggest advantage of this method is that it includes the most types of intangible assets including the very firm-specific human and structural resources, in addition to software, R&D, and artistic originals which have been included into the national accounts. In general, CHS's method includes three categories of business intangibles: (1) computerized information, which a firm's knowledge embedded in the computer programs and computerized databases; (2) innovative property, which is a firm's knowledge acquired through scientific R&D and non-scientific inventive and creative activities; (3) economic competencies, which is a firm's knowledge embedded in firm-specific human and structural resources, including brand equity and on-the-job training.

For the purpose of accounting for the BEA-measured income shares, I target the BEA-measured intangibles (or IPP called by the BEA) on the aggregate. To be consistent with the BEA method on the firm-level, I construct a measure of intangible capital including software and R&D using Compustat data. The difficulty is that the capital that is created by investments in intangible assets such as R&D are only expensed, thus not being reported on firms' balance sheets. Following the method developed by Peters and Taylor (2016) and Falato et al. (2018), the essential idea to overcome this difficulty is to capitalize expenses related to intangible assets consistent with BEA.

More specifically, the replacement cost of knowledge capital is measured by capitalizing

R&D expenditures using perpetual inventory method with depreciation rate of 20%.

Since firm-level expenses on software are not available, I construct an industry-level measure to approximate the intangible assets on them. The BEA classification features 63 industries. I match the BEA data to Compustat firm-level data using SIC codes, assuming that, for a given year, firms in the same industry have the same shares of intangible assets on software. I construct measures of software shares for industry $l$ in year $t$ as

$$software_{l,t} = \frac{IPP_{l,t}^{BEA} \times \frac{software_t^{BEA}}{IPP_t^{BEA}}}{FixedAsset_{l,t}^{BEA}} FixedAsset_{l,t}^{Compustat}$$

where $FixedAsset_{l,t}^{Compustat}$ are total assets in industry $l$ in year $t$. Since data on the fixed assets of software are only available at the aggregate level rather than the industry level but intellectual property products are, I use the economy-wide ratio of software to IPP multiplied by the IPP at the industry level to approximate the industry-level intangible assets on software. The BEA data comes from Fixed Assets Accounts Tables (FAT) Table 3.7I. Informational capital is constructed by capitalizing expenditures on software with a depreciation rate of 31% following BEA.

Knowledge capital (in terms of the replacement cost), information capital, and the on-the-balance-sheet intangibles consists of the intangible capital stock on the firm-level. When a firm purchases an intangible asset externally, for example, by acquiring another firm, the firm typically capitalizes the asset on the balance sheet as part of Intangible Assets, which equals the sum of Goodwill and Other Intangible Assets. The asset is booked in Other Intangible Assets if the acquired asset is separately identifiable, such as a patent, software, or client list[45]. Acquired assets that are not separately identifiable, such as human capital, are in Goodwill. When an intangible asset becomes impaired, firms are required to write down its book value. There is debate about whether on-the-balance sheet "intangibles" should be added into the measure of intangible capital on the firm-level or not. Following Peters and Taylor (2016), I keep Goodwill in Intangible Assets in my main analysis, because Goodwill does include the fair cost of acquiring intangible assets that are not separately identifiable. However, it should be noted that due to the inclusion of goodwill, the item picks up over-payment in mergers & acquisitions (M&As). Hence, I also try excluding Goodwill from external intangibles as a

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[45]But this variable "intangibles-Other" variable in Compustat that is net of goodwill is only available in Compustat since 2000

| coefficient estimate | Data | Model |
|---|---|---|
| $\alpha$ | -0.078*** | -0.043*** |

***$p < 0.01$

Model simulated starting with entrants distribution for 50,000 firms and 100 years

Table 9: Financing patterns

| | Dependent variable: market share | | | |
|---|---|---|---|---|
| | (A) | (B) | (C) | Model |
| **Compustat intangible share** | 0.105*** | 0.025*** | 0.014*** | 0.258*** |
| Industry×Year F.E.. | Yes | No | No | - |
| Firm F.E. | No | Yes | Yes | - |
| Year F.E. | No | No | Yes | - |

***$p < 0.01$

Model simulated starting with entrants distribution for 50,000 firms and 100 years

Table 10: Intangibles and market share

robust check.

