# Announcement-Specific Decompositions of Unconventional Monetary Policy Shocks & Their Macroeconomic Effects

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October 10, 2019

#### Abstract

I propose to identify announcement-specific decompositions of asset price changes into monetary policy shocks using intraday time-varying volatility. This approach is the first to accommodate both changes in the nature of shocks and the state of the economy across announcements, allowing me to explicitly compare shocks across announcements. I compute decompositions with respect to Fed Funds, forward guidance, and asset purchase shocks for 2007-2018. Only a handful of announcements spark significant shocks. Asset purchase shocks lower corporate borrowing costs; both asset purchases and forward guidance increase spreads. Asset purchase shocks have significant expansionary effects on inflation and GDP growth.

*Keywords:* high-frequency identification, time-varying volatility, monetary policy shocks, forward guidance, quantitative easing *JEL codes:* E44, E52, E58, C32, C58

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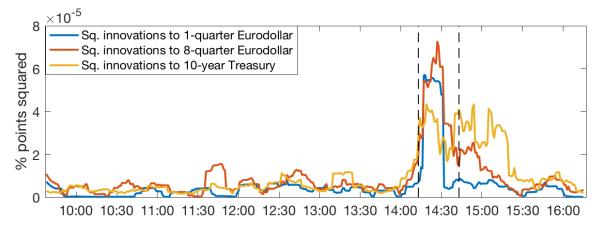
# 1 Introduction

Since the work of Kuttner (2001), high-frequency movements in asset prices have been used extensively to identify monetary policy shocks. However, with the shift towards more open communication starting in the 1990s across many central banks, the possibility of multiple dimensions of policy has complicated the task of identifying such shocks. Existing approaches either assume that each asset price movement considered responds only to a single shock over a certain window (e.g., Krishnamurthy & Vissing-Jorgensen (2011), Gertler & Karadi (2015)), or compute decompositions estimated across announcement dates (e.g., Gurkaynak, Sack & Swanson (2005) (hereafter GSS), Swanson (2017), Nakamura & Steinsson (2018), Inoue & Rossi (2018)). Neither approach is suited to the zero-lower-bound (ZLB) period and unconventional monetary policy.

The former approach either assumes the presence of a single shock or imposes exclusion restrictions across assets or factors (e.g., one price responds to target rate shocks, another to news shocks). The latter approach, computing time-invariant decompositions, follows the highly influential work of Nelson & Siegel (1987) and GSS, extracting factors from a set of asset price movements. However, the loadings on the factors are time-invariant, which means that recovered shocks can differ across announcements only in scale, not in their relative impact across asset prices. For example, this means that the asset purchase shock prompted by the announcement of the first round of quantitative easing, QE1, would be restricted to impact a range of prices in exactly the same manner as that of QE2, despite the fact that the announced measures targeted different assets and occurred at times when the economy, and thus important elasticities, may have been in very different states. The use of factors presents the further challenge of interpretability; factors are identified only up to orthogonal rotations, meaning no "Fed Funds", "forward guidance", or "asset purchase" shocks can be identified without further structural assumptions. Swanson (2017) makes important progress on this last point by using judicious exclusion and event constraints.

I propose to identify announcement-specific decompositions of asset price movements to identify monetary policy shocks without assuming time-invariance across announcements. To do so, I treat all asset price movements over the course of an announcement day as responses to a series of news shocks. In the period following a monetary policy announcement, these news shocks can be interpreted as monetary policy shocks. This means that a full day of intraday data can be used to identify an announcement-specific decomposition of asset price movements into news shocks, and thus monetary policy shocks. Figure 1 plots 15minute moving-averages of squared innovations to three interest rates (based on the model in Section 3) for September 21, 2011, the day of the Federal Open Market Committee (FOMC)





15-minute moving average of squared innovations from my baseline model for September 21, 2011, a VAR(7) with 1-quarter Eurodollar rates, 8-quarter Eurodollar rates, and 10-year Treasury yields. Reference lines indicate the conventional 30-minute event window.

announcement that launched Operation Twist. There are clearly asset price movements besides the change across the usual 30-minute event window (14:13 to 14:43) which may offer previously unexploited identifying variation. To identify the decomposition, I apply results from Lewis (2019a), based on the simple assumption that the shock volatility varies, with some persistence, over the course of the day. The stark intraday volatility patterns evident in Figure 1 motivate this identification approach. The shocks are identified up to labeling, which follows naturally in most cases.

I apply this approach to each scheduled FOMC announcement from 2007-2018. I estimate a triplet of shocks on each day: "Fed Funds", "forward guidance", and "asset purchase". To assess which announcements led to significant monetary policy shocks in each dimension, I compute historical decompositions of asset prices to the end of the day. These decompositions allow comparison of one announcement to the next for the first time. Since historical decompositions are better viewed as random variables than parameters, making inference difficult, I assess the economic significance of shocks by comparing the decompositions to the average daily standard deviation of interest rates on monetary policy announcement days. I find that several shocks that appear significant based on 30-minute windows have no discernible effect by the day's end, possibly distorting the results of studies that have employed such windows. I instead focus on the end-of-day decompositions. Even among the most notable unconventional policy announcements, few spark significant shocks; those that do are generally the launch of policies or their extension, when markets widely believed them to be coming to an end; more subtle revisions appear less important.

For each day, I compute the high-frequency responses of corporate debt and equities to

the shocks, finding some evidence that asset purchases in particular bring down corporate borrowing costs; however, these effects rarely persist to the day's end. I form a time-series of my shock measures, and use these to conduct daily regressions using corporate debt measures, as in Swanson (2017); the findings are qualitatively similar.

While several papers have considered such financial market responses, little work has assessed the lower-frequency response of macroeconomic aggregates, the ultimate variables of interest for central banks, to interpretable unconventional policy shocks. I use the announcement-frequency shocks to compute dynamic responses of both realized inflation and output up to 12 months, and find that asset purchases significantly raise both inflation and output growth. On the other hand, Fed Funds and forward guidance shocks have no significant effects.

Many previous studies have assessed the response of financial variables to unconventional U.S. monetary policy shocks. Among them, the only paper, to my knowledge, to separately identify forward guidance and asset purchase shocks is Swanson (2017). His shock measures and announcement-frequency results for financial variables are broadly comparable with my estimates. For asset purchases shocks, my findings also align with those of Krishnamurthy & Vissing-Jorgensen's event studies (2011, 2013). Other notable studies include Swanson (2011), Campbell, Evans, Fisher, & Justiniano (2012), and Coenen et al (2017).

While relatively little work has estimated the impact of unconventional policy on macroeconomic variables, Baumeister & Bennati (2013), Gambacorta, Hofmann, & Peersman (2014), Lloyd (2018), and Inoue & Rossi (2018) are exceptions. However, none of these papers has separated and simultaneously identified forward guidance and asset purchase shocks, making the present paper, to my knowledge, the first to offer a comparative analysis of the two. The first three papers identify a range of different shocks ("spread compression"; "balance sheet"; "signaling" and "portfolio balance", respectively) in VARs using sign and exclusion restrictions. Inoue & Rossi (2018) estimate local projections for two policy dimensions corresponding to the slope and curvature factors from a Nelson & Siegel (1987) decomposition; they do not have a way to separately identify forward guidance and asset purchase shocks. Baumeister & Bennati (2013) is the only paper to allow for a time-varying nature of shocks (in a parametric sense), using a time-varying parameters model. Gambacorta, Hofmann, & Peersman's (2014) findings for their balance sheet shock align well with the significant effects I find for my asset purchase shock.

The remainder of the paper is organized as follows. Section 2 discusses the identification problem and previous methodologies in more detail before outlining my approach and data. Section 3 presents the results across announcement days, describes the findings for notable FOMC announcements in detail, and characterizes the properties of the time-series of the implied shocks. Section 4 analyzes the high-frequency and daily responses of financial variables to the shocks. Section 5 computes the responses of macroeconomic aggregates to the measures. Section 6 concludes.

# 2 Intraday identification of monetary policy shocks

In this section, I first motivate the use of announcement-specific decompositions and argue that they can, in principle, be identified using intraday data. I then discuss how timevarying volatility can be used to do so. Finally, I briefly sketch my implementation of the identification scheme.

#### 2.1 The case for intraday identification

High-frequency identification of monetary policy shocks draws on the event-study methodology of empirical finance, as described by Campbell, Lo, and MacKinlay (1997). Those authors write abnormal one-period returns,  $\eta_{it}$  for security *i* at time *t*, as

$$\eta_{it} = R_{it} - E\left[R_{it} \mid \mathscr{I}_{t-1}\right],\tag{1}$$

where  $R_t$  is a raw return,  $\mathscr{I}_{t-1}$  is the information set available at t-1 and  $E[R_{it} | \mathscr{I}_{t-1}]$ is based on some model. In typical studies of monetary policy shocks, it is assumed that  $E[R_{it} | \mathscr{I}_{t-1}] = 0$ , treating asset prices as a random walk. This means  $\eta_{it} = R_{it} = P_{it} - P_{it-1}$ . Monetary policy shocks can thus be measured as the change in an interest rate future, Treasury yield, or some basket of such asset prices around an announcement. Much recent work computes the price change from 10 minutes prior to an announcement to 20 minutes following; this measure can be either used directly, originating with Kuttner (2001), or as an instrument for some latent monetary policy shock (e.g., Gertler & Karadi (2015)).

There is much evidence, following GSS, that there is no single monetary policy shock; this dimensionality became more explicit during the Great Recession. Thus, without exclusion restrictions that  $\eta_{it}$  responds only to the shock of interest, a more sophisticated approach is needed. Thus,  $\eta_{it}$  must be explicitly modeled as a combination of different news shocks,  $\epsilon_t$ , and decomposed accordingly. For a vector of n abnormal returns,  $\eta_t$ ,

$$\eta_t = H\epsilon_t, t = 1, \dots, T,\tag{2}$$

where  $\epsilon_t$  is an  $n \times 1$  vector of orthogonal news shocks and H is a constant  $n \times n$  invertible matrix. However, since  $\epsilon_t$  is mean-zero, up to second moments, (2) provides only  $(n^2 + n)/2$ 

identifying equations in  $n^2$  parameters, so additional identifying information is needed to compute the decomposition. This is the SVAR identification problem, which I return to in the next section.

Assuming an identification scheme exists to recover H from (2), doing so has typically required a sample of many monetary policy announcement days, with a single change in asset prices collected for each announcement and pooled to identify H. Thus,  $\eta_t$  is replaced by a series  $\eta_d$ , with a single observed asset price change for each announcement date d (and similarly for  $\epsilon_d$ ) so

$$\eta_d = H\epsilon_d, d = 1, \dots, D. \tag{3}$$

H must be constant for the entirety of the sample for any identification approach based on (3) to be valid. However, this is implausible for the Great Recession. A constant H– the instantaneous effects of shocks — only makes sense if the nature of shocks is the same from one announcement to the next, but during this period, the nature of shocks varied dramatically. For the first time, forward guidance changed from vague to explicitly calendar-based, and again to conditional. The composition of securities purchased through QE changed between MBS and Treasuries, with different maturities targeted. Moreover, even if the nature of the shocks was fixed, the elasticities of financial markets and the economy changed rapidly, likely altering the transmission mechanisms embodied in H. The assumption of a constant H necessarily prevents the comparison of the effects of shocks from one announcement to the next.

For these reasons, I take a novel approach. Rather than viewing each day's monetary policy shock as being reflected in a single change in asset prices across some window, I view intraday asset prices as responses to a continuous stream of news shocks. Monetary policy shocks are a subset of the day's news shocks – those that hit markets as a result of a monetary policy announcement. Under rational expectations and efficient markets, all asset price movements must represent some form of news, and, on days dominated by an FOMC announcement, this news is generally related to the same dimensions of monetary policy as the announcement itself.

