Demographics and Real Interest Rates Across Countries and Over Time

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AEA Session on The Secular Decline in Real Interest Rates

January 3, 2020

*The views expressed in this paper do not necessarily reflect the position of the Central Bank of Brazil.
Three Observations on Global Real Interest Rates

- At very low levels (negative in many countries): Median in 2016 = -0.5%
- Declining for almost three decades: Median above 6% in 1989
- Decreasing dispersion across countries: IQR from 4.27 in 1991 to 0.54 in 2016
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Are Low Real Rates Here to Stay?

Nominal Yields in % as of 29 April 2019

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- Low real rates present challenges and possible opportunities for policy
  - To the extent that they reflect low “natural” rates, constraint on monetary policy
  - Implications for fiscal policy and public finance
Why Are Real Interest Rates So Low?

- Various “structural” explanations

  - Demographic trends (Carvalho, Ferrero and Nechio, 2016)
    - Decline in growth rate of labor force and increase in life expectancy
    - Determine consumption-savings patterns and equilibrium real interest rate
  
  - “Raising the bar”
    - If demographics indeed important factor, should expect patterns across countries and over time to accord with demographics
  
  Assessment complicated by:
    - Other determinants of real rates
    - With (some) capital mobility, a country’s real rate should depend on own and global demographics
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Our paper

- Two contributions
  - Structural model: Relevant channels in open-economy life-cycle model with imperfect capital mobility
  - Empirics: Panel error-correction models, with guidance from structural model
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- Model
  - Demographics can generate significant decline of real interest rates
  - The more financially integrated a country is and the smaller its size:
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    - The less its own real rate determinants matter
  - Convergence of real rates as financial integration increases
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  - Financial integration correlated with decline of cross-country dispersion
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- Implication: Low real rates may be here to stay
Related Literature

- Quantitative models of demographics and real interest rates

- Empirical analysis of real interest rates dynamics

- Other determinants of low real interest rates
  - Caballero, Farhi and Gourinchas (2008), Lo and Rogoff (2015), Caballero and Farhi (2017), Del Negro, Giannone, Giannoni and Tambalotti (2017, 2018), ...
Outline

- Introduction

**Open economy life-cycle model**
- Analytical framework
- Demographics and financial integration
- Other determinants of real interest rates

**Empirical analysis**
- Data description
- Panel ECM regressions
- Dispersion and projections
Model Overview

- Two countries
  - Large and old: Global economy
  - Small and young economy

- In each country, continuum of workers and retirees
  - Face idiosyncratic risk of retirement (for workers) and death (for retirees)
  - Consume one good and can save via three instruments
    - Capital, government bonds, claims on foreign assets

- Standard supply side (labor-augmenting productivity)

- Government funds spending and transfers with taxes and debt

- Friction: Portfolio holding costs
Demographics

- Simple life-cycle structure (Gertler, 1999)
  - Each period, $1 + n_{it} - \omega$ new workers are born in country $i$
  - Remain in labor force with probability $\omega$, retire otherwise
  - Once retired, survive with probability $\gamma_{it}$
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- Growth rate of labor force

\[
N_{it}^w = (1 + n_{it}) N_{it-1}^w
\]
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- Growth rate of labor force

$$N_{it}^w = (1 + n_{it}) N_{it-1}^w$$

- Dependency ratio

$$\psi_{it} \equiv \frac{N_{it}^r}{N_{it}^w} = \frac{(1 - \omega) + \gamma_{it} \psi_{it-1}}{1 + n_{it}}$$
Retirees’ Problem

- Retirees turn their wealth to mutual fund at beginning of each period
  - Mutual fund insures survivors against probability of death (Yaari, 1965)
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\[
V_{it}^r = \max_{C_{it}^r, A_{iit}^r, A_{ijt}^r} \left[ (C_{it}^r)^{\frac{\sigma-1}{\sigma}} + \beta_i \gamma_{it+1} (V_{it+1}^r)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}
\]

subject to

\[
C_{it}^r + \left[ 1 + \frac{\Lambda_{ij}}{2} (\eta_{ijt} - \bar{\eta}_{ij})^2 \right] A_{it}^r = \frac{1}{\gamma_{it}} \left( R_{it-1} A_{iit-1}^r + R_{jt-1} A_{ijt-1}^r \right) + E_{it}^r
\]

with \( j \neq i \) and where

\[
\eta_{ijt} = \frac{A_{ijt}^r}{A_{iit}^r + A_{ijt}^r} = \frac{A_{ijt}^r}{A_{it}^r}
\]
Workers’ Problem

