Origins of International Factor Structures*

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Abstract
We develop and test a model of the global trade network. This model connects international co-movements of quantities and asset prices to a simple measure of network closeness, constructed from observed trade weights. We report three findings: (1) Countries that are closer in the network tend to have more correlated consumption growth rates, more correlated stock returns, and more correlated exchange rate movements. (2) International comovements can be decomposed into a component driven by primitive productivity shocks and a component due to network transmissions. Asset price correlations tend to be explained by the network structure, while consumption correlations by the correlations of primitive shocks. (3) The trade network generates factor structures in equity returns and exchange rate movements. It helps to explain the existence of the dollar and the carry factors, and gives rise to regional factors. These findings offer a network-based account of the origins of factor structures in international economic quantities and asset prices.

*This draft: December 22, 2019. For comments and discussions, we thank Patrick Bolton, Ric Colacito, Max Croce (discussant), Xavier Gabaix, Tarek Hassan, Gill Segal, Andreas Stathopoulos, Adrien Verdelhan (discussant), and seminar participants at Northwestern Kellogg, the Annual Conference in International Finance, the Columbia Workshop on New Methods in Empirical Finance, Hong Kong University of Science and Technology, UNC Kenan-Flagler, CIRANO-Walton Workshop on Networks in Economics and Finance, and the Chicago Booth Asset Pricing Conference. We thank Steven Zheng for excellent research assistance.

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The world is connected through a global trade network that transmits shocks across countries. Depending on the structure of this network, different countries are more or less exposed to particular shocks. Exposure to these shocks influences the behavior of both economic quantities and international asset prices. In this paper, we develop a model of this global trade network and empirically test its implications for international quantities and asset prices. We show how the structure of the network gives rise to related co-movements in consumption growth rates, exchange rates, and equity market returns. In doing so, we connect the behavior of economic fundamentals to the behavior of international asset prices — an important link that has had limited success in the literature.

Our empirical findings consist of three primary results. First, we show that a measure of network closeness explains a great deal of international co-movements in economic quantities and asset prices. Our network closeness measure is a direct implication of our theoretical model and, importantly, is only constructed using quantities — bilateral production and consumption trade shares. Second, we show that international co-movements can be decomposed into a component related to primitive productivity shocks — *primitive closeness* — and a component related to the transmission of shocks through the trade network — *network closeness*. Using this decomposition we show that network closeness is the primary driver of asset price comovements while the primitive shock correlation, as measured by primitive closeness, drives the majority of consumption growth comovements. Third, we demonstrate how structure of the global trade network can give rise to factor structures in asset prices, as are extensively studied in the international finance literature.

All of our empirical findings and measurement are motivated by a tractable model of international trade and asset prices. In our model, each country produces its tradable good using inputs from other countries. This interdependence gives rise to a global production network, which transmits primitive shocks across countries that trade with each other. On top of the production layer, the representative households in each country consume both home and foreign goods with differing weights. As a result, equilibrium consumption and asset prices are determined by the interaction
between the global production network, the consumption weights, and the primitive productivity shocks. This structure allows us to decompose international co-movements into primitive closeness and network closeness. Network closeness accounts for the structure of input-output linkages in the production and consumption networks, whereas primitive closeness is the correlation structure of the primitive productivity shocks that are necessary to match observed consumption growth correlations in the presence of the trade networks.

Figure (1) illustrates the relation between our measure of network closeness and international co-movements. For each pair of countries, we plot quantity and asset price co-movements versus network closeness. The four panels of this figure show that countries with higher network closeness have more correlated consumption growth rates, more correlated equity returns, more correlated currency base factors\(^1\), and less volatile bilateral exchange rates. Because our measure of network closeness is constructed only using trade quantities, these empirical results provide a direct link between the structure of the trade network, business cycles, and international asset prices.

While Figure (1) shows that a portion of consumption growth correlations are explained by the structure of the trade network, a substantial portion remains unexplained. Given this, we show that consumption growth correlations are mainly explained by primitive closeness rather than network closeness. In contrast, stock return correlations and exchange rate correlations have very little relation to primitive closeness, but are explained by network closeness. This helps to reconcile our results with the literature that finds little relation amongst international asset prices and quantities. Our decomposition demonstrates that the cross-country variation in asset prices is related to the global production and consumption networks, which are separate from the correlation structure of the primitive shocks. In this way, we provide a framework to understand the sources of covariation in economic quantities and asset prices across countries.

Our third result builds on the finding that the structure of the trade network explains bilateral correlations in asset prices. Our model demonstrates that, depending the structure of this network, these correlations may be a result of exposure to what appear as common factors. In particular,

\(^1\)We define each country’s currency base factor as the equal-weighted average of its log exchange rate change against all other currencies.
the structure of the network leads to certain combinations of shocks being more or less important for different countries’ asset price movements. These combinations of shocks can be interpreted as common factors in asset prices, with some countries’ asset prices being more or less exposed to particular factors.

Using this insight, we show that the covariation that arises from the structure of the global trade network is related to currency factors such as dollar, carry, and peripheral-minus-central (Lustig et al., 2014, 2011; Richmond, 2019). To show this, we extract principal components from the covariance matrix implied from our measure of network closeness. We find that each country’s loadings on these principal components explain much of the variation in the country’s loadings on standard currency risk factors such as carry and dollar. Currency portfolios constructed from these network-based principal components are also highly correlated with standard currency risk factors. These findings suggest a close link between economic fundamentals, as measured by the structure of the global trade network, and the factor structures in international asset prices.

Finally, we show how the structure of the global trade network can give rise to common regional variation. In our model, a block structure in the international production network will manifest itself as a factor structure in consumption growth and asset prices. In particular, if each country trades only with countries in the same region, then a weighted average of idiosyncratic productivity shocks within each region becomes a systematic factor.

To test this prediction, we use a clustering algorithm to assign countries into blocks based on their network closeness, and construct asset pricing factors as weighted averages across countries in each block. We find that this approximate block structure from the consumption and production network is apparent in the factor structure in global equity market returns and exchange rates. Countries within each block are more exposed to the factor within the block than factors in other blocks.

**Related Literature** Our work is most closely related to the literature on the common factors in international asset prices. Lustig et al. (2011); Verdelhan (2018) show that exchange rate move-
ments are driven by a few common factors. Lustig and Richmond (2019) find that this factor structure is related to bilateral measures of physical, cultural, and institutional distance. Aloosh and Bekaert (2019) use a clustering algorithm on currency base factors and show that cluster-based factors explain variation in currency base factors quite well. They also present evidence that the factor structure is apparent in retail sales growth data across countries. Forbes and Rigobon (2002); Bekaert et al. (2009) also find co-movements in international equity returns. Our paper studies the global trade network and provides a structural way to understand the origins of these factor structures.

Our paper also relates to research in international finance that studies currency risk premia. For example, Richmond (2019) shows how currency risk premia are related to countries’ positions in the global trade network position and how this position affects their consumption growth correlations with global aggregate consumption. Ready et al. (2017) show how each country’s trade composition between final and commodity goods impacts its exchange rate behavior. Corte et al. (2016) discover a priced risk factor based on external imbalances. Jiang (2018) finds that the factor structure in exchange rates is related to the factor structure in government surpluses.

Our work is also related to the literature on the co-movements in international business cycles. Gregory and Head (1999); Kose et al. (2003); Imbs (2004); Calderon et al. (2007); Burstein et al. (2008); Rose and Spiegel (2009) find trade, specialization, and financial integration are relevant for determining the international co-movements in business-cycle fluctuations. Bayoumi and Eichen-green (1998); Devereux and Lane (2003) show that trade links and common economic shocks also affect bilateral exchange rate volatility. Our work structurally relates these co-movements to the input-output linkages in the international production network.

In terms of theory, our model is a simplified version of an international business cycle model (Backus et al. (1992)). Colacito et al. (2018) study a generalization of this framework with recursive preferences. Our model is also related to network and granularity models such as Long Jr and Plosser (1983); Gabaix (2011); Foerster et al. (2011); Acemoglu et al. (2012); Chaney (2014); Acemoglu et al. (2016); Baqae and Farhi (2017, 2018), as well as to the literature that connects as-
set pricing to input-output linkages across firms such as Herskovic et al. (2016); Herskovic (2018); Gofman et al. (2018). We adapt the network model to an international context, and confirm that international comovements align with predictions from a simple network model. In a related paper, Huo et al. (2018) build a production model with a trade network, and show how the distribution of international GDP comovements would change if technology and non-technology shocks are turned off. In comparison, our work proposes a trade-based closeness measure and shows how it explains business cycle and asset price comovements between each pair of countries.

Moreover, the literature on optimal currency areas since Friedman (1953) and Mundell (1961) suggests that countries with correlated business cycles should form currency unions. This is because, on net, countries with more correlated business cycles benefit more from lower transactions costs in international trade. Frankel and Rose (1998) show that business cycles are endogenous: Countries that trade more tend to have more correlated business cycles. Our paper provides a network-based explanation for the relationship between trade and business cycle correlations and connects these findings to asset prices and factor structures.

1 Model

We begin by presenting a general equilibrium model of international trade and asset prices. The model is designed to remain tractable while generating rich implications for co-movements and factor structures. While possible to generalize the model beyond its current setup, most generalizations lead to analytically intractable solutions that are much less intuitive than that expressions which are derived in this setup. The model makes a number of key predictions about comovements of quantities and asset prices which we then take to the data in Section (2).

1.1 Set-Up

Time is discrete and infinite, indexed by $t = \{1, 2, \ldots\}$. There is no storage technology that allows agents to transfer consumption goods across periods.
There are $N$ countries indexed by $i \in \{1, \ldots, N\}$. Each country is populated by a representative household, which produces a distinct tradable good and consumes tradable goods from all countries. The representative household in country $i$ has a Cobb-Douglas production function of the form

$$X_{it} = A_{it} L_{it}^{\theta_i} \left( \prod_{j=1}^{N} X_{ijt}^{w_{ij}} \right),$$

where $A_{it}$ measures the productivity level, $L_{it}$ is the labor input, and $X_{ijt}$ is the quantity of the goods produced by country $j$ that are used as production inputs in country $i$. The parameter $\theta_i$ measures the contribution of country $i$'s labor, and the parameter $w_{ij}$ measures the contribution of country $j$'s input. We assume that

$$\theta_i + \sum_{j=1}^{N} w_{ij} = 1 \text{ and } \theta_i, w_{ij} > 0,$$

so that the production function has constant returns to scale.

