Estimated Dynamic Industry Equilibrium Model with Firing Costs and Subcontracting

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Introduction

- In many countries labor markets are constrained by strict employment protection legislation (EPL).

- EPL raises firms’ labor adjustment costs distorting the efficient allocation of labor across firms while decreasing aggregate productivity.
  
  ▶ Hopenhayn and Rogerson (1993)

- Significant increase of nontraditional staffing arrangements (i.e., subcontracting) with less strict rules regarding dismissals.
  
  ▶ Katz and Krueger (2019); Goldschmidt and Schmieder (2017)

- This adjustment margin has been ignored when assessing the scope of EPL.
Introduction

Why do firms optimally choose to subcontract?

- Subcontracted workers serve as a buffer in times of uncertainty or demand fluctuations
  
  - Firms exposed to higher volatility in idiosyncratic shocks employ subcontracted workers in a larger proportion
  
  - Permanent workers’ fluctuations are smoother and less frequent than fluctuation in subcontracted workers
  
  - Firms subcontract activities regarded as central to the business function
Introduction

But why firms do not subcontract their entire workforce?

In this paper subcontractors’ charges are higher than the firms’ own production costs

- firm pays a fee per worker to the subcontract firm (e.g. recruiting, training, etc.)

- subcontracted workers are paid more to reflect the fact that they bear more risk

- alternatively, they could be less productive (e.g. receive less employer-paid training, put less effort when the prob of becoming permanent is low)
Introduction

Why study the Chilean labor market?

1. Data availability

- Annual Census of Manufacturers (ENIA) provides a complete enumeration at the establishment level of employees working in the manufacturing industry
- Workers are reported in the establishment where they physically perform their task or work independent of their contract status
- Observe how establishments optimally choose the division of labor between permanent and subcontracted workers
- Sample: 2001-2007 (10,906 establishments / 60,938 observations)

2. Significant growth of subcontracting in tandem with an increase in labor adjustment costs for permanent workers
Goal

1. Estimate the real costs of the employment protection regulation in Chile
   - Permanent workers’ firing costs and the “wage premium” on subcontracted workers

2. Analyze the interactions between firing costs and subcontracting

3. Estimate the impact of firing costs on employment and productivity when firms can subcontract to reintroduce flexibility

4. Estimate the costs/benefits of restricting subcontracting or reducing firing costs
This paper

- Build an industry equilibrium model with a dual labor market (Hopenhayn and Rogerson, 1993)
  - 2 types of workers, perfect substitutes in production, but differ in wages and firing costs
  - heterogeneous firms, endogenous entry and exit
  - stationary industry equilibrium

- I use a simulated method of moments (SMM) for the estimation since the model has no closed-form solution
  - choose parameters of the model to reproduce a set of moments that combine time-series employment dynamics and cross-sectional industry characteristics

- Embed estimated model in a general equilibrium framework to perform some policy analysis
Related literature

- Impact of job security provisions on labor markets performance and productivity
  
  Hopenhayn and Rogerson (1993); Alvarez and Veracierto (2001); Poschke (2009); Samaniego (2006); Veracierto (2001)

- Interaction between labor protection policies and temporary workers
  
  ▶ Substitutions between types of workers (Bentolila and Dolado, 1994; Boeri, 2011; Houseman, 2001; Saint-Paul, 1996; Pierre and Scarpetta, 2013)
  
  ▶ Reintroduce flexibility to terminate contracts (Autor, 2003; Dertouzos and Karoly, 1992; Masters and Miles, 2002; Polivka, 1996)

- GE analysis with search frictions and optimal division of labor
  
  Alonso-Borrego, Fernández-Villaverde, and Galdón-Sánchez (2005); Veracierto (2007); Alvarez and Veracierto (2012); Tejada (2017)
Roadmap

1. Description of the model
2. Estimation method
3. Empirical results
4. Policy experiments
5. Conclusions
6. Future work
2 types of workers

- Permanent workers:
  - subject to firing costs that increase with seniority in the job, and receive wage $w_n$
  - only workers with tenure receive severance pay, and $(1 - \lambda)$ probability of getting tenure
  - firms incur in adjustment costs $g(l_t, l_{t-1}) = \max \{0, \tau (l_{t-1} - l_t)\}$

