A Unified Empirical Framework to Study Segregation

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Abstract

We present an empirical framework to study segregation that bridges the empirical literature on residential choice and the theoretical literature on neighborhood segregation. The former literature is based upon equilibrium empirical models of disaggregated choices, whereas the latter literature is concerned with the aggregate phenomenon of segregation, which is often studied theoretically in disequilibrium. Our framework explicitly allows for incomplete information, moving costs, and for the disaggregated households' choices to be observed out of equilibrium. We also propose novel instrumental variables that exploit the logic of a dynamic choice model and can be constructed with no additional data requirements. A simulation procedure aggregates these choices to characterize the dynamic process of segregation. We illustrate our framework with an analysis of racial segregation of White, Black, Hispanic and Asian homeowners in the San Francisco Bay Area from 1990-2004. We find that all homeowners react highly heterogeneously to neighbors of different races. Black and Hispanic segregation would increase by around 17% in the absence of any external shocks to the housing market, but White and Asian segregation would increase by only around 7%. Moving costs play a central role in keeping segregation at bay by maintaining a mismatch between the desired and the current neighborhoods of many households. This mismatch is mostly sustained by neighborhood amenities other than racial composition.

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1 Introduction

There is a growing body of evidence that neighborhood segregation is an engine of socioeconomic inequality.¹ Since similar households tend to have similar preferences, segregation is a natural outcome of residential sorting. Ever since Tiebout (1956), a rich theoretical and empirical literature has developed to study residential sorting and its consequences. Two strands of this literature have been particularly relevant to the study of segregation: empirical models of residential choice that have focused on studying the determinants of segregation in equilibrium (e.g., Bayer, McMillan and Rueben (2004 a)), and disequilibrium models of segregation based on the seminal work of Schelling (1969) that have characterized segregation as an aggregate, dynamic phenomenon. In this paper, we present a unified framework to study segregation that bridges these two strands of the literature. We illustrate how our framework can be applied to analyze racial segregation in the San Francisco Bay Area from 1990 to 2004.

In Figure 1, we show the racial compositions of several neighborhoods over a fifteen year period. These neighborhoods undergo substantial demographic changes that often appear to be serially correlated. A common assumption in models of residential choice is that households are observed in what Bayer and Timmins (2005) define as a *sorting equilibrium*, i.e., in the absence of future amenity shocks, the demographic compositions of the neighborhoods will not change. Under this assumption, the trends shown in Figure 1 would be attributed to serially correlated changes in these neighborhoods. While this interpretation may be appropriate when studying many aspects of residential sorting, it may be less appropriate when studying segregation.

Schelling (1969) has proposed an alternative interpretation of these trends that may be more plausible: the composition of a neighborhood may change endogenously due to the presence of certain amenities that are deemed *endogenous* in the sense that they are affected by residential decisions. When studying segregation, racial composition is the

¹Residential segregation has been linked to a broad set of outcomes including educational attainment and labor market outcomes (Cutler, Glaeser and Vigdor (2008)), infant health (Mason et al. (2009)), friendship formation (Mouw and Entwisle (2006)), crime (Kling, Ludwig and Katz (2005)), intergenerational mobility and economic opportunity (Chetty et al. (2014)) and various measures of subjective well being (Ludwig et al. (2012)). Similarly, school segregation, a close cousin of neighborhood segregation, has been linked to lower educational attainment (Rivkin, Hanushek and Kain (2005)) and wider black-white achievement gaps (Card and Rothstein (2007)). In addition, school desegregation programs have been found to have increased black graduation rates (Guryan (2004)), college attendance and likelihood of arrest (Bergman (2016)).



Figure 1: Racial Composition of Selected Neighborhoods Over Time, 1990-2004

most natural candidate for such an amenity. If, for instance, White households prefer White neighbors relative to non-White households, then an increase in the non-White share of a neighborhood could induce additional relative outflows of White households. This would endogenously lead to a further increase in the non-White share of the neighborhood and generate the observed serial correlation in racial composition even in the absence of other amenity shocks. The dynamics induced by such social interactions has led Schelling (1971) to suggest that neighborhoods are more likely to be observed adjusting along a trajectory to a sorting equilibrium rather than having reached that equilibrium already. While this interpretation is attractive in its simplicity, disequilibrium models of segregation have remained mostly theoretical and have tended to focus on endogenous responses to racial composition to explain segregation while downplaying the potentially important roles of moving costs and other amenities.

This paper unifies these two literatures in a new empirical framework to study segregation. The framework yields four main methodological innovations. (1) Households are allowed to make residential decisions with incomplete information so neighborhoods need not instantaneously converge to a sorting equilibrium. Thus, at any point in time neighborhoods may be observed out of equilibrium. (2) We can identify the location of and rate of convergence to any equilibrium from any initial state. (3) Our framework accommodates a dynamic model of residential choice with moving costs, but crucially, we can avoid many standard assumptions on the structure of state variables and their expected transitions, because we do not need to isolate the flow component of a cumulative utility.² (4) We introduce a novel instrumental variables (IV) approach to identify the causal effects of the racial composition of a neighborhood on the demands for that neighborhood by households of different races. These IVs are created with no additional data requirements.

We demonstrate our framework with an analysis of racial segregation between White, Black, Hispanic and Asian homeowners in the San Francisco Bay Area from 1990-2004 using a recently constructed, high frequency data set on residential moves in the Bay Area (Bayer et al. (2016)). We find that homeowners of different races react

²As Manski (2004) has pointed out, expectations and preferences are indistinguishable using choice data alone. This may, for example, lead to misidentification of households' preferences for neighbors of the same race. A world in which people care strongly for neighbors of the same race generates observationally equivalent choice data to a world in which people care weakly for neighborhoods of the same race but tend to overestimate the proportion of neighbors of the same race. In our framework, we circumvent this issue by focusing on households' choices instead of their preferences since in both worlds, the racial compositions of neighborhoods would evolve identically.

heterogeneously to changes in the racial composition of a neighborhood. Homeowners of all races seek neighbors of the same race, though to differing degrees (e.g., Hispanics seek neighbors of the same race with a higher intensity than homeowners of other races do). However, Whites and Asians react negatively to higher concentrations of Black and Hispanic neighbors, and Whites and Hispanics all react modestly negatively to higher concentrations of Asian neighbors.

These complex responses that characterize household sorting across neighborhoods generate a distinct pattern of increasing segregation. In the absence of external shocks from 2004 onward, segregation in the Bay Area would nevertheless increase in the long run. Much of the adjustment would occur over the first 3-5 years, as the vast majority of neighborhoods would be in flux, turning over several households. After ten years time, roughly 20% of neighborhoods would remain out of equilibrium. However the effects of this adjustment would not be borne equally by all households; Black and Hispanic segregation would increase by 18% and 16% respectively, whereas White and Asian segregation would increase by only 7% and 5% respectively.

With our framework, we can explore the roles of discriminatory responses, moving costs, incomplete information and the initial allocation of households across neighborhoods in shaping the trajectories of segregation. Interestingly, we find that discrimination is not the largest driver of segregation. Instead, we find that a mismatch between the desired and the current levels of neighborhood amenities other than the racial composition is a much greater contributor to future increases in segregation, though this is kept at bay by substantial moving costs and, to a lesser extent, incomplete information.

Relevant Literature

Our paper lies at the nexus of two distinct but related literatures related to neighborhood choice and segregation. We briefly review some of the most relevant studies.

Empirical Models of Residential Choice and Neighborhood Sorting

Because segregation is an outcome of neighborhood sorting, we build upon the prolific literature on the determinants of residential choice.³ This literature is largely interested in estimating the marginal willingness to pay for neighborhood amenities. Three

³See, for example, Epple, Filimon and Romer (1984); Kiel and Zabel (1996); Epple and Sieg (1999); Epple, Romer and Sieg (2001, 2003); Bayer, McMillan and Rueben (2004*a*); Bayer and Timmins (2005, 2007); Bayer, Ferreira and McMillan (2007); Bayer et al. (2016); Caetano (2016). Kuminoff, Smith and Timmins (2013) provide a comprehensive review of the growing literature on neighborhood sorting.

papers in this literature that discuss endogenous amenities are particularly related to our study. Bayer, McMillan and Rueben (2004*a*) develop a framework to estimate horizontal models of neighborhood choice by building on insights from the empirical industrial organization literature (Berry (1994); Berry, Levinsohn and Pakes (1995)). This framework has been widely applied and extended in this literature (e.g., Bayer, McMillan and Rueben (2004*b*); Bayer, Ferreira and McMillan (2007); Bayer, Keohane and Timmins (2009); Ringo (2013); Bayer et al. (2016); Caetano (2016)). They also discuss endogeneity that arises in the presence of an endogenous amenity such as the composition of neighbors. Bayer and Timmins (2005) study the existence and uniqueness of equilibrium in such sorting models with endogenous amenities, and Bayer and Timmins (2007) discuss estimation in empirical models like these and suggest an IV approach for identification based on the logic of a static model of neighborhood choice.

