

# Sovereign Bond Premium and Global Macroeconomic Conditions\*

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## ABSTRACT

This paper studies how global macroeconomic conditions affect sovereign bond prices. Weak and volatile economic performance during recessions increases a country's default probability more than strong and stable performance during expansions reduces it, leading to countercyclical and unconditionally high sovereign credit spreads. We identify the sovereign bond premium arising from this exposure to severe but low-frequency changes in global macroeconomic conditions. Our model predicts that this bond premium is higher for countries that are more exposed to the global business cycle, particularly around recessions. We find support for this prediction using emerging market sovereign bond data over the 1994Q1-2018Q2 period.

JEL Codes: F34, G12, G13, G15, G32

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# 1 Introduction

The Great Recession sparked renewed interest in understanding the causes and consequences of business cycles (e.g., Jurado, Ludvigson, and Ng, 2015; Bloom et al., 2018). Business cycles typically involve sudden shifts between regimes of high expected growth and low uncertainty (expansions), and regimes of reduced expected growth and heightened uncertainty (recessions), thereby suggesting that economic shocks are not linear or homoscedastic (Hamilton, 1989). A recent stream of research shows that accounting for such regime-shifts in macroeconomic conditions help resolve various empirical puzzles in corporate finance and asset pricing.<sup>1</sup> Yet there is limited knowledge about how these time-varying macroeconomic conditions affect sovereign bond valuation in emerging markets. We aim to fill this gap. Specifically, this paper is the first to analyze the sovereign bond premium in a model in which debt and default decisions are endogenously shaped by business cycle fluctuations.

Our model considers a (small) country and a global representative agent with Epstein-Zin-Weil preferences. A global business cycle characterizes the state of the economy, which switches randomly between expansions and recessions via a Markov chain. Recessions are associated with lower expected growth and higher uncertainty for the global agent's consumption and for local output. Hence, the country is economically exposed to the global business cycle. We label the country's exposure to regime-shifts in global macroeconomic conditions as "long-run macro risk". Additionally, innovations on local output and global consumption are conditionally dependent, which we refer to as "short-run macro risk". The country's government accounts for both sources of macroeconomic risk when deciding the indebtedness level and the timing of default. The debt decision trades off the benefits of debt against the rise in default risk, while the default decision trades off the benefit of smaller future debt payments against an immediate output drop associated with default.<sup>2</sup> Security markets are integrated internationally such that sovereign bonds are priced by the global representative agent.

The exposure to the global business cycle has a significant impact on sovereign bond pricing. Long-run macro risk generates sovereign credit spreads that are countercyclical with respect to the global business cycle and unconditionally high, closely matching the data. Calibrating our model with quarterly real GDP data for 40 emerging countries from 1994Q1 to 2018Q2, we find that the sovereign credit

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<sup>1</sup>Corporate finance models with business cycle frequency regime-shifts in macroeconomic conditions include Hackbarth, Miao, and Morellec (2006), Bhamra, Kuehn, and Strebulaev (2010a,b), Chen (2010), Arnold, Wagner, and Westermann (2013), Bhamra, Dorion, Jeanneret, and Weber (2018), and Chen and Strebulaev (2019), among others. See Cochrane (2017) for a review of the role of business cycle fluctuations in asset pricing.

<sup>2</sup>Our model builds on the contingent-claim framework approach developed by Hayri (2000), Gibson and Sundaresan (2001), Andrade (2009), and Jeanneret (2015, 2017, 2018).

spread is 337 bps in recessions and 300 bps in expansions. The unconditional credit spread equals 312 bps, but reduces to 248 bps in the case without long-run macro risk, i.e., when we switch off the country's exposure to the global business cycle. This comparison considers identical unconditional moments of output growth such that both cases are observationally equivalent in terms of risk. In the absence of long-run macro risk, the model sovereign credit spread is not only lower but also becomes mostly acyclical. As a result, the difference between credit spreads with and without macroeconomic risk is countercyclical and economically large.

Long-run macro risk affects sovereign credit risk through two partially offsetting effects. First, the exposure to the global business cycle magnifies the quantity of sovereign risk due to a convexity effect. The relation between output volatility and default probability is strongly convex, which implies that the unconditional default probability is higher in a model in which output growth moments change across regimes than in a model with constant moments. Second, the government responds to the increase in default risk, which raises the country's borrowing costs, by choosing a lower indebtedness level. However, this policy response is not strong enough to reverse the convexity effect and macroeconomic risk thus positively impacts the level of credit risk.

Sovereign bond investors receive a risk premium for bearing macroeconomic risk. The exposure of a country's economic conditions to the global business cycle causes sovereign default risk to be partly systematic, in addition to the instantaneous dependence between local output and global consumption. As both default risk and investors' marginal utility of consumption are higher during recessions, sovereign bond becomes particularly risky. Moreover, investors with a preference for early resolution of uncertainty dislike uncertainty about when recessions arrive and, therefore, price sovereign bonds as if recessions arrive sooner and last longer than under physical probabilities. We find that the price of risk implies that investors overweight the probability of switching from expansion to recession by 53%. Notably, the sovereign bond premium vanishes when investors have power utility, which indicates that Epstein-Zin-Weil preferences play a central role for understanding sovereign bond pricing. Therefore, we show that macroeconomic risk increases a country's sovereign credit spread through both the quantity of risk and the price of risk.

Our model generates a new set of predictions on the sovereign bond premium across countries and over the business cycle. First, we find that the sovereign bond premium equals 28.4bps in expansions and 37.8bps in recessions, which implies that macroeconomic risk plays a greater role for sovereign bond valuation when global economic conditions deteriorate. Second, we show that sovereign

bonds of countries that are more exposed to the global business cycle offer higher excess returns. For this analysis, we compute the sovereign bond premium for different combinations of a country's first and second moments' exposure to the global business cycle. For a given volatility exposure, doubling the expected output growth exposure increases the sovereign bond premium from 31.6 bps to 57.9 bps, while doubling the output volatility exposure increases the sovereign bond premium to 65.5 bps. Combining the two effects, the sovereign bond premium increases to 90.2 bps. Hence, both sources of macroeconomic risk are complementary drivers of the sovereign bond premium, while the exposure to consumption uncertainty has a relatively greater impact on the sovereign bond premium.

We find support for these theoretical predictions using quarterly sovereign bond data.<sup>3</sup> We sort countries according to their estimated exposures of bond excess returns to U.S. consumption moments and compute the performance of long-short portfolios. The average annual excess return of the long-short portfolio based on exposure to U.S. expected consumption growth is 1.52% and its annualized Sharpe ratio is 0.89. The long-short portfolio based on exposure to U.S. consumption growth uncertainty has an average excess return of -1.58% per year and an annualized Sharpe ratio of -0.79. The signs are line with model predictions, since exposure to U.S. expected consumption growth carries a positive price of risk (expected growth is lower in recessions), while exposure to U.S. consumption uncertainty carries a negative price of risk (uncertainty is higher in recessions).

When conditioning the portfolio returns on the business cycle, we find that average long-short portfolio returns are higher during NBER recessions, consistent with our model. Furthermore, the results remain similar whether we individually or jointly estimate the exposure to each U.S. consumption growth moment, which validates the theoretical prediction that sovereign credit spreads embed a sovereign premium reflecting two complementary components of macroeconomic risk. Our results are also robust to controlling for additional risk factors, such as the exposure to U.S. stock market returns and to the exposure to U.S. consumption shocks. Cross-country variation in exposure to the global business cycle thus contributes to explaining differences in the sovereign bond premium across countries.

This paper relates to an extensive literature documenting that sovereign credit risk varies with global financial and economic conditions.<sup>4</sup> The empirical evidence indicates that sovereign credit risk increases

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<sup>3</sup>We consider JP Morgan Emerging Market Bond Indices for 36 countries from 1994Q1 to 2018Q2. We estimate the exposure of sovereign bond excess returns to changes in the first and second moments of expected U.S. consumption growth. We measure the first and second moments of expected consumption growth as the quarterly mean and dispersion (75th-25th percentile) of the U.S. consumption growth forecasts from the Survey of Professional Forecasters.

<sup>4</sup>See, e.g., Pan and Singleton (2008), Remolona, Scatigna, and Wu (2008), Hilscher and Nosbusch (2010), Ang and Longstaff (2013), Benzoni, Collin-Dufresne, Goldstein, and Helwege (2015), Jeanneret (2015), Doshi, Jacobs, and Zurita (2017), and Augustin (2018).

(decreases) with global uncertainty (stock market performance), which tends to increase (decrease) during recessions. Hence, the literature suggests that a country's exposure to regime-shifting global macroeconomic conditions should play a major role in understanding sovereign credit risk premia. We contribute to this literature by quantifying and decomposing the macroeconomic sources of sovereign risk premia, both theoretically and empirically.

Our analysis of sovereign credit spreads relates to previous studies in several dimensions. Pan and Singleton (2008) and Longstaff et al. (2011) propose an affine specification that identifies the default-risk and risk-premium components of sovereign credit spreads. These studies document a significant risk premium embedded in sovereign credit spreads, but remain silent on the fundamental nature of this risk premium, as well as on its variation across time and countries. Augustin and Tédongap (2016) introduce a reduced-form default process into an equilibrium pricing model with recursive preferences, and use the model to highlight the role of macroeconomic risk in explaining the international co-movement in the term structure of sovereign credit default swap (CDS) spreads. We complement their analysis by studying the implications of macroeconomic risk for cross-country differences in sovereign risk premia when governments optimally choose debt and default policies.

Another closely related work is Borri and Verdelhan (2012). The authors propose a model in which a country's economic shocks are instantaneously correlated to consumption shocks, and explore the associated risk premium when investors have habit preferences. Their empirical analysis shows that average bond excess returns increase with the correlation between sovereign bond returns and U.S. corporate bond and stock market returns. We differ from their paper by exploring a fundamentally different source of systematic risk, in addition to using a contingent-claim approach with recursive preferences. While Borri and Verdelhan (2012) analyze the price of risk associated with high-frequency systematic shocks (e.g., daily returns on the S&P500), we focus on larger slow-moving systematic shocks (recessions vs. expansions) as measured by U.S. macroeconomic data. We empirically confirm that the price of macroeconomic risk we uncover is robust to controlling for the exposure to S&P500 returns as well as contemporaneous changes in U.S. consumption. Both papers are thus complementary for understanding how systematic risk drives sovereign spreads.

The present study complements another strand of the literature that explains why countries default and why borrowing costs are countercyclical. For example, this literature includes Aguiar and Gopinath (2006), Arellano (2008), and Yue (2010), who develop dynamic stochastic equilibrium models based on the classic work of Eaton and Gersovitz (1981). Mendoza and Yue (2012) propose a general

equilibrium model of sovereign default and business cycle to explain the fall in economic activity around defaults and the countercyclical nature of sovereign spreads, among other stylized facts. Focusing on unusually severe recessions, Rebelo, Wang, and Yang (2018) explore how disaster risk affects a country’s indebtedness and credit spreads. This theoretical literature focuses on the expected default loss component in sovereign spreads, but remains silent on any risk premia. In contrast, our model sheds light on the role of regime-shifting global macroeconomic conditions and investor preferences in sovereign bond pricing. In particular, we provide new theoretical insights on the magnitude of the macro risk premium and its variation over time and across countries, and we empirically validate such insights.

The remainder of the paper is organized as follows. Section 2 outlines a model to quantify the role of macroeconomic risk for sovereign credit spreads. Section 3 describes the model calibration. Section 4 analyzes the theoretical predictions, which are tested empirically in Section 5. Finally, Section 6 concludes.

## 2 The model

We develop a dynamic asset-pricing model for sovereign bond valuation in the presence of macroeconomic risk, which reflects both a country’s exposure to the global business cycle and the correlation between output and consumption shocks. Each country has a government that sets the debt and default policies optimally, while a global representative agent determines the pricing of the government bonds. All variables are in real terms and information is complete.

### 2.1 Economic environment

We first define the dynamics of global consumption (hereafter “consumption”). The global economy (hereafter “the economy”) can be in expansion or recession, and the conditional moments of consumption growth characterize the global business cycle. We then describe the state-price density of the representative agent.

#### 2.1.1 Consumption

Let  $C_t$  denote the perpetual stream of consumption, with dynamics given exogenously by

$$\frac{dC_t}{C_t} = \mu_{c,s_t} dt + \sigma_{c,s_t} dZ_{c,t}, \quad s_t = \{L, H\}, \quad (1)$$

where  $Z_{c,t}$  is a standard Brownian motion under the physical probability measure  $\mathbb{P}$ . The first and second conditional moments of consumption growth  $\mu_{c,s_t}$  and  $\sigma_{c,s_t}$  take different values depending on the current state of the economy, denoted by  $s_t$ . The economy switches between a recession state ( $s_t = L$ ) and an expansion state ( $s_t = H$ ) according to a two-state Markov chain. Expected consumption growth is procyclical, while consumption growth uncertainty is countercyclical, that is  $\mu_{c,H} > \mu_{c,L}$  and  $\sigma_{c,H} < \sigma_{c,L}$ . We denote the probability under the physical measure of leaving state  $s_t$  by  $\lambda_{s_t}$  so that the expected duration of state  $s_t$  is  $1/\lambda_{s_t}$ . Recessions are shorter than expansions when  $1/\lambda_L < 1/\lambda_H$ .

### 2.1.2 State-price density

The representative agent has Epstein-Zin-Weil preferences with a state-price density  $\pi_t$  given by (see Appendix A):

$$\pi_t = \left( \beta e^{-\beta t} \right)^{\frac{1-\gamma}{1-\frac{1}{\psi}}} C_t^{-\gamma} \left( p_{C,t} e^{\int_0^t p_{C,u}^{-1} du} \right)^{-\frac{\gamma-\frac{1}{\psi}}{1-\frac{1}{\psi}}}, \quad (2)$$

where  $\gamma$  is the coefficient of relative risk aversion,  $\psi$  is the elasticity of intertemporal consumption, and  $\beta$  is the subjective time discount factor. The agent distinguishes between risk aversion and aversion to intertemporal resolution of uncertainty. The value of the claim to one unit of consumption per unit of time is denoted by the price-consumption ratio  $p_{C,t}$ . When  $\psi > 1$ ,  $p_{C,t}$  is procyclical and the state-price density is countercyclical.

The representative agent cares about the risk associated with future consumption growth and prefers early resolution of uncertainty ( $\gamma > 1/\psi$ ).<sup>5</sup> As a result, she uses the risk-neutral probability per unit of time of switching from expansion to recession, which is higher than the physical probability. The conversion of the physical probabilities  $\lambda_L$  and  $\lambda_H$  to their risk-neutral counterparts  $\hat{\lambda}_L$  and  $\hat{\lambda}_H$  depends on a risk distortion factor  $\Delta_H > 1$ , which is defined as the change in the state-price density  $\pi_t$  at the transition time from expansion to recession (see Appendix A). The risk-neutral probabilities of changing state are given by

$$\hat{\lambda}_H = \Delta_H \lambda_H \quad \text{and} \quad \hat{\lambda}_L = \frac{1}{\Delta_H} \lambda_L, \quad (3)$$

which implies that the representative agent prices securities as if recessions last longer ( $\lambda_L > \hat{\lambda}_L$ ) and expansions shorter ( $\lambda_H < \hat{\lambda}_H$ ) than in reality. The risk-neutral rate of news arrival is  $\hat{\rho} = \hat{\lambda}_L + \hat{\lambda}_H$

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<sup>5</sup>The price-consumption ratio claim vanishes from the stochastic discount factor when  $\gamma = \frac{1}{\psi}$ .

and the long-run risk-neutral distribution of the states is given by  $(\hat{f}_L, \hat{f}_H) = \left(\frac{\hat{\lambda}_H}{\hat{p}}, \frac{\hat{\lambda}_L}{\hat{p}}\right)$ . The rate of news arrival reflects the rate at which the distribution for the Markov chain converges in the long run. Therefore, intertemporal risk depends not only on whether news about consumption growth is good or bad, but also on the speed at which such news arrives.

## 2.2 Sovereign bond valuation

We first present the dynamics of government revenue for a small indebted country, which is exposed to various sources of systematic risk, and then derive sovereign bond valuation for given set of default and debt decisions. Later we endogenize these policies.

The level of the country  $i$ 's output is denoted by  $X_{i,t}$  and evolves according to

$$\frac{dX_{i,t}}{X_{i,t}} = \mu_{X,i,s_t} dt + \sigma_{X,i,s_t} dZ_{i,t}, \quad s_t = \{L, H\}, \quad (4)$$

where the conditional expected growth rate is  $\mu_{X,i,s_t}$  and the conditional volatility is  $\sigma_{X,i,s_t}$ . The dynamics of output depend on the global business cycle. Specifically, global recessions imply lower expected output growth rate ( $\mu_{X,i,H} > \mu_{X,i,L}$ ) and higher output volatility ( $\sigma_{X,i,H} < \sigma_{X,i,L}$ ).<sup>6</sup> The instantaneous shocks  $Z_{i,t}$  are correlated with consumption growth shocks  $Z_{c,t}$ , where  $\rho_{i,s_t}$  denotes the level of the correlation in state  $s_t$ . Systematic risk in output is thus associated with high-frequency consumption shocks and with low-frequency changes in the global business cycle determined by the state  $s_t$ . The global business cycle is exogenous to the country's economic performance, that is the country is small relative to the global economy.

The government raises revenue  $Y_{i,t}$  which perfectly correlates with the country's output  $X_{i,t}$ , and therefore is exposed to the same sources of risk. We assume that the dynamics of government revenue satisfy

$$\frac{dY_{i,t}}{Y_{i,t}} = \mu_{Y,i,s_t} dt + \sigma_{Y,i,s_t} dZ_{i,t}, \quad s_t = \{L, H\}, \quad (5)$$

with  $\sigma_{Y,i,s_t} = \eta \sigma_{X,i,s_t}$ , where  $\eta$  amplifies the volatility of government revenue relative to output growth volatility, as in Chen (2013). That is,  $\eta > 1$  and government revenue shocks are more volatile than output shocks.<sup>7</sup> The expected growth rate of government revenue is equal to the expected output

<sup>6</sup>In Section 4.3 we let  $\mu_{X,i,H} - \mu_{X,i,L}$  and  $\sigma_{X,i,L} - \sigma_{X,i,H}$  vary in order to explore cross-country implications of our model.

<sup>7</sup>Essentially, government (net) revenue consist of taxed output net of the public spending component, which tends to

growth rate (i.e.,  $\mu_{Y,i,s_t} = \mu_{X,i,s_t}$ ).

