“Superstitious” Investors

Hongye Guo and Jessica A. Wachter

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The volatility of stock returns on the aggregate market is a puzzle (Campbell & Shiller, 1988).

- US. aggregate stock market volatility is about 20% per annum.
- Riskfree rate volatility is low: 2% per annum is probably an upper bound for the short-term US real rate.
- Consumption volatility is also low: 1–2% per annum in postwar US data.
- An influential research agenda seeks to explain these fluctuations primarily through discount rates.
Problems with the discount-rate-based explanation:

- Decades of empirical research has failed to uncover a robust relation between risk and expected returns.
- Leading candidates predict term structures of returns that are too steep in one direction or another.
We assume that investors hold biased beliefs that are nonetheless reasonable given past data.

Motivation: the classic animal learning study of Skinner (1948)
- Pigeons “learned” to associate certain behaviors with the arrival of food.

The pigeons thought that the something random (food arrival) was predictable.

People, too, tend to place structure on randomness.
Growth rates are iid lognormal.
  Investors, however, believe that they can forecast the growth rate.

We implement biased beliefs in a simple way.
Biased beliefs are isomorphic to prices of risk (if sufficiently flexible), though the interpretation is different.
And extended across asset classes, and to the cross-section.
Unlike previous literature, we do not use belief biases to explain the equity premium.
Model

- Aggregate dividends $D_t$
- Investor’s subjective process for dividend growth:

\[
\begin{align*}
\Delta d_{t+1} &= x_t + u_{t+1} \\
x_{t+1} &= \phi x_t + v_{t+1},
\end{align*}
\]

\[
\begin{bmatrix}
  u_t \\
v_t
\end{bmatrix}
\sim iid \sim N \left(0, \begin{bmatrix}
  \sigma^2_u & 0 \\
0 & \sigma^2_v
\end{bmatrix}\right)
\]

- Value of the aggregate market

\[
P_t = E_t^* \sum_{n=1}^{\infty} \delta^n D_{t+n}
\]

where $\delta$ is the discount factor.
Prices and returns

▶ Price of a dividend strip:

\[ P_{nt} = E_t^* [\delta^n D_{t+n}] = D_t e^{a_n + b_n x_t} \]

▶ Returns up to a constant:

\[
\log(1 + R_{n,t+1}) = \log \frac{P_{n-1,t+1}}{D_{t+1}} - \log \frac{P_{n,t}}{D_t} + \log \frac{D_{t+1}}{D_t} \\
= k + b_{n-1} x_{t+1} - b_n x_t + \Delta d_{t+1} \\
= k + (b_{n-1} \phi - b_n) x_t + b_{n-1} v_{t+1} + \Delta d_{t+1} \\
= k - x_t + b_{n-1} v_{t+1} + \Delta d_{t+1}
\]

▶ If investors are correct, expected returns are constant.
▶ But if \( \Delta d_{t+1} \) is unpredictable, then they contain \( x_t \).

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Where does volatility come from?

- When the physical and the subjective distributions coincide:

\[ \text{Var}(\log(1 + R_{nt})) = b_{n-1}^2 \sigma_v^2 + \sigma_u^2, \]

- When the investors exhibit superstition:

\[ \text{Var}(\log(1 + R_{nt})) = \sigma_x^2 + b_{n-1}^2 \sigma_v^2 + \sigma_u^2, \]

where

\[ \sigma_x^2 \equiv \frac{\sigma_v^2}{1 - \phi^2}. \]

- It turns out that \( \sigma_x^2 << b_{n-1}^2 \sigma_v^2 \), for \( n \) large.
- The model for superstitious investors does not (much) produce more volatility than the full information model.
### Predictive Dividend Growth Regressions

<table>
<thead>
<tr>
<th>Horizon in Years</th>
<th>1</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Panel A: Data 1948-2017</strong></td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>$\beta$</td>
<td>-0.01</td>
<td>-0.01</td>
<td>-0.04</td>
<td>-0.08</td>
<td>-0.09</td>
<td>-0.12</td>
</tr>
<tr>
<td>$t$-stat</td>
<td>[-0.59]</td>
<td>[-0.29]</td>
<td>[-0.72]</td>
<td>[-1.00]</td>
<td>[-0.83]</td>
<td>[-0.86]</td>
</tr>
<tr>
<td>$R^2$</td>
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<td>0.00</td>
<td>0.01</td>
<td>0.04</td>
<td>0.05</td>
<td>0.06</td>
</tr>
</tbody>
</table>

| **Panel B: Disaster Model No Realization** |   |   |   |   |   |    |
| $\beta$          | -0.00 | -0.00 | -0.00 | 0.00 | -0.01 | -0.01 |
| 5th percentile    | -0.07 | -0.15 | -0.29 | -0.41 | -0.52 | -0.61 |
| 95th percentile   | 0.08 | 0.15 | 0.28 | 0.42 | 0.52 | 0.63 |
| $R^2$             | 0.01 | 0.01 | 0.03 | 0.04 | 0.05 | 0.06 |

