Welfare Analysis under Probabilistic Choices in a Rational Expectations Equilibrium Model

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The paper in a nutshell

- More **informed trading** makes price more informative.
- When costly **information acquisition is certain**, this distorts risk-sharing, **reduces** risk and return trade-off and hence **social welfare**.
- However, when **information acquisition is uncertain** and traders make strategic choices about the probability of observing costly information, more informed trading generates a **positive asymmetric-information-effect** on the benefit of informed comparing to uninformed.

Information acquisition uncertainty provides traders an opportunity to improve their ex-ante welfare in more efficient markets.

Introduction

Information acquisition is certain:

- In Grossman and Stiglitz (1980), **informed trading reduces welfare** for two reasons: *Hirshleifer effect* and *risk-return effect*.
- "*The common theme of both channels is that disclosure harms investors through destroying trading opportunities*" (Goldstein & Yang, 2017).
- The only Pareto-efficient equilibrium is the *no-informed-trading equilibrium*.

Information acquisition uncertainty:

- A trader may decide to purchase an analyst report, hoping to obtain some valuable information about the fundamental value of the firm.
- *Ex-post*, the report could turn out to be either informative or completely useless.
- However, *ex-ante*, the trader expects a higher probability of becoming informed by paying more for a more valuable report.
- Therefore information acquisition is uncertain and traders make a decision to increase the probability of observing the information.

Asymmetric-information-effect:

- *information acquisition uncertainty* and *probabilistic choices* (Mattsson & Weibull, 2002) in the standard REE model leads to a positive *asymmetric information effect* on welfare.
- It can overcome the negative risk-return and Hirshleifer effects and improve welfare.

Model and Equilibrium

- A continuum of homogenous traders investing in a risk free asset and a risky asset with payoff $\tilde{D} = d + \tilde{\theta} + \tilde{\epsilon}, \tilde{\theta} \in N(0, v_\theta), \tilde{\epsilon} \in N(0, v_\epsilon)$.
- **Two stages** of the model:
  - Each trader chooses strategically a probability p_i^* to become informed. As a result, a certain (random) fraction λ of traders becomes informed.
  - Each trader forms an optimal portfolio conditional on his information.

$$\max_{p_i} U(p_i;\lambda) = [p_i V_I(\lambda) + (1-p_i)V_U(\lambda)]e^{\alpha\mu c(p_i)};$$

$$V_i(\lambda) = \max_{x_i} E\left(E\left(-e^{-\alpha x_i^*(\theta,P)(D-P)}\middle|\theta, P\right)\right) = -\frac{1}{\sqrt{1+\xi_i(\lambda)}}; \qquad i = I, U.$$

- The ***equilibrium fraction of informed traders*** λ is determined by a Nash equilibrium and the ***equilibrium price*** is determined by market clearing:

$$\lambda = g^{-1}\left(\frac{1}{\alpha\mu}\frac{\gamma(\lambda)}{1-\gamma(\lambda)}\right); \qquad \tilde{P} = d + b_\theta\tilde{\theta} - b_z\tilde{z}; \qquad \gamma(\lambda) = 1 - \frac{V_I(\lambda)}{V_U(\lambda)};$$

$$b_\theta = \frac{\lambda\bar{v}}{v_\epsilon}; b_z = \alpha\bar{v}; \ \frac{1}{\bar{v}} = \frac{\lambda}{v_\epsilon} + \frac{1-\lambda}{v_U}; v_U = v_D\left(1+\frac{n\lambda}{\xi_0}\right); n = \frac{v_\theta}{v_\epsilon}; \xi_0 = \alpha^2 v_z v_D.$$

Welfare Analysis

$$W(\lambda) = U(\lambda,\lambda) = \bar{V}(\lambda)e^{\Phi(\lambda)}, \qquad \bar{V}(\lambda) = \lambda V_I(\lambda) + (1-\lambda)V_U(\lambda); \qquad \Phi(\lambda) = \frac{c(\lambda)}{c'(\lambda)}\frac{\gamma(\lambda)}{1-\lambda\gamma(\lambda)}$$

Welfare improvement decomposition:

risk-return effect + asymmetric-information effect+ marginal cost:

$$\frac{W'(\lambda)}{-W'(\lambda)} = \frac{\lambda V_I'(\lambda) + (1-\lambda)V_U'(\lambda)}{\bar{V}(\lambda)} + \frac{V_I(\lambda) - V_U(\lambda)}{\bar{V}(\lambda)} + [-\Phi'(\lambda)]$$

In Nash equilibrium, when the asymmetric-information-effect dominates the Hirshleifer and risk-return effects, the ex-ante welfare can potentially be improved from the no-informed-trading equilibrium.

