Collateral Runs

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Board of Governors of the Federal Reserve System\textsuperscript{1}

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\textsuperscript{1}The views of this presentation are solely the responsibility of the authors and should not be interpreted as reflecting the views of the Board of Governors of the Federal Reserve System or of any other person associated with the Federal Reserve System.
Motivation

...Thursday had brought an onslaught of new client demands for their cash back, as well as demand for collateral from numerous funding counter parties, leaving the firms with only about $5 billion....

“Okay, where are we in terms of cash we can raise? What collateral can we pledge?”

— Kelly (2009), quoting Sam Molinaro, CFO of Bear Sterns, evening of March 13th 2008.
Motivation – Classic ‘Repo runs’

- Repo contract: Borrower borrows $X from Lender(s) after pledging collateral $T$

- Collateral is risky and may take values between $(T^D, T^U)$

- If low values are likely, lenders may get “spooked” and run to withdraw their cash

- Classic view of repo runs: Run from liabilities’ side

- Solution: Over-collateralization/Zero VaR contracts whereby $X \leq T^D$
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- **Solution**: Over-collateralization/Zero VaR contracts whereby $X \leq T^D$
Motivation – Dealer and Rehypothecation

- Lenders and borrowers do not interact directly: a dealer engages in a repo and a reverse repo, and rehypothecates the collateral

- What if dealer extends fewer funds than the ones received: $X' < X$?

- By investing $\sum_N X - \sum_M X'$ in risky assets, borrowers may get “spooked” and run to withdraw their collateral

- Note: zero VaR contract, $X \leq T^D$ does not protect collateral providers

Dealer's Balance Sheet

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<tr>
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<td>Reverse repos</td>
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\hline
\text{Assets} & \text{Liabilities} \\
\hline
\sum_N X - \sum_M X' & \sum_N X \\
\sum_M X' & \\
\end{array}
\end{equation}

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Motivation – Dealer and Rehypothecation

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- By investing $\sum_N X - \sum_M X'$ in risky assets, borrowers may get “spooked” and *run to withdraw their collateral*.

- **Note:** zero VaR contract, $X \leq T^D$ does not protect collateral providers.
How Important Is Rehypothecation?

Source: Infante, Press, Saravay (2020)
What we do

- Characterize the coordination problem amongst collateral providers, giving rise to a multiplicity of equilibria

- Use traditional global games framework to find the unique threshold equilibrium

- Solve for the optimal contracting terms that result in the unique equilibrium
Main Ingredients for Collateral Runs

1) Rehypothecation of collateral:
   Otherwise bankruptcy regimes could protect the collateral provider

2) Dealers ability to set different contracting terms (market power):
   Dealers need to extract liquidity from matched book activity

3) Dealers discretion to use excess funds for own needs:
   Investment in illiquid (but profitable) projects
Literature Review

- **Collateralized lending**

- **Repo & Rehypothecation**

- **Bank Runs & Global Games**
Model Setup

- 3 periods: \( t \in \{0, 1, 2\} \)

- 2 assets in perfectly elastic supply:
  - 1 safe asset — valued at 1 in all periods, serves as collateral: \( T \)
  - 1 risky asset — payoff in \( t = 2 \) depends on state \( \theta \sim U(0, 1) \) which is realized in \( t = 1 \)
    \[
    \tilde{R} = \begin{cases} 
    R^U > 1 & \text{with prob } \theta \\
    R^D \in [0, 1) & \text{with prob } 1 - \theta 
    \end{cases}
    \]
    valued at 1 in \( t = 0 \) and in \( t = 1 \) has liquidation value \( \lambda \cdot E(\tilde{R}|\theta) \) with \( \lambda \leq 1 \)

- 3 types of agents that consume in \( t = 2 \):
  - Money funds (M): Continuum of “very” risk averse, invest in repo backed by \( T \)
  - Hedge funds (H): Continuum of risk neutral, additional preference for \( T \) between \( t = 1, 2 \):
    \[
    \eta T > T \rightarrow \text{Incentives to take leverage}
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  - Dealer (D): Single intermediary with no initial wealth, who intermediates funds and collateral between H and M
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Timeline Overview

- $t = 0$: Initial financing period
  - D sets one period repo contracting terms for both counterparties & invests in risky asset

- $t = 1$: Intermediate period, $\theta$ is realized
  - H receives individual noisy signal of state $\theta$ & decides whether to roll over or withdraw its collateral
  - Give the fraction of withdrawers, D pays M to recover collateral of withdrawing H & possibly liquidating part of its risky asset
  - If withdrawals are severe, entire risky asset position is liquidated $\longrightarrow$ collateral run!