- Workers start their life with no assets

- No insurance available against probability of retirement (permanent disability)
  - Assume workers are risk-neutral with respect to labor income risk
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\[
V_{it}^w = \max_{C_{it}, A_{itt}^w, A_{ijt}^w} \left\{ \left( C_{it}^w \right)^{\frac{\sigma-1}{\sigma}} + \beta_i [\omega V_{it+1}^w + (1 - \omega) V_{it+1}^r]^{\frac{\sigma-1}{\sigma}} \right\}^{\frac{\sigma}{\sigma-1}}
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subject to

\[
C_{it}^w + \left[ 1 + \frac{\Lambda_{ij}}{2} (\eta_{ij}^w - \bar{\eta}_{ij})^2 \right] A_{it}^w = R_{it-1} A_{itt-1}^w + R_{jt-1} A_{ijt-1}^w + W_{it}^w - T_{it}^w
\]
Portfolio Shares

- Cross-country return differentials only depend on portfolio cost parameters
  - Retirees and workers optimally choose same portfolio shares

\[ \eta_{ijt}^r = \eta_{ijt}^w = \eta_{ijt} \]
**Portfolio Shares**

- Cross-country return differentials only depend on portfolio cost parameters
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    \[ \eta_{ijt}^r = \eta_{ijt}^w = \eta_{ijt} \]

- Let share of asset holdings accruing to retirees be
  \[ \lambda_{it} = \frac{A_{ijt}^r}{A_{ijt}^w + A_{ijt}^r} \]
  - Sufficient statistic to summarize distribution of wealth over life cycle
  - Because of same \( \eta_{ijt} \) between workers and retirees
Aggregate Consumption

- Marginal propensity to consume independent of individual characteristics
  - Can aggregate within each group (retirees and workers)
    - Retirees’ consumption
      \[ C_{it}^r = \xi_{it}^r \left( R_{it-1} A_{iit-1}^r + R_{jt-1} A_{ijt-1}^w + S_{it} \right) \]
    - Workers’ consumption
      \[ C_{it}^w = \xi_{it}^w \left( R_{it-1} A_{iit-1}^w + R_{jt-1} A_{ijt-1}^w + H_{it} + Z_{it} \right) \]

where

- \( S_{it} = \) PDV of pensions for retirees
- \( Z_{it} = \) PDV of pensions for workers
- \( H_{it} = \) PDV of wages net of taxes
Aggregate Consumption

- Marginal propensity to consume independent of individual characteristics
  - Can aggregate within each group (retirees and workers)
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      \[ C_{it}^r = \xi_{it}^r \left( R_{it} - 1 A_{iit}^r - 1 + R_{jt} - 1 A_{ijt}^r - 1 + S_{it} \right) \]
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      \[ C_{it}^w = \xi_{it}^w \left( R_{it} - 1 A_{iit}^w - 1 + R_{jt} - 1 A_{ijt}^w - 1 + H_{it} + Z_{it} \right) \]

- Aggregate consumption

\[ C_{it} = \xi_{it}^w \left\{ \left[ 1 - \left( 1 - \frac{\xi_{it}^r}{\xi_{it}^w} \right) \right] \lambda_{it - 1} \right\} \left( R_{it} - 1 A_{iit} - 1 + R_{jt} - 1 A_{ijt} - 1 \right) + \frac{\xi_{it}^r}{\xi_{it}^w} S_{it} + H_{it} + Z_{it} \]

where \( C_{it} \equiv C_{it}^w + C_{it}^r \)
Distribution of Wealth

- Retirees’ assets

\[
\left[1 + \frac{\Lambda_{ij}}{2} (\eta_{ijt} - \bar{\eta}_{ij})^2\right] A_r^{it} = R_{it-1} A_{r iit-1} + R_{jt-1} A_{r ijt-1} + E_{it} - C_r^{it} + (1 - \omega) \left(R_{it-1} A_w^{iit-1} + R_{jt-1} A_w^{ijt-1} + W_{it} N_{itw} - T_{it} - C_w^{it}\right)
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\]

