Paths of Ideological Conflict: Closing the Gap Between Gamson’s Law and Theory

Julia Belau

TU Dortmund University

Allied Social Science Associations - Annual Meeting
Session: Conflict and Governance
Atlanta, January 5th, 2019
Literature on government formation typically concentrates on governments in “minority legislatures”

Central issue: assignment of ministerial portfolios to parties
cf. [Laver(1998)] and [Laver and Schofield(1998)] for literature review

Most prominent landmark: Gamson’s Law
portfolio payoffs are proportional to relative seat share within the coalition cf. [Gamson(1961)]

Strong empirical evidence, but poor theoretical foundation and conflict with bargaining theory → outside options
Economic theory suggests the use of power-indices as the Banzhaf-Power-Index or the Coalitional Bargaining Solution cf. [Banzhaf(1952)]/[Coleman(1971)]/[Penrose(1946)] and [Compte and Jehiel(2010)]

Approaches based on power indices and bargaining theory stay behind Gamson’s Law w.r.t. explanatory power cf. [Linhart et al.(2008) Linhart, Pappi, and Schmitt]

“[The Power-index approach] should not (even) be considered as part of political science. Viewed as a scientific theory, it is a branch of probability theory and can safely be ignored by political scientists. [...] It has no factual content and can therefore not be used for purposes of prediction or explanation.” cf. [Albert(2003)]
### Example: 2016 State Parliament Election Rhineland-Palatinate

<table>
<thead>
<tr>
<th>party</th>
<th>CDU</th>
<th>SPD</th>
<th>FDP</th>
<th>Grüne</th>
<th>AfD</th>
</tr>
</thead>
<tbody>
<tr>
<td>seats</td>
<td>35%</td>
<td>39%</td>
<td>7%</td>
<td>6%</td>
<td>14%</td>
</tr>
</tbody>
</table>

### Table: Ministerial Positions Cabinett Dreyer II

<table>
<thead>
<tr>
<th>party</th>
<th>SPD</th>
<th>FDP</th>
<th>Grüne</th>
</tr>
</thead>
<tbody>
<tr>
<td># ministers</td>
<td>5 (56%)</td>
<td>2 (22%)</td>
<td>2 (22%)</td>
</tr>
</tbody>
</table>

**Gamson’s Law:**
- 75%, 13%, 12% → 7, 1, 1
- Banzhaf Index:
- 71%, 14%, 14% → 6-7, 1, 1

Both ignore ideological closeness/conflict potential.

**Our Approach:**
- 51-54%, 23-25%, 23-24% → 5, 2, 2
Towards Factual Content

- We suggest portfolio allocation due to relative weakness proportionality (cf. interpretation of Coalitional Bargaining Solution).
- In contrast to CBS, we do not derive weakness by unblocked coalitions, but by election specific ideological closeness → factual content.
- More precisely, we interpret ideological closeness of a coalition as proportional to its materialization probability.
- This yields a measure of weakness for non-member parties.
- Finally, we suggest portfolio allocation to be proportional to relative weakness.
Relative Weakness Proportionality

- Let $N = \{1, \ldots, n\}$ denote the parties in a parliament.
- Let $\{\mu_S\}_{S \subseteq N}$ be a measure of coalitional strength which satisfies $\mu_S \in [0, 1]$ for all coalitions $S \subseteq N$ and $\mu_S = 0$ for all non-winning coalitions.
- Then, $m_i^\mu = \sum_{S \subseteq N \setminus \{i\}} \mu_S$ denotes a party $i$'s weakness.
- We define bargaining power $x_i$ to be proportional to $i$'s relative weakness: for all parties $i, j$ we have

$$x_i = \frac{m_j^\mu}{m_i^\mu} x_j$$

where $\tilde{m}_i^\mu = \begin{cases} 1 + m_i^\mu & \exists \text{ pivotal party} \\ m_i^\mu & \nexists \text{ pivotal party} \end{cases}$

$\rightarrow$ if $i$ is weaker than $j$ (i.e. $m_i^\mu > m_j^\mu$), we have $x_i < x_j$; the weaker $i$ compared to $j$, the lower $x_i$ compared to $x_j$.
Relative Weakness Index

- Normalizing bargaining power by $\sum_{i \in N} x_i = 1$ (index on the unit interval) yields

$$x_i = \frac{1}{\tilde{m}_i^{\mu}} \cdot \left( \sum_{l=1}^{\vert N \vert} \frac{1}{\tilde{m}_l^{\mu}} \right)^{-1}$$

by solving the corresponding system of equations

- Portfolio allocation can be calculated by relative bargaining power within the government coalition

How to measure coalitional strength, i.e. $\{\mu_S\}_{S \subseteq N}$? → ideological closeness via VAA data
Factual Content and VAAs

