Nonparametric Demand Estimation in Differentiated Products Markets

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Motivation

- Many questions in economics hinge on the shape of the market demand functions
- What are the effects of a merger?
- What is the pass-through of a tax?
- What are the sources of firm market power?
Example: tax pass-through

1. Estimate demand from data on quantities, prices and covariates

2. Get firm marginal costs from data or supply & demand

3. Given costs, use supply & demand to get equilibrium after tax
State of the art

- Focus on
  - differentiated products
  - endogenous prices

- The frontier is discrete choice with random coefficients (BLP)

- Convenient, but arbitrary, parametric assumptions might drive results

- Can we avoid these restrictions?
This paper

Idea:

- Minimize use of arbitrary restrictions
- Impose constraints motivated by economics

To do so, I combine:

- large datasets
- economic theory
- frontier econometric tools
The approach is nonparametric
- No distributional assumptions on unobservables
- Relax most functional form restrictions

In addition, the model applies beyond discrete choice:
- Complementarities
- Consumer inattention
- Continuous choices
- Others
Applications

- Using grocery store data, estimate demand in two ways
  - standard mixed logit model
  - my approach

- What is the pass-through of a tax?
  - For one product, I find steeper own-price elasticity function
    ⇒ pass-through is much lower (≈30% vs ≈90% of tax)

- How much competition is internalized by multi-product firm?
  “Portfolio effect” (Nevo, 2001)
  - Two approaches give similar results
  - Not with more restrictive mixed logit model ⇒ model selection
Roadmap

1. General Demand Model
   - Model and identification
   - Nonparametric estimation
   - Monte Carlo simulations

2. Applications
   - BLP demand estimates
   - Nonparametric demand estimates
   - Counterfactual 1: Tax pass-through
   - Counterfactual 2: Effect of two-product retailer
Model

- \( J \) goods plus the outside option
- \( s = (s_1, ..., s_J) \): shares
- \( p = (p_1, ..., p_J) \): endogenous prices
- \( \xi = (\xi_1, ..., \xi_J) \): product- or market-level unobservables
- \( z = (z_1, ..., z_J) \): excluded instruments for price
- \( x = (x^{(1)}, x^{(2)}) \): exogenous demand shifters, with
  \[ x^{(1)} = \left( x_1^{(1)}, ..., x_J^{(1)} \right) \]
Consider the general demand system

\[ s = \sigma (x, \xi, p) \]

I let \( \delta_j = \beta_j x_j^{(1)} + \xi_j \) and require

\[ s = \sigma (\delta, p, x^{(2)}) \]
Model subsumes discrete choice

- This discrete choice model satisfies the index restriction

\[
u_{ij} = \alpha_{p,i} p_j + \alpha_{\delta,i} \delta_j + \alpha_{x,i} x_j^{(2)} + \epsilon_{ij}
\]

\[
\delta_j = \beta_j x_j^{(1)} + \xi_j
\]

- No need to assume distributions for \( \epsilon_{ij} \) nor \((\alpha_{p,i}, \alpha_{\delta,i}, \alpha_{x,i})\)

- \( u_{ij} \) need not be linear in \( p_j, \delta_j, x_j^{(2)} \)
Assuming

- Index restriction
- Strict substitution \textit{under some transformation of demand}
- The instruments \((x, z)\) shift \((s, p)\) ‘enough’,

Berry and Haile (2014) show that

\[ s = \sigma \left( \delta, p, x^{(2)} \right) \]

is nonparametrically identified
Flexibility on consumer behavior

- The model is more general than discrete choice

- Focus on demand vs utility
  
  ⇒ can be more agnostic about what consumers do

- Can accommodate
  
  - Complements
  
  - Consumer inattention
  
  - Consumer loss aversion
  
  - Continuous choice and multiple discrete choices
Berry and Haile (2014) focus on nonparametric identification. What could we learn about demand if we observed the entire population of markets?