- Workers’ assets

\[
\left[1 + \frac{\Lambda_{ij}}{2} (\eta_{ijt} - \bar{\eta}_{ij})^2\right] A_{it}^w = \omega \left(R_{it-1} A_{iit-1}^w + R_{jt-1} A_{ijt-1}^w + W_{it} N_{it}^w - T_{it} - C_{it}^w\right)
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  \]

- Evolution of wealth distribution
  \[
  [\lambda_{it} - (1 - \omega)] \left\{ \left[ 1 + \frac{\Lambda_{ij}}{2} (\eta_{ijt} - \bar{\eta}_{ij})^2 \right] A_{it} \right\} = \omega \left[ (1 - \xi_{it}^r) \lambda_{it-1} (R_{it-1} A_{iit-1} + R_{jt-1} A_{ijt-1}) + E_{it} - \xi_{it}^r S_{it} \right]
  \]
Production

- Perfectly competitive firms produce homogenous consumption good

- Labor-augmenting Cobb-Douglas technology

\[ Y_{it} = (X_{it} N_{it}^w)^\alpha K_{it-1}^{1-\alpha}, \]

where

\[ X_{it} = (1 + x_{it}) X_{it-1} \]

- Law of motion of capital

\[ K_{it} = (1 - \delta) K_{it-1} + I_{it} \]
Fiscal Policy

- Flow budget constraint

\[ G_{it} + E_{it} + R_{it-1}B_{it-1} = B_{it} + T_{it} + \frac{\Lambda_{ji}}{2} (\eta_{jit} - \bar{\eta}_{ji})^2 A_{jt} \]
Fiscal Policy

- Flow budget constraint

$$G_{it} + E_{it} + R_{it-1}B_{it-1} = B_{it} + T_{it} + \frac{\Lambda_{ji}}{2}(\eta_{jit} - \bar{\eta}_{ji})^2A_{jt}$$

- Assume spending, debt, and pensions are exogenous fraction of GDP

$$G_{it} = g_{it}Y_{it} \quad E_{it} = e_{it}Y_{it} \quad B_{it} = b_{it}Y_{it}$$

- Government budget constraint pins down taxes
Balance of Payments

- Domestic assets

\[ A_{iit} = K_{it} + B_{it} - A_{jit} \]
Balance of Payments

- Domestic assets
  \[ A_{iit} = K_{it} + B_{it} - A_{jit} \]

- Net foreign assets
  \[ F_{it} \equiv A_{ijt} - A_{jit} \]
Balance of Payments

- Domestic assets
  \[ A_{iit} = K_{it} + B_{it} - A_{jit} \]

- Net foreign assets
  \[ F_{it} \equiv A_{ijt} - A_{jit} \]

- Evolution of net foreign asset position
  \[
  F_{it} = F_{it-1} + (R_{jt-1} - 1)A_{ijt-1} - (R_{it-1} - 1)A_{jit-1} \\
  + \frac{\Lambda_{ji}}{2}(\eta_{jit} - \bar{\eta}_{ji})^2A_{jt} - \frac{\Lambda_{ij}}{2}(\eta_{ijt} - \bar{\eta}_{ij})^2A_{it} + NX_{it}
  \]  
  (1)

where

\[ NX_{it} = Y_{it} - (C_{it} + I_{it} + G_{it}) \]

with

\[ F_{it} + F_{jt} = 0 \]
Calibration

- Two (fictitious) countries:
  - “Old” (global economy): Relatively low population growth rate and high dependency ratio
  - “Young”: Relatively high population growth rate and low dependency ratio

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<td>$\alpha$</td>
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<td>Elasticity of intertemporal substitution</td>
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<td>Government spending / GDP = 20%</td>
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<td>$b$</td>
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- Period is one year, individuals born workers at 20