The government of country  $i$  issues an infinite maturity sovereign bond that is characterized by a perpetual coupon  $c_i$ . In the absence of default, the bond value equals  $\frac{c_i}{r_{B,s_t}}$  when the current state is  $s_t$ , which is the present value of the continuous stream of coupons  $c_i$  discounted at a riskless perpetuity rate  $r_{B,s_t}$  given by

$$r_{B,s_t} = r_{s_t} + \frac{r_j - r_{s_t}}{\hat{p} + r_j} \hat{p} \hat{f}_j, \quad j \neq s_t, \quad (6)$$

where  $r_{s_t}$  is the equilibrium (instantaneous) risk-free interest rate in state  $s_t$ .<sup>8</sup> The discount rate  $r_{B,s_t}$  captures the expectation that the risk-free rate changes with the global business cycle and thus can differ from the current instantaneous risk-free rate.

The government defaults on its debt when its revenue  $Y_{i,t}$  falls to a threshold that varies with the state of the economy. That is, there are state-dependent default thresholds  $Y_{D,i,s_D}$  for default occurring in state  $s_D = \{L, H\}$ . We assume that default occurs only once. Default may occur either smoothly when the government revenue falls to the threshold  $Y_{D,i,s_D}$ , or discretely when the economy changes state. When the government defaults on its bond, at a time denoted by  $t_{D,i}$ , the coupon  $c_i$  is reduced by a fraction  $\kappa \in (0, 1)$  due to debt restructuring. The sovereign bond value  $B_{i,s_t}$  in state  $s_t$  equals (see Appendix B)

$$B_{i,s_t} = \underbrace{E_t \left[ \int_t^{t_{D,i}} c_i \frac{\pi_u}{\pi_t} du \middle| s_t \right]}_{\text{Before default}} + \underbrace{E_t \left[ \int_{t_{D,i}}^{\infty} (1 - \kappa) c_i \frac{\pi_u}{\pi_t} du \middle| s_t \right]}_{\text{After default}} \quad (7)$$

$$= \frac{c_i}{r_{B,s_t}} \underbrace{\left[ 1 - \sum_{s_D} \kappa \frac{r_{B,s_t}}{r_{B,s_D}} q_{i,s_t s_D} \right]}_{\text{Default risk discount}}, \quad s_t, s_D = \{L, H\}, \quad (8)$$

with

$$q_{i,s_t s_D} = E_t \left[ \frac{\pi_{t_{D,i}}}{\pi_t} \text{Prob}(s_D | s_t) \middle| s_t \right], \quad (9)$$

be stable over time. Hence, the growth rate of government (net) revenue must be more volatile than the growth rate of output. However, we do not aim to micro found the existence of this amplification in the current paper.

<sup>8</sup>Appendix A presents the equation of the instantaneous risk-free interest rate, which is identical to that in Bhamra, Kuehn, and Strebulaev (2010a,b). Higher consumption uncertainty ( $\sigma_{c,H} < \sigma_{c,L}$ ) and lower expected growth ( $\mu_{c,H} > \mu_{c,L}$ ) in recession increases the demand for the risk-free asset, which implies that the instantaneous risk-free interest rate is procyclical in equilibrium ( $r_H > r_L$ ).

where  $q_{i,s_t s_D}$  represents the value of the Arrow-Debreu default claim that pays one unit of consumption at default time  $t_{D,i}$  if the current state is  $s_t$  and the state at the moment of default is  $s_D$  (see Appendix C). The bond value is thus equal to the risk-free bond value  $\frac{c_i}{r_{B,s_t}}$  minus a default risk discount. This default risk discount depends on the value of the Arrow-Debreu default claim, the bond discount rate in both states (at the time of issuance and at the time of default), and the fraction of bond that is reduced in default.<sup>9</sup> The credit spread on the sovereign bond in state  $s_t$  corresponds to  $CS_{i,s_t} = \frac{c_i}{D_{i,s_t}} - r_{B,s_t}$ .

### 2.3 Optimal policies

The government chooses the optimal level of indebtedness and the timing of default.<sup>10</sup> The endogenous level of sovereign debt trades off the economic benefits of debt issuance and the economic cost of default. Issuing debt is beneficial to a country but excessive debt raises the risk of default. Default is detrimental for economic performance by instantaneously reducing the country's level of output by a fraction  $\alpha \in (0, 1)$ .<sup>11</sup> We define the benefits of debt issuance by  $r_g$  per unit of time, such that the total incentive for debt issuance, denoted by  $I_{i,s_t}$ , corresponds to (see Appendix D.1)

$$I_{i,s_t} = E_t \left[ \int_t^\infty r_g B_{i,s_0} \frac{\pi_u}{\pi_t} du \middle| s_t \right] \quad (10)$$

$$= \frac{r_g}{r_{B,s_t}} B_{i,s_0}. \quad (11)$$

We can interpret the debt issuance incentive  $I_{s_t}$  in different ways. The government can use debt proceeds at issuance to finance long-term public investments yielding a return  $r_g$  per unit of time, as in Jeanneret (2015, 2018). In this case,  $I_{s_t}$  would reflect the present economic value generated by public investments. Alternatively, we may view  $r_g$  as the government's private benefits for issuing debt, which could relate to greater political power through more military expenses, rents of larger public budgets under management, or to higher public spending to maintain popularity, among others. While it is beyond the objective of the paper to microfound the existence of the debt benefits, our specification is sufficiently general to encompass various reasons for debt issues.<sup>12</sup>

<sup>9</sup>In case default takes place as a result of a regime change, the level of government revenue immediately changes from  $Y_{t_D}$ , the level just before default, to  $Y_{D,s_D}$ , the level at default. Consequently, we consider a modified set of Arrow-Debreu default claims. See Bhamra, Dorion, Jeanneret, and Weber (2018) for technical details.

<sup>10</sup>In practice, governments also have control, to some extent, over the fiscal policy. We follow Arellano and Bai (2017) and assume that the government faces an unmodeled fiscal constraint in that it cannot raise tax rates to prevent a default.

<sup>11</sup>The presence of economic cost provides the government the motivation for avoiding default, as in Arellano (2008), Andrade (2009), Hatchondo and Martinez (2009), Yue (2010), Borri and Verdelhan (2012), Jeanneret (2015, 2018), or Rebelo, Wang, and Yang (2018). Mendoza and Yue (2012), Hebert and Schreger (2017), and Andrade and Chhaochharia (2018) provide robust empirical evidence on the costs of sovereign defaults.

<sup>12</sup>It is also common to assume that risk-averse governments issue sovereign debt (bought by risk-neutral agents) for

When choosing the debt policy at time  $t = 0$ , the government maximizes the *ex ante* level of sovereign wealth, defined as the debt issuance incentives  $I_{i,s_t}$  plus the present value of the government revenue, which we denote by  $G_{i,s_t}$  in state  $s_t$ . The present value of government revenue is given by (see Appendix D.2)

$$G_{i,s_t} = \underbrace{E_t \left[ \int_t^{t_{D,i}} Y_{i,u} \frac{\pi_u}{\pi_t} du \middle| s_t \right]}_{\text{Before default}} + \underbrace{E_t \left[ \int_{t_{D,i}}^{\infty} (1 - \alpha) Y_{i,u} \frac{\pi_u}{\pi_t} du \middle| s_t \right]}_{\text{After default}} \quad (12)$$

$$= \frac{Y_{i,t}}{r_{Y,i,s_t}} - \underbrace{\alpha \sum_{s_D} \frac{Y_{D,i,s_D}}{r_{Y,i,s_D}} q'_{i,s_t s_D}}_{\text{Costs of default}} \quad (13)$$

with

$$r_{Y,i,s_t} = r_{s_t} - \mu_{Y,i,s_t} + \frac{(r_j - \mu_{Y,i,j}) - (r_{s_t} - \mu_{Y,i,s_t})}{\hat{p} + r_j - \mu_{Y,i,j}} \hat{p} f_j, \quad j \neq s_t, \quad (14)$$

and

$$q'_{i,s_t s_D} = E_t \left[ \frac{\pi_{t_{D,i}}}{\pi_t} \frac{Y_{i,t_{D,i}}}{Y_{D,i,s_D}} \text{Prob}(s_D | s_t) \middle| s_t \right], \quad s_t, s_D = \{L, H\}. \quad (15)$$

The two terms of Equation 12 respectively represent the discounted government revenue before and after default. The value of the Arrow-Debreu claim  $q'_{i,s_t s_D}$  represents the present value of the cost of default (see Appendix C.2). The discount rate  $r_{Y,i,s_t}$  applying to the government revenue conditional on the current state being  $s_t$  is the discount rate for a perpetuity with stochastic risk-free rate  $r_t$  and expected growth rate  $\mu_{Y,i,t}$ , which are currently equal to  $r_{s_t}$  and  $\mu_{Y,i,s_t}$ . If the economy stays in state  $s_t$  forever, the discount rate reduces to the standard expression  $r_{Y,i,s_t} = r_{s_t} - \mu_{Y,i,s_t}$ . In general, however, the economy can change states and thus we need to account for the time spent in recession and in expansion at future times. As default risk increases, under the risk-neutral measure, the present value of the government revenue declines (see Equation 13).

Ex ante sovereign wealth equals  $W_{i,s_t} = G_{i,s_t} + I_{i,s_t}$  when current state is  $s_t$ . The optimal state-dependent coupon at time  $t = 0$  satisfies

$$c_{i,s_0}^* = \arg \max W_{i,s_0}, \quad (16)$$

the purpose of consumption/investment smoothing (e.g., Eaton and Gersovitz, 1981). In this paper, we consider an environment with risk-averse lenders and a risk-neutral government, which precludes any smoothing motives.

where  $s_0$  is the state of the global economy at the moment the debt is contracted.

The government maximizes net sovereign wealth by choosing the optimal state-contingent default thresholds  $Y_{D,i,s_D}$ , which are determined by solving the following two smooth-pasting conditions (see Appendix D.3):

$$\left. \frac{\partial (W_{i,s_t}(Y_{i,t}) - B_{i,s_t})}{\partial Y_{i,t}} \right|_{Y_{i,t}=Y_{D,i,s_t}} = \frac{1 - \alpha}{r_{Y,i,s_t}}, \quad s_t = \{L, H\}. \quad (17)$$

The default thresholds determining the optimal timing of default in each state of the economy reflect the trade-off between the benefits and costs of default. On the one hand, defaulting is beneficial because it reduces future bond coupon payments through debt restructuring, which increases net sovereign wealth. On the other hand, defaulting is costly because it reduces future government revenue due to the economic cost of default, which decreases net sovereign wealth. Similarly, greater debt issuance is beneficial, but too much debt becomes costly due to the increase in default risk. The problem of the government consists of solving Equation 16 subject to Equation 17. A closed-form solution to this optimization problem does not exist and we use standard numerical procedures.

## 2.4 Sovereign bond premium

We now derive the bond pricing implications relative to each source of systematic risk. The sovereign bond premium  $BP_{i,s_t}$  in state  $s_t$ , which captures the expected bond excess return, is equal to

$$BP_{i,s_t} = \gamma \sigma_{s_t} \rho_{i,s_t} \sigma_{i,s_t}^B + \lambda_{s_t} \Theta_{s_t}^P R_{i,s_t}^B, \quad s_t = \{L, H\}. \quad (18)$$

The first term is the compensation for country  $i$ 's exposure to frequent but small unexpected consumption growth shocks, where  $\gamma \sigma_{s_t}$  is the market price of risk associated with the continuous consumption shocks and  $\rho_{i,s_t}$  is the correlation between the country  $i$ 's output shocks and consumption shocks. The last component captures the volatility of sovereign bond returns, which equals

$$\sigma_{i,s_t}^B = \frac{\partial \ln B_{i,s_t}}{\partial \ln Y_{i,s_t}} \sigma_{Y,i,s_t} \quad (19)$$

with  $\frac{\partial \ln B_{i,s_t}}{\partial \ln Y_{i,s_t}} = -\frac{Y_{i,s_t} c_i}{B_{i,s_t} r_{B,s_t}} \sum_{s_D} \phi \frac{r_{B,s_t}}{r_{B,s_D}} q'_{i,s_t s_D}$ . The representative agent dislikes bonds of countries with output growth shocks that correlate positively with consumption growth shocks, i.e.  $\rho_{i,s_t} > 0$ .

A negative unexpected change in consumption increases sovereign default risk, which reduces bond valuation when the agent's marginal utility of consumption increases. Hence, a positive covariation between bond returns and consumption shocks commands a bond risk premium, as predicted by the C-CAPM.

The second term captures the compensation for country  $i$ 's exposure to severe but low-frequency changes in consumption growth. This risk premium component is determined by the probability  $\lambda_{s_t}$  of leaving state  $s_t$ , the price of risk associated with this change of state  $\Theta_{s_t}^P = 1 - \Delta_{s_t}$ , and the change in bond valuation caused by the change of state, given by  $R_{i,s_t}^B = \frac{B_{i,j}}{B_{i,s_t}} - 1$ ,  $s_t \neq j = \{L, H\}$ . Observe that the product  $\Theta_{s_t}^P R_{i,s_t}^B$  is always positive: In recession,  $\Theta_L^P = 1 - \Delta_L > 0$  given that  $\Delta_L = \frac{1}{\Delta_H} < 1$  and  $R_{i,L}^B = \frac{B_{i,H}}{B_{i,L}} - 1 > 0$ , as bond valuation is higher in expansion than in recession. In contrast, the expansion state implies that  $\Theta_H^P = 1 - \Delta_H < 0$  given that  $\Delta_H > 1$  and  $R_{i,H}^B = \frac{B_{i,L}}{B_{i,H}} - 1 < 0$ .

The representative agent cares that bond prices fluctuates over the business cycle, which makes sovereign bonds particularly risky. Specifically, she dislikes high default risk during recessions. As the state of the economy follows a Markov process, recessions also arrive at uncertain times. With preferences for early resolution of uncertainty, the agent prices sovereign bonds using the risk-neutral transition probabilities, assuming that recessions arrive sooner and last longer than under the physical probabilities, which implies that  $\Theta_{s_t}^P > 0$ . The agent thus receives an additional risk premium as a compensation for investing in sovereign bonds, which shows that recursive preferences play a critical role when assessing the impact of macroeconomic risk on the pricing of sovereign bonds.

### 3 Data and model calibration

This section presents the calibration of the model.

#### 3.1 Consumption and output

The conditional moments of consumption growth switch randomly across expansion and recession states according to a Markov chain. We estimate a two-state Markov-regime switching model on quarterly U.S. consumption data. We measure consumption as real non-durables goods plus service consumption expenditures using data from the Bureau of Economic Analysis. The sample period is 1994Q1-2018Q2. The estimation approach is based on Hamilton (1989) and details are provided in Appendix E.

The estimates of the physical probabilities of being in expansion and in recession are respectively  $f_H = 66.58\%$  and  $f_L = 33.42\%$ , while the probabilities per unit of time of leaving the expansion

and recession states are respectively  $\lambda_H = 14.56\%$  and  $\lambda_L = 29.00\%$ . The expected growth rate of consumption is  $\mu_{c,L} = -0.84\%$  in recession and  $\mu_{c,H} = 1.64\%$  in expansion, while the consumption growth uncertainty is  $\sigma_{c,L} = 0.64\%$  in recession and  $\sigma_{c,H} = 0.58\%$  in expansion.

The output dynamics of the debt issuing country is calibrated as follows. We first compute the log growth rate on quarterly real GDP for all emerging countries with data available on Datastream. We use NBER business cycle expansion and recession periods to compute the conditional moments of each country's output growth.<sup>13</sup> We keep the 40 countries for which the quarterly growth rates cover at least one NBER recession. Table 1 reports the conditional moments for each country. The dynamics of output growth for the representative country is determined by averaging the conditional moments across countries. The expected output growth rate is equal to  $\mu_{X,L} = 0.09\%$  in recession and  $\mu_{X,H} = 4.34\%$  in expansion. The output growth volatility is  $\sigma_{X,L} = 4.77\%$  in recession and  $\sigma_{X,H} = 3.46\%$  in expansion. The instantaneous correlation between output growth and consumption growth is  $\rho_L = 0.094$  in recession and  $\rho_H = 0.05$  in expansion.

Table 1 [about here]

### 3.2 Country characteristics and preferences

The parameter capturing the debt issuance benefits  $r_g$  is 1.4%, corresponding to the average of the country-level structural estimates of Jeanneret (2015). The debt haircut fraction  $\phi$  is 75%, which is the ISDA's market convention for pricing credit derivatives in emerging markets. The economic contraction  $\alpha$  (fraction of output) at default is 5%, which is the average estimate reported in Mendoza and Yue (2012) across 23 sovereign default events for the period 1977-2009. Regarding the calibration of the representative agent's preferences, the coefficient of risk aversion is  $\gamma = 10$ , the coefficient of elasticity intertemporal substitution (EIS) is  $\psi = 2$ , and the time discount rate is equal to  $\beta = 4\%$ . The corresponding level of the real risk-free interest rate equals 3.35% in recession and 4.57% in expansion.

Finally, we calibrate the model to generate reasonable levels of default risk. We set the leverage factor  $\eta$  such the 5-year unconditional cumulative default probability and the unconditional credit spread match their empirical counterpart. Moody's (2017) reports a 5-year cumulative default rate of 10.83% for sovereigns issuing speculative grade bonds, while the median credit spread (314.6 bps) in the EMBI

<sup>13</sup>Conditioning the output growth moments on the NBER classification dates is straightforward. By contrast, one needs to specify an ad-hoc filtering rule to classify expansion and recession periods from the recession probabilities obtained from the estimation of the two-state Markov-regime switching model on U.S. consumption growth. Furthermore, Hamilton (1989) concludes that "statistical estimates of the economy's growth state [from a Markov-regime switching model] cohere remarkably well with NBER dating of postwar recessions."

database over the 1994Q1-2018Q2 period. The estimation implies a leverage factor equal to  $\eta = 10.6$ . The unconditional credit spread is 336 bps and the predicted default probability at the 5-year horizon is 10.64%.<sup>14</sup> Table 2 summarizes the parameter values of the baseline calibration. Unconditionally, the government defaults when its revenue (normalized to unity at issuance) fall to a default boundary equal to 0.25, which is higher than the level of debt coupon ( $c = 0.12$ ). Hence, revenue net of debt service always remain positive in our calibration. Unless otherwise mentioned, debt is initially issued in the expansion set and the model predictions are computed for  $Y = 1$ .