This re-framing of the problem as one of identifying high-frequency news shocks, which are present throughout the day, has significant implications. Since a single trading day generally contains many changes in given asset prices, reflecting many news shocks, a dayspecific decomposition,  $H_d$ , can be identified – using only that day's fluctuations in asset prices and an appropriate identification scheme. In particular, I model

$$\eta_{d_t} = H_d \epsilon_{d_t}, t = 1, \dots, T, d = 1, \dots D,$$

$$\tag{4}$$

where t indexes time within a given announcement date, d. Thanks to the infill-asymptotic argument (e.g., Cressie (1993)) common in analysis of intraday financial data, identifying moments can be consistently estimated over the fixed time period of a trading day. This means that, given a valid identification scheme,  $H_d$  can be consistently estimated without assuming that  $H_d \equiv H$  (constant across days), instead assuming that  $H_d$  is constant throughout day d. In the case of the Great Recession and the constantly-changing nature of unconventional monetary policy, this provides the flexibility needed to characterize the potentially time-varying effects of monetary policy via  $H_d$ .

The  $H_d$  identified from (4) is closely related to the conventional event-study object. In particular, define  $H_d^{inf}$  as the infeasible estimator obtained from hypothetical *repeated* samples of

$$\eta_d = H_d^{inf} \epsilon_d$$

for a single day, using some valid identification scheme, where  $\eta_d$  is the *n*-dimensional change in prices from t - k to t, and  $\epsilon_d$  are *n*-dimensional orthogonal shocks over that window. Proposition 1 relates  $H_d$  to  $H_d^{inf}$ :

**Proposition 1.** If the asset prices underlying  $\eta_d$  follow a random walk and  $H_d^{inf}$  is uniquely determined, then  $H_d = H_d^{inf}$ .

This result shows that, under a random walk assumption,  $H_d$  is equivalent to the ideal, but infeasible, event study estimator for a given day. However,  $H_d$  can be consistently estimated. In principle, given the low degree of autocorrelation in financial data, the deviation of  $H_d$ from  $H_d^{inf}$  need not be large.

This approach also does not require the researcher to specify a fixed window over which to compute shocks. The appropriate length of such a window has been a topic of much debate. A full path of intraday shocks can be recovered, and then arbitrary subsets and cumulations of those shocks can be studied, making clear the implications of focusing on a particular window.

On the other hand, this exercise is complicated by the presence of noise and other features of intraday data, which may play less of a role when simple 30-minute windows are used. However, without exploiting intraday time series, it would never be possible to consistently estimate the effects of infrequent events (e.g., the effect of conditional guidance as opposed to calendar-based). Propositions 3 and 4 below address the role of noise and such concerns motivate a number of robustness checks in my eventual analysis.

#### 2.2 Identification via time-varying volatility

I have argued that  $H_d$  can in principle be identified from intraday data, but it remains to propose a suitable identification scheme to do so. It is unappealing to impose assumptions on  $H_d$  (exclusion or sign restrictions) in general as  $H_d$  is the object of interest and in this case in particular because it is hard to argue that some asset prices systematically respond more slowly to forward guidance or asset purchase shocks, for example. Swanson's (2017) clever approach to distinguish forward guidance and asset purchase shocks, based on the absence of asset purchase shocks prior to 2009, is not applicable given all shocks come from a single announcement day, mostly post-2009.

These factors lead me to consider statistical identification, in particular identification based on time-varying volatility. Figure 1 demonstrates strong volatility patterns for a representative announcement date. Identification via heteroskedasticity has proven popular for identifying asset price responses to news and policy shocks, as proposed by Rigobon (2003) and Rigobon & Sack (2003, 2004). However, traditional identification via heteroskedasticity requires the specification of variance regimes. The timing of intraday periods of high volatility varies with the timing of announcements, press conferences, and other events during the day. While Rigobon (2003) contends that misspecification of regimes does not hinder consistent estimation, Lewis (2019b) argues that such misspecification may cause a weak identification problem, with multiple dimensions of monetary policy as a leading example. Lewis (2019a) argues that estimating such regimes may bias estimates.

I therefore identify (4) based on time-varying volatility (TVV-ID), following Lewis (2019a). This result generalizes the parametric arguments for identification based on heteroskedasticity of Rigobon (2003) and Sentana & Fiorentini (2001) to a completely non-parametric argument based on the autocovariance structure of the shock volatilities. Unlike those previous approaches, it does not require the researcher either to specify variance regimes (Rigobon) or recover the full path of volatilities (Sentana & Fiorentini) for identification.

More formally, Assumption 1 lays out assumptions for TVV-ID. I henceforth suppress d subscripts for compactness; all observations and parameters remain date-specific, unless otherwise noted.

Assumption 1. For every  $t = 1, 2, \ldots, T$ ,

- 1. H is fixed, full-rank, and has a unit diagonal,
- 2.  $\sigma_t$  is an  $n \times 1$  stationary stochastic process,
- 3.  $E(\varepsilon_t \mid \sigma_t, \mathcal{F}_{t-1}) = 0$  and  $\operatorname{Var}(\varepsilon_t \mid \sigma_t, \mathcal{F}_{t-1}) = \Sigma_t$ ,

- 4.  $\Sigma_t = diag(\sigma_t^2), \sigma_t^2 = \sigma_t \odot \sigma_t,$
- 5. Var  $(\sigma_t^2) < \infty$ ,

6. Var 
$$(\varepsilon_t \varepsilon'_t) < \infty$$

I assume stationarity of  $\sigma_t$  for clarity and coherence with empirical practice, unlike the more general development in Lewis (2019a). The first assumption is a standard requirement for identification of models of the form (2). The third requires that  $\epsilon_t$  is a martingale difference sequence with respect to the filtration  $\mathcal{F}_{t-1} = \{\varepsilon_1, \ldots, \varepsilon_{t-1}, \sigma_1, \ldots, \sigma_{t-1}\}$  and  $\sigma_t$ , a form of the standard assumption that  $\epsilon_t$  are not serially correlated. The fourth stipulates orthogonality of shocks, and the final two assumptions are regularity conditions.

Under these assumptions, the autocovariance of squared reduced-form innovations provides equations sufficient to identify H as coefficients on the volatility process of the structural shocks  $\epsilon_t$ . Define L and G to be elimination and selection matrices respectively.<sup>1</sup> Lewis (2019a) shows that for  $\zeta_t = vech (\eta_t \eta'_t)$ , Proposition 2 holds.

Proposition 2. Under Assumption 1,

$$Cov\left(\zeta_{t},\zeta_{t-p}\right) = L\left(H\otimes H\right)GM_{p}\left(H\otimes H\right)'L', \ p>0$$

$$(5)$$

where

$$M_{p} = E\left[\sigma_{t}^{2}vec\left(\varepsilon_{t-p}\varepsilon_{t-p}'\right)\right] - E\left[\sigma_{t}^{2}\right]E\left[\sigma_{t}^{2}\right]'G'$$

Define  $\tilde{M}_p = \begin{bmatrix} M_p & E[\sigma_t^2] \end{bmatrix}$ . Using the decomposition (5) of the autocovariance of  $\zeta_t$ , Theorem 1 (Theorem 2 of Lewis (2019a)) shows *H* can be identified from (5):

**Theorem 1.** Under Assumption 1, equation (5) holds. Then H and  $\tilde{M}_p$  are jointly uniquely determined from (5) and  $E(\zeta_t)$  (up to labeling of shocks) provided rank  $(\tilde{M}_p) \geq 2$  and  $\tilde{M}_p$  has no proportional rows.

Briefly, the rank condition will hold provided there is at least one dimension of timevarying volatility in the data.<sup>2</sup> The proportionality condition will hold provided that, for no two dimensions of  $\sigma_t^2$ , say  $\sigma_{it}^2, \sigma_{jt}^2$ , all respective autocovariances with  $vec(\varepsilon_{t-p}\varepsilon'_{t-p})$  are related by the ratio  $E[\sigma_{it}^2]/E[\sigma_{jt}^2]$ . Like other identification arguments based on heteroskedasticity, this implies that at least n-1 dimensions of  $\sigma_t^2$  must vary. H is identified from (5) up to a labeling of the shocks, which I discuss below. Unlike factor models (e.g., GSS, Swanson

<sup>&</sup>lt;sup>1</sup>This means vech(K) = Lvec(K) and  $vec(KDK') = (K \otimes K) Gd$  where d = diag(D).

<sup>&</sup>lt;sup>2</sup>Technically, this is true barring an extremely degenerate case, where all columns of  $M_p$  are proportional to  $E\left[\sigma_t^2\right]$ .

(2017), Inoue & Rossi (2018)), H represents a truly unique decomposition of  $\eta_t$  into orthogonal components  $\epsilon_t$ , and is not only unique up to orthogonal rotations. For additional details and discussion of these results, see Lewis (2019a). Statistical identification requires the recovered shocks to be labeled. I discuss my labeling scheme, specific to the interpretation of shocks to Fed Funds, forward guidance, and asset purchase dimensions of policy, in Section 3.

The presence of microstructure noise is often a concern when analysis depends on highfrequency movements in financial variables, as is the case here. Proposition 3 establishes conditions under which the identification of H in Theorem 1 is largely unaffected.

**Proposition 3.** Suppose  $\bar{\eta}_t = H\epsilon_t + \nu_t$ , where  $\nu_t$  is an  $n \times 1$  vector of noise uncorrelated with  $\epsilon_t$  at all horizons. If the volatility of  $\nu_t$  exhibits zero autocovariance, H is identified from  $Cov(\bar{\zeta}_t, \bar{\zeta}_{t-p})$ , where  $\bar{\zeta}_t = vech(\bar{\eta}_t\bar{\eta}'_t)$ , provided  $M_p$  has no proportional rows.

This result shows that microstructure noise is not an issue, provided its volatility is fixed (white noise) or exhibits heteroskedasticity with no persistence. Such noise contaminates the moments  $E\left[\bar{\zeta}_t\right]$ , but not the autocovariance, which may alone be sufficient for identification if all n shocks exhibit time-varying volatility (Theorem 1 of Lewis (2019a)).

In the current context, where all asset price movements are viewed as responses to news shocks, with those during certain parts of the day interpreted as monetary policy shocks, it may also be of concern that there is some substantial difference between news shocks that take the form of monetary policy shocks around announcements and ordinary news shocks. This might imply a different H for those shocks that are not true monetary policy shocks. These other news shocks will generally be of lower variance, consistent with a possible "noise" interpretation when no meaningful new information is reaching markets. Proposition 4 demonstrates that, provided these other news shocks have relatively lower variance than monetary policy shocks, identification of H is asymptotically unaffected.

**Proposition 4.** Suppose that for news shocks during non-announcement periods,  $\eta_t = H_N \epsilon_t$ , and for monetary policy shocks following monetary policy announcements,  $\eta_t = H_{MP}\epsilon_t$ , with  $H_N \neq H_{MP}$ ; assume that within each period, the  $\sigma_t^2$  process is stationary with respective means  $\sigma_N^2, \sigma_{MP}^2$ . Then if  $\frac{\sigma_{i,N}^2}{\min_j \sigma_{j,MP}^2} \rightarrow 0$ , for all i = 1, ..., n, the H identified by Theorem 1 from full-sample moments is  $H_{MP}$ , provided the monetary policy shocks are not a measure zero share of all shocks.

While only an asymptotic result, this proposition suggests the impact of non-monetary policy shocks on identification will not be fatal provided asset price movements during other periods of the day are relatively low-variance. While news shocks raise concerns about the invertibility of VAR residuals (e.g., Sims (2012), Plagborg-Møller (2018)), this is not a problem for the news shocks I describe above. Intuitively, invertibility fails when observable series do not fully capture the state variables of the economy, for example, unobservable news shocks. However, while the shocks described above include news shocks, if markets are efficient, these shocks are contemporaneously reflected in the observed asset price series, so they can be recovered from the reduced-form innovations.

#### 2.3 Implementation

TVV-ID permits a wide range of estimation approaches. Indeed, any estimator that either explicitly (e.g., GMM estimation of (5)) or implicitly (a quasi-maximum likelihood approach) fits an autocovariance of the volatility process to the  $\eta_t \eta'_t$  data can be used. Lewis (2019a) compares the performance of the entirely non-parametric GMM approach to a quasi-maximum likelihood (QML) approach based on an AR(1) SV model, along with many alternatives in a simulation study, concluding that the AR(1) SV model performs very well and is quite robust to misspecification. Given this finding, I estimate H throughout this paper using QML based on the AR(1) SV model. I adopt the EM algorithm developed in Lewis (2019a), which extends prior work by Chan & Grant (2016) and Bertsche & Braun (2018).