### Parameters and Fixed Exogenous Variables

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Initial Steady State

- Associate two countries with 25\textsuperscript{th} and 75\textsuperscript{th} percentiles of empirical distributions of labor force growth dependency ratios and real rates

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<th>Young Country</th>
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<tr>
<td>(n)</td>
<td>0.59%</td>
<td>1.13%</td>
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<td>(\psi)</td>
<td>24%</td>
<td>21%</td>
</tr>
<tr>
<td>(R)</td>
<td>3.1%</td>
<td>6.2%</td>
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- Fit smooth \(n_{it}, n_{jt}\) paths directly to data
- Pick smooth paths for \(\gamma_{it}, \gamma_{jt}\) to fit paths of dependency ratio in the data

\[
\psi_i = \frac{1 - \omega}{1 + n_i - \gamma_i}
\]

- Calibrate \(\Lambda_{ij} = \Lambda_{ji} = \Lambda\) to yield an average current account deficit for small economy that matches time-average of 25\textsuperscript{th} percentile of current account-to-GDP ratios in the data
- Calibrate \(\beta_i, \beta_j\) to match observed real rates in initial steady state (1990)
Experiment

- What happens in response to changes in demographic variables?
  - Transition driven by changes in population growth and life expectancy
Demographic Trends and Real Interest Rates

- Real rates and financial integration

![Interest Rate (%)]

- Old
- Young
Demographic Trends and Real Interest Rates

- Different degrees of financial integration

![Graphs showing Autarky and High Integration scenarios for demographic and real interest rates](image-url)
Demographic Trends and Real Interest Rates

- Role of country size
Demographic Trends and Real Interest Rates

- Increasing financial integration over time
Demographic Trends and Real Interest Rates

- Sensitivity of small economy’s real rate to global labor force growth
Demographic Trends and Real Interest Rates

- Sensitivity of small economy’s real rate to global life expectancy
Outline

- Introduction

- Open economy life-cycle model
  - Analytical framework
  - Demographics and financial integration

- Empirical analysis
  - Data description
  - Panel ECM regressions
  - Dispersion and projections
Approach

- Empirical analysis informed by lessons from structural model

Demographic variables:
- Labor force growth rate
- Life expectancy

Controls:
- TFP growth
- Fiscal variables (debt, government spending, pensions) % of GDP
- Retirement age
- Others

Complementary role of model and empirics

⇒ Potential missing variables
- Safe assets, private debt, relative price of investment, convenience yield

Introduce as additional controls (although with some data limitations)
Approach

- Empirical analysis informed by lessons from structural model

- Regress real interest rates on demographic variables and controls, interacted with measures of openness and/or size as suggested by model
  - Demographic variables: labor force growth rate and life expectancy
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  - Introduce as additional controls (although with some data limitations)
Data

- Annual frequency, various sources
  - OECD, IFS, WB, UN WPP, PWT, Ameco, National agencies
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- Real interest rate = Nominal interest rate - Expected inflation
  - Nominal interest rate: Short-term yields (O/N to 3m CB/market rates)
  - Inflation: CPI
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  - Life expectancy at 20, converted to an “interest rate equivalent” according to model
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- Controls:
  - Direct measures for TFP growth and fiscal variables
  - Debt/GDP: From WB or Ameco
  - Financial integration: (Foreign Assets + Foreign Liabilities) / GDP
  - Other controls following existing literature
Ex-Ante Real Interest Rates

- Construct following Hamilton et al. (2016)
  - Using rolling windows of 20 years, estimate
    \[ \pi_{it} = \alpha_i + \beta_i \pi_{it-1} + \varepsilon_{it} \]
  - Ex-ante real rate is \[ r_{it} = i_{it} - E_t \pi_{it+1} \], where
    \[ E_t \pi_{it+1} = \hat{\alpha}_i + \hat{\beta}_i \pi_{it} \]
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  - Ex-ante real rate is \( r_{it} = i_{it} - E_t \pi_{it+1} \), where
    \[ E_t \pi_{it+1} = \hat{\alpha}_i + \hat{\beta}_i \pi_{it} \]