Leveraging the identification results, I address estimation and inference. Can we estimate demand nonparametrically on datasets available to economists? Can we obtain informative confidence sets for quantities of interest? How can we test hypotheses on consumer behavior? How do we choose among several parametric models?
\[ s_j = \sigma_j \left( \delta, p, x^{(2)} \right) \quad j = 1, ..., J \]

Under identification assumptions,

\[ x^{(1)}_j + \xi_j = \sigma_j^{-1} \left( s, p, x^{(2)} \right) \]

Also, we assume

\[ \mathbb{E} (\xi_j | x, z) = 0 \quad \text{a.s.} \]

\[ \Rightarrow \text{Approximate } \sigma_j^{-1} \text{ and project predicted residuals onto IVs} \]
I approximate $\sigma_j^{-1}$ using the method of sieves

Basis functions: Bernstein polynomials

Easy to impose a number of economic constraints

I obtain standard errors based on recent results
(Chen and Pouzo, 2015; Chen and Christensen, 2018)
Theorem 1

Let the demand system $\sigma$ be identified. Let $f$ be a scalar functional of $\sigma^{-1}$ and $\hat{\sigma}^{-1}$, and $\hat{v}_T(f)$ be a consistent estimator of the standard deviation of $f(\hat{\sigma}^{-1})$. In addition, let Assumptions 1, 2, 3 and 4 hold. Then,

$$\sqrt{T} \frac{f(\hat{\sigma}^{-1}) - f(\sigma^{-1})}{\hat{v}_T(f)} \xrightarrow{d} N(0, 1).$$

Uses:

- Confidence intervals
- Hypothesis testing
- Model selection
Proofs follow Chen and Christensen (2018), but in my setting there are multiple ($J$) equations and error terms

- need to deal with correlation in the error terms

I provide low-level conditions for functionals of interest:

- Price elasticities
- (Counterfactual) equilibrium prices
Curse of dimensionality

- The functions $\sigma_j^{-1}$ have $2J$ arguments, plus extra covariates
- Number of parameters grows with number of goods and number of covariates

But

- Assumptions based on economics help alleviate that
- Large datasets (e.g. scanner data) are increasingly available
- Several interesting markets are low-dimensional
Constraints

- Exchangeability (Pakes, 1994; BLP)
- Index restriction
- No income effects
- Monotonicity

I do not impose all of them in simulations/application
Computation

- Estimation is based on minimizing a quadratic form in the Bernstein coefficients.

- If constraints are convex, standard algorithms converge to \textit{global} minimizer.

- BLP objective is typically non-convex.
Monte Carlo Simulations

- Given the same data generating process, I estimate demand using
  - my approach
  - random coefficients logit
- Then compare own- and cross-price elasticities
- Show that
  - nonparametric approach works for reasonable sample sizes (3,000)
  - approach is applicable beyond discrete choice
Correctly-specified BLP

Figure: Own-price

Figure: Cross-price
Inattention

- A fraction of consumers ignores good 1
- The fraction of inattentive consumer increases with $p_1$
- Otherwise, same model as before
Inattention

**Figure: Own-price**

**Figure: Cross-price**
Loss Aversion

\[ u_{ij} = -\alpha_i p_j - \alpha_{\text{loss}} (p_j - p_k) + x_j + \xi_j + \epsilon_{ij} \]
Loss Aversion

Figure: Own-price

Figure: Cross-price
Complements

- Exogenous variables and prices are generated as above

- Let
  \[ q_j = 10 \frac{\delta_j}{p_j^2 p_k} \quad j = 1, 2; \quad k \neq j \]

  ⇒ Good 1 and 2 are complements

- Define
  \[ s_j = \frac{q_j}{1 + q_1 + q_2} \quad j = 1, 2 \]

  ⇒ Strict substitution assumption ✓
Complements

Figure: Own-price

Figure: Cross-price
Additional Simulations

- Chi-square random coefficients
- Smaller sample size
- Violation of index restriction
- Sensitivity to tuning parameter
- \( J > 2 \)
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Empirical Setting