Table 2 [about here]

## 4 Model predictions

This section presents the theoretical predictions. We first analyze the impact of macroeconomic risk on the pricing of sovereign bonds under optimal debt and default policies. We then assess the sovereign bond premium associated with the different sources of macroeconomic risk and explore how the corresponding risk premia vary across countries.

### 4.1 Macroeconomic risk, default risk, and optimal policies

Countries face different kinds of macroeconomic risk. First, output shocks correlate with consumption shocks and, second, their output growth moments fluctuate with the state of the economy. Before analyzing the asset pricing implications of macroeconomic risk, it is useful to understand how these sources of risk affect a country's credit risk under endogenous policies.

Table 3 compares the predictions of the baseline model (Column A) with those of an alternative model that shuts down a country's exposure to the global business cycle (Column B). In this special case, the expected growth rate and volatility of the country's output are fixed at their unconditional means. This alternative model's calibration matches the unconditional moments of output growth (i.e., expected growth and volatility) to those implied by the baseline model.<sup>15</sup> That is, both models are observationally equivalent in terms of risk levels.

<sup>14</sup>Throughout the analysis, we compute the unconditional credit spreads, default boundaries, and default probabilities as weighted averages of the state-dependent values with weights given by the long-run distribution of the Markov chain,  $f_L$  and  $f_H$ . See Appendix B for details on the computation of the default probability.

<sup>15</sup>The unconditional expected output growth is simply the weighted average using the long-run probabilities, which given by  $\mu = f_L\mu_L + f_H\mu_H$ . Following Timmermann (2000), the unconditional variance of output growth equals  $\sigma^2 = f_L\sigma_L^2 + f_H\sigma_H^2 + f_L(1 - f_L)(\mu_H - \mu_L)^2$ .

Table 3 [about here]

A country's exposure to expected consumption growth and volatility increases the 5-year default probability from 6.60% to 18.43% in recessions and decreases it from 6.72% to 5.81% in expansions. Note that the optimal default boundary is lower in recessions than in expansions because a government's incentive to exercise the default option decreases with the level of volatility, in line with classical real-option theory (e.g., McDonald and Siegel, 1986).<sup>16</sup> Default risk is thus countercyclical, on average, with respect to the global business cycle, despite the cyclical nature of default boundaries.

Fluctuations in output growth moments amplify a country's default risk during recessions more than it reduces it during expansions. This asymmetry implies that the unconditional default probability is higher in a model in which output growth moments change across regimes than in a model with constant moments. Indeed, the unconditional 5-year default probability increases from 6.68% to 10.03%, although the total level of risk remains observationally equivalent. Therefore, a country's exposure to the global business cycle magnifies sovereign risk, on average.

The higher default probability induced by the business cycle exposure translates into greater borrowing costs for the government. The government chooses a lower debt level to partially offset the asymmetry-induced increase in default risk. Table 3 reports that the optimal debt coupon decreases from 0.17 to 0.11 when a country is exposed to the business cycle. However, this debt policy response is not strong enough to reverse the positive impact on the level of default risk. As a result, the business cycle exposure increases the unconditional credit spread from 247 to 331 bps, thus contributing to a quarter of the total credit spread.

By contrast, an alternative model that assumes independent shocks between output and consumption delivers predictions that are comparable to those of the full model (Column C of Table 3). Hence, the correlation between output and consumption shocks has a negligible impact on a government's policies and sovereign default risk. Our theory shows that a country's exposure to the business cycle is, therefore, the primary source of macroeconomic risk. The role of macroeconomic risk is particularly severe when a country's economic performance deteriorates, as illustrated by Figure 1.

Figure 1 [about here]

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<sup>16</sup>While default risk is naturally higher during recessions (low growth, high volatility), the procyclical default boundaries imply that a poorly-performing country may also instantaneously default when the economy suddenly switches from recession to expansion, should the level of the government revenue be between the boundaries ( $Y_{D,H} > Y_t > Y_{D,L}$ ). Consistent with this prediction, Tomz and Wright (2007) find that defaults are more common in bad times than in good, but they also document many exceptions. In their sample of 175 countries, more than one-third of their 169 default episodes occurred during good economic times.

## 4.2 The sovereign bond premium

This section investigates how macroeconomic risk affects the sovereign bond premium. The total compensation for being exposed to macroeconomic risk, which determines the sovereign bond premium, amounts to 60.3 bps on average. Conditionally, it equals 56.2 bps in expansion and 68.4 bps in recession. We now quantify and analyze the individual components of the risk premium. Table 6 reports the results.

Table 6 [about here]

### 4.2.1 Exposure to long-run consumption shocks

A central source of systematic risk for investors is the effect of the business cycle on sovereign bond returns. Bond valuation tends to decrease during recessions as a result of lower expected output growth rate and higher output volatility. This source of risk reflects the exposure of a country's economic performance to low-frequency but severe changes in expected consumption growth, in contrast to small and transitory consumption shocks. Because changes in the state of the economy are persistent and long-lived, we refer to this source of systematic risk as to "long-run macro risk".

The compensation for long-run macro risk, which we denote by  $BP_{i,st}^{LR}$ , amounts to 59.8 bps in recessions and to 53.4 bps in expansions. This source of systematic risk represents 87% of the bond premium in recessions and 95% in expansions. To understand the drivers of this risk premium, recall that it is given by  $BP_{i,st}^{LR} = \lambda_{st}(1 - \Delta_{st})R_{i,st}^B$ . In recession, the probability per unit of time to leave a recession, expressed by  $\lambda_L$ , is 29%; the price of risk associated with a change of state, expressed by  $(1 - \Delta_L)$ , is around 0.462; and the change in bond valuation caused by the change of state, measured by  $R_{i,L}^B$ , is about 4.46%. Hence, the long-run macro risk premium equals to  $0.29 \times 0.462 \times 0.0446 = 59.8$  bps in recession.

This risk premium component arises because bond prices are sensitive to business cycle variations and investors dislike the fall in bond prices in recessions, when investors' marginal utility of consumption is high. In addition, the state of the economy switches randomly, which introduces uncertainty about future recessions. Investors with a preference for early resolution of uncertainty ( $\gamma > \frac{1}{\psi}$ ) dislike lower bond valuation during an adverse economic state arriving in uncertain times and thus price sovereign bonds with a high risk-neutral probability of staying in recession. Bond investors are thus compensated for this systematic source of risk, even in the case of conditionally independent shocks between a country's output and global consumption.

## 4.2.2 Exposure to short-run consumption shocks

The second source of systematic risk arises from the covariation between a country's output shocks and unexpected changes in aggregate consumption. These changes correspond to small and short-lived shocks that are independent from the low-frequency business cycle fluctuations. Such shocks are expected to be transitory, i.e., expected growth rates and volatilities remain unaffected, and we thus label this source of systematic risk "short-run macro risk". Sovereign bonds tend to perform badly when the global economy experiences bad shocks and investors are averse to such short-run macro risk.

The compensation for short-run macro risk, which we denote by  $BP_{i,s_t}^{SR}$ , equals 2.8 bps in expansion and 8.7 bps in recession. Thus, less than 10% of the unconditional bond premium originates from the correlation between output and consumption. This risk premium component is given by  $BP_{i,s_t}^{SR} = \gamma \sigma_{s_t} \rho_{i,s_t} \sigma_{i,s_t}^B$ , which differs fundamentally from the long-run macro risk premium. In recession, consumption growth volatility  $\sigma_L$  is 0.64%, the correlation between output and consumption shocks  $\rho_{i,L}$  is 9.4%, and the volatility of bond returns  $\sigma_{i,L}^B$  is about 14.5%. Hence, the short-run macro risk premium is equal to  $10 \times 0.0064 \times 0.094 \times 0.145 = 8.7$  bps for a risk aversion  $\gamma = 10$ . This risk premium is negligible, despite high risk aversion, because consumption volatility is low and output growth in emerging markets is only weakly related to consumption shocks. The low risk premium arising from the C-CAPM, which has been extensively studied in the equity market, thus echoes in the sovereign debt market.

One concern is that a regime-switching model underestimates the premium for short-run macro risk. The reason is that the joint fluctuations in output and consumption growth moments drive part of the unconditional output-consumption correlation. Hence, the conditional correlation between output and consumption shocks is lower than the unconditional correlation. To address this concern, we consider a restricted version of the model that turns off a country's exposure to the business cycle. In this case, the unconditional correlation between output and consumption shocks increases to 0.154 and the corresponding bond risk premium amounts to 9.1 bps. This upper bound of the short-run risk premium remains substantially smaller than the total risk premium of the full model (60.3 bps). This analysis confirms that the primary component of the sovereign bond premium is the compensation for long-run macro risk.

### 4.2.3 Investor preferences

We now discuss how investor preferences determine the sovereign bond premium. Figure 2 presents the sovereign bond premium for different levels of relative risk aversion ( $\gamma$ ) and time preference ( $\beta$ ), while Table 5 reports predictions on the level of default risk and the price of risk.

The sovereign bond premium increases when the pricing agent is more risk-averse (higher  $\gamma$ ). The short-run macro risk premium, as given by  $\gamma\sigma_{s_t}\rho_{i,s_t}\sigma_{i,s_t}^B$ , directly increases with the risk aversion coefficient  $\gamma$  through the price of risk, as investors dislike output shocks that covary positively with their consumption. The impact on long-run macro risk is fundamentally different: Higher risk aversion translates into a higher price of risk because investors display a stronger preference for early resolution of uncertainty as the difference between  $\gamma$  and  $\frac{1}{\psi}$  increases.

In addition to these direct effects, higher risk aversion increases the precautionary motives and thus reduces the equilibrium risk-free rate. A lower risk-free rate increases the present value of the debt coupons that the government must service, thereby increasing its default risk. Bonds become riskier and the risk premium increases. Similarly, the sovereign bond premium decreases with investors' preference for time, as a lower  $\beta$  translates into a lower risk-free rate and thus into higher default risk.

Figure 2 [about here]

Table 5 indicates that the price of risk is  $\Delta_H = \hat{\lambda}_H/\lambda_H = 1.86$ , which implies that investors overweight the probability of switching from expansion to recession by 86%. Investors thus price bonds as if recessions arrive sooner (and last longer) than the data suggest. The ratio of the unconditional risk-neutral default probability ( $\mathbb{Q}$ ) over the unconditional physical default probability ( $\mathbb{P}$ ), both computed at a 5-year horizon, reflects how much investors overweight the increase in default probability because of macroeconomic risk.<sup>17</sup> This ratio equals 1.38 in the baseline calibration, which indicates that investors price sovereign bonds as if the unconditional level of default risk were 38% greater than it is in reality.<sup>18</sup> Notably, the sovereign bond premium associated with long-run macro risk vanishes when investors have power utility ( $\Delta_H = 1$  and  $\mathbb{Q} = \mathbb{P}$  when  $\gamma = \frac{1}{\psi}$ ), thereby highlighting the critical role of Epstein-Zin-Weil preferences and long-run macro risk for understanding the pricing of sovereign bonds.

Table 5 [about here]

<sup>17</sup>We compute the unconditional risk-neutral default probability ( $\mathbb{Q}$ ) using the long-run risk-neutral distribution ( $\hat{f}_L = 0.5457, \hat{f}_H = 0.4543$ ) to weight the default probability in each state  $s_t = \{L, H\}$ . Correspondingly, we use the real-world distribution ( $f_L = 0.3342, f_H = 0.6658$ ) to compute the physical default probability ( $\mathbb{P}$ ).

<sup>18</sup>The magnitude of this price of risk compares to Huang and Huang (2012), which report a ratio ranging between 1.1 and 1.7 for corporate bonds.

#### 4.2.4 Time variation of the sovereign bond premium

Figure 3 shows how a country's economic conditions drive the short- and the long-run macro risk premia differently over time. The compensation for short-run macro risk increases when local economic conditions deteriorate. By contrast, the sovereign bond premium associated with long-run macro risk is non-monotonically related to a country's economic conditions. This non-monotonicity is due to two partially offsetting effects. On the one hand, a low performing country has greater default risk, which increases the sensitivity of bond prices to news and, thus, the risk premium. On the other hand, long-run macro risk has less impact on the valuation of bonds with a higher probability of default. Investors become less concerned about the uncertainty regarding future recessions when a bond is expected to default sooner, given that such bond becomes naturally less exposed to future changes in the business cycle.<sup>19</sup> The risk premium for long-run macro risk thus decreases with default risk, and so does the total sovereign bond premium. Hence, the relation between the sovereign bond premium and local economic conditions becomes hump-shaped.

Figure 3 [about here]

In sum, the theory proposed in this paper provides new insights on the premium embedded in sovereign bonds, on the relative importance of the various sources of systematic risk, and on the drivers of each risk compensation.

### 4.3 Cross-sectional implications

We now explore how the sovereign bond premium varies across countries and discuss the asset pricing implications of macroeconomic risk.

#### 4.3.1 Predictions by degree of macroeconomic risk

Our cross-sectional analysis first considers different degrees of short- and long-run macro risk. Panel A of Table 6 shows that the sovereign bond premium associated with short-run macro risk varies substantially with the level of output-consumption correlation. Investors dislike bonds of countries that perform badly when the global economy experiences bad shocks and are thus averse to short-run macro risk when  $\rho_i > 0$ . In contrast, investors favor short-run macro risk when  $\rho_i < 0$ , that is when countries

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<sup>19</sup>Conversely, this prediction implies that the role of long-run macro increases for relatively safer bonds, which have a longer expected time to maturity and thereby are more exposed to future regime shifts. Our model thus rationalizes the empirical finding that the (Q/P) ratio typically increases with better credit ratings (e.g., Augustin and Tédongap, 2016).

perform countercyclically, such that their bonds offer an hedge against adverse consumption shocks. The model generates a positive or a negative short-run risk premium, depending on the sign of output-consumption correlation. The level of short-run risk premium remains, however, below 30 bps, even for high levels of correlation.

Table 6 [about here]

We now explore how the sovereign bond premium varies with a country's level of long-run macro risk. For this analysis, we compute, for each country  $i$ , the degree of exposure to the business cycle as follows:

$$\phi_{\mu,i} = \frac{\mu_{X,i,H} - \mu_{X,i,L}}{\bar{\mu}_{X,H} - \bar{\mu}_{X,L}} \quad (20)$$

$$\phi_{\sigma,i} = \frac{\sigma_{X,i,L} - \sigma_{X,i,H}}{\bar{\sigma}_{X,L} - \bar{\sigma}_{X,H}}, \quad (21)$$

where  $\mu_{X,i,s_t}$  and  $\sigma_{X,i,s_t}$  are the conditional expected growth rate and conditional volatility for country  $i$ , while  $\bar{\mu}_{X,s_t}$  and  $\bar{\sigma}_{X,s_t}$  reflect variations across states for an average country (baseline calibration). When  $\phi_{\mu,i} > 1$  and  $\phi_{\sigma,i} > 1$ , expected output growth rate and output volatility are more exposed to global recessions for country  $i$  than for the baseline country, whereas country  $i$ 's exposures are relatively lower when  $\phi_{\mu,i} < 1$  and  $\phi_{\sigma,i} < 1$ .

Bonds whose valuation fall more in times of lower expected consumption growth and higher consumption growth uncertainty are thus particularly risky to investors and thus offer higher expected excess returns. Panel B of Table 6 shows that the premium associated with long-run macro risk exceeds 100 bps for reasonable calibration values. The sovereign bond premium thus varies meaningfully with the degree of long-run macro risk (through both  $\phi_{\mu,i}$  and  $\phi_{\sigma,i}$ ), much more than what we can observe for short-run macro risk. Our first testable hypothesis is as follows:

**Hypothesis 1:** *Sovereign bonds with greater macroeconomic risk are expected to deliver higher excess returns.*

**Corollary 1a:** *Long-run macro risk dominates short-run macro risk in driving the sovereign bond premium.*

### 4.3.2 Distribution of macroeconomic risk and portfolio sorting

We now exploit the empirical distribution of short-run and long-run macro risk estimated from our sample of emerging economies. For each country, we first estimate the conditional correlation  $\rho_{i,s_t}$  and then compute the state-weighted average. We report the empirical distribution in Column I of Table 7. The correlation between output and consumption shocks varies between -0.26 and 0.51. We also estimate  $\phi_{\mu,i}$  and  $\phi_{\sigma,i}$  for each country and report the distribution in Column II of Table 7. The exposure to the global business cycle varies meaningfully across countries, as  $\phi_{\mu,i}$  ranges between -1.02 and 3.76 while  $\phi_{\sigma,i}$  ranges between -3.65 and 7.01. Emerging markets thus display strong cross-sectional variations in the degree of short-run and long-run macro risk.

Table 7 [about here]

We now assess the performance of a long-short investment strategy that exploits the empirical cross-sectional differences in macroeconomic risk. Panel A of Table 8 reports the short-run macro risk premium for a low and a high level of unconditional output-consumption correlation  $\rho_i$ , based on the 25th and 75th percentile of the distribution, respectively. The high-minus-low (HML) strategy delivers an expected bond excess return of 14.6 bps, which is economically small. Hence, our second hypothesis is as follows:

**Hypothesis 2:** *Investors buying bonds with high short-run macro risk and selling bonds with low short-run macro risk only obtain a modest excess return.*

Table 8 [about here]

Panel B of Table 8 reports the long-run macro risk premium when we focus on fluctuations in expected output growth and output volatility, separately. We compute the results when  $\phi_{\mu,i}$  is equal to the 25th and 75th percentiles of the distribution, and report the expected excess return of the HML strategy. Unconditionally, the HML excess return amounts to 48.2 bps. The same analysis for  $\phi_{\sigma,i}$  yields 102.2 bps. Hence, our final hypothesis is as follows:

**Hypothesis 3:** *Investors buying bonds with high long-run macro risk and selling bonds with low long-run macro risk obtain a sizable excess return.*

**Corollary 3a:** *Both sources of long-run macro risk are complementary, but the performance of the investment strategy is relatively greater when sorting countries based on their exposure to consumption volatility.*

Altogether, these findings indicate that exposures to lower economic growth and higher economic uncertainty in times of global recessions are critical drivers of the sovereign bond premium. In addition, the sovereign bond premium appears to be more sensitive to a change in the volatility exposure  $\phi_{\sigma,i}$  than to the expected growth exposure  $\phi_{\mu,i}$ , although both exposures capture complementary sources of systematic risk. While cross-sectional variations in long-run macro risk induce a sizable sovereign bond premium, the risk premium related to short-run risk, which arises from the output-consumption correlation, is predicted to be economically negligible.