### 3 The intraday shocks

To obtain a vector  $\eta_t$  of unpredictable asset price innovations, I begin with a vector of observed asset prices,

$$y_t = \begin{pmatrix} ED1_t \\ ED8_t \\ T10_t \end{pmatrix},$$

the front-quarter Eurodollar rate, 8-quarter Eurodollar rate, and 10-year Treasury yield. I consider each scheduled FOMC announcement date from January 2007 to December 2018. For each day, the sample spans 9:30am to 4:15pm. These three assets are chosen to plausibly match interest rates associated with the three monetary policy shocks I seek to identify: Fed Funds, forward guidance, and asset purchases, respectively. Additional justifications for this model (in particular as opposed to a factor model), as well as details of the data more generally, are discussed in Section 1 of the Supplement.

I first-difference the data for stationarity due to possible cointegration of rates across

the term structure, as noted by Campbell & Shiller (1987), for example. I then estimate a VAR(p) to obtain the unpredictable innovations to each series. p is chosen separately for each day using the Hannan & Quinn (1979) information criterion, which consistently estimates VAR order. The optimal p ranges from 1 to 15 with a median of 2. I thus obtain  $\eta_t$  as

$$\Delta y_{t} = A_{0} + \sum_{l=1}^{p} A_{l} \Delta y_{t-l} + \eta_{t}.$$
 (6)

Using the estimated residuals  $\hat{\eta}_t$ , I estimate (4), fitting an AR(1) SV process to the variance of  $\hat{\epsilon}_t$ , via QML.

Statistical identification requires the recovered shocks to be labeled *ex post*. The innovations  $\eta_t$  cover short-term and medium term expectations of the Fed Funds rate and a major liquid market impacted by asset purchases. My labeling procedure assumes that the three recovered shocks are the current Fed Funds rate shock, a forward guidance shock, and an asset purchase/QE shock. I label the shocks such that the matrix of  $R^2$  values from the regression of each innovation on each shock (with rows corresponding to  $y_t$  and columns (FF, FG, AP)) is as close as possible to the identity. This implements the assumption that innovations to the front future are best-explained by the Fed Funds shock, those to the 2-year future are best-explained by the forward guidance shock, and those to the 10-year Treasury are best-explained by the asset purchase shock. The first point is standard; the latter two are compatible with the horizon of forward guidance announcements (generally in the two year range), and the type and maturity of assets included in much of QE.<sup>3,4</sup>

#### 3.1 Results

I present three sets of results. First, as an overview, I document the distributions of the  $H_d$  estimates, which represent an instantaneous pass-through to rates. Second, my central results are based on historical decompositions computed daily. These characterize the cumulative causal effect of each type of shock on each interest rate for each day, at all horizons from 10 minutes prior to the FOMC announcements. I plot and discuss historical decompositions for twelve days with notable unconventional monetary policy announcements, along with summary statistics for the 96-day super-sample. Finally, I compute inter-announcement time-series of structural shocks and compare these to a timeline of key historical events.

<sup>&</sup>lt;sup>3</sup>For two announcements, I alter the labeling decision due to announcement-specific factors.

<sup>&</sup>lt;sup>4</sup>For observations prior to the first suggestions of large scale asset purchases in November 2008, the final shock may need to be interpreted slightly differently, as a second dimension of news orthogonal to that which drives medium-term expectations of short rates.

#### The distribution of H across announcements

I first conduct a test for identification for each announcement. I employ the test for the dimension of time-varying volatility proposed in Lewis (2019a). Figure 1 in the Supplement plots the results across announcement. For all announcements, I find evidence of at least n-1 dimensions of time-varying volatility. This finding supports the argument that the model is identified for each announcement date. While not strictly speaking a measure of the dimensionality of monetary policy (but rather the dimensionality of the volatility process underlying monetary policy shocks), the results do suggest lower dimensionality at the heart of the ZLB period, 2011-2014.

Comparing across announcement days shows substantial heterogeneity in the instantaneous response of asset prices to the three monetary policy shocks. Table 1 reports summary statistics for each free element of H, including the frequency with which one-sided tests reject zero. Figure 6 in the Supplement plots histograms of the estimates. Broadly speaking, the results accord with theory. A positive (contractionary) Fed Funds shock, for the most part, raises expectations of future short rates (8-quarter Eurodollar) and 10-year Treasury yields, often to a statistically significant extent; this implies sensible behaviour for expectations of future short rates and accords with any term structure model for the 10-year Treasury. The forward guidance shock on average has zero impact on front Eurodollar rates and a positive impact on 10-year Treasury yields. The presence of a small positive impact on 1-quarter Eurodollar rates for many days is consistent with the fact that for some announcement days, there is a further scheduled announcement before the contract expires, leaving scope for some forward guidance effect. The asset purchase shock has zero effect on average on 1-quarter Eurodollar rates, as expected. The sign and significance of the impact of the asset purchase shock on the 8-quarter Eurodollar rates is quite variable. This is consistent with the possibility that the character of the shocks varied over the course of the Great Recession, or that forward guidance and asset purchases may have at times been seen as complements and at others as substitutes, as discussed in more detail under the correlated shocks robustness check. It is important to remember that these elasticities are only contemporaneous at a high frequency, and, while indicative of short-run dynamics, may not give a complete picture of responses at either a 30-minute window, or by the end of the day, for example.

Figure 8 in the Supplement additionally reports (contemporaneous) variance decompositions for each interest rate with respect to each of the shocks for the 12 key announcement dates listed in Table 3. These further illustrate the nature of the decompositions. First, they show that the majority of fluctuations in short rates, however small, are driven by a factor largely orthogonal to movements in other interest rates during the ZLB period. As predicted by any term-structure model, the forward guidance shock generally has substantial explana-

|              | mean | $q_{10}$ | median | $q_{90}$ | positive | positive<br>sig. 5% | negative | negative<br>sig. 5% |
|--------------|------|----------|--------|----------|----------|---------------------|----------|---------------------|
| $H_{ED8,FF}$ | 0.58 | -0.02    | 0.31   | 1.23     | 78       | 44                  | 15       | 2                   |
| $H_{T10,FF}$ | 0.18 | -0.06    | 0.09   | 0.57     | 70       | 33                  | 23       | 1                   |
| $H_{ED1,FG}$ | 0.00 | -0.02    | 0.00   | 0.05     | 65       | 21                  | 28       | 9                   |
| $H_{T10,FG}$ | 0.32 | -0.00    | 0.24   | 0.35     | 86       | 82                  | 10       | 6                   |
| $H_{ED1,AP}$ | 0.01 | -0.01    | 0.00   | 0.04     | 58       | 10                  | 35       | 4                   |
| $H_{ED,AP}$  | 0.15 | -0.37    | 0.02   | 0.82     | 58       | 20                  | 38       | 11                  |

Table 1: Summary statistics for  $\hat{H}$ 

Estimates of  $\hat{H}$  from AR(1) SV model. Shocks labeled so that the Fed Funds shock best predicts the 1quarter Eurodollar rate, the forward guidance shock the 8-quarter Eurodollar rate, and the asset purchase shock the 10-year Treasury. H is unit-diagonal normalized based on this labeling. Estimates reflect the percentage point response to a shock that increases the reference rate by 1%. Responses of the 10-year Treasury are scaled to the day's constant-maturity Treasury yield. The right panel tabulates the signs and reports one-sided tests. For three dates, the front Eurodollar and Fed Funds shock are dropped due to zero intraday movement in the front contract.

tory power for the 10-year Treasury, while the converse is often not true (asset purchase shocks do not drive 8-quarter Eurodollars, expectations of future short rates). This further supports the validity of the decomposition of the two shocks. Additionally, even though on many dates only one or fewer shocks has a lasting effect in the historical decompositions discussed below, at high frequency all three identified shock series tend to have substantial explanatory power for at least one interest rate series. Table 10 in the Supplement reports the average values for these decompositions, demonstrating that these facts broadly hold across the full 96 announcements. These findings further justify the three-dimensional model I adopt and the dimensionality test results I report above.

#### Historical decompositions

The object of policy interest is not the instantaneous response of interest rates to a single minute's realization of a monetary policy shock, since that is in itself unlikely to play a role in shifting macroeconomic aggregates. To have any meaningful effect on financial conditions or slower-moving macro variables like inflation or unemployment, a response must be persistent and account for the true size of the surprise registered by markets, a process which may take time. In the present intraday setting, with a VAR model for asset prices, this question can only be addressed using historical decompositions. Furthermore, previous work has often used a 30-minute window in the event study framework, but acknowledged that full effects likely take longer. Since I am able to simultaneously estimate  $\epsilon_t$  throughout each day, an additional benefit of computing historical decompositions is that I can assess the extent to

which focusing on a 30-minute window may be misleading, relative to considering end-of-day measures, using a single estimated model.

Historical decompositions are computed based on impulse response functions. The structural impulse response function at horizon h can be computed as

$$\Phi^{h} = H, h = 0,$$
  
$$\Phi^{h} = \sum_{l=1}^{\min(h,p)} A_{l} \Phi^{h-l}, h = 1, 2, \dots.$$

Here, since the data are first-differenced prior to estimating the VAR, the object of interest is the cumulative impulse response function (IRF),  $\tilde{\Phi}^h$ , which is

$$\tilde{\Phi}^h = \sum_{i=0}^h \Phi^i.$$

The responses at time t to a shock realized at t - s can thus be computed as  $\tilde{\Phi}^s \epsilon_{t-s}$ . The historical decomposition,  $\Psi_t$ , takes into account all shocks realized since some start date,  $\tau$ , so

$$\Psi_t = \sum_{s=0}^{t-\tau} \tilde{B}^s H \epsilon_{t-s}$$

These objects are simple to compute once the IRF has been obtained. In the frequentist framework, inference results are not available for historical decompositions, since they are not a parameters in conventional sense, but rather random variables that depends on a sequence of  $\epsilon_t$ . For this reason, instead of using an asymptotically valid test of statistical significance to assess the impact of monetary policy shocks on particular days, I use a measure of *economic* significance. In particular, I compare the decompositions to the average daily standard deviation in the relevant interest rate across my sample of monetary policy announcement days. If a historical decomposition with respect to a given shock exceeds such a measure, it indicates that on that day, interest rates moved by an abnormal amount as a result of that shock. In the tables below, I indicate significance relative to multiples of these standard deviations corresponding to conventional significance levels (1.64, 1.96, 2.58).

