Intertemporal Substitution, Precautionary Saving, and Currency RiskPremium

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- exchange rate $S_t$, home currency price per unit of foreign currency
- home short rate $r_t$ and foreign short rate $r_t^*$
- log exchange rate, $s_t \equiv \ln S_t$
- currency excess return is: $\rho_{t+1} = s_{t+1} - s_t + r_t^* - r_t$
- currency risk premium is: $E_t(\rho_{t+1}) = E_t(s_{t+1} - s_t) + r_t^* - r_t$
Engel’s paradox (AER, 2016), two empirical regularities

- Forward Premium Puzzle (Short Premium Puzzle)
  \[ \text{cov}(E_t[\rho_{t+1}], r^*_t - r_t) > 0 \]

- Excess Co-movement Puzzle or Level Puzzle
  \[ \text{cov}(\sum_{j=0}^{\infty} E_t[\rho_{t+j+1}], r^*_t - r_t) < 0 \]

\[ \Rightarrow \text{(Long Premium Puzzle)} \]
\[ \text{cov}(E_t[\rho_{t+j+1}], r^*_t - r_t) < 0 \text{ for large } j \]

- Engel: these 2 empirical regularities constitute a paradox
forward premium puzzle: \( \text{cov}(E_t(\rho_{t+1}), r^*_t - r_t) > 0 \)

when foreign interest rate is high, the average excess currency return is high

this justifies the carry-trade strategy: borrowing low interest currency to invest in high interest currency

one explanation for high average excess return is compensation for risk (Backus, Foresi and Telmer (2001), Brennan and Xia (2006))
The second empirical regularity is linked to level of exchange rate by telescoping: $\rho_{t+j+1} = s_{t+j+1} - s_{t+j} + r_{t+j}^* - r_{t+j}$

$$s_t - E_t[s_{t+k}] = \sum_{j=0}^{k} E_t[r_{t+j}^* - r_{t+j}] - \sum_{j=0}^{k} E_t[\rho_{t+j+1}]$$

- interest rate parity term
- cumulative risk premium

when $s_t$ is mean reverting, $E_t[s_{t+k}]$ becomes a constant in the limit

$\text{cov}(\sum_{j=0}^{\infty} E_t[\rho_{t+j+1}], r_{t}^* - r_{t}) < 0$ leads to excessive covariance with $s_t$

implies more excessive over-shooting than the classical Dornbusch model and Mundell-Fleming model which assumes UIP
We show that a fairly standard model with time varying risk premium can resolve Engel’s paradox:

- Intertemporal substitution plays an important role.
- Mean consumption growth depends on both consumption volatility and variance.
- Risk can account both forward premium puzzle and the excess co-movement puzzle.
- Did not use recursive utility, long run risk, and bounded rationality.
Existing models can not account these two puzzles simultaneously
Exchange Rate and No Arbitrage

Exchange Rate

- two-agents (country) model
- each country (home and foreign) - a representative agent
- $C_t^i, \ i \in \{h, f\}$, could be interpreted as a quantity index of multiple goods
Exchange Rate

- by no-arbitrage, Backus et. al. (JF, 2001)

\[ s_{t+1} - s_t = \ln \pi^*_{t+1} - \ln \pi_{t+1} \]

where \( \pi^*_{t+1} \) and \( \pi_{t+1} \) are foreign and home country pricing kernels

- we only need to model pricing kernels for each country

- each country has a representative agent, same parameters, independent and identical consumption processes
Expected Utility

- representative agent with expected CRRA utility

\[
\sum_{t=0}^{\infty} E_0 \left[ e^{-\beta t} \frac{C_{t+1}^{1-\gamma}}{1-\gamma} \right].
\]

- \( \beta \) is the subjective discount coefficient
- \( \gamma \) is the risk-aversion coefficient
- home country pricing kernel \( \pi_{t+1} \)

\[
\pi_{t+1} = e^{-\beta} e^{-\gamma(c_{t+1} - c_t)}
\]

- foreign country \( \pi_{t+1}^* \)

\[
\pi_{t+1}^* = e^{-\beta} e^{-\gamma(c_{t+1}^* - c_t^*)}
\]
• log consumption growth

\[ c_{t+1} - c_t = \mu c_t + \sigma c_t \varepsilon_{t+1} \]

• interest rate

\[ r_t = \beta + \gamma \mu c_t - \frac{1}{2} \gamma^2 \sigma^2 c_t \]

