LINEAR CROSS-SECTIONAL MODEL COMPARISONS

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ABSTRACT
This paper evaluates a specification for a conditional beta model following Fama and French (2019, WP). Using a linear-beta model, I show:

- We can reject the Fama and French model that assumes characteristics are conditional betas in favor of a linear conditional beta model following Shanken (1990).
- Model-implied zero-beta rates are particularly sensitive to the specification, and that the linear conditional beta model provides a noticeably lower rate.
- Out-of-sample tests find the linear-beta model has a significantly lower bias, and Clark and West (2007) adjusted-MSPE, but it may come at the cost of a larger variance than the Fama and French model.

INTRODUCTION
Fama and French (FE, 2019) show that stacked Fama-Macbeth (1973) cross-sectional regressions of returns on characteristics price assets well compared to trial-and-error standards (such as the FF5). By using characteristics as factor loadings, this method provides a few benefits:

- Only one round of estimation (no times-series regression is necessary)
- No manual risk-factor calculation (such as replicating the FF5 or FF3 factors)
- Natural time-varying beta (the characteristics)

This paper tests a more flexible Shanken (1990) style linear-beta, where $f_i(X_{t-1}) = b_{i0} + b_{i1}X_{t-1}$.

Comparison of RMW with Linear-Beta OP

MODEL
FF (2019) with a Shanken (1990) style Linear-Beta:

$$R_{it} = R_{it} + f_i(\{MC_{t-1}\}R_{MC,t} + f_i(\{BM_{t-1}\}R_{BM,t} + f_i(\{OP_{1t-1}\}R_{OP,t} + f_i(\{INV_{t-1}\}R_{INV,t} + \epsilon_{it})$$

- Market Capitalization: $MC_{t-1}$
- Book-to-Market: $BM_{t-1}$
- Operating Profitability: $OP_{1t-1}$
- Investment: $INV_{t-1}$

Estimating a stacked cross-sectional regression for each characteristic $X_{t-1}$, zero-beta rate $R_{Z,t}$, and risk-premiums $R_{MC,t}, R_{BM,t}, R_{OP,t}$, and $R_{INV,t}$, Fama and French (2019) take $b_{i0} = 0$ and $b_{i1} = 1$.

Natural Questions:

- Does $\epsilon_{it}$ satisfy time-series requirements?
- Do the characteristics want $b_{i0} = 0$ and $b_{i1} = 1$?
- If not, is the difference meaningful?

DATA
The characteristics and factor terms:

- Asset Weighted for $MC$, $OP$, $INV$, and $BM$
- Include Annual ($BM$) and Monthly ($BMm$) Updated $BM$ for Characteristics and 5x5 Portfolios
- $PM_	ext{PA}$

Repeat the model with the above datasets, with and without cross-sectionally studenising the right-hand-side characteristics.

REDUCED ZERO-BETA RATE

ERROR CORRELATIONS TEST
Does stacking cross-sectionally estimated risk-premiums translate into a time-series regression? Time-series regression should have error term $\epsilon_{it}$ orthogonal to the explanatory variables (stacked risk-premiums for characteristic $k$, $\lambda^k$). Specifically I test,

$$\epsilon^k_t = \lambda^k_t X_{t-1}$$

This setup allows us to test each characteristic to see if the portfolio error terms are, in fact, orthogonal.

MULTIVARIATE REGRESSION TEST
Do the characteristics want more flexibility? I use a multivariate regression test for potential linear-beta style factor loadings. The linear-beta time-series model used for this test includes both characteristics and factor terms:

$$R_{it} = a_i + b_{i1}F_t + b_{i2}(F_{it}X_{i,t-1}) + \epsilon_{it}$$

$X_{t-1}$ denotes the various characteristics ($BM, MC, INV, OP$), and $F_t$ denotes the risk-premiums or factors.

TEST RESULTS

CONCLUSION
In this paper, I use Shanken (90) style linear-betas as loadings on cross-sectionally estimated risk-premiums. The result is a reduction in conditional out-of-sample MSPE, similar to slightly higher 60-Month Average out-of-sample MSPE, and a large reduction in the model-implied zero-beta rate. Overall, it appears that the stacked XS model used by Fama and French (2019, WP) is better suited for forecasting, but the proposed linear-beta model is better for return attribution.

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OUT-OF-SAMPLE PREDICTIONS
Conditional: Average of one-month prediction assuming the RHS risk-premiums are available.

60-Month Average MSPE: Average of portfolio MSPEs with 60-month average of risk-premiums and 60-month calculated $f_i(X)$ used to predict the next month. This is closer to a true forecast.

0 10 20 30 40
0 1 2 3 4
MSPE Variance Bias$^2$

Stacked XS Linear-Beta

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