Table 2 reports summary statistics for the absolute value of historical decompositions at two different horizons. The first panel reports decompositions at 20 minutes following the announcements due to shocks starting from 10 minutes prior to the announcements (the usual 30-minute event-study window). The second panel reports decompositions at 4:15pm due to shocks starting from 10 minutes prior to the announcements. It documents several facts. First, at both horizons, taking simple changes in asset prices (a univariate event-study

| 30-minute window                                    |                                 |                                |   |   |                                      |                                      |                                     |  |  |
|---|---------------------------------|--------------------------------|---|---|--------------------------------------|--------------------------------------|-------------------------------------|--|--|
|   | mean                            | median                         | mean  | median  | 1.64                                 | 1.96                                 | 2.58                                |  |  |
|   | $ \Delta P_{ref} $              | $ \Delta P_{ref} $             | decomp.   | decomp.   | s.d.                                 | s.d.                                 | s.d.                                |  |  |
| ED1, FF   |                                 |                                | 0.02  | 0.01  | 17                                   | 15                                   | 11                                  |  |  |
| ED1, FG   | 0.02                            | 0.01                           | 0.00  | 0.00  | 0                                    | 0                                    | 0                                   |  |  |
| ED1, AP   |                                 |                                | 0.00  | 0.00  | 0                                    | 0                                    | 0                                   |  |  |
| ED8, FF   |                                 |                                | 0.01  | 0.00  | 3                                    | 2                                    | 1                                   |  |  |
| ED8, FG   | 0.05                            | 0.03                           | 0.03  | 0.02  | 20                                   | 11                                   | 6                                   |  |  |
| ED8, AP   |                                 |                                | 0.01  | 0.00  | 3                                    | 2                                    | 2                                   |  |  |
| T10, FF   |                                 |                                | 0.00  | 0.00  | 1                                    | 1                                    | 1                                   |  |  |
| T10, FG   | 0.02                            | 0.01                           | 0.01  | 0.01  | 5                                    | 3                                    | 0                                   |  |  |
| T10, AP   |                                 |                                | 0.01  | 0.01  | 7                                    | 6                                    | 4                                   |  |  |
|   | End-of-day window               |                                |   |   |                                      |                                      |                                     |  |  |
|   |                                 | Ε                              | nd-of-day   | window  |                                      |                                      |                                     |  |  |
|   | mean                            | E<br>median                    | nd-of-day v<br>mean   | window<br>median  | 1.64                                 | 1.96                                 | 2.58                                |  |  |
|   |                                 |                                | -   |   | 1.64<br>s.d.                         | 1.96<br>s.d.                         | 2.58<br>s.d.                        |  |  |
| ED1, FF   |                                 | median                         | mean  | median  |                                      |                                      |                                     |  |  |
| ED1, FF<br>ED1, FG                                  |                                 | median                         | mean<br>decomp.   | median<br>decomp.   | s.d.                                 | s.d.                                 | s.d.                                |  |  |
| ,   | $ \Delta P_{ref} $              | median $ \Delta P_{ref} $      | mean decomp.  | median<br>decomp.<br>0.01   | s.d.<br>20                           | s.d.<br>16                           | s.d.<br>11                          |  |  |
| ED1, FG   | $ \Delta P_{ref} $              | median $ \Delta P_{ref} $      | mean<br>decomp.<br>0.02<br>0.00                                 | median<br>decomp.<br>0.01<br>0.00                                 | s.d.<br>20<br>3                      | s.d.<br>16<br>2                      | s.d.<br>11<br>0                     |  |  |
| ED1, FG<br>ED1, AP                                  | $ \Delta P_{ref} $              | median $ \Delta P_{ref} $      | mean<br>decomp.<br>0.02<br>0.00<br>0.00                         | median<br>decomp.<br>0.01<br>0.00<br>0.00                         | s.d.<br>20<br>3<br>2                 | s.d.<br>16<br>2<br>2                 | s.d.<br>11<br>0<br>1                |  |  |
| ED1, FG<br>ED1, AP<br>ED8, FF                       | $\frac{ \Delta P_{ref} }{0.03}$ | median $ \Delta P_{ref} $ 0.01 | mean<br>decomp.<br>0.02<br>0.00<br>0.00<br>0.01                 | median<br>decomp.<br>0.01<br>0.00<br>0.00<br>0.00                 | s.d.<br>20<br>3<br>2<br>4            | s.d.<br>16<br>2<br>2<br>3            | s.d.<br>11<br>0<br>1<br>2           |  |  |
| ED1, FG<br>ED1, AP<br>ED8, FF<br>ED8, FG            | $\frac{ \Delta P_{ref} }{0.03}$ | median $ \Delta P_{ref} $ 0.01 | mean<br>decomp.<br>0.02<br>0.00<br>0.00<br>0.01<br>0.03         | median<br>decomp.<br>0.01<br>0.00<br>0.00<br>0.00<br>0.02         | s.d.<br>20<br>3<br>2<br>4<br>17      | s.d.<br>16<br>2<br>2<br>3<br>11      | s.d.<br>11<br>0<br>1<br>2<br>7      |  |  |
| ED1, FG<br>ED1, AP<br>ED8, FF<br>ED8, FG<br>ED8, AP | $\frac{ \Delta P_{ref} }{0.03}$ | median $ \Delta P_{ref} $ 0.01 | mean<br>decomp.<br>0.02<br>0.00<br>0.00<br>0.01<br>0.03<br>0.01 | median<br>decomp.<br>0.01<br>0.00<br>0.00<br>0.00<br>0.02<br>0.00 | s.d.<br>20<br>3<br>2<br>4<br>17<br>1 | s.d.<br>16<br>2<br>2<br>3<br>11<br>1 | s.d.<br>11<br>0<br>1<br>2<br>7<br>1 |  |  |

Table 2: Summary statistics for historical decompositions

Summary statistics for the historical decompositions of each rate with respect to the three shocks; the top panel considers the decomposition based on shocks occurring between 10 minutes prior to the announcement and 20 minutes following, and the bottom considers 10 minutes prior until 4:15pm. The units are percentage points. The first two columns summarize the absolute values of the simple change in the reference rate over the window. The next two columns repeat the exercise for the absolute value of the historical decompositions. The entries for the response of the 10-year Treasury are scaled by the ratio of the end-of-day constant-maturity zero–coupon 10-year Treasury yield to the end-of-day value in the data. The final three columns report the frequency with which decompositions with respect to the given shock exceed multiples of the average standard deviation in the interest rate on monetary policy announcement days.

approach), reported in the first two columns, will over-state the size/effect of a particular shock due to the fact that all movements are taken as due to that particular shock of interest, when observed movements are generally due to a combination of realized shocks. Second, the size of the decompositions due to each shock are, on average, generally comparable across the two horizons. However, as the plots for individual dates below make clear, this obscures sometimes substantial differences for a given day.

Table 3 directly reproduces Table 1 of Swanson (2017), adding two additional dates. It reports the selection of highly notable FOMC announcements I address individually, along with key details. Figure 2 plots the historical decompositions for each of these key dates for each interest rate with respect to each of the three shocks. Each column plots responses for a given day, with each panel plotting responses of the indicated rate to the three shocks. For comparison, simple changes, as one would calculate in an event study, are plotted for each rate.<sup>5</sup> Four facts are immediate. First, with the obvious exception of December 2008, when the ZLB was reached, there is virtually no effect of shocks to the current policy rate (conventional monetary policy) on these days, consistent with the rate being at the ZLB. Second, simply computing changes in asset prices would frequently be misleading, to an episode-dependent extent, due to the fact that multiple shocks of meaningful size are generally realized. This may be the case even on days when explicit statements were only made about one dimension of policy, but market expectations were revised on additional dimensions. Third, focusing on only the 30-minute windows around announcements may be misleading. In some cases, as previous work has speculated, effects do continue to grow before the end of trading, but, more often, effects apparent in the 30-minute window do not persist to the end of the day. From a macroeconomic perspective, the 30-minute window thus may *overstate* the relevant monetary policy shocks. Finally, there are very few announcements and shocks for which there are economically significant effects by the end of the day (as measured relative to the average standard deviation of the interest rate on announcement days). I now briefly interpret the results for each announcement in turn. Magnitudes reported are in percentage points at end-of-day, and significance is discussed at the 5% level.

**December 2008** The only shock of note is to the Fed Funds rate, which hit the ZLB for the first time. This results in a significant decrease in all three interest rate series. The suggestion that the Fed may purchase Treasuries ultimately has little effect on interest rates (although the asset purchase shock does explain ample *high frequency* variation in the 10-year Treasury, Figure 8 in the Supplement).

<sup>&</sup>lt;sup>5</sup>Note that the decompositions will not, in general, add up to this path since the regressions are based on first-differenced data.

| December 2008  | FOMC announces that it has cut the FFR to between 0 and 25 basis points (bp), will purchase large quantities of agency debt and will evaluate purchasing long-term Treasuries    |
|----------------|--|
| March 2009     | FOMC announces it expects to keep the federal funds rate between 0 and 25 bp for "an extended period", and that it will purchase \$750B of mortgage-backed securities, \$300B of |
|                | longer-term Treasuries, and \$100B of agency debt (a.k.a. "QE1")   |
| November 2010  | FOMC announces it will purchase an additional \$600B of longer-term Treasuries (a.k.a. "QE2")  |
| August 2011    | FOMC announces it expects to keep the federal funds rate between 0 and 25 bp "at least through mid-2013"   |
| September 2011 | FOMC announces it will sell \$400B of short-term Treasuries and use the proceeds to buy \$400B of long-term Treasuries (a.k.a. "Operation Twist")                                |
| January 2012   | FOMC announces it expects to keep the federal funds rate between 0 and 25 bp "at least through late 2014"  |
| September 2012 | FOMC announces it expects to keep the federal funds rate between 0 and 25 bp "at least   |
|                | through mid-2015", and that it will purchase \$40B of mortgage-backed securities per<br>month for the indefinite future  |
| December 2012  | FOMC announces it will purchase \$45B of longer-term Treasuries per month for the  |
|                | indefinite future, and that it expects to keep the federal funds rate between 0 and 25 bp at   |
|                | least as long as the unemployment remains above 6.5 percent and inflation expectations remain subdued  |
| September 2013 | FOMC announces that it will wait to taper asset purchases  |
| December 2013  | FOMC announces it will start to taper its purchases of longer-term Treasuries and  |
|                | mortgage-backed securities to paces of \$40B and \$35B per month, respectively   |
| December 2014  | FOMC announces that "it can be patient in beginning to normalize the stance of monetary policy"  |
| March 2015     | FOMC announces that "an increase in the target range for the federal funds rate remains<br>unlikely at the April FOMC meeting"   |

Table 3: Key FOMC announcements 2008-2015

This table is replicated from Swanson (2017), with the addition of details on the December 2008 and September 2013 announcements.

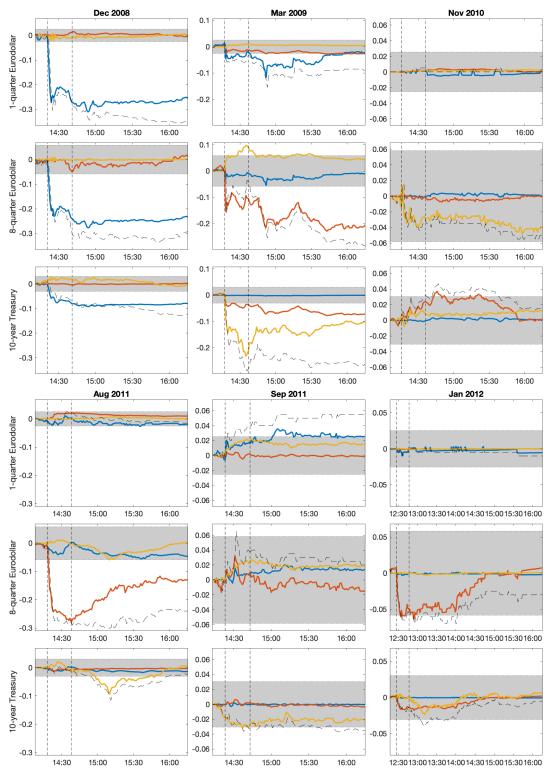


Figure 2: Historical decompositions of key FOMC announcements

Historical decompositions for the rate series indicated in the left margin with respect to each of the three shocks. The shaded interval corresponds to 1.96 times the average standard deviation in the interest rate on monetary policy announcement days. The vertical lines mark the time of the announcement and 20 minutes following the announcement, the end of the conventional analysis window. The black dashed path is the path of the simple change from ten minutes prior to the announcement, the event study estimate. Units are percentage points.

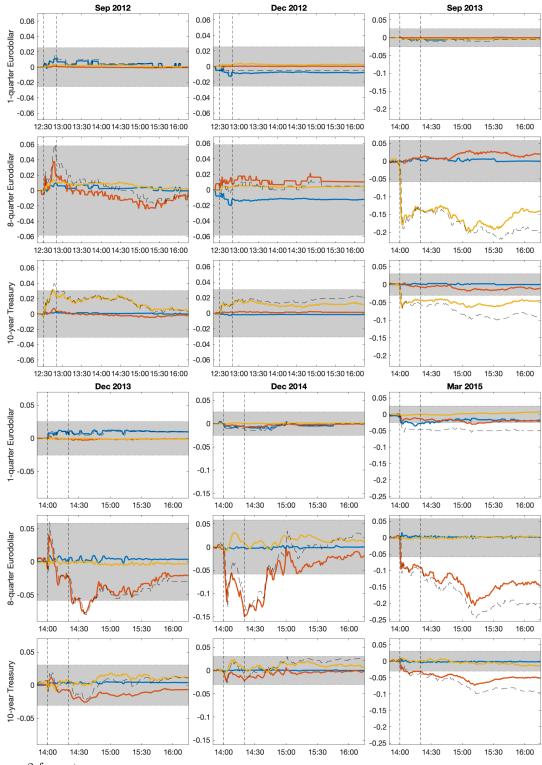


Figure 2b: Historical decompositions of key FOMC announcements (cont'd)

See Figure 2 for notes.

