Investor sentiment, behavioral heterogeneity and stock market dynamics

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1. Introduction

2. The Model

3. Numerical Simulation with Stochastic Model

4. Robustness Checks

5. Conclusion
Outline

1. Introduction

2. The Model

3. Numerical Simulation with Stochastic Model

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5. Conclusion
‘I define a speculative bubble as a situation in which news of price increases spurs investor enthusiasm, which spreads by psychological contagion from person to person, in the process amplifying stories that might justify the price increases and bringing in a larger and larger class of investors, who, despite doubts about the real value of an investment, are drawn to it partly through envy of others’ successes and partly through a gambler’s excitement.’

- Shiller, Irrational Exuberance, 2015
Motivation

- Pioneering works in heterogeneous agent model (HAM)
  - Day and Huang (1990); Lux (1995); Brock and Hommes (1998); Chiarella and He (2003); He and Westerhoff (2005)

- Only a handful of HAM studies have taken into account investor sentiment
  - Lux (2012); Chiarella et al. (2017)
Objective

To analyze interaction between investor sentiment and asset price dynamics in HAM.

- Sentiment indicator captures *memory of sentiment, social interaction and sentiment shock*

- Effect of sentiment on stylized facts, market volatility as well as crises within HAM framework
Sentiment effect on asset pricing

- Theoretical
  - De Long et al. (JEBO, 1990) - *Noise trader model*
  - Lux (EJ, 1995; JEBO, 1998) - *Market mood contagion*

- Empirical
  - Baker and Wurgler (JF, 2006; JEP, 2007) - *Top-down approach*
  - Tetlock (JF, 2007) - *Media effect*

- Experimental
  - Hüsler et al. (JEBO, 2013) - *Over-optimism*
  - Makarewics (Comput. Econ, 2017) - *Friendship network*
Sentiment effect on financial crisis

- Siegel (1992) and Baur et al. (1996) - *U.S. stock market crash of 1987*
- Zouaoui et al. (2011) - *Panel data of international stock markets*
Our contributions are mainly threefold:

- Model heterogeneous responses to sentiment under a fundamentalist-chartist framework
- Investor sentiment is a significant source of market volatility
- The sentiment channel provides an explanation to the mechanism underlying different types of financial crises
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Fundamentalist

- Fundamental value
  \[ \mu_t = \mu + e_t \]  
  where \( e_t \overset{iid}{\sim} \mathcal{N}(0, \sigma^2) \).

- Demand of fundamentalist
  \[ D_t^f = A(x_t)(\mu_t - p_t) \]  
  Where \( x_t = \mu_t - p_t \). The reaction function \( A \) captures the confidence of the fundamentalist.
Fundamentalist

- **Reaction function $A$**

![Diagram of confidence function of fundamentalist](image)

**Figure 1:** Confidence function of fundamentalist

$A(x_t)$

$x_t = p_t - \mu_t$
According to Huang et al. (2010), price domain $\mathbb{P} = [p_{min}, p_{max}]$ can be divided into $n$ mutually exclusive regimes:

$$\mathbb{P} = \bigcup_{j=1}^{n} \mathbb{P}_j = [\bar{p}_0, \bar{p}_1) \cup [\bar{p}_1, \bar{p}_2) \cup \cdots \cup [\bar{p}_{n-1}, \bar{p}_n]$$ (3)

When price falls into a regime, the chartists extrapolate the short run asset value to be in the middle of the regime

$$v_t = (\bar{p}_{j-1} + \bar{p}_j)/2 \quad \text{if } p_t \in [\bar{p}_{j-1}, \bar{p}_j), j = 1, 2 \cdots n$$ (4)
Chartists: Momentum Traders

They chase the price trend and are sensitive to sentiment

\[ D_{t}^{mo} = \beta_{1}m_{t}(p_{t} - v_{t}) \]  \hspace{1cm} (5)

where \( \beta_{1} > 0 \). \( m_{t} \) is the time-varying sentiment factor constructed as

\[ m_{t} = 1 + \tanh(\kappa(p_{t} - v_{t})) \times h_{1} \times S_{t} \]  \hspace{1cm} (6)

where \( S_{t} \) is the market sentiment index, and \( h_{1} \in [0, 1] \) measures sentiment sensitivity. \( \tanh \) function and \( \kappa \) are used to scale the price deviation within \([-1, 1]\).
Chartists: Contrarian Traders