Our framework borrows several insights from these papers. As in this literature, we employ a discrete choice framework that enables us to study the relative importance of racial composition versus other amenities in explaining the sorting patterns that lead to segregation. This also allows us to embed moving costs as an additional friction that prevents sorting. A key departure lies in our weakening of assumptions on households' expectations when residential decisions are made. While these models assume that households possess complete (or nearly complete) information, we build a framework that is agnostic about how expectations are formed. This is crucial, as it renders our approach compatible with residential choices that are observed out of equilibrium. Another important departure is that our framework suggests a different strategy to estimate a dynamic model of residential choice with moving costs. Although this is not the first paper to do so in the context of neighborhood choice (see, for example, Bayer et al. (2016) and Caetano (2016)), we show that many standard assumptions in dynamic demand estimation can be avoided when the goal of researchers is to study segregation (as opposed to estimating the value of amenities as is typical in these studies). Finally, the IV approach that we develop is novel, and it follows from the logic of a dynamic model of neighborhood choice.

Disequilibrium Models of Segregation

A largely theoretical literature based on the seminal Schelling model (Schelling (1969, 1971)), has sought to explore how segregation can arise and evolve when households care about their neighbors (or, in principle, any endogenous amenity). In the Schelling

model, heterogeneous agents select where to live by simple rules of thumb. Although this purely heuristic model is not explicitly based on the optimization of an objective, it generates valuable insight into the fundamental social force that drives segregation: agents of different races react systematically differently to the racial composition of their neighbors. Schelling also makes explicit the role of some friction to ensure that neighborhoods gradually evolve toward an equilibrium state (e.g., myopia as in the original model).

Subsequent theoretical papers have embedded this intuition into a more standard economic framework (e.g., Becker and Murphy (2000); Bayer and Timmins (2005)), and there have been some recent attempts to estimate these models of segregation in reduced-form and structural contexts (e.g., Card, Mas and Rothstein (2008a); Banzhaf and Walsh (2013); Caetano and Maheshri (2017)). Banzhaf and Walsh (2013) discuss the role of exogenous amenities in generating segregation under no moving costs. Caetano and Maheshri (2017) develop an empirical implementation of these models in the context of school segregation. In this paper, we generalize and extend that framework in at least three key directions. First, we make fewer assumptions on households' expectations, thus imposing fewer restrictions on the way racial compositions may evolve. Second, we focus on general equilibria as opposed to partial equilibria.⁴ Third, we explicitly model realistic frictions such as moving costs, which yields novel IVs.

The rest of the paper proceeds as follows. In Section 2, we present a general conceptual framework for the analysis of segregation, and in Section 3, we present a detailed empirical implementation of our framework. In Section 4, we describe a data set of high frequency residential decisions in the San Francisco Bay Area that we use to demonstrate our framework in Section 5. We use our framework to analyze a variety of counterfactuals in Section 6 before concluding in Section 7.

2 Conceptual Framework

We develop a simple model that characterizes the dynamic process of neighborhood segregation. A city is divided into J neighborhoods, each of which are populated by

 $^{^4\}mathrm{Caetano}$ and Maheshri (2018) empirically identify general equilibria in a model of school segregation.

households of R different races.⁵ Let N_{jt}^r represent the number of households of race r who reside in neighborhood j in period t. In each period, each neighborhood possesses a single, multidimensional endogenous amenity: the racial composition of their residents, which we denote with a vector of racial shares $\mathbf{s}_{jt} = (s_{jt}^2, \ldots, s_{jt}^R)'$ where $s_{jt}^r = \frac{N_{jt}^r}{\sum_{r'} N_{jt}^{r'}}$. (Hereafter, we refer to all vectors and matrices in bold type.) The racial compositions of all neighborhoods in the city can be represented by the state matrix \mathbf{s}_t whose jth column is \mathbf{s}_{jt} . At the beginning of each period, households form expectations of their value of residing in each neighborhood and then choose where to reside.

We specify race r specific demand for neighborhood j as

$$N_{jt}^{r} = f_{j}^{r} \left(\boldsymbol{s_{t}^{r,e}}; \boldsymbol{\beta}^{r}, \boldsymbol{\phi_{t}^{r}} \right)$$

$$\tag{1}$$

where $f_j^r(\cdot; \boldsymbol{\beta}^r, \boldsymbol{\phi}_t^r)$ is a function unique to each neighborhood-race combination, and $s_t^{r,e}$ represents the expectations of s_t formed by households of race r. The parameter vector $\boldsymbol{\beta}^r$ represents the marginal effects of $s_t^{r,e}$ on demand, and the $J \times 1$ parameter vector $\boldsymbol{\phi}_t^r$ represents moving costs that households of race r would face if they moved from the house in which they lived in t-1.

Although N_{jt}^r and s_t are typically observable, $s_t^{r,e}$ is not, so it is infeasible to estimate β^r and ϕ_t^r directly from equation (1). To circumvent this issue, we use the actual, observed vector s_t as proxy for $s_t^{r,e}$, which yields

$$N_{jt}^{r} = f_{j}^{r} \left(\boldsymbol{s_{t}}; \boldsymbol{\beta^{r}}, \boldsymbol{\phi_{t}^{r}} \right) + \underbrace{f_{j}^{r} \left(\boldsymbol{s_{t}^{r,e}}; \boldsymbol{\beta^{r}}, \boldsymbol{\phi_{t}^{r}} \right) - f_{j}^{r} \left(\boldsymbol{s_{t}}; \boldsymbol{\beta^{r}}, \boldsymbol{\phi_{t}^{r}} \right)}_{\eta_{jt}^{r}}$$
(2)

where η_{jt}^r is a composite error term. With appropriate restrictions on $f_j^r(\cdot; \boldsymbol{\beta}^r, \boldsymbol{\phi}_t^r)$, the parameters $\boldsymbol{\beta}^r$ and $\boldsymbol{\phi}_t^r$ can be estimated in a dynamic discrete choice model which we describe in our implementation below. The marginal effects $\boldsymbol{\beta}^r$ are identified with novel instrumental variables, and the moving costs $\boldsymbol{\phi}_t^r$ are identified by observing those households who chose not to move.

Given estimates of $\hat{\beta}^r$ and ϕ^r_t , we can analyze how the racial compositions of neighborhoods might evolve under different counterfactual values of $s_t^{r,e}$. The counterfactual

⁵In our empirical application, we restrict our analysis to White, Black, Hispanic and Asian households (R = 4). They constitute well over 95% of the households in our sample. In principle, groups could be defined at more or less aggregate levels and along alternative (non-racial) dimensions per the application.

demand of households of race r for neighborhood j when $s_t^{r,e} = \tilde{s}$ is equal to

$$N_{jt}^{r}\left(\tilde{\boldsymbol{s}}\right) = f_{j}^{r}\left(\tilde{\boldsymbol{s}}; \hat{\boldsymbol{\beta}}^{r}, \hat{\boldsymbol{\phi}}_{t}^{r}\right)$$
(3)

from which we can obtain

$$s_{jt}^{r}\left(\tilde{\boldsymbol{s}}\right) = \frac{N_{jt}^{r}\left(\tilde{\boldsymbol{s}}\right)}{\sum_{r'}N_{jt}^{r'}\left(\tilde{\boldsymbol{s}}\right)}$$
(4)

Calculating equation (4) for each race yields the matrix value function $s_t(\tilde{s})$, whose *j*th column is $(s_{jt}^2(\tilde{s}), \ldots, s_{jt}^R(\tilde{s}))'$. This approach can be repeated for any counterfactual value of \tilde{s} , which allows us to identify the function $s_t(\cdot)$ by simulation. This function is useful, as it relates to equilibrium concepts in the residential sorting literature and allows us to characterize the dynamic properties of equilibria as disequilibrium models of segregation.

Following Bayer and Timmins (2005), we define a *sorting equilibrium* as a state s^* that does not change from period to period in the absence of shocks. This concept relates to our model as follows:

Definition 1. Sorting Equilibrium. State s^* is a sorting equilibrium if $s_t(s^*) = s^*$.

 $s_t(\cdot)$ defines a dynamic system that completely characterizes the evolution of neighborhood segregation from any initial state. By repeatedly evaluating $s_t(\cdot)$ at \tilde{s} , we can construct a *simulated trajectory* $T_t(\tilde{s}) = (\tilde{s}, s_t(\tilde{s}), s_t(s_t(\tilde{s})), \ldots)$, which can be used to evaluate the stability of equilibria. We define two stability concepts below:

Definition 2. Stable Equilibrium. Sorting equilibrium s^* is stable if there exists a neighborhood A of s^* such that for all $s' \in A$, $T_t(s')$ converges to s^* .

Sorting equilibria that are not stable are often referred to as "tipping points."

Definition 3. Stable Trajectory. A simulated trajectory $T_t(\tilde{s})$ is stable if there exists a neighborhood A of \tilde{s} such that for all for all $s' \in A$, $T_t(s')$ converge to the same state.⁶

It is useful to know if the trajectory simulated from neighborhoods as currently observed is stable. If so, modest policies will not affect segregation in the long run, so policymakers can be less worried about unintended consequences of other policies on neighborhood segregation.

⁶Of course, the trajectory $T_t(s^{\star})$ is trivially stable.

Remark 1. In our framework, households can be thought of as players in a game of imperfect information where the action space is the set of possible neighborhoods from which they can choose to reside in each period. In this vein, we first estimate (causal) best response functions, and we subsequently identify subgame perfect Bayesian Nash equilibria by simulation.

