Ambiguity and Information Processing in a Model of Intermediary Asset Pricing

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Introduction

- **Heterogeneity in information processing capacity**
  - Financial intermediaries (specialists) are assumed to possess greater channel capacity (Rational Inattention (Sims, 2003)).
  - Households purchase this capacity by delegating investments to intermediaries.
    - Although households could manage their portfolios themselves, most choose not to do so.
  - Two frictions in financial contract:
    - **Incentive constraint** arises from a moral hazard problem, requires a minimum capital for risk-sharing (He-Krishnamurthy, 2012).
    - **Participation constraint** depends on the heterogeneity in channel capacity.
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    - *Participation constraint* depends on the heterogeneity in channel capacity.

- **Heterogeneity in beliefs**
  - Knightian uncertainty/Ambiguity/Robustness (Hansen-Sargent, 2008))
  - When volatility increases, so does ambiguity, the drift distortions produce *endogenous heterogeneous beliefs*.
  - When volatility is high specialists become *relatively pessimistic*, and this tightens the capital constraint and accelerates the onset of a financial crisis.
**Market Structure**

- **Effective risk sharing constraint:** $\varepsilon_t^h \leq \tilde{m}\varepsilon_t$.
  - $\tilde{m}$ reflects the financial constraint due to agency friction and ambiguity.
- **Participation constraint:** $k_t \leq a_3 (\Sigma - \Sigma^h)$.
  - $a_3 < 0$, $\kappa > \kappa^h \rightarrow \Sigma < \Sigma^h$
Model Structure

- Risky asset dividend is governed by stochastic growth rate $g_t$ and volatility $\sigma_t$,
  \[ \frac{dD_t}{D_t} = g_t dt + \sigma_t dZ_t, \]  
  (1)

- Assume the volatility $\sigma_t$ is a two-state Markov chain with state space $\Sigma_d = \{\sigma_H, \sigma_L\}$, where $\sigma_H > \sigma_L$. The intensity matrix is
  \[ \begin{bmatrix} -\lambda_H & \lambda_H \\ \lambda_L & -\lambda_L \end{bmatrix}. \]  
  (2)

- Unobservable growth rate follows a (known) mean-reverting process
  \[ dg_t = \rho_g (\bar{g} - g_t) dt + \sigma_g dZ_t^u \]  
  (3)

- Agents observe only a noisy signal containing imperfect information
  \[ ds_t = g_t dt + \sigma_s dZ_t^s \]  
  (4)
Capacity-Constrained Kalman Filter

- The Kalman filter of learning is

\[
    d\hat{g}_t = \rho_g (\bar{g} - g_t) \, dt + \frac{\Sigma_t}{\sigma_t} \, d\hat{Z}_t + \frac{\Sigma_t}{\sigma_s} \, d\hat{Z}_s^s
\]

\[
    d\Sigma_t = \left[ \sigma_g^2 - 2\rho_g \Sigma_t - \Sigma_t^2 \left( \frac{1}{\sigma_t^2} + \frac{1}{\sigma_s^2} \right) \right] \, dt
\]  

- \( \Sigma_t \): signal/noise ratio (estimation variance of the unobserved state).
- Investor has a finite information-processing capacity (Sims, 2003)

\[
    \mathcal{H}(g_{t+\Delta t} | \mathcal{I}_t) - \mathcal{H}(g_{t+\Delta t} | \mathcal{I}_{t+\Delta t}) \leq \kappa \Delta t, \tag{7}
\]

- The Kalman gain is constrained by the agent’s channel capacity

\[
    \frac{1}{2} \frac{\Sigma_t}{\sigma_s^2} \leq \kappa. \tag{8}
\]

- Risky asset return

\[
    dR_t = \frac{D_t \, dt + dP_t}{P_t} = \mu_{R,t} \, dt + \sigma_{R,t} \, dZ_t. \tag{9}
\]
Household Robust Consumption/Portfolio Rules

Objective

$$V \left( \tilde{g}_t^h, \Sigma_t^h, W_t^h; Y_t^h \right) = \sup \inf \mathbb{E} \int_0^\infty e^{-\rho^h t} \left[ \ln C_t^h + \frac{1}{2\theta^h} \left( \nu_t^h \right)^2 \right] dt$$

