GHH Preferences on Households’ Portfolio Choices: Theoretical Implications and Empirical Evidence

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Abstract

This paper explores theoretical implications and empirical evidence of GHH preferences [Greenwood et al. (1988)] over portfolio choices. First, we analytically solve a parsimonious life-cycle portfolio choice model with the GHH preferences and endogenous labor-leisure choice. Second, our analytical solution identifies four effects due to the GHH preferences (through endogenous labor-leisure choices) on risky shares; and it shows that two net effects hinge on the value of one key structural parameter. Third, we empirically test main theoretical predictions with the Panel Study of Income Dynamics data. Overall, the estimation results provide empirical evidence in support of certain key implications out of GHH preferences. Thus, our analysis sheds light on one of the most fundamental, yet highly contentious, questions in quantitative macroeconomic analysis: the choice of utility functions in a representative agent model when portfolio choices are of concern.

Keywords: Macro Model Assumption Testing; GHH preferences; Portfolio choice; Labor Income; Household Level Data.

JEL classification: D91; E21; G11.

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1 Introduction

Greenwood et al. (1988) first introduces GHH preferences into their business cycle model to quantitatively analyze the impact of shocks to the marginal efficiency of investment on business cycles. Since GHH preferences neutralize the wealth effect on labor supply, labor effort is determined independent of the intertemporal consumption and saving choices. As a result, Greenwood et al. (1988)’s model generates co-movements among consumption, labor effort, and labor productivity, when responding to the shocks to the marginal efficiency of investment, which are generally supported by empirical evidence. In contrast, other models fail to do so, where the intertemporal consumption and saving choices affect labor supply [see Barro and King (1984)].

GHH preferences have then been widely adopted in models to study various economic issues and these applications have been overwhelmingly successful. For example, models with GHH preferences have effectively explained key business cycles and equity premia in emerging economies [see Mendoza (1991), Garcia-Cicco et al. (2010), and Jahan-Parvar et al. (2013), among others]. Nevertheless, the empirical relevancy of GHH preferences with respect to household level data is still an open question.

To fill this gap in the literature, we explore both the theoretical implications and empirical evidence of GHH preferences on modeling households’ portfolio choices. To derive the theoretical implications on risky shares and financial wealth, we build a discrete-time portfolio choice model with endogenous labor-leisure choices and GHH preferences. To examine empirical evidence, we conduct extensive tests on the key theoretical predictions of GHH preferences concerning portfolio choices with household-level data, the Panel Study of Income Dynamics (hereafter PSID) data.

We aim to make contributions to the literature in four dimensions. First we derive a closed-form solution to models with risky shares; and thus substantially extend the existing portfolio choice problems where exact solutions may be found. In general it is hard to obtain closed-form solutions in portfolio choice problems, while analytical solutions allows for robust comparative static analysis in a way that numerical solutions cannot achieve [see Henderson (2005)]. There are a few papers that provide closed-form solutions to portfolio choice problems. For example, Bodie et al. (2004) derives a closed-form solution to the optimal portfolio in a life-cycle model with habit formation preferences, stochastic opportunity set, stochastic wages, and labor supply flexibility. Henderson (2005) derives closed-form solutions to optimal portfolios in a partial equilibrium model with constant absolute risk aversion preferences. Brunnermeier and Nagel (2008) obtains a closed-form solution to risky shares in a discrete time portfolio model with a fixed external habit. Liu et al. (2016) derives closed-form solutions to risky shares in a discrete time portfolio model with time-varying
habits and time-varying labor income. In this paper, we obtain the closed-form solution to risk shares in a discrete time portfolio choice model with GHH preferences and endogenous labor-leisure choice under simplified assumptions.

Second, we analyze the theoretical implications of GHH preferences on portfolio choices and provide new insights about how households make their portfolio choice decisions. Specifically, our first theoretical finding is as follows. With the analytical solution from a parsimonious model, we show how GHH preferences (through endogenous labor-leisure choices) affect risky shares both directly and indirectly, which adds an important new result in the literature. On the one hand, labor income introduces two effects on the risky shares: the insurance effect and the relaxation effect. With labor income, households are more aggressive in their portfolio choices than in a situation without labor income or with less labor income, and households become less aggressive in adjusting their risky shares in response to wealth accumulations due to the labor income. On the other hand, leisure introduces two opposite effects on risky shares. With leisure (thus no labor income), households are less aggressive in their portfolio choices than the case without leisure; and because of missing labor income, households become more aggressive in adjusting their risky shares in response to wealth accumulations.

We label them as four distinct effects. The first effect is the direct labor income insurance effect on risky shares, an effect that has been initially and formally defined in Bodie et al. (1992). The second effect is the indirect labor income relaxation effect on the relationship between risky shares and wealth, an effect discussed in Liu et al. (2016). The third effect is the direct leisure de-insurance effect, which is opposite to the labor income insurance effect, and also a new addition the literature. The last effect is the indirect leisure aggression effect on the relationship between risky shares and wealth, an effect that is analogous to the one discussed in Stiglitz (1969).

Our second highlighted theoretical finding is that we show both the net effect of GHH preferences (through endogenous labor-leisure choices) on risky shares and their net effect on the relationship between risky shares and wealth are driven by one key structural parameter: the sensitivity of labor input to real wage rates, a parameter uniquely determining the wage elasticity of labor supply. This is another new result in the literature.

The above theoretical findings may help our understanding of two key questions in the portfolio choice literature. The first question is how labor income affects households’ portfolio decisions with respect to their risk preferences, i.e., risky shares. This is a classical research question and has been extensively discussed

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1Liu et al. (2016) derives this effect with habit formation preferences.
in the literature. For example, Bodie et al. (1992) show that the ability to vary labor supply ex post induces an individual to assume greater risks in her investment portfolio ex ante. The second question is how risky shares respond to wealth accumulations, which is another fundamental question. For instance, Brunnermeier and Nagel (2008) show that if households have habit formation preferences (thus decreasing relative risk aversion), risky shares will increase with wealth of conventional form. Liu et al. (2016) show that risky shares will increase, due to decreasing relative risk aversion, with wealth of conventional form only if there are no large negative income shocks.

Third, we make three empirical contributions. First, we contribute to the literature by providing robust empirical evidence in support of GHH preferences about how wealth accumulations affect portfolio choices. We find (robust) negative and statistically significant responses of risky shares to wealth accumulations, which follows the theoretical prediction of GHH preferences under the standard range of the key parameter. This finding provides very strong empirical support of GHH preferences. Second, we contribute the literature by showing the insufficiency of GHH preferences about how labor income affects portfolio choices. In particular, we document overall statistically insignificant responses of risky shares to labor income. Even though they are in line with the literature [see Guiso et al. (1996), Heaton and Lucas (2000), among others], they are not in line with the corresponding theoretical prediction out of our representative agent model with GHH preferences. These two empirical results are quite robust. Our third contribution here is that we document empirical evidence about how labor income risk affects portfolio choices, which are different from the existing literature.

In addition, our empirical results contribute to an important branch of economics literature: testing the theoretical prediction of a utility function with household-level data. For example, the empirical relevance of habit formation preferences has been tested recently with mixed results. The key of these tests is to check whether the theoretical predictions associated with the habit formation preferences are supported by the household-level data. In particular, both Brunnermeier and Nagel (2008) and Liu et al. (2016) derive theoretical predictions about how portfolio choices will respond to the change of wealth and test those predictions with the PSID data. Brunnermeier and Nagel (2008) reject the unconditional theoretical prediction; and Liu et al. (2016) find strong evidence of conditional theoretical prediction given that households are heterogeneous in their income.

Finally, the findings in this paper may help improve our understanding about the choices of utility functions in macroeconomic models, a key assumption that has to be made in every quantitative macroeconomic model. Given that our theoretical model assumes that households are homogeneous and our empirical results
support the range of values of the wage elasticity of labor supply, our analysis reaffirms the validity of a model assuming representative household with GHH preferences [see Mendoza (1991), Garcia-Cicco et al. (2010), and Jahan-Parvar et al. (2013), among others], especially when the impact of wealth accumulations on portfolio choices is of a major concern.

The rest of the paper is organized as follows. Section 2 lays out the theoretical model and obtains the analytical solution. Section 3 provides theoretical discussions and testable hypotheses. The main empirical analyses are conducted in Section 4. In Section 5 we discuss how our findings shed light to the key assumption of utility functions. Our main conclusions are provided in Section 6.

2 The Theoretical Model

2.1 Model Setup

A representative household lives infinitely. The representative household decides consumption, labor effort, and portfolio choices (by investing two assets, a risky asset and a risk-free asset) in each period (a period corresponds to a year in the empirical exercise). Formally, the household chooses consumption, \( C_t \), labor input, \( H_t \), the risky asset holding position, \( S_{t+1} \), and risk-free asset holding position, \( B_t \), to maximize its life-time utility:

\[
U = \mathbb{E} \sum_{t=0}^{\infty} \delta^t \frac{(C_t - H_t^\omega/\omega)^{1-\sigma}}{1-\sigma},
\]

subject to the period budget constraint

\[
C_t + B_t + S_{t+1} = (1 + R_t)S_t + (1 + R_f)B_{t-1} + z_t H_t.
\]

Here \( \mathbb{E} \) denotes the unconditional expectation operator, \( \delta \) denotes the subjective discount factor, \( R_t \) denote the rate of return of holding the risky asset from period \( t-1 \) to period \( t \), \( R_f \) denotes the risk free rate, \( z_t \) denotes total factor productivity, and \( z_t H_t \) denotes the wage income of the household.

2.2 Evolution of Wealth

For our purpose of obtaining the analytical solution, it is important to know how wealth evolves in this model:

- In the beginning of period \( t \), the household inherits wealth, \( W_t \).
- Within period \( t \), the household makes decisions about consumption and labor input. In particular, the household spends \( C_t \) on consumption and receives labor income \( Y_t = z_t H_t \).
• At the end of period $t$, the household’s wealth portfolio is given by $W_t - C_t + Y_t$ and the household determines the fraction of this wealth portfolio, $\alpha_t = \frac{S_{t+1}}{S_t + B_t}$, that is invested in the risky asset.

• The rate of return to this portfolio, $R_{p,t+1}$, is given by $R_{p,t+1} = \alpha_t (R_{t+1} - R_f) + R_f$.

• Thus, we have

$$W_{t+1} = (1 + R_{p,t+1}) (W_t - C_t + Y_t), \tag{2.1}$$

In other words, wealth in the beginning of period $t + 1$, $W_{t+1}$, is the product of the gross rate of return, $1 + R_{p,t+1}$, and wealth at the end of period $t$, $W_t - C_t + Y_t$.