## 5 Empirical tests

This section empirically tests our theoretical predictions. We investigate how the sovereign bond premium varies with short and long-run macro risk, as measured by a country's bond exposure to U.S. consumption. We quantify the pricing implications of the various sources of systematic risk based on portfolio sorts.

### 5.1 Data

Our analysis considers quarterly sovereign bond data using every country's JP Morgan EMBI index. A country's bond index is a market capitalization-weighted aggregate of liquid government bonds denominated in U.S. dollars. Our sample includes 40 countries and spans the period from 1994Q1 to 2018Q2. We obtain the JP Morgan EMBI indices from Datastream and the 3-month Treasury bill rate, which is our measure of the risk-free rate, from the Federal Reserve Bank of St. Louis.

We measure expected consumption growth using the quarterly mean of U.S. consumption growth forecasts from the Survey of Professional Forecasters, which is available from the Federal Reserve Bank of Philadelphia. We use the cross-sectional dispersion in the U.S. consumption growth forecasts to measure consumption (growth) uncertainty. Figure 4 plots the time series of these two state variables. Expected consumption growth decreases and consumption uncertainty increases during NBER recessions. The correlation between the two risk factors is  $-0.39$  in first difference.

Figure 4 [about here]

## 5.2 Methodology

The first step is to compute sovereign bond excess returns for each country. We take the perspective of a U.S. investor who borrows at the risk-free rate  $r_t$  to invest in the country  $i$ 's government bond index. We denote by  $R_{i,t}^B$  the return on the country  $i$ 's bond index, such that the bond excess return at quarter  $t$  is equal to  $R_{i,t}^e = R_{i,t}^B - r_{t-1}$ . Table 9 reports the descriptive statistics for the bond excess returns by country.

Table 9 [about here]

In a second step, we determine the level of short-run and long-run macro risk for each country. We measure systematic risk by the slope coefficients from time-series regressions of the country's bond excess returns on consumption shocks, expected consumption growth, and consumption uncertainty. For each quarter, we use the most recent 20 quarterly observations to estimate the exposure of sovereign bond excess returns to these sources of macroeconomic risk using the following regression:

$$R_{i,t}^e = \alpha_i + \beta_i^c \Delta c_t + \beta_i^\mu \Delta \hat{\mu}_{c,t} + \beta_i^\sigma \Delta \hat{\sigma}_{c,t} + \epsilon_{i,t}, \quad (22)$$

where  $\beta_i^c$  is the risk loading of country  $i$ 's excess bond returns on log consumption growth, while  $\beta_i^\mu$  and  $\beta_i^\sigma$  respectively capture the risk loadings of country  $i$ 's excess bond returns on the first difference of our estimates of consumption growth  $\hat{\mu}_{c,t}$  and consumption uncertainty  $\hat{\sigma}_{c,t}$ , respectively. Table 9 reports the average risk loadings by country. On average, bond excess returns are weakly exposed to consumption shocks ( $\beta^c = -0.43$ ) and to fluctuations in expected consumption growth ( $\beta^\mu = 0.24$ ), but strongly exposed to variations in consumption uncertainty ( $\beta^\sigma = -4.84$ ). Importantly, the average risk loadings vary substantially across countries.

Each quarter we build equally-weighted bond portfolios by sorting countries according to their estimated risk loadings. We construct 4 portfolios ranked from low to high exposure based on loadings with respect to consumption shocks, expected consumption growth, or consumption uncertainty. Portfolio 1 (4) contains countries with the lowest (highest) risk loading. Portfolios are re-balanced quarterly. For each portfolio we compute average betas and excess returns using an holding investment period of one year.

### 5.2.1 Main results

We assess how short-run and long-run macro risk is priced in the cross-section of sovereign bonds. Tables 10 and 11 report the results when we estimate the risk loadings from time-series regressions described in Equation 22. We report, for each of the 4 portfolios, the average risk loading, the average excess return, and the Sharpe ratio. We also report the results of the quarterly-rebalanced long-short strategy that buys Portfolio 4 (high loading) and sells Portfolio 1 (low loading), which we denote the high-minus-low (HML) portfolio. The statistics are reported in U.S. dollars.<sup>20</sup>

Tables 10 and 11 [about here]

The sovereign bond premium associated with short-run risk is negligible in the data. To see that, sorting countries based on their exposure to U.S. consumption shocks does not deliver statistically significant excess returns. The results remain similar whether we estimate  $\beta_i^c$  alone (Column I) or jointly (Column II) with the loadings related to long-run macro risk ( $\beta_i^\mu$  and  $\beta_i^\sigma$ ).

By contrast, long-run macro risk delivers a sizable premium. In Panels A and B of Table 11, portfolios are respectively formed based on loadings with respect to expected consumption growth ( $\beta_i^\mu$ ) and consumption growth uncertainty ( $\beta_i^\sigma$ ). The exposure to expected consumption growth ( $\beta_i^\mu$ ) predicts future bond returns positively, while the exposure to consumption growth uncertainty ( $\beta_i^\sigma$ ) predicts future bond returns negatively. The differences between the high and low-beta portfolios are economically and statistically significant in both cases, whether we estimate the risk loadings separately (Column I) or jointly (Column II). In the first case, the average excess returns of the HML portfolios are 1.52% ( $t$ -stat of 3.36) when sorting on  $\beta_i^\mu$  and -1.58% ( $t$ -stat of -2.46) when sorting on  $\beta_i^\sigma$ . The corresponding Sharpe ratios are 0.89 and -0.79, respectively.

The exposure to expected consumption growth carries a positive price of risk, while the exposure to consumption uncertainty carries a negative price of risk. Both sources of long-run macro risk help explain cross-sectional differences in sovereign bond excess returns. When we jointly estimate the risk loadings  $\beta_i^\mu$  and  $\beta_i^\sigma$  from time-series regressions, the exposure to consumption uncertainty appears to be economically more important (excess return of -1.46% vs. 1.05%) for explaining the cross-sectional variation in sovereign bond premium

Overall, confirming our theoretical predictions, we find that sovereign bonds featuring greater macroeconomic risk deliver higher excess returns and that long-run macro risk dominates short-run

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<sup>20</sup>We multiply the means of quarterly returns by 4 and standard deviations by 2. The Sharpe ratio is the ratio of the annualized mean to the annualized standard deviation.

macro risk in driving the sovereign bond premium (Hypothesis 1). Specifically, the premium for short-run macro risk is statistically negligible (Hypothesis 2). By contrast, the premium for long-run risk is economically and statistically sizable and arises through a country's exposure to both expected consumption growth and consumption uncertainty (Hypothesis 3). The data therefore provide strong empirical support for the properties of the sovereign bond premium predicted by our theory.

### **5.2.2 Robustness analysis**

We conduct additional tests to assess the robustness of our empirical findings. First, we ensure that the pricing implications of long-run macro risk do not depend on whether or not we control for short-run macro risk in the estimation. We reproduce the portfolio sorts when the main specification accounts for the exposure to quarterly (log) changes in U.S. consumption. Table 12 confirms the main findings that the price of expected consumption growth and consumption uncertainty are respectively positive and negative, and moreover statistically significant. Table 12 also reports the results when we augment the main specification (22) with the exposure to the excess returns on the S&P 500 index, following the CAPM analysis of Borri and Verdelhan (2012). These authors show that bond excess returns compensate investors for the covariance with the U.S. stock market performance. We find that the results regarding the role of long-run macro risk remain similar when controlling for this complementary risk factor. Note that, in this case, the exposure to consumption uncertainty dominates the exposure to expected consumption growth in explaining bond excess returns. This result provides further evidence regarding the critical role of a country's exposure to global uncertainty for understanding the sovereign bond premium.

Table 12 [about here]

Second, we ensure that the results are robust to alternative specifications. Column I of Table 13 reproduces the Table 11 but considering a different holding investment period, using one quarter instead of one year. Column II of Table 13 reports the results excluding NBER recessions. In both cases, the results remain similar to those of the baseline case. Hence, our findings are not driven by the choice of the investment strategy or by specific periods of financial stress, such as the latest 2007-9 financial crisis.

Table 13 [about here]

Finally, we consider an alternative measure of consumption uncertainty to measure long-run macro

risk. We replace the consumption growth forecast dispersion by the macroeconomic uncertainty index of Jurado, Ludvigson, and Ng (2015), following the cross-sectional analysis of Bali, Brown, and Tang (2017) for U.S. stocks. Table 14 shows that a country's exposure to changes in the macroeconomic uncertainty index is priced negatively and significantly, confirming our baseline results, even after controlling for different sources of short-run macro risk.

Table 14 [about here]

This robustness analysis confirms that the risk premium embedded in sovereign bonds compensates investors for a country's exposure to the slow-moving but large changes in global business conditions rather than to frequent but smaller changes in consumption or stock market prices. The results are robust to the econometric specification, are not driven by the Great Recession, and hold for different measures of global uncertainty. Our empirical study, therefore, provides robust evidence that the risk premium arising from long-run macro risk helps explain the cross-sectional differences in sovereign bond excess returns.

## **6 Concluding remarks**

This paper provides new insights on how regime-changing global macroeconomic conditions affect sovereign bond pricing. We uncover a new sovereign bond premium arising from a country's exposure to the global business cycle, which differs from the exposure to higher-frequency global economic shocks. Countries experiencing lower economic growth and more volatile shocks during global recessions have a higher default probability, not only during recessions but also unconditionally. Moreover, investors dislike the rise in sovereign default risk during recessions and adjust their pricing of default risk accordingly. As a result, a country's exposure to global macroeconomic conditions increases the quantity and the price of risk, both of which contribute meaningfully to our understanding of sovereign credit spreads.

We provide empirical support for the prediction that sovereign bonds of countries that are more exposed to the U.S. business cycle offer a higher risk premium. The exposure to both expected consumption growth and consumption uncertainty in the U.S. contributes to explain cross-sectional differences in sovereign bond excess returns, especially during NBER recessions.

## References

- Aguiar, M., and Gopinath, G., 2006, Defaultable Debt, Interest Rates and the Current Account, *Journal of international Economics* 69(1), 64–83.
- Andrade, S. C., 2009, A Model of Asset Pricing under Country Risk, *Journal of International Money and Finance* 28(4), 671–695.
- Andrade, S. C., and Chhaochharia, V., 2018, The Costs of Sovereign Default: Evidence from the Stock Market. *Review of Financial Studies* 31(5), 1707–1751.
- Ang, A., and Longstaff, F. A., 2013, Systemic Sovereign Credit Risk: Lessons from the U.S. and Europe, *Journal of Monetary Economics* 60(5), 493–510.
- Arellano, C., 2008, Default Risk and Income Fluctuations in Emerging Economies, *American Economic Review* 98(3), 690–712.
- Arellano, C., and Bai, Y., 2017, Fiscal Austerity during Debt Crises, *Economic Theory* 64(4), 657–673.
- Arnold, M., Wagner, A. F., and Westermann, R., 2013. Growth Options, Macroeconomic Conditions, and the Cross Section of Credit Risk, *Journal of Financial Economics* 107(2), 350–385.
- Augustin, P., 2018, The Term Structure of CDS Spreads and Sovereign Credit Risk, *Journal of Monetary Economics* 96, 53–76.
- Augustin, P., and Tédongap, R., 2016, Real Economic Shocks and Sovereign Credit Risk, *Journal of Financial and Quantitative Analysis* 51(2), 541–587.
- Bali, T. G., Brown, S. J., and Tang, Y., 2017, Is Economic Uncertainty Priced in the Cross-section of Stock Returns?, *Journal of Financial Economics* 126(3), 471–489.
- Benzoni, L., Collin-Dufresne, P., Goldstein, R. S., and Helwege, J., 2015, Modeling Credit Contagion via the Updating of Fragile Beliefs, *Review of Financial Studies* 28(7), 1960–2008.
- Bhamra, H. S., Dorion, C., Jeanneret, A., Weber, M., 2018, Low Inflation: High Default Risk AND High Equity Valuations, NBER Working paper No. 25317.
- Bhamra, H. S., Kuehn, L.-A., and Strebulaev, I. A., 2010a, The Aggregate Dynamics of Capital Structure and Macroeconomic Risk, *Review of Financial Studies* 23(12), 4187–4241.
- Bhamra, H. S., Kuehn, L.-A., and Strebulaev, I. A., 2010b, The Levered Equity Risk Premium and Credit Spreads: A Unified Framework, *Review of Financial Studies* 23(2), 645–703.
- Bloom, N., Floetotto, M., Jaimovich, N., Saporta-Eksten, I., and Terry, S. J. 2018, Really Uncertain Business Cycles, *Econometrica* 86(3), 1031–1065.
- Borri, N., and Verdelhan, A., 2012, Sovereign Risk Premia, Working Paper, MIT.
- Chen, H., 2010, Macroeconomic Conditions and the Puzzles of Credit Spreads and Capital Structure, *Journal of Finance* 65(6), 2171–2212.
- Chen, H., 2013, Comment on "Systemic Sovereign Credit Risk: Lessons from the U.S. and Europe" by Ang and Longstaff, *Journal of Monetary Economics* 60(5), 511–516.

- Chen, Z., and Strebulaev, I. A., 2019. Macroeconomic Risk and Idiosyncratic Risk-taking, *Review of Financial Studies* 32(3), 1148–1187.
- Cochrane, J. H., 2017. Macro-finance. *Review of Finance* 21(3), 945–985.
- Doshi, H., Jacobs, K., and Zurita, V., 2017, Economic and Financial Determinants of Credit Risk Premiums in the Sovereign CDS Market, *Review of Asset Pricing Studies* 7, 43–80.
- Duffie, D., and Epstein, L. G., 1992, Stochastic Differential Utility, *Econometrica* 60(2), 353–94.
- Duffie, D., and Skiadas, C., 1994, Continuous-Time Security Pricing: A Utility Gradient Approach, *Journal of Mathematical Economics* 23(2), 107–131.
- Dumas, B., 1991, Super Contact and Related Optimality Conditions, *Journal of Economic Dynamics and Control* 15(4), 675–685.
- Eaton, J., and Gersovitz, M., 1981, Debt with Potential Repudiation: Theoretical and Empirical Analysis, *The Review of Economic Studies* 48(2), 289–309.
- Gibson, R., and Sundaresan, S., 2001, A Model of Sovereign Borrowing and Sovereign Yield Spreads, Working Paper, Columbia University.
- Hackbarth, D., Miao, J., and Morellec, E., 2006, Capital Structure, Credit Risk, and Macroeconomic Conditions, *Journal of Financial Economics* 82(3), 519–550.
- Hamilton, J. D., 1989, A New Approach to the Economic Analysis of Nonstationary Time Series and the Business Cycle, *Econometrica* 57(2), 357–384.
- Hatchondo, J. C., and Martinez, L., 2009, Long-duration Bonds and Sovereign Defaults, *Journal of International Economics* 79(1), 117–125.
- Hayri, A., 2000, Debt Relief, *Journal of International Economics* 52(1), 137–152.
- Hebert, B., and Schreger, J., 2017, The Costs of Sovereign Default: Evidence from Argentina, *American Economic Review* 107(10), 3119–3145.
- Hilscher, J., and Nosbusch, Y., 2010, Determinants of Sovereign Risk: Macroeconomic Fundamentals and the Pricing of Sovereign Debt, *Review of Finance* 4(2), 235–262.
- Huang, J. Z., and Huang, M., 2012, How Much of the Corporate-Treasury Yield Spread is due to Credit Risk? *The Review of Asset Pricing Studies* 2(2), 153–202.
- Jeanneret, A., 2018, Sovereign Credit Spreads under Good/Bad Governance, *Journal of Banking and Finance*, 93, 230–248.
- Jeanneret, A., 2017, Sovereign Default Risk and the U.S. Equity Market, *Journal of Financial and Quantitative Analysis*, 52(1), 305–339.
- Jeanneret, A., 2015, The Dynamics of Sovereign Credit Risk, *Journal of Financial and Quantitative Analysis*, 50(5), 963–985.
- Jurado, K., Ludvigson, S. C., and Ng, S., 2015, Measuring Uncertainty, *American Economic Review* 105(3), 1177–1216.

- Longstaff, F. A., Pan, J., Pedersen, L. H., and Singleton, K. J., 2011, How Sovereign Is Sovereign Credit Risk, *American Economic Journal: Macroeconomics* 3(2), 75–103.
- Mendoza, E. G., and Yue, V. Z., 2012, A General Equilibrium Model of Sovereign Default and Business Cycles, *Quarterly Journal of Economics* 127(2), 889–946.
- Merton, R. C., 1973, The Theory of Rational Option Pricing, *Bell Journal of Economics* 4(1), 141–183.
- Moody's, 2017, Sovereign Default and Recovery Rates, 1983-2016, Special Comment, Global Credit Policy, New York.
- Pan, J., and Singleton, K. J., 2008, Default and Recovery Implicit in the Term Structure of Sovereign CDS Spreads, *Journal of Finance* 63, 2345–2384.
- Rebelo, S., Wang, N., Yang, J., 2018, Rare Disasters, Financial Development, and Sovereign Debt, NBER Working paper No. 25031.
- Remolona, E., Scatigna, M., and Wu, E., 2008, The Dynamic Pricing of Sovereign Risk in Emerging Markets: Fundamentals and Risk Aversion, *Journal of Fixed Income* 17(4), 57–71.
- Tédongap, R., 2015, Consumption Volatility and the Cross-Section of Stock Returns, *Review of Finance* 19(1), 367–405.
- Timmermann, A., 2000, Moments of Markov Switching Models, *Journal of Econometrics* 96(1), 75–111.
- Yue, V. Z., 2010, Sovereign Default and Debt Renegotiation, *Journal of International Economics* 80(2), 176–187.