- I use scanner data from CA supermarkets
- A market is a store/week
- Look at sales of
  - non-organic strawberries (=1)
  - organic strawberries (=2)
  - other fruit (=0)
- Assume retailer is a monopolist \textit{wrt} strawberries
Perishability simplifies framework
Empirical Model

\[ s_1 = \sigma_1 \left( \delta_{str}, \delta_{org}, p_0, p_1, p_2, x^{(2)} \right) \]

\[ s_2 = \sigma_2 \left( \delta_{str}, \delta_{org}, p_0, p_1, p_2, x^{(2)} \right) \]

where

- \( x^{(2)} = \text{Income} \)
- \( p_0, p_1, p_2 = \text{Prices} \)
- \( \delta_{str}, \delta_{org} = \text{Quality indices} \)
Exogenous Demand Shifters

\[
\delta_{str} = \beta_{0,str} - \beta_{1,str}x_{str}^{(1)} + \xi_{str}
\]

\[
\delta_{org} = \beta_{0,org} + \beta_{1,org}x_{org}^{(1)} + \xi_{org}
\]

where

- \( x_{str}^{(1)} \) is a proxy for richness of outside option
  - Captures substitution between inside and outside goods

- \( x_{org}^{(1)} \) is a proxy for taste for organic products
  - Captures substitution between the two inside goods

- \((\xi_{str}, \xi_{org})\) = Unobservables varying across markets

Microfoundation
Endogenous Prices

- I instrument for retail prices using wholesale spot prices
- I also use retail prices of same products in other marketing areas (Hausman IVs)
  - Valid if unobservable demand shocks are independent across marketing areas, but retailer costs are not
For comparison, I also fit a logit model with a random coefficient on price.

I take a two-point distribution for the random coefficient

\[ u_{i,1} = \beta_1 + \left( \beta_{p,i} + \beta_2 x^{(2)} \right) p_1 + \beta_{p,0} p_0 + \beta_{str} x^{(1)}_{str} + \xi_1 + \epsilon_{i,1} \]

\[ u_{i,2} = \beta_2 + \left( \beta_{p,i} + \beta_2 x^{(2)} \right) p_2 + \beta_{p,0} p_0 + \beta_{str} x^{(1)}_{str} + \beta_{org} x^{(1)}_{org} + \xi_2 + \epsilon_{i,2} \]
<table>
<thead>
<tr>
<th>Variable</th>
<th>Type I</th>
<th>Type II</th>
</tr>
</thead>
<tbody>
<tr>
<td>Price</td>
<td>$-7.58$</td>
<td>$-89.85$</td>
</tr>
<tr>
<td></td>
<td>(0.07)</td>
<td>(6.53)</td>
</tr>
<tr>
<td>Price×Income</td>
<td>0.89</td>
<td>0.06</td>
</tr>
<tr>
<td>Price other fruit</td>
<td>8.70</td>
<td>0.23</td>
</tr>
<tr>
<td>Other fruit</td>
<td>$-0.37$</td>
<td>(0.01)</td>
</tr>
<tr>
<td>Taste for organic</td>
<td>0.08</td>
<td>(0.06)</td>
</tr>
<tr>
<td>Fraction of consumers</td>
<td>0.82</td>
<td>0.18</td>
</tr>
<tr>
<td></td>
<td>(0.00)</td>
<td>(0.00)</td>
</tr>
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Roadmap

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Counterfactual 1: Per-unit Tax

- Per-unit tax equal to 25% of the price
- Results depend on curvature of the demand function
- The faster the elasticity increases with price, the lower the pass-through
Significant difference in pass-through for organic

<table>
<thead>
<tr>
<th></th>
<th>NPD</th>
<th>Mixed Logit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Non-organic</td>
<td>0.84 (0.17)</td>
<td>0.53 (5 \cdot 10^{-3})</td>
</tr>
<tr>
<td>Organic</td>
<td>0.33 (0.23)</td>
<td>0.91 (5 \cdot 10^{-4})</td>
</tr>
</tbody>
</table>

**Table:** Median changes in prices as a percentage of the tax
Steeper own-price elasticity is consistent with lower pass-through

Figure: Estimated own-price elasticity function
Counterfactual 2: Portfolio Effect

- In each market, one retailer sells both types of strawberries
- Suppose there were two competing single-product retailers
- How much lower would markups be?
Portfolio Effect: Choice of parametric model matters