Welfare Analysis

Proposition. In equilibrium, the **welfare is increasing**, $\boldsymbol{W'(\lambda) \geq 0}$,**if and only if**

$$\frac{V_U'(\lambda)}{V_U(\lambda)} = \frac{-\xi_U'(\lambda)}{1+\xi_U(\lambda)} \leq \frac{1}{2}\frac{[1-2\lambda\gamma(\lambda)][\gamma(\lambda)+\lambda\gamma'(\lambda)]}{[1-\lambda\gamma(\lambda)]^2};$$

In particular, $\boldsymbol{W'(0) \geq 0}$,**if and only if**

$$\frac{V_U'(0)}{V_U(0)} = \frac{n\xi_0}{1+\xi_0} \leq \frac{1}{2}\gamma(0) = \frac{1}{2}\left(1 - \frac{1}{\sqrt{1+n}}\right).$$

- The expected utility and Sharpe ratio decrease faster when the initial Sharpe ratio ξ_0 in the no-informed-trading equilibrium is relatively high.
- The risk-return effect must be weak (when n and $\xi_0$are small).
- Lower ξ_0 and less precise signal n weaken the risk-return effect, improving welfare.

Corollary. In equilibrium, (i) if $\xi_0 < \frac{2}{13}$, then $W'(0) > 0$; (ii) if $\xi_0 > \frac{1}{3}$, then $W'(0) < 0$.

- The trading opportunities can be measured by the Sharpe ratio ξ_0.
- Informed trading improves the welfare for low ξ_0, but worsens it for high ξ_0
- The positive asymmetric-information-effect is more likely to dominate the risk-return effect at low level of informed trading, thus improving welfare.
- The relationship between welfare and λ is hump-shaped, leading to a unique Pareto-optimal state, $\lambda \in (0,1)$, where traders' welfare is maximized.
- When the noise demand is endogenized by introducing trader-specific endowment shocks, there can be multiple Pareto-optimal equilibria
  - information acquisition is welfare-reducing for traders with large endowment shocks, i.e., hedger, because the Hirshleifer effect dominates.
  - information acquisition can be welfare improving for speculators with small endowment shocks, if asymmetric-information effect dominates.

