The Macroeconomic Implications of Limited Arbitrage

Ally Quan Zhang

Lancaster University Management School

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Motivations I

- real sector disturbance $\Rightarrow$ arbitrage crashes: GFC
  - 2007 subprime: collateral value collapse $\Rightarrow$ arbitrageurs unwind
  - price gap of similar assets $\uparrow$, arbitrage crashes
- arbitrage failure $\Rightarrow$ real contractions: European banking crisis
  - “carry trade” by Eurozone banks: high-yield GIPSI & low-yield German sovereign bond (Acharya & Steffen (2015))
    - yield diverge — 70% bank losses — firm lending and output plummet
- slow, incomplete recoveries in real and financial sectors
  - mispricing skyrocketed and remained large after crises
  - e.g., violation of CIP, CDS-bond basis
Literature on Financial Frictions and Crises

- finance: limits of arbitrage in financial markets

- macro: limits of arbitrage in production
  - e.g., Kiyotaki & Moore (1997), Bernanke, Gertler & Gilchrist (1999), Brunnermeier & Sannikov (2014), Kiyotaki & Gertler (2015)

- links between arbitrage trading & macroeconomy, role in crises
  - ???
Overview

- unified and tractable framework
  - link real investments & mispricing in segmented markets
- macroeconomic impacts of limited arbitrage
  - boost aggregate investments and output
  - increase systemic risk
- analytical solutions to multiple equilibria
  - regime shifts: crisis & policy indications
  - slow & incomplete recovery from Great Recession
1. Baseline Model

2. Model Implications

3. Crises and Recovery
Baseline Model

Market A
- HH's Natural Endowment
- Financial Asset $\theta_t$

The Intermediaries
- Capital
- Production Process

Market B
- HH's Natural Endowment
- Financial Asset $\theta_t$

Endowment Shock $+u\theta_t$

Endowment Shock $-u\theta_t$
HH’s natural endowment

\[ y_{i,t} = b + u_i \theta_t, \quad i \in \{A, B\}, \quad t \in \{1, 2, \ldots\} \]

- \( \theta_t \) follows a symmetric distribution around zero on \([-\bar{\theta}, \bar{\theta}]\)
- shock intensities: \( u_A = -u_B =: u \)

- opposite shocks, opposite hedging demand
Intermediaries

- both arbitrageurs and entrepreneurs
  - take identical but opposite positions $x_{A,t} = -x_{B,t} = x_t$
  - convert perishable goods one-to-one into durable goods
  - invest capital & hire HH as labor

$$Y_t = F(K_{t-1}) + (1 - \delta)K_{t-1}$$

$$= a K_{t-1}^{\alpha} L^{1-\alpha} + (1 - \delta)K_{t-1}$$
Financial Assets

  - long-lived, in zero net supply
  - settlement of previous positions: $x_{t-1}(P^A_t - P^B_t)$
  - IM’s liability—net payment from IM to HH
Collateral Constraints

- post capital input as collateral
  - cover IM’s next period liability in case of default
  - depreciated capital as limit: $(1 - \delta)K_t$

- real-world securitization
  - securitized products as collateral
IM’s Optimization Problem

\[
\max_{c_s^{IM}, x_s, K_s} \mathbb{E} \left[ \sum_{s=t}^{\infty} \rho^s \log \left( c_s^{IM} \right) \right],
\]

subject to

\[
c_t^{IM} + K_t = -x_{t-1}(P_t^B - P_t^A) + x_t(P_t^B - P_t^A) + F(K_{t-1}) + (1 - \delta)K_{t-1},
\]

\[
-x_t(P_{t+1}^B - P_{t+1}^A) + (1 - \delta)K_t \geq 0.
\]
HH’s Optimization Problems

\[
\max_{c^i_s, y^i_t} \mathbb{E} \left[ \sum_{s=t}^{\infty} \beta^s \log \left( c^i_s \right) \right], \quad i \in \{A, B\},
\]

subject to

\[
c^i_t = y^i_{t-1}(P^i_t + \theta_t) - y^i_t P^i_t + a(1 - \alpha)K^{\alpha}_{t-1}L^{-\alpha} + (b + u_i\theta_t).
\]

- income from trading assets
- labor income
- endowment
Market Liquidity & Mispricing in Equilibrium

- $\rho > \bar{\rho}$, patient IM
  - full liquidity, no price discrepancy.
  - neoclassical growth model with frictionless financial markets

- $0 < \rho \leq \bar{\rho}$, impatient IM, collateral constrained
  - mispricing with limited arbitrage

\[
x_t \in (0, u) \quad \text{and} \quad \phi_t =: P_t^B - P_t^A = \frac{(1 - \delta)K_{t-1}}{x_{t-1}} > 0.
\]
Dynamics of IM’s Wealth, Capital Accumulation and Consumption

