The Pricing of Market and Idiosyncratic Jump and Volatility Risks
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1. Market and Idiosyncratic Risks

Summary
Extending the method of Cremers, Halling, Weinbaum (2015), I use option portfolio returns that have a constant exposure to either jump or diffusive risk to decompose the total volatility risk into four components: market volatility risk, idiosyncratic volatility risk, market jump risk, and idiosyncratic jump risk. Returns on these option portfolios load directly on changes in the corresponding risk premium. The decomposition helps in explaining contemporaneous and future returns. While all four components are at play when stocks earn contemporaneously negative returns, the idiosyncratic components are able to explain most of the cross-sectional variation in positive returns. In addition, stocks that have higher idiosyncratic jump risk earn higher subsequent returns. This relation is robust to various stock characteristics and cannot be explained by the low beta anomaly.

Relation of Jump and Volatility Risk
Total volatility may stem from continuous (volatility) or discontinuous (jump) stock price movements.

- Market jump and volatility risks are two important risk channels (e.g., Bates (1991)):
  $$\frac{dS_t}{S_t} = \alpha_t dt + \sqrt{\text{VOL}_t}\, dW_t^V + \sqrt{\text{JUMP}_t}\, dW_t^J$$

- On single stock level stock prices might be subject to jump and volatility risk stemming from the market or being purely idiosyncratic:
  $$\frac{dS_t}{S_t} = \alpha_t dt + \sqrt{\text{VOL}_t}\, dW_t^V + \text{JUMP}_t\, dW_t^J$$

- If risks are priced, stocks that have a higher exposure towards these risks should earn higher expected returns.
- How are these different risks priced in the cross-section of stock returns?

Measure for Jump and Volatility Risk
Options embedded forward looking information, thus natural proxy for measuring expected jump and volatility risk (e.g., Martin (2017)).

2. Data and Methodology

Option Portfolio Returns
Assume the stock price follows a double stochastic volatility model:
$$\frac{dS_t}{S_t} = \alpha_t dt + \sqrt{\text{VOL}_t}\, dW_t^V + \sqrt{\text{JUMP}_t}\, dW_t^J$$

- $\alpha_t$ = drift coefficient
- $\text{VOL}_t$ = volatility risk
- $\text{JUMP}_t$ = idiosyncratic volatility risk
- $dW_t^V$ = Brownian motion
- $dW_t^J$ = jump process

The instantaneous excess option returns of the volatility portfolios are then given by:
$$\frac{dV_t}{V_t} = 0.01 dt + \sqrt{\text{VOL}_t}\, dW_t^V + \sqrt{\text{JUMP}_t}\, dW_t^J$$

- Returns on the market volatility portfolio are proportional to the vega of the portfolio and changes in the market volatility risk premium.
- Returns on individual volatility portfolios are proportional to changes in the market volatility risk premium, the idiosyncratic volatility risk premium and the sensitivities towards these risks (where $\text{VOL}_t = \text{VOL}_t - \text{VOL}_t^M$).

I impose constant vega (gamma) of 0.00 (0.01) to construct VOL (JUMP) portfolios
- Allows to estimate the component stemming from market volatility risk in a linear fashion
- (Some approach for jumps)

Construction of VOL and JUMP Portfolio
Every day I select two option pairs with different maturities between 7 and 90 days.

- Take these two pairs that are closest to the money.
- If multiple option pairs are closest to the money, I use the ones with shortest and longest maturity (largest spread in vega and gamma).

I construct the option portfolio using one long and one short position in two straddles (for VOL (JUMP) short (long) in short maturity and long (short) in long maturity).

$$\text{arg min}_u \sum_{i} w^O_i \left( \alpha_i - \alpha^O_i \right)$$

s.t.

- 4 options to hedge 3 weeks
- 1 re-run a numerical optimization and minimize the relative portfolio weights for all options (minimize potential data noise)
- for VOL portfolio: set $\omega^O = 0$ and $\omega^V = 100$
- for JUMP portfolio: set $\omega^V = 0$ and $\omega^J = 100$

I measure the portfolio return over one trading day.

- JUMP and VOL portfolios could be zero cost portfolios
- Calculate returns relative to the absolute amount invested (Tosi and Zeglot (2018))
- (Portfolio returns are always well defined)

If options are not observable the next day I interpolate using a kernel smoother, smooth over moneyness and put-call identifier.
- Same approach for SPX options and equity options

Data
- I use full cross-section of stock option data.
  - Sample period: 01/1996 - 04/2016, daily frequency
  - Stock data: CRSP all stocks on NYSE, AMEX and NASDAQ
  - Option data: OptionMetrics

I filter the data similar to Goyal and Saretto (2009) and exclude options with:
- zero open interest
- no available implied volatility
- violations against standard no-arbitrage bounds

Quoted options are American styled. Given an implied volatility I calculate synthetic European option prices, to account for the early exercise premium:
- Replicate all American options with a Cox et al (1979) (CRR) tree, using 1000 steps and incorporating expected discrete dividends
- discard all options with a pricing error of more than 1%
- use the same CRR tree to price the corresponding European option.

3. Empirical Results

Investors willing to pay a premium to hedge market volatility and jump risk
- Positive correlation of VOL and JUMP portfolio returns
- VOL and JUMP highly correlated with changes in expected risk-neutral total variance

Large changes in expected risk-neutral total variance go along with large returns of the VOL or JUMP portfolios
- Changes in expected risk-neutral total variance load positive on both measures
- Market returns load negative on both, JUMP and VOL returns.
- Only market jumps are able to explain positive market returns

Cross-Sectional JUMP and VOL Portfolios
Investors willing to pay a premium to hedge idiosyncratic and market risk
- Negative correlation of VOL and JUMP portfolio returns
- VOL and JUMP highly correlated with changes in stock's expected risk-neutral total variance

Estimates the market and idiosyncratic part:
$$\gamma^* = \alpha + \frac{1}{\omega^O} + \frac{1}{\omega^V} + \frac{1}{\omega^J}$$

Forbidden is to hedge idiosyncratic and market risk
- Changes in risk-neutral total variance of single stocks load positive on all four measures
- Stock returns load negative on market JUMP and VOL, only
- Diverse effect of idiosyncratic components:
  - Positive returns load positive on idiosyncratic VOL and JUMP
  - Negative returns load negative on idiosyncratic VOL and JUMP

Cross-Sectional Predictive Analysis
In order to infer the pricing, I ran a cross-sectional predictive analysis. Every month I use the daily observation over the last month to estimate the market and idiosyncratic part of the portfolio return. I sort stocks into quintile portfolios and hold them for one month.

- Contemporaneous returns are monotonically increasing
- If no, market returns statistically significant
- Next month returns/alphas are monotonically decreasing
- For low portfolios, changes in (expected) physical jumps smaller than changes in expected risk-neutral jumps, no risk of dislike jumps and demand a premium for holding these stocks
- Positive price of idiosyncratic jump risk

Results are robust to stock characteristics, risk-neutral moments and the low beta anomaly.