In general, my resulting estimate for the ratio of intangible to tangible capital over the past decades is comparable to the estimate based on BEA.

Next, I discuss two empirical patterns I generate from Compustat and compare the corresponding patterns generated from my model with them, which serve as validations.

First, I find that intangible-intensive firms tend to rely less on external debt finance. More specifically, I run the following OLS regression:

$$\frac{Net\,Debt_{it}}{Asset_{it}} = \alpha\frac{Intangible\,Capital_{it}}{Asset_{it}} + \beta X_{it} + Year_{dummy} + Industry_{dummy} + \varepsilon_{it}$$

where $X$: a set of control variables including Tobin's Q, physical investment/assets ratio, firm size, dummy variables for positive dividend payout. I report the results in Table 9 with comparison between model and data.

Second, I find that a firm's market share in its industry is higher when its intangible capital to total assets is higher, and this relationship holds between firms of the same industry, within firms over time, and controlling for year effects. This is in line with Crouzet and Eberly (2019) although they measure intangible capital at the firm-level different from mine [46]. I

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[46]Crouzet and Eberly (2019) don't capitalize R&D expenses and software expenses. That is, they only consider intangibles on the balance sheet.

| Data Source | Number of Employees |
|---|---|
| BDS 2007 | 119,627,020 |
| SBO PUMS 2007 | 30,465,820 |
| Compustat 2007 | 73,434,000 |
| BEA 2007 (all private domestic industries) | 118,944,000 |

| Data Source | Employer Firms (number) | Employer Firms' receipts ($1000) | Nonemployer Firms (number) | Nonemployer Firms' receipts ($1000) |
|---|---|---|---|---|
| SBO 2007 | 5,287,344 | 10,015,051,752 | 21,104,893 | 932,487,829 |
| BDS 2007 | 5,240,019 | - | 0 | 0 |

Table 11: Coverage of Firms in Multiple Datasets

Note: The SBO covers all nonfarm businesses filling IRS tax forms as individual proprietorships, partnerships, or any type of corporation with receipts of $1,000 or more. However, businesses classified in the SBO as publicly owned are not included in the PUMS version.

report results in Table 10.

### A.2.2 Other Firm-level Moments

For firm distribution, I focus on all the employer firms in U.S. business sector. For statistics related to firm entry, exit, job creation and destruction, firm size and age distribution, I calculate directly from the Business Dynamic Statistics (BDS), which are compiled from the Longitudinal Business Database (LBD). The LBD is a confidential longitudinal database of business establishments and firms starting from 1976. For any moments I need to rely on LBD, I draw them from the existing literature with corresponding years.

As I have mentioned in the main text, I admit that there exists discrepancy between the coverage of BEA-business sector, which covers both employer and non-employer firms and the coverage of BDS/LBD, which only covers employer firms. However, as shown in Table 11, non-employer firms do not contribute to the total employment and only takes a very small portion of total sales (which is 2.48% based on SBO 2007), I assume BEA-business sector and BDS/LBD cover the same firms.

| Change in Category | Data (pp.) | Model (pp.) |
|---|---|---|
| Labor share (post-2013-revision) | -4.6 | -2.7 |
| Labor share (pre-1999-revision) | -2.4 | -1.1 |
| Saving flows/GVA | +6.0 | +5.0 |
| Annual Firm Entry rate | -4.5 | -2.9 |
| Employment Share of Large Firms (500+) | +4.7 | +4.2 |
| Employment Share of Mature Firms (11+) | +13.7 | +6.9 |
| Top 10% concentration (Sales) | +5.3 | +4.4 |
| Average firm size | +12.2 | +8.9 |
| Labor productivity dispersion | +8.0 | +5.5 |
| Labor productivity gap between frontier and lagged firms | +20.00 | +7.20 |

Table 12: Aggregate Implications: Additional Results

# Appendix B: Empirical Facts

In this section, I summarize the empirical facts, mainly declined labor share and increased concentration, that my paper attempts to explain, together with some additional facts regarding business dynamism that my model can account for as well.