2.3 Closed-Form Solution to Labor Income

We take two parts to derive the solution to $\alpha_t$. In this section, we derive the solution to the equilibrium labor input in the model. It is straightforward to show that in the equilibrium, we have

$$H_t = z_t^{\frac{1}{\omega - 1}},$$

$$Y_t = z_t H_t = z_t^{\frac{1}{\omega - 1}} = H_t^{\omega - 1}.$$

It is clear that both $H_t$ and $Y_t$ are solely determined by $z_t$. As a result, we can decompose the above question into two sub-questions. The first sub-question is to determine $H_t$ and $Y_t$ in the equilibrium, as we have shown in the above. The second sub-question is to determine $\alpha_t$ for the given $H_t$ and $Y_t$ in the equilibrium, i.e., to solve a portfolio choice question for the give $H_t$ and $Y_t$, as what we do in the next.

2.4 Closed-Form Solution under Simplified Assumptions

Define $X_t = H_t^{\omega} / \omega - Y_t = \frac{1-\omega}{\omega} Y_t = \frac{(1-\omega)}{\omega} z_t^{\frac{1}{\omega - 1}}$. To obtain the analytical solution to $\alpha_t$, we impose the following two conditions:

1. We assume that $X_t$ follows an AR(1) process such as

$$X_t - X = \kappa (X_{t-1} - X),$$

where $\kappa$ is a constant. Throughout the paper, a variable without the time-subscript denotes the mean of this variable; for example, $X$ denotes the mean of $X_t$. One justification for this AR(1) process assumption is as follows. The literature typically assumes that the logarithm of $z_t$ follows an AR(1) process; thus, up to the first order approximation, $X_t$ will also follow an AR(1) process since $X_t = \frac{(1-\omega)}{\omega} z_t^{\frac{1}{\omega - 1}}$. 
2. We assume that the expected return and the standard deviation are constant, an assumption that is also imposed in Samuelson (1969).

Under these two conditions, the solution to $\alpha_t$ is given by:

$$\alpha^*_t = \alpha^S \left[ 1 - \frac{X}{(W_t - C_t + Y_t - \frac{X_t - X}{Z + R_f}) R_f} \right] \left[ 1 - \frac{X_t - X}{(W_t - C_t + Y_t)(Z + R_f)} \right].$$

(2.2)

Here $X = H^\omega / \omega - Y = \frac{1 - \omega}{\omega} (1 + X_t) Y_t$; $\tilde{C}_t = C_t - \left( \frac{Y_t}{\omega} \right) = C_t - Y_t$; and $Z = (1 + R_f) / \kappa - (1 + R_f)$. Besides, $\alpha^S$ is the solution to $\alpha$ in the Samuelson (1969) and its value is very close to 1. Next is the derivation of Eq. (2.2).

We take 3 steps to obtain a closed-form solution to $\alpha_t$ (not $C_t$) with two assumptions mentioned in the above. One thing worth mentioning is that we follow the same logic as in Liu et al. (2016) to partially solve the portfolio choice problem here, i.e., to obtain a closed-form solution to $\alpha_t$. The key (of using this approach to partially solve the portfolio choice problem) is to manipulate the objective function and wealth in the beginning of period $t + 1$; to transform the present portfolio choice problem into one that has the analytical solution; and then to back up the solution to $\alpha_t$ in the original portfolio choice problem accordingly.

1. Let $W_t$ denotes the wealth portfolio in the beginning of period $t$. The wealth portfolio at the end of period $t$ is given by

$$W_t + Y_t - C_t.$$

Suppose that the household invests a fraction $\alpha_t$ of this period-end wealth portfolio in the risky assets and the rest in the risk-free asset; and the return rate to this wealth portfolio, $R_{p,t+1}$, is:

$$R_{p,t+1} = \alpha_t (R_{t+1} - R_f) + R_f.$$

(a) The portfolio choice problem is to choose $C_t$ and $\alpha_t$ to maximize

$$U = \mathbb{E} \sum_{t=0}^{\infty} \delta^t \frac{(C_t - Y_t/\omega)^{1-\sigma}}{1-\sigma},$$

subject to the law of motion of wealth portfolio:

$$W_{t+1} = (1 + R_{p,t+1}) (W_t + Y_t - C_t),$$

where $W_{t+1}$ denotes the wealth portfolio at the beginning of period $t + 1$. Here we have used $Y_t = H^\omega_t$.

2. Define $\tilde{C}_t = C_t - Y_t$. 

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(a) The portfolio choice problem becomes to choose $\tilde{C}_t$ and $\alpha_t$ to maximize

$$U = \mathbb{E} \sum_{t=0}^{\infty} \delta^t \left( \frac{C_t - Y_t - \frac{(1-\omega)Y_t}{1-\sigma}}{1-\sigma} \right) \left( \frac{\tilde{C}_t - (1-\omega)Y_t}{1-\sigma} \right),$$

subject to the law of motion of wealth portfolio:

$$W_{t+1} = (1 + R_{p,t+1}) \left( W_t - \tilde{C}_t \right).$$

3. Define $X_t = \frac{(1-\omega)Y_t}{\omega}$.

(a) The portfolio choice problem becomes to choose $\tilde{C}_t$ and $\alpha_t$ to maximize

$$U = \mathbb{E} \sum_{t=0}^{\infty} \delta^t \left( \frac{\tilde{C}_t - X_t}{1-\sigma} \right),$$

subject to the law of motion of wealth portfolio:

$$W_{t+1} = (1 + R_{p,t+1}) \left( W_t - \tilde{C}_t \right).$$

(b) This portfolio choice problem is exactly the same portfolio choice problem as the model discussed in Liu et al. (2016) (see pages 244-245). Thus, if $X_t$ follows an AR(1) process such as

$$X_t - X = \kappa \left( X_{t-1} - X \right),$$

where $\kappa$ is a constant, and if expected return and the standard deviation are constant, the solution to $\alpha_t$ will be:

$$\alpha_t^* = \alpha^S \left[ 1 - \frac{X}{(W_t - \tilde{C}_t - X/W) R_f} \right] \left[ 1 - \frac{X_t - X}{(W_t - \tilde{C}_t) (Z + R_f)} \right]. \quad \text{(2.3)}$$

where $\alpha^S$ is the solution to risky shares in Samuelson (1969) and $Z = (1 + R_f)/\kappa - (1 + R_f)$. Replace $\tilde{C}_t$ with $C_t - Y_t$ in Eq. (2.3) to obtain Eq. (2.2).

Eq. (2.2) enables us to discuss how the risky shares respond to post consumption wealth and how the GHH preferences (through endogenous labor-leisure choices) affect the portfolio choices.

3 Theoretical Discussions

With the closed-form solution, Eq. (2.2), we discuss the intuitions behind the impact of GHH preferences (through endogenous labor-leisure choices) on risky shares, theoretical implications of GHH preferences on
risky shares, and our contributions to the literature in this section.

3.1 Intuitions: Four Partial Equilibrium Effects

We present the intuitions by discussing the simplest case in which the total factor productivity is a constant, $z_t \equiv z$. In other words, we present the intuition by discussing the static relationships among variables of interest. In this simplest case, we have $X_t \equiv X = \frac{1}{z} (H)^{\omega} - Y$ and $\alpha_t^*$ is simplified as

$$
\alpha^* = \alpha^S \left[ 1 + \frac{Y}{(W - C + Y) R_f} - \frac{\frac{1}{z} (H)^{\omega}}{(W - C + Y) R_f} \right]. 
$$

(3.1)

Note that we have $Y = z^{\frac{1}{1-\omega}}$ and $H = z^{\frac{1}{1-\omega}}$ in the equilibrium. With Eq. (3.1), we discuss four partial equilibrium effects of GHH preferences (through endogenous labor-leisure choices) on risky shares. We use the term “partial equilibrium” because we do hold the ceteris paribus assumption when we discuss each of these four effects.

3.1.1 The Impact of Labor Income on Risky Shares

To see the impact of labor income on risky shares, we shut down the leisure channel, i.e., letting $H = 0$ in Eq. (3.1), and obtain the simplified $\alpha^*$ in this case as:

$$
\alpha_t^* = \alpha^S \left[ 1 + \frac{Y}{(W - C + Y) R_f} \right].
$$

(3.2)

From Eq. (3.2), labor income introduces two effects on risky shares. The first effect is the direct labor income insurance effect discussed in Bodie et al. (1992) and it is the direct effect of labor income on risky shares. According to Bodie et al. (1992), labor income provides an insurance to the households against adverse investment outcomes; and thus households will become more aggressive in their portfolio choices with a higher labor income for the given financial wealth. For example, this effect means that the ability to have labor income induces the individual to assume greater risks in her investment portfolio (for the given financial wealth), i.e., $\alpha_t^* > \alpha^S$ because $Y > 0$ (note that $\alpha^S$ is associated with the case in which labor income is zero). For another example, this effect implies that higher labor income may induce an individual to assume greater risks in her investment portfolio (for the given financial wealth), i.e., $\frac{\partial \alpha_t^*}{\partial Y} > 0$ for the given financial wealth. Bodie et al. (1992) argues that this effect helps explain “why the young (with greater labor flexibility over their working lives) may assume greater investment risks than the old.”

The second effect is the effect of labor income on the relationship between risky shares and financial
wealth and it is an indirect effect. In particular, with the given positive labor income, households will actually become less aggressive (more relaxing) in their portfolio choices, i.e., decrease their risky shares, when their financial wealth accumulates. Mathematically, this effect implies that \( \partial \alpha^*_t / \partial (W - C) < 0 \) when \( Y > 0 \). The intuition behind this effect is provided as follows. Everything else being equal, the occurrence of constant labor income will automatically increase the size of risky risk-free assets and lower risky shares before the household re-balance their portfolio. This is the same effect through the income channel as discussed in Liu et al. (2016). We define this effect as the indirect labor income relaxation effect. Nevertheless, Farhi and Panageas (2007) shows cases in which an agent may increase her risky shares as she accumulates assets when her income and the investment opportunity set are constant.

### 3.1.2 The Impact of Leisure on Risky Shares

To see the impact of leisure, the difference between the time endowment of households and \( H \), on risky shares, we shut down the labor income channel, i.e., letting \( Y = 0 \) in Eq. (3.1). As a result, we obtain

\[
\alpha^*_H = \alpha S \left[ 1 - \frac{(H)^\omega / \omega}{(W - C) R_f} \right].
\]

(3.3)

From Eq. (3.3), leisure also introduces two distinct effects on risky shares, which are qualitatively opposite to the two effects due to labor income. The first effect is the direct leisure de-insurance effect. When households enjoy leisure time, they will lose labor income and they will become less aggressive in their portfolio choices. In other words, the ability to have leisure induces the individual to assume less risks in her investment portfolio. Mathematically, we have \( \partial \alpha^*_H / \partial H < 0 \). This result is intuitive as labor and leisure are substitute to each other.