The representative household in country $i$ assembles its aggregate consumption bundle $\overline{C}_{it}$ from each country's tradable good:

$$\overline{C}_{it} = \prod_{j=1}^{N} C_{ijt}^{v_{ij}},$$

where the parameters satisfy

$$\sum_{j=1}^{N} v_{ij} = 1 \text{ and } v_{ij} > 0$$

and $C_{ijt}$ is the consumption by household $i$ of country $j$'s good at time $t$.

The households have log preferences over their aggregate consumption, and discount future utility at rate $\beta$. Markets are complete. In each period, households in all countries share risk before any shock is realized.
Country \( i \) imports country \( j \)’s tradable goods not only for consumption, but also for country \( i \)’s production of its tradable goods. Therefore, the market clearing condition for country \( i \)’s tradable good is

\[
\bar{X}_{it} = \sum_{j=1}^{N} (C_{jit} + X_{jit}).
\]

Labor supply is fixed. The market clearing condition for country \( i \)’s labor is

\[
L_{it} = \bar{T}_{i}.
\]

We use lower case letters to represent the logs of their uppercase counterparts. A variable with its country subscript omitted is a vector. For example, \( \bar{c}_{t} \) is the vector where each element is \( \bar{c}_{it} = \log \bar{C}_{it} \). A capitalized parameter with two country indices omitted is a matrix. For example, \( W \) is the matrix with each element being \( w_{ij} \).

Lastly, the log productivity follows a random walk:

\[
a_{it+1} = a_{it} + \varepsilon_{it+1}.
\]

These shocks \( \varepsilon_{t+1} \) are normally distributed with mean zero. They are mutually independent across time, but they can be correlated across countries. Their covariance matrix is defined as

\[
\mathbb{E}_t[\varepsilon_{t+1}\varepsilon'_{t+1}] = \Omega.
\]

1.2 Quantities and the Trade Network

We focus on the competitive equilibrium defined in the usual fashion: All representative households maximize their utilities taking prices as given and market clearing conditions for each good and labor are satisfied. Since markets are complete, the competitive equilibrium can be characterized by the solution to a social planner’s problem. The social planner assigns Pareto weights \( \lambda_i \) to
country $i$, and maximizes:

$$
\sum_{i=1}^{N} \lambda_i \sum_{t=1}^{\infty} \exp(-\beta t) \log C_{it}.
$$

The following lemma characterizes the real quantities.

**Lemma 1** (Real Quantities). *In equilibrium, the vectors of each country’s log production growth rate and log consumption growth rate are*

$$
\Delta \overline{x}_{t+1} = (I - W)^{-1} \varepsilon_{t+1},
$$

$$
\Delta \overline{c}_{t+1} = V(I - W)^{-1} \varepsilon_{t+1}.
$$

The equilibrium consumption is determined by each country’s productivity shock $\varepsilon_{t+1}$ and the trade network $V$ and $W$. The term $(I - W)^{-1}$ is commonly known as the Leontief inverse, which summarizes the transmission of the productivity shocks. Notice that $(I - W)^{-1} = I + W + W^2 + W^3 + \ldots$, where $W$ reflects the transmission from the exporting country to the importing country, and $W^k$ reflects the transmission through higher orders of linkages.

We also derive a simple mapping from the observed trade quantities to the network parameters. Let $q_{jt}$ denote the price of the tradable good in country $j$. Then, $q_{jt}X_{ijt}$ is the total value of inputs of intermediate goods from country $j$ to country $i$, and $q_{jt}C_{ijt}$ is the total value of inputs of consumption goods from country $j$ to country $i$.

**Lemma 2** (Input-Output Network). *(a) Input shares of intermediate goods reflect matrix $W$:

$$
\frac{q_{jt}X_{ijt}}{q_{kt}X_{ikt}} = \frac{\omega_{ij}}{w_{ik}}.
$$

*(b) Input shares of consumption goods reflect matrix $V$:

$$
\frac{q_{jt}C_{ijt}}{q_{kt}C_{ikt}} = \frac{v_{ij}}{v_{ik}}.
$$

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(c) The share of value-added, defined as the total output minus the input cost as a fraction of total output, reflects the labor share $\theta_i$:

$$\frac{q_{it}X_{it} - \sum_{j=1}^{N} q_{jt}X_{ijt}}{q_{it}X_{it}} = \theta_i.$$ 

In the empirical section, we use the above expressions to map observed trade flows between countries to the underlying model parameters. In our model with no capital, a country’s value-added is entirely attributed to the labor share. Moreover, the Cobb-Douglas aggregation of consumption goods implies that the price of good $q_{it}$ is inversely proportional to the aggregate quantity $X_{it}$. Under certain parametric restrictions, the gains from having complete markets are minimal (Cole and Obstfeld (1991)). Real exchange rates and stock returns are nevertheless still well-defined, as shown below.

### 1.3 Real Exchange Rates and Stock Returns

We define the bilateral log real exchange rate between countries $i$ and $j$, $e_{ijt}$, as the price of country $i$’s consumption bundle per unit of country $j$’s consumption bundle. An increase in $e_{ijt}$ implies an appreciation of country $j$’s real exchange rate relative to country $i$’s. For country $i$, we define its currency base factor $\Delta \bar{e}_{it}$ as its equal-weighted average real exchange rate change against all countries, including itself:

$$\Delta \bar{e}_{it} = \frac{1}{N} \sum_{j=1}^{N} \Delta e_{ijt}.$$ 

This currency base factor measures the increase of the real exchange rate in country $i$. For example, it is closely related to the change in the dollar index in the real world, which measures the dollar’s nominal or real exchange rate change as a weighted average of bilateral exchange rate changes.

We define each country’s stock market as the claim to the future consumption stream. Let $P_{it}^{eq}$

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denote the ex-dividend stock price in units of the local consumption bundle. Let \( r_{it}^e \) denote the log cum-dividend return:

\[
    r_{it}^e = \log \frac{C_{it+1} + P_{it+1}^e}{P_{it}^e}.
\]

The following proposition characterizes the behavior of currency base factors and stock market returns in terms of the dynamics of consumption:

**Lemma 3 (Asset Prices).** (a) The currency base factor is

\[
    \Delta e_{it+1} = 1 - \sum_{j=1}^{N} \Delta c_{jt+1} - \Delta e_{it+1}.
\]

(b) The log cum-dividend return of the stock market is

\[
    r_{it+1}^e = \beta + \Delta e_{it+1}.
\]

The above lemma shows that a key quantity which drives comovements in the model is the co-variance of consumption growth across countries. It turns out that the co-variance of consumption across countries can be characterized in an intuitive analytical expression. To do so, we begin by defining the network profile \( H \) as the vector

\[
    H \equiv V(I - W)^{-1},
\]

The network profile captures the way which shocks propagate through the trade network. Using the network profile, we define a general measure of *closeness* between two countries \( i \) and \( j \) as

\[
    C(i, j) \equiv \{HH^\prime\}_{ij}.
\]

Closeness between countries measures how similar the shocks are which drive countries’ quantities. In particular, the following proposition shows that closeness is directly related to consumption
growth covariances and equity market correlations.

**Proposition 1** (Consumption Growth and Equity Returns). *Closer countries have more correlated consumption growth rates and more correlated stock returns:*

\[
\text{cov}(\Delta c_{it}, \Delta c_{jt}) = C(i, j), \\
\text{cov}(r_{it}^{eq}, r_{jt}^{eq}) = C(i, j).
\]

To characterize the behavior of exchange rates and currency base factors, we define the *average closeness* of country \(i\) as the average closeness between country \(i\) and all countries:

\[
\bar{C}(i) = \frac{1}{N} \sum_{j=1}^{N} C(i, j).
\]

Using this, we have the following proposition which relates closeness to the behavior of exchange rates:

**Proposition 2** (Exchange Rate Movements). *(a) Controlling for country-level fixed effects, closer countries have more correlated currency base factors and less volatile bilateral real exchange rate movements:*

\[
\text{cov}(\Delta e_{it}, \Delta e_{jt}) = C(i, j) - \bar{C}(i) - \bar{C}(j) + \kappa^e, \\
\text{var}(\Delta e_{ijt}) = -2\bar{C}(i, j) + C(i, i) + C(j, j),
\]

where \(\kappa^e\) is a constant that applies to all countries:

\[
\kappa^e = \frac{1}{N^2} \sum_{k=1}^{N} \sum_{\ell=1}^{N} C(k, \ell).
\]

*(b) The variance of country \(i\)’s currency base factor is*

\[
\text{var}(\Delta e_{it}) = -2\bar{C}(i) + C(i, i) + \kappa^e.
\]
Fixing its closeness to itself $C(i, i)$, the country’s currency base factor is less volatile if it has a higher average closeness.

1.4 Factor Structures

The prior propositions relate the behavior of bilateral asset prices and quantities to the structure of the production and consumption networks. The rich structure of these networks also generate what appear as common factors in asset prices and quantities. While it is not possible to analytically characterize the factor structure embedded in the networks for arbitrary structures $V$ and $W$, we can illustrate how factor structures arise in two special cases. Our empirical implementation takes these ideas seriously using the full structure of the global trade network.

For this section, we assume the home bias in consumption tends to 1 in the limit:

$$V \rightarrow I,$$

which allows us to abstract away the consumption trade network $V$ and focus on the production trade network $W$. In the data, households have high degrees of home bias in consumption.

Example 1: A Common Global Factor

The network structure could give rise to common global factors. Consider the following production network:

$$W = (1 - \theta) \begin{bmatrix} 1 & 0 & 0 & 0 \\ \gamma_2 & 1 - \gamma_2 & 0 & 0 \\ \gamma_3 & 0 & 1 - \gamma_3 & 0 \\ \gamma_4 & 0 & 0 & 1 - \gamma_4 \end{bmatrix}.$$

In this network, the first country is the central country and exports to all other countries. Other countries, which we refer to as peripheral, do not export. The labor share is fixed at $1 - \theta$ for all countries, but each peripheral country has a different import share from country 1 given by $\gamma_2, \gamma_3, \gamma_4$. 
Suppose the log productivity shocks are i.i.d. across countries, $\Omega = I$. Then, the consumption growth of each country can be characterized as having a factor structure. By Lemma 1,

$$
\Delta \bar{c}_{t+1} = (I - W)^{-1}\varepsilon_{t+1} = \begin{bmatrix}
a_{11} & 0 & 0 & 0 \\
a_{21} & a_{22} & 0 & 0 \\
a_{31} & 0 & a_{33} & 0 \\
a_{41} & 0 & 0 & a_{44}
\end{bmatrix} \begin{bmatrix}
\varepsilon_{t+1}^{(1)} \\
\varepsilon_{t+1}^{(2)} \\
\varepsilon_{t+1}^{(3)} \\
\varepsilon_{t+1}^{(4)}
\end{bmatrix},
$$

where the $a$’s are functions of $\theta$ and the $\gamma$’s. They satisfy $a_{11} > a_{i1} > 0$ and $a_{ii} > 0$ for $i = 2, 3, 4$. This formula shows that the central country’s productivity shock $\varepsilon_{t+1}^{(1)}$ becomes the common factor in the cross-section of countries. All other (peripheral) countries load on this common factor, and have no further pairwise comovements beyond that induced by their exposure to the common factor. The consumption shocks $\varepsilon_{t+1}^{(2)}, \varepsilon_{t+1}^{(3)},$ and $\varepsilon_{t+1}^{(4)}$ are idiosyncratic in the sense that only one country is exposed to each of them.