To summarize, I focus on the following regularities, most of which occur during the the past three decades:

1. The labor share of gross value-added has gone down.

2. The employment share of large firms (500+ employees) has risen.

3. The employment share of mature firms (11+ years) has risen.

4. Market concentration in terms of top 10% has risen.

5. Firm entry rate has declined.

6. Productivity dispersion of firms has risen. Similarly, the labor productivity gap between frontier and laggard firms has widened.

7. The corporate saving flows relative to gross value-added has risen, together with weaker physical investment.

The rise of corporate saving is due to the non-collateralizability of intangible capital. Since the IISTC leads to greater importance of intangible capital in firms' production, firms

A. intangible capital to tangible assets ratio
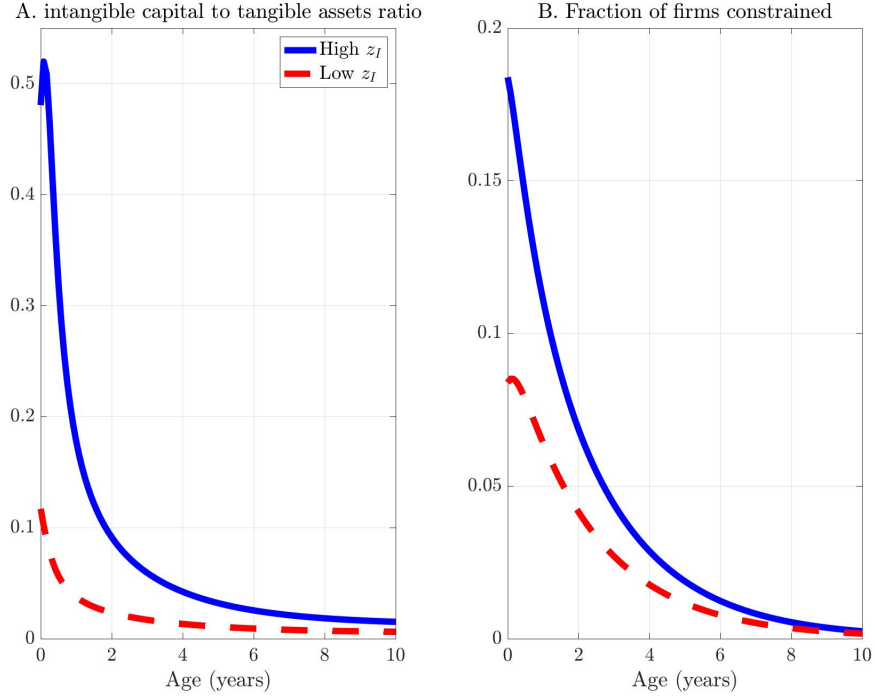
B. Fraction of firms constrained

Figure 4: Life Cycle Dynamics of Firms: Additional Results

accumulate more intangible capital. On the other hand, since intangible capital cannot be used as collateral for firms to borrow, firms need to rely more on internal financing rather than external financing. This is the channel where the IISTC results in increased firms' saving flows relative to the total income on the aggregate level. Additional results on the cross-sectional life cycle dynamics of firms can also shed light on this point. In Figure 4, we can see that firms with high productivity in producing intangible investment goods are more intangible-capital intensive and are also more financial constrained. As firms become mature, they can gradually get rid of the collateral constraints.