The second effect is that of leisure on the relationship between risky shares and financial wealth and it is an indirect effect. With leisure (thus less labor income), households will become more aggressive (less relaxing) in their portfolio choices, i.e., increase their risky shares, when their financial wealth accumulates. Mathematically, this effect means that \( \partial \alpha^*_{t,H} / \partial (W - C) > 0 \) for the given leisure. This effect is actually quite general. Stiglitz (1969) shows that an important implication of nonhomothetic utility is decreasing relative risk aversion. GHH preferences are nonhomothetic. Thus it is not surprising that households with GHH preferences will increase their risky shares when their financial wealth accumulates. This effect is similar to the effect that is due to the existence of habit as discussed in both Brunnermeier and Nagel (2008) and Liu et al. (2016). We label this indirect effect as the indirect leisure aggression effect.
3.1.3 Time Varying $z_t$

When $z_t$ becomes time-varying, the situation will become much more complicated. All the net effects discussed in Section 3.2 will be modified by the second term in Eq. (2.2), $1 - \frac{X_t - X}{(W_t - C_t)(Z + R_f)}$. However, given that $X_t = H_t/\omega - Y_t = \frac{(1-\omega)}{\omega} z^{-\frac{1}{\omega}}$, $X = H/\omega - Y = \frac{1}{\omega} z^{-\frac{1}{\omega}} - z^{-\frac{1}{\omega}}$, the term $X_t - X$ may be regarded as a difference-in-difference term and its impact on $\alpha^*_t$ is likely to be of the third-order magnificence. This is especially true since we focus on the mean of those variables of interest. As a result, we ignore the impact of $X_t - X$ in our theoretical discussions.

3.2 Theoretical Predictions: Two General Equilibrium Effects

In Section 3.1, we have shown four partial equilibrium effects (thus theoretical predictions) of GHH preferences (through endogenous labor-leisure choices), two direct effects on risky shares and two indirect effects on the relationship between risky shares and financial wealth. In this section, we discuss two general equilibrium effects, i.e., the net effects, of GHH preferences (through endogenous labor-leisure choices) on risky shares. We use the term “general equilibrium” here because we have used $Y = zH$, which holds in the equilibrium, in the discussion of theoretical predictions. As a result, we can derive the general equilibrium effect, i.e., the net effect, of labor income on $\alpha^*$ using the two direct effects and the general equilibrium effect, i.e., the net effect of labor income on the relationship between financial wealth and $\alpha^*$ using the two indirect effects.

3.2.1 Net Effect of Labor Income on Risky Shares

The direct labor income insurance effect implies that for the given positive financial wealth, households will increase their risky shares when their labor income becomes higher, i.e., $\partial \alpha^*_Y/\partial Y > 0$. The direct leisure de-insurance effect implies that for the given positive financial wealth, households will increase their risky shares when their labor effort (thus labor income) becomes higher, i.e., $\partial \alpha^*_H/\partial H < 0$. Since $Y = zH$, the net effect of labor income on risky shares depends on the size of two direct effects and the weights associated with each direct effect. Formally, we have

$$\alpha^* = \alpha^* \left[ 1 + \frac{Y}{(W - C + Y) R_f} \right] = \alpha^* \left[ 1 + \frac{(1 - \frac{1}{\omega}) Y}{(W - C + Y) R_f} \right]. \quad (3.4)$$

In the second equalization step, we have used the solution, $Y = z^{-\frac{1}{\omega}}$ and $H = z^{-\frac{1}{\omega-1}}$, in the equilibrium. Eq. (3.4) implies the following

$$\frac{\partial \alpha^*}{\partial Y} = \left( 1 - \frac{1}{\omega} \right) \times Q. \quad (3.5)$$
where \( Q = \frac{\alpha^S(W-C)}{(W-C+Y)^2R_f} \) and is positive. Eq. (3.5) implies that whether \( \frac{\partial \alpha^*}{\partial Y} \) is positive, zero, or negative, crucially depending on the value of \( \omega \). From Eq. (3.5), when labor income, \( Y \), increases, risky shares, \( \alpha^* \), will increase if \( \omega > 1 \); will decrease if \( \omega < 1 \); and will not change if \( \omega = 1 \). This is a new result in the literature. One contribution here is we present the net effect of labor income on risky shares in such a simple, clean and net way. Another nice contribution of our work is that we show the net effect crucially hinges on the size of this key structural parameter \( \omega \).

### 3.2.2 Net Effect of Labor Income on the Relationship between Financial Wealth on Risky Shares

The indirect labor income relaxation effect implies that for the given positive financial wealth, households will increase their risky shares when their labor income becomes higher, i.e., \( \frac{\partial \alpha^*}{\partial Y} > 0 \). The indirect leisure aggression effect implies that for the given positive financial wealth, households will increase their risky shares when their labor effort (thus labor income) becomes higher, i.e., \( \frac{\partial \alpha^*}{\partial H} < 0 \). Again, since \( Y = zH \), the net effect of financial wealth on risky shares depends on the size of two indirect effects and the weights associated with each indirect effect. From Eq. (3.4), we have the following

\[
\frac{\partial \alpha^*}{\partial (W-C)} = - \left( 1 - \frac{1}{\omega} \right) \times P_t.
\]  

where \( P = \frac{\alpha^S Y}{(W-C+Y)^2R_f} \) and it is positive. Eq. (3.6) implies that whether \( \frac{\partial \alpha^*}{\partial (W-C)} \) is positive, zero, or negative crucially depends on the value of \( \omega \). From Eq. (3.6), when post-consumption financial wealth, \( (W-C) \), increases, risky shares, \( \alpha^* \), will decrease if \( \omega > 1 \); will increase if \( \omega < 1 \); and will not change if \( \omega = 1 \). This is a new result in the literature and its significance is provided as following: (1) the net effect of labor income on the relationship between risky shares and financial wealth is presented in a simple, yet effective, way; and (2) we show the net effect crucially relies on the magnitude of the key structural parameter \( \omega \).

### 3.3 Contributions to the Literature

In Section 3.1, we presents four partial equilibrium effects. The direct labor income insurance effect has also been discussed in Bodie et al. (1992) and by others. The direct leisure de-insurance effect is a new addition but may be regarded as a straightforward extension of the direct labor income insurance effect. The indirect labor income relaxation effect was first discussed in Liu et al. (2016). The indirect leisure aggression effect is a new and important result in the literature. In addition to the last partial equilibrium effect, we note that (1) all four partial equilibrium effects are derived and presented in a simple but effective fashion; and
we show all these four effects in a very parsimonious model with GHH preferences.

In Section 3.2, we present two general equilibrium effects, including the net effect of the labor income on risky shares, and the net effect of the labor income on the relationship between risky shares and financial wealth. We also illustrate that the net effects may be produced in such a parsimonious model; and we show the net effects crucially hinge on the key structural parameter $\omega$. More importantly, our findings regarding these two net effects introduce new insights to the understanding of two classical questions: how labor income affects risky shares and how financial wealth affects risky shares.

The first classical question is how labor income affects optimal portfolio choices, in particular, risky shares. It has been extensively studied [see Bodie et al. (1992), Danthine and Donaldson (2002), Bodie et al. (2004), Munk and Sorensen (2010), among many others]. For example, Bodie et al. (1992) shows the labor income insurance effect; Henderson (2005) studies the optimal portfolio choice problem of an investor with negative exponential utility and facing imperfectly hedgeable stochastic income; Franke et al. (2011) studies how uncertain labor income affects optimal portfolio choice. The consensus is that the inclusion of labor income has dramatically effects on optimal portfolio choices in theoretical models; and the exact way about how labor income changes optimal portfolio choices depends on many factors. Our finding of the first net effect shows whether risky shares will increase with wealth depends on the key structural parameter, $\omega$, with a closed-form solution to risky shares in a parsimonious model with GHH preferences. Our result holds even if labor income is constant, i.e., no labor income risks at all.

The second classical question is how financial wealth affects risky shares. The conventional wisdom indicates that households with constant relative risk aversion, risky shares will not change as financial wealth ($W_t - C_t$) increases [see Samuelson (1969)]; and if households have decreasing relative risk aversion, risky shares will increase with wealth [see Brunnermeier and Nagel (2008)]. The modified version as discussed in Liu et al. (2016) suggests that, risky shares will increase, due to decreasing relative risk aversion, with wealth ($W_t - C_t$) only if there are no large negative income shocks. Our finding of the second net effect is in sharp contrast to the existing convention and thus could be an important contribution to the literature. In particular, we show with GHH preferences that risky shares may decrease with financial wealth, even though at the presence of decreasing relative risk aversion and even if there are no large negative income shocks.

Our findings provide alternative mechanisms for various “puzzles” identified in the literature. For example, Farhi and Panageas (2007) finds that “(c)ontrary to common intuition, an investor might find it optimal to increase the proportion of financial wealth held in stocks as she ages and accumulates assets, even when
her income and the investment opportunity set are constant.” Based on our findings, this outcome could be likely caused by the fact that households have GHH preferences; and for details please see Section 3.2.1. For another example, Wachter and Yogo (2010) shows that “(f)or a given household, the portfolio share can fall in response to an increase in wealth, even though the model implies decreasing relative risk aversion.” An reasonable explanation based on our findings might also be the fact that household has GHH preferences. Further details are provided in Section 3.2.2.

4 Empirical Analysis

In this section, we present data, testable hypotheses, regression specifications, econometric strategies, and regression results.

4.1 Data

PSID is a national study of socioeconomics over lifetimes and across generations. The study began in 1968 with a nationally representative sample of over 18,000 individuals living in 5,000 families in the United States. The data cover many aspects of households, such as employment, income, wealth, expenditure, etc. The households’ asset holdings are not measured every year. Instead they are measured in years 1984, 1989, 1994, 1999, 2001, 2003, 2005, 2007, 2009, 2011, 2013, and 2015. Thus, we divide the data into two subsamples: the 1984-1999 \((k = 5)\) subsample and the 1999-2015 \((k = 2)\) subsample. We apply similar sampling criteria as in Brunnermeier and Nagel (2008) to the PSID data to obtain our samples. Detailed descriptions about sampling criterion will be provided below along with the benchmark regression equation, Eq. (4.4).

4.1.1 Key Variables

There are three key variables in our empirical models, financial wealth, risky shares, and labor income. Financial wealth is defined as the sum of liquid wealth, equity in a private business, and home equity. Liquid wealth is herein defined as the difference between liquid assets (which are the sum of risk-free assets and the holdings of stocks and mutual funds) and liquid liabilities, while risk-free assets are defined as the sum of cash-like assets and holdings of bonds. Define risky wealth as the summation of liquid wealth (taking risk-free assets away), equity in a private business, and home equity, then the risky share is defined as the ratio of risky wealth over financial wealth.

Labor income is defined as the labor income of an household. In particular, the value for “labor income” represents the sum of household head’s labor income and spouse’s labor income. The value for head’s labor income comprises labor part of farm income, labor part of business income, head’s wages income, head’s
bonuses, overtime, commissions, head’s income from professional practice or trade, labor part of market gardening income, and labor part of roomers and boarders income. If the spouse had any income from farming, business, market gardening, or roomers and boarders, labor-asset splits were made following the same rules as those for the head. The labor portion of such income is included in spouse’s labor income. We focus on the labor income because wealth in our theoretical model is defined as financial wealth.