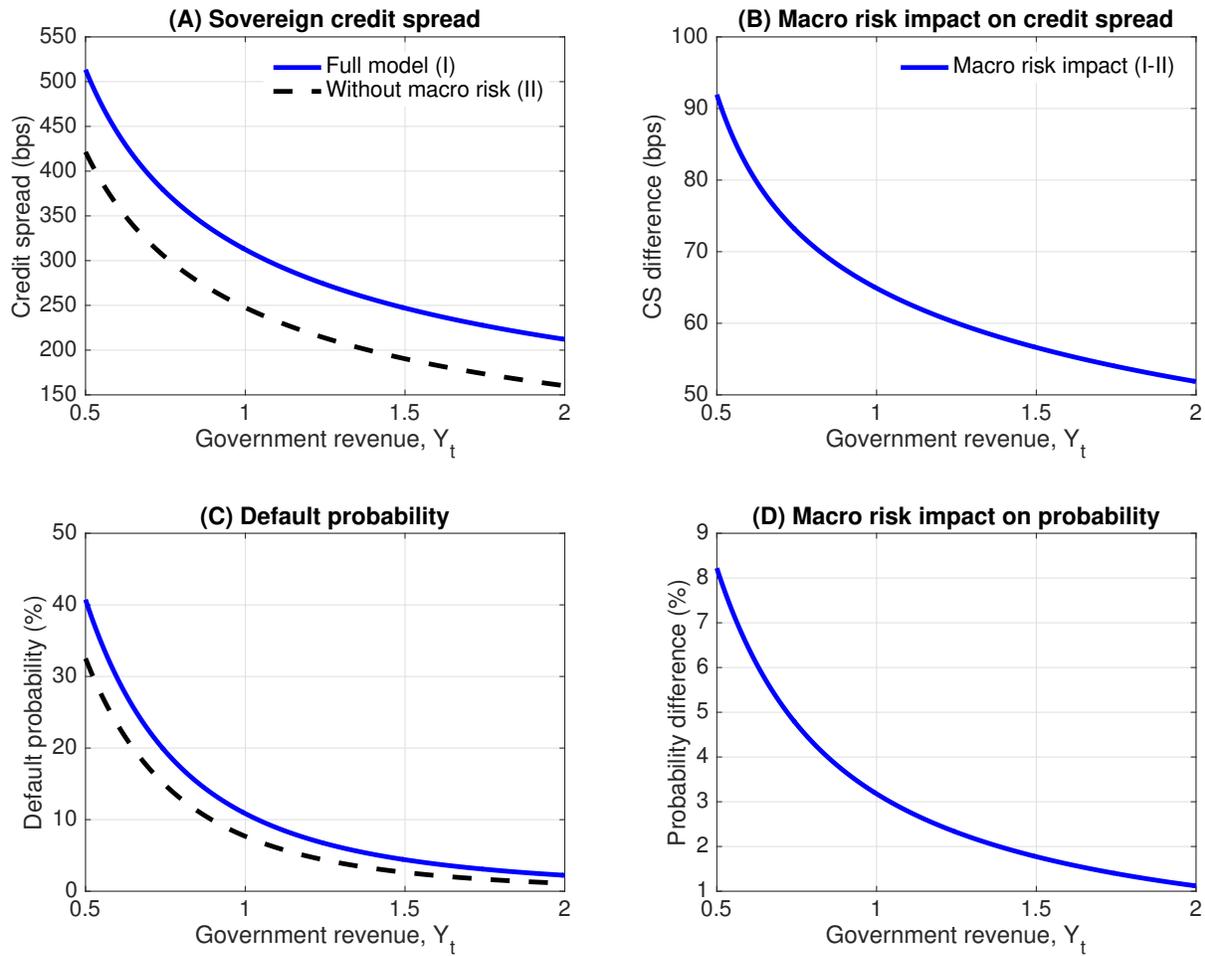


Figure 1: **Sovereign credit spread, default risk, and local economic conditions.**

This figure illustrates the impact of macroeconomic risk on the sovereign credit spread and the default probability. Predictions are reported for different levels of government revenue, as a measure of local economic conditions. Panel A compares sovereign credit spreads with and without macroeconomic risk. Panel B displays the difference in sovereign credit spreads. Panel C compares the results for the 5-year physical default probability with and without macroeconomic risk, while Panel D reports the difference in default probability. The predictions of the model without macroeconomic risk are obtained by switching-off the country's exposure to the global business cycle (i.e., constant mean and volatility of output growth). Both cases are observationally equivalent in terms of risk and imply identical unconditional output growth moments. Unless otherwise specified, we use the parameters of the baseline calibration (see Table 2).

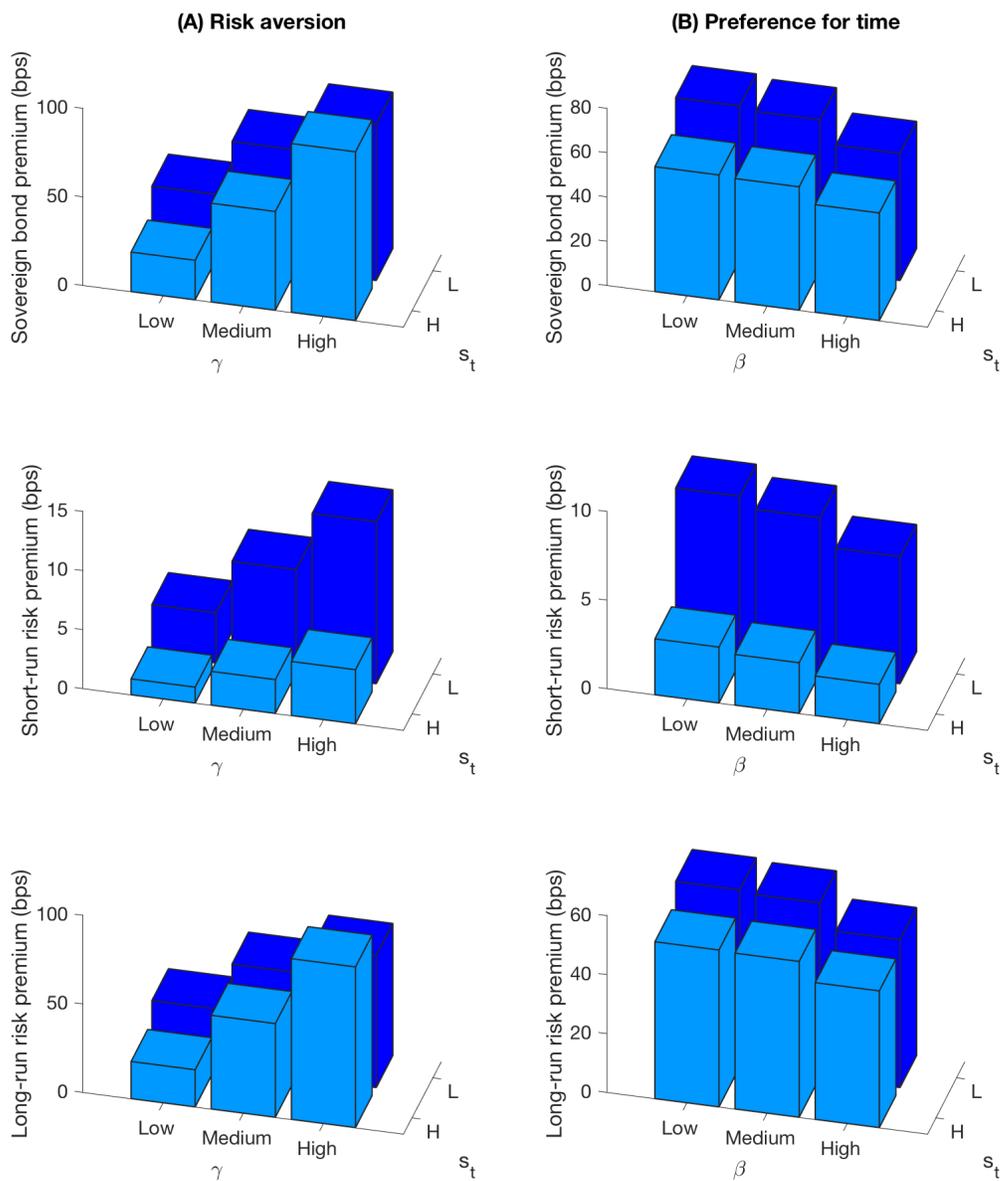


Figure 2: **Sovereign bond premium by investor preferences.**

This figure illustrates how the sovereign bond premium varies with investor preferences. Panel A reports predictions for different levels of relative risk aversion, while Panel B reports predictions for different levels of preference for time. Low and high exposures correspond to risk aversion  $\gamma = 6$  and  $\gamma = 10$  and to preference for time  $\beta = 0.03$  and  $\beta = 0.05$ , respectively. The figure compares predictions when the current state  $s_t$  is in recession ( $L$ ) or expansion ( $H$ ). Unless otherwise specified, we use the parameters of the baseline calibration (see Table 2) and report predictions at issuance time ( $Y = 1$ ).

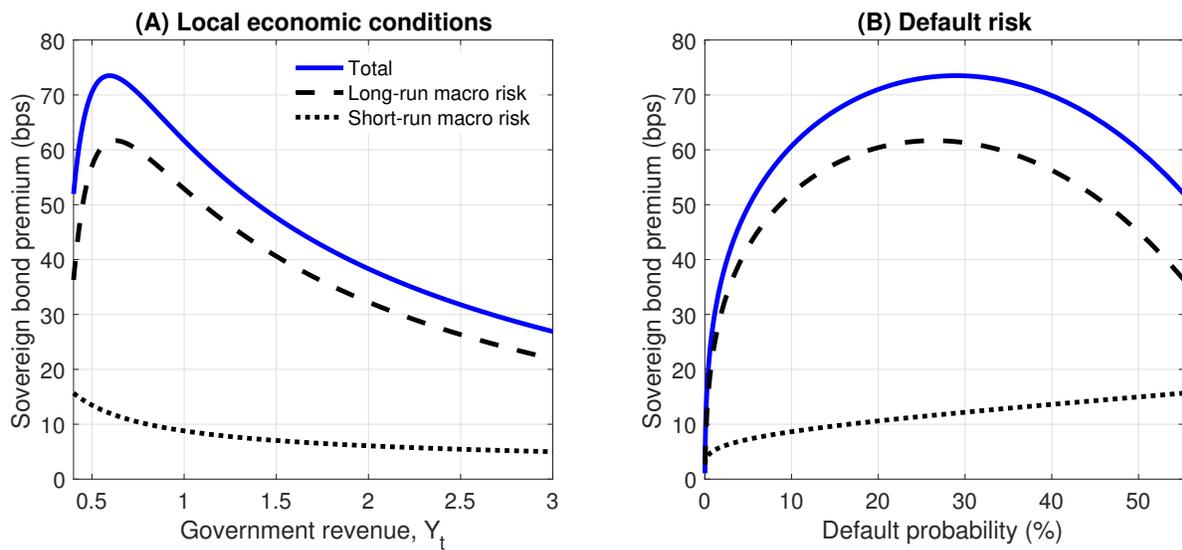


Figure 3: **Sovereign bond premium over time.**

This figure illustrates how the sovereign bond premium varies with local economic conditions. Panel A reports predictions for different levels of government revenue, while Panel B reports predictions by physical default probability computed at a 5-year horizon. The bond premium is decomposed into the compensations for short-run and long-run macro risk. Unless otherwise specified, we use the parameters of the baseline calibration (see Table 2).

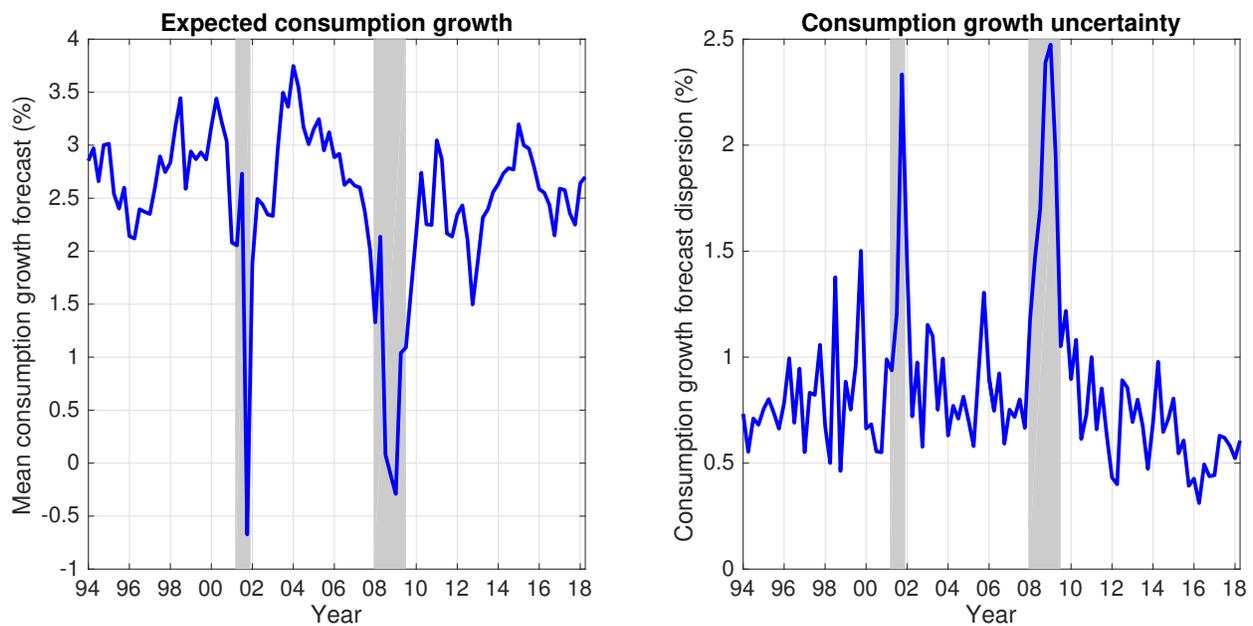


Figure 4: **Global risk factors.**

This figure illustrates the time variation of the global risk factors. Panel A displays the U.S. expected consumption growth, computed as the quarterly mean of consumption growth forecasts. Panel B displays the U.S. consumption growth uncertainty, as measured by the quarterly cross-sectional dispersion in consumption growth forecasts. Both series are annualized and from the Survey of Professional Forecasters, as available at the Federal Reserve Bank of Philadelphia. Grey areas denote NBER recession periods. Data span the period 1994Q1-2018Q2.

Table 1 : **Conditional output growth moments by country.**

This table displays the annualized mean and volatility of output growth for the emerging countries considered in the calibration. We compute the output growth moments with quarterly real GDP data obtained from Datastream and condition the moments on NBER recession/expansion periods. Data span the period 1994Q1-2018Q2.

	<b>Output growth rate (%)</b>		<b>Output growth volatility (%)</b>	
	Recession	Expansion	Recession	Expansion
Algeria	-5.60	4.25	5.08	3.29
Bahrain	5.57	3.21	7.05	4.18
Bolivia	4.04	4.11	1.54	2.13
Brazil	0.24	2.51	3.82	2.22
Bulgaria	1.08	3.87	4.67	1.76
Chile	0.04	4.20	2.10	2.09
Colombia	1.01	4.34	3.37	1.53
Croatia	-0.38	2.16	4.72	2.35
Czech Republic	-1.52	2.88	2.85	1.37
Dominican Republic	5.82	5.01	3.71	3.89
Ecuador	3.27	3.76	2.31	2.14
Estonia	-8.18	5.20	8.02	3.37
Greece	-0.89	1.02	4.23	2.77
Hungary	-1.85	2.78	3.85	1.31
India	5.08	6.74	3.36	2.46
Kazakhstan	-0.78	10.70	20.01	19.69
Latvia	-5.51	4.81	7.07	3.24
Lithuania	-11.36	4.63	11.09	1.89
Malta	0.69	3.95	3.95	3.36
Malaysia	-0.68	5.51	5.00	1.14

Continued.

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	<b>Output growth rate (%)</b>		<b>Output growth volatility (%)</b>	
	Recession	Expansion	Recession	Expansion
Mexico	-3.82	3.03	3.58	2.40
Morocco	4.82	3.59	3.12	3.45
Mozambique	3.49	6.69	3.72	3.80
Namibia	-5.80	4.34	3.49	7.43
Peru	4.42	4.76	3.08	2.68
Philippines	2.36	5.09	2.69	1.61
Poland	2.44	3.99	1.54	1.84
Romania	0.51	3.21	6.16	3.14
Russia	-5.17	3.72	6.18	2.21
Slovakia	-1.55	4.44	7.36	2.53
Slovenia	-2.56	3.19	4.39	1.66
South Africa	0.41	2.95	1.65	1.11
South Korea	1.97	4.68	3.11	2.65
Taiwan	-3.19	4.87	4.29	2.92
Tanzania	3.75	6.83	5.44	3.12
Thailand	0.09	3.85	3.51	3.93
Turkey	-6.23	5.77	7.35	3.80
Uganda	9.24	4.90	2.75	3.01
Venezuela	-1.01	2.28	3.28	8.06
Vietnam	9.30	5.86	6.20	10.70
<i>Average</i>	<i>0.09</i>	<i>4.34</i>	<i>4.77</i>	<i>3.46</i>

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Table 2 : **Model calibration.**

This table reports the parameter values used in the calibration of the model. The state of the global economy  $s_t = H$  refers to expansion while  $s_t = L$  corresponds to recession. The frequency of the data is quarterly and the values are annualized when applicable.

Variable	Notation	Conditional values		Source
State of the global economy	$s_t$	L	H	
<i>Panel A: Global environment</i>				
Expected consumption growth (%)	$\mu_{c,s_t}$	-0.839	1.637	Estimates of a Markov-switching model on U.S. consumption data (Bureau of Economic Analysis), 1994Q1-2018Q2
Consumption growth volatility (%)	$\sigma_{c,s_t}$	0.636	0.584	
Long-run probability (%)	$f_{s_t}$	33.42	66.58	
Convergence rate	$p$	0.436	0.436	
<i>Panel B: Agent preferences</i>				
Time preference	$\beta$	0.04	0.04	
Relative risk aversion	$\gamma$	10	10	
Elasticity of intertemporal substitution	$\psi$	2	2	
<i>Panel C: Country characteristics</i>				
Expected output growth (%)	$\mu_{s_t}$	0.089	4.341	Real GDP data (Datastream), 1994Q1-2018Q2
Output growth volatility (%)	$\sigma_{s_t}$	4.766	3.455	
Correlation with consumption	$\rho_{s_t}$	0.094	0.05	
Volatility leverage factor	$\eta$	10.40	10.40	Calibrated to match default risk
Return on public investment	$r_g$	0.014	0.014	Jeanneret (2015)
Default costs (fraction of output)	$\alpha$	0.050	0.050	Mendoza and Yue (2012)
Debt reduction in default	$\kappa$	0.750	0.750	ISDA's CDS pricing convention

Table 3 : **Sovereign default risk and optimal policies.**

This table reports the sovereign credit spread, debt coupon value, default threshold, and the 5-year physical default probability under endogenous debt policies. Column A contains predictions of the full model, while Column B contains the predictions of a model that switches off a country's exposure to the global business cycle (i.e.,  $\mu_{X,L} = \mu_{X,H}, \sigma_{X,L} = \sigma_{X,H}$ ). Both cases are observationally equivalent in terms of risk and imply identical unconditional output growth moments. Column C displays the predictions of a model in which output shocks are independent from consumption shocks (i.e.,  $\rho = 0$ ). Panel A reports the unconditional results, while Panels B and C present the conditional results when the global economy is respectively in recession and expansion. We use the parameters of the baseline calibration (see Table 2) and report predictions at issuance time ( $Y=1$ ).