- Subcontracted workers:
  - no firing costs
  - wage: $w_s = w_n(1 + f)$, where $w_n$ is permanent workers’ wage, and $f$ is the fee firms paid per worker to the subcontract firm
Dynamic optimization: incumbent firm

Bellman equation incumbent firm:

\[ V(l_{t-1}, z_t) = \max_n \{ R(n_t, z_t) - g(l_t, l_{t-1}) + \beta \max[E_{z_{t+1}} V(l_t, z_{t+1}), -g(0, l_t)] \} \]  

(1)

- Profit function for an active firm:

\[ R(n, z) = \{ pz (n + s(n, z))^\alpha - n - w_s s - p c_f \} \]  

(2)

- Optimal subcontracted labor choice at state \((n, z)\):

\[ s(n, z) = \begin{cases} 
\left( \frac{\alpha p z}{w_s} \right)^{\frac{1}{1-\alpha}} - n, & \text{if } \alpha p z n^{\alpha-1} > w_s \\
0, & \text{if } \alpha p z n^{\alpha-1} < w_s 
\end{cases} \]  

(3)

- \(c_f\) is a fixed operating costs
Dynamic optimization: incumbent firm

- Law of motion for permanent workers with tenure:
  \[
  l_t = \begin{cases} 
  l_{t-1} + (1 - \lambda) o_t, & \text{if } o_t > 0 \\
  l_{t-1} + o_t, & \text{if } o_t \leq 0.
  \end{cases}
  \] (4)
  
  - Permanent workers:
    \[
    n_t = l_{t-1} + o_t
    \] (5)

  - \( o_t \) number of workers hired/fired

- Two decisions of an incumbent firm:
  
  i) optimal labor demand, \( n_t = L(l_{t-1}, z_t) \), and \( s_t = S(n_t, z_t) \),

  - no firing costs
  - no subcontracting
  - full model

  ii) optimal exit decision, \( x_{t+1} = X(l_t, z_t) \in \{0, 1\} \) (\( X = 1 \) for exit)
Estimation method

- Model has no closed-form solution, is solved using standard numerical techniques
- Full set of parameters necessary to compute the model are:

\[
\theta = \{\beta, \alpha, c_f, c_e, \rho, \mu, \sigma_\varepsilon, \tau, f, \lambda\}
\]  

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>Discount rate</td>
<td>0.965</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Curvature production function</td>
<td>0.85</td>
</tr>
<tr>
<td>$c_f$</td>
<td>Operating fixed cost</td>
<td>estimated</td>
</tr>
<tr>
<td>$c_e$</td>
<td>Entering fixed cost</td>
<td>model solution</td>
</tr>
<tr>
<td>$w_n$</td>
<td>Wage permanent workers</td>
<td>normalized</td>
</tr>
<tr>
<td>$f$</td>
<td>Premium on subcontracted labor</td>
<td>estimated</td>
</tr>
<tr>
<td>$1 - \lambda$</td>
<td>Probability of getting tenure</td>
<td>estimated</td>
</tr>
<tr>
<td>$\tau$</td>
<td>Fixed firing cost</td>
<td>estimated</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Persistence</td>
<td>estimated</td>
</tr>
<tr>
<td>$\mu$</td>
<td>Mean</td>
<td>estimated</td>
</tr>
<tr>
<td>$\sigma_\varepsilon$</td>
<td>Std. dev. productivity shock</td>
<td>estimated</td>
</tr>
</tbody>
</table>
Selection of moments

- Choose parameters to reproduce a set of moments

- 2 sets of moments: cross-sectional industry characteristics, and time-series employment dynamics