4.1.2 Sample Selection

Detailed information on our sample selection for year $t$ is given as follows. First, we keep every household that the marital status of the head was the same from year $t-k$ to year $t$. Second, we keep every household that had not moved from year $t-k$ to year $t$. Third, we keep every household whose head did not retire in year $t$. Fourth, we keep every household that participated in the stock market in year $t$. Fifth, we keep every household whose labor income was positive in year $t$. Sixth, we keep every household that had sufficient financial wealth (> $10,000) in year $t-k$.

4.1.3 Descriptive Statistics, Outliers, and Scatter Plots

Table 1 presents some descriptive statistics of risky shares, financial wealth, labor income of households. The column under $\alpha$ shows the descriptive statistics of risky shares in both subsamples. From the table, the average of risky shares in the 1984-1999 subsample is around 74.4%, which is slightly higher than that (74.0%) in the 1999-2015 subsample. The median of risky shares is about 78.8% in the 1984-1999 subsample, which is slightly lower than that (79.2%) in the 1999-2015 subsample. These statistics suggest that, on average, households invest about 3 quarters of their financial wealth in risky assets (such as stocks) over time. The column under $W$ shows the descriptive statistics of financial wealth. The average of financial wealth in the 1984-1999 subsample is $676,210, which is smaller than that ($738,389) in the 1999-2015 subsample. The median of financial wealth is $288,034 in the 1984-1999 subsample, which is also smaller than that ($332,996) in the 1999-2015 subsample. The column under $Y$ shows the descriptive statistics of labor income. The average of labor income in the 1984-1999 subsample is $98,816, which is smaller than that ($117,189) in the 1999-2015 subsample. The median of financial wealth is $78,924 in the 1984-1999 subsample, which is also smaller than that ($84,728) in the 1999-2015 subsample. On average, households become richer and make more labor income over time. And the masses of households are concentrated on the left of the distributions of both financial wealth and labor income over time as well. The other columns present the descriptive statistics of the changes of these three variables in both subsamples.

It is clear that all variables have outliers when one compares the minimums and maximums to the mean or
the median. For example, the distributions of risky shares is over an extreme large spectrum. The minimum risky share is as low as -2.5 (i.e., -250%) and as high as 700% in the 1984-1999 subsample. The distribution becomes even more volatile in the 1999-2015 subsample, i.e., the minimum is as low as -49.0 (i.e., -4,900%) and the maximum is as high as 20.3 (i.e., 2,030%). Those extreme values exist for various reasons. One reason is that households’ home equity became negative. For example, the value of one household’s house was below -$110,000, while its financial wealth was barely over $2,000. As a result, the household’s risky share was -4,900%. Another reason is that households borrowed too much so that their financial wealth became negative. For example, a household had about $9,000 risky assets, while its financial wealth was -$830. This implies that the household had a total liabilities about $9,830. Together, this household’s risky share was about -1,100%. For another example, a household’s financial wealth was about $19,200 while risky assets were about $390,000. Thus, the household’s risky share was about 2,030%. Clearly, all examples mentioned here are extreme cases and all of them lie outside three standard errors from the mean. Other two variables, financial wealth and labor income, contain outliers as well.

The variables of interest are the changes of risky shares, $\Delta_k \alpha$ which is given by $\alpha_t - \alpha_{t-k}$, the changes of financial wealth, $\Delta_k w$ which is given by $\log(W_t) - \log(W_{t-k})$, and the changes of labor income, $\Delta_k y$ which is given by $\log(Y_t) - \log(Y_{t-k})$. Here the subscript $t$ denotes year $t$ and the subscript $t-k$ denotes year $t-k$. Table 1 presents some descriptive statistics of these changes. There are interesting observations of these statistics. For example, the top 10% households increased their risky shares by 41.0% from year $t-k$ to year $t$ in the 1984-1999 subsample and by 30.8% in the 1999-2015 subsample. Both increases are far larger than the corresponding changes made by the mode household. However the bottom 25% households actually decreased their risky shares. Similar patterns emerge in the changes of financial wealth, $\Delta_k w$, and the changes of labor income, $\Delta_k y$. One may view these observations as a piece of evidence showing the gap among households had been getting wider and wider from 1984 to 2015.

Figures 1 shows scatter plots of $\Delta_k \alpha$ against $\Delta_k w$ and $\Delta_k y$ in the two subsamples. In all panels, the vertical axis represents the absolute value of the change of risky shares. The horizontal axes in Panel (a) and (c) represent the change of financial wealth and the horizontal axes in Panel (b) and (d) represents the change of labor income. Panel (a) is the scatter plot of $\Delta_k \alpha$ and $\Delta_k w$ in the 1984-1999 subsample. Panel (b) is the scatter plot of $\Delta_k \alpha$ and $\Delta_k y$ in the 1984-1999 subsample. Panel (c) is the scatter plot of $\Delta_k \alpha$ and $\Delta_k w$ in the 1999-2015 subsample. And Panel (d) is the scatter plot of $\Delta_k \alpha$ and $\Delta_k y$ in the 1999-2015 subsample. The extreme distributions of $\Delta_k \alpha$, $\Delta_k w$ and $\Delta_k y$ in two subsamples are presented in these four panels. Given the existences of outliers of risky shares, financial wealth, and labor income, it is not surprising
that the changes of these three variables also contain outliers. We further check the data. The proportions
of observations that lie outside three standard errors from the mean are less than 2.0% with respect to $\Delta_k \alpha$, $\Delta_k w$ and $\Delta_k y$. Given the low proportions, we treat observations that lie outside three standard errors from
the mean as outliers.\footnote{According to the settings of our theory model, we expect that the minimum of $\Delta_k \alpha$ be -100% and the maximum be 100%. Further check shows that $\Delta_k \alpha$ below -100% or above 100% is less than 1.4% in the 1984-1999 subsample and less than 2.0% in the 1999-2015 subsample. Given the low proportion, if we treat observations whose $\Delta_k \alpha$ below -100% or above 100% as outliers (an alternative way to define outliers), our main regression results remain unchanged.}

We report our regression results with and without outliers.

\section*{4.2 Testable Hypothesis}

Based on Eqs. (3.5) - (3.6), we can derive two relationships: one is between $\alpha_t$ and $w_t$, where $w_t = \log(W_t - C_t)$; and the other one is between $\alpha_t$ and $y_t$, where $y_t = \log(Y_t)$ (and $Y_t$ denotes labor income in
year $t$). Mathematically, the two relationships are given as follows:

\begin{align*}
\Delta \alpha_t &\approx \rho \Delta w_t, & (4.1) \\
\Delta \alpha_t &\approx \gamma \Delta y_t, & (4.2)
\end{align*}

where $\rho = \frac{(1-\frac{1}{\omega})Ye_w}{(e^w+Y)^2 R_f}$, $\Delta \alpha_t = \alpha_t - \alpha_{t-1}$, $\Delta w_t = \log(W_t) - \log(W_{t-1})$, and $\Delta y_t = \log(Y_t) - \log(Y_{t-1})$.

Eq. (4.1) is derived from the linear relationship between $\alpha_t$ and $w_t$. We present the relationship between risky shares and labor income in the form of Eq. (4.1) in order to take advantage of the difference method to control the impact of changes of other factors on risky shares. Eq. (4.2) implies that the value of household’s risky share is positively correlated with the deviation of labor income from the mean if $\gamma > 0$ and vice verse.

The signs of $\rho$ and $\gamma$ depend on the value of $\omega$. If $\omega > 1$, we will have $\rho < 0$ and $\gamma > 0$. If $\omega < 1$, we will have $\rho > 0$ and $\gamma < 0$. The exact value of $\omega$ is not known in the literature and it is typically assumed
that $\omega > 1$ [see Mendoza (1991), Schmitt-Grohé and Uribe (2003), Neumeyer and Perri (2005), among many
others]. Such a calibration is in line with the empirical estimates of the wage elasticity of labor supply, $\eta$, in the literature. Table 1 in Chetty (2006) show that the estimates of $\eta$ ranges from 0.033 to 1.040. Given
that $\omega = \frac{1}{\eta} + 1$, it is reasonable to argue that empirical results in the literature imply that $\omega > 1$. This
means that $\rho < 0$ and $\gamma > 0$. Thus, given the literature about $\eta$ and $\omega$ and given Eqs. (4.1)-(4.2), we have
the following testable hypotheses:

$$\rho < 0 \ & \ \& \ \gamma > 0,$$

We test these hypotheses with the PSID data from 1984 to 2015.
4.3 Regression Specification

We consider the following regression equation:

\[
\Delta_k \alpha_t = \rho \times \Delta_k w_t + \gamma \times \Delta_k y_t + \beta \times q_{t-k} + \Delta_k h_t + \varepsilon_t
\]  

(4.4)

In the above equation, \( \alpha_t \) denotes the risky share and is defined as the proportion of financial wealth invested in stocks, mutual funds, equity in a private business, and home equity, in year \( t \). As we have already defined in the above, \( \Delta_k \alpha_t \) denotes the change of risky shares of the household \( j \) over the \( K \) years; \( \Delta_k w_t \) denotes the change of log of financial wealth over the \( K \) years; and finally \( \Delta_k y_t \) denotes the change of log of labor income over the \( K \) years.

\( q_{t-k} \) is a vector of household characteristics and the fixed time effects for the household. For example, it includes a broad range of variables related to the life cycle, background, and financial situation of the household. The vector \( \Delta_k h_t \) contains variables that capture major changes in household characteristic or asset ownership. For example, it includes: changes in family size, changes in the number of children, and sets of dummies for house ownership, business ownership, and nonzero labor income at \( t \) and \( t-k \).

We estimate Eq. (4.4) and its various versions, in both the 1984-1999 (\( K = 5 \)) subsample and the 1999-2015 (\( K = 2 \)) subsample, to test hypothesis (4.3).

4.4 Regression Strategy

As emphasized in Bettermier et al. (2012), a major challenge for empirical analyses on this topic is that events may exist, which cause both changes of income/wealth and risky shares. For example, it is possible that both \( \alpha_t \) and \( w_t/y_t \) have some correlated deterministic pattern over the life-cycle [Brunnermeier and Nagel (2008)]. We take three steps to account for this challenge. In the first step, we impose the following sample criterion (as shown in Section 4.1.2): the marital status of the family unit head remained unchanged, the household head did not retire, and the household did not move, from year \( t-k \) to year \( t \). In the second step, we introduce control variables such as \( q_{t-k} \) and \( \Delta_k h_t \) in the regression model. The inclusion of these additional variables serves the purpose of controlling for some important variables, such as life-cycle effects and preference shifters, and idiosyncratic versus aggregate wealth changes, that may cause changes of both \( \alpha_t \) and \( w_t/y_t \). In the last step, we use the difference method; and for example, we use \( \Delta_k \alpha_t \) instead of \( \alpha_t \).

