Uncertainty and Economic Activity: Identification Through Cross-section Correlations

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*The views expressed in this paper do not necessarily reflect the position of the Bank of England.
Strong and robust association between measures of uncertainty and economic activity

- Uncertainty proxy: realized volatility equity market volatility
- Economic activity proxy: Quarterly real GDP growth
- Data for 32 countries, covering about 90 percent of world GDP
But difficult to interpret

- Uncertainty dampens activity
  - Precautionary savings [Kimball (1990)], irreversible investments [Bernanke (1983), Bloom (2009)], and financial frictions [Christiano et al. (2014), Gilchrist et al. (2014)]
  - Pricing frictions and ZLB can amplify these effects [Basu and Bundick (2017), Fernandez-Villaverde et al. (2011)]

- Recessions can also increase uncertainty
This paper

- Takes a novel multi-country perspective and models the relation between uncertainty and economic activity without restricting the direction of causation.

- Identify two factors, a real and financial factor, exploiting different patterns correlation of volatility and growth across countries.

NOTE: Average pairwise correlation of volatility (yellow bars) and GDP growth (blue bars) series.
Main findings

1. For most countries, the real common factor accounts for the bulk of the correlation between volatility and growth

2. The "Endogenous" component of volatility is quite small (< 5%)
   - Innovations to the real factor, to country growth, and growth in other countries explain a very small share of volatility variance
   - The financial common factor and the idiosyncratic components of volatility explain a large share of volatility variance

3. Idiosyncratic components of volatility is small (or well diversified)
   - Only the common components of volatility explain a significant share of growth variance and can have deep impact on country growth when it hits
   - Idiosyncratic components of volatility explain very little growth variance
(Large) Related literature

- Volatility does respond to the business and financial cycles [Ludvigson, Ma, and Ng (2015), Carriero, Clark, Marcellino (2016)]
- First vs Second moments factors[ e.g., Gorodnichenko and Ng (2017)]: we identify a pure second-moment factor and quantify its importance for and dynamic impact on growth
- International dimension [Carriere-Swallow and Cespedes (2013), Baker and Bloom (2013), Hirata, Kose, Otrok, and Terrones (2012)]
  - Multi-country framework, as opposed to a set of countries considered in isolation.
  - We do not assume volatility is exogenous
Outline

1. Factor model for volatility and growth
2. Data & Empirical Results
3. Conclusions
A static factor model

- For each country $i$ assume that both volatility and GDP growth load on a common factor ($f_t$) as follows

$$v_{it} = \lambda_i f_t + u_{it}$$
$$\Delta y_{it} = \gamma_i f_t + \epsilon_{it}$$

- Growth equation easily derived from stochastic RBC/Solow growth model
Consumption-based CAPM interpretation of volatility equation

\[ r \approx \log(1 + r) = \delta + \rho f - \frac{\rho^2 \sigma_f^2}{2}. \tag{1} \]

\[ (E_t r_{i,t+1} - r) = \rho \text{Cov} [\Delta y_{w,t+1}, r_{i,t+1}] = \rho \text{Corr} [\Delta y_{w,t+1}, r_{i,t+1}] \sigma_f \sigma_{ir} \tag{2} \]

\[ \sigma_i = \left| \frac{(E_t r_{i,t+1} - r)}{\rho \sigma_f} \right| = \left| \frac{E_t r_{i,t+1} - \delta - \rho f + \frac{\rho^2 \sigma_f^2}{2}}{\rho \sigma_f} \right|. \tag{3} \]
A static factor model (Cont.)