	<b>Full model</b>	<b>No business cycle exposure</b>	<b>Independent output shocks</b>
	(A)	(B)	(C)
<i>Panel A: Unconditional</i>			
Credit spread (bps)	331.0	246.9	326.0
Coupon	0.108	0.169	0.117
Default threshold	0.25	0.209	0.249
Default probability (%)	10.03	6.68	9.95
<i>Panel B: Recession</i>			
Credit spread (bps)	358.4	243.5	352.8
Default threshold	0.230	0.208	0.229
Default probability (%)	18.43	6.60	18.26
<i>Panel C: Expansion</i>			
Credit spread (bps)	317.2	248.7	312.5
Default threshold	0.260	0.210	0.260
Default probability (%)	5.81	6.72	5.78

Table 4 : **Sovereign bond premium decomposition.**

This table reports predictions for the sovereign bond premium and its decomposition by source of risk. Column A displays the risk premium associated with a country's exposure to small but frequent consumption growth shocks (short-run macro risk). Column B displays the risk premium associated with a country's exposure to low-frequency but severe changes in consumption growth (long-run macro risk). Column C shows the total bond risk premium of the full model, while Column D reports the bond risk premium in absence of long-run macro risk. This restricted model is obtained by switching-off a country's exposure to the global business cycle (i.e.,  $\mu_{X,L} = \mu_{X,H}, \sigma_{X,L} = \sigma_{X,H}$ ) and by setting the output-consumption correlation to the unconditional value in the data. Panel A shows the predictions for the unconditional case, Panel B when the economy is in recession, and Panel C when the economy is in expansion. Unless otherwise specified, we use the parameters of the baseline calibration (see Table 2) and report predictions at issuance time ( $Y=1$ ).

	I. Full model			II. Restricted model
	Short-run macro risk (A)	Long-run macro risk (B)	Total (C)	Short-run macro risk (D)
<i>Panel A: Unconditional</i>				
Risk premium (bps)	4.76	55.53	60.29	9.14
Fraction of total risk premium (%)	7.90	92.10		
<i>Panel B: Recession</i>				
Risk premium (bps)	8.66	59.77	68.43	9.66
Fraction of total risk premium (%)	12.66	87.34		
<i>Panel C: Expansion</i>				
Risk premium (bps)	2.80	53.40	56.20	8.87
Fraction of total risk premium (%)	4.99	95.01		

Table 5 : **Sovereign bond premium, price of risk, and investor preferences.**

This table presents the model predictions by investor preferences. Column A reports the sovereign bond premium, Column B the level of default risk measured by the 5-year physical default probability ( $\mathbb{P}$ ), Column C the ratio of the risk-neutral default probability ( $\mathbb{Q}$ ) over the physical default probability ( $\mathbb{P}$ ), and Column D the price of risk  $\Delta_H$ . Panel A reports predictions for the baseline calibration, Panel B reports predictions for different levels of relative risk aversion  $\gamma$ , while Panel C reports predictions for different levels of preference for time  $\beta$ . Unless otherwise specified, we use the parameters of the baseline calibration (see Table 2) and report predictions at issuance time ( $Y=1$ ).

	<b>Sovereign bond premium (bps)</b>	<b>Default probability (%)</b>	<b>Risk-neutral over physical default probability (<math>\mathbb{Q}/\mathbb{P}</math>)</b>	<b>Price of risk (<math>\Delta_H</math>)</b>
	(A)	(B)	(C)	(D)
<i>Panel A: Baseline case</i>				
	60.29	10.03	1.38	1.86
<i>Panel B: Relative risk aversion</i>				
High ( $\gamma = 15$ )	94.92	13.79	1.38	2.53
Low ( $\gamma = 5$ )	27.09	7.69	1.20	1.34
<i>Panel C: Preference for time</i>				
High ( $\beta = 0.045$ )	57.66	7.46	1.44	1.87
Low ( $\beta = 0.035$ )	45.73	11.47	0.82	1.84

Table 6 : **Sovereign bond premium and cross-sectional predictions.**

This table reports cross-sectional predictions on the sovereign bond premium. Each value arises from a different combination of exposure to the various sources of macroeconomic risk. Rows report the sovereign bond premium by exposure of expected output growth rate to global expected consumption growth ( $\phi_\mu$ ), whereas the columns report the sovereign bond premium by exposure of output growth volatility to global consumption volatility ( $\phi_\sigma$ ). We use the parameters of the baseline calibration (see Table 2) and report predictions at issuance time ( $Y=1$ ).

*Panel A: Premium for short-run macro risk*

		<b>Output-consumption correlation, <math>\rho</math></b>				
		-0.5	-0.25	0	0.25	0.5
		-29.29	-14.66	0	14.68	29.39

*Panel B: Premium for long-run macro risk*

		<b>Volatility exposure, <math>\phi_\sigma</math></b>				
		0	0.5	1	1.5	2
<b>Expected growth exposure, <math>\phi_\mu</math></b>	0	-26.87	-0.09	26.02	51.19	75.42
	0.5	-9.51	17.12	43.07	67.90	91.49
	1	8.21	34.67	60.29	84.78	107.75
	1.5	26.17	52.43	77.73	101.80	124.12
	2	44.45	70.39	95.34	118.91	140.50

Table 7 : **Empirical distribution of macroeconomic risk.**

This table reports statistics on macroeconomic risk across countries. Column I reports statistics on the output-consumption correlation. Column II reports statistics on the degree of exposure the business cycle, whose computation is discussed in Section 4.3. Data span the period 1994Q1-2018Q2.

	<b>I. Output-consumption correlation</b>	<b>II. Exposure to the business cycle</b>	
	$\rho$	$\phi_{\mu}$	$\phi_{\sigma}$
Minimum	-0.256	-1.021	-3.646
10th percentile	-0.060	-0.241	-0.388
25th percentile	0.047	0.420	-0.025
Median	0.121	0.759	0.969
75th percentile	0.295	1.533	2.132
90th percentile	0.406	2.564	2.989
Maximum	0.507	3.761	7.014

Table 8 : **Asset pricing implications.**

This table reports the expected excess return of an investment strategy that exploits the cross-sectional differences in sovereign bond premium. Panel A reports the sovereign bond premium for different levels of output-consumption correlation. The low (high) correlation level equals the average of the bottom (top) quartile of the empirical distribution of the output-consumption correlation. Panel B reports predictions for different exposures of expected output growth and output volatility to the global business cycle. The low (high) exposure level equals the average of the bottom (top) quartile of the empirical distribution of  $\phi_\mu$  and  $\phi_\sigma$ . Each panel also reports predictions for the expected excess return of the high-minus-low (HML) strategy. Results are presented unconditionally and conditionally. We use the parameters of the baseline calibration (see Table 2) and report predictions at issuance time ( $Y=1$ ).

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*Panel A: High vs. low short-run macro risk*

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	<b>Output-consumption correlation, <math>\rho</math></b>		
	Low (A)	High (B)	HML (B) - (A)
Unconditional	45.99	75.57	29.58
Recession	49.91	86.81	36.90
Expansion	44.02	69.93	25.90

*Panel B: High vs. low long-run macro risk*

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	<b>Expected growth exposure, <math>\phi_\mu</math></b>			<b>Volatility exposure, <math>\phi_\sigma</math></b>		
	Low (A)	High (B)	HML (B) - (A)	Low (C)	High (D)	HML (D) - (C)
Unconditional	24.64	72.81	48.17	21.47	128.99	107.52
Recession	26.11	79.05	52.94	23.39	152.52	129.13
Expansion	23.91	69.67	45.77	20.51	117.18	96.67

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Table 9 : **Descriptive statistics.**

This table reports descriptive statistics for the sovereign bond excess returns and the estimated risk loadings by country. Bond excess returns are computed for each country as the quarterly log return on the country's JP Morgan EMBI index less the 3-month Treasury-bill rate. Bond excess returns are annualized and reported in percentage points. The country-average risk loadings  $\beta_i^c$ ,  $\beta_i^\mu$ , and  $\beta_i^\sigma$  respectively determine the exposure of a country  $i$ 's bond excess returns to U.S. consumption shocks, expected consumption growth, and consumption uncertainty. Section 5.1 presents the data while Section 5.2 details the methodology to compute the risk loadings. The sample covers the period 1994Q1-2018Q2.

	Bond excess returns (%)				Risk loadings		
	Mean	Std	p10	p90	$\beta_i^c$	$\beta_i^\mu$	$\beta_i^\sigma$
Argentina	1.68	14.05	-10.53	17.87	8.90	2.05	-13.10
Belize	2.46	17.9	-23.98	18.7	7.05	18.82	-25.83
Brazil	2.27	7.87	-6.64	11.07	1.14	-2.49	-2.85
Bulgaria	2.60	7.67	-5.48	9.86	0.33	2.55	-2.58
Chile	1.28	3.03	-2.13	4.65	-0.21	-0.35	-1.29
China	1.09	3.04	-1.49	3.54	-1.01	-1.07	0.65
Colombia	1.79	5.6	-4.56	8.03	0.49	0.61	-0.53
Cote d'Ivoire	1.76	14.16	-14.11	17.83	-7.94	-0.41	-4.90
Croatia	0.27	5.92	-1.61	4.94	-0.79	-1.94	2.13
Dominican Republic	2.42	10.04	-5.49	9.30	0.59	-0.14	-5.89
Ecuador	3.10	14.1	-15.17	20.17	1.95	3.69	-10.76
Egypt	1.56	4.43	-3.00	7.98	1.57	-2.63	0.29
El Salvador	1.76	6.67	-4.25	7.02	1.75	1.09	-4.68
Gabon	2.39	10.10	-3.89	10.87	-2.29	0.80	-4.76
Ghana	2.86	10.45	-2.91	11.28	-0.40	0.28	-4.96
Hungary	1.18	3.68	-2.21	4.83	-0.30	-0.17	1.09
Indonesia	2.11	7.47	-3.88	8.73	0.16	0.74	-7.27
Iraq	3.00	11.32	-6.05	14.63	-3.87	1.26	0.06
Kazakhstan	2.05	9.27	-4.48	9.06	-1.89	0.62	-3.33
Lebanon	1.55	3.25	-2.23	5.24	-0.07	0.05	-0.84

Continued.

	<b>Bond excess returns (%)</b>				<b>Risk loadings</b>		
	Mean	Std	p10	p90	$\beta_i^c$	$\beta_i^\mu$	$\beta_i^\sigma$
Malaysia	1.17	4.38	-3.70	5.51	-1.29	-1.18	-1.65
Mexico	1.62	5.33	-3.30	7.5	-0.92	-0.96	-1.80
Morocco	0.34	8.63	-3.04	6.77	1.04	-4.52	0.27
Pakistan	2.01	11.88	-3.05	9.27	-4.24	5.66	-13.65
Panama	2.38	7.61	-4.02	8.94	-1.79	-2.04	-4.29
Peru	2.55	8.49	-7.98	12.72	-1.53	-3.77	-2.94
Philippines	1.82	5.46	-3.69	7.66	-1.86	-2.52	-3.92
Poland	1.60	5.14	-3.04	6.35	-0.94	-2.40	-1.17
Russia	3.77	14.1	-4.29	15.41	-4.13	-0.96	-9.40
Serbia	1.65	6.75	-2.74	7.39	-0.04	1.23	-4.33
South Africa	1.59	4.59	-2.68	6.40	-1.80	-1.62	-1.89
Sri Lanka	2.43	9.56	-2.85	7.89	-3.76	-1.20	-0.40
Thailand	1.42	9.06	-1.71	4.13	-3.84	-1.31	-2.58
Trinidad and Tobago	0.39	4.80	-2.51	3.85	1.18	1.37	-3.37
Tunisia	0.74	14.72	-15.52	18.3	5.76	0.95	-16.37
Turkey	2.00	7.36	-6.14	8.84	0.42	-1.11	-3.35
Ukraine	3.83	17.65	-7.18	14.85	-6.65	3.72	-15.76
Uruguay	2.22	9.94	-6.79	12.16	2.99	1.04	-2.19
Venezuela	2.54	11.80	-10.23	17.48	-0.74	-3.57	-10.72
Vietnam	1.48	4.87	-2.81	5.67	-0.08	-0.75	-4.60
<i>Average</i>	<i>1.92</i>	<i>8.55</i>	<i>-5.54</i>	<i>9.82</i>	<i>-0.43</i>	<i>0.24</i>	<i>-4.84</i>

Table 10 : **Main results – portfolios formed on short-run macro risk.**

This table reports statistics on the risk loadings, sovereign bond excess returns, and Sharpe ratio for each portfolio formed on short-run macro risk. Risk loadings are obtained from time-series regressions of quarterly sovereign bond excess returns on U.S. consumption shocks. Columns I and II report results when the risk loadings are estimated without or with long-run macro risk exposures. Panels A and B report the results of 4 portfolios, which are formed quarterly by sorting countries based on the estimated risk loadings. The column “HML” shows returns of a zero investment portfolio that is long in the high exposure portfolio and short in the low exposure portfolio. The holding investment period is four quarters. Excess returns are annualized and reported in percentage points. Sharpe ratios are computed as ratios of annualized means to annualized standard deviations. Reported standard errors are based on a Newey and West (1987)’s correction with 4 lags. The sample covers the period 1994Q1-2018Q2.

	I. Without long-run macro risk					II. With long-run macro risk				
	Low			High	HML	Low			High	HML
$\beta^c$	-6.44	-2.21	-0.26	5.20	11.63	-7.09	-1.74	0.11	5.44	12.53
Std	4.56	3.24	2.72	3.82	4.00	3.50	2.50	2.30	4.30	4.44
Exc. return	2.65	0.90	1.27	2.33	-0.32	2.76	1.35	1.24	1.85	-0.92
t-stat	3.99	2.94	4.30	4.01	-0.35	4.03	4.12	4.13	3.49	-1.01
Sharpe ratio	1.89	0.73	1.01	1.15	-0.15	1.86	1.13	1.24	0.89	-0.43

Table 11 : **Main results – portfolios formed on long-run macro risk.**

This table reports statistics on the risk loadings, sovereign bond excess returns, and Sharpe ratio for each portfolio formed on long-run macro risk. Risk loadings are obtained from time-series regressions of quarterly sovereign bond excess returns on (the first difference of) U.S. expected consumption growth and U.S. consumption uncertainty. Columns I and II report results when the risk loadings are estimated individually or jointly. Panels A and B report the results of 4 portfolios, which are formed quarterly by sorting countries based on the estimated risk loadings. The column “HML” shows returns of a zero investment portfolio that is long in the high exposure portfolio and short in the low exposure portfolio. The holding investment period is four quarters. Excess returns are annualized and reported in percentage points. Sharpe ratios are computed as ratios of annualized means to annualized standard deviations. Reported standard errors are based on a Newey and West (1987)’s correction with 4 lags. The sample covers the period 1994Q1-2018Q2.

	I. Individual estimation					II. Joint estimation				
<i>Panel A: Portfolios sorted by exposure to expected consumption growth</i>										
	Low			High	HML	Low			High	HML
$\beta^\mu$	-5.09	-2.64	-1.04	4.51	9.60	-5.32	-2.71	-1.04	4.16	9.48
Std	6.40	5.43	4.69	3.42	5.30	5.18	4.24	3.31	2.24	5.19
Exc. return	1.36	0.90	2.07	2.88	1.52	1.78	0.91	1.64	2.83	1.05
t-stat	4.03	1.89	4.24	5.58	3.36	5.18	1.99	4.41	4.97	2.39
Sharpe ratio	1.14	0.57	1.54	1.55	0.89	1.33	0.54	1.39	1.72	0.72
<i>Panel B: Portfolios sorted by exposure to consumption uncertainty</i>										
	Low			High	HML	Low			High	HML
$\beta^\sigma$	-14.48	-5.63	-2.46	1.79	16.27	-13.94	-5.01	-2.00	2.38	16.32
Std	9.61	6.07	5.13	4.72	6.95	8.79	5.96	4.94	4.88	5.45
Exc. return	3.00	1.50	1.35	1.42	-1.58	2.95	1.26	1.51	1.49	-1.46
t-stat	4.68	3.93	3.96	3.73	-2.46	4.31	2.72	5.05	4.10	-2.07
Sharpe ratio	1.40	1.11	1.21	1.52	-0.79	1.41	0.84	1.53	1.47	-0.74

Table 12 : **Robustness analysis – controlling for short-run risk.**

This table reports statistics on the risk loadings and sovereign bond excess returns for each portfolio, when controlling for various sources of short-run macro risk. Columns I and II report results associated with long-run macro risk when controlling for a country’s exposure to U.S. consumption shocks or excess returns on the S&P 500 index. Risk loadings are obtained from time-series regressions of quarterly sovereign bond excess returns on (the first difference of) U.S. expected consumption growth and U.S. consumption uncertainty. Panels A and B report the results of 4 portfolios, which are formed quarterly by sorting countries based on the estimated risk loadings. The column “HML” shows returns of a zero investment portfolio that is long in the high exposure portfolio and short in the low exposure portfolio. The holding investment period is four quarters. Excess returns are annualized and reported in percentage points. Sharpe ratios are computed as ratios of annualized means to annualized standard deviations. Reported standard errors are based on a Newey and West (1987)’s correction with 4 lags. The sample covers the period 1994Q1-2018Q2.