The second challenge is that data may contain measurement errors. We use a two-stage least square estimator to account for potential measurement errors. The identification requirement is that the instruments, IVs, are (partially) correlated with \( \Delta_k w_t \) (and/or \( \Delta_k y_t \)), but not correlated with the error terms. In this
paper, we adopt three instrumental variables for changes in wealth and labor-income growth, includes one from Liu et al. (2016), and two other instruments from Brunnermeier and Nagel (2008).

The instrumental variable from Liu et al. (2016) is given by
\[ \frac{dlabfw}{llabfw} = \log\left(\frac{labfw}{llabfw}\right), \]
where \( labfw = hdlabinc5( fw + svodbt) \) and \( llabfw = lhdlabinc5/ (lfw + lsvodbt) \). Here \( hdlabinc5 \) denotes household head’s labor income in the current year, \( fw \) denotes liquid wealth in the current year, \( svodbt \) denotes the dollar value of other debts in the current year (other debt is comprised of non-mortgage debt such as credit card debt and consumer loans), and \( (fw + svodbt) \) denotes liquid assets in the current year. And \( lhdlabinc5, lfw, lsvodbt, \) and \( (lfw + lsvodbt) \) are the lagged \( hdlabinc5, fw, svodbt, \) and \( (fw + svodbt) \) by \( k \) years. By definition, it is reasonable to assume that this instrument is partially correlated with financial wealth fluctuations and labor income changes.

The two instrument variables from Brunnermeier and Nagel (2008) are dummy variables for income growth that is measured independently of wealth, \( dincd1 \) and \( dincd2 \). In particular, \( dincd1 = 1 \) if the household’s income growth \( (k\text{-years ago}) \) is in the lowest decile, \( 0 \) otherwise; and \( dincd2 = 1 \) if the household’s income growth \( (k\text{-years ago}) \) is in the top decile, \( 0 \) otherwise. As emphasized in Brunnermeier and Nagel (2008), the values of these three instrument variables are based upon survey questions that are different from those for \( w_t \) and it is thus reasonable to assume that they are uncorrelated with the error terms. The performance of these key instrument variable are discussed below in Section 4.5.

### 4.5 Benchmark Regression Results

In this section, we report the regression results (as shown in Table 2, Table 4, and Table 5) in the following order. First, we present the performance assessment of the instrumental variables. Second, we discuss the results about how financial wealth affects risky shares. Third, we present the results about how labor income affects risky shares. And finally we do some sensitivity analysis.

#### 4.5.1 The Performance of the Instruments

According to the estimation results, it is evident that our instrumental variables are correlated to the financial wealth fluctuations and labor income changes. The results in panel (A) of Table 2 show that the instruments explain a reasonable fraction of variation in financial wealth fluctuations. The instruments are highly significant, with \( p \)-values smaller than 1\%. The partial \( R^2 \) of the instruments is about 0.410 for the 1984-1999 \((k = 5)\) subsample; and is about 0.379 for the 1999-2015 \((k = 2)\) subsample.

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\(^3\)The income here is not the labor income.
Based on the estimation results, our instrumental variables are also correlated to the labor income changes. The results in panel (B) of Table 2 show that the instruments have a significant partial correlation with changes in log labor income. The instruments are highly significant, with $p$-values smaller than 1%. The partial $R^2$ of the instruments is about 0.427 for the 1984-1999 ($k = 5$) subsample; and is about 0.419 for the 1999-2015 ($k = 2$) subsample.

The results in panel (C) of Table 2 shows that our instruments passes the weak-instrument test. In particular, the value of the Cragg-Donald Wald $F$ statistic is 95 in the 1984-1999 ($k = 5$) subsample, and is 656 in the 1999-2015 ($k = 2$) subsample, respectively. Since we estimate by clustering data with family IDs, the robust statistic is the Kleibergen-Paap Wald $F$ statistic [see Kleibergen and Paap (2006)].$^4$ The value of the Kleibergen-Paap Wald $F$ statistic is 53 in the 1984-1999 ($k = 5$) subsample, and is 295 in the 1999-2015 ($k = 2$) subsample, respectively. All of them are way larger the 10% Stock-Yogo weak ID test critical value, 13.43. Thus, we reject the hypothesis that our instruments are weak.

### 4.5.2 Responses to Financial Wealth Accumulations

How risky shares respond to financial wealth accumulations is a classical question. According to the classical economic theory, if a household has constant relative risk aversion (hereafter CRRA) preferences, risky shares will be a constant and thus will not respond to wealth accumulations [see Samuelson (1969)]. Sahm (2012) finds evidence of CRRA preferences in the 1992-2002 Health and Retirement Study data. Brunnermeier and Nagel (2008) show that if a household has habit-formation preferences, its risky shares will increase when its wealth accumulates. However, Brunnermeier and Nagel (2008) find a negative response of risky shares to wealth accumulation in the 1984-2003 PSID, which deviates from the prediction of habit formation preferences. Liu et al. (2016) prove that a household with habit formation preferences may reduce its risky shares in responding to wealth accumulation if the household experiences a large negative income shock. Liu et al. (2016) find predicted heterogeneous responses in the 1984-1999 PSID data, which is in line with their heterogeneous prediction.

With the instruments, the results panel (C) of Table 2 find significant negative responses of risky shares to financial wealth accumulations in both the 1984-1999 ($k = 5$) subsample and the 1999-2015 ($k = 2$) subsample. For example, the estimates of the responses, $\hat{\rho}$, are -0.178 and -0.163 in 1984-1999 ($k = 5$) subsample and the 1999-2015 ($k = 2$) subsample, respectively. Both of them are statistically significantly

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$^4$According the STATA manual: “When the i.i.d. assumption is dropped and ivreg2 is invoked with the robust, bw or cluster options, the Cragg-Donald-based weak instruments test is no longer valid. ivreg2 instead reports a correspondingly-robust Kleibergen-Paap Wald $F$ statistic.”
different from zero at the 1% significance level. These estimates are economically significant as well and they are close in terms of magnitude to the estimates in related studies, such as Brunnermeier and Nagel (2008) and Liu et al. (2016).

With respect to this classical question, we make substantial contributions. First, we make theoretical contributions. The main theoretical contribution, as we have shown in Section 3.2.2, is that under the standard calibration of the key parameter $\omega$, a household with GHH preferences will decrease (as the net effect) its risk tolerance (i.e., decrease its risky shares) when it accumulates wealth. This theoretical finding is in sharp contrast to the existing ones outlined in the above. Second, we obtain new empirical results about this classical question. Our 2SLS estimates are (on average) negative. The estimates are comprehensive in the sense they hold in the 1984-2015 PSID data. In contrast, Liu et al. (2016) only use the 1984-1999 PSID data while Brunnermeier and Nagel (2008) only uses 1984-2003 PSID data. And the estimates are both statistically and economically significant. Third, more importantly, our comprehensive and significant negative estimates are in line with the hypothesis that $\rho < 0$ as predicted with our theory. Put it differently, our empirical results about how risky shares respond to wealth fluctuations provide very strong empirical support of GHH preferences. Overall, we obtain new empirical results and provide the underlying theory which is compatible with the empirical results with respect to this classical question.

4.5.3 Responses to Labor Income Changes

How risky shares respond to labor income changes remains another classical question. According to the theoretical discussions in Bodie et al. (1992), labor income provides an insurance to the households against adverse investment outcome; and thus households will increase their risky shares with higher labor income. Empirical studies with aggregate data (with noisy measurements) find mixed evidence [see Fama and Schwert (1977), Lustig and Nieuwerburgh (2008), among many others]. Existing empirical studies with household level data have found flat responses of risky shares to labor income [see Guiso et al. (1996), Heaton and Lucas (2000), Betermier et al. (2012), among others].

With the instrument, the results panel (C) of Table 2 show (insignificant) responses of risky shares to labor income changes in both the 1984-1999 ($k = 5$) subsample and the 1999-2015 ($k = 2$) subsample. For example, the estimates of the responses, $\hat{\gamma}$, are -0.020 in the 1984-1999 ($k = 2$) subsample and 0.051 in the 1999-2015 ($k = 2$) subsample, respectively. The estimates are statistically insignificant. The results from the 1999-2015 ($k = 2$) subsample provide very weak empirical evidence, if there is, in support of GHH preferences.
With respect to this second classical question, we also make contributions. First, we bring new insight to the literature. The main theoretical contribution, as we have shown in Section 3.2.1, is that under the standard calibration of the key parameter $\omega$, a household with GHH preferences will increase (as the net effect) its risk tolerance (i.e., increase its risky shares) when its labor income increases. Second, we document new empirical results about how labor income affects risky shares. In particular, our 2SLS estimate using the 1984-2015 PSID data finds that risky shares do not respond to the labor income growth. Even though our estimates about how labor income affects risky shares are in line with existing literature [see Guiso et al. (1996), Heaton and Lucas (2000), Betermier et al. (2012), among others], they are different from our theoretical predictions. We take this separation of the empirical results and the theoretical implications as the indicator that the modeling of labor income should be improved.

4.5.4 Impact of Outliers/High (Low) Quantile

From the data presented in Section 4.1.3, it is clear that there are extreme cases of risky shares changes, financial wealth fluctuations, and labor income growth changes, i.e., there are outliers. The benchmark results may be distorted by those extreme values. In this section, we use two different approaches to analyze the impact of the extreme values on our results. With the first approach, we re-estimate Eq. (4.4) after we remove outliers, which are defined in a naive way: any observation that lies outside three standard errors from the mean. With this naive definition of outliers, we remove outliers of both $\Delta k_{\alpha_t}$, $\Delta k_{w_t}$ and $\Delta k_{y_t}$.

With the second approach, we estimate Eq. (4.4) using quantile regressions, a natural way to test the impact of outliers, or more generally the cases at the highest or lowest quantile, on the relationships of interest.

Panel (A) of Table 3 shows the second stage regression results of running the 2SLS regression of Eq. (4.4) without outliers. The responses of risky shares to financial wealth fluctuations are pretty much the same as in the case which includes outliers. For example, the estimates of the responses, $\hat{\rho}$, are -0.175 and -0.157 in 1984-1999 ($k = 5$) subsample and the 1999-2015 ($k = 2$) subsample, respectively. These estimates are statistically significant at the 1% level and their magnitudes are very close to -0.178 and -0.163 (the estimates associated with the case which includes outliers), respectively. Thus, outliers do not affect the relationship between risky shares and financial wealth fluctuations in the data. The responses of risky shares to labor income changes are pretty much the same as in the case which includes outliers. For example, the estimates of the responses, $\hat{\gamma}$, are -0.050 and -0.001 in 1984-1999 ($k = 5$) subsample and the 1999-2015 ($k = 2$) subsample, respectively. These estimates are statistically insignificant as in the case which include outliers. Outliers have no effect on the relationship between risky shares and labor income in the data as
well. Put together, outliers in the PSID data do not have a fundamentally effect on households’ portfolio choices.