(10)

s.t. $dW_t^h = \left[ \varepsilon_t^h (\pi^{R,t} - k_t) + r_t W_t^h - C_t^h \right] dt + \sigma_{W,t}^h \left( \nu_t^h dt + d\hat{Z}_t \right)$,

(11)

$$d\tilde{g}_t^h = \rho_g (\bar{g} - g_t) dt + \frac{\Sigma_t^h}{\sigma_t^h} d\hat{Z}_t + \frac{\Sigma_t^h}{\sigma_s^h} d\hat{Z}^s_t$$

(12)

$$d\Sigma_t^h = \left[ \sigma_g^2 - 2\rho_g \Sigma_t^h - \frac{(\Sigma_t^h)^2}{\sigma_t^2} - 2\kappa^h \left( \Sigma_t^h \right)^2 \right] dt$$

(13)

Optimal rules

$$\nu_t^{h*} = -\frac{\theta^h}{\rho^h} \varepsilon_t^h \sigma_{R,t}^h W_t^h$$

(14)

$$C_t^{h*} = \rho^h W_t^h$$

(15)

$$\varepsilon_t^{h*} = \frac{\pi_{R,t} - k_t}{\gamma^h \sigma_{R,t}^2} W_t^h$$

(16)

Effective HH risk aversion $\gamma^h = 1 + \frac{\theta^h}{\rho^h}; \theta^h$: HH ambiguity aversion degree.
Specialist Robust Consumption/Portfolio Rules

- **Objective**

\[
J(\hat{g}_t, \Sigma_t, W_t; Y_t) = \sup_{\{C_t, \varepsilon_t\}} \inf_{\nu_t} \mathbb{E} \int_0^\infty e^{-\rho t} \left[ \ln C_t + \frac{1}{2\theta} (\nu_t)^2 \right] dt
\]  

(17)

s.t. \(dW_t = [\varepsilon_t \pi_{R,t} + (q_t + r_t) W_t - C_t] dt + \sigma_{W,t} (\nu_t dt + d\hat{Z}_t)\)

(18)

- **Optimal rules:**

\[
\nu_t^* = -\frac{\theta \varepsilon_t \sigma_{R,t}}{\rho W_t}
\]

(21)

\[
C_t^* = \rho W_t
\]

(22)

\[
\varepsilon_t^* = \frac{\pi_{R,t}}{\gamma \sigma^2_{R,t}} W_t.
\]

(23)

- **Effective specialist risk aversion** \(\gamma = 1 + \frac{\theta}{\rho}; \ \theta: \text{specialist's ambiguity aversion.}\)
Equilibrium

- Intermediation market clears,

\[ \varepsilon_t^{h*} = \frac{1 - \beta_t^*}{\beta_t^*} \varepsilon_t^*. \]  

(24)

- Stock market clears,

\[ \varepsilon_t^* + \varepsilon_t^{h*} = P_t. \]  

(25)

- Goods market clears,

\[ C_t^* + C_t^{h*} = D_t. \]  

(26)
Risk Sharing Constraint

- In *unconstrained* region,
  - Slack risk sharing constraint
    
    \[ \varepsilon^h_t \big|_{k_t=0} < m \varepsilon_t \iff \frac{\pi_{R,t}}{\gamma^h \sigma^2_{R,t}} W^h_t < m \frac{\pi_{R,t}}{\gamma \sigma^2_{R,t}} W_t \]
    
    \[ \iff T^h_t = W^h_t < \tilde{m} W_t. \]

- In *constrained* region,
  - Binding risk sharing constraint
    
    \[ \varepsilon^h_t = m \varepsilon_t \iff W^h_t \geq \tilde{m} W_t = T^h_t. \]

Define scaled specialist wealth as the unique state variable

\[ x_t = \frac{W_t}{D_t}. \]

When the risk sharing constraint just starts to bind,

\[ x_c = \frac{1}{\tilde{m} \rho^h + \rho}. \]
Risk Sharing Constraint

- In *unconstrained* region,
  - Slack risk sharing constraint
    \[
    \varepsilon^h_t|_{k_t=0} < m\varepsilon_t \iff \frac{\pi_{R,t}}{\gamma^h \sigma^2_{R,t}} W^h_t < m \frac{\pi_{R,t}}{\gamma \sigma^2_{R,t}} W_t
    \]
    \[
    \iff T^h_t = W^h_t < \tilde{m} W_t.
    \]

