ASSA 2020 Annual Meeting

Liquidity and Default: A Continuous Time Approach

Theofanis V. Papamichalis
Dimitrios P. Tsomocos

January 3-5, 2020
Outline

- Motivation
- Related Literature
- Model
- Results
Motivation

- Bankruptcy laws impose a penalty to individuals, firms and even countries that decide to default.
- Question: Is there any trade-off between Monetary Policy and default penalties (Regulatory Policy)?
- If the answer is yes, are they complements or substitutes and how does this affect optimal welfare? What is the optimal policy mix?
- Why is this question important?
Related Literature

One salient feature of this class of models is that they can analyze the full macroeconomic dynamics. In particular, they show that the long run behavior of an economy facing aggregate shocks under the presence of financial frictions can be in stark contrast with the one predicted by the steady state analysis that is typically used in mainstream linearized macroeconomic models. The main reason for this divergence is the presence of system generated (endogenous) risk that may drive the equilibrium dynamics far away from the state in which the system would end up in the absence of such risk. Taking into account this feature is crucial, if one intends to assess welfare distortions or the impact of policy actions.
Dubey and Geanakoplos (1992)

- Provides microfoundations for the positive value of money by establishing a role for banks.
- The "banking sector" issues both inside and outside money.
- The value of money to the agents is derived from the transaction technology (see, Gurley and Shaw (1956))
- It also allows for endogenous default to be compatible with the orderly function of markets.
Heterogeneous Households

- 2 Types of Agents: Experts (Agents A) and Households (Agents B).
- Both types are endowed with outside money $m^A_t$ and $m^B_t$ each period.
- A (dummy) Central Bank. The objective of the CB is to withdraw all liquidity in the end of each period.
- Both types of agents can own capital, but experts have higher productivity.
- Denote the aggregate amount of capital in the economy by $K_t$ and capital held by an individual agent by $k_t$
- The production function of experts is:

$$y_t = ak_t$$
Model (2)

Heterogeneous Households (cont.)

- When held by an expert, capital evolves according to:
  \[ dk_t = (\phi(i_t) - \delta)k_t dt + \sigma k_t dZ_t \]

- The productivity function of households is:
  \[ y_t = a k_t \]

  with \( a \leq a \)

- When held by an households, capital evolves according to:
  \[ \bar{dk}_t = (\phi(i_t) - \bar{\delta})\bar{k}_t dt + \sigma \bar{k}_t dZ_t \]

  with \( \bar{\delta} > \delta \)
Model (3)

Monetary flows

**Figure: The Economy**
Model (4)

Preferences

- Experts and less productive households are risk averse with logarithmic utilities.
- Discount rates are $b_1$ and $b_2$ respectively.
- Household utility function:
  \[ E[\int_0^\infty e^{-b_2 t} \ln c_t^B dt] \]
- We assume that $b_1 > b_2$.
- Experts utility function:
  \[ E[\int_0^\infty e^{-b_1 t} \ln c_t^A dt] - E[\int_0^\infty e^{-b_1 t} \max(0, \frac{\tau(1 - u_t)\mu_t}{q_t a_t}) dt] \]
Model (5)

Market for Capital (A)

- Individual experts and households trade physical capital in a fully liquid market.
- We denote the equilibrium market price of capital $q_t$ and postulate that its law of motion is of the form:

$$dq_t = \mu^q_t q_t dt + \sigma^q_t q_t dZ_t$$

- When an expert buys and holds $k_t$ units of capital at price $q_t$, by Ito’s Lemma the value of this capital evolves as follows:

$$\frac{dk_t q_t}{k_t q_t} = (\phi(i_t) - \delta + \mu^q_t + \sigma^q_t) dt + (\sigma + \sigma^q_t) dZ_t$$

Go to Appendix
Model (6)

Market for Capital (B)

Thus, the total return that experts earn from capital (per unit of wealth invested) is:

\[
dr_t^k = \frac{a - i_t}{q_t} dt + (\phi(i_t) - \delta + \mu_t + \sigma q_t) dt + (\sigma + \sigma_t) dZ_t
\]

- **dividend yield**
- **capital gains rate**

Similar for (less productive) Households.

\[
dr_t^k = \frac{a - i_t}{q_t} dt + (\phi(i_t) - \delta + \mu_t + \sigma q_t) dt + (\sigma + \sigma_t) dZ_t
\]

- **dividend yield**
- **capital gains rate**
Model (7)

Optimization Problems

► Experts and Households can trade physical capital in a fully liquid market.