The bilateral exchange rate changes also load on the shocks:

$$
\Delta e_{1,2,t+1} = (a_{21} - a_{11})\varepsilon_{t+1}^{(1)} + a_{22}\varepsilon_{t+1}^{(2)},
$$

$$
\Delta e_{1,3,t+1} = (a_{31} - a_{11})\varepsilon_{t+1}^{(1)} + a_{33}\varepsilon_{t+1}^{(3)},
$$

$$
\Delta e_{2,3,t+1} = (a_{31} - a_{21})\varepsilon_{t+1}^{(1)} + a_{33}\varepsilon_{t+1}^{(3)} - a_{22}\varepsilon_{t+1}^{(2)},
$$

and the central country’s shock $\varepsilon_{t+1}^{(1)}$ becomes the common factor in the cross-section of exchange rates. Since the loadings $(a_{21} - a_{11}), (a_{31} - a_{11}),$ and $(a_{31} - a_{21})$ can be different depending on the values of the $\gamma$’s, these currencies have different loadings on the common factor.

This expression demonstrates how our general equilibrium model produces exchange rate dynamics like those found in factor models of exchange rates such as Lustig et al. (2011). In these
factor models, the exchange rate change between two countries can be expressed as

\[ \Delta e_{i,j,t+1} = (\delta^i - \delta^j) f_{t+1} + \eta^i_{t+1} - \eta^j_{t+1}, \]

where \( f_{t+1} \) represents one or multiple common factors and \( \eta^i_{t+1} \) and \( \eta^j_{t+1} \) represent idiosyncratic shocks. In particular, our example offers a mechanism that generates the common factor \( f_{t+1} \) through network propagation of independent shocks across countries. This example could also be generalized to allow for multiple factors once we impose a richer dependence structure in the production network.

Furthermore, the currency base factors can be expressed as the sum of the loading on the common factor \( \varepsilon^{(1)}_{t+1} \), the average of peripheral countries’ consumption shocks, and (for peripheral countries) an idiosyncratic component:

\[
\Delta \varepsilon_{1,t+1} = \left( \frac{a_{11} + a_{21} + a_{31} + a_{41}}{4} - a_{11} \right) \varepsilon^{(1)}_{t+1} + \frac{a_{22}\varepsilon^{(2)}_{t+1} + a_{33}\varepsilon^{(3)}_{t+1} + a_{44}\varepsilon^{(4)}_{t+1}}{4},
\]

\[
\Delta \varepsilon_{2,t+1} = \left( \frac{a_{11} + a_{21} + a_{31} + a_{41}}{4} - a_{21} \right) \varepsilon^{(1)}_{t+1} + \frac{a_{22}\varepsilon^{(2)}_{t+1} + a_{33}\varepsilon^{(3)}_{t+1} + a_{44}\varepsilon^{(4)}_{t+1}}{4} - a_{22}\varepsilon^{(2)}_{t+1},
\]

with variances determined by the exposures to the common factor and the magnitude of the idiosyncratic shocks:

\[
\text{var}(\Delta \varepsilon_{1,t+1}) = \left( \frac{a_{11} + a_{21} + a_{31} + a_{41}}{4} - a_{11} \right)^2 + \frac{a_{22}^2 + a_{33}^2 + a_{44}^2}{16},
\]

\[
\text{var}(\Delta \varepsilon_{2,t+1}) = \left( \frac{a_{11} + a_{21} + a_{31} + a_{41}}{4} - a_{21} \right)^2 + \frac{9a_{22}^2 + a_{33}^2 + a_{44}^2}{16}.
\]

The currency base factors offer a simple way to compare different currencies’ strengths and volatilities. In particular, when the common factor \( \varepsilon^{(1)}_{t+1} \) is negative, the central country’s base factor experiences the greatest appreciation, and a peripheral country \( i \)'s base factor appreciates more if its exposure \( a_{i1} \) is higher. A peripheral country \( i \)'s base factor is more volatile if its exposure to the common factor \( a_{i1} \) is more different from the average exposure \( (a_{11} + a_{21} + a_{31} + a_{41})/4 \) and if its
loading $a_{ii}$ on the idiosyncratic shock is higher.

While the structure of this example is intentionally stylized to demonstrate analytically how common factors can arise through the trade network, the intuition generalizes. In particular, the structure of the network can give rise to common factors when countries tend to be exposed to similar sets of shocks. In our general model with dense input-output linkages in $W$ and $V$, there are no truly idiosyncratic shocks, but some shocks are more important for the variation of quantities and asset prices. Therefore, in Section 4 we apply principal component analysis to extract the common factors embedded in the structure of the trade network which contribute to a large proportion of the variation.

**Example 2: Regional Factor Structure** We can further understand the origins of international co-movements by specifying the following block structure:

**Assumption 1.** (a) The productivity shocks load on systematic shocks $u_t$ and idiosyncratic shocks $\eta_t$:

$$
\varepsilon_t = \Psi u_t + \Xi \eta_t,
$$

which $u_t$ is a $K$-by-1 vector of standard normal random variables and $\eta_t$ is a $N$-by-1 vector of standard normal random variables. $u_t$ and $\eta_t$ are mutually independent. $\Xi$ is a diagonal matrix with elements $\xi_{ii}$, so other countries’ idiosyncratic shocks $\eta_{jt}$ do not affect $\varepsilon_{it}$.

(b) The production trade network has a block structure. There are $M$ regions, and each country only imports from other countries in the same region for production inputs. Let $R(i)$ denote the set of countries in the same region as country $i$. We assume

$$
w_{ij} = \begin{cases} 
\omega_0, & \text{if } i = j, \\
\omega_1, & \text{if } i \neq j \text{ and } j \in R(i), \\
0, & \text{if } j \notin R(i),
\end{cases}
$$
where the parameters satisfy $\omega_0 + (|R(i)| - 1)\omega_1 = 1 - \theta_i$. The parameters $\omega_0$ and $\omega_1$ can vary across regions, but we drop the index of the region for notational simplicity.

Assumption 1(a) allows the productivity shocks to have $K$ systematic components. The covariance matrix $\Omega$ for productivity shocks can be expressed as

$$\Omega = \Psi \Psi' + \Xi \Xi'.$$

Since $\eta$ is a country-specific shock, $\Xi \Xi'$ is a diagonal matrix. So, only $\Psi \Psi'$ generates co-movements between different countries’ productivity shocks. However, later we will show that the trade network propagates the idiosyncratic shocks within the region.

Assumption 1(b) highlights the fact that trade partners are related through geographical or industrial proximity. For example, United States, Canada and Mexico mainly trade with each other because they are close. Similarly, countries linked by supply chains also import each other’s goods.

Under these simplifying assumptions, the following proposition characterizes how the aggregate factor structures in quantities and asset returns come from productivity shocks and the trade network.

**Proposition 3 (Regions and the Factor Structure).** (a) The consumption growth in each country is driven by the systematic shocks $v_k$, a regional shock that is a weighted average of the idiosyncratic shocks from countries in the same region, and the country-specific shock $\eta_i$:

$$\Delta c_{it+1} = \sum_{k=1}^{K} \kappa^h \left( \theta_i \psi_{ik} + \sum_{j \in R(i)} \omega_1 \psi_{jk} \right) u_{kt+1} + \kappa^h \omega_1 \left( \sum_{j \in R(i)} \xi_{jj} \eta_{jt+1} \right) + \kappa^h \theta_i \xi_{ii} \eta_{it+1},$$

where $\kappa^h > 0$ is a constant that depends on $\omega_0$ and $\omega_1$.

(b) The closeness between country $i$ and country $j$ is

$$c(i, j) = \begin{cases} (\kappa^h)^2 \left( \theta_i \psi_i + \sum_{k \in R(i)} \omega_1 \psi_k \right) \left( \theta_i \psi_j + \sum_{k \in R(i)} \omega_1 \psi_k \right)' + (\kappa^h \omega_1)^2 \sum_{k \in R(i)} \xi_{kk}, & \text{if } j \in R(i), \\ (\kappa^h \theta_i)^2 \psi_i \psi_j', & \text{if } j \notin R(i). \end{cases}$$
(c) If all countries have the same idiosyncratic variance $\xi_{ii}^2$ and the same loadings on the systematic shock $\psi_i$, then the first $K + M$ principal components in consumption growth are the systematic shocks $u_{kt+1}$ and the regional shocks $\sum_{j \in R(i)} \xi_{jj} \eta_{jt+1}$. Countries are closer to countries in the same region than to countries in other regions.

This proposition shows that a single regional factor arises from the production network within each region. Within each region, a country with a higher productivity volatility $\xi_{jj}$ has a higher weight in the regional factor. Each country’s consumption growth $\Delta \bar{c}_{it+1}$ loads on the systematic productivity shocks $u_{kt+1}$, this regional factor, and its own idiosyncratic productivity shock $\eta_{it+1}$. Both the systematic shocks and the regional factors affect aggregate fluctuations in real quantities and asset returns: Two countries are close if they have similar loadings on the systematic shocks or if they are in the same region.

2 Data

2.1 Data Sources

We consider two samples. Our primary sample contains developed countries as classified by MSCI: Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Ireland, Italy, Japan, Netherlands, Norway, Portugal, Spain, Sweden, Switzerland, United Kingdom, and United States. The second sample of developed and emerging countries adds Brazil, China, Czech Republic, Greece, Hungary, India, Indonesia, Mexico, Poland, Russia, South Korea, and Turkey. The sample of countries is primarily limited by the coverage of the World Input Output Database.