<table>
<thead>
<tr>
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<th>MLog (II)</th>
<th>MLog (III)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Non-organic</td>
<td>0.10</td>
<td>0.08</td>
<td>0.20</td>
<td>0.21</td>
</tr>
<tr>
<td></td>
<td>(3·10^{-3})</td>
<td>(1·10^{-3})</td>
<td>(8·10^{-4})</td>
<td>(2·10^{-3})</td>
</tr>
<tr>
<td>Organic</td>
<td>0.43</td>
<td>0.42</td>
<td>0.54</td>
<td>0.55</td>
</tr>
<tr>
<td></td>
<td>(6·10^{-3})</td>
<td>(2·10^{-3})</td>
<td>(9·10^{-4})</td>
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**Table:** Median decrease in prices as a percentage of markups
Portfolio Effect: Choice of parametric model matters

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Table: Median decrease in prices as a percentage of markups
Conclusion

- First nonparametric approach to estimate demand for differentiated products

- Approach is applicable to data available to economists

- Flexibility matters for questions of interest
Related Literature

- Random coefficients discrete choice [Berry, Levinsohn and Pakes (1995); Nevo (2001); Petrin (2002); Berry, Levinsohn and Pakes (2004)]

- Discrete choice with inattention [Goeree (2008); Abaluck and Adams (2017)]

- Continuous and multiple discrete choice [Dubin and McFadden (1984); Hendel (1999); Dubé (2004)]

- Neoclassical demand [Deaton and Muellbauer (1980); Banks, Blundell and Lewbel (1997); Blundell, Chen and Kristensen (2007)]

- Complements [Gentzkow (2007)]

- Incomplete pass-through [Nakamura and Zerom (2010); Goldberg and Hellerstein (2013)]

- Multi-product firms and market power [Nevo (2001)]
Illustration

Nonparametric Demand Estimation in Differentiated Products Markets
Bernstein Polynomials

- The $k$–th Bernstein polynomial of degree $m$ is defined as

$$b_{k,m}(u) \equiv \binom{m}{k} u^k (1 - u)^{m-k}$$

for $u \in [0, 1]$ and $k = 0, \ldots, m$

- A continuous univariate function may be approximated by

$$\sum_{k=0}^{m} \theta_k b_{k,m}(u)$$

- For continuous multivariate functions, we may take the tensor product of the univariate Bernstein polynomials
Bernstein Polynomials

Theorem (Uniform Approximation)

Let $g$ be a bounded real-valued function on $[0, 1]^N$ and define

$$B_m[g] = \sum_{v_1=0}^m \cdots \sum_{v_N=0}^m g \left( \frac{v_1}{m}, \cdots, \frac{v_N}{m} \right) b_{v_1,m}(u_1) \cdots b_{v_N,m}(u_N)$$

Then,

$$\sup_{u \in [0,1]^N} |B_m[g](u) - g(u)| \to 0$$

as $m \to \infty$. 

Assumptions for Theorem 1

**Assumption 1**

For all \( j, k \in J, j \neq k \):

1. \( \sup_{w \in \mathcal{W}} \mathbb{E} \left( \xi_j^2 | w \right) \leq \bar{\sigma}^2 < \infty \);
2. \( \inf_{w \in \mathcal{W}} \mathbb{E} \left( \xi_j^2 | w \right) \geq \sigma^2 > 0 \);
3. \( \sup_{w \in \mathcal{W}} \mathbb{E} \left( |\xi_j \xi_k| | w \right) \leq \bar{\sigma}_{cov} < \infty \);
4. \( \sup_{w \in \mathcal{W}} \mathbb{E} \left[ \xi_j^2 \mathbb{I} \left\{ \sum_{i=1}^{J} |\xi_i| > \ell(T) \right\} | w \right] = o(1) \) for any positive sequence \( \ell(T) \uparrow \infty \);
5. \( \mathbb{E} \left( |\xi_j|^{2+\gamma^{(1)}} \right) < \infty \) for some \( \gamma^{(1)} > 0 \);
6. \( \mathbb{E} \left( |\xi_j \xi_k|^{1+\gamma^{(2)}} \right) < \infty \) for some \( \gamma^{(2)} > 0 \).
Assumptions for Theorem 1

Assumption 2

1. $\tau M \zeta \sqrt{M \log M} / T = o(1)$;
2. $\zeta^{(2+\gamma(1))}/\gamma(1) \sqrt{(\log K) / T} = o(1)$ and $\zeta^{(1+\gamma(2))}/\gamma(2) \sqrt{(\log K) / T} = o(1)$, where $\gamma(1), \gamma(2) > 0$ are defined in Assumption 1;
3. $K \asymp M$.