• intertemporal substitution component: \( i.s. = \gamma \mu c_t \)

• precautionary saving component: \( p.s. = -\frac{1}{2} \gamma^2 \sigma^2 c_t \)
currency premium of the simple return, \( \frac{S_{t+1}}{S_t} e^{r^f_t} \), is

\[
E_t \left( \frac{S_{t+1}}{S_t} e^{r^f_t - r^h_t} \right) = e^{\gamma^2 \sigma_{ct}^2},
\]

depends on home consumption variance \( \gamma^2 \sigma_{ct}^2 \) but not on foreign consumption variance.

only home risk is priced.
currency premium for the log return is

$$E_t[\rho_{t+1}] = E_t \left( \ln \left[ \frac{S_{t+1}}{S_t} - r^h_t + r^f_t \right] \right)$$

$$= \gamma^2 \sigma^h_{ct} - \frac{\gamma^2}{2} (\sigma^h_{ct} + \sigma^f_{ct})$$

Jensen's effect

compensation for risk

can be written as the differential of home and foreign country premiums:

$$E_t[\rho_{t+1}] = \frac{\gamma^2}{2} (\sigma^h_{ct} - \sigma^f_{ct})$$

log return makes it symmetric

from now on, we will focus on one country.
risk premium: $\nu^h_t = \frac{\gamma^2}{2} \sigma^2_{ct}$

interest rate: $r_t = i.s. + p.s.$
- intertemporal substitution component: $i.s. = \gamma \mu_{ct}$
- precautionary saving component: $p.s. = -\frac{1}{2} \gamma^2 \sigma^2_{ct}$ negatively proportional to risk premium

positive correlation between the risk premium and interest rates has to come from $\mu_{ct}$, this channel is ignored in existing literature
Our Model
Consumption Process

- log consumption growth

\[ c_{t+1} - c_t = \lambda \sigma_{ct} + (h - 1/2)\sigma_{ct}^2 + \sigma_{ct} \varepsilon_{t+1}^c \]

- conditional volatility

\[ \sigma_{ct} = x_t + \theta \]

\[ x_{t+1} = \varphi x_t + \sigma \varepsilon_{t+1}^x \]

OU process, Stein and Stein (1991) and Constantinides (1992)

- conditional mean

\[ \mu_{ct} = \lambda \sigma_{ct} + (h - 1/2)\sigma_{ct}^2 \]

depends on both consumption volatility and variance

  (new to literature)
the expected future risk premium $\nu_t = \frac{\gamma^2}{2} \sigma_{ct}^2$

$$E_t[\sigma_{ct+j}^2] = E_t[(x_{t+j} + \theta)^2] = 2\theta x_t \varphi^j + x_t^2 \varphi^{2j} + \cdots$$

the expected future risk premium depends on both consumption variance $x_t^2$ and consumption volatility $x_t$

the term with consumption volatility dominates the term with consumption variance when $j$ is large

existing (Affine) models only have consumption variance, thus only 1 decay mode
the interest rate is

\[ r_t = \beta + \gamma (\lambda (x_t + \theta) + (h - 1/2)(x_t + \theta)^2) - \frac{1}{2} \gamma^2 (x_t + \theta)^2 \]

Interpretation:

- Positively depends on the consumption volatility \( x_t + \theta \) through the intertemporal substitution effect.
- Negatively depends on the consumption variance \( (x_t + \theta)^2 \) through the precautionary saving effect.
when $x_t + \theta$ is large, $(x_t + \theta)^2$ dominates, precautionary effect dominates

interest rate decreases with consumption variance

when $x_t + \theta$ is small, $x_t + \theta$ dominates, intertemporal substitution effect dominates

interest rate increases with consumption volatility
the competing mechanism of these two effects makes the interest rate a nonmonotonic function of $x_t + \theta$

- key to resolving Engel’s paradox

- existing models only have conditional variance
Resolution of Engel’s Paradox
Covariance Between Interest Rate and Risk Premium

- interest rate

\[ r_t = \beta + \gamma \lambda (x_t + \theta) - \gamma \left( \frac{1 + \gamma}{2} - h \right) (x_t + \theta)^2 \]

- expected future premium

\[ \nu_t^h = \frac{\gamma^2}{2} E_t [\sigma_{t+j}^2] = \frac{\gamma^2}{2} (2\theta x_t \varphi^j + x_t^2 \varphi^{2j} + \cdots) \]