Data are annual, 1947–2017
### Predictive Regressions: Excess Stock Market Returns

#### Panel A: Data 1948-2017

<table>
<thead>
<tr>
<th>Horizon in Years</th>
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<th>8</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \beta )</td>
<td>0.10</td>
<td>0.20</td>
<td>0.28</td>
<td>0.40</td>
<td>0.51</td>
<td>0.59</td>
</tr>
<tr>
<td>( t )-stat</td>
<td>2.27</td>
<td>2.59</td>
<td>2.90</td>
<td>2.91</td>
<td>2.87</td>
<td>2.71</td>
</tr>
<tr>
<td>( R^2 )</td>
<td>0.07</td>
<td>0.13</td>
<td>0.16</td>
<td>0.22</td>
<td>0.27</td>
<td>0.31</td>
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#### Panel B: Disaster Model No Realization

<table>
<thead>
<tr>
<th></th>
<th>1</th>
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<th>6</th>
<th>8</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \beta )</td>
<td>0.12</td>
<td>0.24</td>
<td>0.44</td>
<td>0.62</td>
<td>0.77</td>
<td>0.89</td>
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<tr>
<td>5th percentile</td>
<td>0.03</td>
<td>0.06</td>
<td>0.11</td>
<td>0.13</td>
<td>0.16</td>
<td>0.16</td>
</tr>
<tr>
<td>95th percentile</td>
<td>0.30</td>
<td>0.55</td>
<td>0.95</td>
<td>1.28</td>
<td>1.53</td>
<td>1.74</td>
</tr>
<tr>
<td>( R^2 )</td>
<td>0.05</td>
<td>0.09</td>
<td>0.18</td>
<td>0.24</td>
<td>0.30</td>
<td>0.34</td>
</tr>
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▶ Data are annual, 1947–2017
Prices and dividends in the data (postwar)

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“Superstitious” Investors
How irrational are these beliefs?

- The investor’s beliefs imply dividend growth is predictable.
- If an econometrician started in 1927 with the beliefs that we assign to our investors, what would she think at the end of the sample?
- Consider the following predictive system:

\[
\Delta d_{t+1} = \beta \hat{x}_t + u_{t+1}
\]

\[
\hat{x}_{t+1} = \phi \hat{x}_t + \hat{v}_{t+1},
\]

where \( \hat{x}_t = p_t - d_t \), the log price-dividend ratio, and where

\[
\begin{bmatrix}
  u_t \\
  \hat{v}_t
\end{bmatrix}
\sim iid \mathcal{N}
\left(0, \begin{bmatrix}
  \sigma_u^2 & 0 \\
  0 & \hat{\sigma}_v^2
\end{bmatrix}\right).
\]
Notes: We regress log dividend growth on the log of the dividend-price ratio. $g$ represents the strength of the prior. Shaded
Extensions

1. Value premium
2. Violations of the expectations hypothesis of interest rates [bond return predictability]
3. Violations of uncovered interest rate parity [predictability in currency returns]

▶ There are many examples of time series and cross-sectional predictability. The predictability appears to be asset-specific.
Explaining the Value Premium

- Sort stocks on the basis of book-to-market, earnings-to-price, or similar scaling.
- The value premium is the finding that assets with high values of these ratios (namely prices are low relative to fundamentals) have high expected returns.
- What makes the value premium into a puzzle is that expected returns are not related to beta.
Explaining the value premium (cont.)

- Asset-specific dividend growth:

\[
\Delta d_{j,t+1} = x_t + \beta_{zj} z_t + u_{j,t+1},
\]

where

\[
\begin{align*}
x_{t+1} &= \phi_x x_t + v_{x,t+1} \\
z_{t+1} &= \phi_z z_t + v_{z,t+1},
\end{align*}
\]

- Assume all shocks are iid with variance \(\sigma_u^2\), \(\sigma_{vx}^2\) and \(\sigma_{vz}^2\).

- So that \(x_t\) has the interpretation of the market shock, \(\sum_j \beta_{z,j} = 0\).
Explaining the value premium (cont.)

Prices on a dividend strip:

\[ P_t^j = D_t^j e^{a_j,n+b_x,n x_t+\beta_{z,j}b_z,n z_t}, \]

Assume \( z_t > 0 \): High PD (growth firms) ⇔ firms with high \( \beta_{z,j} \)

Returns up to a constant

\[
\log(1 + R_{n,t+1}^j) = \log \frac{P_{n-1,t+1}^j}{D_{j,t+1}} - \log \frac{P_{n,t}^j}{D_{jt}} + \log \frac{D_{j,t+1}}{D_{jt}} \\
= k - x_t - \beta_{z,j}z_t + b_{x,n-1}v_{x,t+1} + \beta_{z,j}b_{z,n-1}v_{z,t+1}
\]