Assumption 3

The basis used for the instrument spaces is the same across all goods, i.e. $K_j = K_k$ and $a_{K_j}^{(j)}(\cdot) = a_{K_k}^{(k)}(\cdot)$ for all $j, k \in J$. 

Return
Assumptions for Theorem 1

Assumption 4

Let $\mathcal{H}_T \subset \mathcal{H}$ be a sequence of neighborhoods of $h_0$ with $\hat{h}, \tilde{h} \in \mathcal{H}_T$ wpa1 and assume $\nu_T(f) > 0$ for every $T$. Further, assume that:

1. $v \mapsto Df(h_0)[v]$ is a linear functional and there exists $\alpha$ with $|\alpha| \geq 0$ s.t. $|Df(h_0)[h - h_0]| \lesssim ||\partial^\alpha h - \partial^\alpha h_0||_\infty$ for all $h \in \mathcal{H}_T$;

2. There are $\alpha_1, \alpha_2$ with $|\alpha_1|, |\alpha_2| \geq 0$ s.t.

   1. $\left| f(\hat{h}) - f(h_0) - Df(h_0)[\hat{h} - h_0] \right| \lesssim ||\partial^{\alpha_1} \hat{h} - \partial^{\alpha_1} h_0||_\infty ||\partial^{\alpha_2} \hat{h} - \partial^{\alpha_2} h_0||_\infty$;

   2. $\frac{\sqrt{T}}{\sigma_T(f)} \left( ||\partial^{\alpha_1} \hat{h} - \partial^{\alpha_1} h_0||_\infty ||\partial^{\alpha_2} \hat{h} - \partial^{\alpha_2} h_0||_\infty + ||\partial^\alpha \tilde{h} - \partial^\alpha h_0||_\infty \right) = O_p(\eta_T)$ for a nonnegative sequence $\eta_T$ such that $\eta_T = o(1)$;

3. $\frac{1}{\nu_T(f)} \left| \left( Df(\hat{h})[\psi_M]' - Df(h_0)[\psi_M]' \right) \left( G_A^{-1/2} S \right)_i \right| = o_p(1)$. 

[Return]
Assumption 5

1. \( P \) has bounded support and \((P, S)\) have densities bounded away from 0 and \( \infty \);

2. The basis used for both the sieve space and the instrument space is tensor-product Bernstein polynomials. Further, for the sieve space, the univariate Bernstein polynomials all have the same degree \( M^{1/4} \);

3. \( h_0 = [h_{0,1}, h_{0,2}] \) where \( h_{0,1} \) and \( h_{0,2} \) belong to the Hölder ball of smoothness \( r \geq 4 \) and finite radius \( L \), and the order of the tensor-product Bernstein polynomials used for the sieve space is greater than \( r \);

4. \( M^{2 + \gamma(1) \frac{\log T}{T}} = o(1) \) and \( M^{1 + \gamma(2) \frac{\log T}{T}} = o(1) \);

5. \( \frac{\sqrt{T}}{v_T(f_\epsilon)} \times \left( M^{\frac{3-r}{4}} + \tau_M M^{9/4 \frac{\log M}{T}} \right) = o(1) \).
Equilibrium Price Functional

Assumption 6

1. \( P \) has bounded support and \((P, S)\) have densities bounded away from 0 and \( \infty \);

2. The basis used for both the sieve space and the instrument space is tensor-product Bernstein polynomials. Further, for the sieve space, the univariate Bernstein polynomials all have the same degree \( M^{1/4} \);

3. \( h_0 = [h_{0,1}, h_{0,2}] \) where \( h_{0,1} \) and \( h_{0,2} \) belong to the Hölder ball of smoothness \( r \geq 5 \) and finite radius \( L \), and the order of the tensor-product Bernstein polynomials used for the sieve space is greater than \( r \);

4. \( \frac{2 + \gamma^{(1)}}{2 \gamma^{(1)}} \sqrt{\frac{\log T}{T}} = o(1) \) and \( \frac{1 + \gamma^{(2)}}{2 \gamma^{(2)}} \sqrt{\frac{\log T}{T}} = o(1) \);