<table>
<thead>
<tr>
<th>Description</th>
<th>Baseline Moments</th>
<th>Elasticities of model moments with respect to the model parameters:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$c_f$</td>
</tr>
<tr>
<td>Average firm size</td>
<td>72.0</td>
<td>1.094</td>
</tr>
<tr>
<td>Exit rate</td>
<td>0.091</td>
<td>1.502</td>
</tr>
<tr>
<td>Fraction of plants in each bin:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10-19 emp.</td>
<td>0.39</td>
<td>-1.225</td>
</tr>
<tr>
<td>20-99 emp.</td>
<td>0.45</td>
<td>0.903</td>
</tr>
<tr>
<td>100-499 emp.</td>
<td>0.15</td>
<td>1.219</td>
</tr>
<tr>
<td>+ 500 emp.</td>
<td>0.02</td>
<td>1.331</td>
</tr>
<tr>
<td>Share of employment in each bin:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10-19 emp.</td>
<td>0.06</td>
<td>-1.944</td>
</tr>
<tr>
<td>20-99 emp.</td>
<td>0.26</td>
<td>-0.127</td>
</tr>
<tr>
<td>100-499 emp.</td>
<td>0.42</td>
<td>0.151</td>
</tr>
<tr>
<td>+500 emp.</td>
<td>0.26</td>
<td>0.296</td>
</tr>
</tbody>
</table>
Selection of moments

<table>
<thead>
<tr>
<th>Description</th>
<th>Baseline Moments</th>
<th>Elasticities of model moments with respect to the model parameters:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$c_f$</td>
</tr>
<tr>
<td>Volatility $g_I$</td>
<td>0.69</td>
<td>0.838</td>
</tr>
<tr>
<td>Volatility $g_s$</td>
<td>2.16</td>
<td>-0.456</td>
</tr>
<tr>
<td>Kurtosis $g_I$</td>
<td>5.14</td>
<td>-0.952</td>
</tr>
<tr>
<td>Kurtosis $g_s$</td>
<td>1.97</td>
<td>0.182</td>
</tr>
<tr>
<td>Inaction rate $g_I$</td>
<td>0.181</td>
<td>-0.302</td>
</tr>
<tr>
<td>Share of subcontracting</td>
<td>0.247</td>
<td>0.406</td>
</tr>
</tbody>
</table>

Nota: la tabla reporta las elasticidades de los momentos del modelo con respecto a los parámetros del modelo.

- To pin down $\lambda$, $\tau$ and $f$ match the volatility and kurtosis of permanent and subcontracted employment growth, and the inaction rate [Figure](#).
- Share of subcontracting provides an independent source of information on $\tau$ and $f$.
Empirical results: model without subcontracting

<table>
<thead>
<tr>
<th>Moments</th>
<th>Data</th>
<th>S.E.</th>
<th>Simulated Moments</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Slow tenure</td>
</tr>
<tr>
<td>Average firm size</td>
<td>66.76</td>
<td>1.7310</td>
<td>67.97</td>
</tr>
<tr>
<td>Exit rate</td>
<td>0.091</td>
<td>0.0012</td>
<td>0.100</td>
</tr>
<tr>
<td>Fraction of plants in each bin:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10-19 employees</td>
<td>0.402</td>
<td>0.0049</td>
<td>0.418</td>
</tr>
<tr>
<td>20-99 employees</td>
<td>0.440</td>
<td>0.0049</td>
<td>0.434</td>
</tr>
<tr>
<td>100-499 employees</td>
<td>0.139</td>
<td>0.0038</td>
<td>0.130</td>
</tr>
<tr>
<td>+ 500 employees</td>
<td>0.019</td>
<td>0.0015</td>
<td>0.018</td>
</tr>
<tr>
<td>Share of employment in each bin:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10-19 employees</td>
<td>0.071</td>
<td>0.0023</td>
<td>0.076</td>
</tr>
<tr>
<td>20-99 employees</td>
<td>0.272</td>
<td>0.0084</td>
<td>0.283</td>
</tr>
<tr>
<td>100-499 employees</td>
<td>0.423</td>
<td>0.0121</td>
<td>0.368</td>
</tr>
<tr>
<td>+ 500 employees</td>
<td>0.234</td>
<td>0.0177</td>
<td>0.274</td>
</tr>
<tr>
<td>Volatility $g_I$</td>
<td>0.688</td>
<td>0.0160</td>
<td>0.833</td>
</tr>
<tr>
<td>Kurtosis $g_I$</td>
<td>5.144</td>
<td>0.0606</td>
<td>3.035</td>
</tr>
<tr>
<td>Inaction rate $g_I$</td>
<td>0.181</td>
<td>0.0026</td>
<td>0.153</td>
</tr>
<tr>
<td>Criterion, $\Gamma(\theta)$</td>
<td></td>
<td>1,524.4</td>
<td>2,937.9</td>
</tr>
</tbody>
</table>