They bet on the price reverting to the short-term asset price $v_t$ and are sensitive to sentiment

$$D_t^{co} = \beta_2 c_t (p_t - v_t)$$

(7)

where $\beta_2 < 0$. $c_t$ is the time-varying sentiment factor constructed as

$$c_t = 1 - \tanh(\kappa (p_t - v_t)) \ast h_2 \ast S_t$$

(8)

where $S_t$ is the market sentiment index, and $h_2 \in [0, 1]$ measure sentiment sensitivity.
Agents are allowed to switch their belief type conditional on the performance of three rules measured as

\[ U_{n,t} = \varphi U_{n,t-1} + \pi_{n,t} \]  \hspace{1cm} (9)

where \(0 \leq \varphi \leq 1\) represents the strength of memory, \(n\) is the type of trader.

Profit can be calculated as

\[ \pi_{n,t} = (p_t - p_{t-1})D_{t-1}^n \] \hspace{1cm} (10)
Belief Switching Regime

Market fractions $\omega_{i,t}$ are updated according to performance $U_{n,t}$ by following a discrete choice probability

$$\omega_{i,t}(p_t) = \frac{\exp(\rho U_{i,t}(p_t))}{\sum_k \exp(\rho U_{k,t}(p_t))}$$

(11)

$\rho$ measures the intensity of the choice as in Brock and Hommes (1998).
Sentiment Indicator

- Three main sources of sentiment: last-period sentiment, investor mood from social interaction, and sentiment shock.

\[ S_t = \eta_1 S_{t-1} + \eta_2 SI_t + \eta_3 \epsilon_t \]  

where \( \eta_1, \eta_2, \eta_3 \) are the weights. \( \epsilon_t \sim U(-1, 1) \).

- Social interaction measurement based on majority opinion formation in Kirman (1993), Lux (1995)

\[ SI_t = \tanh [\kappa (\mu_t - p_t)] \times \omega_t^f + \tanh [\kappa (p_t - v_t)] \times (\omega_t^{mo} - \omega_t^{co}) \]
We assume net zero supply of the risky asset, and the market price is determined by a market maker as

\[ p_{t+1} = p_t + \gamma (\omega^f_t D^f_t + \omega^{mo}_t D^{mo}_t + \omega^{co}_t D^{co}_t) \]  

(14)

where \( \gamma \) represents the speed of price adjustment by the market maker.
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We calibrate our model based on some well-documented stylized facts of financial markets as summarized by Westerhoff and Dieci (2006):

1. price distortions in the forms of bubbles and crashes
2. excess returns
3. leptokurtic distribution of returns
4. negligible autocorrelation of daily returns
5. strong autocorrelation of absolute daily returns
Figure 2: Daily S&P500 index between Jan 3, 2000 and Feb 12, 2019
### Stylized Facts: Standard Parameter Setting

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Definition</th>
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</thead>
<tbody>
<tr>
<td>$\mu$</td>
<td>1014</td>
<td>Mean of fundamental prices</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>1</td>
<td>SD of fundamental prices</td>
</tr>
<tr>
<td>$\beta_1$</td>
<td>1.75</td>
<td>Momentum extrapolation rate</td>
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<td>$\beta_2$</td>
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<td>Contrarian extrapolation rate</td>
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<tr>
<td>$\phi$</td>
<td>0.1</td>
<td>Performance memory strength</td>
</tr>
<tr>
<td>$\rho$</td>
<td>0.5</td>
<td>Intensity of choice</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.845</td>
<td>Speed of price adjustment</td>
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<tr>
<td>$\eta_1$</td>
<td>0.4</td>
<td>Last-period sentiment weight</td>
</tr>
<tr>
<td>$\eta_2$</td>
<td>0.5</td>
<td>Social interaction weight</td>
</tr>
<tr>
<td>$\eta_3$</td>
<td>0.1</td>
<td>Sentiment shock weight</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>12</td>
<td>Support and resistance interval</td>
</tr>
<tr>
<td>$a$</td>
<td>$1.11 \times 10^{-5}$</td>
<td>Confidence function factor</td>
</tr>
<tr>
<td>$b$</td>
<td>$1 \times 10^{-8}$</td>
<td>Confidence function factor</td>
</tr>
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<td>$\kappa$</td>
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<td>Scaling factor</td>
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<tr>
<td>$h = h_1 = h_2$</td>
<td>0/1</td>
<td>Without sentiment/with sentiment</td>
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Figure 3: Dynamics of model without sentiment ($N = 17000$).
Figure 4: Dynamics of model with sentiment ($N = 17000$).
To check the robustness, we run our models 1000 times by using Monte Carlo simulation for a range of sentiment sensitivity.