- the covariance

\[ \text{cov}[\nu_t^h, -r_t] = \gamma^3 \left[ 2 \left( 2\theta^2 \left( \frac{1 + \gamma}{2} - h \right) - \lambda \theta \right) \varphi^j \text{Var}[x_t] \right. \\
\quad \quad \quad \left. + \left( \frac{1 + \gamma}{2} - h \right) \varphi^{2j} \text{Var}[x_t^2] \right] \]
The Covariance

- the covariance

\[ \text{cov}[\nu_t^h, -r_t] = \gamma^3 \left[ 2 \left( 2\theta^2 \left( \frac{1 + \gamma}{2} - h \right) - \lambda \theta \right) \varphi^j \text{Var}[x_t] + \left( \frac{1 + \gamma}{2} - h \right) \varphi^{2j} \text{Var}[x_{t^2}] \right] \]

- first term due to conditional volatility \( x_t \) and second term due to conditional variance

- when \( j \) is large, the first term dominates
Short Premium Puzzle

Intertemporal Substitution, Precautionary Saving, and Currency Risk Premium
\begin{itemize}
  \item \( r(x_t) \) and \( \nu(x_t) \) are both non-monotone in \( x_t \)
  
  \item for \( x_t \in (-\theta, x_r) \), \( x_r = -\theta + \frac{\lambda}{\gamma+1-2h} \), both \( r(x_t) \) and \( \nu(x_t) \) increase with \( x_t \) and thus increase with each other
  
  \item for \( x_t > x_r \) or \( x_t < -\theta \), \( r(x_t) \) decrease with \( \nu(x_t) \)
  
  \item the unconditional covariance between the two is negative if \( \lambda \) is small enough (place an upper bound on \( \lambda \))
  
  \item Note that the region \( x_t < -\theta \) has negligible probability mass if \( \theta \gg 0 \)
\end{itemize}
Long Premium Puzzle

\[ r_t \]

\[ x_t \]

\[ r \]

\[ -\theta \]

\[ 0 \]

\[ x_r \]
• long country premium is proportional to $x_t$

• $r(x_t)$ is non-monotone in $x_t$

• $r(x_t)$ increases with $x_t$ for $x_t \leq x_r$ and decrease for $x_t > x_r$, 
  $$x_r = -\theta + \frac{\lambda}{\gamma + 1 - 2h}$$

• over all correlation between $r(x_t)$ and $x_t$ is positive if $x_r > 0$ (place an lower bound on $\lambda$)
Cumulative Premium Puzzle

\[ \sum_{j=0}^{\infty} E_t [\nu_{t+j}^i - \Theta_1] \]
• $r(x_t)$ and $\sum_{j=0}^{\infty} E_t[\nu_{t+j}](x_t)$ are both non-monotone in $x_t$

• both $r(x_t)$ and $\sum_{j=0}^{\infty} E_t[\nu_{t+j}](x_t)$ increase with $x_t$, and thus, increase with each other if $-\theta(1 + \varphi) < x_t < x_r$,

$\quad x_r = -\theta + \frac{\lambda}{\gamma + 1 - 2h}$

• for $x_t > x_r$ or $x_t < -\theta(1 + \varphi)$, $r(x_t)$ decreases with $\sum_{j=0}^{\infty} E_t[\nu_{t+j}](x_t)$

• the unconditional covariance between the two is positive if $\lambda$ is large enough.
for the short premium, the precautionary saving effect (higher consumption variance causes investors to save more) dominates on average

places an upper bound on \( \lambda \)

for the long premium, the intertemporal substitution effect (higher consumption volatility implies higher consumption growth) dominates on average

provides a lower bound on \( \lambda \)

there is a range for \( \lambda \) such that both bounds are satisfied, thus resolving Engel’s paradox.
Term structure of $\text{Cov}(E_t[\rho_{t+j+1}], r_t^* - r_t)$

Figure: $\theta = 0.05$, $\sigma = 0.04$, $\varphi = 0.9$, $\gamma = 15$, $\lambda = 5$ and $h = -20$
Extension

- recursive utility
- stationary currency level
- both in closed form
We provide a risk based rational model to resolve Engel’s paradox.

Our model is parsimonious model with stochastic volatility and variance in mean consumption growth.

The intertemporal substitution account for excess co-movement puzzle while the precautionary saving account for the forward premium puzzle.

Our results point to new features for asset pricing models.