5. \( \frac{\sqrt{T}}{\nu_T(f_{p_1})} \times \left( M^{\frac{4-r}{4}} + \tau^2 M^{9/4 \log \frac{M}{T}} \right) = o(1) \).
Low-dimensional Settings

- International economics (Adao, Costinot and Donaldson, 2017)
- Market for news (Gentzkow, 2007)
- US presidential elections
Exchangeability

Given a permutation \( \pi : \{1, ..., J\} \to \{1, ..., J\} \), assume that \( \forall j \)

\[
\sigma_j \left( \delta, p, x^{(2)} \right) = \sigma_{\pi(j)} \left( \delta_{\pi(1)}, ..., \delta_{\pi(J)}, p_{\pi(1)}, ..., p_{\pi(J)}, x_{\pi(1)}^{(2)}, ..., x_{\pi(J)}^{(2)} \right)
\]

- In words, only the products’ attributes—not their names—matter
- E.g. for \( J = 3 \) and no \( x^{(2)} \),

\[
\sigma_1 \left( \delta_1, \bar{\delta}, p_1, \bar{p}, \bar{p} \right) = \sigma_1 \left( \delta_1, \bar{\delta}, \bar{\delta}, p_1, \bar{p}, \bar{p} \right)
\]

- Implicit in most IO demand models
- Systematic differences between goods can be captured by product fixed effects
Exchangeability

• This assumption greatly reduces the number of parameters to be estimated. E.g. for univariate polynomials of degree 2 (and no $x^{(2)}$):

<table>
<thead>
<tr>
<th>$J$</th>
<th>exchangeability</th>
<th>no exchangeability</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>324</td>
<td>729</td>
</tr>
<tr>
<td>4</td>
<td>900</td>
<td>6,561</td>
</tr>
<tr>
<td>5</td>
<td>2,025</td>
<td>59,049</td>
</tr>
<tr>
<td>10</td>
<td>27,225</td>
<td>3.4bn</td>
</tr>
</tbody>
</table>

• It can be shown that exchangeability of $\sigma$ implies exchangeability of $\sigma^{-1}$

• Exchangeability of $\sigma^{-1}$ can be imposed through linear restrictions on the Bernstein coefficients
Index Restriction

- $x^{(2)}$ enters demand flexibly

\[ s = \sigma \left( \delta, p, x^{(2)} \right) \]

\[ \delta_j = \beta_j x^{(1)}_j + \xi_j \]

- $x^{(2)}$ enters demand through $\delta \Rightarrow$ number of parameters drops

\[ s = \sigma \left( \delta, p \right) \]

\[ \delta_j = \beta^{(1)}_j x^{(1)}_j + \beta^{(2)}_j x^{(2)}_j + \xi_j \]

- Same logic applies to $p$
Berry, Gandhi and Haile (2013) show that the Jacobian of the demand function, $\mathbb{J}_\sigma^\delta$ is an $M$–matrix.

This encapsulates restrictions from economic theory:
- e.g. shares increase (decrease) in own (competitors') $\delta$

By the implicit function theorem,

$$\mathbb{J}_\sigma^s - 1 = \left[ \mathbb{J}_\sigma^\delta \right]^{-1}$$

i.e. $\mathbb{J}_\sigma^s - 1$ is an inverse $M$–matrix.

There is a large literature in linear algebra on properties of inverse $M$–matrices.

A number of them can be imposed through linear restrictions.
With no income effects, then Hicksian and Walrasian demands coincide
\[ J^p_\sigma \text{ is symmetric} \]

By the implicit function theorem,
\[ J^p_\sigma = - \left[ J^s_{\sigma-1} \right]^{-1} J^p_{\sigma-1} \]

These are nonlinear constraints \( \Rightarrow \) Use KNITRO
Correctly-specified BLP

- Two goods with utility
  \[ u_{ij} = -\alpha_i p_j + x_j + \xi_j + \epsilon_{ij} \]

- Plus an outside option with utility \( u_{i0} = \epsilon_{i0} \)