\[
\begin{align*}
\text{Quick tenure} & : \quad 7.756 & - & 0.871 & 0.048 & 0.144 & - & 0.133 \\
(\lambda = 0) & : \quad (0.0263) & - & (0.0092) & (0.0032) & (0.0068) & - & (0.0048) \\
\text{Slow tenure} & : \quad 5.654 & 0.684 & 0.915 & 0.016 & 0.133 & - & 0.285 \\
& : \quad (0.0546) & (0.0234) & (0.0283) & (0.0017) & (0.0247) & - & (0.0268)
\end{align*}
\]
### Empirical results: full model

<table>
<thead>
<tr>
<th>Moments</th>
<th>Data</th>
<th>S.E.</th>
<th>Simulated Moments</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Slow tenure</td>
</tr>
<tr>
<td>Average firm size</td>
<td>71.95</td>
<td>1.8782</td>
<td>71.53</td>
</tr>
<tr>
<td>Exit rate</td>
<td>0.091</td>
<td>0.0012</td>
<td>0.098</td>
</tr>
<tr>
<td>Fraction of plants in each bin:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10-19 employees</td>
<td>0.386</td>
<td>0.0049</td>
<td>0.398</td>
</tr>
<tr>
<td>20-99 employees</td>
<td>0.447</td>
<td>0.0049</td>
<td>0.436</td>
</tr>
<tr>
<td>100-499 employees</td>
<td>0.145</td>
<td>0.0038</td>
<td>0.148</td>
</tr>
<tr>
<td>+ 500 employees</td>
<td>0.022</td>
<td>0.0016</td>
<td>0.018</td>
</tr>
<tr>
<td>Share of employment in each bin:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10-19 employees</td>
<td>0.064</td>
<td>0.0021</td>
<td>0.062</td>
</tr>
<tr>
<td>20-99 employees</td>
<td>0.260</td>
<td>0.0081</td>
<td>0.264</td>
</tr>
<tr>
<td>100-499 employees</td>
<td>0.417</td>
<td>0.0118</td>
<td>0.398</td>
</tr>
<tr>
<td>+ 500 employees</td>
<td>0.260</td>
<td>0.0173</td>
<td>0.275</td>
</tr>
<tr>
<td>Volatility $g_I$</td>
<td>0.688</td>
<td>0.0160</td>
<td>0.781</td>
</tr>
<tr>
<td>Volatility $g_s$</td>
<td>2.161</td>
<td>0.0618</td>
<td>2.118</td>
</tr>
<tr>
<td>Kurtosis $g_I$</td>
<td>5.144</td>
<td>0.0606</td>
<td>3.141</td>
</tr>
<tr>
<td>Kurtosis $g_s$</td>
<td>1.973</td>
<td>0.0273</td>
<td>1.645</td>
</tr>
<tr>
<td>Inaction rate $g_I$</td>
<td>0.181</td>
<td>0.0026</td>
<td>0.231</td>
</tr>
<tr>
<td>Share of subcontracting</td>
<td>0.247</td>
<td>0.0053</td>
<td>0.253</td>
</tr>
<tr>
<td>Criterion, $\Gamma(\theta)$</td>
<td>1,342.52</td>
<td></td>
<td>5,265.9</td>
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</table>

<table>
<thead>
<tr>
<th>$c_f$</th>
<th>$\lambda$</th>
<th>$\rho$</th>
<th>$\mu$</th>
<th>$\sigma_\epsilon$</th>
<th>$f$</th>
<th>$\tau$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quick tenure</td>
<td>4.807</td>
<td>-</td>
<td>0.903</td>
<td>0.023</td>
<td>0.139</td>
<td>0.095</td>
</tr>
<tr>
<td>$(\lambda = 0)$</td>
<td>(0.0353)</td>
<td>-</td>
<td>(0.0197)</td>
<td>(0.0047)</td>
<td>(0.0198)</td>
<td>(0.0027)</td>
</tr>
<tr>
<td>Slow tenure</td>
<td>6.384</td>
<td>0.758</td>
<td>0.913</td>
<td>0.029</td>
<td>0.129</td>
<td>0.101</td>
</tr>
<tr>
<td>$(\lambda = 0)$</td>
<td>(0.0403)</td>
<td>(0.0284)</td>
<td>(0.0113)</td>
<td>(0.0025)</td>
<td>(0.0121)</td>
<td>(0.0025)</td>
</tr>
</tbody>
</table>
General equilibrium framework