Panel (B) of Table 3 shows the quantile regression results of Eq. (4.4) without removing outliers. The responses of risky shares to financial wealth fluctuations across all quantiles are pretty much the same as the corresponding second stage 2SLS regression results: statistically significant at the 1% level and their magnitudes are very close to -0.178 and -0.163 (the estimates associated with the 50 percentile), respectively. In the 1984-1999 \((k = 5)\) subsample, the estimate of the responses, \(\hat{\rho}\), is -0.132 at the lowest 10 percentile, becomes more negative as the quantile climbs up, and reaches the highest, -0.211 at the 90 percentile. The quantile regression results about \(\hat{\rho}\) in the 1999-2015 \((k = 2)\) subsample are very close to what those in the 1984-1999 \((k = 5)\) subsample. The estimate of the responses, \(\hat{\rho}\), is -0.106 at the lowest 10 percentile, becomes more negative as the quantile climbs up, and reaches the highest, -0.176 at the 90 percentile. Thus, cases at the highest or lowest quantile (including outliers about portfolio choice adjustments) do not affect the relationship between risky shares and financial wealth fluctuations in the data.

The responses of risky shares to labor income changes are also pretty much the same as the corresponding second stage 2SLS regression results: statistically insignificant and quantitatively close to zero. In the 1984-1999 \((k = 5)\) subsample, the estimate of the responses, \(\hat{\gamma}\), is -0.016 at the 10 percentile, and it is statistically insignificant. Across all quantiles, the estimates, \(\hat{\gamma}\), are statistically insignificant. In the 1999-2015 \((k = 2)\) subsample, the estimates are slightly different in the sense that estimates at the the highest or lowest quantile are statistically significant. In this subsample, the estimate at the 10 percentile is -0.025 and statistically significant at the 1% level; and the estimate at the 10 percentile is 0.023 and also statistically significant at the 1% level. Overall, the estimates are statistically insignificant as in the case which include outliers. Thus, cases at the highest or lowest quantile (including outliers about portfolio choice adjustments) do not affect the relationship between risky shares and labor income in the data.

Put together, cases at the highest or lowest quantile (including outliers) do not have a fundamentally effect on households’ portfolio choices.

4.6 More Numerical Results

4.6.1 Labor Income and Labor Income Risks

Overall, the estimates, \(\hat{\gamma}\), that are associated with the change of labor income \((\Delta_k y_t)\) in Eq. (4.4) are statistically insignificant. In realizing this, we run additional regressions with two other measures that are related to labor income. The first measure is about the level of income. The exact measure we use is
\( y_t - \bar{y}_t \) (here \( \bar{y}_t \) denotes the cross-sectional mean of \( y_t \) conditional on the information available in year \( t \)). In particular, we run the following regression with the first new measure:

\[
\Delta_k \alpha_t = \rho \times \Delta_k w_t + \gamma \times (y_t - \bar{y}_t) + \beta \times q_{t-k} + \gamma \times \Delta_k h_t + \varepsilon_t. \tag{4.5}
\]

The panel (A) of Table 4 shows the results. With the replacement of \( \Delta_k y_t \) with \( (y_t - \bar{y}_t) \), \( \hat{\gamma} \) from the regressions of Eq. (4.5) in both subsamples are also insignificant. The insignificant results hold when we use different measures of \( \bar{y}_t \).

We consider the second measure, the risk of labor income, for the following reasons. Betermier et al. (2012) find that wage volatility significantly reduces risky shares while labor income does not affect risky shares using the Longitudinal Individual Data for Sweden database from 1999 to 2002. This raises a challenge to our results. In particular, it may be the case that the negative estimate of \( \hat{\rho} \) reported in Section 4.5.2 is actually due to the labor income risks instead of wealth accumulations; and the insignificant estimate of \( \hat{\gamma} \) reported in Section 4.5.3 may become significant if we replace labor income with its risk. Note that we do not include labor income risk in Eq. (4.4) because our theory is silent about how labor income risk affects risky shares. We explore these issue about how labor income risk affects risky shares by running the following regressions

\[
\Delta_k \alpha_t = \rho \times \Delta_k w_t + \gamma \times \hat{V}^y_t + \beta \times q_{t-k} + \gamma \times \Delta_k h_t + \varepsilon_t. \tag{4.6}
\]

\( \hat{V}^y_t \) denotes labor income risks. We define the labor income risks are defined as the square of fitted residuals. We obtain the residuals by running the the first-stage regression of labor income growths on those explanatory variables.

The panel (B) of Table 4 shows the second-stage regression results. The responses of risky shares to labor income risks are modest. Specifically, the estimates of the responses, \( \hat{\gamma} \), are -0.104 and 0.089 in 1984-1999 (\( k = 5 \)) subsample and the 1999-2015 (\( k = 2 \)) subsample, respectively. These estimates are statistically insignificant. The response of risky shares to financial wealth fluctuations are pretty much the same as in the benchmark case. For example, the estimates of the responses, \( \hat{\rho} \), are -0.154 and -0.178 in 1984-1999 (\( k = 5 \)) subsample and the 1999-2015 (\( k = 2 \)) subsample, respectively. These estimates are statistically significant at the 1% level and their magnitudes are very close to -0.178 and -0.163 (the benchmark estimates), respectively.

The literature on the issue how labor income risk affects portfolio choices is extensive. For example, it has been shown that labor income risk is likely to reduce risky shares [Kimball (1993), Bertaut and Haliassos (1997), among others]. For another example, Angerer and Lam (2009) document that permanent income...
risk significantly reduces risky shares, while transitory income risk does not with the Current Population Survey data and the NLSY79 data. Our contribution is that we document new empirical evidence on this issue. In particular, with our naive measure of labor income risk, our empirical results show that the negative responses of risky shares are not caused by labor income risks; and labor income risk does not affect risky shares in the PSID data.\(^5\) We defer further study on this issue to our future research.

4.6.2 Great Recession

The 2007-2009 Great Recession is the most severe economic downturn since the Great Depression. In our PSID data, households’ financial wealth decreased substantially in the recession. For example, the mean went down from $872,243 in 2007 to $715,040 in 2009, an 18% decrease. It is of interest to check how households’ portfolio choices responded to the Great Recession. For this purpose, we run Eq. (4.4) using the 2007-2009 PSID data. Panel (A) of Table 5 presents the results.

The responses of risky shares to financial wealth fluctuations during the Great Recession are even stronger than in the 1999-2015 subsample. For example, the estimate of \(\rho\), \(\hat{\rho}\), during the Great Recession is -0.218 and is 0.061 points larger than -0.157, the estimate of \(\rho\) in the 1999-2015 \((k = 2)\) subsample. The estimates are statistically significant at the 1% level. The responses of risky shares to labor income changes are statistically significant now. For example, the estimates of the responses, \(\hat{\gamma}\), are 0.125 and are statistically significant at the 1% level. Both responses are the same as predicted by our theory. Thus, our empirical results about households’ portfolio choices in the Great Recession provides the strongest support of the GHH preferences.

4.6.3 Life Cycle

The portfolio choice over the life cycle has been discussed extensively. Empirically, it has been documented that risky shares have been low at the young ages, and then either increasing or hump-shaped later; and theoretically, it has bee argued that risky shares will decrease with ages [Benzoni et al. (2007)]. Even though our theoretical model is silent about the this issue, it is still of interest to show the empirical evidence about portfolio choices over the life cycle in the PSID data. Figure 2 presents the scatter plots of risky shares against ages. Panel (a) shows the plot in the 1984-1999 subsample, and panel (b) shows the plot in the 1999-2015 subsample. It is clear that there is no clear statistical relationship between risky shares and ages in both samples. The regression results shown in Table 5 show similar results. In particular, the estimates of the coefficient of \(\text{age}\) in the benchmark regression using the 1999-2015 subsample data are small and

\(^{5}\)Letendre and Smith (2001) show that background income risk does affect optimal portfolios but that this effect may be difficult to detect empirically.
We further break down the 1999-2015 subsample into three age groups, the younger-than-40 group (the young age group), the between-40-and-60 group (the middle age group), and the older-than-60 group (the old age group). The break-down regression results are shown in Panel (B) in Table 5. With this break-down, the portfolio choices of households whose heads are younger than 40 are in line with the choices of the whole subsample. On the one hand, households in this group, following the theoretical predictions of GHH preferences, will decrease their risky shares when financial wealth accumulates. On the other hand, their risky shares will not respond when labor income is higher. These empirical results do not depend on the cut-off age of 40: they hold when we change the upper age limit of the young age group from 40 to a value in the range of [34, 42]. The estimates of \( \rho \) and \( \gamma \) for the middle age group have the expected signs but not significant. The estimates of \( \rho \) and \( \gamma \) for the old age group are also not significant. However, when we change the bottom age limit of the old age group from 60 to a lower value in the range of [50, 59], the estimates of \( \rho \) become negative and statistically significant.

Overall, our regression results from the 1999-2015 subsample show a significant-insignificant-significant pattern of the estimates of \( \rho \) and a insignificant-insignificant-insignificant pattern of the estimates of \( \gamma \) over the young-middle-old life cycle. These results are the new results we obtain using the 1999-2015 PSID data.

### Contributions

We contribute three empirical regularities about portfolio choices. First, we document the robust empirical finding that wealth accumulations significantly reduce risky shares in the 1984-2015 PSID data. Second, we document the robust empirical finding that labor income increase does not have statistically significant impacts on risky shares in the 1984-2015 PSID data. Third, we also document that labor income risks (with our naive measure) does not have statistically significant impacts on risky shares in the 1984-2015 PSID data, a finding that implies further analysis on this issue is required.

This is a major contribution to the literature. On the other hand, we show that if the impact of labor income on portfolio choices is important to consider in a representative agent model, GHH preferences are insufficient. This is because our unconditional theoretical predictions about labor income affects portfolio choices (from a representative agent model with GHH preferences) are not supported by the 1984-2015 PSID data. How to make improvement over the existing representative agent model with GHH preferences is clearly an important issue to explore in the future. Our second contribution here is to show the insufficiency of the representative agent model with GHH preferences in understanding how labor income affects portfolio choices.
choices.

There is a clear room for the improvement because the paper is silent on many important while related issues. For example, the theoretical analysis of the paper focuses on the static relationships among the three key variables, thus ignoring the impact of the short-run dynamics of labor income on these variables. For another example, the empirical analysis of the paper focuses on the linear relationships among the three key variables, thus ignoring the non-linear relationships. Above all, the key limitation of the paper is about the estimation of $\omega$, a parameter that is related to the wage elasticity of labor supply, $\eta$, through the equation, $\eta = \frac{1}{\omega - 1}$. The wage elasticity of labor supply is clearly one of many important structural parameters. It would have been nice if our paper can shed some light on the estimate of $\eta$. Given the benchmark estimates of $\rho$ and $\omega$, we cannot back out reasonable estimates of $\eta$. We view the incapability of estimating $\eta$ as the consequence of our simple theoretical model. In other words, to have reasonable estimates of $\eta$, our current model should be at least modified.