Expected return differential if dividends were unpredictable:

\[
\log E_t \left[ 1 + R_{n,t+1}^j \right] - \log E_t \left[ 1 + R_{n,t+1}^k \right] = (\beta_{z,k} - \beta_{z,j})z_t
\]
Return statistics for value and growth portfolios

<table>
<thead>
<tr>
<th></th>
<th>1 (Low)</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5 (High)</th>
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<tbody>
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<td>Panel A: Data 1952–2017</td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>$E[R]$</td>
<td>6.46</td>
<td>7.61</td>
<td>8.96</td>
<td>11.34</td>
<td>13.65</td>
<td>7.19</td>
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<tr>
<td>$t$-stat</td>
<td>[2.72]</td>
<td>[3.73]</td>
<td>[4.25]</td>
<td>[4.86]</td>
<td>[4.79]</td>
<td>[3.46]</td>
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<tr>
<td>$\sigma(R)$</td>
<td>19.29</td>
<td>16.60</td>
<td>17.13</td>
<td>18.97</td>
<td>23.17</td>
<td>16.87</td>
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<tr>
<td>$\alpha$</td>
<td>-2.05</td>
<td>-0.05</td>
<td>1.20</td>
<td>2.96</td>
<td>3.77</td>
<td>5.82</td>
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<tr>
<td>$t$-stat</td>
<td>[-1.99]</td>
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<td>[1.59]</td>
<td>[2.74]</td>
<td>[2.72]</td>
<td>[2.58]</td>
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<tr>
<td>$\beta_{mkt}$</td>
<td>1.03</td>
<td>0.93</td>
<td>0.94</td>
<td>1.01</td>
<td>1.19</td>
<td>0.17</td>
</tr>
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<td>Panel B: Model</td>
<td></td>
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<td></td>
</tr>
<tr>
<td>$E[R]$</td>
<td>-0.14</td>
<td>-0.14</td>
<td>0.39</td>
<td>1.37</td>
<td>2.67</td>
<td>2.83</td>
</tr>
<tr>
<td>$\sigma(R)$</td>
<td>21.63</td>
<td>17.65</td>
<td>16.19</td>
<td>17.00</td>
<td>19.51</td>
<td>25.18</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>-1.01</td>
<td>-1.01</td>
<td>-0.42</td>
<td>0.57</td>
<td>1.89</td>
<td>2.93</td>
</tr>
<tr>
<td>$\beta_{mkt}$</td>
<td>1.07</td>
<td>1.02</td>
<td>0.99</td>
<td>0.97</td>
<td>0.95</td>
<td>-0.12</td>
</tr>
</tbody>
</table>

Portfolios are formed by sorting on earnings-to-price ratios. Data are annual, 1952–2017
Abnormal returns relative to a two-factor model

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<td>-0.05</td>
<td>0.95</td>
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<td>[0.57]</td>
<td>[0.12]</td>
<td>[-0.09]</td>
<td>[1.47]</td>
<td>[0.57]</td>
</tr>
<tr>
<td>$\beta_{mkt}$</td>
<td>1.10</td>
<td>0.93</td>
<td>0.90</td>
<td>0.96</td>
<td>1.10</td>
</tr>
<tr>
<td>$\beta_{hml}$</td>
<td>-0.40</td>
<td>-0.02</td>
<td>0.22</td>
<td>0.35</td>
<td>0.60</td>
</tr>
<tr>
<td>Panel B: Model</td>
<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.49</td>
<td>-0.30</td>
<td>-0.48</td>
<td>-0.18</td>
<td>0.49</td>
</tr>
<tr>
<td>$\beta_{mkt}$</td>
<td>1.01</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
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</tr>
<tr>
<td>$\beta_{hml}$</td>
<td>-0.52</td>
<td>-0.24</td>
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<td>0.26</td>
<td>0.48</td>
</tr>
</tbody>
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- Portfolios are formed by sorting on earnings-to-price ratios. Data are annual, 1952–2017
Valuation versus forecasted earnings growth

Forecast ROE Growth versus Valuation, S&P 500 Index

Correlation = 0.8
Valuation versus realized earnings growth

Realized ROE Growth versus Valuation, S&P 500 Index

- Correlation = 0.3
Valuation versus forecasted earnings growth: Cross-section

Mean ROE Growth Forecast

-0.05 0 0.05 0.1 0.15 0.2 0.25 0.3 0.35
1 (Growth) 2 3 4 5 6 7 8 9 10 (Value)
Conclusions

- Like the pigeons in Skinner’s classic (1948) experiment, investors discover meaning in randomness.
- We show that this simple insight has far-reaching consequences for asset pricing.
- When incorrect information is embedded into prices, prices adjust to meet cash flows, rather than the other way around.
- We find evidence for this in IBES analyst forecasts.
- We apply this insight to explain:
  - Excess volatility and predictability in aggregate stock returns
  - The value puzzle
  - The failure of the expectations hypothesis of interest rate
  - The failure of uncovered interest rate parity (the forward premium puzzle)