A Life Cycle Model with Unemployment Traps

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Motivation

- Stock market disasters display life-cycle effects (Fagereng and Guiso, 2017).

- We study Personal Disaster Risk (PDR): rare but large reduction in the permanent component of individual earnings.

- We calibrate the model to unemployment, rather than bankruptcy, since most workers face a small risk of falling in an unemployment trap (UT).

- Unemployment by duration (2014)

<table>
<thead>
<tr>
<th>&gt;27 weeks</th>
<th>&gt;52 weeks</th>
<th>&gt;99 weeks</th>
</tr>
</thead>
<tbody>
<tr>
<td>33.5%</td>
<td>22%</td>
<td>11%</td>
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</table>

- Only 11% of the long-term unemployed finds a job a year later; exit from labor force is likelier (Krueger et al., 2014).

- The risk is small, but uniform across education groups (Katz et al., 2016).

- Earnings losses are persistent (Jacobson et al., 2005) and increase with unemployment duration (Keane and Wolpin, 1997; Arulampalam et al., 1993).
Model in a Nutshell

- No unemployment risk (Cocco et al., 2005): Permanent and transitory earnings shocks

- Unemployment risk (Bremus and Kuzin, 2014): Three-state Markov chain
  - A young, employed agent either remains employed or becomes unemployed;
  - Next, if she stays unemployed, her earnings fall;
  - .....

- Unemployment Trap: % loss in the permanent component of earnings.
  - Deterministic: set to 0.6, including losses due to exit from the labor force.
  - Stochastic: expected loss at 0.2 delivers same results.
    - Beta distribution, with shape parameter putting most probability mass on low realizations of this loss
  - Transition matrix conservatively matches unemployment by duration
Unemployment Trap versus Other Cases

A rare personal disaster risk

- increases optimal savings and cautiousness when young: grandma’s advice!
- flattens the optimal investment profile, due to higher uncertainty when young
- reduces the average skewness of consumption growth

- shrinks heterogeneity in optimal portfolios despite unequal career histories
- amplifies welfare losses of sub-optimal default investment rules (3-10 times as large), due to excess (insufficient) consumption when young (old)
- dampens sensitivity to (both inter-temporal and across assets) correlation due to skewness-inducing disaster
Contribution

- Normative analysis of the economics behind negative skewness in earnings
  - relevant in the data (Guvenen, Karahan, Ozkan and Song, 2015; Catherine, 2018; Galvez, 2017; Shen, 2018).

- Average implied skewness of consumption growth becomes negative, without reinforcing change in the labor income process
  - this improves asset pricing in Constantinides and Ghosh, 2014, and Schmidt, 2016

- Portfolio choice with non-Gaussian returns to human capital
  - instead of non-Gaussian financial returns (Guidolin and Timmerman, 2008)

- We add the rare personal disaster dimension to the following insights:
  - Resolution of uncertainty over working years
    - Bagliano et al., (2014); Hubener, Maurer and Mitchell (2016); Chang, Hong and Karabarounis (2017)
  - Precautionary savings and employment insurance (Low, Meghir and Pistaferri, 2010)
The Model

- Finite horizon with uncertain lifespan

\[
\frac{C_{it_0}^{1-\gamma}}{1 - \gamma} + E_{t_0} \left[ \sum_{j=1}^{T} \beta^j \left( \prod_{k=0}^{j-2} p_{t_0+k} \right) \left( p_{t_0+j-1} \frac{C_{it_0+j-1}^{1-\gamma}}{1 - \gamma} + (1 - p_{t_0+j}) b \frac{(X_{it_0+j}/b)^{1-\gamma}}{1 - \gamma} \right) \right]
\]

- \( C_{it} \) level of consumption at time \( t \); \( X_{it} \) wealth the investor leaves as bequest
- \( b \geq 0 \) strength of the bequest motive; \( \beta < 1 \) discount factor; \( \gamma \) CRRA.