Our World Input Output Tables (WIOT) are from World Input-Output Database (Timmer et al. (2015)). The data are annual from 2000 to 2014.

Our spot exchange rate data are monthly from Global Financial Data from April 1973 to December 2014. We end the sample in 2014 to align with the last available year of the World Input-Output Database. We define the currency base factor of country $i$ as in Verdelhan (2018) and Lustig and Richmond (2019) as the average log exchange rate change with respect to all foreign
countries:

\[ \Delta \bar{e}_{it} = \frac{1}{N - 1} \sum_{j \neq i} \Delta e_{ijt}; \]  \hspace{1cm} (1)

a positive value means an appreciation currency in country \( i \) relative to other currencies.

The consumption data are from the World Bank’s World Development Indicators from 1973 until 2014. Equity return data are from MSCI.

### 2.2 Estimating the Trade Network

We use the WIOT to measure the bilateral consumption and production weights that our model takes as parameters. Following Lemma 2, we recover the production matrix \( W \) and the labor share \( \theta_i \) from the input shares of intermediate goods and final consumption shares.

For our main results we take the average of \( \theta, W \) and \( V \) across all years in the WIOT. This is equivalent to assuming that the structure of global production remained stable over our sample period. Then, we use these average parameters from WIOT to explain asset prices and quantities in our sample period that start before the WIOT data are available. While the time-series evolution of the global structure of production and consumption is interesting, we leave the study of this for future work.

Figure (A1) and Figure (A2) visualize the intermediate trade network \( W \) and the consumption trade network \( V \). Countries that are closer in these figures have stronger connections in the matrices \( W \) and \( V \). We notice that the intermediate trade network and the consumption trade network do not necessarily coincide.
2.3 Estimating the Covariances of Productivity Shocks

We estimate the covariance matrix $\Omega$ of productivity shocks from the covariance between consumption growth rates. According to Proposition 1,

$$ cov(\Delta \tilde{c}_{it}, \Delta \tilde{c}_{jt}) = C(i, j) \equiv \{H \Omega H'\}_{ij} \quad (2) $$

where $H = V(I - W)^{-1}$.

Given our estimate of the trade networks $W$ and $V$, we solve for $\Omega$ using the covariance matrix of consumption growth rates. Figure (A3) depicts the covariance matrix $\Omega$. Countries that are closer in this figure have larger covariances in $\Omega$.

2.4 Measures of Closeness

As closeness is defined as $C(i, j) = \{H \Omega H'\}_{ij}$, two countries can be close because of either the structure of the trade network, as summarized by $H$, or the covariance of primitive shocks, $\Omega$. To understand empirically which of these components drive asset price and quantity correlations, we examine five different measures of closeness.

Our first measure of closeness is the covariance in consumption growth, which is given by

$$ C_{\text{Consumption}}(i, j) = \{H \Omega H'\}_{ij}. $$

Second, we examine the closeness as driven by only the covariances of primitive shocks:

$$ C_{\text{Primitive}}(i, j) = \{\Omega\}_{ij}, $$

which amounts to assuming autarky in consumption and production, $H = I$.

Third, we examine the closeness as driven by the trade network:

$$ C_{\text{Network}}(i, j) = \{HH'\}_{ij}. $$
which amounts to assuming the productivity shock in each country is i.i.d., $\Omega = I$.

Fourth, we further reduce the trade network to just the production network by assuming perfect home bias in consumption, i.e. $V = I$. Since $H = V(I - W)^{-1}$, this assumption gives rise to a closeness measure that is only based on the production trade network:

$$C^{Production}(i, j) = \{(I - W)^{-1}((I - W)^{-1})^\prime\}_{ij}.$$ 

Lastly, we apply a first-order approximation to $C^{Production}(i, j)$. Noticing that

$$(I - W)^{-1} = I + W + W^2 + \ldots,$$

we define

$$C^{Production FO}(i, j) = \{I + W + W^\prime\}_{ij} = w_{ij} + w_{ji} \text{ for } i \neq j.$$ 

All these measures of closeness are covariances. To compare across countries with different volatilities, we normalize them by the two countries’ corresponding standard deviations. We define correlation based closeness for each type of closeness (network, primitive, etc.) as

$$\tilde{C}^{Type}(i, j) = \frac{C^{Type}(i, j)}{\sqrt{C^{Type}(i, i)}\sqrt{C^{Type}(j, j)}}.$$ 

### 3 Empirical Tests

#### 3.1 Closeness and International co-movements

Proposition 1 predicts that countries that have higher closeness have more correlated production growth rates, more correlated consumption growth rates, more correlated real exchange rate movements, more correlated stock returns, and less volatile bilateral real exchange rate movements. In this section, we study which of our five measures of closeness as defined in Section (2.4) explains
the co-movements of these quantities in asset prices.

We begin by studying consumption correlations and stock market returns. Following Proposition 1(a), we run regressions of the correlations in consumption, \( \text{corr}(\Delta \bar{c}_{it}, \Delta \bar{c}_{jt}) \), and stock returns in local currency, \( \text{corr}(r_{eq}^{it}, r_{eq}^{jt}) \), on the correlation based measures of bilateral closeness, \( \tilde{C}(i, j) \):

\[
\text{corr}(\Delta \bar{c}_{it}, \Delta \bar{c}_{jt}) = \alpha + \beta \cdot \tilde{C}(i, j) + \varepsilon_{ij}, \tag{3}
\]

\[
\text{corr}(r_{eq}^{it}, r_{eq}^{jt}) = \alpha + \beta \cdot \tilde{C}(i, j) + \varepsilon_{ij}. \tag{4}
\]

Table (1) reports the results in the sample of developed countries and Table (2) for the sample of developed and emerging countries. For all bilateral regressions we only select the set of unique country pairs. That is, we keep \((i, j)\), but do not include \((j, i)\), \((i, i)\), and \((j, j)\).

The first 4 columns explain consumption correlations. We do not include total closeness when explaining consumption correlations because by construction it is exactly equal to the bilateral consumption correlation. Our first finding in this table is that cross-sectional variation in consumption correlations is primarily related to primitive closeness rather than network closeness. This can be seen by comparing the R-squared in the first and second columns. Primitive closeness has and R-squared of 43% versus 8% for network closeness in the developed sample. The third and fourth columns explain consumption correlations with production based closeness measures. The R-squared in both of these columns is about 8%, similar to that of the network based closeness measure and substantially lower than primitive closeness.

The last 5 columns of Table (1) and Table (2) explain bilateral equity market correlations. In column 5 we regression bilateral equity market correlations on total closeness, or equivalently on bilateral consumption correlations. The R-squared in this regression is 11% which suggests a very minor relation between consumption correlations and equity market correlations. Moving to column 7 we see that network closeness has the highest explanatory power for equity market correlations with an R-squared of 32% and 23% for the developed and the full samples respectively.
This suggests that correlation in equity markets is mostly related to the structure of the global production and consumption network rather than the correlation of primitive shocks as measured by primitive closeness.

The last two columns of Table (1) and Table (2) present results using production closeness and its first order approximation. These have similar explanatory power of about 29% in the developed sample and 18% in the developed and emerging sample. This suggests that first order connections in the production network capture much of the explanatory power for equity market correlations. That said, the R-squared using production based closeness measures is lower than when using network closeness. This implies that it is important to take into account the structure of the global consumption network as well as the production network when explaining equity market correlations.

We next turn to studying the how the behavior of exchange rates is related to our various measures of closeness. Following Proposition 2 (a), we run regressions of the correlations of currency base factors and the volatility of the bilateral exchange rate movements on our measures of closeness, but also including country fixed effects as is implied by the proposition:

$$corr(\Delta \pi_{it}, \Delta \pi_{jt}) = \delta_i + \delta_j + \beta \cdot \tilde{C}(i, j) + \epsilon_{ij},$$

$$std(\Delta \pi_{i,j,t}) = \delta_i + \delta_j + \beta \cdot \tilde{C}(i, j) + \epsilon_{ij}.$$  

The results are presented in Table (3) for the developed sample and Table (4) for the developed and emerging sample. We find similar results for exchange rates to those that we found for equity market correlations. Network closeness has the highest explanatory power for currency base factor correlations with an R-squared of 22% and 17% in the developed and full samples respectively. In comparison, the primitive closeness and the consumption correlation capture much smaller variations in these exchange rate moments. Production based closeness measures explain more than primitive closeness, but again not as much as network closeness.

Finally, we present evidence for Proposition 2 (b) in Figure (2). We plot the variance of
each country’s currency base factor versus variances implied by network closeness. As the figure illustrates, there is a strong positive relation between actual currency base factor variances and those that are implied by the global production and consumption network. This suggests that the variance of a country’s exchange rate relative to all other exchange rates is closely related to where a country is positioned in the global trade network relative to others. This finding is consistent with the findings of Lustig and Richmond (2019) who study how base factor variances are related to the gravity effect in the exchange rate factor structure. Our findings here provide an explanation for how base factor variances are related to the underlying structure of the global trade network. We further explore this in the next section.

Overall, we conclude that by taking into account the structure of global consumption and production, our network closeness measure explains variation across countries that drives co-movement in their asset prices. This is in contrast to primitive closeness, which is the primary driver of variation in consumption correlations across countries.

4 International Factor Structures

We next study whether the covariance structure embedded in our measures of network closeness is related to the factor structures in international asset prices. Following the two examples in Section (1.4) we do this in two parts. First, we apply a principal component analysis to the covariances implied by network closeness. We then show that the principal components embedded in our network closeness measure are related to common currency factors such as dollar and carry (Lustig et al., 2011). Second, we apply a clustering algorithm to approximate block structures and show that these blocks correspond to common regional factors.

4.1 The Trade Network Origins of Common Currency Factors

Our first example in Section (1.4) illustrated that common factors in asset prices can arise if a particular shock or set of shocks are important globally due to the network structure. While this
example was stylized to provide analytical solutions, the intuition generalizes. In particular, we want to extract common factors that are apparent in the covariance structure that is generated by the trade network or, potentially, by the correlation of the primitive shocks. To do so, we apply a principal component analysis to the covariance matrices from network closeness, $C^{Network}$, and primitive closeness, $C^{Primitive}$.