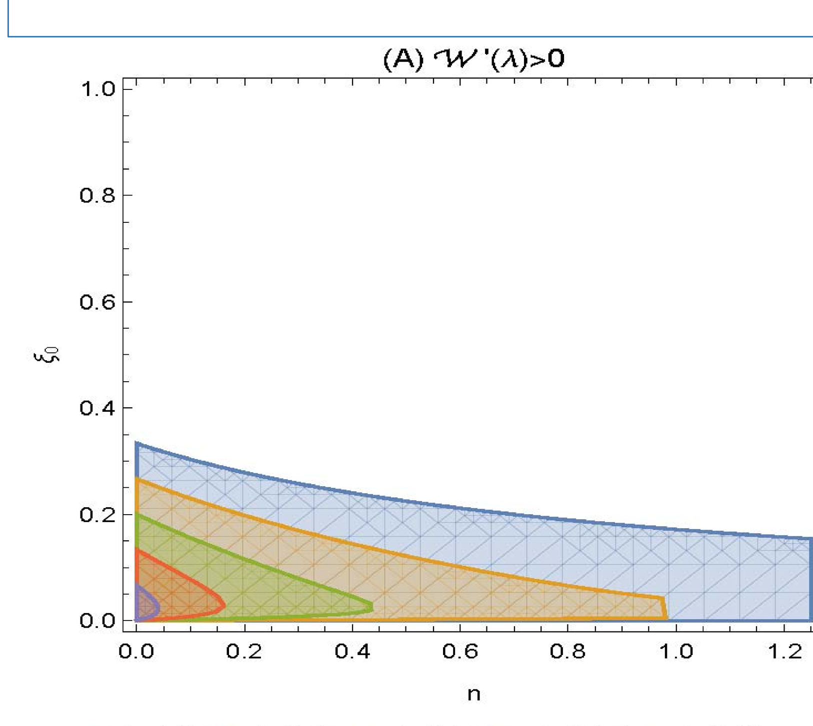

Figure 1. the region for $W'(\lambda) > 0$.

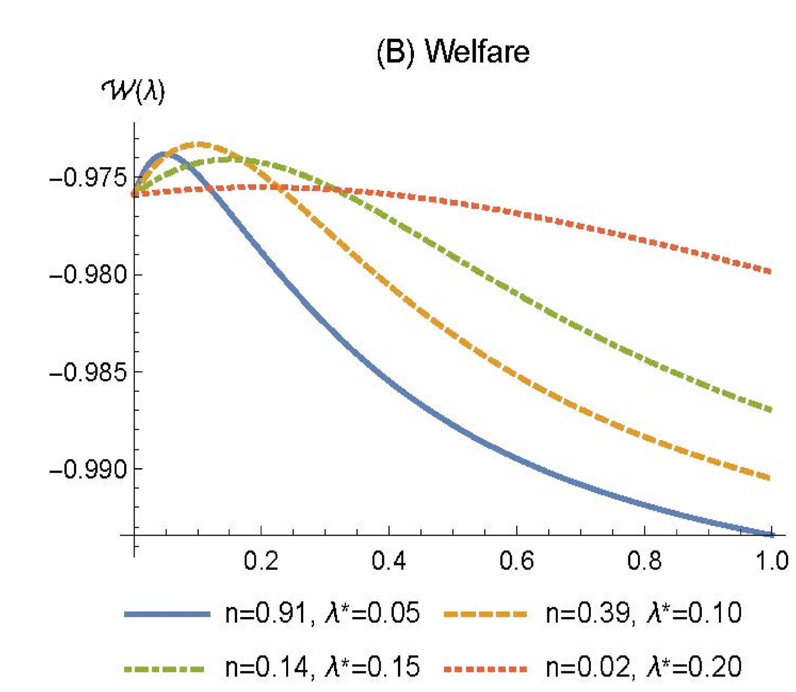

Figure 2. welfare $W(\lambda)$

Policy Implications

- By levelling the playing field, i.e., reducing information asymmetry by making information acquisition more costly, is not always Pareto-optimal, especially for speculators who provide liquidity.
- No-informed-trading equilibrium is more likely to be Pareto-optimal in markets with relatively high Sharpe ratios (e.g., developing and emerging markets).
- Informed-trading equilibrium is more likely to be Pareto-optimal in markets with relatively low Sharpe ratios (e.g., developed markets).
- Information acquisition as a probabilistic choice can have a positive social value.

Conclusions

- ✓ Investors facing information acquisition uncertainty make strategic probabilistic choices about observing a costly private signal about the risky asset.
- ✓ More informed trading, by resolving payoff uncertainty, makes price more informative but reduces the Sharpe ratio and distorts risk-sharing.
- ✓ However, due to information acquisition uncertainty, traders who become informed receive a net benefit, which can dominate the aforementioned negative effects.
- ✓ Therefore, with information acquisition uncertainty, more informed trading can lead to an overall welfare improvement in the economy.

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References

1. Goldstein & Yang (2015), Information disclosure in financial markets, Annual Review of Financial Economics, 9, 101-125.
2. Grossman & Stiglitz (1980), On the impossibility of information efficiency markets, American Economic Review 70(3), 393-408
3. Hirshleifer (1971), The private and social value of information and the reward to inventive activity, American Economic Review 61(4), 561-574.
4. Mattsson & Weibull (2002), Probabilistic choice and procedurally bounded rationality, Games and Economic Behavior, 41, 61-78.