For each country $i$ assume that both volatility and GDP growth load on a common factor ($f_t$) as follows

$$v_{it} = \lambda_i f_t + u_{it}$$
$$\Delta y_{it} = \gamma_i f_t + \varepsilon_{it}$$

If we consider only one country in isolation, the model is not identified, even assuming $\varepsilon_{it}$ and $u_{it}$ are uncorrelated

- Four unknown parameters $\lambda_i, \gamma_i, \sigma^2_{u,i}, \sigma^2_{\varepsilon,i}$ (normalizing $\sigma_f^2 = 1$)
- But covariance matrix of $v_{it}$ and $\Delta y_{it}$ provides only three independent restrictions
- Identification usually achieved with an exclusion restriction

If we take a multi-country approach, we can identify $f_t$ from restrictions implicit in the pattern of correlation the two shocks across countries, even leaving the correlation between $\varepsilon_{it}$ and $u_{it}$ unrestricted
Some notation & Identifying assumptions

Notation

- Define global volatility \((\bar{\omega},t)\) and GDP growth \((\Delta \bar{y},t)\) as weighted (\(w_i\)) averages over a large number of countries

\[
\bar{\omega},t = \sum_{i=1}^{N} w_i \omega_{it}, \quad \Delta \bar{y},t = \sum_{i=1}^{N} w_i \Delta y_{it}
\]

Identifying assumptions

1. Loadings: factor \(f_t\) is strong (or pervasive) for both volatility and activity
2. Weights: granularity, i.e. weights (\(w_i\)) are not dominated by a few cross-section units (can be partially relaxed)
3. Cross-sectional correlations: volatility innovations are strongly correlated across countries (pairwise correlation does not tend to zero), while GDP growth innovations are weakly correlated across countries (pairwise correlation tends to zero)
Identification of the real factor \((f_t)\) by aggregation

**Proposition 1** Under these assumptions, for \(N\) large enough, \(f_t\) can be identified by \(\bar{y}_{\omega,t}\) up to a constant

**Proof** Consider the weighted average of the country systems

\[
\begin{align*}
\bar{v}_{\omega,t} &= \lambda f_t + \bar{u}_{\omega,t}, \\
\Delta\bar{y}_{\omega,t} &= \gamma f_t + \bar{\varepsilon}_{\omega,t},
\end{align*}
\]

where \(\bar{u}_{\omega,t} = w'u_t\) and \(\bar{\varepsilon}_{\omega,t} = w'\varepsilon_t\). For \(N\) sufficiently large, we can show that have

\[
f_t = \frac{\Delta\bar{y}_{\omega,t}}{\gamma} + \frac{\bar{\varepsilon}_{\omega,t}}{\gamma}.
\]

And thus the last term becomes negligible as the sample size increase.
Proof (cont.)

This is because:

$$\text{var} (\varepsilon_{\omega,t}) = w'\Sigma\varepsilon w \leq (w'w) \rho_{\text{max}} (\Sigma\varepsilon).$$ \(4\)

But under the assumptions made:

$$\text{var} (\varepsilon_{\omega,t}) = O (w'w) = O \left( N^{-1} \right),$$ \(5\)

and hence:

$$\varepsilon_{\omega,t} = O_p \left( N^{-1/2} \right).$$ \(6\)

QED
Remarks

Remark

(Interpretation of $f_t$) Because $f_t$ is the same as world growth rescaled, we label it a “real” or “macroeconomic” factor.

Remark

(Estimation of $f_t$) As $f_t$ is pervasive or strong, we can estimate it with either principal component techniques of cross-section averages of $\Delta y_{it}$ (for $i = 1, 2, \ldots, N$).

Remark

(Identification of $f_t$) Nonetheless, $f_t$ cannot be identified from the cross-section average or the principal component of the panel of volatilities series $\nu_{it}$. 
The financial factor \((g_t)\)

- By assumption, \(u_{it}\) must share at least one more factor than \(\varepsilon_{it}\).
- Assume for simplicity that, conditional on \(f_t\), \(u_{it}\) share only one additional strong factor.

\[u_{it} = \theta_i \rho_t + \eta_{it}\]

- The model becomes:

\[
\begin{align*}
  v_{it} &= \lambda_i f_t + \theta_i g_t + \eta_{it} \\
  \Delta y_{it} &= \gamma_i f_t + \varepsilon_{it}
\end{align*}
\]