Because the closeness matrices are already measuring implied covariances, we can simply apply an eigenvalue decomposition to these matrices to extract the common factors which arise due to the network structure. Specifically, for any covariance matrix $C$, we can apply the eigenvalue decomposition:

$$C = U \Lambda U'$$

where $U$ is an orthogonal matrix, and $\Lambda = diag(\lambda_1, \ldots, \lambda_N)$ is diagonal. The scalars $\lambda_j$ are the eigenvalues of the matrix $C$. The associated eigenvectors are denoted by $PC_k(C)$, and are the columns of $U$. We order these eigenvectors, or principal components, in descending order by their associated eigenvalues.

Table (5) reports the fractions of variance explained by the first 4 principal components for four covariance matrices: network closeness ($C^{Network}$), primitive closeness ($C^{Primitive}$), the covariance matrix of consumption growth ($C^{Consumption}$), and the covariance matrix of currency base factor movements ($C^{FX}$). The first 4 PCs explain the majority of variation in $C^{Primitive}$, $C^{Consumption}$, and $C^{FX}$. In comparison, the first 4 PCs explain very little variation in $C^{Network}$. This low explanatory power of the first four principal components for network closeness is a result of the large degree of home bias in consumption. Since each country’s consumption loads heavily on domestic goods, it is largely affected by its own productivity shocks. In this case, the covariance matrix based on network closeness is largely diagonal, with each principal component explaining a small amount of the variation. That said, the common variation that does arise from network closeness explains variation in currencies exposure to common factors, as we show next.
This eigenvalue decomposition allows us to study the relation between the principal components in primitive and network closeness and common currency factors studied in the literature. To do so, we first note that currency base factors are simply linear combinations of log exchange rate movements. As a result of this linearity, the change in the value of a portfolio of currencies can be expressed as a linear combination of currency base factors\(^2\). For example, the dollar and the carry factors (Lustig et al., 2011, 2014) can be expressed as some linear combinations of currency base factors.

A particularly interesting set of portfolios in our context are those derived from principal components of our closeness measures. Given a covariance matrix \(C\), each principal component \(PC_n(C)\) can be interpreted as a currency portfolio. The \(i\)-th element \(PC_n,i(C)\) indicates the relative weight on currency base factor \(i\), which itself is a linear combination of all exchange rates versus currency \(i\). The currency portfolio based on the first principal component \(PC_1,i(C)\) then represents the combination of currencies that account for the largest fraction of variance in implied exchange rate movements.

To study how these portfolios relate to standard currency factors, we first obtain loadings of currency base factors on factors. To do so we run regressions of the following form:

\[
\Delta \varepsilon_{it} = \alpha + \beta_i f_{ac_t} + \epsilon_{it},
\]  

(7)

where \(f_{ac_t}\) is a standard currency risk factor such as carry or dollar. We then regress each currency’s factor loading from the above regressions on the portfolio loadings based on the first three principal components:

\[
\beta_i = \alpha + \beta_1 PC_{1,i}(C) + \beta_2 PC_{2,i}(C) + \beta_3 PC_{3,i}(C) + \epsilon_i.
\]  

(8)

These principal components are obtained from either \(C^{Network}\) or \(C^{Primitive}\). \(\beta_i\), as an example,

\(^2\)This follows directly from application of triangular arbitrage on log exchange rate changes. For useful discussions of the spanning properties of currency base factors see Appendix A of Lustig and Richmond (2019) and Aloosh and Bekaert (2019).
is the loading of currency $i$’s base factor $\Delta \bar{e}_i$ with respect to the dollar factor. We regress each countries’ base currency dollar factor loading, $\beta_i$, on that country’s loadings on the first 3 principal components of network closeness: $PC_{1,i}(C^{\text{Network}})$, $PC_{2,i}(C^{\text{Network}})$, and $PC_{3,i}(C^{\text{Network}})$. The R-squared in these regressions answers the following question: how much of the cross-sectional variation in currency factor loadings can be explained by variation in common exposures that arises in our measures of closeness?

Table (7) reports the results for 4 currency factors: dollar, carry (HML), unconditional carry (UHML), and peripheral-minus-central (PMC as in Richmond, 2019). The currency loadings based on the first 3 PCs implied from $C^{\text{Network}}$ explain 64% of the variation in each currency’s dollar beta. In other words, the major currency comovements implied from $C^{\text{Network}}$ align with the dollar beta. These currency loadings from $C^{\text{Network}}$ also explain 9% of the variation in each currency’s conditional carry beta, 21% of the variation in each currency’s unconditional carry beta, and 38% of the variation in each currency’s peripheral-minus-central beta. In comparison, the currency loadings based on the first 3 PCs implied from $C^{\text{Primitive}}$ explain much smaller variations in currency betas.

Figure (3) illustrates the above point graphically. In this figure we plot the actual loading of each currency base factor versus those predicted from the principal components of network and primitive closeness. In the top panel we use network closeness and in the bottom panel we use primitive closeness. For the dollar, PMC, and UHML factors, we see that network closeness based loadings explain a great deal of variation in currency base factors loadings on the common factors, as is consistent with the high R-squareds in Table (7). On the other hand, primitive closeness based principal components tend to explain very little of the variation in loadings on the common factors.

Next, we study how variation in the actual portfolios correlates with standard currency factors. Given that the vector of currency base factors is given by $\Delta \bar{e}_t$, the exchange rate movement on the currency portfolio based on $PC_n(C)$ can be expressed as

$$PC_n(C)' \Delta \bar{e}_t.$$
We regress each currency risk factor $f_t$ (dollar, conditional carry, unconditional carry, and peripheral-minus-central) directly on the exchange rate movements of the currency portfolios based on the first 3 PCs implied from $C^{Network}$:

$$f_t = \alpha + \beta_1 PC_1(C)' \Delta \bar{e}_t + \beta_2 PC_2(C)' \Delta \bar{e}_t + \beta_3 PC_3(C)' \Delta \bar{e}_t + \epsilon_t.$$  \hspace{1cm} (9)

Table (6) reports the results. The currency portfolios based on the first 3 PCs implied from $C^{Network}$ explain 88% of the variation in the dollar factor, 22% of the variation in the conditional carry factor, 42% of the variation in the unconditional carry factor, and 44% of the variation in the peripheral-minus-central factor. In comparison, the currency loadings based on the first 3 PCs implied from $C^{Primitive}$ explain much smaller variations in these currency factors.

In summary, we show that each currency’s loadings on the first 3 principal components of network closeness explain its exposures with respect to currency risk factors. As a result of this, the currency portfolios based on these 3 principal components are highly correlated with standard currency risk factors. This is especially true for risk factors which tend to represent unconditional exposures such as unconditional carry and peripheral minus central. We conclude that the international risk factors are mostly related to the structure of the trade network, rather than the covariance structure of primitive shocks.

### 4.2 Regional Factors

In addition to the standard risk factors, there may exist other factor structures in the covariance matrix based on the trade network. From our model, Proposition 3 shows that if there is a block structure in the world trade network, then this block structure will also manifest itself in consumption correlations, equity market correlations, and in FX correlations. Additionally, this block structure would lead to a factor structure in these asset prices and quantities. In this section, we link the regional comovements to factor structure in the trade network.

We begin by extracting the block structure in the intermediate and consumption trade network.
for developed countries, as suggested by Proposition 3. To do so, we start with our matrix of network based closeness measures. We then apply a hierarchical clustering algorithm (Johnson (1967)) to these closeness measures using the inverse of closeness as a measure of distance. Hierarchical clustering finds approximate block structures in network matrices by minimizing the within block distance and maximizing the across block distance of countries in each block. To illustrate our points in this section, we partition countries into 3 clusters.

We present our first set of clustering results in Figure (4). This figure presents average correlations or volatilities of countries asset prices and quantities with other countries, conditional on being in the same cluster (blue bars) or different clusters (red bars). Countries are grouped into 3 panels from left to right which represent the clusters. The first cluster has Australia, Canada, Japan, and the US. The second cluster has mostly mainland European countries. The third cluster has Scandinavian countries.

The top panel presents consumption growth correlations. Notably, only countries in the second block have higher within correlations than outside correlations. This is not particularly surprising given our finding that the majority of variation in consumption growth correlations is explained by primitive closeness rather than network closeness. We could conduct a similar exercise constructing clusters using primitive closeness, but we focus on network closeness to emphasize the implications of our theoretical model.

The third and fourth panels present results for equity market correlations and currency base factor correlations. For almost all countries, correlations are higher within the cluster than outside of the cluster. Interestingly, for currency base factor correlations, most of the outside cluster correlations are positive. The most noticeable exception to this is the US, which has a negative base factor correlation with respect to base factors of countries outside of cluster 1. This is likely consistent with Lustig et al. (2014); Verdelhan (2018); Jiang et al. (2018) that show that the US dollar

\footnote{Aloosh and Bekaert (2019) also apply a clustering algorithm. In their case, they start with currency base factors (currency baskets in their terminology) and apply a clustering algorithm. This is in contrast to our clustering which is done on the world input output network. Interestingly, there are numerous similarities to the clusters which we observe, which is consistent with the gravity in the exchange rate factor structure findings of Lustig and Richmond (2019).}
has unique properties relative to most other currencies. A similar result holds for FX volatility in second panel, where the within cluster volatility being lower than the outside of cluster volatility.

It is important to note that the block structure implied by clustering network closeness is only approximate. Proposition 3 has an exact block structure and thus would imply that the correlation outside of each countries block would be zero. As seen in Figure (4), the correlation outside of each countries block is non-zero, which implies that the block structure is only approximate or that there is cross-sectional correlation due to the primitive shocks. Nevertheless, the structure of the global trade network still gives rise to phenomena that is consistent Proposition 3, which we continue to explore in this section.

We next turn to understand how this commonality within clusters can generate a regional factor structure. For each cluster and for each quantity or asset price, we construct a factor that is the average change in that quantity or asset price across countries within the cluster. We then regress each country’s quantity or asset price on each these factors to obtain factor loadings. When we regress country i’s quantity and asset price, we omit this country from the cluster factor so as to not have any mechanical link between the country and the cluster factor. For example, when we regress the Australian dollar base factor on cluster 1’s currency base factor, we construct cluster 1’s currency base factor as the average change in all countries in cluster 1 except Australia. We also regress the Australian dollar base factor on factors constructed from cluster 2 and cluster 3.

Figure (5) presents the factor loadings for each country across clusters. For consumption, there is not much variation in the loadings. This is again consistent with our finding that the majority of variation in consumption is explained by primitive closeness and not by network closeness. For equity return loadings we find that countries’ within cluster loadings are almost always higher than the loadings on cluster factors other than their own. The same is true for the FX base factors. These results show that when currencies within a cluster systematically appreciate, they tend to do so in tandem. Due to the construction of currency base factors, this also implies that currencies outside of the cluster tend to systematically depreciate on a relative basis.