- March 2009 The first iteration of forward guidance significantly lowers medium-term expectations of short rates (-0.21) and long term rates (-0.07). The announcement of QE1 lowers long-term rates (-0.10); perhaps puzzlingly, it increases medium-term expectations of short rates. The impacts of these two shocks on medium-term expectations of short rates and long-term rates respectively are of comparable magnitude. Note that the former would have been understated (-0.10 instead of -0.21) by using a 30-minute window, while the latter would been have overstated (-0.21 instead of -0.10).
- November 2010 The announcement was dominated by the launch of QE2, which (insignificantly) lowers medium-term expectations of short rates (-0.04), but does not appear in longer-term rates, which actually rose.<sup>6</sup> Examining contemporary market commentary, it appears that the \$600B pledged was towards the upper end of market expectations, but the rate of purchases, \$75B per month, was somewhat low relative to expectations; a perceived focus on medium-term securities may also have been disappointing (Anderson & Englander (2010)). Moreover, the apparent non-response of long rates may reflect a trading strategy of "buying the rumour and selling the fact", discussed by commentators prior to the announcement (e.g., Capo McCormick (2010)).
- August 2011 The first case of calendar-based guidance ("mid-2013") has a significant effect on medium-term expectations of short rates (-0.13). Using a 30-minute window would inflate this effect by a factor of two (-0.28).
- September 2011 The asset purchase shock of "Operation Twist" has a modest (insignificant) downward effect on long-term rates (-0.02). Surprisingly (given the ZLB), the Fed Funds shock has a significant positive effect on the front Eurodollar contract. However, the announcement directly follows a Eurodollar settlement date. This means the front contract expires in mid-December, and movement in expectations of short rates over the next *three months* is plausible (even if current rates were rooted at zero).
- January 2012 The extension of guidance to "late-2014" initially causes a dramatic fall in medium-term expectations of short rates, which does not persist to the end of the day (in fact reversing); using a 30-minute window would estimate an effect of -0.05. This is consistent with the fact that many analysts expected language to be extended to some point in 2014 (Blackden (2012), Crutsinger (2012)).

<sup>&</sup>lt;sup>6</sup>Accordingly, this date is one of the two in which the labeling rule appears unreliable, and the reported results reflect a switch of forward guidance and asset purchase shocks relative to the rule; this is necessitated by the fact that long-term rates (over 7 years) actually rose following the (expansionary) announcement. This labeling adjustment is supported by the variance decomposition in Figure 8 in the Supplement, which shows that the shock labelled as "asset purchases" does drive fluctuations in the 5-year Treasury, even if not in the 10-year.

- September 2012 Neither the extension of guidance to "mid-2015" nor the announcement of \$40B purchases of mortgage-backed securities per month has a significant effect. The extension of guidance was expected, possibly out to *late-2015* (Kucukreisoglu (2012)). Of course, a purchase of MBS need not lower risk-free rates, so the non-response to the asset purchase shock may be unsurprising. The announcement was also widelyexpected, with some sources reporting its magnitude fell short of expectations, while others found it larger than expected, (Klein (2012), Kucukreisoglu (2012), Popper (2012)).
- **December 2012** Neither the replacement of calendar-based guidance with conditional guidance nor the announcement of \$45B in Treasury purchases for the indefinite future has a significant impact on markets, with the latter actually raising long-term rates slightly. While the former was unanticipated, market expectations may have translated the given numbers to the calendar-based horizon already in place (Goldfarb (2012)); the latter was anticipated (Irwin (2012)).
- September 2013 The announcement that the Fed would wait to taper asset purchases leads to a significant decrease in long-term rates (-0.05) and a significant decrease in medium-term expectations of short rates (-0.14).<sup>7</sup>
- **December 2013** The announcement that asset purchases would be tapered has only a minor, non-significant positive effect on long-term rates; a clarification of conditional guidance that the target rate is unlikely to change until "well past" the time that unemployment falls past 6.5%, leads to a non-significant fall in medium-term expectations of short rates. The former is consistent with the relatively small scale of the tapering (\$10B) and the fact that some analysts anticipated the move (Appelbaum (2013)).
- December 2014 The announcement that the Fed would be "patient" in normalizing monetary policy ultimately has a minimal impact (-0.02) on medium-term expectations of short rates. Focusing on a 30-minute window would risk overstating the effect (-0.15). This is consistent with contemporary discourse, with many analysts expecting some revision to the "considerable time" language (Chen & McMahon (2014)).

<sup>&</sup>lt;sup>7</sup>This date is one of the two in which the labeling rule appears unreliable since one shock similarly drives both 8-quarter Eurodollar and 10-year Treasury rates, and the reported results reflect a switch of forward guidance and asset purchase shocks. This labeling adjustment is supported by the variance decomposition in Figure 8 in the Supplement, which shows that the third shock additionally drives 5-year Treasury rates, confirming the existence of a single shock affecting long rates and expected future short rates, which I label as the asset purchase shock based on the importance of the "no-taper" announcement and relative lack of forward guidance news.

March 2015 The announcement that rates would stay at the ZLB through at least the April FOMC meeting significantly reduces both medium-term expectations of short rates (-0.15) and long-term rates (-0.05). Using a 30-minute window would substantially underestimate the effect.

While the correct bar to measure significance of these movements in interest rates is open to debate, the subset of episodes that do meet the bar I adopt is interesting. In particular, the first forward guidance announcement in March 2009, ("extended period"), the launch of calendar-based guidance in August 2011, and the final March 2015 announcement of an additional FOMC cycle at the ZLB pass the bar. On the asset purchase side, the QE1 announcement of March 2009 and the September 2013 decision to delay tapering led to significant decreases in long-term rates.

For forward guidance, this suggests that the revision of calendar-based guidance, once introduced, did not convey significant new information that markets did not already anticipate in 2012, nor did the switch to conditional guidance change this relationship. Rather, the introduction of forward guidance, an unprecedented move, and its extension beyond the point where markets expected rates to "lift-off" are the two episodes that stand out. The latter accords with the finding of Akkaya, Gürkaynak, Kısacıkoğlu, & Wright (2015) that the potency of forward guidance grows as the distance of the shadow rate from zero shrinks. In a cross-country study, Coenen et al (2017) find that the nature of guidance can imply significantly different effects on bond yields, but their result is not robust to omitting observations confounded by simultaneous asset purchase policies.

For asset purchases, effects were not limited to long rates (e.g., September 2013), and not always most emphatic at the longer end of the yield curve (e.g., QE2, November 2010). The launch of the policy, as well as its continuation (when markets expected a taper), along with announcements signaling a change in the focus of purchases, are among the most impactful moves by the FOMC.

Bauer, Lakdawala, & Mueller (2019) study the effects of monetary policy uncertainty, and argue that changes in uncertainty around monetary policy shocks can explain why some strongly impact asset prices, while others do not. Lower uncertainty amplifies the effects of shocks. Among the dates in Table 3, the announcements that I find to be associated with significant shocks are precisely those that the authors associate with large falls in monetary policy uncertainty. This suggests that their story of uncertainty explaining which shocks are most impactful is consistent with my results.

Finally, I investigate whether there are any announcements not considered "notable" above that sparked particularly significant shocks (exceeding 2.58 standard deviations). For the Fed Funds rate, there are several additional dates, since Table 3 focuses on unconventional policy announcements. They are September 2007 (50 bp cut), December 2007 (25 bp cut, but more expected), Jan 2008 (50 bp cut), March 2008 (75 bp cut, but at least 100 bp expected according to e.g., Goodman & Pan (2008)), April 2008 (25 bp cut), June 2008 (no change in face of rising inflation), September 2008 (no further cut), October 2008 (no further cut), September 2015 (no lift-off), and March 2016 (no additional hike). For forward guidance, September 2008 (no Fed Funds cut), June 2013 ("downside risks diminished") and March 2014 (6.5% unemployment reached and "considerable time" language dropped) were contractionary, while March 2017 (rate hike, but no revision to further anticipated hikes in medium-term, see e.g., Riccadonna, Shulyatyeva, & Yamarone (2017)) was expansionary. Finally, for asset purchases, an expansionary shock is registered in December 2007, prior to the launch of such policies in November 2008, making it difficult to interpret. This may be due to considerable discussion about deteriorating financial conditions and uncertainty over economic prospects in the FOMC statement (consistent with some sort of "Fed information effect"). Contractionary shocks occurred in June 2009 (talking down expectations of expanded purchases, e.g., Lanman (2009)), and December 2016 (rate hike, but no change to asset purchases). For the most part, these findings align with important revisions to the relevant dimensions of FOMC statements.

#### A new monetary policy shock series

While comparison of the decompositions for these notable announcements presents interesting results in its own right, many questions can only be answered by aggregating these findings into a time-series of inter-announcement shocks to be used in further analysis. To do so requires a stance on first the horizon at which effects will be measured and second the units by which shocks will be scaled. For macroeconomic purposes, I adopt a series defined by the end-of-day horizon, based on the fact that to pass through to the macroeconomy, effects must be at least somewhat persistent; however, I additionally present a series based on 30-minute windows in this section for comparison. I normalize each daily shock by using the historical decomposition of the 1-quarter Eurodollar for  $\epsilon_{FFt}$ , 8-quarter Eurodollar for  $\epsilon_{FGt}$ , and the 10-year Treasury for  $\epsilon_{APt}$ . Together, these values form a time series of 96 announcement dates.

Table 4 reports the correlation of the shocks constructed using these decompositions with simple changes in the relevant asset prices for the 30-minute window and the end-ofday window. On one hand, the Fed Funds shock series appear to be fairly consistent across all measures, likely due to the fact that most Fed Funds shocks occur prior to the ZLB period, when the other shocks are less active (and event study assumptions are broadly valid). The similarity across horizons also suggests that such shocks are quite persistent. Because the

|                    | 30-n                | nin deco            | omp.                | End-o               | End-of-day decomp.  |                     |  |  |
|--------------------|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|--|--|
|                    | $\varepsilon_{FFt}$ | $\varepsilon_{FGt}$ | $\varepsilon_{APt}$ | $\varepsilon_{FFt}$ | $\varepsilon_{FGt}$ | $\varepsilon_{APt}$ |  |  |
| End-of-day decomp. | 0.97                | 0.76                | 0.75                | —                   | _                   | _                   |  |  |
| 30-min. change     | 0.98                | 0.85                | 0.87                | 0.96                | 0.64                | 0.71                |  |  |
| End-of-day change  | 0.96                | 0.72                | 0.65                | 0.98                | 0.86                | 0.79                |  |  |

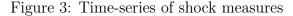
 Table 4: Correlation of shock measures

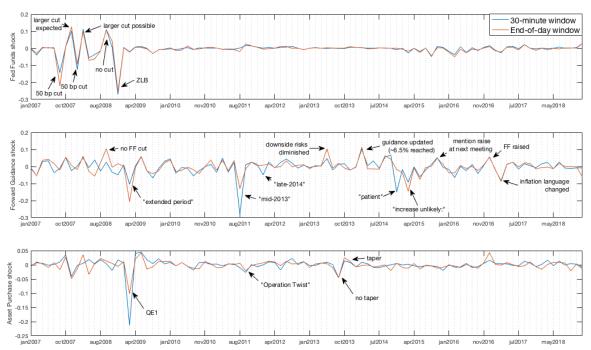
Shock measures are computed by forming time-series of the historical decompositions of the reference rates at either the 30-minute or end-of-day horizon. The 30-minute and End-of-day changes are simply the change in the reference rate over the specified window.

forward guidance and asset purchase shocks are more likely to appear in conjunction, there is more discrepancy between simple changes, ignoring the need to decompose asset price movements in the face of multiple contemporaneous shocks, and decomposition measures. The greater discrepancy across horizons for these shocks also suggests that they are more likely to either wash out over the course of the day or take some time to be fully incorporated by markets. This may be reflective of the fact that these shocks (and the language that triggers them) are of a more complex nature than a comparatively "up or down" change in the Fed Funds rate.