Investment opportunities, with short-sales and borrowing constraints:

Portfolio return:

\[
R_{it}^P = \alpha_{it}^s R_{it}^s + (1 - \alpha_{it}^s) R_f^f
\]  

- \( R_f^f \) one-period risk-free return; \( \alpha_{it}^s \) share invested in stocks; stock return:

\[
\tilde{R}_t^s = R_t^f + \mu_t^s + \nu_t^s; \nu_t^s \sim N(0, \sigma_s^2)
\]

Cash on hand

\[
X_{it+1} = (X_{it} - C_{it}) R_{it}^P + Y_{it+1}
\]
Labor and Retirement Income

- Labor income process

\[ Y_{it} = H_{it} N_{it} \quad t_0 \leq t \leq t_0 + K \]  

- \( H_{it} = F(t, Z_{it}) P_{it} \) permanent income component
- \( F(t, Z_{it}) \equiv F_{it} \) deterministic trend component
- \( \log(N_{it}) \) is \( N(0, \sigma_E^2) \)
- Stochastic permanent component:

\[ \log P_{it} = \log P_{it-1} + \omega_{it} \]  

- \( \omega_{it} \) is \( N(0, \sigma_\omega^2) \)

- Retirement income

\[ Y_{it} = \lambda F(t, Z_{it0+l}) P_{it0+l} \quad t_0 + K < t \leq T \]  

- \( t_0 + l \) last working period ; \( t_0 + K \) retirement age
- \( \lambda \) of the permanent component of labor income in the last working year
Labor Market Dynamics and Income

- Transition matrix

\[
\begin{bmatrix}
\pi_{e,e} & 1 - \pi_{e,e} & 0 \\
\pi_{u_1,e} & 0 & 1 - \pi_{u_1,e} \\
\pi_{u_2,e} & 0 & 1 - \pi_{u_2,e}
\end{bmatrix}
\]

- Labor income depends on past working history, \(0 \leq \Psi_j \leq 1\):

\[
H_{it} = \begin{cases}
H_{it} & \text{if } s_t = e \text{ and } s_{t-1} = e \\
(1 - \Psi_1)H_{it-1} & \text{if } s_t = e \text{ and } s_{t-1} = u_1 \\
(1 - \Psi_2)H_{it-1} & \text{if } s_t = e \text{ and } s_{t-1} = u_2
\end{cases}
\]

\(t = t_0, \ldots, t_0 + K\) \hspace{1cm} (6)

- Cocco et al: \(\pi_{e,e} = 1\); Bremus et al: \(\Psi_j = 0\)

- Unemployment benefit

\[
Y_{it} = \begin{cases}
\xi_1 H_{it-1} & \text{if } s_t = u_1 \text{ and } s_{t-1} = e \\
0 & \text{if } s_t = u_2 \text{ and } s_{t-1} = u_1 \text{ and } s_{t-2} = e
\end{cases}
\]

\(t = t_0, \ldots, t_0 + K\) \hspace{1cm} (7)
Maximization problem

individual problem    value function

Value function in each possible labor market state

maximization problem
Transition matrix between labor market states implies conservative short (4.7%) and long-term (0.8%) unconditional probability of being unemployed:

\[
\begin{bmatrix}
\pi_{e,e} & 1 - \pi_{e,e} & 0 \\
\pi_{u1,e} & 0 & 1 - \pi_{u1,e} \\
\pi_{u2,e} & 0 & 1 - \pi_{u2,e}
\end{bmatrix} =
\begin{bmatrix}
0.96 & 0.04 & 0 \\
0.85 & 0 & 0.15 \\
0.85 & 0 & 0.15
\end{bmatrix}
\]

Unemployment benefits: \( \xi_1 = 0.3 \) Average before 26 weeks. After: \( \xi_2 = 0 \)

\( \psi_1 = 0 \); \( \psi_2 = 60\% \)

- Persistent earning losses: 43-66\% (Jacboson et al., 2005)
- After 24 months: 40 \% probability of finding a job; and 88\% of exiting the labor force (Katz et al., 2016)

Stochastic earnings loss (expected value 10%-20\%, st.dev 20%-30\%)
Table 1. Calibration parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Working life (max)</td>
<td>20 - 65</td>
</tr>
<tr>
<td>Retirement (max)</td>
<td>65 - 100</td>
</tr>
<tr>
<td>Discount factor (β)</td>
<td>0.96</td>
</tr>
<tr>
<td>Risk aversion (γ)</td>
<td>5</td>
</tr>
<tr>
<td>Replacement ratio (λ)</td>
<td>0.68</td>
</tr>
<tr>
<td>Variance of permanent shocks to labour income (σ₂ε)</td>
<td>0.0106</td>
</tr>
<tr>
<td>Variance of transitory shocks to labour income (σ₂n)</td>
<td>0.0738</td>
</tr>
<tr>
<td>Riskless rate</td>
<td>2%</td>
</tr>
<tr>
<td>Excess returns on stocks (μ^σ)</td>
<td>4%</td>
</tr>
<tr>
<td>Variance of stock returns innovations (σ₂σ)</td>
<td>0.025</td>
</tr>
<tr>
<td>Stock ret./permanent lab. Income shock correlation (ρστ)</td>
<td>0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>No unemployment risk</th>
<th>Unemployment risk with no traps</th>
<th>Unemployment traps</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Unemployment benefit</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Short term unemployed (ξ₁)</td>
<td>-</td>
<td>0.3</td>
<td>0.3</td>
</tr>
<tr>
<td>Long term unemployed (ξ₂)</td>
<td>-</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td><strong>Human capital erosion</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Short term unemployed (Ψ₁)</td>
<td>-</td>
<td>-</td>
<td>0</td>
</tr>
<tr>
<td>Long term unemployed (Ψ₂)</td>
<td>-</td>
<td>-</td>
<td>0.6</td>
</tr>
</tbody>
</table>
Optimal stock shares - all models