I summarize the results on how my model accounts for the additional empirical facts driven by the IISTC besides labor share and concentration in Table 12.
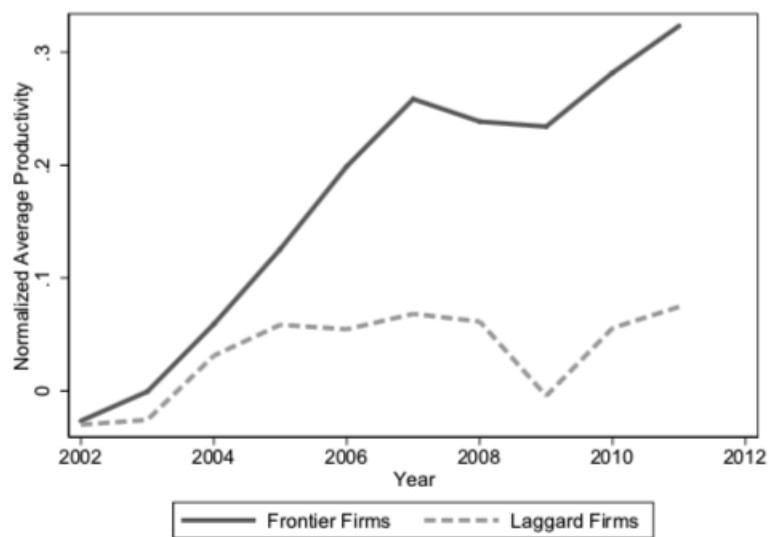
Figure 5: Productivity gap between frontier and laggard firms
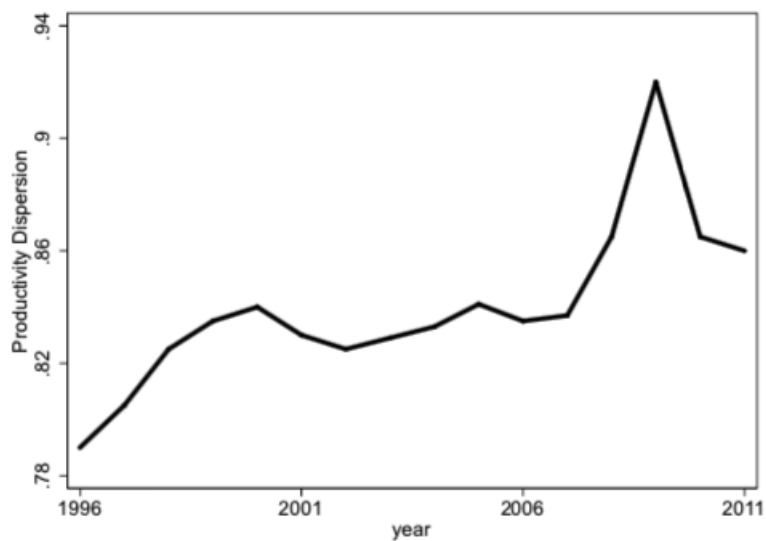Source: Andrews, Criscuolo, and Gal (2015)



Figure 6: Labor productivity dispersion
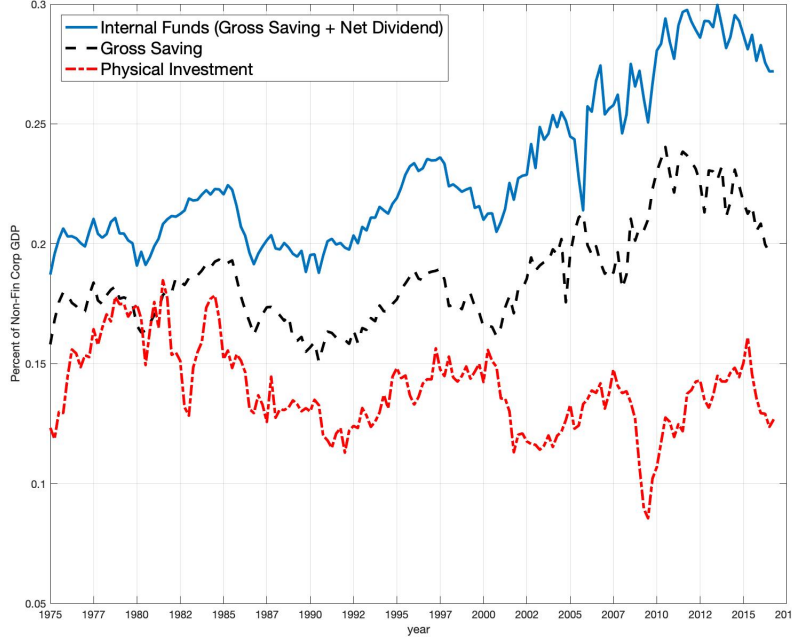Source: Decker et al. (2018)