- $t = 2$: Final period, if collateral run does not occur
  - Asset outcome realized & contracts settled
  - D protected by limited liability: consider contracts were D can default
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Contracting Terms & Rehypothecation

- D offers take-it-or-leave-it repo contracting terms

- Repo contract at time $t \in \{0, 1\}$ for agent $i \in \{H, M\}$:
  - Margin: degree of overcollateralization $m_t^i \rightarrow$ initial loan amount $T - m_t^i$
  - Repurchase price: final repayment to repo $F_t^i$

- Relevant cash flows for dealer:
  - $\Delta m_t := m_t^H - m_t^M$: net inflow in the initial leg
  - $\Delta F_t := F_t^H - F_t^M$: net “inflow” in the closing leg
D’s balance sheet at \( t = 0 \)

- **\( t = 0 \) : Initial financing period**
  - D sets one period repo contracting terms such that \( m_0^H > m_0^M \)
  - \( (T - m_0^M) - (T - m_0^H) = \Delta m_0 > 0 \)
  - D buys risky asset with liquidity windfall \( \Delta m_0 \)
D’s balance sheet at $t = 1$

- H receives noisy signal about $\theta$ & decides whether to roll over its repo
  - If H withdraws, D receives $\Delta F_0$
  - If H rolls over, D receives $\Delta F_0 + \Delta m_1$ (case $\geq 0$)

- Depending on the portion of H withdrawing, $\mu \in [0, 1]$:
  (a) $\mu$ “small”: D does not need to sell: $\mu \Delta F_0 + (1 - \mu)(\Delta F_0 + \Delta m_1) \geq 0$
  (b) $\mu$ “intermediate”: D must sell $\xi$ of risky asset position:
      $\mu \Delta F_0 + (1 - \mu)(\Delta F_0 + \Delta m_1) + \xi \lambda E(\hat{R} | \theta) = 0$
  (c) $\mu$ “severe”: Full liquidation, M seize collateral & H (sequentially) served with probability $f$:
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Small withdrawals

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Infante & Vardoulakis

Collateral Runs

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Rolling over:
- Upside proportional to levered position $\eta T - F$
- But exposure to run risk

Early withdrawal:
- Reduces exposure to run risk
- But upside proportional to unlevered position $\eta(T - F)$

H’s decision to withdraw depends on signal over $\theta$ and beliefs over number of other hedge funds withdrawing → Coordination problem!
Global Game

- At beginning of $t = 1$ H receive a noisy signal $x_i = \theta + \epsilon_i$, $\epsilon_i \sim U(-\epsilon, \epsilon)$, i.i.d.

- With complete information about state ($\epsilon_i = 0$) $\rightarrow$ multiple equilibria

  \[ \begin{align*}
  \theta = 0 & \quad \theta = \theta^{LD} & \quad \theta = \theta^{UD} & \quad \theta = 1 \\
  \text{Withdraw} & \quad \text{Multiple equilibria} & \quad \text{Roll over} & \\
  \end{align*} \]

  - If signal very bad, H knows D will likely default $\rightarrow$ H withdraws
  - If signal very good, H knows liquidation value pays all $\rightarrow$ H rolls over
  - If signal in-between, coordination failure $\rightarrow$ individual action depends on others

- With incomplete information ($\epsilon > 0$ but small) $\rightarrow$ unique threshold equilibrium

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Existence and Uniqueness of Collateral Run

Proposition

Given $0 < \theta^{LD} < \theta^{UD} < 1$, there exists a unique threshold strategy determined by $x^*$ and a unique threshold state $\theta^*$ such that a run occurs only for $\theta < \theta^*$

- Define $\nu(\mu, \theta)$ the utility differential between rolling over and withdrawing

- $\theta^*$ is the level of fundamentals that makes $H$ indifferent

\[
V(\theta^*, \Delta m_t, \Delta F_t) = \int_0^1 \nu(\mu, \theta^*) d\mu = 0
\]
Technical point

Usual proof requires global strategic complementarities and state monotonicity

- Bank-run models exhibit one-sided strategic complementarities
  - Given that a run occurs, the incentives to withdraw are lower as $\mu$ increases

- Existing literature relied on fixed liquidation value

- We endogenize the liquidation value at the cost of losing global state monotonicity
  - Given that a run occurs, the incentives to withdraw are higher as $\theta$ increases
Threshold Equilibria

