Optimal Currency Exposure Under Risk and Ambiguity Aversion

Urban Ulyrch \(^{a,b}\) and Nikola Vasiljevic \(^a\)

\(^a\)Department of Banking and Finance, University of Zürich
\(^b\)Swiss Finance Institute

Motivation
- Investors tend to hold portfolios with global exposure primarily for diversification benefits.
- Recent studies of foreign currency exposure show that full hedging is not optimal.
- In addition to market risk, agents face model uncertainty of the probability laws governing the stochastic processes of asset and currency returns.
- This paper:
  - Explores the implications of currency exposure under ambiguity and sheds new light on optimal currency allocations.
  - Builds a bridge between the literatures on currency hedging and ambiguity aversion.

Model
- For a fully hedged portfolio return \(\pi_{h,i}^a\), currency exposure \(\phi_{i}\), foreign exchange rate return \(\Delta e_{i+1}\) and forward premium \(f_{i}\), we derive
  \[\pi_{h,i}^a = \phi_{i} \Delta e_{i+1} + f_{i} \phi_{i+1} \]
  - This expression is model-free.
- No underlying dynamics for asset or currency returns are assumed.
- Model uncertainty: The situation in which an investor is uncertain about the true probabilistic model governing the occurrence of different states.
- For a coefficient of risk aversion \(\lambda\) and a coefficient of ambiguity aversion \(\alpha\), a risk and ambiguity aversion investor maximizes her utility
  \[U(\pi_{h,i}^a) = V(\pi_{h,i}^a) - \frac{\lambda}{2} \pi_{h,i}^a - \frac{\alpha}{2} \phi_{i}^2 \]
  - The argument \(\phi_{i}\) which maximizes the above expression is the optimal currency exposure in the presence of risk and ambiguity and is given by
  \[\phi_{i} = \left( -\lambda V''(\pi_{h,i}^a) - 2\alpha \phi_{i} \right) \left( -\lambda V''(\pi_{h,i}^a) + 2\alpha \phi_{i} \right) \]
  - In the limit when \(\lambda \to \infty\), the optimal currency exposure converges to the minimum variance case.
  - When \(\lambda \to \infty\), the optimal currency exposure converges to zero (full hedging) and the entire currency exposure is kept solely in the domestic currency.
  - The puzzle of insufficient currency diversification (home-currency bias) may be driven by investors’ ambiguity aversion.

Empirical Analysis

- Aim: Investigate historical optimality and the role of sampling error in the construction of the ex-post efficient currency exposures.
- Here, we work with the demeaned historical returns and define a loss function as \(L_i(\pi_{h,i}^a) = -\pi_{h,i}^a\).
- For a matrix of demeaned currency excess returns \(X\), vector of demeaned fully hedged portfolio returns \(y\), weighting matrix \(\Pi\), ambiguity matrix \(\Phi\), ambiguity currency exposure \(\phi_{i}\), and a weighted \(L_i\)-norm squared
  \[\phi_{i} = -\frac{1}{2} \left( y' \Pi X' \Phi X' \Pi y \right) \]
  - We prove that the in-sample efficient currency exposure can be found as a generalized ridge regression
  \[\phi_{i} = \left( -X' \Pi X \right)^{-1} \left( -X' \Pi y \right) \]
  - Ambiguity induces shrinkage (regularization) towards the infinitely ambiguity averse optimal exposure \(\phi_{i}\) distorted by the level and structure of uncertainty from matrix \(Z\).
- The optimal in-sample currency weights produce a pure currency exposure which is closest in terms of penalized least squares distance to the fully hedged portfolio returns.
- The generalized penalty term corresponds to the utility loss arising from model uncertainty. It geometrically implies a non-zero centered, ellipsoid parameter constraint.

Example:
- Solve an optimal currency allocation problem by looking at the domestic assets position as purely risky and an exposure to foreign currencies as ambiguous.
- The optimal currency exposure is obtained as
  \[\phi_{i} = \left( -\lambda V''(\pi_{h,i}^a) - 2\alpha \phi_{i} \right) \left( -\lambda V''(\pi_{h,i}^a) + 2\alpha \phi_{i} \right) \]
  - In the limit when \(\lambda \to \infty\), the optimal currency exposure converges to the minimum variance case.
- When \(\lambda \to \infty\), the optimal currency exposure converges to zero (full hedging) and the entire currency exposure is kept solely in the domestic currency.
- The puzzle of insufficient currency diversification (home-currency bias) may be driven by investors’ ambiguity aversion.

References

Main Results
- Closed form expressions of optimal currency exposure for a risk and ambiguity averse investor are derived in a model-free setting.
- The in-sample efficient currency exposure capturing agent’s dislike for risk as well as model uncertainty are found by a generalized ridge regression.
- The penalty term corresponds to the utility loss arising from model uncertainty.
- Empirically, ambiguity induces a bias-variance trade-off which leads to an improved in-sample estimator of optimal currency exposure.
- Realized volatility and Sharpe ratios for the ambiguity adjusted currency volatility strategy lie between the minimum variance and mean-variance cases.
- The investigated link between model uncertainty and penalized regression formally connects the areas of financial economics (asset allocation) and statistical learning (regularization).

Data
- The empirical analysis employs the data of exchange rates, short-term interest rates, equity broad market indices, and fixed income total return indices (for various maturities).
- The data series for seven developed economies: Australia, Canada, Switzerland, Eurozone, United Kingdom, Japan and United States, are available at a daily frequency.
- The sample period starts in January 1999, when the euro was introduced to the world financial markets, and ends in June 2018.

Contact Information
Email: urban.ulrych@bf.uzh.ch
Phone: +41 76 667 31 92

In-Sample Analysis

In-Sample Optimal Currency Exposures

Optimal Currency Exposure with Risk and Ambiguity Aversion

Figure 1: Optimal currency exposure in CHF (for a EUR based investor) in dependence of risk and ambiguity aversion parameters is plotted here. We assume independent prediction models and the uncovered interest rate parity to hold.

Figure 3: Optimal currency exposure and the corresponding bootstrapped 95% confidence intervals for CHF and EUR (for a USD based investor) in dependence of risk and ambiguity aversion parameters are plotted here.

Table 1: This table reports annualized standard deviations and Sharpe ratios of portfolios featuring different uses of currencies for risk management. An equally weighted global equity portfolio and hedging at a quarterly horizon are assumed.

Variance and Sharpe ratios of Hedged Global Equity Portfolios with Ambiguity Aversion

<table>
<thead>
<tr>
<th>Currency</th>
<th>Variance</th>
<th>Sharpe</th>
</tr>
</thead>
<tbody>
<tr>
<td>CHF</td>
<td>0.01</td>
<td>0.03</td>
</tr>
<tr>
<td>EUR</td>
<td>0.02</td>
<td>0.04</td>
</tr>
<tr>
<td>USD</td>
<td>0.03</td>
<td>0.05</td>
</tr>
</tbody>
</table>

Figure 2: Bootstraped distribution of optimal currency exposure in CHF (for a USD based investor) for different values of risk and ambiguity aversion parameters is plotted here.