Figure 3 plots the time series for these shocks for the full sample, annotated with important historical events. Broadly speaking, the behaviour of these shocks accords with narrative evidence and expectations. There are large realizations for the Fed Funds shock prior to the ZLB, and then minimal movement until lift-off in December 2015. The largest forward guidance shocks generally correspond with the most notable episodes. The most puzzling feature is some fluctuations in the asset purchase shock prior to the introduction of that measure in the policy discourse in the fall of 2008, as mentioned above.

The paths of the forward guidance and asset purchase shocks can be compared to the 2009-2015 paths reported in Figure 1 of Swanson (2017). For forward guidance, the Swanson series notably allocates most of the first announcement, in March 2009, to asset purchases instead. One of his largest forward guidance shocks is associated with the announcement of a 1-quarter extension of QE1 (September 2009); that shock is much more moderate in my series. The results agree on a substantial forward guidance shock with the introduction of calendar guidance (August 2011), but my series do not pick up Swanson's puzzling contractionary shock at the next meeting, which was dominated by Operation Twist. Swanson's series also picks up a puzzling large contractionary guidance shock following the "taper tantrum" (June 2013). The series agree with a contractionary shock with the updated guidance following unemployment reaching 6.5% in March 2014, with similar shocks at subsequent meetings. Finally, the "patient" and "increase unlikely" shocks at the turn of 2014-2015 appear across





Shock measures are computed by forming time-series of the historical decompositions of the reference rates at either the 30-minute or end-of-day horizon. Units are percentage points of the reference rates. Large fluctuations that correspond to notable announcements or statement features are labeled.

series (although the end-of-day "patient" shock is much smaller).

Turning to asset purchase shocks, all series agree that the announcement of QE1 was the most significant episode. Operation Twist is also notable across series. Swanson picks up a large contractionary "taper tantrum" shock in 2013, which is puzzling given that Bernanke's testimony that provoked the tantrum occurred on May 22nd, while the shock registers at the time of the June 19th FOMC announcement. My series have no such shock. Finally, the series agree on an expansionary shock with the announcement that there would be no immediate taper in September 2013, with contractionary shocks through the confirmation of a taper two meetings later.

#### 3.2 Robustness checks

I first consider a simple placebo test based on non-announcement days. I choose a sample of 12 days, consisting of each of the 12 dates 7 days prior to my 12 key announcements. For each of these "placebo" dates, I estimate my baseline model, and compute end-of-day historical decompositions. I report these decompositions in Table 11 in the Supplement. The vast majority of the responses are zero, to the nearest basis point. No responses meet any level of significance. This suggests that the sizable and significant responses I highlight above are abnormal and representative of true economic events, and not an artifact of the econometric method or random error.

I consider five principal robustness checks to assess the identification of monetary policy shocks from the data. The first allows for the possibility that surprises to multiple dimensions of monetary policy are correlated. To do so, I propose a new identification scheme, which is, to my knowledge, the first approach to allow for possible correlation of structural shocks. Section 4 of the Supplement outlines the argument in detail. Since identification is based on lower frequency moments (covariance of asset price innovations across periods of the day), it also provides a foil for the possibility that the baseline results, which rely on minute-by-minute variation, are contaminated by the noise of intraday data, in spite of the results of Propositions 3 & 4. The second approach uses a simple 2-regime version of identification via heteroskedasticity, where the regimes are 9:30am to 10 minutes prior to the announcement and 10 minutes prior to the announcement to 4:15pm. This provides an additional check on the parametric volatility model (with continuous variance process) adopted in the baseline and concerns over high-frequency noise. The third alternative model adds an additional dimension, S&P 500 returns, in an attempt to capture possible macroeconomic news shocks contained in announcements, otherwise known as "Delphic" forward guidance or "Fed information", in the spirit of Matheson & Stavrev (2014) and Jarocinski & Karadi (2019). The fourth alternative addresses the concern of non-stationarity between the preand post-announcement periods by simply discarding pre-announcement data and estimating the model on data beginning 10 minutes prior to the announcement. The final check assesses whether the use of Eurodollar contracts confounds the analysis by introducing credit risk in the period where the TED spread was both elevated and volatile (my first 21 announcements, January 2007 to August 2009) by replacing the 1-quarter Eurodollar contract with the 2month Fed Funds future contract. All robustness checks are discussed comprehensively in Section 2 of the Supplement. Table 5 reports the correlation of the baseline shock measures with those from all alternatives at both 30-minute and end-of-day horizons. The Fed Funds shocks identified are incredibly close across approaches. The forward guidance and asset purchase shocks are generally closely related; lower correlations are often due to one or two outlying announcements, as discussed in detail in the Supplement.

# 4 High-frequency effects on financial markets

Having computed minute-by-minute measures of monetary policy shocks for each announcement and an aggregate time-series for the period 2007-2018, I now assess the effects of policy on a range of variables of economic interest, starting in this section with financial variables,

|                         | 30-m                | nin dec             | omp.                | End-o               | End-of-day decomp.  |                     |  |
|-------------------------|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|--|
|                         | $\varepsilon_{FFt}$ | $\varepsilon_{FGt}$ | $\varepsilon_{APt}$ | $\varepsilon_{FFt}$ | $\varepsilon_{FGt}$ | $\varepsilon_{APt}$ |  |
| Correlated shocks model | 0.97                | 0.83                | 0.95                | 0.97                | 0.85                | 0.84                |  |
| Regimes model           | 0.97                | 0.88                | 0.95                | 0.95                | 0.88                | 0.87                |  |
| Macro shocks model      | 1.00                | 0.97                | 0.98                | 0.99                | 0.96                | 0.89                |  |
| Post-announcement only  | 0.99                | 0.95                | 0.88                | 0.99                | 0.93                | 0.72                |  |
| Fed Funds future        | 0.99                | 0.98                | 0.99                | 1.00                | 0.99                | 0.98                |  |

Table 5: Robustness of shock measures

Correlation between the baseline shock time series and those of the five robustness checks, for measures based on both the 30-minute and end-of-day horizons. The sample for the Fed Funds future model consists of only the first 21 announcements.

which are available at higher frequencies. First, I consider variables available at the same frequency as the identifying data, which allows for announcement-specific estimates; I then turn to daily data.

#### 4.1 Intraday responses

To conduct announcement-specific regressions, the dependent variable must be available at high enough frequency to make daily estimation reliable. This means only financial variables may be assessed, while the intended effects of monetary policy are generally macroeconomic. However, one aim of the large scale asset purchases was to stimulate the economy by lowering corporate borrowing costs. Unfortunately, corporate debt is not liquid enough to conduct high-frequency analysis using specific bonds. A potential proxy is an investment-grade corporate debt ETF, the price of which aims to track the prices of a basket of Aaa corporate bonds. I consider the minute-by-minute returns of the iShares IBoxx \$ Invest Grade Corporate Bond ETF (LQD), the most liquid such ETF throughout the period in question; an *increase* in this variable suggests a *fall* in corresponding bond yields. In the Supplement, I also consider the spread of this return over the minute-by-minute return on the 10-year Treasury. The other dependent variable I consider is the S&P 500 return, proxying for equity markets and capturing some measure of market sentiment. Unfortunately, no ETF or similar index related to MBS is suitably liquid during much of the sample.

I conduct simple regressions of the relevant return on the contemporaneous and possibly lagged values of the three identified shocks at the minute frequency, according to

$$r_t = \omega + \gamma_0 \epsilon_t + \sum_{l=1}^p \gamma_l \epsilon_{t-l} + u_t.$$
(7)

|                        | mean  | $q_{10}$ | median | $q_{90}$ | positive    | positive<br>sig. 5% | negative | negative<br>sig. 5% |  |  |
|------------------------|-------|----------|--------|----------|-------------|---------------------|----------|---------------------|--|--|
| Corporate return proxy |       |          |        |          |             |                     |          |                     |  |  |
| $\epsilon_{FF}$        | 1.38  | -0.97    | 0.65   | 4.57     | 65          | 33                  | 28       | 3                   |  |  |
| $\epsilon_{FG}$        | 2.46  | -0.04    | 1.68   | 2.85     | 84          | 66                  | 12       | 2                   |  |  |
| $\epsilon_{AP}$        | 2.09  | 0.56     | 1.84   | 3.79     | 94          | 83                  | 2        | 0                   |  |  |
|                        |       |          |        | S&P      | 500 returns |                     |          |                     |  |  |
| $\epsilon_{FF}$        | -0.12 | -4.39    | 0.88   | 4.87     | 56          | 17                  | 37       | 11                  |  |  |
| $\epsilon_{FG}$        | -1.61 | -4.95    | -0.48  | 1.75     | 37          | 13                  | 59       | 34                  |  |  |
| $\epsilon_{AP}$        | -2.12 | -5.02    | -1.59  | 0.54     | 16          | 3                   | 80       | 50                  |  |  |

Table 6: Summary statistics: contemporaneous coefficients for external regressors

The corporate debt return proxy is described in the text. Coefficients are estimated by simple regressions of the respective measure on the current and possibly lagged values of the shocks, plus a constant, equation (7). The units are percentage return per expansionary shock (leading to a 1% *fall* in the reference rate). HAC standard errors are computed following Lazarus, Lewis, & Stock (2019).

The number of lags, p, is selected day-by-day using the Hannan-Quinn criterion. I compute HAC standard errors using the equal-weighted-periodogram estimator with 8 degrees of freedom, following Lazarus, Lewis, & Stock (2019). Table 6 reports summary statistics for the estimated coefficients on contemporaneous *expansionary* shocks. Since the shocks are generated regressors, affected by estimation error, these coefficients are attenuated and estimated effects should be seen as a lower bound. It is clear that, on average, all three shocks move corporate returns, and thus yields, in the anticipated direction: the positive coefficients imply that an expansionary shock raises prices, implying lower yields. The magnitude of the effect is comparable for forward guidance and asset purchases and lower for Fed Funds shocks. This makes sense as forward guidance likely concerns a longer portion of the time to maturity, and asset purchases either directly targeted corporate debt (QE1), or comparable assets of similar maturity. For both forward guidance and asset purchases, the effect of the majority of announcements is both positive and statistically significant. Turning to the S&P 500, the average effect of Fed Funds shocks is more ambiguous, but the majority of estimates are positive (with some significant), indicating expansionary shocks raise returns, as expected. For the unconventional policy shocks, the majority of coefficients are negative (with some statistically significant); however, the  $R^2$  with respect to each shock is 0.05 or lower, so these do not represent particularly economically significant effects.

However, these results indicate only instantaneous elasticities; the end-of-day historical decompositions are more informative of economically meaningfully effects. Table 7 summarizes these decompositions. The end-of-day responses of the corporate return proxy follow

|                        | mean decomp.    | median decomp. | $1.64  \mathrm{s.d.}$ | 1.96 s.d. | 2.58 s.d. |  |  |  |  |  |
|------------------------|-----------------|----------------|-----------------------|-----------|-----------|--|--|--|--|--|
| Corporate return proxy |                 |                |                       |           |           |  |  |  |  |  |
| $\epsilon_{FF}$        | 0.02            | 0.01           | 0                     | 0         | 0         |  |  |  |  |  |
| $\epsilon_{FG}$        | 0.06            | 0.03           | 3                     | 2         | 2         |  |  |  |  |  |
| $\epsilon_{AP}$        | 0.07            | 0.04           | 3                     | 3         | 2         |  |  |  |  |  |
|                        | S&P 500 returns |                |                       |           |           |  |  |  |  |  |
| $\epsilon_{FF}$        | 0.07            | 0.01           | 2                     | 1         | 1         |  |  |  |  |  |
| $\epsilon_{FG}$        | 0.09            | 0.03           | 2                     | 2         | 1         |  |  |  |  |  |
| $\epsilon_{AP}$        | 0.07            | 0.05           | 1                     | 0         | 0         |  |  |  |  |  |

Table 7: Summary statistics for historical decompositions of external regressors

Historical decompositions are computed using the contemporaneous and possibly lag coefficients estimated in equation (7) and the intraday time-series of shocks. Mean and median decompositions are computed based on absolute value. Units are percentage points. The final three columns report the frequency with which decompositions with respect to to the given shock exceed multiples of the average standard deviation in the dependent variable on monetary policy announcement days.

the same pattern as the contemporaneous coefficients, ranging from 2 bp on average for Fed Funds shocks to 7 bp for asset purchase shocks. Very few are significant. Turning to the S&P 500, the effects are comparable across shocks (7-9 bp), with similarly few significant.