- Different pattern of correlation across countries of volatility and growth innovations implicitly provides a restriction on the factor loadings:

\[
\begin{bmatrix}
  v_{it} \\
  \Delta y_{it}
\end{bmatrix} =
\begin{bmatrix}
  \lambda_i & \theta_i \\
  \gamma_i & 0
\end{bmatrix}
\begin{bmatrix}
  f_t \\
  g_t
\end{bmatrix} +
\begin{bmatrix}
  \eta_{it} \\
  \varepsilon_{it}
\end{bmatrix}
\]
Identification of the financial factor \( (g_t) \) by aggregation

- If one factor is enough, volatility innovations \( \eta_{it} \) are cross-sectionally weakly correlated
  - That is, similarly to \( \varepsilon_{it} \), we have that \( \bar{\eta}_{\omega,t} = O_p \left( N^{-1/2} \right) \)

- **Proposition 2** Conditional on \( f_t \), for \( N \) large enough, \( g_t \) is given by

\[
g_t = \frac{\bar{v}_{\omega,t}}{\theta} - \frac{\lambda}{\theta \gamma} \Delta \bar{y}_{\omega,t} + \frac{\bar{\eta}_{\omega,t}}{\theta} O_p \left( N^{-1/2} \right)
\]

- Factors \( f_t \) and \( g_t \) can then be estimated up to a scalar and a rotation of the coefficients in the expression for \( g_t \)
Remarks

Remark

We label $g_t$ “financial” factor to stress the idea that $g_t$ must be capturing time variation in either systematic risk or time and risk preferences not affecting the growth series contemporaneously.

Remark

(Relation to structural models) We are agnostic: some models are consistent others are not with the triangular factor structure identified. Approach similar to APT applied to second moments.
Additional results

Proposition

Denote with $\tilde{f}_t$ and $\tilde{g}_t$ a consistent, orthogonalized estimate of estimate of $f_t$ and $g_t$, respectively. We can obtain $\tilde{f}_t$ by rescaling $\Delta \bar{y}_{\omega,t}$ so that its equal to 1, while $\tilde{g}_t$ can be obtained for $t = 1, 2, ..., T$ as the standardized residual of a regression of $\bar{v}_{\omega,t}$ on $\Delta \bar{y}_{\omega,t}$.

Remark

(Equivalent models) The derived empirical model is equivalent to a factor augmented VAR (FAVAR) model in which $\tilde{f}_t$ and $\tilde{g}_t$ have been orthogonalized with a Cholesky decomposition of the variance-covariance matrix of the global variables $\bar{v}_{\omega,t}$ and $\Delta \bar{y}_{\omega,t}$, ordering world GDP growth first, but is not consistent with a FAVAR model in which $\tilde{f}_t$ and $\tilde{g}_t$ have been orthogonalized with a Cholesky decomposition and the opposite ordering of the global variables.
Dynamic model (Factor-augmented large VAR)

- Theoretical results carry through a fully heterogeneous dynamic version of the model
  - Country interactions and spillovers through unrestricted variance-covariance matrix and the factors

- Country-specific model with orthonormal factors

\[
\begin{bmatrix}
v_{it} \\
\Delta y_{it}
\end{bmatrix} = \begin{bmatrix}
\phi_{i,11} & \phi_{i,12} \\
\phi_{i,21} & \phi_{i,22}
\end{bmatrix} \begin{bmatrix}
v_{i,t-1} \\
\Delta y_{i,t-1}
\end{bmatrix} + \begin{bmatrix}
\beta_{i,11} & \beta_{i,12} \\
\beta_{i,21} & 0
\end{bmatrix} \begin{bmatrix}
\tilde{f}_t \\
\tilde{g}_t
\end{bmatrix} + \begin{bmatrix}
\psi_{vi,11} & \psi_{vi,12} \\
\psi_{\Delta y_{i,11}} & \psi_{\Delta y_{i,12}}
\end{bmatrix} \begin{bmatrix}
\tilde{v}_{\omega,t-1} \\
\Delta y_{\omega,t-1}
\end{bmatrix} + \begin{bmatrix}
\eta_{it} \\
\varepsilon_{it}
\end{bmatrix}
\]