- In *constrained* region,
  - Binding risk sharing constraint
    \[
    \varepsilon^h_t = m\varepsilon_t \iff W^h_t \geq \tilde{m} W_t = T^h_t.
    \]

- Effective financial constraint:
  \[
  \tilde{m} \equiv \frac{\gamma^h}{\gamma} m = \frac{1 + \theta^h / \rho^h}{1 + \theta / \rho} m
  \]
  \[
  \rho^h \geq \rho, \theta^h = \theta \Rightarrow \gamma^h \leq \gamma \Rightarrow \tilde{m} \leq m
  \]

  - Define scaled specialist wealth as the unique state variable \( x_t = W_t / D_t \).
  - When the risk sharing constraint just starts to bind, \( x^c_t = \frac{1}{\tilde{m} \rho^h + \rho} \).
Steady State Solution

- In the steady state,
  \[ \Sigma = \bar{\sigma}^2 \left[ - (\kappa + \rho_g) + \sqrt{(\kappa + \rho_g)^2 + \left(\sigma_g/\bar{\sigma}\right)^2} \right] \] (29)
  \[ \frac{d\Sigma}{d\kappa} < 0 \] (30)

- Value function
  \[ J(\hat{g}_t, \Sigma_t, W_t; Y_t) = \frac{1}{\rho} \ln W_t + a_0 + a_1 \hat{g}^2 + a_2 \hat{g} + a_3 \Sigma + Y(x_t), \quad a_3 < 0. \] (31)
  \[ \frac{dJ}{d\kappa} = \frac{dJ}{d\Sigma} \frac{d\Sigma}{d\kappa} = a_3 \frac{d\Sigma}{d\kappa} > 0. \] (32)

- Agents with higher channel capacity have higher steady state welfare.
  \[ \kappa > \kappa^h \rightarrow \bar{k} \equiv J - V = a_3 \left(\Sigma - \Sigma^h\right) > 0. \] (33)

- Participation Constraint: \( k_t \leq a_3(\Sigma - \Sigma^h) \).
  - Households will remain in the contract as long as the channel capacity difference is sufficiently greater than the intermediation fee.
Stationary Wealth Distribution (Constant Volatility)

- **Endogenous Wealth Evolution**

\[
\frac{dx_t}{x_t} = \mu_{x,t} dt + \sigma_{x,t} dZ_t.
\]

- **left:** \( \sigma_H = 0.15 \)
- **right:** \( \sigma_L = 0.09 \)
Simulated Wealth Distribution

Scaled Expert Wealth Distribution ($\theta = 0.04$, $\theta^h = 0.04$, $\rho = 0.005$, $\rho^h = 0.011$)
Probability of Constraint Binds

- Probability of Sharpe Ratio Exceed Twice of the Mean: 0.32%
Asset Prices

Table 1.1: Measurements

<table>
<thead>
<tr>
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<th>Model</th>
</tr>
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<tbody>
<tr>
<td>$\theta$</td>
<td>0.0001</td>
</tr>
<tr>
<td>$\theta^h$</td>
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<tr>
<td>$\gamma$</td>
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<tr>
<td>$\gamma^h$</td>
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<tr>
<td>Risk Premium (%)</td>
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</tr>
<tr>
<td>Sharpe Ratio (%)</td>
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<tr>
<td>Interest Rate (%)</td>
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<tr>
<td>Interest Rate Volatility (%)</td>
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<tr>
<td>Return Volatility (%)</td>
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<tr>
<td>Portfolio Share</td>
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</tr>
<tr>
<td>Probability of Sharpe Ratio Exceed Twice of the Mean (%)</td>
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</tr>
</tbody>
</table>

This table reports the unconditional simulated results. We simulate 5000 years and 5000 sample paths with quarterly frequency. To match the data from 1970-2017, we report 47 years simulated results in stationary distribution.
Conclusion

- **Heterogeneity in information processing capacity**
  - Two frictions in financial contract:
    - Participation constraint depends on the heterogeneity in channel capacity.
    - Incentive constraint requires a minimum capital for risk-sharing, subjected to effective financial constraint.

- **Endogenous heterogeneous beliefs due to ambiguity**
  - When volatility is high specialists become relatively pessimistic, and this tightens the capital constraint and accelerates the onset of a financial crisis.