► Formally, each expert solves:

$$\max_{x_t, c^A_t, i_t, \mu_t, v_t, b_t} E\left[\int_0^\infty e^{-b_1 t} \ln c^A_t dt\right] - E\left[\int_0^\infty e^{-b_1 t} \tau (1 - u_t) \mu_t dt\right]$$

► Subject to her budget constraint (wealth accumulation):

$$\frac{dn_t}{n_t} = x_t dr^k_t + (x_t - 1) rt - \frac{p^c_t c^A_t dt}{n_t}$$

and the Cash-in-Advance Constraints:

$$n_t (x_t - 1) dt \leq (e^{-r_t} \mu_t + m_t^A) dt$$
Model

Optimization Problems (cont.)

\[ v_t \mu_t dt \leq \frac{b_t(a - i)p_t^c n_t x_t}{q_t} dt \]

and

\[ p_t^c c_t^A dt \leq \frac{(1 - b_t)(a - i)p_t^c n_t x_t}{q_t} dt \]

Likewise for Households.
Model

Markets

- Capital

\[ \int_{I} k_i^i di + \int_{J} k_i^j dj = K_t \]

- Money

\[ e^{-r_t} \mu_t dt = M^{CB} dt \]

- Commodity

\[ \int_{I} c_i^i di + \int_{J} c_i^j dj = (\int_{I} (a - i_t^i) k_i^i di + \int_{J} (a - i_t^j) k_i^j dj) dt \]
Model

Equilibrium

For any initial endowments of capital \( \{k_i^0, k_j^0 : i \in I, j \in J\} \) such that:

\[
\int_I k_i^0 di + \int_J k_j^0 dj = K_0
\]

an equilibrium is described by stochastic processes on the filtered probability space defined by the Brownian Motion \( \{Z_t, t \geq 0\} \): the price processes of consumption capital \( p_c^i \) and \( q_t \), net worths \( \{n_i^i, n_i^j \geq 0\} \), capital holdings \( \{k_i^i, k_j^j \geq 0\} \), investment decisions \( \{i_t^i, i_t^j \in \mathbb{R}\} \), consumption choices \( \{c_t^A, c_t^B\} \) of individual agents \( i \in I, j \in J \) and repayment rate \( v_t \) and borrowing \( \mu_t \), such that

(i) initial net worths satisfy \( n_i^0 = k_i^0 q_0 \) and \( n_j^0 = k_j^j q_0 \)

(ii) each expert \( i \in I \) and each household \( j \in J \) solve their problems given prices and

(iii) markets for consumption, capital and money clear.
Results

1) Given parameters, there exists $\tau_{min} \gg 0$ such that a Monetary Equilibrium exists only for values $\tau \geq \tau_{min}$. Furthermore, there exists a $\tau_{max}$ such that for every $\tau \geq \tau_{max}$ the repayment rate is always 1 (corner solution).

2) Interest rates decrease with increased penalty (Default Premium decreases).

3) Default suppresses investment and decreases capital accumulation of agent A.

4) For any given penalty, there exists an optimal money supply that maximizes welfare.

5) For any given money supply, there exists a finite positive default penalty, that maximizes Welfare.
1st Result

1) Monetary equilibrium exists ONLY for positive and finite values of the default penalty $\tau$.

**Figure:** Different Regions
1st Result

Figure: Optimal Welfare for different Regulatory and Monetary Policies
2nd Result

2) Interest rates decrease with increased penalty (Default Premium decreases).

Figure: Interest Rate for different Regulatory and Monetary Policies
3rd Result

3) Default suppresses investment and decreases capital accumulation of agent A.

Figure: Leverage for different default penalties
3rd Result

3) Default suppresses investment and decreases capital accumulation of agent A.

Figure: Leverage for different default penalties
4th Result

4) For any given penalty, there exists an optimal money supply that maximizes welfare.

Figure: Optimal Welfare for given $\tau$
5) For any given money supply, there exists a finite positive default penalty, that maximizes Welfare.
Final Remarks

Figure: Trade-off between $M$ and $\tau$
Thank you!
Appendix

We use Ito’s product rule, i.e. If \( \frac{dX_t}{X_t} = \mu_t X_t dt + \sigma_t X_t dZ_t \) and \( \frac{dY_t}{Y_t} = \mu_t Y_t dt + \sigma_t Y_t dZ_t \) then

\[
d(X_t Y_t) = Y_t dX_t + X_t dY_t + (\sigma_t X_t \sigma_t Y_t)(X_t Y_t) dt
\]

Applying this to

\[
dk_t = (\phi(i_t) - \delta)k_t dt + \sigma k_t dZ_t
\]

and to

\[
dq_t = \mu_t q_t dt + \sigma_t q_t dZ_t
\]

we obtain:

\[
\frac{dk_t q_t}{k_t q_t} = (\phi(i_t) - \delta + \mu_t q_t + \sigma \sigma_t q_t) dt + (\sigma + \sigma_t q_t) dZ_t
\]