Optimal policy functions for stock investing at age 40

- Blue line: no unemployment risk
- Orange line: unemployment risk no traps
- Green line: unemployment risk with traps

Cash on hand
Optimal Life Cycle Profiles

**Average stock share**

- **no unemployment risk**
- **unemployment risk with no traps**
- **unemployment risk with traps**

**Average financial wealth**

age: 20 30 40 50 60 70 80 90 100

value: 0 5 10 15 20 25 30 35 40

value: 0 0.2 0.4 0.6 0.8 1
Average Skewness of Consumption Growth

Skewness of consumption growth

- traps
- notraps
Stochastic loss due to long-term unemployment
Skewness induces Robustness

- age-dependent unemployment risk
  - In 2015, U.S. overall unemployment rate 5.7%, LTU 1.7%
  - 20% 16-24 years old
  - 35% 25-55 years old
  - 41% over 55 years old

- Correlation between earnings and stock returns

- Epstein Zin preferences
Age-dependent long-term unemployment

![Graph showing age-dependent long-term unemployment](image)
Correlation between earnings and stock returns
Default Investment Rules

Optimal and suboptimal life-cycle portfolio share profiles

![Graph showing optimal and suboptimal life-cycle portfolio share profiles with various lines representing different scenarios such as unemployment traps, TDF, AGE RULE, and unaware of traps.]
## Welfare Analysis of Default Investment Rules

### a. Distribution of welfare gains (% points)

<table>
<thead>
<tr>
<th></th>
<th>Age Rule</th>
<th>TDF rule</th>
<th>Unaware of Traps</th>
</tr>
</thead>
<tbody>
<tr>
<td>mean</td>
<td>3.3</td>
<td>12.0</td>
<td>642.5</td>
</tr>
<tr>
<td>median</td>
<td>3.3</td>
<td>11.8</td>
<td>215.8</td>
</tr>
<tr>
<td>5th</td>
<td>1.5</td>
<td>8.0</td>
<td>-40.5</td>
</tr>
<tr>
<td>95th</td>
<td>5.4</td>
<td>17.0</td>
<td>573.6</td>
</tr>
</tbody>
</table>

### b. Welfare gains conditional on income at age 64 (% points)

<table>
<thead>
<tr>
<th></th>
<th>Age Rule</th>
<th>TDF rule</th>
<th>Unaware of Traps</th>
</tr>
</thead>
<tbody>
<tr>
<td>Below 5\textsuperscript{th} percentile</td>
<td>1.6</td>
<td>9.5</td>
<td>1024.0</td>
</tr>
<tr>
<td>Above 95\textsuperscript{th} percentile</td>
<td>2.4</td>
<td>12.3</td>
<td>218.2</td>
</tr>
</tbody>
</table>
Optimal and Default Consumption Profiles
Optimal and Default Wealth Profiles
Conclusions

- We show the life-cycle implications of a rare personal disaster risk during working life (uninsured bankruptcy or unemployment)
  - inducing skewness in labor income and consumption
  - robustly changings the optimal savings/ risk-taking profile

- Calibrations to US long-term unemployment show that common Default Investment Rules may generate large welfare losses.
LTU Share and Education, 2000-13

Source: Katz et al., 2016
Maximization Problem

Individual’s optimal program

\[
\max_{\{C_{it}\}_{t_0}^{T-1}, \{\alpha_{it}^s\}_{t_0}^{T-1}} \left( \frac{C_{it_0}^{1-\gamma}}{1-\gamma} + E_{t_0} \left[ \sum_{j=1}^{T} \left( \prod_{k=0}^{j-2} p_{t_0+k} \right) \left( p_{t_0+j-1} \frac{C_{it_0+j}^{1-\gamma}}{1-\gamma} + (1 - p_{t_0+j-1}) b \left( \frac{X_{it_0+j}/b}{1-\gamma} \right) \right) \right] \right)
\] (8)

s.t. \( X_{it+1} = (X_{it} - C_{it}) (\alpha_{it}^s R_t^s + (1 - \alpha_{it}^s) R^f) + Y_{it+1} \) (9)