5 Assumptions about Preferences

One important assumption that has to be made in any quantitative macroeconomic analyses is about the utility function. This assumption is important because models with the deliberately chosen utility function have successfully explained, to certain level, the important targeted economic phenomena. For example, models with habit formation preferences have successfully explained the hump-shaped responses of aggregate variables to monetary shocks [Fuhrer (2000)] and other empirical facts. For another example, models with the GHH preferences [Greenwood et al. (1988)] have successfully explained business cycle and equity premium in emerging economies [Mendoza (1991), Garcia-Cicco et al. (2010), and Jahan-Parvar et al. (2013)].

To our best knowledge, existing structural studies about how preferences affect risky shares are far from being satisfactory. Some of them reject theoretical predictions out of certain preferences. For example, the unconditional predictions on portfolio choices (from a representative agent model with habit formation preferences) are rejected by the data [see the results in Brunnermeier and Nagel (2008) and the strong-form test results in Liu et al. (2016)]. One implication about economic modeling from these studies is to introduce non-habit-formation preferences with a representative agent model. Some of them find strong empirical evidence of theoretical predictions of certain preferences. For example, the conditional predictions (conditional on heterogeneous agents) are strongly supported by the PSID data [see the weak-form test results in Liu et al. (2016)]. However, an implication of these results is to assume habit-formation preferences with heterogeneous agent models.
We make two contributions about economic modeling when portfolio choices are of the major concern. On the one hand, we show that if the impact of wealth accumulations on portfolio choices is important to consider in a representative agent model, GHH preferences are a good choice. This is because our unconditional theoretical predictions about how wealth accumulations affect portfolio choices (from a representative agent model with GHH preferences) are strongly supported by the 1984-2015 PSID data. This is a major contribution to the literature. On the other hand, we show that if the impact of labor income on portfolio choices is important to consider in a representative agent model, GHH preferences are insufficient. This is because our unconditional theoretical predictions about labor income affects portfolio choices (from a representative agent model with GHH preferences) are not supported by the 1984-2015 PSID data. How to make improvement over the existing representative agent model with GHH preferences is clearly an important issue to explore in the future. Our second contribution here is to show the insufficiency of the representative agent model with GHH preferences in understanding how labor income affects portfolio choices.

6 Conclusion

In this paper, we examine the theoretical and empirical implications of GHH preferences for households portfolio choices. We contribute to the macroeconomic and finance literature over four dimensions. First, we obtain a closed-form solution to risky shares. Second, we derive clear theoretical predictions of portfolio choices when household preferences are GHH. Third, we test the theoretical implications to the data. Our empirical results strongly support the predictions about how wealth accumulations affect portfolio choices; while they do not support the predictions about how labor income affects portfolio choices. Our empirical results raise concern about how labor income risks affect portfolio choices. Finally, our results show that GHH preferences are a reasonable functional form to use in a representative agent model if the impact of wealth accumulations on portfolio choices is of concern; while the existing representative agent model with GHH preferences needs to be further modified if the impact of labor income on portfolio choices is important to consider.

Based on our work in this paper, there are several extensions worth being explored in the future. The first extension is how to obtain reliable estimate of a key deep structural parameter about labor supply. The second extension is how to modify the representative agent model with GHH to account for the impact of labor income on portfolio choices. The third extension is how labor income risks affects portfolio choices. The fourth extension is to have joint discussions with both portfolio choices and consumption.
References


Figures and Tables

Figure 1: Scatter Plots of $\Delta_k \alpha$ against $\Delta_k w$ and $\Delta_k y$

Notes:

1. $\alpha$, $w$, and $Y$ denote risky shares, financial wealth, and labor income, respectively. Their definitions are given in Section 4.1.1.

2. $\Delta_k \alpha$ denotes the change of risky shares. In particular, $\Delta_k \alpha = \alpha_t - \alpha_{t-k}$. The subscript $t$ denotes the current year; and the subscript $t - k$ denotes the year that is $k$ years lagged behind the current year.

3. $\Delta_k w$ denotes the change of logarithms of financial wealth. In particular, $\Delta_k w = \log(W_t) - \log(W_{t-k})$.

4. $\Delta_k y$ denotes the change of logarithms of labor income. In particular, $\Delta_k y = \log(Y_t) - \log(Y_{t-k})$.

5. In all panels, the vertical axis represents the absolute value of the change of risky shares. The horizontal axes in Panel (a) and (c) represent the change of financial wealth and the horizontal axes in Panel (b) and (d) represents the change of labor income.

6. Panel (a) is the scatter plot of $\Delta_k \alpha$ and $\Delta_k w$ in the 1984-1999 subsample. Panel (b) is the scatter plot of $\Delta_k \alpha$ and $\Delta_k y$ in the 1984-1999 subsample.

7. Panel (c) is the scatter plot of $\Delta_k \alpha$ and $\Delta_k w$ in the 1999-2015 subsample. Panel (d) is the scatter plot of $\Delta_k \alpha$ and $\Delta_k y$ in the 1999-2015 subsample.
Figure 2: Scatter Plots of Risky Shares and Ages

Notes:

1. Panel (a) is the scatter plot of risky shares and ages in the 1984-1999 subsample. Panel (b) is the scatter plot of risky shares and ages in the 1999-2015 subsample.

2. In Panel (a) and (b), the vertical axis represents the absolute value of risky shares; and the horizontal axis in Panel (a) and (b) represents the absolute value of ages.
### Table 1: Descriptive Statistics

The 1984-1999 Subsample (k=5)

<table>
<thead>
<tr>
<th>Variables Statistics</th>
<th>α</th>
<th>Δₖα</th>
<th>W</th>
<th>Δₖw</th>
<th>Y</th>
<th>Δₖy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>0.744</td>
<td>0.047</td>
<td>$676,210</td>
<td>0.315</td>
<td>$98,816</td>
<td>-0.432</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>0.362</td>
<td>0.392</td>
<td>$1,809,472</td>
<td>0.848</td>
<td>$103,917</td>
<td>0.726</td>
</tr>
<tr>
<td>25%</td>
<td>0.581</td>
<td>-0.122</td>
<td>$141,101</td>
<td>-0.110</td>
<td>$51,207</td>
<td>-0.691</td>
</tr>
<tr>
<td>50%</td>
<td>0.788</td>
<td>0.030</td>
<td>$288,034</td>
<td>0.323</td>
<td>$78,924</td>
<td>-0.405</td>
</tr>
<tr>
<td>75%</td>
<td>0.932</td>
<td>0.196</td>
<td>$626,881</td>
<td>0.742</td>
<td>$112,945</td>
<td>0.001</td>
</tr>
<tr>
<td>90%</td>
<td>0.993</td>
<td>0.410</td>
<td>$1,216,057</td>
<td>1.188</td>
<td>$163,918</td>
<td>0.228</td>
</tr>
<tr>
<td>Maximum</td>
<td>7.000</td>
<td>6.134</td>
<td>$3.0e+7</td>
<td>4.684</td>
<td>$8,650,498</td>
<td>6.640</td>
</tr>
<tr>
<td>N</td>
<td>1,416</td>
<td>1,416</td>
<td>1,417</td>
<td>1,407</td>
<td>1,417</td>
<td>1,412</td>
</tr>
</tbody>
</table>

The 1999-2015 Subsample (k=2)

<table>
<thead>
<tr>
<th>Variables Statistics</th>
<th>α</th>
<th>Δₖα</th>
<th>W</th>
<th>Δₖw</th>
<th>Y</th>
<th>Δₖy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>0.740</td>
<td>0.008</td>
<td>$738,389</td>
<td>0.090</td>
<td>$117,189</td>
<td>0.023</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>1.082</td>
<td>1.353</td>
<td>$2,002,355</td>
<td>0.738</td>
<td>$199,364</td>
<td>0.654</td>
</tr>
<tr>
<td>Minimum</td>
<td>-49.000</td>
<td>-49.405</td>
<td>$-664,557</td>
<td>-5.941</td>
<td>$1.962</td>
<td>-9.018</td>
</tr>
<tr>
<td>25%</td>
<td>0.600</td>
<td>-0.113</td>
<td>$149,680</td>
<td>-0.211</td>
<td>$51,211</td>
<td>-0.129</td>
</tr>
<tr>
<td>50%</td>
<td>0.792</td>
<td>0.005</td>
<td>$332,996</td>
<td>0.101</td>
<td>$84,728</td>
<td>0.029</td>
</tr>
<tr>
<td>75%</td>
<td>0.936</td>
<td>0.129</td>
<td>$709,021</td>
<td>0.427</td>
<td>$132,082</td>
<td>0.226</td>
</tr>
<tr>
<td>90%</td>
<td>0.997</td>
<td>0.308</td>
<td>$1,411,008</td>
<td>4.684</td>
<td>$8,650,498</td>
<td>6.640</td>
</tr>
<tr>
<td>Maximum</td>
<td>20.304</td>
<td>49.696</td>
<td>$4.3e+7</td>
<td>4.684</td>
<td>$8,650,498</td>
<td>6.640</td>
</tr>
<tr>
<td>N</td>
<td>6,152</td>
<td>6,152</td>
<td>6,154</td>
<td>6,081</td>
<td>6,154</td>
<td>6,110</td>
</tr>
</tbody>
</table>

Notes:

1. α, w, and Y denote risky shares, financial wealth, and labor income, respectively. Their definitions are given in Section 4.1.1.

2. Δₖα denotes the change of risky shares. In particular, Δₖα = αₜ - αₜ₋ₖ. The subscript t denotes the current year; and the subscript t – k denotes the year that is k years lagged behind the current year.

3. Δₖw denotes the change of logarithms of financial wealth. In particular, Δₖw = log(wₜ) – log(wₜ₋ₖ).