Figure 7: Corporate saving flows & Physical Investment
Source: Flow of Funds; BEA-NIPA

## Appendix C: Computational details

I essentially follow an algorithm that is similar to Gavazza, Mongey, and Violante (2018) and solve value and policy functions using a method developed by Mongey (2015).[47]

### C.1 Algorithm

I describe the solution method for my long-run GE economy in this section. The usual nested fixed point approach is extended in order to accommodate the additional features of my model. That is, the essence of our approach is to guess a set of prices, compute decision rules (given prices) to simulate the economy, and finally verify whether those are the equilibrium prices.

Specifically, I execute the following steps:

1. Make an initial guess for the wage $\tilde{w}$. With specifying the upper bound and lower bound for wage, $w_0^u, w_0^l$, the initial guess can be $\frac{w_0^u + w_0^l}{2}$. Due to the risk neutrality of

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[47]This method involves programming in Matlab with MEX, which relies on the $COMPECON$ toolbox developed by Miranda and Fackler (2002). The toolbox can be found at http://www4.ncsu.edu/~pfackler/compecon/toolbox.html

households, the real interest rate is $r = \frac{1}{\beta} - 1$ at the steady state.

2. Solve both incumbent firms' and entrants' dynamic programming problem described in Section 3 at the prices $\tilde{w}, r$. The general procedure involves using a nested vectorized golden search method to solve the optimization problems and find the policy functions $\left(k'_I(s), a'(s)\right)$. More specifically, I solve the optimization problem in two steps: (1) solve the policy function for $a'(s)$ given $k'_I(s)$. I first solve $(k_{T1}, l_1)$ which is the solution to a static problem of firms. I then define a cash-in-hand variable $cih = y - (r + \delta_T)k_{T1} - \tilde{w}l_1 + (1+r)a - \kappa_o$, and solve $k_{T2}, l_2$ according to the production technology for intangible investment goods (equation 2) given $k'_I, z_I$, and prices $R_T$ and $\tilde{w}$. Note that the effective rental rate of physical capital can be different since some firms' collateral constraint are more binding than the other, so I also need to solve the value of the Lagrange multiplier of the collateral constraint, call it $\zeta$ faced by each individual firm using a bisection method. Next, check two things: whether $cih - wl_2 - R_T k_{T2} \geq 0$ or not and whether $\zeta > 0$ or not. Depending on the answers to the two checking questions, we are going to discuss three cases. Case I: $cih - wl_2 < 0$. In this case, we simply set $a' = 0$ and $d = cih - wl_2 - R_T k_{T2}$. Negative dividend payment serves as a penalty. Case II: $cih - wl_2 - R_T k_{T2} \geq 0$ and $\zeta > 0$. In this case, firms are constrained, thus paying zero dividend, i.e. $d = 0$. Then $a' = min\left(cih - wl_2 - R_T k_{T2}, amax\right)$. Case III: $cih - wl_2 - R_T k_{T2} \geq 0$ and $\zeta = 0$. In this case, firms are NOT constrained and thus need to pay the dividend. Use golden search to solve for $\left(a', d\right)$. (2) after expressing $a'$ in terms of $k'_I$, solve for $k'_I$ using golden search again. More details about the method I used to solve the policy functions will be discussed in section C.2.

3. Using the policy functions for both incumbents and entrants to find the stationary distribution using the method discussed in section C.3.