- Country-specific models can be combined in a large model of the global economy
Volatility measurement

- We compute the realized volatility for country $i$ in quarter $t$ as:

$$\sigma_{it} = \sqrt{D_t^{-1} \sum_{\tau=1}^{D_t} (r_{it}(\tau) - \bar{r}_{it})^2}$$

(7)

where $r_{it}(\tau) = \Delta \ln P_{it}(\tau)$, and $\bar{r}_{it} = D_t^{-1} \sum_{\tau=1}^{D_t} r_{it}(\tau)$ is the average daily price changes in the quarter $t$, and $D_t$ is the number of trading days in quarter $t$.

- We work with log of $\sigma_{it}$
Data & Empirical Results
Data & Empirical Results

Data

- Balanced panel data for 32 countries from 1993:Q1 to 2011:Q2
- Results robust to
  - Using a slightly longer sample with fewer countries (from 1988:Q1, excluding China, Indonesia, Brazil, and Peru)
  - Using a significantly longer sample in an unbalanced panel data set of the same 32 countries (some empirical results gets hard to compute)

Empirical results

- Factors estimates
- Evidence in support of identifying assumptions
- Within-country identification
- IRFs and FEVDs to factors and country-specific shocks
Estimated orthogonal factors ($\tilde{f}$ and $\tilde{g}$)

(A) Real factor $\tilde{f}_t$

(B) Financial factor $\tilde{g}_t$
Interpreting the $\tilde{g}$ factor

- Correlation between $\tilde{g}$ and Excess Bond Premium (Gilchrist and Zakrajsek, 2012) is 0.34
Interpreting the $\tilde{g}$ factor

- Correlation between $\tilde{g}$ and Ludvigson, Ma and Ng (2017)'s financial uncertainty measure is 0.4
Is the identifying assumption on cross-sectional dependence consistent with the data?

- Estimate country models with $\tilde{f}_t$ only:

\[
\begin{align*}
    v_{it} &= \beta_{i,11}\tilde{f}_t + lags + u_{it} \\
    \Delta y_{it} &= \beta_{i,21}\tilde{f}_t + lags + \epsilon_{it}
\end{align*}
\]

Note. Average pairwise correlation of the $u_{it}$ (yellow bars) and the $\epsilon_{it}$ (blue bars).
Is one additional strong factor sufficient to model country volatilities?

- Estimate country models with $\tilde{f}_t$ and $\tilde{g}_t$:

\[
\begin{align*}
\nu_{it} &= \beta_{i,11}\tilde{f}_t + \beta_{i,12}\tilde{g}_t + \text{lags} + \eta_{it} \\
\Delta y_{it} &= \beta_{i,21}\tilde{f}_t + \text{lags} + \epsilon_{it}
\end{align*}
\]

NOTE. Average pairwise correlation of the $\eta_{it}$ (yellow bars) and the $\epsilon_{it}$ (blue bars).
Tests of cross-sectional dependence don’t reject identifying assumptions

- CD and Exponent of cross-sectional dependence tests [Pesaran, 2015 and Bailey et al, 2016]

- Results in accordance with assumptions of
  - Weak/strong cross-sectional dependence of $\varepsilon_{it}/u_{it}$, respectively
  - Weak cross-sectional dependence of both $\varepsilon_{it}$ and $\eta_{it}$

<table>
<thead>
<tr>
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<th>$\alpha$</th>
<th>$\alpha_{0.95}$</th>
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<td>$\Delta y_{it}$</td>
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<td>$u_{it}$</td>
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<td>$\varepsilon_{it}$</td>
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<td>0.50</td>
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<tr>
<td>$\varepsilon_{it}$</td>
<td>5.40</td>
<td>0.73</td>
<td>0.79</td>
<td>0.85</td>
</tr>
</tbody>
</table>
Within-country correlations between volatility and growth