Finally, I focus on the key announcement dates in Table 8. Broadly speaking, the results accord with intuition; on the most stimulatory announcement days (as determined in Section 3), the launch of unconventional policy (March 2009), the taper delay (September 2013), and the final extension of zero-rate guidance (March 2015), there are sizable (and significant) positive effects, up to nearly a full percentage point for March 2009, suggesting a substantial fall in yields. Turning to the S&P 500, while the signs of instantaneous effects, were, on average, surprising across the sample, for the announcements of importance the evidence appears more in-line with intuition. The launch of forward guidance (March 2009), the delay of tapering (September 2013), and the delay of "lift-off" (March 2015) all see sizable positive effects. The only significant effects are due to forward guidance, at its launch and the lift-off delay, and asset purchases at the taper delay. Overall, there is clearer evidence of unconventional policy having the desired effect on corporate debt markets than a stimulatory effect on equities, which accords with the objectives of the Federal Reserve. Almost all of the significant shocks noted in Table 7 correspond to those key dates documented in Table 8. My results display considerable heterogeneity in responses across announcements. This type of evidence was previously available only in non-parametric analysis like that of Krishnamurthy & Vissing-Jorgensen (2011, 2013) for asset purchases, whose results also demonstrate this variation. Such results, however, are unable to separate forward guidance

|                 | Dec<br>2008            | Mar<br>2009  | Nov<br>2010 | Aug<br>2011 | Sep<br>2011 | Jan<br>2012 | Sep<br>2012 | Dec<br>2012 | Sep<br>2013 | Dec<br>2013 | Dec<br>2014 | Mar<br>2015  |
|-----------------|------------------------|--------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|--------------|
|                 | Corporate return proxy |              |             |             |             |             |             |             |             |             |             |              |
| $\epsilon_{FF}$ | 0.11                   | 0.02         | -0.02       | 0.25        | 0.06        | 0.01        | 0.00        | 0.01        | 0.00        | -0.01       | 0.00        | 0.04         |
| $\epsilon_{FG}$ | -0.04                  | $0.90^{***}$ | -0.02       | -0.05       | 0.01        | -0.01       | 0.01        | -0.01       | 0.12        | 0.08        | 0.02        | $0.49^{***}$ |
| $\epsilon_{AP}$ | 0.00                   | $1.13^{***}$ | $-0.22^{*}$ | * -0.07     | 0.14        | -0.06       | -0.03       | -0.11       | 0.65***     | -0.18       | -0.06       | 0.12         |
|                 | S&P 500                |              |             |             |             |             |             |             |             |             |             |              |
| $\epsilon_{FF}$ | 0.40                   | 0.03         | 0.03        | -0.01       | -0.04       | 0.03        | 0.00        | -0.04       | 0.00        | 0.09        | 0.00        | 0.14         |
| $\epsilon_{FG}$ | 0.05                   | $0.81^{**}$  | 0.01        | -0.06       | -0.13       | -0.01       | 0.00        | 0.00        | 0.08        | 0.03        | 0.02        | $0.44^{***}$ |
| $\epsilon_{AP}$ | -0.09                  | 0.19         | -0.09       | 0.11        | -0.25       | 0.03        | 0.03        | 0.10        | $0.58^{*}$  | -0.28       | 0.03        | -0.05        |

Table 8: End-of-day responses of external regressors on key dates

For each dependent variable, end-of-day historical decomposition values are reported for the 12 key announcement dates detailed in Table 3. Results are starred relative to the average standard deviation in that asset price on monetary policy announcement days.

effects from contemporaneous asset purchase effects (for example, the joint announcement of March 2009). Tables 18 & 19 in the Appendix report similar results for the spread of Corporate returns over the 10-year Treasury. These results show that spreads *rise* in response to expansionary unconventional policy shocks, with the same key announcements found to be most important. These results align with existing announcement-frequency regressions, as in Swanson (2017).

#### 4.2 Daily responses of financial variables

Turning to the inter-day time-series of shocks, I now consider the daily impact of the shocks on corporate debt yields and spreads and TIPS spreads. The simple regression takes the form

$$\Delta r_d = \nu + \psi \epsilon_d + u_d,\tag{8}$$

where d indexes the announcement dates, with HAC standard errors. Table 9 reports the results. Recall that the shock series is aggregated from end-of-day shock measures, which may exhibit considerable estimation error resulting from the cumulation of reduced form and structural sampling error in the historical decompositions; this means that estimated effects are likely attenuated. I find that yields fall significantly in response to both Fed Funds and asset purchase shocks, but less so in response to forward guidance shocks. This partially aligns with Swanson's (2017) finding that asset purchases and not forward guidance matter for yields during the ZLB period (although he does not report responses to Fed Funds shocks). The asset purchase coefficients (relative to movements in the 10-year Treasury) are

|                 | Aaa yield     | Baa yield     | Aaa spread   | Baa spread   | TIPS spread |
|-----------------|---------------|---------------|--------------|--------------|-------------|
| $\epsilon_{FF}$ | -0.02         | -0.01         | -0.04        | -0.04        | 0.13**      |
| $\epsilon_{FG}$ | $-0.25^{**}$  | -0.23         | $0.46^{***}$ | $0.48^{***}$ | 0.30***     |
| $\epsilon_{AP}$ | $-1.44^{***}$ | $-1.69^{***}$ | $1.12^{***}$ | $0.86^{**}$  | -0.12       |

Table 9: Corporate debt responses to monetary policy

Coefficients are estimated from equation (8). Coefficients can be interpreted as the response in percentage points to an expansionary shock leading to a 1% fall in the reference rate. HAC standard errors are calculated following Lazarus, Lewis, & Stock (2019). Significant results are starred at the 10%, 5% and, 1% levels.

larger, at -1.44 and -1.69, compared to his (normalized by the estimated impact on the 10year Treasury, 4.51/6.49 = 0.69 and 5.25/6.49 = 0.80), and larger than those for Fed Funds shocks. The larger asset price coefficients I obtain here may be related to the fact that Swanson considers 30-minute windows, which, as argued above, may lead to larger shocks and thus smaller coefficients. Both forward guidance and asset purchase shocks increase spreads, with asset purchases having larger coefficients. This again aligns with Swanson (2017), as well as Krishnamurthy & Vissing-Jorgensen (2011, 2013) and Swanson (2011). The TIPS spread, proxying market expectations of inflation, rises significantly in response to both Fed Funds and forward guidance shocks, signaling looser monetary conditions.

# 5 Low-frequency effects on the macroeconomy

While financial series are available at high frequencies, the macroeconomic aggregates of ultimate importance to central bankers are only available at lower frequencies. As a result, little previous work has examined the real effects of unconventional policy shocks in a unified manner. In this section, I compute the dynamic responses of key macroeconomic variables to unconventional policy shocks. Recall that the shock series is aggregated from end-ofday shock measures, which were shown in Section 3.1 to have considerable estimation error, meaning that the reported effects are likely considerably biased towards zero, and constitute lower bounds.

In particular, I focus my analysis on PCE inflation and real GDP growth. To this point, relatively little work has assessed these impacts, with Baumeister & Bennati (2013), Gambacorta, Hofmann, & Peersman (2014), Lloyd (2018), and Inoue & Rossi (2018) being notable exceptions. However, as discussed in the introduction, none of these papers has separated and simultaneously identified interpretable forward guidance and asset purchase shocks, making my analysis the first of its kind.

I merge my shock measures into a monthly time series with PCE inflation, real GDP growth (based on the Macroeconomic Advisers monthly measure), and the Federal Funds

target rate. This yields a time-series of 144 observations. I compute impulse response functions using local projections of the form

$$x_{m+h} = \mu^h + \pi_0^h \epsilon_m + \sum_{l=1}^6 \pi_l^h \epsilon_{m-l} + \sum_{s=1}^3 \kappa^h X_{m-s} + u_m^h, h = 0, 1, \dots, 12,$$
(9)

controlling for the previous six months' worth of monetary policy shocks and the prior quarter's macro aggregates in  $X_{m-s}$  (inflation, real GDP growth, and the Fed Funds rate). The coefficient of interest is  $\pi_0^h$  – the effect of a month m shock on  $x_{m+h}$ . I consider only horizons up to one year, seeing as, given the limited data, the need for additional leads starts to severely limit the sample, and the imprecision that often affects local projection results is pronounced. I again compute HAC standard errors, and cumulate both responses and standard errors across horizons to obtain cumulative impulse responses.

Figure 4 plots the dynamic response of inflation and real GDP growth to a one standard deviation impulse to each expansionary shock, with 68% and 90% confidence intervals. Inflation displays a small, significant positive response to an expansionary Fed Funds shock on impact, with no significant response thereafter. Forward guidance has no significant impact within the year. On the other hand, a one standard deviation asset purchase shock (one that raises 10-year Treasury yields by 1.8 bp) raises inflation by up to about 10 bp from 8 months onward, statistically significant after the first month or so. Turning to real GDP growth, it appears at first that an expansionary Fed Funds shock significantly *lowers* growth over the first few months; however, this perplexing result is almost entirely driven by the December 2008 observation, and vanishes if this shock is "zeroed out". Again, forward guidance has no discernible effect, except for a marginal increase on impact. However, the asset purchase shock raises GDP growth by 20 bp on impact and about 50 bp at its peak. The response is statistically significant through 2 months and again further along the horizon, depending on the significance level considered.

Figures 12 and 13 in the Supplement compare the responses to those that would be obtained using event study measures instead. These alternative estimates replace the shocks in (9) with simple 30-min changes in the three interest rates or a recursively orthogonalized version of these event study measures. The first imposes strong exclusion restrictions (each rate reacts only to one shock) and the second adopts a common orthogonalization for the short-rate shock and a more arbitrary decomposition between forward guidance and asset purchases. The responses are dramatically different. The simple event study measure finds expansionary effects for Fed Funds shocks and contractionary effects for forward guidance, with ambiguous effects for asset purchases. The recursive version finds near-zero effects for all three shocks. These results demonstrate the value of the decompositions I propose

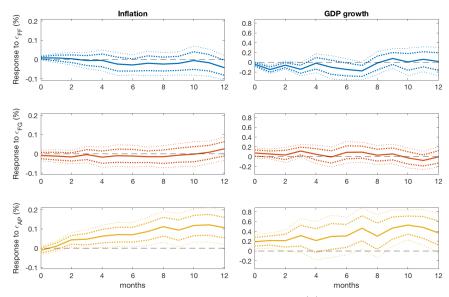


Figure 4: Dynamic response of macroeconomic aggregates

Impulse responses are calculated via local projection as in equation (9) using monthly data and the full sample, January 2007 to December 2018. Responses are cumulated to obtain the dynamic responses. Responses are scaled to a one standard deviation expansionary impulse for each shock. 68% and 90% HAC confidence intervals are calculated following Lazarus, Lewis, & Stock (2019).

compared to two variants of the event study approach, not out of line with existing empirical practice, which cannot as credibly disentangle the dimensions of monetary policy.

In their recent paper, Inoue and Rossi (2018) do not report mean responses for the "unconventional" period, instead plotting responses for selected announcements. They break down the overall effects of monetary policy as responses to their identified slope and curvature shocks. For both output and inflation, they find that the slope factor drives responses, except in 2012, when the influence of the curvature factor increases. The authors argue that the curvature factor can be seen as a forward guidance shock. These findings do not align with my results, which indicate that, over the same period, a single shock (the asset purchase shock) has pronounced economic effects, while the others do not. It is difficult to compare the results further (for example, to examine the impact of the authors' use of a time-invariant decomposition), since their statistically-identified factors do not have clear economic interpretations along the lines of the three dimensions of monetary policy I consider.