Remark 2. In this paper, we assume that the racial composition of neighborhoods is the only endogenous amenity of interest. With the appropriate data, one might propose to allow for households to consider additional endogenous amenities (e.g., home prices, the incomes of neighbors, etc.). This is not a good idea because these variables are post-determined from the perspective of $s_t^{r,e}$. To see this, suppose that we considered neighborhood home prices, P_t , as an explicit argument of f_j^r . Changes in $s_t^{r,e}$ may affect the demands of households of all races, and P_t may in turn change to accommodate excess supply or demand. For example, if more White households wanted to move into a neighborhood than households of other races wanted to leave, the neighborhood would experience excess demand at the original price. If we wanted to explicitly model this causal channel, we would not only have to estimate how $s_t^{r,e}$ affects s_t (holding P_t constant), we would also have to estimate how $s_t^{r,e}$ (for each race) affects P_t and how P_t affects s_t . Estimating these additional parameters would require further assumptions related to the price elasticities of supply and demand in addition to assumptions on the adjustment path of prices that balances supply and demand.

We avoid making these assumptions by not explicitly controlling for P_t when estimating β^r . In doing so, our estimates of $\hat{\beta}^r$ capture the full impact of $s_t^{r,e}$ on s_t either directly or indirectly (via prices or changes to other neighborhood amenities). This also ensures that the simulation described in equation (4) is well-defined. Thus, we select the set of endogenous amenities parsimoniously by focusing on a single *primitive* dimension along which households sort. For a study of racial segregation, this dimension is naturally the racial composition of neighborhoods.

To be sure, our strategy of not controlling for price or other post-determined variables affects our interpretation of β^r . If we find that Whites (Blacks) respond to a reduction in the share of Black neighbors by entering (leaving) a neighborhood, then this finding is compatible with many scenarios, including one in which only Whites are biased against Blacks and one in which Whites and Blacks are biased against Blacks with the same intensity but Whites are less price sensitive than Blacks (so Whites outbid Blacks for the houses in that neighborhood). For our purposes, we need not distinguish between these different channels because we simply want to understand how the racial compositions of neighborhoods endogenously evolve over time.

Remark 3. A key difference between our approach and other empirical approaches to study segregation is that ours allows for the racial composition of neighborhoods to be observed out of equilibrium. This is intimately related to assumptions on the expectations of households. Note that $s_t(s_t^{r,e}) = s_t$ by construction since the actual choices that are observed are made when $\tilde{s} = s_t^{r,e}$. Thus, assuming $s_t^{r,e} = s_t$ implies assuming data are observed in equilibrium since $s_t(s_t^e) = s_t = s_t^e$. Similarly, small deviations from that assumption (e.g., information set is the same across households *i* apart from zero mean private information) will imply data to be observed in equilibrium as well. Therefore, it is crucial that we do not constrain the formation of households' expectations too restrictively if we wish to study segregation.

Remark 4. Equation (2) may raise some concern of simultaneity when estimating β^r since s_t^r and N_{jt}^r are jointly determined by construction. However, the simultaneity is broken by the fact that $s_t^{r,e}$ causes N_{jt}^r and N_{jt}^r causes s_t^r (trivially, via equation (4)). Although in practice we use s_t^r as proxy for $s_t^{r,e}$, we instrument for s_t^r with data prior to period t to avoid these issues.

3 Empirical Implementation

In this section we formalize a model of demand underlying equation (1) and impose some restrictions that allow for the feasible estimation of the function $f_j^r(\cdot; \boldsymbol{\beta}^r, \boldsymbol{\phi}_t^r)$ with data that is typically available. We then describe the empirical implementation of our approach in three stages.

A Dynamic Model of Residential Choice

At the beginning of period t, households choose whether or not to move. Conditional on deciding to move, they then choose an option j = 0, ..., J. Options j = 1, ..., Jcorrespond to residing in neighborhood j. Option j = 0 corresponds to the outside option of residing outside of the city.⁷ Households face a moving cost of ϕ_t^r , and j_{it-1} denotes where the household lived in t - 1. Following Bayer et al. (2016), we simplify

⁷As in Bayer et al. (2016), we only observe data on homeowners, so in our application, j = 0 also corresponds to the outside option of renting within the city.

notation and index the option of staying in the same house in neighborhood j = 1, ..., Jas option J + 1. For each j = 1, ..., J + 1, r and t, we observe N_{jt}^r , the total number of race r households who choose option $j_{it} = j$. For j = 0, ..., J, we observe n_{jt}^r , which represents the number of inflows into neighborhood j from t - 1 to t.

We define the choice-specific cumulative utility of household i of race r from owning a house in neighborhood j in period t as $v_{jt}^r + \epsilon_{ijt}^r$. In each period, household i of race r observes the vector $(\boldsymbol{v}_t^r, \phi_t^r, j_{it-1}, \boldsymbol{\epsilon}_{it}^r)$ and chooses j in order to maximize their cumulative utility given by

$$V_{ijt}^{r}\left(\boldsymbol{v_{t}^{r}}, \phi_{t}^{r}, j_{it-1}, \boldsymbol{\epsilon_{it}^{r}}\right) = \mathbb{I}_{\{j=J+1\}}\left(v_{j_{it-1}t}^{r} + \epsilon_{iJ+1t}^{r}\right) + \mathbb{I}_{\{j\in0,\dots,J\}}\max_{k\in\{0,\dots,J\}}\left(v_{kt}^{r} - \phi_{t}^{r} + \epsilon_{ikt}^{r}\right)$$
(5)

where \mathbb{I} is the indicator function. The error term ϵ_{ijt}^r is assumed to be i.i.d. extreme value 1.

Our approach unfolds in three stages: we first estimate v_{jt}^r and ϕ_t^r for all j, t and r (stage 1) and then we estimate the causal effect of the expected endogenous amenity (racial composition of j) on v_{jt}^r (stage 2). Finally, we use these estimates to simulate the evolution of the racial compositions of neighborhoods under different counterfactuals (stage 3).

Remark 5. The buying or selling of a house may impact household wealth. Despite its undeniable importance when studying the behavior of homeowners, we do not explicitly model the effects of moving on wealth, and we do not allow for household heterogeneity by wealth. In our context, doing so would not only substantially increase the number of types of households rendering the estimation infeasible (particularly for non-White households, as discussed in Bayer et al. (2016)), but it would require additional assumptions on the joint formation of expectations of the racial compositions and wealth of all neighborhoods (see Remark 2).

Stage 1: Estimating Cumulative Household Utilities

First, we consider households who move. Their choices are used to estimate the cumulative utilities v_{jt}^r . Having decided to move, race r households solve the following optimization problem:

$$\max_{k \in \{0,\dots,J\}} v_{kt}^r - \phi_t^r + \epsilon_{ikt}^r \tag{6}$$

Following the logit formula, the choice-specific probabilities are

$$P(j_{it} = j \mid j \in \{0, ..., J\}, j_{it-1}) = \frac{\exp\left(v_{jt}^r - \phi_t^r\right)}{\sum_{k=0}^J \exp\left(v_{kt}^r - \phi_t^r\right)} = \frac{\exp\left(v_{jt}^r\right)}{\sum_{k=0}^J \exp\left(v_{kt}^r\right)}$$
(7)

Because moving costs do not vary by option, they cancel out.⁸ The data analog to the choice-specific probability is simply $\frac{n_{jt}^r}{\sum_{k=0}^J n_{kt}^r}$. We estimate \hat{v}_{jt}^r for $j \in \{0, \ldots J\}$ as

$$\hat{v}_{jt}^r = \log\left(n_{jt}^r\right) - \log\left(n_{J0t}^r\right).$$
(8)

Next, we consider households who do not move. Their decisions, along with estimates of \hat{v}_{jt}^r , can be used to identify the moving cost parameters ϕ_t^r . For a race rhousehold, the choice-specific probability for option J + 1 is

$$P(j_{it} = J + 1 | j_{it-1} = j) = P(v_{jt}^r + \epsilon_{iJ+1t}^r > v_{kt}^r - \phi_t^r + \epsilon_{ikt}^r | j_{it-1} = j)$$

=
$$\frac{\exp(v_{jt}^r)}{\exp(v_{jt}^r) + \sum_{k=0}^J \exp(v_{kt}^r - \phi_t^r)}$$
(9)

where the first line must hold for all k = 1, ..., J+1, and the second line follows from the logit formula. If we define stayers^{*r*}_{*jt*} to be the number of households who lived in neighborhood *j* in *t* - 1 and did not move, then the data analog to $P(j_{it} = J + 1 | j_{it-1} = j)$ is simply $\frac{\text{stayers}^{r}_{jt}}{N^{r}_{jt-1}}$; hence, equation (9) yields the *J* moment restrictions

$$g_j\left(\phi_t^r; \boldsymbol{v}_t^r\right) = \frac{\text{stayers}_{jt}^r}{N_{jt-1}^r} - \frac{\exp\left(v_{jt}^r\right)}{\exp\left(v_{jt}^r\right) + \sum_{k=0}^J \exp\left(v_{kt}^r - \phi_t^r\right)}$$
(10)

for each j = 1, ..., J. By plugging in our estimates of \hat{v}_{jt}^r from equation (8) into the moment condition for v_{jt}^r , we can estimate the moving costs parameters ϕ_t^r by GMM.⁹ *Remark* 6. For our purposes, we are interested in identifying only the causal effects of expected endogenous amenities on households' *choices*, which are linked to cumulative

⁸This insight is due to Bayer et al. (2016).