- Estimate country models conditional on $\tilde{f}_t$ and $\tilde{g}_t$ factor

\[
v_{it} = \beta_{i,11}\tilde{f}_t + \beta_{i,12}\tilde{g}_t + \text{lags} + \eta_{it}
\]
\[
\Delta y_{it} = \beta_{i,21}\tilde{f}_t + \text{lags} + \varepsilon_{it}
\]

- Remarks
  - Important result: Country VCM approximately diagonal
  - Result robust to conditioning on fundamental factor $\tilde{f}_t$ only

Uncertainty and Economic Activity — Data & Empirical Results
Within-country correlations between volatility and growth conditioning on $\tilde{f}$ only

- Estimate country models conditional on $\tilde{f}_t$ factor only

$$v_{it} = \beta_{i,1} \tilde{f}_t + \text{lags} + u_{it}$$

$$\Delta y_{it} = \beta_{i,2} \tilde{f}_t + \text{lags} + \epsilon_{it}$$
### Statistically significant pairwise correlations

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<td>BEL VOL NLD VOL 0.60</td>
<td>BEL VOL CHE VOL 0.51</td>
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<td>BEL GDP CHN GDP</td>
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<td>BRA VOL MEX VOL 0.56</td>
<td>CAN VOL NOR VOL 0.40</td>
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<td></td>
<td>FRA VOL NLD VOL 0.63</td>
<td></td>
<td>FRA VOL NLD VOL 0.63</td>
</tr>
</tbody>
</table>

### Notes
- **Between-county correlations**
- **Within-county correlations**
Real factor (purple areas) and country specific growth innovations (green and light blue areas) explain less than $< 5\%$ of country volatilities.

Financial factor (dark blue areas) explains a significant share of growth variance (about $\sim 10\%$), but country-specific volatility shocks (orange and yellow areas) diversified away.
Average FEVD: Block Diagonal covariance matrix

Volatility ($v_i$), average

Real GDP ($\Delta y_i$), average

Uncertainty and Economic Activity — Data & Empirical Results
Average Generalized FEVD: Threshold covariance matrix

Volatility ($v_i$), average

Real GDP ($\Delta y_i$), average
Shocks to the factors have expected effects

- Countercyclical volatility response to $\tilde{f}_t$ shock and recession induced by $\tilde{g}_t$ shock
- Size of volatility responses to $\tilde{g}_t$ shock larger than responses to $\tilde{f}_t$ shock, but comparable growth responses
Conclusions

- Paper takes a multi-county approach to model the relation between volatility and growth without imposing restrictions on the direction of causation.

- Paper exploits the different cross-country correlation structure of volatility and growth innovations to identify a "real" and a "financial" factor.

Main take-aways

- Much of the unconditional correlation between volatility and growth is driven by the real factor.
- Endogenous component of volatility small.
- Country volatility shocks wash away and only shocks to financial factor explains some share of growth variance with impact comparable to real factor when they realize.
Appendix
Assumption 1: Loadings

- The factor loadings, $\lambda_i$ and $\gamma_i$, are distributed independently across $i$ and the common factors $f_t$, for all $i$ and $t$, with non-zero means $\lambda$ and $\gamma$.

- Assume that

\[ N^{-1} \sum_{i=1}^{N} \lambda_i^2 = O(1) \quad \text{and} \quad N^{-1} \sum_{i=1}^{N} \gamma_i^2 = O(1), \]

\[ \lambda = \sum_{i=1}^{N} \hat{w}_i \lambda_i \neq 0 \quad \text{and} \quad \gamma = \sum_{i=1}^{N} w_i \gamma_i \neq 0, \]

for all $N$, and as $N \to \infty$.

- **Interpretation** Factor $f_t$ is strong (or pervasive) for both volatility and growth.
  
  - Standard in the factor literature (see Bai and Ng 2002).
  