4. Given the stationary distribution and policy functions of firms, compute the aggregate labor demand $LD$.

5. Compare the aggregate labor demand $LD$ at prices $\tilde{w}, r$ with the inelastic labor supply $\bar{N}$. If $|LD - \bar{N}| < tolerance$, then we are done. Otherwise, we employ a **bisection price updating scheme**. More specifically, we update the guess for wage based on the following rule: if $LD > \bar{N}$, set the lower bound equal to the lower bound the same

as before, $w^l = w^l{}_{old}$ and the upper bound equal to the wage in the current iteration, $w^u = \tilde{w}$; if $LD < \bar{N}$, set $w^l = \tilde{w}$ and $w^u = w^u{}_{old}$. Then a new guess for wage is $\frac{w^u + w^l}{2}$ and return to step 2.

## C.2 Value functions and policy functions

I use collocation methods to solve the firm's value function problem (4), (5), (6), and (7). Let $s \equiv (k_I, a, z)$ be the firm's idiosyncratic state, abstracting from heterogeneity in $(\eta, z_I)$ which are both permanent. I solve for an approximant of the expected value function $V^e\left(k_I', a', z'\right)$ which gives the firm's expected value conditional on current decisions for intangible capital, tangible assets, and productivity:

$$V^e\left(k_I', a', z'\right) = \int_{\mathcal{Z}} V\left(k_I', a', z'\right) d\Gamma\left(z, z'\right)$$

where the integrand is the value given in (5).

I set up a grid of collocation nodes $S = K_I \times A \times Z$ with $Nk_I = Na = Nz = 10$. Following Gavazza, Mongey, and Violante (2018), I construct $Z$ by first creating equi-spaced nodes from 0.001 to 0.999, which I then invert through the cumulative distribution function of the stationary distribution implied by the $AR(1)$ process for $z$ to obtain $Z$. This ensures better coverage in the higher probability regions for $z$. I also choose $K_I, A$ to have a higher density at lower values. The upper bound for intangible capital, $\bar{k}_I$, is chosen so that the optimal size of the highest productivity firm $k_I^*(\bar{z})$ is less than $\bar{k}_I$. I choose the upper bound for tangible assets, $\bar{a}$, so that the maximum optimal physical capital $k_{T1}^*(\bar{z})$ can be financed, that is, $k_{T1}^*(\bar{z}) \leq \lambda \bar{a}$. Note that $K_I, A$, and $Z$ are parameter dependent, and therefore recomputed for each new vector of parameters considered in estimation.

I approximate the expectation of value function $V^e(s)$ on $S$ using a linear spline[48] with $Nk_I \times Na \times Nz$ coefficients. Given a guess for the spline's coefficients $\left(c_j, c_j^e\right)_{j=1,\cdots,Ns}$, I iterate towards a vector of coefficients that solve the system of $Ns$ Bellman equations, which are linear in the $Ns$ unknown coefficients. More specifically, I have:

$$V(s_j) = \sum_{j=1}^{Ns} \phi(s_j) c_j, \quad V^e(s_j) = \sum_{j=1}^{Ns} \phi(s_j) c_j^e$$

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[48] Alternatively, you may use cubic spline or Chebyshev polinomials.

where $\phi(\cdot)$ is a basis function.

Each iteration proceeds as follows. Given the spline coefficients, I use vectorized golden search to compute the optimal policies for all states $s \in S$ and the value function $V(s)$. I then fit another spline to $V(s)$, which facilitates the integration of productivity shocks $\varepsilon \sim N(0, \sigma_z)$. To compute $V^e(s)$ on $S$, I approximate the integral by

$$V^e(k_I, a, z) = \sum_{i=1}^{N_\varepsilon} w_i V^e(k_I, a, exp(\rho_z log(z) + \varepsilon_i))$$

Here, $N_\varepsilon = 80$, and the values of $\varepsilon_i$ are constructed by creating a grid of equi-spaced nodes between 0.001 to 0.999, then using the inverse cumulative distribution function of the shocks (normal) to create a grid in $\varepsilon$. The weights $w_i$ are given by the probability mass of the normal distribution centered on each $\varepsilon_i$.