⁹We allow for heterogeneity in moving costs by race and by year. In our application, it is infeasible to allow for heterogeneity in moving costs by neighborhood for races other than Whites.

utilities. In contrast, most empirical analyses of residential choice seek to identifying household *preferences*, which are typically understood as parameters of households' flow utilities (Bayer et al. (2016); Caetano (2016)). Hence, to study segregation we can estimate a dynamic choice model with less structure on the state variables and on their expected transitions over time since there is no need to separate the flow utility from the cumulative utility.

Stage 2: Estimating the Causal Effect of Endogenous Amenities on the Choice of Neighborhood

We decompose the average cumulative utility that households of race r obtain from living in neighborhood j in period t as

$$v_{jt}^r = \boldsymbol{\beta}^{r's} \boldsymbol{s}_{jt}^{r,e} + \gamma_t^r + \tilde{\xi}_{jt}^r, \quad j = 1, ..., J$$

$$(11)$$

The parameters of interest, β^r , represent the causal effects of $s_{jt}^{r,e}$ on v_{jt}^r . γ_t^r is a raceperiod fixed effect, and $\tilde{\xi}_{jt}^r$ is an error term that includes all remaining unobserved determinants of v_{jt}^r .

Because we do not observe v_{jt}^r or $s_{jt}^{r,e}$, we use \hat{v}_{jt}^r and s_{jt} as proxies for them. Accordingly, we rewrite equation (11) as

$$\hat{v}_{jt}^r = \boldsymbol{\beta}^{r'} \boldsymbol{s}_{jt} + \gamma_t^r + \xi_{jt}^r, \quad j = 1, ..., J$$
(12)

where the error term ξ_{jt}^r is equal to

$$\xi_{jt}^{r} = \tilde{\xi}_{jt}^{r} + \boldsymbol{\beta}^{r'} \left(\boldsymbol{s}_{jt}^{r,e} - \boldsymbol{s}_{jt} \right) + \left(\hat{v}_{jt}^{r} - v_{jt}^{r} \right)$$
(13)

Equation (12) should be understood as a transformation of equation (2) from the framework above. The first term on the right-hand-side of equation (13) corresponds to unobserved determinants of households' cumulative utilities. The second term corresponds to errors in households' expectations. The third term corresponds to any bias in the estimation of households' cumulative utilities that arose in the first stage. ξ_{jt}^r is potentially correlated to s_{jt} , which would bias an OLS estimate of β^r . We address this endogeneity problem with synthetic instrumental variables that follow from the logic of the dynamic choice model.

Instrumental Variables

To identify β^r , we exploit the idea that \hat{v}_{jt}^r is a *flow* variable whereas s_{jt} is a *stock* variable. While \hat{v}_{jt}^r only reflects the value of neighborhood j in period t, s_{jt} also reflects how the neighborhood was valued in $t - 1, t - 2, \ldots$ when other residents previously moved into or out of the neighborhood. We leverage this asymmetry to construct an IV that plausibly affects \hat{v}_{jt}^r only through s_{jt} by isolating the transitory components of past valuations of the neighborhood. In short, we identify β^r off of transitory shocks that affected s_{jt} through previous choices that households made that no longer directly influence choices in t.

Specifically, we use s_{jt-2} as an IV for s_{jt} in the following equation

$$\hat{v}_{jt}^{r} = \boldsymbol{\beta}^{r'} \boldsymbol{s_{jt}} + \gamma_{t}^{r} + h^{r} \left(v_{jt-1}^{W}, v_{jt-1}^{B}, v_{jt-1}^{H}, v_{jt-1}^{A} \right) + \mu_{jt}^{r}$$
(14)

where $\mu_{jt}^r = \xi_{jt}^r - h^r(\cdot)$, and $h^r\left(v_{jt-1}^W, v_{jt-1}^B, v_{jt-1}^H, v_{jt-1}^A\right)$ is a flexible control function.

Following the choice model, s_{jt}^r and s_{jt-2}^r are correlated because of shocks in $v_{kt-2}^{r'}, v_{kt-3}^{r'}, ...$ that either (a) persist until t or (b) do not persist until t but nevertheless affected households who moved into (out of) neighborhood j as of t-2 and chose to remain in (stay out of) the neighborhood until t due to moving costs. Shocks of type (a) are problematic, as they may be correlated to μ_{jt}^r , but shocks of type (b) are, by construction, uncorrelated to μ_{jt}^r . By holding v_{jt-1} constant through the use of the control function $h^r(\cdot)$, we absorb the variation in our instruments that is due to shocks of type (a), thus isolating variation from shocks of type (b) alone. Put another way, our identifying assumption is that shocks to households' valuations of neighborhood j are Markov(1), i.e., shocks in $t-2, t-3, \ldots$ are uncorrelated to shocks in t conditional on shocks in $t-1.^{10}$

The logic of our IV can be leveraged to weaken the identifying assumption. For any $T \ge 2$, we can use the component of s_{jt-T} that is orthogonal to v_{jt-1} as an IV for s_{jt} . Larger choices of T imply a plausibly weaker identifying assumption (amenity shocks affecting decisions in t - T that are irrelevant to decisions in t - 1 are also irrelevant to decisions in t).

¹⁰Because we use the cumulative utilities rather than the flow utilities as controls in $h^r(\cdot)$, our identifying assumption is actually weaker than it seems. The only instance in which it will be violated is if a component of s_{jt-2} happens to show up again in v_{jt}^r as a surprise to all races (i.e., households of all races as of t-1 were not able to predict that shock as of t-1). It has to be a surprise, as the predictable component of v_{jt}^r (as of t-1) is already included in v_{jt-1} if households are forward looking.

Remark 7. It is worth discussing why our IV is plausibly uncorrelated to $\beta^{r'} (s_{jt}^{r,e} - s_{jt})$, the second component of ξ_{jt}^r in equation (13). Note that v_{jt}^r actually represents $v_{jt}^{r,e}$, the expectation of the value of neighborhood j in t formed by households of race r. Because $v_{jt}^{r,e}$ and $s_{jt}^{r,e}$ influence decisions made at the same time by the same households, these expectations were formed with the same information. Thus, any component of s_{jt} affecting $v_{jt}^{r,e}$ must also affect $s_{jt}^{r,e}$. It follows that any valid IV of s_{jt} would affect $v_{jt}^{r,e}$ only through $s_{jt}^{r,e}$ and not through $s_{jt} - s_{jt}^{r,e}$.

Stage 3: Identifying Sorting Equilibria by Simulation

Once we obtain estimates of \hat{v}_{jt}^r , $\hat{\phi}_t^r$ and $\hat{\boldsymbol{\beta}}^r$, we can identify how the racial composition of each neighborhood evolves from any initial state in the absence of external shocks. We denote the population distribution of the entire city with race-specific population vectors $\boldsymbol{N}_t^r = (N_{1t}^r, \dots, N_{Jt}^r)$ which imply share vectors $\boldsymbol{s}_t^r = (s_{1t}^r, \dots, s_{Jt}^r)$ and a racial composition matrix $\boldsymbol{s}_t = (\boldsymbol{s}_t^{B\prime}, \boldsymbol{s}_t^{H\prime}, \boldsymbol{s}_t^{A\prime})'$.

Equation (12) represents the causal relationship between the expected racial composition of a neighborhood and \hat{v}_{jt}^r . For any given counterfactual matrix of expected racial compositions of neighborhoods $\tilde{s} = (\tilde{s}_1, ..., \tilde{s}_J)$, we write the implied expected valuation for neighborhood j of race r households in period t as

$$v_{jt}^{r}\left(\tilde{\boldsymbol{s}}\right) = \hat{v}_{jt}^{r} + \hat{\boldsymbol{\beta}}^{r\prime}\left(\tilde{\boldsymbol{s}}_{\boldsymbol{j}} - \boldsymbol{s}_{\boldsymbol{jt}}\right), \quad \boldsymbol{j} = 1, ..., J$$
(15)