- \( \alpha_i \sim N (1, 0.15^2) \)
- \( \epsilon_{ij} \) is extreme value
- \( x_j \sim U [0, 2] \) independently across \( j \)
- \( \xi_j \sim N (1, 0.15^2) \)
- \( z_j \sim U [0, 1] \)
- \( p_j = 2(z_j + \eta_j) + \xi_j \), where \( \eta_j \sim U [0, 0.1] \)
- Constraints: symmetry, \( M\)-matrix properties and exchangeability
Chi-Square Random Coefficients

Figure: Own-price

Figure: Cross-price
Complements: $T=500$

**Figure: Own-price**

**Figure: Cross-price**
Violation of index restriction: \( st. \text{dev.} = 0.10 \)

**Figure:** Own-price

**Figure:** Cross-price
Violation of index restriction: \( st.\, dev. = 0.50 \)

**Figure:** Own-price

**Figure:** Cross-price
Violation of index restriction: $st.\ dev. = 1.50$

**Figure: Own-price**

**Figure: Cross-price**

Nonparametric Demand Estimation in Differentiated Products Markets
Sensitivity: complements, degree=20

Figure: Own-price

Figure: Cross-price
Sensitivity: complements, degree=16

**Figure:** Own-price

**Figure:** Cross-price
Sensitivity: complements, degree=12

![Own-price diagram](image1)
![Cross-price diagram](image2)

**Figure:** Own-price  
**Figure:** Cross-price
Sensitivity: complements, degree=8

Figure: Own-price

Figure: Cross-price
Sensitivity: complements, degree = 6

Figure: Own-price

Figure: Cross-price
Sensitivity: complements, degree = 4

Figure: Own-price

Figure: Cross-price
Mixed Logit: $J = 3$

Figure: Own-price

Figure: Cross-price
Mixed Logit: $J = 5$

Figure: Own-price

Figure: Cross-price
Mixed Logit: \( J = 7 \)

**Figure: Own-price**

**Figure: Cross-price**
Nonparametric Estimation

- Each of the functions to be estimated has six arguments ⇒ a lot of parameters
- I impose $M$–matrix restrictions on the Jacobian
- Overall, I have 648 parameters
- The number of parameters could be reduced by including income and/or prices in the linear indices $\delta$
Table: Two-Fold Cross-Validation Results

<table>
<thead>
<tr>
<th></th>
<th>NPD</th>
<th>Mixed Logit</th>
</tr>
</thead>
<tbody>
<tr>
<td>MSE</td>
<td>0.93</td>
<td>2.38</td>
</tr>
</tbody>
</table>

Fit
## Median Nonparametric Elasticities

<table>
<thead>
<tr>
<th></th>
<th>Non-organic</th>
<th>Organic</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Own-price elasticity</strong></td>
<td>−1.402 (0.032)</td>
<td>−5.503 (0.672)</td>
</tr>
<tr>
<td><strong>Cross-price elasticity</strong></td>
<td>0.699 (0.044)</td>
<td>1.097 (0.177)</td>
</tr>
</tbody>
</table>
Micro-foundation 1: discrete choice

\[ u_{i1} = \theta_{str} \delta_{str}^* + \alpha_i p_1 + \epsilon_{i1} \]

\[ u_{i2} = \theta_{str} \delta_{str}^* + \theta_{org} \delta_{org}^* + \alpha_i p_2 + \epsilon_{i2} \]

\[ u_{i0} = \theta_{0,str} \chi_{str}^{(1)} + \theta_{0,org} \delta_{org} + \alpha_i p_0 + \epsilon_{i0} \]

where

\[ \delta_{str}^* = \xi_{str} \]

\[ \delta_{org}^* = \theta_{1,org} \chi_{org}^{(1)} + \xi_{org} \]
Micro-foundation 2: continuous choice

\[
\max_{q_0, q_1, q_2} q_0^{d_{0i,0}} q_1^{d_{1i,1}} q_2^{d_{2i,2}}
\]

s.t. \( p_0 q_0 + p_1 q_1 + p_2 q_2 \leq y_i^{inc} \)

where

\[
d_0 = \exp \left\{ \theta_{0,org} \delta^*_{org} + \theta_{0,\text{str}} x_{\text{str}}^{(1)} \right\}
\]

\[
d_1 = \exp \left\{ \theta_{\text{str}} \delta^*_{\text{str}} \right\}
\]

\[
d_2 = \exp \left\{ \theta_{\text{str}} \delta^*_{\text{str}} + \theta_{\text{org}} \delta^*_{\text{org}} \right\}
\]
### Descriptive Statistics