- HH preferences:
  \[
  \sum_{t=1}^{\infty} \beta_t [\log(c_t) - B \frac{n_t^{1+\phi}}{1+\phi}],
  \tag{7}
  \]

- Resource constraint:
  \[
  C = Y - Me - F,
  \tag{8}
  \]
  and output is
  \[
  Y = \int_{z^*} [f(L(l, z; p), S(n, z; p), z) - c_f] d\mu(z, l) + M \int_{z^*} f(L(0, z; p), S(n, z; p), z) d\nu(z),
  \tag{9}
  \]
  and the wage premium on subcontracted workers is
  \[
  F = fw \left[ \int_{z^*} S(n, z; w) d\mu(z, l) + M \int_{z^*} S(n, z; w) d\nu(z) \right]. \tag{10}
  \]

- Labor market clearing condition is
  \[
  N^s(\mu, M; w) = \int_{z^*} [L(l, z; w) + S(n, z; w)] d\mu(z, l) + M \int_{z^*} [L(0, z; w) + S(n, z; w)] d\nu(z).
  \tag{11}
  \]
### Steady-state effect of eliminating firing costs

<table>
<thead>
<tr>
<th></th>
<th>Full model</th>
<th>No subcontracting</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Slow tenure</td>
<td>Quick tenure</td>
</tr>
<tr>
<td>Output</td>
<td>3.54</td>
<td>4.20</td>
</tr>
<tr>
<td>Consumption</td>
<td>3.59</td>
<td>2.90</td>
</tr>
<tr>
<td>Average labor productivity</td>
<td>1.02</td>
<td>2.49</td>
</tr>
<tr>
<td>Total employment</td>
<td>2.49</td>
<td>1.67</td>
</tr>
<tr>
<td>Permanent</td>
<td>3.73</td>
<td>1.67</td>
</tr>
<tr>
<td>Wage permanent workers</td>
<td>5.75</td>
<td>4.34</td>
</tr>
<tr>
<td>Layoff costs/wage bill (before)</td>
<td>0.061</td>
<td>0.034</td>
</tr>
<tr>
<td>Subcontracting costs/wage bill (before)</td>
<td>0.092</td>
<td>-</td>
</tr>
</tbody>
</table>

Note: The table reports the steady-state percentage change if the firing costs are eliminated starting from each of the different estimated models.
### Steady-state effect of eliminating subcontracted workers

<table>
<thead>
<tr>
<th></th>
<th>Quick tenure</th>
<th>Slow tenure</th>
</tr>
</thead>
<tbody>
<tr>
<td>Output</td>
<td>-0.15</td>
<td>-0.08</td>
</tr>
<tr>
<td>Average labor productivity</td>
<td>-0.10</td>
<td>-0.02</td>
</tr>
<tr>
<td>Mass of firms</td>
<td>-0.62</td>
<td>-0.23</td>
</tr>
<tr>
<td>Layoff costs/wage bill</td>
<td>1.74</td>
<td>0.67</td>
</tr>
<tr>
<td>Total employment</td>
<td>-0.05</td>
<td>-0.06</td>
</tr>
<tr>
<td>Permanent</td>
<td>1.07</td>
<td>1.15</td>
</tr>
<tr>
<td>Wage permanent workers</td>
<td>-0.06</td>
<td>-0.02</td>
</tr>
<tr>
<td>Layoff costs/wage bill</td>
<td></td>
<td></td>
</tr>
<tr>
<td>before</td>
<td>0.04</td>
<td>0.06</td>
</tr>
<tr>
<td>after</td>
<td>0.09</td>
<td>0.09</td>
</tr>
<tr>
<td>Subcontracting costs/wage bill</td>
<td>0.09</td>
<td>0.09</td>
</tr>
</tbody>
</table>

Notes: The table reports the steady-state percentage change if subcontracted work was eliminated from both of models or, equivalently, if the wage premium on subcontracted workers was prohibitively high.
Conclusions

- Estimate the ‘real’ costs of employment protection legislation using an industry equilibrium model and plant-level data for Chile.