<table>
<thead>
<tr>
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<td>kurtosis</td>
<td>2.155</td>
<td>2.075</td>
<td>3.396</td>
<td>4.615</td>
<td>4.730</td>
<td>5.804</td>
</tr>
<tr>
<td>skewness</td>
<td>-0.010</td>
<td>-0.015</td>
<td>-0.026</td>
<td>-0.036</td>
<td>-0.037</td>
<td>-0.040</td>
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<tr>
<td>AC $r_1$</td>
<td>0.010</td>
<td>0.041</td>
<td>0.166</td>
<td>0.219</td>
<td>0.189</td>
<td>0.166</td>
</tr>
<tr>
<td>AC $r_5$</td>
<td>-0.006</td>
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<td>0.007</td>
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<td>-0.004</td>
<td>-0.002</td>
<td>-0.003</td>
<td>-0.002</td>
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<td>-0.003</td>
<td>0.172</td>
<td>0.344</td>
</tr>
<tr>
<td>AC $</td>
<td>r_5</td>
<td>$</td>
<td>0.070</td>
<td>0.075</td>
<td>0.179</td>
<td>0.252</td>
</tr>
<tr>
<td>AC $</td>
<td>r_{10}</td>
<td>$</td>
<td>0.039</td>
<td>0.046</td>
<td>0.167</td>
<td>0.241</td>
</tr>
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To check the robustness, we run the three-type model 1000 times by using Monte Carlo simulation for a range of sentiment sensitivity

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</tr>
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<td>-0.004</td>
<td>-0.002</td>
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<td>0.241</td>
</tr>
</tbody>
</table>
We use the standard deviation of the market prices from the fundamental values as a quantitative measure of excess volatility

\[ SD_{p-\mu} = \sqrt{\frac{1}{T} \sum_{t=1}^{T} (p_t - \mu_t)^2} \]
Figure 5: Average standard deviation of the market prices from the fundamental values for 1000 simulations
Following Huang et al. (2010), we replicate 3 different types of crisis
  - Sudden crisis
  - Smooth crisis
  - Disturbing crisis

Differences between our model and Huang’s model
  - 3 agent types vs 2 agent types
  - with sentiment vs without sentiment
  - stochastic vs deterministic
Figure 6: Sudden crisis modelling of S&P500 index from 1987/8/3 to 1987/12/22
Smooth Crisis

Figure 7: Smooth crisis modelling of S&P500 index from 1932/1/20 to 1932/6/13
Disturbing Crisis

Figure 8: Disturbing crisis modelling of S&P500 index from 1929/8/1 to 1929/12/26
To identify the crisis in financial market, we adopt a crisis indicator called $CMAX$ used in Patel and Sarkar (1998) and Zouaoui (2011)

$$CMAX_t = \frac{p_t}{\max(p_{t-T}, \ldots, p_t)}$$

usually $T$ is 12 to 24 months
A crisis is identified if

(1) \( CMAX_t < \overline{CMAX} - 2\sigma \)

(2) \( p_t < \tau \ast \mu_t \quad (\tau < 1) \)

A crisis is eliminated if detected twice over \( T \) periods
Crisis Identification with S&P500

Figure 9: Crises detected from 1950 using real S&P500 monthly data, $\tau = 0.9$
Figure 10: Crisis identified in simulated monthly data with sentiment (left) and without sentiment (right), $\tau = 0.9$
The magnitude of a crisis is defined as the percentage drop in price from the peak to the trough

- Peak: maximum price over $T$ periods prior to crisis identification
- Trough: minimum price during the crisis
Crisis Identification & Magnitude

Figure 11: Average number & average magnitude of crisis with different sentiment sensitivity for 1000 simulations
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# Alternative Fundamentalist Chance Functions