We simultaneously simulate implied race-specific demands for all neighborhoods from any given starting counterfactual \tilde{s}_0 as

$$\tilde{N}_{jt+1}^{r}\left(\tilde{\boldsymbol{s}}_{t}\right) = \tilde{N}_{jt}^{r}\left(\tilde{\boldsymbol{s}}_{t}\right) \left(\frac{\exp\left(v_{jt}^{r}\left(\tilde{\boldsymbol{s}}_{t}\right)\right)}{\exp\left(v_{jt}^{r}\left(\tilde{\boldsymbol{s}}_{t}\right)\right) + \sum_{k'=1}^{J}\exp\left(v_{k't}^{r}\left(\tilde{\boldsymbol{s}}_{t}\right) - \hat{\phi}_{t}^{r}\right) + \exp\left(-\hat{\phi}_{t}^{r}\right)}\right) + (16)$$

$$+ \sum_{k=1}^{J} \tilde{N}_{kt}^{r}\left(\tilde{\boldsymbol{s}}_{t}\right) \left(\frac{\exp\left(v_{kt}^{r}\left(\tilde{\boldsymbol{s}}_{t}\right)\right) - \hat{\phi}_{t}^{r}\right)}{\exp\left(v_{kt}^{r}\left(\tilde{\boldsymbol{s}}_{t}\right)\right) + \sum_{k'=1}^{J}\exp\left(v_{k't}^{r}\left(\tilde{\boldsymbol{s}}_{t}\right) - \hat{\phi}_{t}^{r}\right) + \exp\left(-\hat{\phi}_{t}^{r}\right)}\right)$$

with initial condition $\tilde{N}_t^r(\tilde{s}_0) = N_t^r$ and the identity $\tilde{s}_{jt}^r = \frac{\tilde{N}_{jt}^r(\tilde{s}_{t-1})}{\sum_{r'} \tilde{N}_{jt}^{r'}(\tilde{s}_{t-1})}$. The first term on the right-hand side of equation (16) corresponds to the simulated number of households who resided in neighborhood j in t-1 and remained in their house, incurring

no moving costs. The second term represents the simulated number of households who chose option k in t-1 and then moved to neighborhood j (households with k = jmoved within neighborhood j). Because our simulation explicitly holds fixed all factors that affect households' propensity to choose the outside option (i.e., $v_{0t}^r(\tilde{s}_t)$), we only consider the re-sorting of households who chose neighborhoods $j \in 1, ..., J$ in t.¹¹

Putting these together, we can define the implied racial composition of a neighborhood as

$$s_{jt+1}^{r}\left(\tilde{\boldsymbol{s}}_{t}\right) = \frac{\tilde{N}_{jt+1}^{r}\left(\tilde{\boldsymbol{s}}_{t}\right)}{\sum_{r'\in R}\tilde{N}_{jt+1}^{r'}\left(\tilde{\boldsymbol{s}}_{t}\right)}$$
(17)

$$=\tilde{s}_{t+1} \tag{18}$$

This empirical analog to equation (4) can be collected into a well-defined matrix-valued function $\mathbf{s}_t(\tilde{\mathbf{s}}) : [0,1]^{R-1 \times J} \to [0,1]^{R-1 \times J}$ whose (r,j) element is equal to $s_{jt}^r(\tilde{\mathbf{s}})$. The function $\mathbf{s}_t(\tilde{\mathbf{s}})$ is implicitly defined by equations (15), (16) and (17). Note that the counterfactual manipulation concerns households' expectations just before households make their decisions in t (i.e., we set $\mathbf{s}_t^{r,e} = \tilde{\mathbf{s}}$, not $\mathbf{s}_t^r = \tilde{\mathbf{s}}$).¹²

In principle, we could identify all sorting equilibria as fixed points of the function $s_t(\cdot)$ by conducting a grid search of all possible states \tilde{s} and computing $s_t(\tilde{s})$ for each counterfactual using equations (15), (16) and (17). Given a sufficiently fine grid and tolerance δ , those states \tilde{s} for which $\|s_t(\tilde{s}) - \tilde{s}\| < \delta$ for all $t > \bar{t}$ can be interpreted as sorting equilibria. Because the domain of the grid search is very large ($[0, 1]^{3 \times J}$), it may be computationally infeasible to identify all such equilibria.

Instead, we compute the simulated trajectory from s_t under several specific counterfactuals. Our baseline counterfactual that we discuss in Section 5 assumes discriminatory responses and moving costs equal to those that we estimate, and racial compositions of neighborhoods as observed at the end of our sample period. A simulation

¹¹Because we do not want to allow for movement into or out of the outside option in our simulation, we re-weight our simulated $\tilde{N}_{jt'}^r$ to ensure that $\sum_{j=1}^J \tilde{N}_{jt'}^r = \sum_{j=1}^J N_{jt}^r$ for all t'. In practice, this does not affect our baseline results.

¹²In principle, when choosing different counterfactuals, we could allow these expectations to vary by race: $s_t^{r,e} \neq s_t^{r',e}$. However, the dimensionality of the counterfactual – and hence the computational complexity of the simulation procedure – would increase by a factor of 4. Thus, in our implementation of this approach, we consider only counterfactuals where different races share the same expectations. This does not restrict our search for equilibria, since at any equilibrium, $s_t^{r,e} = s_t^{r',e}$ for all r, r' must occur. Nonetheless, this restriction does constrain the deviations from equilibria that we can analyze.

under this counterfactual yields the equilibrium s^* to which the system would converge solely because of the endogenous discriminatory process. This is particularly meaningful, as it describes how neighborhoods will evolve in the absence of outside shocks.¹³ If $s_t \neq s^*$, we can conclude that the data are not observed in equilibrium.¹⁴ Moreover, this provides a useful benchmark to evaluate the alternative counterfactuals that we consider in Section 6.

4 Data

We apply our framework to a monthly sample of all San Francisco Bay Area neighborhoods from January 1990 to November 2004. We define the San Francisco Bay Area as the six core counties (Alameda, Contra Costa, Marin, Santa Clara, San Francisco and San Mateo counties) that comprises the major cities of San Francisco, Oakland and San Jose and their surroundings, and we divide the sample region into 224 neighborhoods. Neighborhoods are defined by merging contiguous Census tracts until each resulting neighborhood contains approximately 10,000 households. Those neighborhoods with fewer than six annual home sales in our sample period are dropped.

For each neighborhood in each month, we compute estimates of their racial composition following the approach described in Bayer et al. (2016).¹⁵ Because high frequency data on the racial composition of neighborhoods is unavailable from standard sources (e.g., the Census) we must merge information from two main sources in order to construct these variables. The first source is Dataquick Information Services, a national real estate data service. Dataquick provides a detailed listing of *all* real estate transactions in the Bay Area including buyers' and sellers' names, buyer's mortgage information and property locations. The second source is a a dataset on mortgage applications published in accordance with the Home Mortgage Disclosure Act (HMDA) of 1975. Notably, HMDA data contains demographic information on mortgage applicants and the locations of properties that the applicants are buying. By linking these datasets on buyer's mortgage information and property locations, we can estimate how

¹³It is possible that the simulated trajectory from s_t will perpetually oscillate and never reach an equilibrium. This is unlikely if households are homophilic (i.e., they tend to choose similar race neighbors).

¹⁴By perturbing s^* and simulating trajectories from the perturbed states, we could ascertain if s^* was a stable equilibrium. Similarly, by perturbing s_t and simulating trajectories from the perturbed states, we could ascertain the stability of the simulated trajectory from s_t .

¹⁵We gratefully acknowledge Bayer et al. (2016) for sharing their raw data and programs to convert this raw data into a usable dataset for our analysis.

Variable	White	Black	Hispanic	Asian
Average Number of Homeowners per Neighborhood	4,104 (3,075)	284 (457)	517 (578)	841 (1068)
Average Monthly Neighborhood Inflows	11.08 (12.66)	0.68 (1.36)	2.18 (3.79)	4.72 (7.91)
Average Monthly Neighborhood Outflows	3.52 (6.04)	$\begin{array}{c} 0.19 \\ (0.59) \end{array}$	$0.57 \\ (1.54)$	1.22 (3.06)
Total Number of Observations	40,096			

Table 1: Summary Statistics

Note: Standard deviations are presented in parentheses.

the demographics of neighborhoods change with each real estate transaction. With neighborhood-level estimates of the flows of households of different races, we estimate the actual demographic composition of each neighborhood by anchoring our flow estimates to the actual racial composition of each neighborhood per the 1990 US Census. ¹⁶

We conduct this procedure for each month of our sample period in each Bay Area neighborhood separately for White, Black, Hispanic and Asian households, which yields the total number of homeowners, inflows to each neighborhood, and outflows from each neighborhood. We summarize our data in Table 1. The majority of homeowners in the Bay Area are White, although there are sizable Asian and Hispanic populations as well. The high variance in the race specific populations reflects substantial crosssectional heterogeneity in the racial composition of neighborhoods. This composition also changes over time in our sample as reflected in net monthly inflows (inflows minus outflows) on the order of approximately 0.2%-0.4%.

 $^{^{16}}$ Bayer et al. (2016) report the results of multiple diagnostic tests that ensure the validity of this estimation procedure.

		White	Black	Hispanic	Asian
Responses to:	s^B_{jt}	-2.92^{***} (0.35)	5.61^{***} (0.45)	-0.12 (0.37)	-2.12^{***} (0.41)
	s^H_{jt}	-3.67^{***} (0.51)	-0.61 (0.69)	9.60^{***} (0.55)	-1.81^{***} (0.57)
	s^A_{jt}	-1.48^{***} (0.25)	-0.73 (0.47)	-2.23^{***} (0.46)	5.70^{***} (0.35)
R^2			0.48		
Num. Obs.			$124,\!544$		

Table 2: IV Estimation Results - Responses to the Racial Compositions of Neighborhoods (β^r)

Notes: This specification includes race-month fixed effects and control variables (v_{jt-1}^r) through v_{jt-12}^r for r = W, B, H, A, with s_{jt-13}^r for r = B, H, A as instrumental variables. All standard errors clustered by race-month. * - 90% significance, ** - 95% significance, *** - 99% significance. The p-value for a joint test of whether all instruments are relevant in the first stage is 0.00.