- In $t = 0$, $D$ sets contracting terms $(\Delta m_t, \Delta F_t)$ to maximize

\[
\int_{\theta^*}^{1} \theta \cdot u(R^U \Delta m_0 + \Delta F_0 + \Delta m_1 + \Delta F_1) d\theta
\]

subject to how contracting terms affect the probability of a run, $\theta^*$, through hedge funds’ indifference condition:

\[
V(\theta^*, \Delta m_t, \Delta F_t) = 0
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- Contracting terms are pinned down implicitly, but some intuition:
  - $D$ internalizes that a higher liquidity windfall, $\Delta m_0$, increases the probability of a run
  - $D$ would like to increase liquidity in the interim period, $\Delta m_1$, but only impacts agents who were already rolling over
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Case Study: Bear Stearns Windfall as a Fraction of Repo Book

Fraction of Repo Book

- Bear Stearns
- All Primary Dealers Excluding Bear Stearns

March 12th 2008

Note: Fraction calculated by (Securities sold - Securities held - Net position) / Securities held

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Comparing Collateral Runs and Bank Runs

▶ Similarities
agents have an unsecured claim on a financial intermediary and liquidation falls short of repaying all agents.

▶ Important Differences
1) Conceptual:
Cash borrowers (asset side) can be the source of fragility rather than cash lenders (liability side)—notion of collateral liability

2) Incentives for intermediation:
Traditional bank runs are liquidity provision (risk sharing) while collateral runs are for investor to take levered positions—no maturity mismatch

3) Policy implications:
Newly regulation does little to reduce collateral run risk & monitor asset side contracting terms and rehypothecation activity
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Monitoring & Impact of Recent Regulation

► Monitoring:
- In equilibrium collateral runs and repo runs happen simultaneously
  → Hard to disentangle
- Dynamics before run event are different
  - Repo runs: increase haircuts for cash providers for riskier asset classes
  - Collateral runs: increase in haircuts for collateral providers with stable haircuts for cash providers in safe asset classes

► Regulatory framework:
- FSB proposes introduction of haircut floors to make repo markets more resilient
- Model extension shows haircut floors increases the probability of a run
  → Even if H receives M's overcollateralization directly, haircut floors increases incentives to get it sooner
Monitoring & Impact of Recent Regulation

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    - Collateral runs: increase in haircuts for collateral providers with stable haircuts for cash providers in safe asset classes

- **Regulatory framework:**
  - FSB proposes introduction of haircut floors to make repo markets more resilient
  - Model extension shows haircut floors *increases* the probability of a run
    -→ Even if H receives M’s overcollateralization directly, haircut floors increases incentives to get it sooner
Additional Policy Proposals

- Main ingredients for collateral runs are:
  1) Rehypothecation of collateral
  2) Dealers ability to set different contracting terms (market power)
  3) Dealers discretion to use excess funds for own needs

- Policy proposals to reduce collateral run risk (working progress)
  P1) Limits to rehypothecation:
      Cannot raise secured funds to finance H’s repo \( \rightarrow \) part of the windfall used to finance it
  P2) Reinvestment of \( \Delta m_0 \) in UST:
      No pooling of H claims \( \rightarrow \) no coordination problem
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Concluding Remarks

- Paper highlights symmetry in dealer balance sheet:
  - Loss of securities introduces runs risk from the asset side

- Use of global games to refine equilibria to eliminate multiplicity
  - Characterize the existence of panic-based collateral runs

- Policy prescription:
  - Regulators should consider stability threats from collateral runs
    - limits to haircuts, reinvestment, and rehypothecation
  - Comprehensive cost benefit analysis of limits to rehypothecation
Back-up slides
Institutional Details

- Two main segments of the U.S. repo market:
  - Tri-party repo market (≈ $ 2.5 trillion for all collateral, $ 1.2 trillion for UST):
    Cash investors (i.e., money funds, securities lenders, corporate cash managers) provide secured funding to dealers
  - Bilateral repo market (≈ $ 1.8 trillion for all collateral)\(^2\):
    Securities dealers provide secured funding to investors and source specific collateral

- Dealers borrow funds to finance inventory and conduct “matched book”

- Regulatory treatment of repo:
  - Exempt from automatic stay: cash lenders has immediate access to collateral
  - No limits to repo rehypothecation: SEC Rule 15c3-3 does not apply