The findings in this section suggest that the structure of the world trade network is an important
determinant of the factor structure that we observe in global equity returns and exchange rate movements. Specifically, factors implied from the world’s input-output linkages are able to explain how exchange rates and equity markets co-move.

5 Conclusion

In this paper, we develop a model of the global production and consumption network and show that a measure of network closeness explains numerous co-movements in economic quantities and asset prices. We empirically measure our theoretically motivated network closeness measure and show that countries that are closer in this network tend to have more correlated consumption growth, stock returns, and exchange rate movements. The network also generates factor structures in equity returns and exchange rates as found in the data. These results offer a network-based account of the origins of factor structures in international asset prices and economic quantities.
References


Milton Friedman. The case for flexible exchange rates. 1953.


Figure 1: Asset Price and Consumption Correlations versus Network Implied Correlations

Note: Plots of equity market correlations, exchange rate volatilities, and currency base factor correlations versus network closeness. Consumption growth correlations are constructed from yearly consumption growth. Equity market correlations are correlations equity market returns in local currency. Exchange rates are nominal, and currency base factors are the average appreciation of each currency against all other currencies in the sample. Network closeness measures the implied correlation assuming primitive shocks are uncorrelated across countries. Exchange rate regressions include home country and foreign country fixed effects. Consumption data are from the World Bank’s World Development Indicators. Equity market return data are from MSCI. Spot rate data are monthly from Global Financial Data from April 1973 to December 2014 for 19 developed countries as classified by MSCI. World Input Output Tables are from World Input-Output Database.
Figure 2: Base Factor Variance versus Implied from Network

Note: Plot of base factor variances versus those implied by network closeness. Currency base factors are the average appreciation of each currency against all other currencies in the sample. Network closeness measures the implied correlation assuming primitive shocks are uncorrelated across countries. Spot rate data are monthly from Global Financial Data from April 1973 to December 2014 for 19 developed countries as classified by MSCI. World Input Output Tables are from World Input-Output Database.
**Figure 3:** Actual versus Predicted Currency Factor Loadings

Note: Plots of actual loadings of currency base factors on the dollar, unconditional carry, and peripheral-minus-central exchange rate factors versus predicted loadings from network and exposure closeness as in Equation (8). Currency base factors are the average appreciation of each currency against all other currencies in the sample. Network closeness measures the implied correlation assuming primitive shocks are uncorrelated across countries. Exposure closeness is the correlation of primitive shocks. Actual loadings are estimated from regressions of currency $i$’s base factor $\Delta \pi_i$ on the dollar, peripheral-minus-central, and unconditional carry factors, respectively. Predicted loadings are the projection of actual loadings on the first three principal components extracted from the network and exposure closeness covariance matrices, respectively. Spot rate data are monthly from Global Financial Data from April 1973 to December 2014 for 19 developed countries and 12 emerging countries, as classified by MSCI. World Input Output Tables are from World Input-Output Database. Consumption data are from the World Bank’s World Development Indicators.
Figure 4: Network-Implied Clusters: Correlations and Volatilities

Note: Plots of average consumption growth correlations, equity market correlations, exchange rate volatilities, and base factor correlations, within and across clusters. Consumption growth correlations are constructed from yearly consumption growth. Equity market correlations are correlations of equity market returns in local currency. Exchange rates are nominal, and currency base factors are the average appreciation of currencies against all other currencies in the sample. Network closeness measures the implied correlation assuming primitive shocks are uncorrelated across countries. Three clusters of countries are constructed from a hierarchical clustering algorithm (Johnson (1967)) using the inverse of network closeness as a measure of distance. From left to right panel: the first cluster has Australia, Canada, Japan and the US; the second cluster has mostly mainland European countries; and the third cluster has Scandinavian countries. Red denotes across cluster averages while blue denotes averages within each cluster. Spot rate data are monthly from Global Financial Data from April 1973 to December 2014 for 19 developed countries as classified by MSCI. World Input Output Tables are from World Input-Output Database. Consumption data are from the World Bank’s World Development Indicators. Equity return data are from MSCI.

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Figure 5: Network-Implied Clusters: Factor Loadings

Note: Plots of loadings on cluster factors for consumption, equity market returns, and exchange rate base factors. Three clusters of countries are constructed using hierarchical clustering with the inverse of network closeness as a measure of distance. Cluster factors are constructed as the average across all countries quantity or asset price within each cluster. Loadings on these factors are estimated by regressing each country’s quantity on each cluster factor, omitting itself in the construction of the factor. Currency base factors are the average appreciation of each currency against all other currencies in the sample. Network closeness measures the implied correlation assuming primitive shocks are uncorrelated across countries. Consumption data are from the World Bank’s World Development Indicators. Spot rate data are monthly from Global Financial Data from April 1973 to December 2014 for 19 developed countries as classified by MSCI. World Input Output Tables are from World Input-Output Database. Equity return data are from MSCI.
Table 1: Explaining Consumption and Equity Correlations (Developed)

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Within $R^2$   0.43 0.08 0.08 0.08 0.11 0.01 0.32 0.29 0.28
Num. obs.      171 171 171 171 171 171 171 171 171

$*** p < 0.01, ** p < 0.05, * p < 0.1$

Note: Regressions $\text{corr}(\Delta c_{it}, \Delta c_{jt}) = \alpha + \beta \cdot \tilde{C}(i,j) + \varepsilon_{ij}$ and $\text{corr}(r_{eqit}, r_{eqjt}) = \alpha + \beta \cdot \tilde{C}(i,j) + \varepsilon_{ij}$ of bilateral consumption growth correlations and equity market correlations, respectively, on the consumption, exposure, network, production and production FO measures of closeness, as in Equation (3) and Equation (4). Bilateral regressions only contain the set of unique country pairs for 19 developed countries, as classified by MSCI. Consumption growth correlations are constructed from yearly consumption growth. Equity market correlations are correlations of equity market returns in local currency. Consumption closeness is bilateral consumption correlation. Network closeness measures the implied correlation assuming primitive shocks are uncorrelated across countries. Exposure closeness is the correlation of primitive shocks. Production closeness is based on only the production trade network and assumes perfect home bias in consumption. Production FO closeness is its first-order approximation using the Leontief Inverse. Consumption data are from the World Bank’s World Development Indicators. Equity return data are from MSCI. World Input Output Tables are from World Input-Output Database.
### Table 2: Explaining Consumption and Equity Correlations (Developed and Emerging)

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<tr>
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<th>Cons Cor</th>
<th>Cons Cor</th>
<th>Cons Cor</th>
<th>Cons Cor</th>
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<th>Eq</th>
<th>Eq</th>
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<td>0.17***</td>
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<td>0.49***</td>
<td>0.49***</td>
<td>0.49***</td>
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<td>0.49***</td>
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<tr>
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<td>2.87**</td>
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<tr>
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<td>7.03**</td>
<td>7.03**</td>
<td>7.03**</td>
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<td>12.10**</td>
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<td>0.04</td>
<td>0.03</td>
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<td>0.12</td>
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<td>0.18</td>
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Note: Regressions $\text{corr}(\Delta c_{it}, \Delta c_{jt}) = \alpha + \beta \cdot \bar{C}(i, j) + \epsilon_{ij}$ and $\text{corr}(r^e_{it}, r^e_{jt}) = \alpha + \beta \cdot \bar{C}(i, j) + \epsilon_{ij}$ of bilateral consumption growth correlations and equity market correlations, respectively, on the consumption, exposure, network, production and production FO measures of closeness, as in Equation (3) and Equation (4). Bilateral regressions only contain the set of unique country pairs for 19 developed countries and 12 emerging countries, as classified by MSCI. Consumption growth correlations are constructed from yearly consumption growth. Equity market correlations are correlations of equity market returns in local currency. Consumption closeness is bilateral consumption correlation. Network closeness measures the implied correlation assuming primitive shocks are uncorrelated across countries. Exposure closeness is the correlation of primitive shocks. Production closeness is based on only the production trade network and assumes perfect home bias in consumption. Production FO closeness is its first-order approximation using the Leontief Inverse. Consumption data are from the World Bank’s World Development Indicators. Equity return data are from MSCI. World Input Output Tables are from World Input-Output Database.
### Table 3: Explaining FX Correlations and Volatilities (Developed)

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<th>Base FX</th>
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<td>(1.81)</td>
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<td>(2.02)</td>
<td>-0.02**</td>
<td>(2.09)</td>
<td></td>
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<tr>
<td>Network</td>
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<td>(2.82)</td>
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</tr>
<tr>
<td>Production</td>
<td>13.46***</td>
<td>(2.64)</td>
<td>-0.98***</td>
<td>(2.72)</td>
<td></td>
<td></td>
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</tr>
<tr>
<td>Production FO</td>
<td>22.15**</td>
<td>(2.58)</td>
<td>-1.61***</td>
<td>(2.68)</td>
<td></td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td><strong>Within $R^2$</strong></td>
<td>0.06</td>
<td>0.11</td>
<td>0.22</td>
<td>0.17</td>
<td>0.16</td>
<td>0.07</td>
<td>0.14</td>
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<td>0.18</td>
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<td>Num. obs.</td>
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<td>171</td>
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</table>

**Note:** Regressions $corr(\Delta \tau_{it}, \Delta \tau_{jt}) = \delta_i + \delta_j + \beta \cdot \tilde{C}(i,j) + \varepsilon_{ij}$ and $std(\Delta \epsilon_{ij}) = \delta_i + \delta_j + \beta \cdot \tilde{C}(i,j) + \varepsilon_{ij}$ of currency base factor correlations and exchange rate volatility, respectively, on the consumption, exposure, network, production and production FO measures of closeness, as in Equation (5) and Equation (6). Bilateral regressions only contain the set of unique country pairs for 19 developed countries, as classified by MSCI, and include home country and foreign country fixed effects. Exchange rates are nominal, and currency base factors are the average appreciation of each currency against all other currencies in the sample. Consumption closeness is bilateral consumption correlation. Network closeness measures the implied correlation assuming primitive shocks are uncorrelated across countries. Exposure closeness is the correlation of primitive shocks. Production closeness is based on only the production trade network and assumes perfect home bias in consumption. Production FO closeness is its first-order approximation using the Leontief Inverse. Spot rate data are monthly from Global Financial Data from April 1973 to December 2014 for 19 developed countries as classified by MSCI. World Input Output Tables are from World Input-Output Database.
Table 4: Explaining FX Correlations and Volatilities (Developed and Emerging)