<table>
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<th>Linear chance function</th>
<th>Day &amp; Huang (1990)</th>
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<td>3.377</td>
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<tr>
<td>skewness</td>
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<td>-0.109</td>
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<tr>
<td>AC $r_5$</td>
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<td>0.004</td>
</tr>
<tr>
<td>AC $r_{10}$</td>
<td>-0.001</td>
<td>-0.002</td>
</tr>
<tr>
<td>AC $r_{20}$</td>
<td>0.000</td>
<td>-0.001</td>
</tr>
<tr>
<td>AC $</td>
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<td>$</td>
</tr>
<tr>
<td>AC $</td>
<td>r_{10}</td>
<td>$</td>
</tr>
<tr>
<td>AC $</td>
<td>r_{20}</td>
<td>$</td>
</tr>
<tr>
<td>$SD_{\mu-\mu}$</td>
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<td>293.900</td>
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<tr>
<td># of crisis</td>
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<td>7.353</td>
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<td>crisis magnitude (%)</td>
<td>20.166</td>
<td>38.883</td>
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## Alternative Chartist Strategies

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<tr>
<td>AC ( r_5 )</td>
<td>0.021</td>
<td>0.086</td>
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<td>AC ( r_{10} )</td>
<td>0.000</td>
<td>0.017</td>
</tr>
<tr>
<td>AC ( r_{20} )</td>
<td>-0.003</td>
<td>-0.006</td>
</tr>
<tr>
<td>AC (</td>
<td>r_5</td>
<td>)</td>
</tr>
<tr>
<td>AC (</td>
<td>r_{10}</td>
<td>)</td>
</tr>
<tr>
<td>AC (</td>
<td>r_{20}</td>
<td>)</td>
</tr>
<tr>
<td>( SD_{p-\mu} )</td>
<td>169.035</td>
<td>196.864</td>
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<td>crisis magnitude (%)</td>
<td>39.815</td>
<td>49.425</td>
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</table>
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The key findings of this paper are

- Investor sentiment contributes to more realistic stylized facts and excess market volatility.
- With presence of investor sentiment, financial crisis can be triggered even without mean-reverting action of fundamentalist.
Thank you!
Appendix: Deterministic Models

- 2-type models
  - Fundamentalist versus momentum traders.
  - Fundamentalist versus contrarian traders.
  - Momentum versus contrarian traders.

- 3-type model
  - Fundamentalist, momentum and contrarian traders.
The system can be modelled as a three-dimensional nonlinear map:

\[
\begin{align*}
  p_{t+1} &= p_t + \gamma [\omega^f_t A_t (\mu_t - p_t) + \beta_1 \omega^{mo}_t m_t (p_t - \nu_t)] \\
  U_{t+1} &= \varphi U_t + (p_{t+1} - p_t) [(A_t (\mu_t - p_t) - \beta_1 m_t (p_t - \nu_t)] \\
  S_{t+1} &= \eta_1 S_t + \eta_2 \left[ \tanh (\kappa (\mu_t - p_{t+1})) \omega^f_{t+1} + \tanh (\kappa (p_{t+1} - \nu_{t+1})) \omega^{mo}_{t+1} \right]
\end{align*}
\]

where

\[
\begin{align*}
  A_t &= \frac{a(\mu_t - p_t)^2}{1 + b(\mu_t - p_t)^4}, \\
  \omega^f_t &= \frac{\exp (\rho U_t)}{\exp (\rho U_t)+1}, \quad \omega^{mo}_t = \frac{1}{\exp (\rho U_t)+1}, \\
  m_t &= 1 + \tanh (\kappa (p_t - \nu_t)) \ast h_1 \ast S_t, \\
  U_t &= U_t^f - U_t^{mo}
\end{align*}
\]
Proposition 1

The system has

1. **an unstable fundamental steady state (FSS) with**
   \[(p^*, U^*, S^*) = (\mu, 0, 0) \text{ if } \mu = \nu \] ; **two types of non-fundamental steady states (NFSS) with the form**
   \[(p, U, S) = (p_1^*, 0, 0) \] ,
   \[(p, U, S) = (p_2^*, 0, 0) \text{ and } p_1^* < \mu, p_2^* > \mu \text{ if } \mu = \nu \].