4. Δₖy denotes the change of logarithms of labor income. In particular, Δₖy = log(yₜ) – log(yₜ₋ₖ).
### Table 2: Benchmark Regression Results

(A) First Stage Results

<table>
<thead>
<tr>
<th>Subsample</th>
<th>Dependent variable: $\Delta_k w_t$</th>
<th>$k = 5$</th>
<th>$k = 2$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Key explanatory variables of interest:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>IV: dlabfw</td>
<td>-0.348*** (0.022)</td>
<td>-0.373*** (0.012)</td>
<td></td>
</tr>
<tr>
<td>IV: dincd1</td>
<td>0.485*** (0.093)</td>
<td>-0.421*** (0.032)</td>
<td></td>
</tr>
<tr>
<td>IV: dincd2</td>
<td>0.388*** (0.069)</td>
<td>0.346*** (0.034)</td>
<td></td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.410</td>
<td>0.379</td>
<td></td>
</tr>
<tr>
<td>Sample size</td>
<td>1,407</td>
<td>6,081</td>
<td></td>
</tr>
</tbody>
</table>

(B) First Stage Results

<table>
<thead>
<tr>
<th>Subsample</th>
<th>Dependent variable: $\Delta_k y_t$</th>
<th>$k = 5$</th>
<th>$k = 2$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Key explanatory variables of interest:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>IV: dlabfw</td>
<td>0.116*** (0.015)</td>
<td>0.124*** (0.010)</td>
<td></td>
</tr>
<tr>
<td>IV: dincd1</td>
<td>-0.751*** (0.083)</td>
<td>-0.731*** (0.032)</td>
<td></td>
</tr>
<tr>
<td>IV: dincd2</td>
<td>0.398*** (0.065)</td>
<td>0.655*** (0.029)</td>
<td></td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.427</td>
<td>0.419</td>
<td></td>
</tr>
<tr>
<td>Sample size</td>
<td>1,412</td>
<td>6,110</td>
<td></td>
</tr>
</tbody>
</table>

(C) Second Stage Results

<table>
<thead>
<tr>
<th>Subsample</th>
<th>Dependent variable: $\Delta_k \alpha_t$</th>
<th>$k = 5$</th>
<th>$k = 2$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Key explanatory variables of interest:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Delta_k w_t$ ($\hat{\rho}$):</td>
<td>-0.178*** (0.025)</td>
<td>-0.163*** (0.048)</td>
<td></td>
</tr>
<tr>
<td>$\Delta_k y_t$ ($\hat{\gamma}$):</td>
<td>-0.020 (0.036)</td>
<td>0.051 (0.045)</td>
<td></td>
</tr>
<tr>
<td>Sample size</td>
<td>1,402</td>
<td>6,038</td>
<td></td>
</tr>
</tbody>
</table>

Weak instrument test

- Cragg-Donald Wald F statistic: 95 (656)
- Kleibergen-Paap rk Wald F statistic: 53 (295)
- 10% significance critical value: 13.43 (13.43)

**Notes:**
1. The benchmark regression equation is given by Eq. (4.4).
2. All IVs are defined in Section 4.4.
3. The values of standard deviations are shown in the parenthesis.
4. Heteroskedasticity and autocorrelation-robust standard errors are used to judge the significance of estimates. *** (**, *) means that the estimate is statistically significantly different from 0 at the 1% (5%, 10%) significance level, respectively.
### Table 3: Impact of Outliers

#### (A) Eq. (4.4) without outliers

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>($k = 5$)</td>
<td>($k = 2$)</td>
</tr>
<tr>
<td>Dependent variable:</td>
<td>$\Delta_k \alpha_t$</td>
<td></td>
</tr>
<tr>
<td>Key explanatory</td>
<td>$\Delta_k w_t (\hat{\rho})$: -0.175***</td>
<td>-0.157***</td>
</tr>
<tr>
<td>variables of interest:</td>
<td>(0.028)</td>
<td>(0.012)</td>
</tr>
<tr>
<td>$\Delta_k y_t (\hat{\gamma})$:</td>
<td>-0.050</td>
<td>-0.001</td>
</tr>
<tr>
<td></td>
<td>(0.040)</td>
<td>(0.015)</td>
</tr>
<tr>
<td>Sample size</td>
<td>1,343</td>
<td>5,795</td>
</tr>
<tr>
<td>Weak instrument test</td>
<td>Cragg-Donald Wald F statistic</td>
<td>102</td>
</tr>
<tr>
<td></td>
<td>Kleibergen-Paap rk Wald F statistic</td>
<td>61</td>
</tr>
<tr>
<td></td>
<td>10% significance critical value</td>
<td>13.43</td>
</tr>
</tbody>
</table>

#### (B) Eq. (4.4) quantile regressions

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>($k = 5$)</td>
<td>($k = 2$)</td>
</tr>
<tr>
<td>Dependent variable:</td>
<td>$\Delta_k \alpha_t$</td>
<td></td>
</tr>
<tr>
<td>Key explanatory</td>
<td>$\Delta_k w_t (\hat{\rho})$:</td>
<td></td>
</tr>
<tr>
<td>variables of interest:</td>
<td>(10%) -0.132***</td>
<td>-0.106***</td>
</tr>
<tr>
<td>(25%)</td>
<td>-0.142***</td>
<td>-0.139***</td>
</tr>
<tr>
<td>(50%)</td>
<td>-0.179***</td>
<td>-0.138***</td>
</tr>
<tr>
<td>(75%)</td>
<td>-0.189***</td>
<td>-0.150***</td>
</tr>
<tr>
<td>(90%)</td>
<td>-0.211***</td>
<td>-0.176***</td>
</tr>
<tr>
<td>$\Delta_k y_t (\hat{\gamma})$:</td>
<td>(10%) -0.016</td>
<td>-0.025***</td>
</tr>
<tr>
<td>(25%)</td>
<td>-0.011</td>
<td>0.004</td>
</tr>
<tr>
<td>(50%)</td>
<td>-0.024</td>
<td>-0.005</td>
</tr>
<tr>
<td>(75%)</td>
<td>-0.043</td>
<td>0.003</td>
</tr>
<tr>
<td>(90%)</td>
<td>-0.022</td>
<td>0.023***</td>
</tr>
<tr>
<td>Sample size</td>
<td>1,402</td>
<td>6,038</td>
</tr>
</tbody>
</table>

**Notes:**

1. All the results in panel (A) are the second stage regression results.
2. All the results in panel (B) are the quantile regression results.
3. Heteroskedasticity and autocorrelation-robust standard errors are used to judge the significance of estimates. *** (**, *) means that the estimate is statistically significantly different from 0 at the 1% (5%, 10%) significance level, respectively.
Table 4: Labor Income and Labor Income Risk


<table>
<thead>
<tr>
<th>Dependent variable: $\Delta_k \alpha_t$</th>
<th>$(k = 5)$</th>
<th>$(k = 2)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Key explanatory variables of interest:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Delta_k w_t (\hat{\rho})$:</td>
<td>-0.172***</td>
<td>-0.174***</td>
</tr>
<tr>
<td></td>
<td>(0.024)</td>
<td>(0.041)</td>
</tr>
<tr>
<td>$y_t - \bar{y}_t (\hat{\gamma})$:</td>
<td>-0.012</td>
<td>0.053</td>
</tr>
<tr>
<td></td>
<td>(0.012)</td>
<td>(0.050)</td>
</tr>
<tr>
<td>Sample size</td>
<td>1,407</td>
<td>6,081</td>
</tr>
<tr>
<td>Weak instrument test</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cragg-Donald Wald F statistic</td>
<td>46</td>
<td>34</td>
</tr>
<tr>
<td>Kleibergen-Paap rk Wald F statistic</td>
<td>24</td>
<td>27</td>
</tr>
<tr>
<td>10% significance critical value</td>
<td>13.43</td>
<td>13.43</td>
</tr>
</tbody>
</table>

(B) Eq. (4.6) The 1984–1999 Subsample $(k = 5)$ The 1999–2015 Subsample $(k = 2)$

<table>
<thead>
<tr>
<th>Dependent variable: $\Delta_k \alpha_t$</th>
<th>$(k = 5)$</th>
<th>$(k = 2)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Key explanatory variables of interest:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Delta_k w_t (\hat{\rho})$:</td>
<td>-0.154***</td>
<td>-0.178***</td>
</tr>
<tr>
<td></td>
<td>(0.033)</td>
<td>(0.039)</td>
</tr>
<tr>
<td>$\hat{V}_y (\hat{\gamma})$:</td>
<td>-0.104</td>
<td>0.089</td>
</tr>
<tr>
<td></td>
<td>(0.119)</td>
<td>(0.104)</td>
</tr>
<tr>
<td>Sample size</td>
<td>1,402</td>
<td>6,038</td>
</tr>
<tr>
<td>Weak instrument test</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cragg-Donald Wald F statistic</td>
<td>5.37</td>
<td>36.0</td>
</tr>
<tr>
<td>Kleibergen-Paap rk Wald F statistic</td>
<td>4.09</td>
<td>9.30</td>
</tr>
<tr>
<td>10% significance critical value</td>
<td>13.43</td>
<td>13.43</td>
</tr>
<tr>
<td>20% significance critical value</td>
<td>3.95</td>
<td>6.40</td>
</tr>
</tbody>
</table>

Notes:
1. The values of standard deviations are shown in the parenthesis.
2. We use two instruments, $dlabfw$ and $dincd2$, in estimating Eq. (4.6) with the 1984-1999 subsample. For any other estimations, we use the three instruments discussed in Section 4.4.
3. All the results in the table are the second stage regression results.
4. Heteroskedasticity and autocorrelation-robust standard errors are used to judge the significance of estimates. *** (**, *) means that the estimate is statistically significantly different from 0 at the 1% (5%, 10%) significance level, respectively.
### Table 5: Great Recession and Life Cycle

(A) Eq. (4.4) Great Recession

<table>
<thead>
<tr>
<th>Dependent variable: $\Delta_k \alpha_t$</th>
<th>The 2007–2009 Subsample $(k = 2)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Key explanatory variables of interest:</td>
<td></td>
</tr>
<tr>
<td>$\Delta_k w_t$ ($\hat{\rho}$):</td>
<td></td>
</tr>
<tr>
<td>$\Delta_k y_t$ ($\hat{\gamma}$):</td>
<td></td>
</tr>
<tr>
<td>Sample size</td>
<td></td>
</tr>
<tr>
<td>Weak instrument test</td>
<td></td>
</tr>
<tr>
<td>Cragg-Donald Wald F statistic</td>
<td></td>
</tr>
<tr>
<td>Kleibergen-Paap rk Wald F statistic</td>
<td></td>
</tr>
<tr>
<td>10% significance critical value</td>
<td></td>
</tr>
</tbody>
</table>

(B) Eq. (4.4) Life Cycle

<table>
<thead>
<tr>
<th>Dependent variable: $\Delta_k \alpha_t$</th>
<th>The 1999–2015 Subsample $(k = 2)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Key explanatory variables of interest:</td>
<td></td>
</tr>
<tr>
<td>$\Delta_k w_t$ ($\hat{\rho}$):</td>
<td></td>
</tr>
<tr>
<td>$\Delta_k y_t$ ($\hat{\gamma}$):</td>
<td></td>
</tr>
<tr>
<td>Age</td>
<td></td>
</tr>
<tr>
<td>Sample size</td>
<td></td>
</tr>
<tr>
<td>Weak instrument test</td>
<td></td>
</tr>
<tr>
<td>Cragg-Donald Wald F statistic</td>
<td></td>
</tr>
<tr>
<td>Kleibergen-Paap rk Wald F statistic</td>
<td></td>
</tr>
<tr>
<td>10% significance critical value</td>
<td></td>
</tr>
</tbody>
</table>

Notes:
1. We use the three instruments discussed in Section 4.4. We do not report the results from the regressions where the weak instrument hypothesis has not been rejected.
2. All the results in the table are the second stage regression results.
3. Heteroskedasticity and autocorrelation-robust standard errors are used to judge the significance of estimates. *** (**, *) means that the estimate is statistically significantly different from 0 at the 1% (5%, 10%) significance level, respectively.
4. The 1984-1999 subsample does not have sufficient observations to check the estimates over the life cycle.