<table>
<thead>
<tr>
<th></th>
<th>Base FX</th>
<th>Base FX</th>
<th>Base FX</th>
<th>Base FX</th>
<th>Base FX</th>
<th>FX Vol</th>
<th>FX Vol</th>
<th>FX Vol</th>
<th>FX Vol</th>
</tr>
</thead>
<tbody>
<tr>
<td>Consumption</td>
<td>0.25***</td>
<td>(2.72)</td>
<td></td>
<td></td>
<td></td>
<td>-0.05**</td>
<td>(-2.44)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Exposure</td>
<td>0.11*</td>
<td>(1.74)</td>
<td></td>
<td></td>
<td></td>
<td>0.02**</td>
<td>(-2.12)</td>
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<td></td>
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<tr>
<td>Network</td>
<td>6.86***</td>
<td>(5.00)</td>
<td>16.39***</td>
<td>(3.35)</td>
<td></td>
<td>-0.64***</td>
<td>(-3.60)</td>
<td></td>
<td>-1.53***</td>
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<tr>
<td>Production</td>
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<td></td>
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<td>-2.56***</td>
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<td>Production FO</td>
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<td></td>
<td></td>
<td>27.28***</td>
<td>(3.24)</td>
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<td></td>
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<tr>
<td>Within R²</td>
<td>0.04</td>
<td>0.02</td>
<td>0.17</td>
<td>0.11</td>
<td>0.11</td>
<td>0.06</td>
<td>0.02</td>
<td>0.06</td>
<td>0.04</td>
</tr>
</tbody>
</table>

**Note:** Regressions $corr(\Delta e_{it}, \Delta e_{jt}) = \delta_i + \delta_j + \beta \cdot \tilde{C}(i, j) + \varepsilon_{ij}$ and $std(\Delta e_{i,j,t}) = \delta_i + \delta_j + \beta \cdot \tilde{C}(i, j) + \varepsilon_{ij}$ of currency base factor correlations and exchange rate volatility, respectively, on the consumption, exposure, network, production and production FO measures of closeness, as in Equation (5) and Equation (6). Bilateral regressions only contain the set of unique country pairs for 19 developed countries and 12 emerging countries, as classified by MSCI, and include home country and foreign country fixed effects. Exchange rates are nominal, and currency base factors are the average appreciation of each currency against all other currencies in the sample. Consumption closeness is bilateral consumption correlation. Network closeness measures the implied correlation assuming primitive shocks are uncorrelated across countries. Exposure closeness is the correlation of primitive shocks. Production closeness is based on the production trade network and assumes perfect home bias in consumption. Production FO closeness is its first-order approximation using the Leontief Inverse. Spot rate data are monthly from Global Financial Data from April 1973 to December 2014 for 19 developed countries as classified by MSCI. World Input Output Tables are from World Input-Output Database.

Table 5: Percent of variance explained by principal components

<table>
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<tr>
<th>PC</th>
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<th>Exposure</th>
<th>Consumption</th>
<th>Base Factors</th>
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<td>1</td>
<td>4.5</td>
<td>58.6</td>
<td>31.5</td>
<td>24.8</td>
</tr>
<tr>
<td>2</td>
<td>3.5</td>
<td>19.6</td>
<td>17.9</td>
<td>19.7</td>
</tr>
<tr>
<td>3</td>
<td>3.1</td>
<td>9.8</td>
<td>11.3</td>
<td>9.2</td>
</tr>
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<td>4</td>
<td>3.0</td>
<td>5.5</td>
<td>8.7</td>
<td>8.6</td>
</tr>
</tbody>
</table>

**Note:** Percent of variance explained by the first four principal components extracted from network closeness, exposure closeness, covariance matrix of consumption growth rates, and covariance matrix of currency base factors. Network closeness measures the implied correlation assuming primitive shocks are uncorrelated across countries. Exposure closeness is the correlation of primitive shocks. Currency base factors are the average appreciation of each currency against all other currencies in the sample. For each of the four covariance matrices, $C$, the eigenvalue decomposition produces $C = U \Lambda U'$, where $U$ is an orthogonal matrix, and $\lambda = diag(\lambda_1, \ldots, \lambda_N)$ is a diagonal matrix containing the eigenvalues or principal components of $C$. Consumption data are from the World Bank’s World Development Indicators. World Input Output Tables are from World Input-Output Database. Spot rate data are monthly from Global Financial Data from April 1973 to December 2014 for 19 developed countries as classified by MSCI.
<table>
<thead>
<tr>
<th>Network</th>
<th>Dollar</th>
<th>HML</th>
<th>UHML</th>
<th>PMC</th>
<th>Dollar</th>
<th>HML</th>
<th>UHML</th>
<th>PMC</th>
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</thead>
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<td>2.04</td>
<td>4.10*</td>
<td>4.92**</td>
<td>(2.73)</td>
<td>(1.14)</td>
<td>(1.84)</td>
<td>(2.21)</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(−0.56)</td>
<td>(0.37)</td>
<td>(0.58)</td>
<td>(0.39)</td>
</tr>
<tr>
<td>Network 2</td>
<td>−0.37</td>
<td>0.28</td>
<td>0.54</td>
<td>0.37</td>
<td>(−0.68)</td>
<td>(0.69)</td>
<td>(1.17)</td>
<td>(2.59)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(−0.28)</td>
<td>(−0.29)</td>
<td>(−0.07)</td>
<td>(0.10)</td>
</tr>
<tr>
<td>Network 3</td>
<td>−1.69***</td>
<td>−0.28</td>
<td>−0.59</td>
<td>−1.33**</td>
<td>(−4.68)</td>
<td>(−0.69)</td>
<td>(−1.17)</td>
<td>(−2.59)</td>
</tr>
<tr>
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<td>(0.94)</td>
<td>(−0.20)</td>
<td>(−0.27)</td>
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<tr>
<td>Expos 1</td>
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<td>(−0.28)</td>
<td>(−0.29)</td>
<td>(−0.07)</td>
<td>(0.10)</td>
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<td>(0.47)</td>
<td>(0.01)</td>
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<td>(0.16)</td>
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<td>Expos 3</td>
<td></td>
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<td>0.21</td>
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<td>0.05</td>
<td>0.01</td>
<td>0.01</td>
<td>0.02</td>
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<tr>
<td>Within R²</td>
<td>0.64</td>
<td>0.09</td>
<td>0.21</td>
<td>0.38</td>
<td>0.05</td>
<td>0.01</td>
<td>0.01</td>
<td>0.02</td>
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Note: Regressions $\beta_i = \alpha + \beta_1 PC_{1,i}(C) + \beta_2 PC_{2,i}(C) + \beta_3 PC_{3,i}(C) + \varepsilon_i$ of actual loadings of currency base factors on the dollar, unconditional carry, and peripheral-minus-central exchange rate factors on the loadings of the first three principal components implied by network and exposure closeness as in Equation (8). Currency base factors are the average appreciation of each currency against all other currencies in the sample. Network closeness measures the implied correlation assuming primitive shocks are uncorrelated across countries. Exposure closeness is the correlation of primitive shocks. Actual loadings are estimated from regressions of currency $i$’s base factor $\Delta\tau_i$ on the dollar, peripheral-minus-central, and unconditional carry factors, respectively. Predicted loadings are the fitted values. Spot rate data are monthly from Global Financial Data from April 1973 to December 2014 for 19 developed countries and 12 emerging countries, as classified by MSCI. World Input-Output Tables are from World Input-Output Database. Consumption data are from the World Bank’s World Development Indicators.
Table 7: Explaining FX Factors with Principal Components of Closeness

<table>
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<tr>
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<th>UHML</th>
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<th>HML</th>
<th>UHML</th>
<th>PMC</th>
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<td>1.32***</td>
<td>1.19***</td>
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<td></td>
<td>(6.28)</td>
<td>(4.44)</td>
<td>(5.58)</td>
<td>(6.06)</td>
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<tr>
<td>Network 2</td>
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<td>0.35</td>
<td>0.20</td>
<td>0.29**</td>
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<td>(−7.47)</td>
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<td>−0.03</td>
<td>0.03</td>
<td>−0.13**</td>
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<td>(2.54)</td>
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<tr>
<td>Exposure 2</td>
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<td>−0.24</td>
<td>−0.29</td>
<td>−0.33</td>
<td>−0.32**</td>
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<tr>
<td></td>
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<td>(−1.54)</td>
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<tr>
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<td></td>
<td>0.10</td>
<td>0.02</td>
<td>0.06</td>
<td>0.16</td>
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<td></td>
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<tr>
<td></td>
<td></td>
<td>(0.78)</td>
<td>(0.10)</td>
<td>(0.40)</td>
<td>(1.44)</td>
<td></td>
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<td></td>
</tr>
<tr>
<td>Within R²</td>
<td>0.88</td>
<td>0.22</td>
<td>0.42</td>
<td>0.44</td>
<td>0.10</td>
<td>0.02</td>
<td>0.03</td>
<td>0.04</td>
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<td>179</td>
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</table>

***p < 0.01, **p < 0.05, *p < 0.1

Note: Regressions $f_t = \alpha + \beta_1 PC_1(C)\Delta \bar{e}_i + \beta_2 PC_2(C)\Delta \bar{e}_i + \beta_3 PC_3(C)\Delta \bar{e}_i + \varepsilon_t$ of the dollar, carry, unconditional carry, and peripheral-minus-central exchange rate factors on the exchange rate movements of the currency portfolios based on the first three principal components implied by from network and exposure closeness, as in Equation (8). Currency base factors are the average appreciation of each currency against all other currencies. Network closeness measures the implied correlation assuming primitive shocks are uncorrelated across countries. Exposure closeness is the correlation of primitive shocks. Spot rate data are monthly from Global Financial Data from April 1973 to December 2014 for 19 developed countries and 12 emerging countries, as classified by MSCI. World Input Output Tables are from World Input-Output Database. Consumption data are from the World Bank’s World Development Indicators.
A Proof Appendix

Lemma 1

For proof of the first lemma we omit time subscripts because there is no storage technology and therefore the model can be solved period by period. The social planner’s Lagrangian is

$$\sum_{i=1}^{N} \lambda_i \left( \sum_{j=1}^{N} v_{ij} \log C_{ij} \right) + \varphi_i \left( A_i L_i^\theta \left( \prod_{j=1}^{N} X_{ij}^{w_{ij}} \right) - \sum_{j=1}^{N} (C_{ji} + X_{ji}) \right) + \chi_i (\bar{L}_i - L_i) \quad (10)$$

FOCs are

w.r.t. $C_{ji}$: \[ \lambda_j v_{ji} C_{ji}^{-1} = \varphi_i \quad (11) \]

w.r.t. $X_{ji}$: \[ \varphi_j X_{ji}^\theta w_{ji} X_{ji}^{-1} = \varphi_i \quad (12) \]

w.r.t. $L_i$: \[ \varphi_i \bar{X}_i \theta L_i^{-1} = \chi_i \quad (13) \]

Substitute into the market clearing condition:

$$\varphi_i \bar{X}_i = \sum_{j=1}^{N} \left( \lambda_j v_{ji} + \varphi_j \bar{X}_j w_{ji} \right). \quad (14)$$

Define $\Gamma_i = \varphi_i \bar{X}_i$. Then

$$\Gamma = (I - W')^{-1} V' \lambda \quad (15)$$

is determined by the primitive parameters. In particular, it is not determined by the productivity shocks $A_i$.