	<b>I. Controlling for consumption shocks</b>					<b>II. Controlling for stock market returns</b>						
<i>Panel A: Portfolios sorted by exposure to expected consumption growth</i>												
	Low			High		HML	Low			High		HML
$\beta^\mu$	-5.16	-2.57	-0.98	4.65	9.81	-4.93	-2.63	-1.22	3.24	8.17		
Std	6.18	5.15	4.44	3.44	5.31	5.02	3.94	2.98	1.90	5.22		
Exc. return	1.67	1.05	1.73	2.73	1.06	1.75	1.32	1.54	2.60	0.85		
t-stat	4.42	2.09	4.43	5.19	2.32	3.94	3.06	4.93	4.12	1.12		
Sharpe ratio	0.98	0.62	1.66	1.81	0.59	1.08	0.75	1.42	1.67	0.47		
<i>Panel B: Portfolios sorted by exposure to consumption uncertainty</i>												
	Low			High		HML	Low			High		HML
$\beta^\sigma$	-14.53	-5.53	-2.25	2.26	16.79	-9.68	-3.43	-1.26	2.32	12.00		
Std	9.60	6.23	5.43	5.62	7.13	4.67	4.26	4.30	4.55	3.08		
Exc. return	2.83	1.50	1.50	1.45	-1.39	2.84	1.88	1.25	1.38	-1.46		
t-stat	4.57	3.23	4.94	3.88	-2.40	4.90	4.26	3.03	4.71	-2.99		
Sharpe ratio	1.42	1.01	1.43	1.54	-0.76	1.49	1.60	0.83	1.50	-0.95		

Table 13 : **Robustness analysis – alternative specifications.**

This table reports statistics on the risk loadings and sovereign bond excess returns for each portfolio under different specifications. Column I reports results when the holding investment period is one quarter. Column II reports results when excluding NBER recessions. Risk loadings are obtained from time-series regressions of quarterly sovereign bond excess returns on (the first difference of) U.S. expected consumption growth and U.S. consumption uncertainty. Panels A and B report the results of 4 portfolios, which are formed quarterly by sorting countries based on the estimated risk loadings. The column “HML” shows returns of a zero investment portfolio that is long in the high exposure portfolio and short in the low exposure portfolio. The holding investment period is four quarters in Column II. Excess returns are annualized and reported in percentage points. Sharpe ratios are computed as ratios of annualized means to annualized standard deviations. Reported standard errors are based on a Newey and West (1987)’s correction with 4 lags. The sample covers the period 1994Q1-2018Q2.

	I. One-quarter holding period					II. Excluding recessions				
<i>Panel A: Portfolios sorted by exposure to expected consumption growth</i>										
	Low			High	HML	Low			High	HML
$\beta^\mu$	-5.09	-2.64	-1.04	4.51	9.60	-5.08	-2.55	-0.98	4.73	9.81
Std	6.40	5.43	4.69	3.42	5.30	6.51	5.49	4.79	3.47	5.39
Exc. return	1.36	1.31	2.41	2.80	1.44	1.45	1.40	1.97	2.73	1.28
t-stat	4.35	4.16	3.62	5.50	2.67	3.34	6.01	4.31	7.02	2.79
Sharpe ratio	0.55	0.68	0.93	0.71	0.39	1.49	1.75	1.74	2.19	0.94
<i>Panel B: Portfolios sorted by exposure to consumption uncertainty</i>										
	Low			High	HML	Low			High	HML
$\beta^\sigma$	-14.48	-5.63	-2.46	1.79	16.27	-14.49	-5.46	-2.18	2.11	16.60
Std	9.61	6.07	5.13	4.72	6.95	9.93	6.24	5.25	4.88	7.10
Exc. return	3.30	1.75	1.50	1.29	-2.00	2.84	1.80	1.49	1.40	-1.44
t-stat	4.39	4.19	4.47	3.22	-2.38	5.27	5.14	5.01	4.20	-2.58
Sharpe ratio	0.80	0.73	0.69	0.65	-0.54	1.99	1.89	1.50	1.76	-0.94

Table 14 : **Robustness analysis – alternative measure of long-run macro risk.**

This table reports statistics on the risk loadings and sovereign bond excess returns for each portfolio based on a country's exposure to macroeconomic uncertainty. Columns I and II report results when controlling for a country's exposure to U.S. consumption shocks and to excess returns on the S&P 500 index. Risk loadings are obtained from time-series regressions of quarterly sovereign bond excess returns on the (first difference of the) macroeconomic uncertainty index of Jurado, Ludvigson, and Ng (2015). Panels A and B report the results of 4 portfolios, which are formed quarterly by sorting countries based on the estimated risk loadings. The column "HML" shows returns of a zero investment portfolio that is long in the high exposure portfolio and short in the low exposure portfolio. The holding investment period is four quarters. Excess returns are annualized and reported in percentage points. Sharpe ratios are computed as ratios of annualized means to annualized standard deviations. Reported standard errors are based on a Newey and West (1987)'s correction with 4 lags. The sample covers the period 1994Q1-2018Q2.

	I. Controlling for consumption shocks					II. Controlling for stock market returns				
	Low		High		HML	Low		High		HML
$\beta^\sigma$	-1.64	-0.68	-0.41	0.32	1.96	-1.24	-0.44	-0.22	0.43	1.67
Std	1.40	0.88	0.77	0.38	1.37	1.06	0.57	0.47	0.43	1.10
Exc. return	2.84	1.78	0.94	1.72	-1.12	2.90	1.32	1.46	1.57	-1.33
t-stat	5.46	3.61	2.30	5.83	-2.88	5.21	3.72	3.96	4.83	-3.03
Sharpe ratio	1.62	1.41	0.67	1.52	-0.74	1.55	1.27	1.15	1.31	-0.89

## Appendix

This appendix provides details on the model derivation. We determine all claims and endogenous variables according to the state of the economy, which can be in expansion ( $s_t = H$ ) or in recession ( $s_t = L$ ).

### A State-price density and equilibrium risk-free rate

In this section, we describe the state-price density and the equilibrium risk-free rate, which closely follow Bhamra, Kuehn, and Strebulaev (2010b). The state-price density is initially derived by Duffie and Skiadas (1994) for the general class of stochastic differential utility function proposed by Duffie and Epstein (1992). This type of utility function incorporates not only the agent's risk aversion but also the aversion for intertemporal resolution of the uncertainty. We denote the coefficient of relative risk aversion by  $\gamma$ , the elasticity of intertemporal substitution by  $\psi$ , and the subjective time discount factor by  $\beta$ .

The representative agent's state-price density  $\pi_t$ , in the case  $\psi \neq 1$ , is given by

$$\pi_t = \left( \beta e^{-\beta t} \right)^{\frac{1-\gamma}{1-\frac{1}{\psi}}} C_t^{-\gamma} \left( p_{C,s_t} e^{\int_0^t p_{C,s_u}^{-1} du} \right)^{-\frac{\gamma-\frac{1}{\psi}}{1-\frac{1}{\psi}}}, \quad (23)$$

where  $C_t$  is the agent's consumption and  $p_{C,s_t}$  is the price-consumption ratio that satisfies the following implicit non-linear equation:

$$p_{C,s_t}^{-1} = \bar{r}_{s_t} - \mu_{c,s_t} + \gamma \sigma_{c,s_t}^2 - \left( 1 - \frac{1}{\psi} \right) \lambda_{s_t} \left( \frac{\left( \frac{p_{C,\bar{s}_t}}{p_{C,s_t}} \right)^{\frac{1-\gamma}{1-\frac{1}{\psi}}} - 1}{1-\gamma} \right), \quad s_t, \bar{s}_t \in \{L, H\}, \bar{s}_t \neq s_t \quad (24)$$

with

$$\bar{r}_{s_t} = \beta + \frac{1}{\psi} \mu_{c,s_t} - \frac{1}{2} \gamma \left( 1 + \frac{1}{\psi} \right) \sigma_{c,s_t}^2. \quad (25)$$

The dynamics of the state-price density  $\pi_t$  follow the following stochastic differential equation

$$\frac{d\pi_t}{\pi_t} = -r_{s_t}dt - \frac{dM_t}{M_t} \quad (26)$$

$$= -r_{s_t}dt - \Theta_{s_t}^B dB_t - \Theta_{s_t}^P dN_{s_t,t}, \quad (27)$$

where  $M$  is a martingale under the physical measure,  $N_{s_t,t}$  a Poisson process which jumps upward by one whenever the state of the global economy switches from  $s_t$  to  $\bar{s}_t \neq s_t$ ,  $\Theta_{s_t}^P = 1 - \Delta_{s_t}$  is the market price of risk due to Poisson shocks when the economy switches out of state  $s_t = \{L, H\}$ , and  $\Theta_{s_t}^B = \gamma\sigma_{c,s_t}^2$  is the market price of risk due to Brownian shocks in state  $s_t$ . The risk distortion factors are such that  $\Delta_H = \Delta_L^{-1}$ , with  $\Delta_H$  the solution to  $G(\Delta_H) = 0$  where

$$G(x) = x^{-\frac{1-\frac{1}{\psi}}{\gamma-1}} - \frac{\bar{r}_H + \gamma\sigma_{c,H}^2 - \mu_{c,H} + \lambda_H \frac{1-\frac{1}{\psi}}{\gamma-1} \left( x^{\frac{\gamma-1}{\gamma-1\psi}} - 1 \right)}{\bar{r}_L + \gamma\sigma_{c,L}^2 - \mu_{c,L} + \lambda_L \frac{1-\frac{1}{\psi}}{\gamma-1} \left( x^{-\frac{\gamma-1}{\gamma-1\psi}} - 1 \right)}, \quad \psi \neq 1. \quad (28)$$

Finally,  $r_{s_t}$  represents the equilibrium instantaneous risk-free rate, which is given by

$$r_{s_t} = \begin{cases} \bar{r}_L + \lambda_L \left[ \frac{\gamma-\frac{1}{\psi}}{1-\gamma} \left( \Delta_H^{-\frac{\gamma-1}{\gamma-1\psi}} - 1 \right) - (\Delta_H^{-1} - 1) \right] & , s_t = L \\ \bar{r}_H + \lambda_H \left[ \frac{\gamma-\frac{1}{\psi}}{1-\gamma} \left( \Delta_H^{\frac{\gamma-1}{\gamma-1\psi}} - 1 \right) - (\Delta_H - 1) \right] & , s_t = H. \end{cases} \quad (29)$$

## B Sovereign bond valuation and credit spread

The sovereign bond value, denoted by  $D_{s_t}(Y_t)$  when the current state is  $s_t$ , is determined by

$$D_{s_t}(Y_t) = E_t \left[ \int_t^{t_D} c \frac{\pi_u}{\pi_t} du \middle| s_t \right] + E_t \left[ \int_{t_D}^{\infty} (1 - \kappa) c \frac{\pi_u}{\pi_t} du \middle| s_t \right] \quad (30)$$

$$= E_t \left[ \int_t^{\infty} c \frac{\pi_u}{\pi_t} du \middle| s_t \right] - E_t \left[ \frac{\pi_{t_D}}{\pi_t} \int_{t_D}^{\infty} \kappa c \frac{\pi_u}{\pi_{t_D}} du \middle| s_t \right], \quad (31)$$

where  $c$  is the perpetual debt coupon,  $\kappa$  is the debt haircut in default, and  $t_D$  is the unknown default time. The first term of Equation 31 represents a risk-free claim that delivers  $c$  in every period. It

corresponds to the value of a perpetual risk-free bond, which equals

$$E_t \left[ \int_t^\infty c \frac{\pi_u}{\pi_t} du \mid s_t \right] = \frac{c}{r_{B,s_t}}, \quad (32)$$

where  $r_{B,s_t}$  is the discount rate for a riskless perpetuity, when the current state is  $s_t$ , which is given by

$$r_{B,s_t} = r_{s_t} + \frac{r_j - r_{s_t}}{\hat{p} + r_j} \hat{p} f_j, \quad j \neq s_t; \quad j, s_t = \{L, H\} \quad (33)$$

which indicates that the discount rate  $r_{B,H}$  is lower than the corresponding instantaneous risk-free rate  $r_H$  (and  $r_{B,L}$  is higher than  $r_L$ ) because the risk-free rate is expected to change in the future with the state of the economy.

The second part of Equation 31 is given by

$$E_t \left[ \frac{\pi_{t_D}}{\pi_t} \int_{t_D}^\infty \kappa c \frac{\pi_u}{\pi_{t_D}} du \mid s_t \right] = \sum_{s_D} E_t \left[ \Pr(s_D \mid s_t) \frac{\pi_{t_D}}{\pi_t} \int_{t_D}^\infty \kappa c \frac{\pi_u}{\pi_{t_D}} du \mid s_t \right] \quad (34)$$

$$= \sum_{s_D} E_t \left[ \Pr(s_D \mid s_t) \frac{\pi_{t_D}}{\pi_t} \mid s_t \right] E_t \left[ \int_{t_D}^\infty \kappa c \frac{\pi_u}{\pi_{t_D}} du \mid s_{t_D} \right] \quad (35)$$

$$= \sum_{s_D} \frac{\kappa c}{r_{B,s_D}} q_{s_t s_D}(Y_t). \quad (36)$$

where  $s_D \in \{L, H\}$  is the state at the time of default and the summation over  $s_D$  indicates that a default can occur in either state,  $s_D = L$  or  $s_D = H$ . Equation 34 can be separated into two parts (Equation 35), given the state-price density is Markovian. The first term of Equation 35 is equal to

$$E_t \left[ \Pr(s_D \mid s_t) \frac{\pi_{t_D}}{\pi_t} \mid s_t \right] = q_{s_t s_D}(Y_t), \quad (37)$$

which is the claim that pays one unit of consumption at the default time  $t_D$ , when the current state is  $s_t$ , which corresponds to the Arrow-Debreu claim  $q_{s_t s_D}(Y_t)$ . The second term of Equation 35,  $E_t \left[ \int_{t_D}^\infty \kappa c \frac{\pi_u}{\pi_{t_D}} du \mid s_{t_D} \right]$ , is the value of a claim at default time, which pays  $\kappa c$  in perpetuity and whose discount rate is  $r_{B,s_D}$ . It is thus equal to  $\frac{\kappa c}{r_{B,s_D}}$ .

Combining the different parts, the sovereign bond value is finally equal to

$$D_{s_t}(Y_t) = \frac{c}{r_{B,s_t}} - \sum_{s_D} \frac{\kappa c}{r_{B,s_D}} q_{s_t s_D}(Y_t), \quad s_t, s_D = \{L, H\}. \quad (38)$$

The sovereign credit spread that the agent requires for holding the country's government bond,

when the current state is  $s_t$ , is determined as follows:

$$CS_{s_t}(Y_t) = \frac{c}{D_{s_t}(Y_t)} - r_{B,s_t} \quad (39)$$

$$= \frac{1}{\left[ r_{B,s_t} - \sum_{s_D} \frac{\kappa}{r_{B,s_D}} q_{s_t s_D}(Y_t) \right]} - r_{B,s_t} \quad (40)$$

$$= r_{B,s_t} \left[ \frac{1}{1 - \sum_{s_D} \frac{\kappa}{r_{B,s_D}} q_{s_t s_D}(Y_t)} - 1 \right], \quad s_t, s_D = \{L, H\}. \quad (41)$$

The probability of sovereign default over a time period  $T$  and within a given state  $s_t$ , is given by:

$$P \left( \inf_{0 \leq t \leq T} Y_t \leq Y_{D,s_t} \mid Y_t > Y_{D,s_t} \right) = \Phi \left( \frac{\ln\left(\frac{Y_{D,s_t}}{Y_t}\right) - \left(\mu_{s_t} - \frac{\sigma_{s_t}^2}{2}\right) T}{\sigma_{s_t} \sqrt{T}} \right) \quad (42)$$

$$+ \left( \frac{Y_{D,s_t}}{Y_t} \right)^{\frac{2\mu_{s_t}}{\sigma_{s_t}^2} - 1} \Phi \left( \frac{\ln\left(\frac{Y_{D,s_t}}{Y_t}\right) + \left(\mu_{s_t} - \frac{\sigma_{s_t}^2}{2}\right) T}{\sigma_{s_t} \sqrt{T}} \right),$$

where  $\Phi(\cdot)$  is the cumulative density of a standard normal distribution.

## C Arrow-Debreu default claims

This section derives the two kinds of Arrow-Debreu default claims that are used to discount risky government revenue. The first kind of Arrow-Debreu claims captures the default triggered by the country's government revenue continuously falling below a default threshold within a given state, which is given by

$$q_{s_t s_D} = E_t \left[ \frac{\pi_{tD}}{\pi_t} Prob(s_D \mid s_t) \mid s_t \right]. \quad (43)$$

The second kind of Arrow-Debreu claims additionally accounts for the instantaneous default related to a change in the state of the global economy, although the country's government revenue may remain unchanged. This situation can occur when the global economy is in the economic state with the lower default threshold and switches to the other state, such that the default threshold instantaneously increases to a higher level. If the level of the country's government revenue was above the initial default

threshold, but below the new default threshold, there is a sudden unexpected default. This second kind of Arrow-Debreu claims is given by

$$q'_{s_t s_D} = E_t \left[ \frac{\pi_{t_D} Y_{t_D}}{\pi_t Y_{s_t}} \text{Prob}(s_D | s_t) \middle| s_t \right]. \quad (44)$$

### C.1 First kind

The Arrow-Debreu default claim  $q_{s_t s_D}$  is the time- $t$  value of a security that pays one unit of consumption at the moment of default  $t_D$ , where  $s_t$  represents the present state of the global economy, and  $s_D$  the state at the default time. The time of default is the first time that the government revenue of the country falls to the threshold  $Y_{D,s_D}$ . By definition, this Arrow-Debreu claim is given by

$$q_{s_t s_D} = E_t \left[ \frac{\pi_{t_D}}{\pi_t} \text{Prob}(s_D | s_t) \middle| s_t \right], \quad (45)$$

which is solution of the two ordinary differential equations (ODE):

$$\frac{1}{2} \sigma_{s_t}^2 Y^2 \frac{d^2 q_{s_t s_D}}{dY^2} + \mu_{s_t} Y \frac{dq_{D,s_t s_D}}{dY} + \hat{\lambda}_{s_t} (q_{j s_D} - q_{s_t s_D}) - r_{s_t} q_{s_t s_D} = 0, \quad s_t = \{L, H\}, \quad (46)$$

where  $\mu_{s_t}$  and  $\sigma_{s_t}$  denote the expected growth rate and the volatility of government revenue in state  $s_t$ , and  $\hat{\lambda}_{s_t}$  is the risk-neutral probability of leaving state  $s_t$ .