The major advantages of having continuously distributed productivity shocks rather than discretizing it by employing Tauchen or Rouwenhorst are threefold. First, I can compute the integral more precisely. Second, in a model that features endogenous entry and exit, continuously distributed productivity shocks ensures much easier convergence in the value function iteration, compared to discretization. Another important advantage is that by having continuously distributed productivity shocks, you can have a sufficiently dense grid of productivities for the potential entrants, while in the case of discretization, the number of grids of productivities must be equal to $Nz = 10$. When the number of grids of productivities is too small, changing in prices may lead to either too much entry or too little entry, which makes it much harder to find the equilibrium. Having a sufficiently dense grid of productivities is also important to calibration since it ensures that entry does not jump across parameters.

## C.3 Stationary distribution

To construct the stationary distribution, as Gavazza, Mongey, and Violante (2018), I use the method of non-stochastic simulation from Young (2010), modified to accommodate a continuously distributed stochastic state. I create a new, fine grid of points $Sf$ on which I approximate the stationary distribution using a histogram, setting $Nk_If = Naf = Nzf = 100$. Given our approximation of the expected continuation value, I solve for the policy functions $k_I'(s^f)$ and $a'(s^f)$ on the new grid and use these to create two transition matrices $\mathbf{Q}_{k_I}, \mathbf{Q}_a$ which

determine how mass shifts from points $s^f \in S^f$ to points in $K_I f, Af$, respectively. I construct $\mathbf{Q}_x$ as follows for $x \in \{k_I, a\}$:

$$\mathbf{Q}_x[i, j] = \left[ \mathbf{1}_{x'\left(s_i^f\right) \in \left[X_{j-1}^f, X_j^f\right]} \frac{x'\left(s_i^f\right) - X_j^f}{X_j^f - X_{j-1}^f} + \mathbf{1}_{x'\left(s_i^f\right) \in \left[X_j^f, X_{j+1}^f\right]} \frac{X_{j+1}^f - x'\left(s_i^f\right)}{X_{j+1}^f - X_j^f} \right],$$

for $i = 1, \ldots, N_s^f$ and $j = 1, \ldots, N_x^f$. The transition matrix for the process for $z$ is computed by $\mathbf{Q}_z = \sum_{i=1}^{N_\varepsilon} w_i \mathbf{Q}_z^i$, where $\mathbf{Q}_z^i$ is computed as above under $z'\left(s^f\right) = exp\left(\rho_z log\left(z\right) + \varepsilon_i\right)$. The overall incumbent transition matrix $\mathbf{Q}$ is simply the tensor product $\mathbf{Q} = \mathbf{Q}_z \otimes \mathbf{Q}_a \otimes \mathbf{Q}_{k_I}$.

To compute the stationary distribution, I still need the distribution of entrants. To allow for entry cutoffs to move smoothly, I compute entrant policies on a dense grid of $Nz^0 = 100$ productivities. The grid $Z^0$ is constructed by taking an equally spaced grid in $[0.01, 0.99]$ and inverting it through the cumulative distribution function of potential entrant productivities (exponential). Let the corresponding vector of weights be given by $\mathbf{P}_0$. Given the approximation of the continuation value $V^e\left(s_0\right)$, I can solve the potential entrant's policies conditional on entry. I can then solve the firm's discrete entry decision. Finally, I compute an equivalent transition matrix $\mathbf{Q}_0$ using these policies, where non-entry results in a row of zeros in $\mathbf{Q}_0$.

The discretized stationary distribution $\mathbf{L}$ on $S^f$ is then found by the following approximation to the law of motion

$$\mathbf{L} = \left(\mathbf{1} - \pi_{\mathbf{d}}\right) \mathbf{Q}'\mathbf{L} + M_0 \mathbf{Q}_0'\left(\mathbf{P}_0\right),$$

which is a contraction on $\mathbf{L}$, solved by iterating on a guess for $\mathbf{L}$. The final stationary distribution is found by choosing $M_0$ such that $\sum_{i=1}^{N_s^f} L_i = 1$.