\(^2\)Source: Baklanova et. al (2019)
Hedge Fund Payoffs

\[ U^H(\mu, \theta; \text{roll over}) = \begin{cases} 
\theta(\eta T - F^H_1) + (1 - \theta)G^D_S(\mu, \theta) + \\
\eta(T - m^H_0 - F^H_0 - m^H_1) & \mu \in [0, \mu_S) \\
\theta(\eta T - F^H_1) + (1 - \theta)G^D_I(\mu, \theta) + \\
\eta(T - m^H_0 - F^H_0 - m^H_1) & \mu \in [\mu_S, \mu_I) \\
\theta G^U_I(\mu, \theta) + (1 - \theta)G^D_I(\mu, \theta) + \\
\eta(T - m^H_0 - F^H_0 - m^H_1) & \mu \in [\mu_I, \mu_R) \\
\eta(T - m^H_0 - F^H_0 - m^H_1) & \mu \in [\mu_R, 1] 
\end{cases} \]

\[ U^H(\mu, \theta; \text{withdraw}) = \begin{cases} 
\eta(T - F^H_0) - \eta m^H_0 & \mu \in [0, \mu_R) \\
\eta(f(\mu, \theta) \cdot (T - F^H_0)) - \eta m^H_0 & \mu \in [\mu_R, 1] 
\end{cases} \]

where

- \( G^D_S \) value of defaulted portfolio with no sales when \( \tilde{R} = R^D \)
- \( G^D_I \) value of defaulted portfolio with sales when \( \tilde{R} = R^D \)
- \( G^U_I \) value of defaulted portfolio when \( \tilde{R} = R^U \)
- \( f(\mu, \theta) \) pro-rata probability of getting paid conditional on a run

- Infante & Vardoulakis Collateral Runs A.2 / A.6
\( \nu(\mu, \theta) = \begin{cases} 
\theta [(\eta - 1) T - \Delta F_1] + (1 - \theta) G^D_S(\mu, \theta) - \eta \Delta m_1 & \mu \in [0, \mu_S) \\
\theta [(\eta - 1) T - \Delta F_1] + (1 - \theta) G^D_I(\mu, \theta) - \eta \Delta m_1 & \mu \in [\mu_S, \mu_I) \\
\theta G^U_I(\mu, \theta) + (1 - \theta) G^D_I(\mu, \theta) - \eta \Delta m_1 & \mu \in [\mu_I, \mu_R) \\
-\eta \frac{\lambda \nu R_0 \Delta m_0 + \Delta F_0 + \Delta m_1}{\mu} & \mu \in [\mu_R, 1] 
\end{cases} \)


**Dealer Optimization**

\[
\max_{\{\Delta m_0, \Delta F_0, \Delta m_1, \Delta F_1\}} \frac{1}{2} (1 - \theta^*^2) u^D (R^U \Delta m_0 + \Delta F_1 + \Delta F_0 + \Delta m_1)
\]

s.t.

\[
\begin{align*}
\Delta m_1 + \Delta F_0 & \geq 0 \quad \text{(Positive Liquidity Injection)} \\
R^D \Delta m_0 + \Delta F_0 + \Delta m_1 + \Delta F_1 & \leq 0 \quad \text{(Dealer default in bad state)} \\
PC_0 & \geq 0 \quad \text{(HF Participation in } t = 0) \\
PC_1 & \geq 0 \quad \text{(HF Participation in } t = 1) \\
\Delta m_0, \Delta m_1 & \in [0, T] \quad \text{(Feasible Haircuts)} \\
\Delta F_0, \Delta F_1 & \leq 0
\end{align*}
\]

where

\[
\begin{align*}
PC_0 & : \int_{\theta^*}^{1} \eta \cdot (T - F_0^H) d\theta + \int_{0}^{\theta^*} \eta \cdot f(1, \theta) \cdot (T - F_0^H) d\theta - \eta \cdot m_0^H \geq 0 \\
PC_1 & : \theta(\eta \cdot T - F_1^H) + (1 - \theta) G_D^S(0, \theta) - \eta \cdot m_1^H \geq 0,
\end{align*}
\]
Comparative Statics when $R^D = 0$ and r.n. $D$

- If $R^D = 0$ and r.n. $D$ then $\theta^*$ only depends on the liquidation value of the asset

- $\frac{\partial \theta^*}{\partial \lambda R^U} > 0$:
  - For given contracting terms, a higher $\lambda R^U$ increases the payoff from running

- $D$ reacts by reducing $\Delta m_0$, but not enough to counterbalance $H$’s increased incentive to run

- Result is not robust to a risk averse $D$

- $\frac{\partial \theta^*}{\partial \eta} = 0$:
  - Increase in $\eta$ do not change the probability of a run, but increases $\Delta m_0$
Estimate of Liquidity Windfall: FR 2004

- FR 2004 reports funds paid or received in short-term financing transactions

- Estimate of cash windfall:

\[ CW = (R - m_R) - (RR - m_{RR}) + (L - S) = -"BoxConstraint" + \Delta m_0 \leq \Delta m_0 \]

- Box Constraint: budget constraint for securities that must be positive (Huh & Infante, 2018)