- Voting Advice Applications (VAA) are a commonly used tool in Europe and “slowly but surely are gaining ground in other parts of the world”
  - Garzia and Marschall (2012), Marschall and Garzia (2014), Van Camp et al. (2014)
- VAAs provide yes/no/neutral positions for the “most important” election statements of potential parliament parties
- We use data from the German “Wahl-o-Mat”
- Equivalent use with “StemWijzer” (Netherlands), “Smartvote” (Switzerland), “Vote Compass” (Canada, USA, Australia, New Zealand), ...
Measuring Consensus via VAA data

- For each statement $s = 1, \ldots, S$, the parties $i = 1, \ldots, N$ self-position by choosing “agree”/“not agree”/“neutral”

- For each two parties $i, j$ and each statement $s$, we define the consensus value $c_{ij}^s$ according to

<table>
<thead>
<tr>
<th></th>
<th>agree</th>
<th>neutral</th>
<th>not agree</th>
</tr>
</thead>
<tbody>
<tr>
<td>agree</td>
<td>2</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>neutral</td>
<td>1</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>not agree</td>
<td>0</td>
<td>1</td>
<td>2</td>
</tr>
</tbody>
</table>

$\Rightarrow c_{ij}^s \in \{0, 1, 2\}$
**Motivation Relative Weakness VAAs Coalitional Closeness Application**

---

**Bilateral Closeness and Example Revisited**

**Definition (Bilateral Closeness between parties $i$ and $j$)**

\[
\text{bilclos}_{ij} := \left( \sum_{s=1}^{S} c_{ij}^s \right) \frac{1}{2 \cdot S} \in [0, 1]
\]

**Table: Closeness Matrix RP 2016**

<table>
<thead>
<tr>
<th></th>
<th>CDU</th>
<th>SPD</th>
<th>FDP</th>
<th>Grüne</th>
</tr>
</thead>
<tbody>
<tr>
<td>CDU</td>
<td>1</td>
<td>0.645</td>
<td>0.697</td>
<td>0.395</td>
</tr>
<tr>
<td>SPD</td>
<td>0.645</td>
<td>1</td>
<td>0.579</td>
<td>0.750</td>
</tr>
<tr>
<td>FDP</td>
<td>0.697</td>
<td>0.579</td>
<td>1</td>
<td>0.408</td>
</tr>
<tr>
<td>Grüne</td>
<td>0.395</td>
<td>0.750</td>
<td>0.408</td>
<td>1</td>
</tr>
</tbody>
</table>

Closeness of SPD and FDP: 57.9%  
Closeness of SPD and Green: 75%  
Closeness of FDP and Green: 40.8%  
\[\Rightarrow\]  
Closeness of Traffic-Light Coalition?
Instead of consensus, the statement specific conflict value can be measured by $2 - c_{ij}^s$. This yields distance between each two parties:

$$d_{ij} := \left( \sum_{s=1}^{S} 2 - c_{ij}^s \right) \frac{1}{2 \cdot S} = 1 - \text{biloc}_{ij}$$

**Figure: Distance Network RP 2016**
Coalitional Closeness: Possible vs Actual Conflict

- Centrality approach: Closeness by inverting the length of paths of least distance (cf. [Freeman(1978)])

- Problems regarding scales, relative differences and since distance = 1 – bilclos is cumulated across statements

- We use differences to possible conflict on negotiation paths w.r.t. statement-specific consensus/conflict

Possible vs. Actual Conflict

We interpret closeness as the difference between possible and actual conflict on a conflict path.
Coalitional Closeness: Sequential Negotiation on Paths

- Let $K \subseteq N$ be a coalition of parties
- Let $p^K := \{ij | i, j \in K, i \neq j\}$ be the link set of the complete graph with node-set $K$
  \[\rightarrow \text{set of all (bilateral) negotiation possibilities within } K\]

1. **Average conflict across complete negotiation graph**
   \[\rightarrow \text{concides with average closeness of coalition}\]

2. **Overall conflict across complete negotiation graph**
   \[\rightarrow \text{conflict potential of coalition}\]

3. **Least possible conflict across connecting negotiation path**
   \[\rightarrow \text{path of least conflict in coalition}\]
**Possible vs. Actual Conflict: Average Conflict**

**Definition (Average Conflict)**

Average conflict closeness of $K$ is given by

$$\text{AVCclos}_K := \sum_{s=1}^{S} \left( 2 - \sum_{i,j \in K: \atop i \neq j} (2 - c_{ij}^s) \left( \frac{|K|}{2} \right)^{-1} \right) \left( 2S \right)^{-1}$$

where we normalize by maximal average conflict across statements (index on unit interval).

**Lemma:** we have $\text{AVCclos}_K = \sum_{i,j \in K: \atop i \neq j} \text{bilclos}_{ij} \left( \frac{|K|}{2} \right)^{-1}$. 

---

J. Belau  
*Paths of Ideological Conflict: Closing the Gap Between Gamson’s Law and Theory*
**Possible vs. Actual Conflict: Conflict Potential**

**Theorem:** For each statement $s = 1, \ldots, S$, the maximal overall conflict within $K \subseteq N$ is $\left\lfloor \frac{|K|^2}{2} \right\rfloor$.