Under binding collateral constraints, IM’s consumption and capital evolves according to

\[ C_t = (1 - \alpha \rho) W_t, \quad K_t = \alpha \rho W_t S_t. \]

where \( W_t \) is IM’s wealth at the beginning of \( t \),

\[ W_t := F(K_{t-1}) + (1 - \delta) K_{t-1} - x_{t-1} \phi_t = F(K_{t-1}) \]

and the leverage ratio: \( S_t := \frac{\phi_{t+1}}{\phi_{t+1} - (1 - \delta) \phi_t} > 1. \)
arbitrage gain serves as leverage to production

\[ K_t = \alpha \rho W_t + x_t \phi_t = \alpha \rho W_t S_t \]

negative interest loan to IM

loan: immediate arbitrage gains

repayment: next period obligated settlement

capital’s collateral premium, marginal return ↑
Steady States With Binding Collateral Constraints

- steady states: $K_t = K^*$, $x_t = x^*$, $\phi_t = \phi^*$

- collateral premium boosts capital: $K^* = F'^{-1}\left(\frac{\delta}{\rho}\right) > F'^{-1}\left(\frac{1}{\rho}\right)$
  - depreciation $\delta$, inverse measure of collateral value

- fixed “loan” size: $x^*\phi^* = x_t\phi_t = x_{t-1}\phi_t$
  - zero-interest, roll over infinitely
Steady States With Binding Collateral Constraints

- binding collateral constraints

\[
(1 - \delta) K^* = x^* \phi^*
\]

- trading volume \( x^* \uparrow \), price gap \( \phi^* \downarrow \)

- given unique \( K^* \), 2 equilibria: bad vs good regime
  - small (big) trading vol \( x^* \), large (small) price gap \( \phi^* \)
  - market microstructure: transaction costs, market-making rebate;
    collateral policy: re-use limits, eligibility scope, velocity, etc
  - heavily (lightly) regulated trading environment
Two Steady States with Binding Collateral Constraints

- **IM indifferent:** $C_{IM}^* = (1 - \alpha \rho)F(K^*)$

- **HH prefers the good regime**
  - higher trading volume $x^*$, better risk sharing
Multiple Equilibria and Asset Demand $u$

All else equal, shock intensity $u_1 < u_2$, binding collateral constraint:

- $K^* [u_1] = K^* [u_2]$;
- $x_1^* [u_1] > x_1^* [u_2]$, $\phi_1^* [u_1] < \phi_1^* [u_2]$;
- $x_2^* [u_1] < x_2^* [u_2]$, $\phi_2^* [u_1] > \phi_2^* [u_2]$
1. Baseline Model

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3. Crises and Recovery
Crisis from Regime Shifts

- **regime shifts**
  - sudden changes in regulation, trading platform, market sentiment, macro/micro factors, etc

- **crises arise when shifting from good to bad**
  - price gap widens to fit the bad regime
  - large initial positions inherited from the good
  - financial distress or insolvency
Markets panic at the good regime:

- immediate reaction

- price gap $\uparrow$ & big initial position $\rightarrow$ IM’s obligation $\uparrow$

- financial distress $\rightarrow$ $K \downarrow$ & liquidity $\downarrow$
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Crisis Scenario & Incomplete Recovery I

Markets panic at the good regime:

- long term

- IM: slowly recovered; HH: slow & incomplete recovery
Crisis from Regime Shifts II

- crises unavoidable even when switching to a good regime
  - as long as new regime features a bigger price gap
  - example: sudden drop in asset demand $u$

![Graph showing the relationship between $x\phi$ and $x$, with two curves labeled $u-10$ and $u-12$, and a line labeled steady state collateral.]

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• crises unavoidable even when switching to a good regime

  ▶ as long as new regime features a bigger price gap

  ▶ example: sudden drop in asset demand $u$
Crisis Scenario & Incomplete Recovery II

switch to a good regime

- price gap $\phi_t \uparrow$ & big initial position $x_{t-1} \rightarrow$ IM’s liability $x_{t-1} \phi_t \uparrow$
  - financial distress $\rightarrow$ $K \downarrow$ & liquidity $\downarrow$, crisis unavoidable
Crisis Scenario & Incomplete Recovery II

switch to a good regime

- price gap $\phi_t \uparrow$ & big initial position $x_{t-1} \rightarrow$ IM’s liability $x_{t-1}\phi_t \uparrow$
- financial distress $\rightarrow$ $K \downarrow$ & liquidity $\downarrow$, crisis unavoidable
Policy Trade-off

**Welfare vs vulnerability**

Given the sudden shock & post-shock regime, the bad-regime economy is (weakly) better off than the good one, with higher post-shock $K_t$ and liquidity $x_t$ before converging to new steady states.

- good regime
  - more vulnerable to systemic risk
  - more negative impact on real sectors and liquidity supply

- policy trade-off: bad to good regime
  - pareto improvement: liquidity, risk sharing & price discovery
  - financial instability, slow recovery & severe contagion to real sectors
Take-away

- interactions of arbitrage and real activities boost production
  - by giving capital investment extra collateral premium
- also increase systemic risks
  - regime shifts trigger crises
- may derail full & fast recoveries
  - policy trade-off