5 Empirical Results

In Table 2, we present estimates of the β^r parameter vectors from Equation (14) that represent the causal responses to endogenous amenities for each race. Each of the endogenous amenities is instrumented by the racial compositions of each neighborhood in t-2, and the control variables v_{it-1}^r are specified linearly.

Homeowners of all races respond positively to neighborhoods with a greater share of residents of their own race, i.e., they are more likely to move into such neighborhoods. Such homophilic forces contribute to sorting patterns that increase racial segregation. White and Asian homeowners react negatively to greater shares of Black and Hispanic residents, whereas Black and Hispanic homeowners have neutral reactions to greater shares of Hispanic and Black residents respectively. Whites and Hispanics react slightly negatively to greater shares of Asian residents. These repelling forces can also contribute to sorting patterns that increase residential segregation.

In Appendix A, we reestimate β^r using instrumental variables from more distant earlier periods to check the validity of our IV strategy, demonstrating the robustness of our results (Figure 11). We also present OLS estimates of β^r (Table 3). These are much larger in magnitude than our IV estimates as expected since there are many other



Figure 2: Estimated Moving Costs Over Time

Notes: Race- and year- specific moving costs are estimated by GMM with moment conditions given in Equation (10). Moving costs denominated in units of utils. Standard errors of all parameter estimates are below 0.6.

(confounding) reasons why similar homeowners would choose the same neighborhoods, all of which would bias the OLS estimates upwards. Indeed, the OLS bias is most pronounced in the within-race parameter estimates (e.g., s_{jt}^B for Black homeowners), which is reassuring.

In Figure 2, we present estimates of moving costs over time that vary by race and year.¹⁷ White, Hispanic and Asian homeowners' moving costs are of similar magnitude, and they decrease moderately by about 15% over the period from 1991-2004. Black homeowners have systematically lower moving costs (roughly 10% lower) than homeowners of other races, but these costs do not decrease over the sample period, so by the end of the sample period, all homeowners have roughly the same moving costs.

With these estimates, we can simulate how the racial compositions of neighborhoods would evolve in the absence of external shocks. For narrative purposes, we focus on the results of a simulation that begins in the final month of our sample, November 2004. In Figure 3, we present a graph of the number of neighborhoods that experience at least 1, 2, 5 or 10 simulated inflows of homeowners in a given future month. We describe such neighborhoods as "in flux." Initially, nearly all neighborhoods are in flux. From this result, we can conclude that neighborhoods are actually not observed to be in equilibrium. Even with substantial moving costs, the amenities of the neighborhood where households currently reside are sufficiently unattractive to enough households that most

¹⁷Note that the estimates of moving costs are denominated in utils, not dollars. Hence, they should only be interpreted relative to each other. Confidence intervals for moving costs have been omitted for clarity, but each estimate of ϕ_t^r is statistically significantly different from zero at the 99% level.

Figure 3: Number of Neighborhoods In Flux (Simulated)



Notes: Figure shows the number of neighborhoods with at least one, two, five or ten monthly inflows + outflows of all races (out of a total of 224 neighborhoods). Simulation begins in November 2004.

neighborhoods experience turnover. Over time, changes in the racial compositions of these neighborhoods feedback and also spill over to other neighborhoods, which in turn changes their relative attractiveness to homeowners of all races. After ten years, all neighborhoods are out of "extreme" flux with ten or more monthly moves, but many still experience two or more moves. Indeed, it is not until twenty years pass that the Bay Area effectively reaches sorting equilibrium.

The outcome of this pattern of sorting is a change in the levels of segregation in the Bay Area. In Figure 4, we present the evolution of the dissimilarity index for each race across all Bay Area neighborhoods.¹⁸ We choose this widely used measure of segregation because it is easy to interpret. For instance, a dissimilarity index of 0.34 for White homeowners, indicates that 34% of White homeowners would have to be relocated in order to generate an equal distribution of Whites across all Bay Area neighborhoods. Thus, the index ranges from zero to one, and a higher value means that homeowners of a given race are more concentrated in certain neighborhoods.

Over the course of the simulation, White segregation increases by only three percentage points (a 8% increase), which is likely due to the fact that they seek same-race neighbors less strongly than other races.¹⁹ Black homeowners start off much more seg-

¹⁸The dissimilarity index for race r households is defined as $\frac{1}{2} \sum_{j} \left| \frac{N_{rj}}{\sum_{j} N_{r}} - \frac{N_{r'j}}{\sum_{j} N_{r'j}} \right|$ where $N_{r'j} = \sum_{s \neq r} N_{sj}$.

¹⁹Areas with a higher concentration of Whites tend to be more expensive, so Whites may seek samerace neighbors less strongly than other races do purely because it is more expensive to do so for Whites than for other races. This is in line with our broader interpretation of β^r as discussed in Remark 2.



Figure 4: Trajectories of Segregation Levels by Race (Simulated)

Notes: Simulated Dissimilarity Index for households of each race. A White dissimilarity index for of, say, 0.34, means that 34% of White homeowners would have to be relocated in order to generate an equal distribution of Whites across all Bay Area neighborhoods. Simulation begins in November 2004.

regated than other homeowners, and over the course of the simulation, their segregation increases by roughly ten percentage points (an 18% increase). Similarly, Hispanic segregation rises by almost six percentage points (a 17% increase). These large increases can be explained by the fact that both groups of homeowners seek same-race neighbors very strongly, and both elicit large negative demand responses from other races. Finally, Asian segregation rises by roughly two percentage points (a 6% increase). Although Asians seek Asian neighbors very intensely, non-Asian homeowners do not mind having them as neighbors very much, which mitigates their isolation.

6 Determinants of Long-Run Segregation

In this section we study the roles of discrimination, moving costs, incomplete information and the initial allocations of households in explaining the long-run levels of segregation that we found in Section 5. We weigh the importance of these determinants by leveraging the various moving parts of our framework to simulate several relevant counterfactuals. This ensures that we allow for complex sorting patterns to emerge that would otherwise be difficult to predict but are nonetheless integral to the dynamic process of segregation. Indeed, the discriminatory responses ($\hat{\beta}^r$) that we estimate may not necessarily increase segregation as one may naively expect. For instance, White homeowners fleeing a neighborhood that is becoming more Black will, all else constant, increase not only the Black share of neighbors, but also the Hispanic and Asian shares of neighbors. That in turn may lead to further inflows of not only Blacks, but also Hispanics and Asians.²⁰ The complexity of this sorting pattern grows over time not only because all races continue to respond endogenously to each of these changes in a given neighborhood, but also because they respond to concomitant changes in other neighborhoods.

6.1 The Roles of Discriminatory Responses

Our estimates of $\hat{\beta}^r$ reveal systematic discriminatory responses for homeowners of all races. To isolate their roles in explaining the patterns of segregation dynamics presented in Figures 3-4, we consider a series of counterfactuals in which certain races are "race-blind", i.e. are indifferent about the racial composition of their neighbors ($\beta^r = 0$ for certain races r). We present the simulated increase in segregation under each of these counterfactuals in Figure 5, where the label on each bar denotes the races that are assumed to be race-blinded under each counterfactual. Each panel shows the long-run increase (in percentage terms relative to its value in November 2004) in the dissimilarity index for households of each race.

In the first bar, all homeowners respond to changes in the racial compositions of neighborhoods as reported in Table 2. The results correspond to the long-run increases in segregation found in Figure 4. If only Whites did not discriminate (second bar), we would find an almost equivalent long-run increase in segregation. As more and more races became indifferent to the racial compositions of the neighbors, the simulated long-run increase in segregation would barely change. Indeed, if all homeowners were race-blind (as in the last bar), we would still find that the long-run increase in segregation is effectively unchanged. Overall, this suggests that discriminatory responses by themselves contribute very little to segregation in the Bay Area. Black homeowners are an important exception, as Black segregation would decrease by roughly 30% if Black homeowners were race-blind (see the B, WB, BH, WBH, WBA and WBHA counterfactuals).

Why do we find that segregation would still increase if no homeowners discriminated (last bar in each panel)? The allocation of households across Bay Area neighborhoods in November 2004 reflects residential choices made by discriminatory homeowners; during our simulation they adjust their residential choices to align with their newly preferred

²⁰As shown in Table 2, households tend to respond more positively to an increase in the share of same-race households than negatively to an increase in the shares of other races.

Figure 5: Long-Run Increases in Segregation When Certain Households Do Not Discriminate



Notes: Each panel shows the long-run increase (in percentage relative to its value in November 2004) in the dissimilarity index for households of each race. The first bar corresponds to the baseline counterfactual (i.e., no change in any parameter) and the subsequent bars correspond to different counterfactuals whereby we assume that home-owners of certain races are indifferent to the races of their neighbors. For example, the bar labeled "W" indicates a counterfactual in which only White homeowners are race-blind, but all other homeowners respond to changes in racial compositions of the neighborhoods as in Table 2. Analogously. the bar labeled "WB" indicates a counterfactual in which only White and Black homeowners are race-blind, etc. All simulations begin in November 2004.