2. **two types of non-fundamental steady states (NFSS) with the form**
   \[(p^*, U^*, S^*) = (p_1^*, 0, S_1^*) \] ,
   \[(p, U, S) = (p_2^*, 0, S_2^*) \text{ and } p_1^* < \mu, p_2^* > \mu \text{ if } \mu \neq \nu \].
Lemma 1

For NFSS, the system could achieve positive sentiment equilibria ($S^* > 0$) if $\mu - \nu > 0$, negative sentiment equilibria ($S^* < 0$) if $\mu - \nu < 0$, zero sentiment equilibria ($S^* = 0$) if $\mu - \nu = 0$ and $\beta_1 \leq \frac{1}{2} ab^{-\frac{1}{2}}$. 
The system can be modelled to a five-dimensional dynamic map

\[
\begin{aligned}
    p_{t+1} &= p_t + \gamma \left[ \omega_f^t A_t \left( \mu_t - p_t \right) + \beta_1 \omega^m_t m_t + \beta_2 \omega^c_t c_t \left( p_t - v_t \right) \right] \\
    u_{t+1}^f &= \varphi u_t^f + \left( p_{t+1} - p_t \right) A_t \left( \mu_t - p_t \right) \\
    u_{t+1}^m &= \varphi u_t^m + \left( p_{t+1} - p_t \right) \beta_1 m_t \left( p_t - v_t \right) \\
    u_{t+1}^c &= \varphi u_t^c + \left( p_{t+1} - p_t \right) \beta_2 c_t \left( p_t - v_t \right) \\
    S_{t+1} &= \eta_1 S_t + \eta_2 \left[ \tanh \left( \kappa \left( \mu_{t+1} - p_{t+1} \right) \right) \omega_{t+1}^f + \tanh \left( \kappa \left( p_{t+1} - v_{t+1} \right) \right) \left( \omega_{t+1}^m - \omega_{t+1}^c \right) \right]
\end{aligned}
\]

where

\[
A_t = \frac{a(\mu_t - p_t)^2}{1 + b(\mu_t - p_t)^4},
\]

\[
\omega_{h,t} = \frac{\exp \left( \rho U_{h,t} \right)}{\sum_{h=1}^{2} \exp \left( \rho U_{h,t} \right)},
\]

\[
m_t = 1 + \tanh(\kappa(p_t - v_t)) \ast h_1 \ast S_t,
\]

\[
c_t = 1 - \tanh(\kappa(p_t - v_t)) \ast h_2 \ast S_t
\]
Proposition 2

The system has

1. a unique FSS with \((p^*, u^*_f, u^*_m, u^*_c, S^*) = (\mu, \frac{1}{3}, \frac{1}{3}, \frac{1}{3}, 0)\) if \(\beta_1 = -\beta_2\). The Jacobean matrix of this system has five eigenvalues with \(\lambda_1 = 1, \lambda_2 = \varphi\). FSS is asymptotically stable for \(|\lambda_3|, |\lambda_4|, |\lambda_5| < 1\).

2. a FSS with \((p^*, u^*_f, u^*_m, u^*_c, S^*) = (\mu, \frac{1}{3}, \frac{1}{3}, \frac{1}{3}, 0)\) if \(\beta_1 \neq -\beta_2\) and \(\mu = \nu\). FSS is asymptotically stable for \(-6 < \gamma (\beta_1 + \beta_2) < 0\); Two types of NFSS with the form \((p^*, u^*_f, u^*_m, u^*_c, S^*) = (p^*_1, \frac{1}{3}, \frac{1}{3}, \frac{1}{3}, S^*_1)\), \((p^*, u^*_f, u^*_m, u^*_c, S^*) = (p^*_2, \frac{1}{3}, \frac{1}{3}, \frac{1}{3}, S^*_2)\), and \(p^*_1 < \mu, S^*_1 > 0; p^*_2 > \mu, S^*_2 < 0\) if \(\beta_1 \neq -\beta_2\) and \(\mu \neq \nu\).
Lemma 2

For NFSS, the system can achieve both positive and negative sentiment equilibria. If $\beta_1 > -\beta_2$, positive (negative) sentiment equilibrium exists at $p^* < (>) \mu$ and $p^* < (>) v$. If $\beta_1 < -\beta_2$, positive (negative) sentiment equilibrium exists at $p^* < (>) \mu$ and $p^* > (<) v$. 