Abstract

In a risk-neutral world, the equilibrium exchange rate is determined by the uncovered interest rate parity (UIP) condition. This condition implies that the regression of realized excess returns of investing in foreign short-term bonds on current interest rate differential should produce a coefficient of zero. Nevertheless, Fama (1984) and many other subsequent empirical tests rejected a coefficient of zero, and these lead to several well-documented major exchange rate puzzles. Our paper develops a new behavioral monetary model with privately informed heterogeneous agents to study equilibrium exchange rate behavior. Motivated by the empirical evidence of the dynamic responses of survey forecasts to the interest rate shocks, we allow the agents in our model to perceive the interest rate process differently from the truth process and misunderstand the precision of their private signals. Our model produces the empirical pattern of interest rate forecasts we observe in the data: under-shooting (under-reaction) early on, but overshooting (over-reaction) later on in response to monetary shocks. It is also helpful in explaining several major exchange puzzles. We also use survey forecast data to uncover a range of model parameters and conduct some quantitative exercise.

Exchange Rate Anomalies

The UIP condition is $E_t[\pi_{t+1}] = x_t + x_r$, where $x_t$ is the log of the nominal exchange rate defined as home currency units per one unit of foreign currency and $x_r = i_t - i_t^*$ is the interest rate differential defined as the difference between the nominal and foreign nominal interest rates. We define the excess return in period $t + k$ as $\varepsilon_t^{\pi_{t+k}} = x_{t+k} - x_t + \pi_{t+k}$, where we have $E_t[\varepsilon_t^{\pi_{t+k}}] = 0$ by the UIP condition. Equivalently, if we run the following Fama regressions,

$$\varepsilon_t^{\pi_{t+k}} = \alpha_k - \beta_k x_t + \varepsilon_{t+k}$$

we should expect $\beta_k = 0$ for all $k$. However, the empirical tests of this excess return regression show strong deviations and some puzzling facts (Valchev, forthcoming).

![Fig 1. Fama Regressions on G-10 Countries](image)

We see the sign of $\beta_k$ is a function of $k$ (time), initially positive and decreases to negative values. This finding leads to the following well-documented puzzles in exchange rate market:

1. **Forward discount puzzle**: higher interest rate currencies have higher expected returns over the near future i.e. $\beta_k > 0$
2. **Predictability reversal puzzle**: higher interest rate currencies have lower expected returns over some periods of time i.e. $\beta_k < 0$ for all $k > K$
3. **Engel puzzle**: high interest rate currencies are stronger than implied by UIP such that $\sum \beta_k < 0$
4. **Delayed overshooting puzzle**: a monetary contraction that raises the interest rate leads to a prolonged period of appreciation, followed by gradual depreciation

IRF of Forecast Errors

Following Kucinskas and Peters (2019), we estimate the impulse response function of forecast errors

![Fig 2. IRF of Forecast Errors of U.S. Interest Rates (1985-2018)](image)

The IRF of the forecast errors of the U.S. short-term interest rates is consistent with the findings of Angelos, Hoover, and Sastry (2020) under a Taylor-rule monetary policy: under-shooting (under-reaction) early on but overshooting (over-reaction) later on in response to shocks.

The Model

The interest rate differential process $x_t$ has a persistent and a temporary component:

$$x_t = x_t + \sigma_k x_t + \sigma_t$$

where $x_t$ and $\sigma_t$ are independently distributed $N(0,1)$ and $x_t$ is the underlying latent variable which is not observable to any agents. However, each agent observes a private signal of the fundamental variable:

$$y_t = x_t + \sigma_t$$

where $\sigma_t \sim N(0,1)$. There is a unit mass of agents and they learn the underlying variable upon seeing the current realization of interest rate differential and signals. The equilibrium exchange rate is determined by aggregated investor beliefs:

$$s_t = s_t + E_t[\pi_{t+1}] = x_t + \sum E_t[\pi_{t+1}] x_t y_t$$

whose difference from the standard UIP conditions is that the representative agent’s expectation term is replaced by the aggregated market belief.

To reconcile this empirical finding, we assume that the agents in our model are not fully rational. Instead, we allow the agents to perceive the interest rate process differently from the true process and mis-interpret the precision of their private signals. Agents regard the AR coefficient of the latent variable as $\beta$ and the variance of the private signal as $\sigma^2$.

Proposition 1. The equilibrium exchange rate is $s_t = h_1(t) v_t + h_2(t) f_t$ where $h_1$ and $h_2$ are functions of model parameters and lag operator.

We simulate the model and the following plots the exchange rate path and UIP regression coefficients

![Fig 3a. Simulated Exchange Rates](image) ![Fig 3b. UIP Regression Coefficients](image)

Calibrations

We calibrate the model parameters to match the UIP regression coefficients using the U.S. interest rate data

![Fig 4a. Calibrated UIP Regression Coefficients and Forecast Error IRF](image)

References


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