Figure 6: Number of Neighborhoods In Flux - No Discrimination (Simulated)



Notes: Figure shows the number of neighborhoods with at least one, two, five or ten monthly inflows + outflows of all races (out of a total of 224 neighborhoods) under the counterfactual of the last bar of Figure 5 (WBHA). Simulation begins in November 2004.

neighborhoods having been race-blinded. Importantly, this suggests that amenities other than the racial compositions of neighborhoods seem to play an important role in generating segregation. Because of moving costs, this realignment is a gradual process, as shown in Figure 6.

6.2 The Role of Moving Costs

Figure 6 suggests that moving costs play an important role in the dynamics of segregation. To explore this further, we consider a counterfactual in which all homeowners incur zero moving costs. As shown in the first panel of Figure 7, the Bay Area converges to a sorting equilibrium much faster. This is not surprising, but it is important to note that it still takes almost two years for neighborhoods to stabilize. This owes to the fact that homeowners may not perfectly and instantaneously predict the racial compositions of all neighborhoods. To confirm this, in the second panel of Figure 7 we present an analogous figure for the scenario in which all races incur no moving costs *and* are race-blind. In this scenario, the two frictions that play a role on the dynamics of segregation in this model – moving costs and expectations – are shut off. As expected, the convergence to the sorting equilibrium is instantaneous.

We explore the interaction between moving costs and discrimination in Figure 8, which is the analog to Figure 5 when moving costs are assumed to be zero. Compared to our baseline results (the dashed horizontal line), eliminating moving costs results in

Figure 7: Number of Neighborhoods In Flux (Simulated) - No Moving Costs



Notes: Figure shows the number of neighborhoods with at least one, two, five or ten monthly inflows + outflows of all races (out of a total of 224 neighborhoods). The first panel is under the counterfactual of no moving costs, and the second panel is under the counterfactual of both no moving costs and no discrimination (WBHA). Simulation begins in November 2004.

dramatically higher levels of segregation in the long run. We would expect long run increases in segregation ranging from 75% for Black homeowners to nearly 200% for Hispanic homeowners. An interesting finding is that the segregation for all minorities races (Blacks, Hispanics and Asians) shrinks considerably whenever that race does not discriminate, which mirrors our findings for Black homeowners in Figure 5. Furthermore, if all homeowners are race-blind (last bar), we would expect a larger long run increase in segregation with no moving costs, which reflects the fact that moving costs in November 2004 were sufficiently high to dissuade some homeowners from moving out of their sub-optimal neighborhood. This further reinforces our finding that amenities other than racial composition play an important role in segregation.

6.3 The Role of Mismatched Amenities

We now consider a counterfactual that plausibly increases the amount of mismatch in amenities between the current and the most desired neighborhoods for many homeowners. We implement it by re-allocating households so that all neighborhoods have the exact same initial racial compositions (a policy generating full integration of all races). Because the racial compositions of neighborhoods as observed in November 2004 had likely arisen from an ongoing process of convergence to some equilibrium, a



Figure 8: Long-Run Increases in Segregation When Certain Households Do Not Discriminate - No Moving Costs

(c) Hispanic Index

(d) Asian Index

Notes: Each panel shows the long-run increase (in percentage relative to its value in November 2004) in the dissimilarity index for households of each race. The dashed line corresponds to the baseline counterfactual (i.e., no change in any parameter) and the bars correspond to different counterfactuals whereby we assume that homeowners of certain races are indifferent to the races of their neighbors. For example, the bar labeled "W" indicates a counterfactual in which only White homeowners are race-blind, but all other homeowners respond to changes in racial compositions of the neighborhoods as in Table 2. Analogously. the bar labeled "WB" indicates a counterfactual in which only White and Black homeowners are race-blind, etc. In all counterfactuals except the baseline one, moving costs for all homeowners are assumed to be zero. All simulations begin in November 2004.

Figure 9: Number of Neighborhoods In Flux (Simulated) - Full Integration



Notes: Figure shows the number of neighborhoods with at least one, two, five or ten inflows + outflows of all races (out of a total of 224 neighborhoods). The first panel is under the counterfactual of full integration with actual moving costs, and the second panel is under the counterfactual of full integration with no moving costs. Simulation begins in November 2004.

fully integrated counterfactual would likely increase amenity mismatch. The first panel of Figure 9 plots the number of neighborhoods in flux after the full integration policy, to be compared to Figure 3. Given this re-arrangement of households, it takes longer for turnover to decrease in most neighborhoods. This is intuitive, as this policy likely leads to a major misalignment that takes longer to reach equilibrium because of moving costs. Moreover, neighborhoods that experience only mildly influx may suddenly experience larger changes in racial compositions later on (see the non-monotonic range in the "10+ moves" curve around months 120 to 150 in Figure 9). This happens for two reasons: First, changes in one neighborhood in one month end up propagating to other neighborhoods in future months, as households reallocate in the Bay Area. Second, small systematic changes in racial compositions must accumulate for many people to be compelled to incur moving costs to leave a neighborhood. In the second panel of Figure 9 we show what happens when households act in a fully integrated Bay Area with no moving costs. Adjustment occurs very quickly (as quickly as in the baseline case without moving costs in Figure 7), but still not monotonically. This suggests that the first reason mentioned above plays an important role in the process of convergence.

We explore the interaction between initial racial compositions and discrimination in Figure 10, which shows the long-run change in overall segregation under various counterfactuals. Starting from a fully integrated Bay Area leads to long-run outcomes that are less segregated than what we observe in November 2004, but the magnitude of this difference varies considerably by race. Under this counterfactual, White and Asian homeowners would be about 70% less segregated, while Black and Hispanic homeowners would be roughly 40% less segregated. Because full integration effectively creates a 100% initial reduction in segregation, the effects of this policy would be partly undone by sorting, particularly for Blacks and Hispanics. Such sorting is not motivated by discriminatory responses to the racial compositions of the neighborhoods; after all, the effects shown in Figure 10 do not change as more races are assumed to be race-blind. Instead, it is motivated chiefly by other amenities that might make a neighborhood relatively more attractive to homeowners of a particular race.

The existence of substantial moving costs implies that the current racial distribution in San Francisco is highly responsible for future trends in segregation. A full integration policy would have majorly impacted segregation in the long-run, as many households would be more mismatched to their neighborhood (relative to the neighborhood that they actually resided in as of November 2004), but this mismatch would still not be large enough to surmount moving costs and persuade some of them to move.²¹

We conclude that moving costs are extremely important in explaining segregation. If they remain at their current levels, they will keep Bay Area segregation in check by preventing homeowners from sorting to their favorite neighborhoods. However, differences in the racial compositions of neighborhoods are not primarily responsible for this latent desire for sorting. Rather, our findings suggest that households seek their desired neighborhoods mostly because of other amenities that they offer.

An important caveat in our application is that we do not observe the racial composition of renters over time, so we are likely painting an incomplete picture of the patterns of neighborhood segregation in San Francisco. This drawback may be less damaging to our conclusions if the aspects of the expected racial composition of neighborhoods that are most relevant to sorting decisions are the ones proxied by the actual racial composition of homeowners (e.g., different allocations of local public goods spending depending on the racial composition of local taxpayers). In contrast, this may be of concern in neighborhoods with lower rates of homeownership if the aspects of the expected racial composition of neighborhoods that are most relevant to sorting decisions are the racial

 $^{^{21}}$ With no moving costs, we find that the long-run increases in segregation do not change depending on whether we started from a fully integrated initial state or from the state as observed in November 2004, as expected.



Figure 10: Long-Run Increases in Segregation When Certain Households Do Not Discriminate - Full Integration

(c) Hispanic Index

(d) Asian Index

Notes: Each panel shows the long-run increase (in percentage relative to its value in November 2004) in the dissimilarity index for households of each race. The dashed line corresponds to the baseline counterfactual (i.e., no change in any parameter or initial allocation of households) and the bars correspond to different counterfactuals whereby we assume that homeowners of certain races are indifferent to the races of their neighbors. For example, the bar labeled "W" indicates a counterfactual in which only White homeowners are race-blind, but all other homeowners respond to changes in racial compositions of the neighborhoods as in Table 2. Analogously. the bar labeled "WB" indicates a counterfactual in which only White and Black homeowners are race-blind, etc. In all counterfactuals except the baseline one, we re-allocate households so that all neighborhoods have the exact same racial compositions (full integration). All simulations begin in November 2004.

compositions of the people that *use* public goods and, at the same time, landlords' race is a poor predictor of tenant's race. In any case, because renters face relatively lower moving costs than homeowners, we would expect to find patterns of segregation somewhere in between our estimates in Section 5 and our estimates under no moving costs. In particular, our conclusions regarding the small role of racial composition relative to other neighborhood amenities as determinants of segregation are likely to prevail as they do not depend on the size of moving costs. Future research with access to better data is needed to address these issues.