- Alternative: Use inflation expectations from Consensus Forecast
  - Limitation: Much shorter sample (early-mid 1990s)
Ex-Ante Real Interest Rates

- Construct following Hamilton et al. (2016)
  - Using rolling windows of 20 years, estimate
    \[ \pi_{it} = \alpha_i + \beta_i \pi_{it-1} + \epsilon_{it} \]
  - Ex-ante real rate is
    \[ r_{it} = i_{it} - \mathbb{E}_t \pi_{it+1}, \text{ where} \]
    \[ \mathbb{E}_t \pi_{it+1} = \hat{\alpha}_i + \hat{\beta}_i \pi_{it} \]

- Alternative: Use inflation expectations from Consensus Forecast
  - Limitation: Much shorter sample (early-mid 1990s)

- Unbalanced panel of 20 OECD countries between 1980 and 2016
  - Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Ireland, Italy, Japan, Luxembourg, Netherlands, New Zealand, Norway, Spain, Sweden, Switzerland, United Kingdom, and United States
  - Exclude countries with inflation above 25% between 1970 and 2016
Ex-Ante vs. Ex-Post Real Rates

Real Interest Rate

Percentage points

Median Ex-Post Interquartile Range (Ex-Ante)

-4 -2 0 2 4 6 8

Carvalho, Ferrero, Mazin & Nechio
Demographics and RIRs Across Countries and Time
January 3, 2020
Empirical Specification

- Panel error-correction model

\[ \Delta r_{m,t} = \alpha_m + \gamma r_{m,t-1} + \theta \Theta_{m,t-1} r^*_{m,t-1} + \sum_j \psi_j (1 - \Theta_{m,t-1}) Dmg_{m,j,t-1} \]
\[ + \sum_k \Psi_k (1 - \Theta_{m,t-1}) X_{m,k,t-1} \]
\[ + \lambda \Delta (\Theta_{m,t} r^*_m, t) \]
\[ + \sum_j \phi_j \Delta ((1 - \Theta_{m,t}) Dmg_{m,j,t}) + \sum_k \chi_k \Delta ((1 - \Theta_{m,t}) X_{m,k,t}) + \epsilon_{m,t}, \quad (2) \]

where \( \alpha_m \) is country fixed effect, \( r_{m,t} \) is ex-ante real interest rate of country \( m \), \( \Theta_{m,t} \) is financial openness, \( Dmg_{m,\bullet,t} \) are demographic variables, \( r^*_{m,t-1} \) is foreign real interest rate faced by country \( m \), and \( X_{m,\bullet,t} \) are control variables and other potential determinants of real rates. Regressions weighted regressions by working age population, robust standard errors (cluster at country level)
Table 3: Panel Error-Correction regressions. Global Rates weighted by population size and openness. Lane and Milesi-Ferretti (2017) measure

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<td>0.052</td>
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<td>-0.792*</td>
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<td><strong>Long Term Bonds over GDP</strong></td>
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<td>0.225</td>
<td>0.345</td>
<td>0.063</td>
<td>0.358</td>
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</table>

| **R2**                  | 0.333     | 0.350     | 0.361     | 0.589     | 0.618     | 0.627     | 0.728     | 0.765     | 0.691     | 0.765       |
| **Observations**         | 683       | 683       | 663       | 430       | 219       | 217       | 126       | 103       | 142       | 103         |
| **Clusters**             | 20        | 20        | 20        | 20        | 17        | 17        | 7         | 7         | 7         | 7            |
| **RMSE**                | 1.215     | 1.201     | 1.210     | 0.915     | 0.792     | 0.784     | 0.598     | 0.618     | 0.863     | 0.618       |
Demographic Projections and Real Rates

How do real interest rate will look based on demographic projections?

- Population growth projected to remain relatively stable
- But life expectancy projected to continue to increase
Demographic Projections and Real Rates

- How do real interest rate will look based on demographic projections?
  - Population growth projected to remain relatively stable
  - But life expectancy projected to continue to increase

- Further downward pressure on real interest rates