The above ODEs are obtained by applying Ito's Lemma to the classical non-arbitrage condition

$$E_t^Q [dq_{s_t s_D} - r_{s_t} q_{s_t s_D}] = 0. \quad (47)$$

The Arrow-Debreu claim payoffs are such that:

$$q_{s_t s_D}(Y) = \begin{cases} 1, & s_t = s_D, \quad Y \leq Y_{D,s_t} \\ 0, & s_t \neq s_D, \quad Y \leq Y_{D,s_t} \end{cases}, \quad s_t, s_D = \{L, H\} \quad (48)$$

Therefore, each state of the global economy is characterized by a specific default threshold. The Arrow-Debreu claims are derived in two distinct cases:  $Y_{D,H} < Y_{D,L}$  or  $Y_{D,H} > Y_{D,L}$ .

In the first case, when the default barriers are higher in recession and lower in expansion, that is  $Y_{D,H} < Y_{D,L}$ , then each of the four Arrow-Debreu claims is determined over three separate intervals:  $Y \geq Y_{D,L}$ ,  $Y_{D,L} \geq Y \geq Y_{D,H}$ , and  $Y \leq Y_{D,H}$ .

From the payoff equations, we can infer the values of the four Arrow-Debreu claims in the interval  $Y \leq Y_{D,H}$ . For the interval  $Y \geq Y_{D,L}$ , we are looking for a solution of the following general form:

$$q_{s_t s_D}(Y) = h_{s_t s_D} Y^k, \quad (49)$$

which implies that  $k$  must be a root of the quartic equation

$$\left[ \frac{1}{2} \sigma_L^2 k(k-1) + \mu_L k + (-\hat{\lambda}_L - r_L) \right] \left[ \frac{1}{2} \sigma_H^2 k(k-1) + \mu_H k + (-\hat{\lambda}_H - r_H) \right] - \hat{\lambda}_L \hat{\lambda}_H = 0. \quad (50)$$

The Arrow-debreu claims can be written as

$$q_{s_t s_D}(Y) = \sum_{m=1}^4 h_{s_t s_D m} Y^{k_m} \quad (51)$$

with  $k_1, k_2 < 0$  and  $k_3, k_4 > 0$ . However, when  $Y$  goes to infinity the Arrow-Debreu claims must be null, which indicates that we should have  $h_{s_t s_D,3} = h_{s_t s_D,4} = 0$ . We then obtain

$$q_{L s_D}(Y) = \sum_{m=1}^2 h_{L s_D, m} Y^{k_m} \quad (52)$$

$$q_{H s_D}(Y) = \sum_{m=1}^2 h_{H s_D, m} \varepsilon(k_m) Y^{k_m}, \quad (53)$$

where

$$\varepsilon(k_m) = -\frac{\hat{\lambda}_H}{\frac{1}{2} \sigma_H^2 k(k-1) + \mu_H k - (\hat{\lambda}_H + r_H)} = -\frac{\frac{1}{2} \sigma_L^2 k(k-1) + \mu_L k - (\hat{\lambda}_L + r_L)}{\hat{\lambda}_L}. \quad (54)$$

Finally, over the interval  $Y_{D,L} \geq Y \geq Y_{D,H}$ , both  $q_{D,LL}$  and  $q_{D,LH}(Y)$  are known from the payoffs equations and are respectively equal to 1 and 0. Then,

$$q_{HL}(Y) = \frac{\hat{\lambda}_H}{r_H + \hat{\lambda}_H} + \sum_{m=1}^2 s_{L,m} Y^{j_m} \quad (55)$$

$$q_{HH}(Y) = \sum_{m=1}^2 s_{H,m} Y^{j_m}, \quad (56)$$

where

$$\frac{1}{2}\sigma_H^2 j(j-1) + \mu_H j - (\hat{\lambda}_H + r_H) = 0 \quad (57)$$

with  $j_1 < j_2$ .

To summarize, the four Arrow-Debreu claims can be written as follows

$$q_{LL} = \begin{cases} \sum_{m=1}^2 h_{LL,m} Y^{k_m}, & Y \geq Y_{D,L} \\ 1, & Y_{D,L} \geq Y \geq Y_{D,H} \\ 1, & Y \leq Y_{D,H} \end{cases} \quad (58)$$

$$q_{LH} = \begin{cases} \sum_{m=1}^2 h_{LH,m} Y^{k_m}, & Y \geq Y_{D,L} \\ 0, & Y_{D,L} \geq Y \geq Y_{D,H} \\ 0, & Y \leq Y_{D,H} \end{cases} \quad (59)$$

$$q_{HL} = \begin{cases} \sum_{m=1}^2 h_{LL,m} \varepsilon(k_m) Y^{k_m}, & Y \geq Y_{D,L} \\ \frac{\hat{\lambda}_H}{r_H + \hat{\lambda}_H} + \sum_{m=1}^2 s_{L,m} Y^{j_m}, & Y_{D,L} \geq Y \geq Y_{D,H} \\ 0, & Y \leq Y_{D,H} \end{cases} \quad (60)$$

$$q_{HH} = \begin{cases} \sum_{m=1}^2 h_{LH,m} \varepsilon(k_m) Y^{k_m}, & Y \geq Y_{D,L} \\ \sum_{m=1}^2 s_{H,m} Y^{j_m}, & Y_{D,L} \geq Y \geq Y_{D,H} \\ 1, & Y \leq Y_{D,H}. \end{cases} \quad (61)$$

The eight constants are determined by eight threshold conditions, which are

$$\begin{aligned} \lim_{Y \rightarrow Y_{D,L}^-} q_{LL} &= 1, & \lim_{Y \rightarrow Y_{D,L}^+} q_{LH} &= 0 \\ \lim_{Y \rightarrow Y_{D,L}^+} q_{HL} &= \lim_{Y \rightarrow Y_{D,L}^-} q_{HL}, & \lim_{Y \rightarrow Y_{D,L}^+} q_{HH} &= \lim_{Y \rightarrow Y_{D,L}^-} q_{HH} \\ \lim_{Y \rightarrow Y_{D,L}^+} \dot{q}_{HL} &= \lim_{Y \rightarrow Y_{D,L}^-} \dot{q}_{HL}, & \lim_{Y \rightarrow Y_{D,L}^+} \dot{q}_{HH} &= \lim_{Y \rightarrow Y_{D,L}^-} \dot{q}_{HH} \\ \lim_{Y \rightarrow Y_{D,H}^-} q_{HL} &= 0, & \lim_{Y \rightarrow Y_{D,H}^-} q_{HH} &= 1. \end{aligned}$$

In the second case, when the default barriers are higher in expansion and lower in recession, that is  $Y_{D,H} > Y_{D,L}$ , then each of the four Arrow-Debreu claims is determined over three separate intervals:

$Y \geq Y_{D,H}$ ,  $Y_{D,H} \geq Y \geq Y_{D,L}$ , and  $Y \leq Y_{D,L}$ . We then obtain

$$q_{LL} = \begin{cases} \sum_{m=1}^2 h_{LL,m} Y^{k_m}, & Y \geq Y_{D,H} \\ \sum_{m=1}^2 s_{L,m} Y^{j_m}, & Y_{D,H} \geq Y \geq Y_{D,L} \\ 1, & Y \leq Y_{D,L} \end{cases} \quad (62)$$

$$q_{LH} = \begin{cases} \sum_{m=1}^2 h_{LH,m} Y^{k_m}, & Y \geq Y_{D,H} \\ \frac{\hat{\lambda}_L}{r_L + \hat{\lambda}_L} + \sum_{m=1}^2 s_{H,m} Y^{j_m}, & Y_{D,H} \geq Y \geq Y_{D,L} \\ 0, & Y \leq Y_{D,L} \end{cases} \quad (63)$$

$$q_{HL} = \begin{cases} \sum_{m=1}^2 h_{LL,m} \varepsilon(k_m) Y^{k_m}, & Y \geq Y_{D,H} \\ 0, & Y_{D,H} \geq Y \geq Y_{D,L} \\ 0, & Y \leq Y_{D,L} \end{cases} \quad (64)$$

$$q_{HH} = \begin{cases} \sum_{m=1}^2 h_{LH,m} \varepsilon(k_m) Y^{k_m}, & Y \geq Y_{D,H} \\ 1, & Y_{D,H} \geq Y \geq Y_{D,L} \\ 1, & Y \leq Y_{D,L}. \end{cases} \quad (65)$$

The eight constants are determined by eight threshold conditions, which are

$$\begin{aligned} \lim_{Y \rightarrow Y_{D,L}} q_{LL} &= 1, & \lim_{Y \rightarrow Y_{D,L}} q_{LH} &= 0 \\ \lim_{Y \rightarrow Y_{D,H}^+} q_{LL} &= \lim_{Y \rightarrow Y_{D,H}^-} q_{LL}, & \lim_{Y \rightarrow Y_{D,H}^+} q_{LH} &= \lim_{Y \rightarrow Y_{D,H}^-} q_{LH} \\ \lim_{Y \rightarrow Y_{D,H}^+} \dot{q}_{LL} &= \lim_{Y \rightarrow Y_{D,H}^-} \dot{q}_{LL}, & \lim_{Y \rightarrow Y_{D,H}^+} \dot{q}_{LH} &= \lim_{Y \rightarrow Y_{D,H}^-} \dot{q}_{LH} \\ \lim_{Y \rightarrow Y_{D,H}} q_{HL} &= 0, & \lim_{Y \rightarrow Y_{D,H}} q_{HH} &= 1. \end{aligned}$$

## C.2 Second kind

We use the same approach to derive the second kind of Arrow-Debreu default claims, which account for the possibility that a default can happen when the state of the global economy changes. In the case when  $Y_{D,H} < Y_{D,L}$ , the only claim that is different from that of the first kind is  $q_{HL}$ , and its expression

becomes

$$q'_{HL} = \begin{cases} \sum_{m=1}^2 h_{LL,m} \varepsilon(k_m) Y^{k_m}, & Y \geq Y_{D,L} \\ \frac{\hat{\lambda}_H}{r_H + \hat{\lambda}_H - \mu_H} \frac{Y}{Y_{D,L}} + \sum_{m=1}^2 s_{L,m} Y^{j_m}, & Y_{D,L} \geq Y \geq Y_{D,H} \\ 0, & Y \leq Y_{D,H}. \end{cases} \quad (66)$$

Now, in the case when  $Y_{D,H} > Y_{D,L}$ , the only claim that is different from that of the first kind is  $q_{LH}$ , and its expression becomes

$$q'_{LH} = \begin{cases} \sum_{m=1}^2 h_{LH,m} Y^{k_m}, & Y \geq Y_{D,H} \\ \frac{\hat{\lambda}_L}{r_L + \hat{\lambda}_L - \mu_L} \frac{Y}{Y_{D,H}} + \sum_{m=1}^2 s_{H,m} Y^{j_m}, & Y_{D,H} \geq Y \geq Y_{D,L} \\ 0, & Y \leq Y_{D,L}. \end{cases} \quad (67)$$

## D Government

This section derives the debt issuance benefits, the present value of the country's government revenue, and the country's sovereign wealth.

### D.1 Debt issuance benefits

The government's motivation for issuing debt is to invest in the country the amount of capital raised at time of debt issuance ( $t = 0$ ). Financing public investments yields a return  $r_g$ . The government's incentives for issuing debt, denoted by  $I_{s_t}(Y_t)$  when the state is  $s_t$  at time  $t$ , equal

$$I_{s_t}(Y_t) = E_t \left[ \int_t^\infty r_g \frac{\pi_u}{\pi_t} du \middle| s_t \right] D_{s_0}(Y_0) \quad (68)$$

$$= r_g E_t \left[ \int_t^\infty \frac{\pi_u}{\pi_t} du \middle| s_t \right] D_{s_0}(Y_0) \quad (69)$$

$$= \frac{r_g}{r_{B,s_t}} D_{s_0}(Y_0) \quad (70)$$

## D.2 Discounted government revenue

The present value of the country's government revenue, denoted by  $G_{s_t}(Y_t)$  when the current state is  $s_t$ , can be written as

$$G_{s_t}(Y_t) = E_t \left[ \int_t^{t_D} Y_u \frac{\pi_u}{\pi_t} du \middle| s_t \right] + E_t \left[ \int_{t_D}^{\infty} (1 - \alpha) Y_u \frac{\pi_u}{\pi_t} du \middle| s_t \right] \quad (71)$$

$$= E_t \left[ \int_t^{\infty} Y_u \frac{\pi_u}{\pi_t} du \middle| s_t \right] - \alpha E_t \left[ \int_{t_D}^{\infty} Y_u \frac{\pi_u}{\pi_t} du \middle| s_t \right]. \quad (72)$$

The first term of Equation 72 is determined by

$$E_t \left[ \int_t^{\infty} Y_u \frac{\pi_u}{\pi_t} du \middle| s_t \right] = Y_t E_t \left[ \int_t^{\infty} \frac{\pi_u}{\pi_t} \frac{Y_u}{Y_t} du \middle| s_t \right] \quad (73)$$

$$= Y_t \frac{1}{r_{Y,s_t}}, \quad (74)$$

where  $r_{Y,s_t}$  is the discount rate related to risky government revenue, which is determined by

$$r_{Y,s_t} = r_{s_t} - \mu_{s_t} + \frac{(r_j - \mu_j) - (r_{s_t} - \mu_{s_t})}{\hat{p} + r_j - \mu_j} \hat{p} \hat{f}_j, \quad j \neq s_t; \quad j, s_t = \{L, H\}. \quad (75)$$

From the strong Markov property, we can solve for the second part of Equation 72, which yields

$$E_t \left[ \int_{t_D}^{\infty} Y_u \frac{\pi_u}{\pi_t} du \middle| s_t \right] = \sum_{s_D} q'_{s_t s_D} (Y_t) \frac{Y_{D,s_D}}{r_{Y,s_D}}. \quad (76)$$

Eventually, the present value of the country's government revenue is given by

$$G_{s_t}(Y_t) = \frac{Y_t}{r_{Y,s_t}} - \alpha \sum_{s_D} \frac{Y_{D,s_D}}{r_{Y,s_D}} q'_{s_t s_D} (Y_t). \quad (77)$$

## D.3 Sovereign wealth and smooth pasting conditions

Sovereign wealth is defined as the present value of government revenue,  $G_{s_t}(Y_t)$ , plus the benefits of issuing debt,  $I_{s_t}(Y_t)$ . From the derivation above, sovereign wealth  $W_{s_t}(Y_t)$ , at time  $t$  and for current

state  $s_t$ , is given by

$$W_{s_t}(Y_t) = G_{s_t}(Y_t) + I_{s_t}(Y_t) \quad (78)$$

$$= \frac{Y_t}{r_{Y,s_t}} - \alpha \sum_{s_D} \frac{Y_{D,s_D}}{r_{Y,s_D}} q'_{s_t s_D}(Y_t) + \frac{r_g}{r_{B,s_t}} D_{s_0}(Y_0) \quad (79)$$

We now derive the smooth-pasting conditions that ensure continuity in the objective function at the time of default (see Merton, 1973; Dumas, 1991). For convenience, let us denote the value of sovereign wealth after debt payments have been made by  $\bar{W}_{s_t}(Y_t) \equiv W_{s_t}(Y_t) - D_{s_t}(Y_t)$ . Combining Equations (38) and (79),  $\bar{W}_{s_t}(Y_t)$  is given by

$$\begin{aligned} \bar{W}_{s_t}(Y_t) &= \frac{Y_t}{r_{Y,s_t}} - \alpha \sum_{s_D} \frac{Y_{D,s_D}}{r_{Y,s_D}} q'_{s_t s_D}(Y_t) + \frac{r_g}{r_{B,s_t}} D_{s_0}(Y_0) \\ &\quad - \left[ \frac{c}{r_{B,s_t}} - \sum_{s_D} \frac{c\kappa}{r_{B,s_D}} q_{s_t s_D}(Y_t) \right]. \end{aligned} \quad (80)$$

The smooth-pasting conditions must satisfy the following equations:

$$\left. \frac{\partial \bar{W}_{s_t}(Y_t)}{\partial Y_t} \right|_{Y_t=Y_{D,s_t}} = \frac{\partial}{\partial Y_{D,s_t}} \left( \bar{W}_{s_t}(Y_t) \Big|_{Y_t=Y_{D,s_t}} \right), \quad s_t = \{L, H\}. \quad (81)$$

From the definition of the Arrow-Debreu claims (48),  $\bar{W}_{s_t}(Y_t)$  at default time is given by

$$\bar{W}_{s_t}(Y_t) \Big|_{Y_t=Y_{D,s_t}} = Y_{D,s_t} \frac{1-\alpha}{r_{Y,s_t}} + \frac{r_g}{r_{B,s_t}} D_{s_0}(Y_0) - \frac{(1-\kappa)c}{r_{B,s_t}} \quad (82)$$

and the right-hand side of Equation 81 is thus determined by

$$\frac{\partial}{\partial Y_{D,s_t}} \left( \bar{W}_{s_t}(Y_t) \Big|_{Y_t=Y_{D,s_t}} \right) = \frac{1-\alpha}{r_{Y,s_t}}. \quad (83)$$

Hence, the smooth-pasting conditions satisfy the pair of equations given by

$$\left. \frac{\partial \bar{W}_{s_t}(Y_t)}{\partial Y_t} \right|_{Y_t=Y_{D,s_t}} = \frac{(1-\alpha)}{r_{Y,s_t}}, \quad s_t = \{L, H\}. \quad (84)$$

## E Estimation of the transition probabilities

This section describes the estimation of the transition probabilities considered in the calibration. We estimate a Markov regime-switching model with two regimes on U.S. consumption growth over the period 1994Q1-2018Q2. The transition probability matrix, which is obtained by maximum likelihood using the Hamilton (1989)'s approach, is given by

$$T = \begin{bmatrix} T_{HH} & T_{HL} \\ T_{LH} & T_{LL} \end{bmatrix} = T = \begin{bmatrix} 0.9655 & 0.0345 \\ 0.0687 & 0.9313 \end{bmatrix} \quad (85)$$

where  $T_{ij}$  denotes the probability of a switch from state  $i$  to state  $j$ .

Following Bhamra, Kuehn, and Strebulaev (2010a,b), the actual long-run probability  $f_{s_t}$  to be in the state  $s_t \in \{L, H\}$  is determined by  $f_H = \left(1 + \frac{T_{HL}}{T_{LH}}\right)^{-1}$  and  $f_L = 1 - f_H$ . The probability  $\lambda_{s_t}$  that the global economy leaves the state  $s_t \in \{L, H\}$  is then given by  $\lambda_L = p f_H$  and  $\lambda_H = p f_L$ , with  $p = -4 \ln \left(1 - \frac{T_{LH}}{1 - f_L}\right)$ .