7 Conclusion

Neighborhoods constantly evolve: their amenities are not static and their residents are in flux. Disequilibrium models of racial segregation tend to attribute this evolution to endogenous changes in neighborhood residents arising from racial discrimination, while disaggregated models of residential choice tend to attribute this evolution to exogenous changes in other amenities arising from serially correlated external shocks. In this paper, we develop an empirical framework that bridges these two approaches and provides new and interesting perspectives on how the aggregate phenomenon of segregation arises from the accumulation of disaggregate residential choices. We find evidence of discriminatory sorting: all else constant homeowners tend to reside around other homeowners of the same race. However, we also find that non-discriminatory sorting (in response to amenities other than the racial compositions of neighbors) plays a more important role in explaining patterns of segregation than discriminatory sorting. All of these adjustments are heavily mitigated by moving costs and by incomplete information.

Because segregation is a complex, nonlinear, dynamic process, the strengths of our framework are apparent. By clearly delineating the interconnected roles of discriminatory sorting, non-discriminatory sorting, incomplete information and moving costs, we can exploit our framework to better understand the underlying forces that drive segregation in the real world through counterfactual analysis. Indeed, we find that discrimination alone provides a very incomplete explanation for segregation in the San Francisco Bay Area, but it does give rise to a gradual adjustment process for the racial compositions of neighborhoods. This process is gradual primarily due to moving costs, although incomplete information plays a discernible role. Meanwhile, the mismatch between residents' current and most desired neighborhoods (that is primarily driven by amenities other than the racial compositions of neighbors) is shown to be a strong motivator for homeowners to resettle in different neighborhoods in spite of substantial moving frictions. The interplay of all of these forces contribute to a metropolitan area that is observed to be on the path to further segregation.

We view our framework as a platform for the empirical analysis of determinants of segregation that can be directly adapted to various contexts. For example, with appropriate data, our approach is well suited to study gentrification – neighborhood sorting between income groups. Moreover, our approach can be used to explore higher dimensional social interactions. In addition to being of interest *per se*, a comparison of the degree of sorting along different socio-economic dimensions could prove valuable in revealing the importance of different cleavages in our society.

References

- Banzhaf, H. Spencer and Randall P. Walsh. 2013. "Segregation and Tiebout sorting: The link between place-based investments and neighborhood tipping." Journal of Urban Economics 74(0):83 – 98.
- Bayer, P. and C. Timmins. 2005. "On the equilibrium properties of locational sorting models." *Journal of Urban Economics* 57(3):462–477.
- Bayer, P. and C. Timmins. 2007. "Estimating Equilibrium Models Of Sorting Across Locations." The Economic Journal 117(518):353–374.
- Bayer, P., F. Ferreira and R. McMillan. 2007. "A Unified Framework for Measuring Preferences for Schools and Neighborhoods." *Journal of Political Economy* 115(4):588– 638.
- Bayer, P., N. Keohane and C. Timmins. 2009. "Migration and hedonic valuation: The case of air quality." Journal of Environmental Economics and Management 58(1):1– 14.
- Bayer, P., R. McMillan and K. Rueben. 2004*a*. An equilibrium model of sorting in an urban housing market. Technical report National Bureau of Economic Research.

- Bayer, Patrick, Robert McMillan, Alvin Murphy and Christopher Timmins. 2016. "A Dynamic Model of Demand for Houses and Neighborhoods." *Econometrica* 84(3):893– 942.
- Bayer, Patrick, Robert McMillan and Kim S. Rueben. 2004b. "What drives racial segregation? New evidence using Census microdata." Journal of Urban Economics 56(3):514 – 535.

URL: http://www.sciencedirect.com/science/article/pii/S0094119004000634

- Becker, G.S. and K.M. Murphy. 2000. Social Economics: Market Behavior in a Social Environment. Harvard University Press.
- Bergman, Peter. 2016. "The Effects of School Integration: Evidence from a Randomized Desegregation Program." *Working Paper*.
- Berry, S., J. Levinsohn and A. Pakes. 1995. "Automobile Prices in Market Equilibrium." *Econometrica* 63(4):841–890.
- Berry, S.T. 1994. "Estimating Discrete-Choice Models of Product Differentiation." *The RAND Journal of Economics* pp. 242–262.
- Caetano, Gregorio. 2016. "Neighborhood Sorting and the Valuation of Public School Quality." *Mimeo*.
- Caetano, Gregorio and Vikram Maheshri. 2017. "School segregation and the identification of tipping behavior." *Journal of Public Economics* 148:115–135.
- Caetano, Gregorio and Vikram Maheshri. 2018. "Explaining Trends in US School Segregation: 1988-2014 and Beyond." Unpublished Manuscript.
- Card, D., A. Mas and J. Rothstein. 2008a. "Tipping and the Dynamics of Segregation." Quarterly Journal of Economics 123(1):177–218.
- Card, David and Jesse Rothstein. 2007. "Racial segregation and the black-white test score gap." *Journal of Public Economics* 91(11):2158–2184.
- Chetty, Raj, Nathaniel Hendren, Patrick Kline and Emmanuel Saez. 2014. Where is the land of opportunity? The geography of intergenerational mobility in the United States. Technical report National Bureau of Economic Research.

- Cutler, David M, Edward L Glaeser and Jacob L Vigdor. 2008. "When are ghettos bad? Lessons from immigrant segregation in the United States." Journal of Urban Economics 63(3):759–774.
- Epple, D. and H. Sieg. 1999. "Estimating Equilibrium Models of Local Jurisdictions." Journal of Political Economy 107(4).
- Epple, D., T. Romer and H. Sieg. 2001. "Interjurisdictional sorting and majority rule: An empirical analysis." *Econometrica* 69(6):1437–1465.
- Epple, D., T. Romer and H. Sieg. 2003. "Interjurisdictional sorting and majority rule: An empirical analysis." *Econometrica* 69(6):1437–1465.
- Epple, Dennis, Radu Filimon and Thomas Romer. 1984. "Equilibrium among local jurisdictions: toward an integrated treatment of voting and residential choice." *Journal of Public Economics* 24(3):281–308.
- Guryan, Jonathan. 2004. "Desegregation and black dropout rates." The American Economic Review 94(4):919–943.
- Kiel, Katherine A and Jeffrey E Zabel. 1996. "House price differentials in US cities: Household and neighborhood racial effects." *Journal of housing economics* 5(2):143– 165.
- Kling, Jeffrey R, Jens Ludwig and Lawrence F Katz. 2005. "Neighborhood effects on crime for female and male youth: Evidence from a randomized housing voucher experiment." *The Quarterly Journal of Economics* pp. 87–130.
- Kuminoff, Nicolai V, V Kerry Smith and Christopher Timmins. 2013. "The new economics of equilibrium sorting and policy evaluation using housing markets." *Journal* of Economic Literature 51(4):1007–1062.
- Ludwig, Jens, Greg J Duncan, Lisa A Gennetian, Lawrence F Katz, Ronald C Kessler, Jeffrey R Kling and Lisa Sanbonmatsu. 2012. "Neighborhood effects on the long-term well-being of low-income adults." *Science* 337(6101):1505–1510.
- Manski, Charles F. 2004. "Measuring expectations." *Econometrica* 72(5):1329–1376.

- Mason, Susan M, Lynne C Messer, Barbara A Laraia and Pauline Mendola. 2009. "Segregation and preterm birth: the effects of neighborhood racial composition in North Carolina." *Health & Place* 15(1):1–9.
- Mouw, Ted and Barbara Entwisle. 2006. "Residential Segregation and Internacial Friendship in Schools1." *American Journal of sociology* 112(2):394–441.
- Ringo, Daniel. 2013. "Residential Clustering by Race and Income: What Drives the Sorting?" *Mimeo*.
- Rivkin, Steven G, Eric A Hanushek and John F Kain. 2005. "Teachers, schools, and academic achievement." *Econometrica* 73(2):417–458.
- Schelling, Thomas C. 1969. "Models of Segregation." *The American Economic Review* 59(2):488–493.
- Schelling, Thomas C. 1971. "Dynamic Models of Segregation." Journal of Mathematical Sociology 1:143–186.
- Tiebout, Charles. 1956. "A Pure Theory of Local Expenditures." The Journal of Political Economy 64(5):416–424.

A Appendix Tables and Figures



Figure 11: Estimates of $\pmb{\beta^r}$ Under Alternative Specifications

Notes: All specifications include race-month fixed effects and control variables (v_{jt-1}^r) . Instrumental variables are used from period (s_{jt-T}^r) , All standard errors clustered by race-month. *** - 99% significance. For all values of T, the p-value for a joint test of whether instrumental variables are relevant in the first stage is 0.00.

		White	Black	Hispanic	Asian
Responses to:	s^B_{jt}	-10.72^{***} (1.41)	17.97^{***} (1.80)	2.23^{*} (1.21)	-6.75^{***} (1.14)
	s^H_{jt}	-11.69^{***} (1.83)	10.96^{***} (3.22)	38.51^{***} (3.76)	-1.02 (2.47)
	s^A_{jt}	-5.15^{***} (1.17)	-1.08 (2.31)	-2.58 (2.62)	22.35^{***} (1.89)
R^2			0.15		
Num. Obs.			$156,\!800$		

Table 3: OLS Estimation Results - Responses to the Racial Compositions of Neighborhoods (β^r)

Notes: This specification includes race-month fixed effects. All standard errors clustered by race-month. * - 90% significance, ** - 95% significance, *** - 99% significance.