- Study the interaction between firing costs and subcontracting as a way to substitute for hiring workers.

- Allowing firms to subcontract workers increases output, employment and productivity.

- Effect is stronger on output as subcontracting firms can respond more aggressively to productivity shocks, enhancing the allocation of labor across firms, and hence TFP.

- When firms can subcontract, the negative effects of firing costs in aggregate outcomes are less than previously estimated in the literature.
Future work

- Hopenhayn and Rogerson (1993) model is not appropriate for welfare analysis
  - frictionless economy with perfect insurance markets
  - firing costs have no potential benefits, only distort job turnover process

- Alvarez and Veracierto (2001) nice start
  - reallocation process is costly; unemployed must search to find new employment
  - no insurance markets are available but agents can save and accumulate an interest-bearing asset
  - since agents are risk averse and there are no insurance markets, firing costs improve welfare (workers transit fewer times through unemployment)
Note: the figures show the percentage of total workforce by year (on the left) and the share of subcontracted workers as a percentage of total workforce by plant (on the right). Source: Author’s calculations using data from ENIA.
Establishments more constrained by the regulation use more subcontracting.

Note: Only plants that use subcontracting are considered.

Note: the figure shows the average share of subcontracted workers in an establishment by percentile of sales volatility.

Source: Author’s calculations using data from ENIA.
Adjustments in employment using subcontracted workers are more frequent.

Note: the figure represents the fraction of plants expanding (contracting) at different growth rate intervals (as measured in the horizontal axis). Growth rate is computed according to the standard Davis and Haltiwanger (1992) definitions: 

$$ g_{it} = (x_{it} - x_{it-1}) / (0.5 * (x_{it} + x_{it-1})) $$

where $x_{it}$ is the number of employees (subcontracted or permanent) in plant $i$ at time $t$. The bars to the right of the origin correspond to job creation and to the left to job destruction. At the center, the proportion of plants for which employment remains unchanged, and death (births) correspond to the left (right) endpoint.

Source: Author’s calculations using data from ENIA.
Subcontracting is present in key value-adding functions in the firms.

Note: the figures show the total number of subcontracted workers by occupation (on the left) and the share of subcontracted workers as a percentage of the plant’s workforce by occupation (on the right).

Source: Author’s calculations using data from ENIA.
Notes: the “job security” index measures in monthly wages the expected cost of dismissing a full-time indefinite worker at the time the worker is hired. Source: Heckman and Pagés (2000) for 24 countries in OECD and Latin America; updated for Chile from 1960-1996 by Montenegro and Pagés (2005), and from 1996-2005 by Alvarez and Fuentes (2011).
After economic liberalization process of the 1970s and 1980s, several changes were introduced to the regulation on permanent workers to increase job protection during the 1990s and 2000s. The upper limit on severance payments was raised from five to eleven month wages; penalties for firms that do not prove just cause increase (from 0% to 20% to a range 30-100%), and the causes for just dismissal.

Instead, for years subcontracted work remained practically deregulate; lack of clarity regarding which employer is legally responsible for obligations towards subcontracted workers.

- late-1970s complete liberalized in the use of subcontracted workers
- the counter-reform process of the 1990s-2000s did not reach them
Labor decisions: no firing costs

- No role for subcontracted workers
- Firms choose permanent workers: $n_t = (\alpha p z_t / w)^{1/(1-\alpha)}$
- High productivity firms hire permanent workers and fire them if productivity is low
Firms hire permanent workers only if productivity shock is high enough

- Employment decision (s,S) rule
Firms use subcontracting to buffer stock of permanent workers and avoid firing costs

High productivity increases subcontracted workers; only if shock is large enough, hire permanent workers
Timing of the model

- **Incumbents:**
  1. Enter period $t$ with $(z_{t-1}, l_{t-1})$
  2. Exit decision. If exit, pays $g(0, l_{t-1})$, zero profits, and avoids $c_f$
  3. If stays, it pays $c_f$ and receives $z_t$
  4. Chooses labor demand, produces output and receives profits
  5. Enter period $t + 1$ with $(z_t, l_t)$