**Definition (Conflict Potential Closeness)**

Conflict Potential closeness of $K$ is given by

$$CPclos_K := \sum_{s=1}^{S} \left( \left\lfloor \frac{|K|^2}{2} \right\rfloor - \sum_{i,j \in K, i \neq j} \left( 2 - c_{ij}^s \right) \right) \left( \left\lfloor \frac{|K|^2}{2} \right\rfloor \cdot S \right)^{-1}$$

where we normalize by the maximal conflict value across statements to obtain an index on a normalized scale.
Possible vs. Actual Conflict: Path of Least Conflict

Interpret coalitional negotiation as a *minimal* sequential process of bilateral negotiations. The conflict value on a path of least conflict (PLC) w.r.t. statement $s$ is given by

$$plc^s_K := \min \left\{ \sum_{ij \in p} (2 - c_{ij}^s) \mid p \text{ connects } K \right\}$$

**Definition (Path of Least Conflict Closeness)**

*Path of Least Conflict closeness of $K$* is given by

$$\text{PLCclos}_K := \sum_{s=1}^{S} \left( 2(|K| - 1) - plc^s_K \right) \left( 2(|K| - 1) \cdot S \right)^{-1}$$

where we normalize by the maximal *bilateral* conflict on a minimal connecting path across statements.
## Comparison of Closeness Measures

<table>
<thead>
<tr>
<th>Coalition</th>
<th>CDU &amp; SPD</th>
<th>SPD, FDP &amp; Grüne</th>
</tr>
</thead>
<tbody>
<tr>
<td>AVC-Closeness</td>
<td>64.47</td>
<td>57.89</td>
</tr>
<tr>
<td>CP-Closeness</td>
<td>64.47</td>
<td>36.84</td>
</tr>
<tr>
<td>PLC-Closeness</td>
<td>64.47</td>
<td>68.42</td>
</tr>
</tbody>
</table>

CP rules out certain consensus in a group of 3 parties: at least 2 parties have equal or not strongly opposite positions!
**Application: Parliament Elections Germany**

- Always adjust for incompatibilities!
- Analysis on ministerial positions (w/o prime minister)
- 31 Parliament Elections, comparison to Gamson’s Law
  - Baden-Wuerttemberg 06 ✓, 11 ✓, 16 ✓
  - Bavaria 18 ✓
  - Berlin 06 ✓, 11 ✓, 16 ✓
  - Bremen 07 ✓, 11 ✓, 15 ✓
  - Federal Parliament 05 ✓, 09 ✓, 13 ✓, 17 ✓
  - Hamburg 08 ✓, 15 ✓
  - Lower Saxony 08 ✓, 13 ✓
  - North-Rhine-Westphalia 05 ✓, 10 ✓, 12 ✓, 17 ✓
  - Rhineland-Palatinate 12 ✓, 16 ✓
  - Saarland 12 ✗, 17 ✓
  - Schleswig-Holstein 12 ✓, 17 ✓
  - Saxony 14 ✓
  - Saxony-Anhalt 16 ✗
  - Thuringia ✓

- ✓: confirm (20), ✗: not confirm but better (9)
- confirmation rate for Gamson’s Law:
  - 91% (20 out of 31-9=22) / 93.5% (20+9=29/31)
**Table : Proxy-Performance for Portfolio Allocation**

<table>
<thead>
<tr>
<th></th>
<th>Correct Proxy</th>
<th>Best Proxy</th>
<th>Gamson’s Law</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gamson</td>
<td>18</td>
<td>20</td>
<td>8</td>
</tr>
<tr>
<td>Banzhaf</td>
<td>4</td>
<td>7</td>
<td>9</td>
</tr>
<tr>
<td>NoWeight</td>
<td>4</td>
<td>8</td>
<td></td>
</tr>
<tr>
<td>AVCclos</td>
<td>15</td>
<td>18</td>
<td>14 (22)</td>
</tr>
<tr>
<td>CPclos</td>
<td>19</td>
<td>21</td>
<td>15 (23)</td>
</tr>
<tr>
<td>PLCclos</td>
<td>16</td>
<td>20</td>
<td>12 (22)</td>
</tr>
<tr>
<td>Merged</td>
<td>25</td>
<td>29</td>
<td>20 (29)</td>
</tr>
</tbody>
</table>

J. Belau

Paths of Ideological Conflict: Closing the Gap Between Gamson’s Law and Theory
Conclusion

- We suggest portfolio allocation due to relative weakness proportionality: Closeness interpreted as materialization probability → weakness for non-member parties
- Ideological closeness is derived by conflict path analysis / consensus and conflict from VAA data
- Analysis of 31 elections in Germany → 91% / 93.5% confirmation rate
- Further research
  - More data ("StemWijzer"/"Smartvote"/"Vote Compass")
  - Hybrid between CP and PLC → SMC
  - Centrality/Distance analysis

Thank you for your attention


References II


### References III

<table>
<thead>
<tr>
<th>Motivation</th>
<th>Relative Weakness</th>
<th>VAAs</th>
<th>Coalitional Closeness</th>
<th>Application</th>
</tr>
</thead>
</table>