Then, the log production is

$$\log \bar{X}_i = a_i + \theta \ell_i + \sum_{j=1}^{N} w_{ij} \log \left( \frac{\Gamma_i \bar{X}_j}{\Gamma_j} w_{ij} \right), \quad (16)$$
which implies

\[ \bar{x} = \kappa x + a + W \bar{x} \] (17)

\[ = (I - W)^{-1}(\kappa x + a). \] (18)

The log consumption is

\[ \bar{c}_i = \sum_{j=1}^{N} v_{ij} \log \left( \frac{\lambda_i v_{ij} \bar{x}_j}{\Gamma_j} \right) \] (19)

\[ \bar{c} = \kappa^c + V \bar{x} \] (20)

\[ \bar{c} = \kappa^c + V(I - W)^{-1}(\kappa x + a) \] (21)

Lemma 2

Pick any numeraire. Let \( q_{it} \) denote the price of country \( i \)'s tradable goods at time \( t \) in the numeraire. We return from the social planner's solution to the competitive equilibrium. The optimization of country \( i \)'s representative household is

\[ \sum_{t=1}^{\infty} \sum_{j=1}^{N} v_{ij} \log C_{ijt}. \] (22)

Since the markets are complete, the household can pick any consumption path that satisfies the infinite-horizon budget constraint:

\[ W_0 + \sum_{t=1}^{\infty} q_{it} A_{it} \theta \left( \prod_{j=1}^{N} X_{ijt}^{w_{ij}} \right) - \sum_{t=1}^{\infty} \sum_{j=1}^{N} q_{jt} (C_{ijt} + X_{ijt}) = 0, \] (23)

where \( W_0 \) is the initial transfer that is consistent with the Pareto weights \( \lambda_i \). The Lagrangian is

\[ \sum_{t=1}^{\infty} \sum_{j=1}^{N} v_{ij} \log C_{ijt} + M_i \left( W_0 + \sum_{t=1}^{\infty} q_{it} A_{it} \theta \left( \prod_{j=1}^{N} X_{ijt}^{w_{ij}} \right) - \sum_{t=1}^{\infty} \sum_{j=1}^{N} q_{jt} (C_{ijt} + X_{ijt}) \right) + \sum_{t=1}^{\infty} N_{it} (L_i - L_{it}) \] (24)
The FOC w.r.t. $C_{ijt}$ is
\[ v_{ij}C_{ijt}^{-1} = M_i q_{jt} \] (25)

which implies $q_{it} = \xi \varphi_{it}$ where $\xi$ is a constant. Then Lemma 2(a) and 2(b) directly follow from the social planner’s FOCs:
\[ \lambda_i v_{ij} = q_{jt} C_{ijt} / \xi \] (26)
\[ q_{it} X_{it} w_{ij} = q_{jt} X_{ijt} \] (27)

Lemma 2(c) follows from $\theta_i + \sum_j w_{ij} = 1$, which implies
\[ q_{it} X_{it} (1 - \theta_i) = q_{it} X_{it} \sum_j w_{ij} = \sum_j q_{jt} X_{ijt}. \] (28)

**Lemma 3**

Let $p_{it}$ denote the price of country $i$’s consumption basket. The household maximizes
\[ p_{it} \left( \prod_{j=1}^{N} C_{ijt}^{w_{ij}} \right) - \sum_{j=1}^{N} q_{jt} C_{ijt} \] (29)

The zero-profit condition implies
\[ p_{it} = \frac{\sum_{j=1}^{N} q_{jt} C_{ijt}}{C_{it}} \] (30)
Then, the log real exchange rate between countries $i$ and $j$ is

$$e_{ij} = \log \frac{p_{it}}{p_{jt}}$$  \hspace{1cm} (31)$$

$$= \log \frac{\sum_{k=1}^{N} q_{kt} C_{ikt}}{\sum_{k=1}^{N} q_{kt} C_{jkt}} + \tau_{jt} - \tau_{it}$$  \hspace{1cm} (32)$$

$$= \log \frac{\sum_{k=1}^{N} \lambda_{i} v_{ik}}{\sum_{k=1}^{N} \lambda_{j} v_{jk}} + \tau_{jt} - \tau_{it}$$  \hspace{1cm} (33)$$

Then the change in the currency base factor is

$$\Delta \tau_{it+1} = \frac{1}{N} \sum_{j=1}^{N} \Delta \tau_{jt+1} - \Delta \tau_{it+1}. \hspace{1cm} (34)$$

The price of the claim to period $t + k$ consumption is

$$\mathbb{E}_t \left[ e^{-k\beta - \tau_{it+k} + \tau_{it} C_{it+k}} \right] = e^{-k\beta} C_{it}. \hspace{1cm} (35)$$

So the price of the ex-dividend claim to the consumption stream is

$$P_{it}^{eq} = \sum_{k=1}^{\infty} e^{-k\beta} C_{it} = \frac{e^{-\beta}}{1-e^{-\beta}} C_{it}. \hspace{1cm} (36)$$

The log cum-dividend return is

$$r_{it+1}^{eq} = \log \frac{C_{it+1} + \frac{e^{-\beta}}{1-e^{-\beta}} C_{it+1}}{\frac{e^{-\beta}}{1-e^{-\beta}} C_{it}} = \beta + \Delta \tau_{it+1}. \hspace{1cm} (37)$$

**Proposition 1 and 2**

The covariance of consumption growth and equity returns follows directly from Lemma 1:

$$\text{cov}(\Delta \tau_{it}, \Delta \tau_{jt}) = \text{cov}(r_{it}^{eq}, r_{jt}^{eq}) = H' \Omega' = \mathcal{C}(i, j).$$

The moments of exchange rates follow from the covariance of consumption growth. In partic-
ular, the covariance between the changes in currency base factors is

\[
\text{cov}(\Delta e_{it+1}, \Delta e_{jt+1}) = \text{cov} \left( \frac{1}{N} \sum_{k=1}^{N} \Delta \bar{e}_{kt+1} - \Delta \bar{e}_{it+1}, \frac{1}{N} \sum_{k=1}^{N} \Delta \bar{e}_{kt+1} - \Delta \bar{e}_{jt+1} \right) = \mathcal{C}(i, j) - \overline{\mathcal{C}}(i) - \overline{\mathcal{C}}(j) + \kappa^e. \tag{38}
\]

**Proposition 3**

If two matrices \( A \) and \( B \) have the same block structure,

\[
(AB)_{ij} = \sum_{k} A_{ik} B_{kj} \tag{40}
\]

which is non-zero only if \( k \in R(i) \) and \( k \in R(j) \). So, not only does \( AB \) have the same block structure, the results in each block of \( AB \) are the product between the corresponding blocks of \( A \) and \( B \).

Because

\[
(I - W)^{-1} = I + W + W^2 + \ldots, \tag{42}
\]

then the inverse \((I - W)^{-1}\) in each block is also the inverse of the corresponding block of \((I - W)\).

Next, we consider a \( \ell \)-by-\( \ell \) matrix \( U \) such that \( U_{ij} \) is \( \omega_0 \) if \( i = j \) and \( \omega_1 \) otherwise. We conjecture

\[
\{(I - U)^{-1}\}_{ij} = \begin{cases} 
\kappa^h(1 - \omega_0 - (\ell - 2)\omega_1), & \text{if } i = j, \\
\kappa^h \omega_1, & \text{if } i \neq j.
\end{cases} \tag{43}
\]
Then

$$\{(I - U)^{-1}(I - U)\}_{ij} = \sum_{k=1}^{\ell} \{(I - U)^{-1}\}_{ik} \{(I - U)\}_{kj}$$

$$= \begin{cases} 
\kappa^h(1 - \omega_0 - (\ell - 2)\omega_1) \cdot (1 - \omega_0) + (\ell - 1)\kappa^h\omega_1 \cdot (-\omega_1), & \text{if } i = j, \\
\kappa^h(1 - \omega_0 - (\ell - 2)\omega_1) \cdot (-\omega_1) + \kappa^h\omega_1 \cdot (1 - \omega_0) + (\ell - 2)\kappa^h\omega_1 \cdot (-\omega_1), & \text{if } i \neq j \text{ and } j \in R(i). 
\end{cases}$$

Thus, we confirm our conjecture, and $\kappa^h$ can be solved from $\kappa^h(1 - \omega_0 - (\ell - 2)\omega_1) \cdot (1 - \omega_0) + (\ell - 1)\kappa^h\omega_1 \cdot (-\omega_1) = 1$. It then follows that $\kappa > 0$ and

$$\{(I - U)^{-1}H\}_{ij} = \begin{cases} 
\kappa^h\omega_1\xi_{jj} + \kappa^h\theta\xi_{jj}, & \text{if } i = j, \\
\kappa^h\omega_1\xi_{jj}, & \text{if } i \neq j.
\end{cases} \quad (44)$$

Similarly,

$$\{(I - U)^{-1}\Psi\}_{ik} = \kappa^h\theta\psi_{ik} + \sum_{j=1}^{\ell} \kappa^h\omega_1\psi_{jk} \quad (45)$$

### B Empirical Appendix
Figure A1: World Production Network

Note: Plot of the bilateral world production network as implied by the World Input Output Table.
Figure A2: World Consumption Network

Note: Plot of the bilateral world consumption network as implied by the World Input Output Table.
Figure A3: Covariances between Productivity Shocks

Note: Plot of the implied exposure network from consumption growth and the world input output network. Exposure shocks are the implied shock structure that is necessary to explain bilateral consumption growth correlations using the world input output network.