- **Potential entrants:**
  1. Pay $p_t c_e$, and draw $z_t$ from $\nu(z_0)$ (which is independent across firms)
  2. Exit decision. If $z_t$ is above exit threshold firm stays and produces as in (4)
Entry decision and stationary distribution

- Potential entrant operates if:

\[ V^e = \int V(0, z) d\nu(z) \geq p c_e, \tag{12} \]

where \( V(0, z_t) \) is given by eq. (1)

- Distribution of state variables \((z, l)\) for all firms evolves:

\[ \mu'(z, l) = \int_{z'} \int_z \left[ 1 - X(l, z) \right] F(z' / z) d\mu(z, l) + \int_{z'} M' d\nu(z) \tag{13} \]

where \( M \) corresponds to the mass of new firms.

- Stationary equilibrium such that the distribution reproduces itself, i.e. \( \mu' = \mu \)
Definition of equilibrium

- A *stationary industry equilibrium* with positive entry is a set of value functions and decisions rules, and a list \( \{p^*, \mu^*, M^*\} \) such that:

1. Given prices, firms’ value function and policy functions are optimal
2. Markets clear:
   \[
   Q^* = \int_{z^*} f(n(l, z), s(n, z), z)d\mu(z, l) + M \int_{z^*} f(n(0, z), s(n, z), z)d\nu(z)
   \]  
   (14)
3. There is an invariant distribution over firms: \( \mu^* = T(\mu^*, M^*; p^*) \)
4. The free entry condition is satisfied: \( V^e(p^*) = p^* c_e \)
Simulated method of moments

- SMM works as follows:

\[ \hat{\theta} = \arg \min_{\theta \in \Theta} [\Psi^A - \Psi^S(\theta)]' \ W \ [\Psi^A - \Psi^S(\theta)] \]  

(15)

where \( \Psi^A \) data moments, \( \Psi^S(\theta) \) simulated moments, \( W = \text{diag}(V^{-1}) \) weighting matrix, and \( V \) covar matrix data moments.

- Use Nelder-Mead simplex algorithm starting from 1,000 initial values to minimize criterion function.

- Standard errors:

\[ SE(\hat{\theta}) = [(J'WJ)^{-1}]^{1/2}, \]  

(16)

where \( J = E(\partial \Psi^S(\theta)/\partial \theta) \) of dimension \( p \ (#\text{moments}) \times q \ (#\text{parameters}) \).
Solution method

Algorithm: 2 “Do Loops”

- The model has no closed-form solution, is solved numerically
- Iterate over \( p_i \) until the entry condition is satisfied at \( p^* \):
  1. For each \( p_i \), compute \( V_i(n, z; p_i) \) and \( V_i(0, z; p_i) \)
  2. Let \( EC(p_i) \equiv \int V(0, z; p_i)d\nu(z)/p_i - c_e \). If \( EC(p_i) > 0 \), then set \( p_{i+1} < p_i \), otherwise set \( p_{i+1} > p_i \).
- Iterate over \((\mu_i, M_i)\) until \( Q^d = Q^s \) at \((\mu^*, M^*)\):
  1. Letting \( M_0 = 1 \), solve for the stationary distribution \( \mu_{0ss}(M_0 = 1) \) using equation (13)
  2. Let \( EQ(\mu_i, M_i) \equiv Q^d - Q^s(\mu_i(M_i), M_i; p^*) \). If \( EQ(\mu_i, M_i) > 0 \), then set \( M_{i+1} > M_i \), otherwise set \( M_{i+1} < M_i \). When \( EQ(\mu_i, M_i) \approx 0 \) then \((\mu_{i+1}, M_{i+1}) = (\mu^*, M^*)\)
Solution method

- To approximate the distribution of the idiosyncratic shocks use the quadrature-based method developed in Rouwenhorst (1995).

- For state variable $l$ (permanent employment with tenure), I assign a log-linear grid of size $g_l = 300$. The discretized stochastic process for $z$ varies over a grid of size $g_z = 30$.

- Iso-elastic industry demand: $p = Q^{-\frac{1}{\eta}}$, where $p$ is output price, $Q$ is the industry output, and $\eta > 0$ is the price elasticity of demand elasticity.

- The model period is one year.
References 1


References II