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Median</th>
<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quantity non-organic</td>
<td>735.33</td>
<td>581.00</td>
<td>6.00</td>
<td>5,729.00</td>
</tr>
<tr>
<td>Quantity organic</td>
<td>128.91</td>
<td>78.00</td>
<td>1.00</td>
<td>2,647.00</td>
</tr>
<tr>
<td>Price non-organic</td>
<td>2.97</td>
<td>2.89</td>
<td>0.93</td>
<td>4.99</td>
</tr>
<tr>
<td>Price other fruit</td>
<td>3.95</td>
<td>3.80</td>
<td>1.30</td>
<td>13.88</td>
</tr>
<tr>
<td>Hausman non-organic</td>
<td>3.00</td>
<td>2.98</td>
<td>2.09</td>
<td>4.05</td>
</tr>
<tr>
<td>Hausman organic</td>
<td>4.28</td>
<td>4.07</td>
<td>2.95</td>
<td>5.50</td>
</tr>
<tr>
<td>Hausman other fruit</td>
<td>4.50</td>
<td>3.79</td>
<td>1.19</td>
<td>13.33</td>
</tr>
<tr>
<td>Spot non-organic</td>
<td>1.46</td>
<td>1.35</td>
<td>0.99</td>
<td>2.32</td>
</tr>
<tr>
<td>Spot organic</td>
<td>2.38</td>
<td>2.17</td>
<td>1.25</td>
<td>4.88</td>
</tr>
<tr>
<td>Quantity other fruit (per capita)</td>
<td>0.83</td>
<td>0.82</td>
<td>0.60</td>
<td>1.08</td>
</tr>
<tr>
<td>Share organic lettuce</td>
<td>0.08</td>
<td>0.06</td>
<td>0.00</td>
<td>0.41</td>
</tr>
<tr>
<td>Income</td>
<td>82.54</td>
<td>72.61</td>
<td>33.44</td>
<td>405.09</td>
</tr>
</tbody>
</table>
### First-Stage Regressions

<table>
<thead>
<tr>
<th></th>
<th>Non-organic</th>
<th></th>
<th>Organic</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Price</td>
<td>Share</td>
<td>Price</td>
<td>Share</td>
</tr>
<tr>
<td><strong>Spot price (own)</strong></td>
<td>0.12**</td>
<td>−0.68**</td>
<td>0.35**</td>
<td>−0.26**</td>
</tr>
<tr>
<td><strong>Spot price (other)</strong></td>
<td>0.04**</td>
<td>0.10**</td>
<td>−0.21**</td>
<td>0.22**</td>
</tr>
<tr>
<td><strong>Hausman (own)</strong></td>
<td>0.70**</td>
<td>−1.30**</td>
<td>0.46**</td>
<td>−0.19**</td>
</tr>
<tr>
<td><strong>Hausman (other)</strong></td>
<td>−0.01</td>
<td>0.25**</td>
<td>0.13**</td>
<td>0.22**</td>
</tr>
<tr>
<td><strong>Hausman (out)</strong></td>
<td>−0.01**</td>
<td>0.11**</td>
<td>−0.10**</td>
<td>0.04**</td>
</tr>
<tr>
<td><strong>Availability other fruit</strong></td>
<td>−0.01**</td>
<td>−0.07**</td>
<td>−0.02**</td>
<td>−0.01**</td>
</tr>
<tr>
<td><strong>Share organic lettuce</strong></td>
<td>0.08**</td>
<td>−0.20**</td>
<td>−0.01**</td>
<td>0.10**</td>
</tr>
<tr>
<td><strong>Income</strong></td>
<td>−0.02**</td>
<td>0.00**</td>
<td>0.01**</td>
<td>0.04**</td>
</tr>
<tr>
<td><strong>R^2</strong></td>
<td>0.46</td>
<td>0.27</td>
<td>0.52</td>
<td>0.16</td>
</tr>
</tbody>
</table>
Figure: Own-price