  - Factor can be estimated using principal components or the cross-section averages.
Assumption 2: Weights

- Let $\mathbf{w} = (w_1, w_2, ..., w_N)'$ and $\hat{\mathbf{w}} = (\hat{w}_1, \hat{w}_2, ..., \hat{w}_N)'$ be $N \times 1$ vectors of non-stochastic weights with $\sum_{i=1}^{N} w_i = 1$ and $\sum_{i=1}^{N} \hat{w}_i = 1$

- Weights $\mathbf{w}$ and $\hat{\mathbf{w}}$ are granular, in the sense that, for instance:

  \[ \frac{\| \mathbf{w} \|}{\| \mathbf{w} \|} = O(N^{-1}), \quad \frac{\mathbf{w}_i}{\| \mathbf{w} \|} = O(N^{-\frac{1}{2}}), \quad \forall i, \]

- **Interpretation** Granularity rules our dominance of one or more cross-section units
  - Could be problematic for realized volatility
  - We can relax this assumption to derive $f$, leaving volatility weights $\hat{\mathbf{w}}$ unrestricted, but cannot make certain statements about the financial factor in the US case.
(Key) Assumption 3: Cross-section correlations

Let the variance-covariance matrices of the \( N \times 1 \) error vectors
\( \varepsilon_t = (\varepsilon_{1t}, \varepsilon_{2t}, ..., \varepsilon_{Nt})' \) and \( u_t = (u_{1t}, u_{2t}, ..., u_{Nt})' \) be \( \Sigma_\varepsilon = \text{Var}(\varepsilon_t) \) and \( \Sigma_u = \text{Var}(u_t), \) respectively.

Assume that

\[
\varrho_{\text{max}}(\Sigma_u) = O(N), \\
\varrho_{\text{max}}(\Sigma_\varepsilon) = O(1).
\]

where \( \varrho_{\text{max}}(A) \) denotes the largest eigenvalue of matrix \( A \).

**Interpretation** Weak cross-country correlation means that, as \( N \) becomes large, the average pairwise correlations of growth innovations tends to zero, since the largest eigenvalue is bounded in \( N \).
Estimating observable and orthogonal factors

- **Issue** Factors $f_t$ and $g_t$ are unobservable, and even if known, would be correlated with each other.

- For ease of interpretation it is standard to work with the orthogonalized version of the factors:
  - This task is simplified due to the triangular way the factors affect the global variables, $\Delta \bar{y}_{\omega,t}$ and $\bar{v}_{\omega,t}$.

- Proceed recursively:
  - Factor $f_t$ can be identified up to a constant:
    \[
    f_t = \frac{\Delta \bar{y}_{\omega,t}}{\gamma} \implies \tilde{f}_t = \Delta \bar{y}_{\omega,t}
    \]
  - Factor $g_t$ can then be approximated by the residuals of a regression of world volatility $\bar{v}_{\omega,t}$ on world growth:
    \[
    g_t = \frac{\bar{v}_{\omega,t}}{\theta} - \frac{\lambda}{\theta \gamma} \Delta \bar{y}_{\omega,t} \implies \bar{v}_{\omega,t} = \hat{\beta} \Delta \bar{y}_{\omega,t} + \tilde{g}_t
    \]
Comparison between VIX and US realized volatility

Uncertainty and Economic Activity — Appendix: Additional charts
Volatility shocks in the United States have similar recessionary impacts

- Shocks to $\tilde{\eta}_t$ larger impact than shocks to $\eta_{US}$

- However, we need to be cautious with interpretation of this split
  - US might not be granular in global financial markets
Growth shocks in the United States

- Both shocks to $\tilde{f}_t$ and $\varepsilon_{US}$ lead to a fall in volatility, but global component has a larger effect.
- Shock to $\tilde{f}_t$ has smaller impact of country specific one.
Variance decomposition in the United States is similar.

![Graph showing Volatility ($v_i$), USA and Real GDP ($\Delta y_i$), USA](image-url)
Country-specific response to the factors

(A) \( v_i \) to a shock to \( \tilde{f} \)

(B) \( \Delta y_i \) to a shock to \( \tilde{f} \)

(C) \( v_i \) to a shock to \( \tilde{g} \)

(D) \( \Delta y_i \) to a shock to \( \tilde{g} \)