Dynamic Programming Form

\[
V_{it} (X_{it}, P_{it}, s_{it}) = \max_{\{C_{it}\}_{t_0}^{T-1}, \{\alpha_{it}^s\}_{t_0}^{T-1}} \left( \frac{C_{it}^{1-\gamma}}{1-\gamma} + \beta E_{t} [p_{t} V_{it+1} (X_{it+1}, P_{it+1}, s_{it+1}) \right.
\]
\[
\left. + (1 - p_{t}) b \left( \frac{X_{it+1}/b}{1-\gamma} \right) \right] \)
\]
Value Function

\[ V_{it}(X_{it}, P_{it}, s_{it}) = \max \{ C_{it} \}_{t_0}^{T-1}, \{ \alpha_{it} \}_{t_0}^{T-1} \left( \frac{C_{it}^{1-\gamma}}{1-\gamma} + \beta \sum_{s_{it+1}=e_1, u_2} p_t \pi(s_{it+1}|s_{it}) \right) \]

\[ \tilde{E}_t V_{it+1}(X_{it+1}, P_{it+1}, s_{it+1}) = (1 - p_t) b \sum_{s_{it+1}=e_1, u_2} \pi(s_{it+1}|s_{it}) \frac{(X_{it+1}/b)^{1-\gamma}}{1-\gamma} \]
Value function in each possible labor market state

\[ V(X_{it}, P_{it}, e) = u(C_{it}) + \beta p_{t} \]

\[ \begin{align*}
\text{with prob. } \pi_{e,e} & \quad \begin{cases} \bar{E}_t V(X_{it+1}, P_{it+1}, e) \\
\text{with } P_{it+1} = P_{it} e^{\omega_{it+1}} \quad \text{and} \\
X_{it+1} = (X_{it} - C_{it}) R_{it}^p + F_{it+1} P_{it+1} e^{\epsilon_{it+1}}
\end{cases} \\
\text{with prob. } \pi_{e,u_1} & \quad \begin{cases} \bar{E}_t V(X_{it+1}, P_{it+1}, u_1) \\
\text{with } P_{it+1} = (1 - \Psi_1) P_{it} \quad \text{and} \\
X_{it+1} = (X_{it} - C_{it}) R_{it}^p + \xi_1 F_{it} P_{it}
\end{cases}
\end{align*} \]

\[ V(X_{it}, P_{it}, u_1) = u(C_{it}) + \beta p_{t} \]

\[ \begin{align*}
\text{with prob. } \pi_{u_1,e} & \quad \begin{cases} \bar{E}_t V(X_{it+1}, P_{it+1}, e) \\
\text{with } P_{it+1} = (1 - \Psi_1) P_{it-1} e^{\omega_{it+1}} P_{it} e^{\omega_{it+1}} = P_{it} e^{\omega_{it+1}} \quad \text{and} \\
X_{it+1} = (X_{it} - C_{it}) R_{it}^p + F_{it-1} P_{it+1} e^{\epsilon_{it+1}}
\end{cases} \\
\text{with prob. } \pi_{u_1,u_2} & \quad \begin{cases} \bar{E}_t V(X_{it+1}, P_{it+1}, u_2) \\
\text{with } P_{it+1} = (1 - \Psi_2)(1 - \Psi_1) P_{it-1} = (1 - \Psi_2) P_{it} \quad \text{and} \\
X_{it+1} = (X_{it} - C_{it}) R_{it}^p
\end{cases}
\end{align*} \]

\[ V(X_{it}, P_{it}, u_2) = u(X_{it}) + \beta p_{t} \]

\[ \begin{align*}
\text{with prob. } \pi_{u_2,e} & \quad \begin{cases} \bar{E}_t V(X_{it+1}, P_{it+1}, e) \\
\text{with } P_{it+1} = (1 - \Psi_2)(1 - \Psi_1) P_{it-1} e^{\omega_{it+1}} = P_{it} e^{\omega_{it+1}} \quad \text{and} \\
X_{it+1} = (X_{it} - C_{it}) R_{it}^p + F_{it-2} P_{it+1} e^{\epsilon_{it+1}}
\end{cases}
\end{align*} \]