Gambacorta, Hofmann, & Peersman (2014) focus on identifying the effects of balance sheet size shocks in a cross-country panel VAR. Their findings indicate significant stimulatory effects for the asset purchase dimension of policy, peaking around six months. The output response is about three times that of inflation, roughly according with my finding of an up to five-times larger response of output. Finally, Gertler & Karadi (2015) find suggestive evidence that forward guidance serves to amplify shocks to the current policy rate. They do so by comparing responses using the front Fed Funds future as an instrument for the Fed Funds rate to their baseline, which uses three-month ahead futures to instrument for the 1-year Treasury yield. However, their sample runs from 1991-2012, so is dominated by observations outside of the ZLB. Thus, their evidence that forward guidance can offer additional stimulus may be compatible with my finding that it did not have a pronounced impact during the Great Recession. Indeed, since they argue that forward guidance may be effective by *augmenting* policy rate shocks, this distinction accords with the fact that the Fed Funds rate was at the ZLB, so policy rate shocks were not forthcoming.

Previous work has additionally examined in detail the effect of unconventional policy shocks on the expectations of professional forecasters; the expectations channel is theoretically important to the transmission of unconventional monetary policy (see e.g., Eggertsson & Woodford (2003), McKay, Nakamura, & Steinsson (2016)). I conduct similar analysis, focused instead on consumer sentiment, but also considering professional expectations, in a companion paper, Lewis, Makridis, & Mertens (2019).

# 6 Conclusion

I use intraday data on interest rate movements to recover high frequency time series of monetary policy shocks on announcement days using announcement-specific decompositions. I identify the decompositions based on time-varying volatility. Because I am able to identify different decompositions for each announcement, I can compare the effects of shocks directly from one announcement to the next. I find that a small handful of notable FOMC announcements of unconventional measures sparked significant monetary policy shocks. In particular, the leading announcements are the launch of forward guidance (March 2009), the introduction of calendar-based guidance (August 2011), the prolonging of forward guidance (March 2015), the launch of QE1 (March 2009), and the decision to delay tapering (September 2013). The fact that these announcements are dominated by the launch or unexpected extension of the policies indicates that the usage of these tools, as opposed to subtle refinements of statement language or adjustments of purchases, is what matters to markets. I additionally find that conclusions based on standard 30-minute changes in asset prices may be unreliable, on some days overstating effects, and on some days understating them.

At high frequency, many announcements, particularly on the asset purchase dimension, raise a proxy for corporate debt returns, and thus lower yields, but the cumulative effects rarely persist by day's end. At the daily frequency, corporate yields also fall significantly with Fed Funds and asset purchase shocks, but spreads rise in response to both forward guidance and asset purchase shocks.

Most importantly, I find important macroeconomic effects. The dynamic responses of both realized inflation and GDP growth display significant responses to asset purchase shocks, but not to Fed Funds or forward guidance shocks. Taken together, these results offer some of the first evidence on the macroeconomic effects of the Federal Reserve's unconventional monetary policy broken down by policy dimension. They suggest that asset purchase policies in particular were effective with regard to a number of policy objectives.

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# A Proofs

**Proposition 1.** If the asset prices underlying  $\eta_d$  is a random walk and  $H_d^{inf}$  is uniquely determined, then  $H_d = H_d^{inf}$ .

*Proof.* If the asset prices underlying  $\eta_d$  follow a random walk, then  $\eta_d = \sum_{s=0}^k \eta_{t-s}$ ; the same holds for  $\epsilon_d$ . This means that

$$\eta_d = \sum_{s=0}^k \eta_{t-s} = \sum_{s=0}^k H_d \epsilon_{t-s} = H_d \sum_{s=0}^k \epsilon_{t-s} = H_d \epsilon_d,$$

so if  $H_d^{inf}$  is unique, then  $H_d = H_d^{inf}$ .

**Proposition 3.** Suppose  $\eta_t = H\epsilon_t + \nu_t$ , where  $\nu_t$  is an  $n \times 1$  vector of noise uncorrelated with  $\epsilon_t$  at all horizons. Provided the volatility of  $\nu_t$  does not exhibit non-zero autocovariance, H is still identified from  $Cov(\bar{\zeta}_t, \bar{\zeta}_{t-p})$ , where  $\bar{\zeta}_t = vech(\bar{\eta}_t \bar{\eta}'_t)$ , provided  $M_p$  has no proportional rows.

*Proof.*  $\bar{\eta}_t \bar{\eta}'_t$  can be rewritten as

$$\bar{\eta}_t \bar{\eta}'_t = \eta_t \eta'_t + \eta_t \nu'_t + \nu_t \eta'_t + \nu_t \nu'_t$$
$$= (H\Sigma_t H' + V_t) + \eta_t \nu'_t + \nu_t \eta'_t + \nu_t \nu'_t,$$

where  $V_t = H\left(\epsilon_t \epsilon_t' - \Sigma_t\right) H'$ . Then

$$\bar{\zeta}_t = (vech (H\Sigma_t H') + vech (V_t)) + vech (\eta_t \nu'_t + \nu_t \eta'_t + \nu_t \nu'_t)$$
  
=  $(L (H \otimes H) vec (\Sigma_t) + v_t) + vech (\eta_t \nu'_t + \nu_t \eta'_t + \nu_t \nu'_t)$   
=  $(L (H \otimes H) G\sigma_t^2 + v_t) + vech (\eta_t \nu'_t + \nu_t \eta'_t + \nu_t \nu'_t),$ 

where  $v_t = vech(V_t)$ . Note that

$$Cov\left(vech\left(\eta_{t}\nu_{t}'+\nu_{t}\eta_{t}'+\nu_{t}\nu_{t}'\right), vech\left(\eta_{t-p}\nu_{t-p}'+\nu_{t-p}\eta_{t-p}'+\nu_{t-p}\nu_{t-p}'\right)\right) = 0,$$

since neither  $\eta_t$  nor  $\nu_t$  is serially correlated, and  $\nu_t \nu'_t$  has zero autocovariance by assumption. Similarly,

$$Cov\left(L\left(H\otimes H\right)G\sigma_{t}^{2}+v_{t}, vech\left(\eta_{t-p}\nu_{t-p}'+\nu_{t-p}\eta_{t-p}'+\nu_{t-p}\nu_{t-p}'\right)\right)=0,$$

and the same holds with the lag reversed. Thus, all  $\nu_t$  terms drop out of  $Cov(\bar{\zeta}_t, \bar{\zeta}_{t-p})$ , so

 $Cov(\bar{\zeta}_t, \bar{\zeta}_{t-p}) = Cov(\zeta_t, \zeta_{t-p}).$  The identification result then follows directly from Theorem 1 of Lewis (2019a).

**Proposition 4.** Suppose that for news shocks during non-announcement periods,  $\eta_t = H_N \epsilon_t$ , and for monetary policy shocks following monetary policy announcements,  $\eta_t = H_{MP} \epsilon_t$ , with  $H_N \neq H_{MP}$ ; assume that within each period, the  $\sigma_t^2$  process is stationary with respective means  $\sigma_N^2, \sigma_{MP}^2$ . Then if  $\frac{\sigma_{i,N}^2}{\min_j \sigma_{j,MP}^2} \rightarrow 0$ , for all i = 1, ..., n, the H identified by Theorem 1 from full-sample moments is  $H_{MP}$ , provided the monetary policy shocks are not a measure zero share of all shocks.

*Proof.* Define  $W_{MP} \in (0, 1]$  as the share of time periods corresponding to monetary policy shocks and  $\bar{\sigma}^2 \equiv \min_j \sigma_{j,MP}^2$ . Without loss of generality I now work with the re-scaled  $\zeta_t/\bar{\sigma}^2$  as the "data". Then

$$E\left[\zeta_t/\bar{\sigma}^2\right] = vech\left(\left(1 - W_{MP}\right)H_N\frac{\Sigma_N}{\bar{\sigma}^2}H'_N + W_{MP}H_{MP}\frac{\Sigma_{MP}}{\bar{\sigma}^2}H'_{MP}\right)$$

Observe that, as  $\frac{\sigma_{i,N}^2}{\min_j \sigma_{j,MP}^2} \to 0 \ \forall i = 1, \dots, n$ ,

$$E\left[\zeta_t/\bar{\sigma}^2\right] \to \frac{W_{MP}}{\bar{\sigma}^2}H_{MP}\Sigma_{MP}H'_{MP},$$

since  $\sigma_N^2/\bar{\sigma}^2 \to 0$ .

Turning now to  $Cov(\zeta_t, \zeta_{t-p})$ , for t and/or t-p not in the monetary policy shock period,

$$E\left(\frac{\zeta_t}{\bar{\sigma}^2}\frac{\zeta'_{t-p}}{\bar{\sigma}^2} \mid t \lor t - p \in N\right)$$
  
=  $L\left(H_N \otimes H_N\right) G \frac{E\left[\sigma_t^2 vec\left(\varepsilon_{t-p}\varepsilon'_{t-p}\right) \mid t \lor t - p \in N\right]}{\bar{\sigma}^2 \times \bar{\sigma}^2} \left(H_N \otimes H_N\right)' L'$   
 $\rightarrow L\left(H_N \otimes H_N\right) G \times 0 \times \left(H_N \otimes H_N\right)' L' = 0,$ 

since  $\sigma_t^2/\bar{\sigma}^2 \to 0$  and/or  $(\varepsilon_{t-p}\varepsilon'_{t-p}) \to 0$ ,  $E\left[\sigma_t^2 vec\left(\varepsilon_{t-p}\varepsilon'_{t-p}\right)\right] \to 0$ . On the other hand, for both t, t-p in the monetary policy shock period,

$$E\left(\frac{\zeta_t}{\bar{\sigma}^2}\frac{\zeta'_{t-p}}{\bar{\sigma}^2} \mid t \wedge t - p \in MP\right)$$
  
=  $L\left(H_{MP} \otimes H_{MP}\right)G\frac{E\left[\sigma_t^2 vec\left(\varepsilon_{t-p}\varepsilon'_{t-p}\right) \mid t \wedge t - p \in MP\right]}{\bar{\sigma}^2 \times \bar{\sigma}^2}\left(H_{MP} \otimes H_{MP}\right)'L'.$ 

Then since  $\sigma_{i,MP}^2 \ge \bar{\sigma}^2 \ \forall i$ ,

$$E\left(\frac{\zeta_t}{\bar{\sigma}^2}\frac{\zeta'_{t-p}}{\bar{\sigma}^2}\right) \to L\left(H_{MP}\otimes H_{MP}\right)G\frac{W_{MP}}{\bar{\sigma}^4}E\left[\sigma_t^2 vec\left(\varepsilon_{t-p}\varepsilon'_{t-p}\right) \mid t\wedge t-p\in MP\right]\left(H_{MP}\otimes H_{MP}\right)'L'.$$

Further, since  $E[\zeta_t/\bar{\sigma}^2] \rightarrow vech(H_{MP}W_{MP}/\bar{\sigma}^2\Sigma_{MP}H_{MP})$ ,

$$E\left[\frac{\zeta_t}{\bar{\sigma}^2}\right] E\left[\frac{\zeta_t}{\bar{\sigma}^2}\right]' \to L\left(H_{MP} \otimes H_{MP}\right) G\frac{W_{MP}^2}{\bar{\sigma}^4} \sigma_{MP}^2 \sigma_{MP}^{2'} G'\left(H_{MP} \otimes H_{MP}\right)' L'.$$

Thus,  $Cov(\zeta_t, \zeta_{t-p}) = L(H_{MP} \otimes H_{MP}) GM_{p,MP} (H_{MP} \otimes H_{MP})' L'$ , where

$$M_{p,MP} = \frac{W_{MP}}{\bar{\sigma}^4} E\left[\sigma_t^2 vec\left(\varepsilon_{t-p}\varepsilon_{t-p}'\right) \mid t \wedge t - p \in MP\right] - \frac{W_{MP}^2}{\bar{\sigma}^4}\sigma_{MP}^2\sigma_{MP}^{2'}G'.$$

This identifying equation, along with  $E\left[\frac{\zeta_t}{\bar{\sigma}^2}\right]$ , has the exact same form as in Theorem 1. Thus, the result guaranteeing a unique solution holds, with the conditions stated there for  $\tilde{M}_p$  being applied instead to  $\tilde{M}_{p,MP} = \left[ M_{p,MP} \quad \frac{W_{MP}}{\bar{\sigma}^2} \sigma_{MP}^2 \right]$ .

# Supplemental Materials

Supplemental materials can be found on my personal website, here.