Spatial dynamic models with intertemporal optimization: specification and estimation

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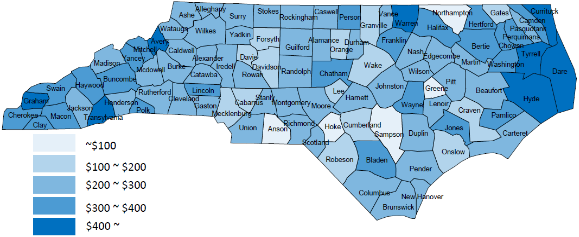
0. Preliminaries (What?)

- Regional policy interdependence



Public safety spending (per capita, 2015): county governments in NC

  - public expenditure, tax rates, etc.

- Tool: spatial econometrics
- This research: spatial econometric model specification-structural spatial econometric model

0. Preliminaries

- Theoretical foundation $\Rightarrow$ a game setting
  - n agents, an $n \times n$ spatial network matrix W_n (zero diagonals)
  - e.g., LQ payoff

$$u_i(Y_n) = \underbrace{\eta_i}_{=i\text{'s exo. char.}} \underbrace{y_i}_{=i\text{'s decision}} + \underbrace{\lambda_0}_{=\text{parameter}} y_i \underbrace{w_{i\cdot}}_{i-th \text{ row of } W_n} Y_n - \frac{1}{2} y_i^2$$

  - complete information

- A spatial autoregressive (SAR) model : $Y_n = (y_1, \cdots, y_n)'$ and $X_n = (x_1, \cdots, x_n)'$

$$Y_n = \lambda_0 W_n Y_n + X_n \beta_0 + \mathcal{E}_n.$$

  - exogenous characteristics? $\boldsymbol{\eta}_n = (\eta_1, \cdots, \eta_n)'$
  - regression function: $\boldsymbol{\eta}_n = X_n \beta_0 + \mathcal{E}_n$

1. Introduction (Why?)

Motivation

- Economic explanation of spatial/time dependencies from a spatial panel data set
- Considerations
  - a panel data set has id's $\Rightarrow$ dynamics of individual actions
    - ★ e.g., agent=county government, its action=public safety spending
  - multi decision-making periods
  - rational economic agents $\Rightarrow$ forward-looking agents' behaviors.

$\Rightarrow$ Corresponding econometric model specification?

1. Introduction (Overview - How?)

1. A new model specification
   - n agents, innate locations ⇒spatial network W_n
   - continuous type action, parametric LQ payoff for choices of agents' actions
   - conventional SDPD model: myopic behaviors
   - maximization of agent's lifetime payoff: stable economic environment ⇒ time-invariant optimal policy functions
     - ★ LQ value functions ⇒ a linear system ⇒ correlation structure
2. QML method, asymptotic properties, bias correction, Monte Carlo simulations
3. Case study: counties' public safety spending in NC
   - agent=county government, action=its public safety spending
   - two policy functions: (i) myopic (conventional) v.s (ii) forward-looking

2. A spatial dynamic game with intertemporal optimization

2.1 Literature review: spatial dynamic panel models and myopic choices

- Data environment
  - panel data set : $\left\{ \underbrace{Y_{nt}}_{\text{dependent V. (action)}} , \underbrace{X_{nt}}_{\text{indepenent V.}} \right\}_{t=0}^{T}$ and given W_n.
- SDPD model :

$$Y_{nt} = \lambda_0 W_n Y_{nt} + \gamma_0 Y_{n,t-1} + \rho_0 W_n Y_{n,t-1} + \boldsymbol{\eta}_{nt}. \quad (1)$$

  - $\left(\underbrace{\lambda_0}_{\text{current competition}} , \underbrace{\gamma_0}_{\text{persistency}} , \underbrace{\rho_0}_{\text{diffusion}} \right)$: main parameters
  - $\boldsymbol{\eta}_{nt} = (\eta_{1t}, \cdots, \eta_{nt})'$ at time t: observable + unobservable characteristics

2. A spatial dynamic game with intertemporal optimization

2.1 Literature review: spatial dynamic panel models and myopic choices

- Justification: agent i's t^{th}-period payoff u_{it}

$$\eta_{it} y_{it} + \rho_0 y_{it} \underbrace{w_{i.} Y_{n,t-1}}_{=\text{previous neighbors' actions}} + \lambda_0 y_{it} \underbrace{w_{i.} Y_{nt}}_{=\text{current neighbors' actions}} - c\left(y_{it}, y_{i,t-1}\right) \quad (2)$$

where $w_{i.}$=i^{th}-row of W_n,

$$\underbrace{c\left(y_{it}, y_{i,t-1}\right)}_{=\text{cost}} = \underbrace{\frac{\gamma_0}{2}\left(y_{it} - y_{i,t-1}\right)^2}_{=\text{adjustment cost}} + \underbrace{\frac{1-\gamma_0}{2} y_{it}^2}_{=\text{cost of selecting } y_{it}}, \ (0 < \gamma_0 < 1).$$

  - η_{it}: exogenous characteristics, η_i^{iv} (innate) and η_{it}^{v} (time-variant)
  - η_{it}^{v}: stationary first-order linear Markov process, exogenous

- Lifetime problem:
  - time-discounting factor: $\delta \in [0, 1)$
  - complete information setting up to $t \Rightarrow E_t(\cdot)$ is defined.
  - agent i's t^{th}-period problem: given $(Y_{n,t-1}, \boldsymbol{\eta}_{nt})$, maximizes

$$u_i\left(y_{it}, Y_{-i,t}, Y_{n,t-1}, \eta_{it}\right) + \sum_{s=1}^{\infty} \delta^s E_t\left(u_i\left(Y_{n,t+s}, Y_{n,t+s-1}, \eta_{i,t+s}\right)\right) \tag{3}$$

    by selecting y_{it}.
  - stable economic environment

- The stable system of NE is

$$Y_{nt}^* = (\lambda_0 W_n + \delta Q_n^*) Y_{nt}^* + (\gamma_0 I_n + \rho_0 W_n) Y_{n,t-1} + (I_n + \delta L_n^* \Pi_n) \boldsymbol{\eta}_{nt}. \tag{4}$$

  - δ: prespecified parameter
  - Q_n^* and L_n^*: functions of parameters and W_n
  - Π_n: nuisance parameters for $\boldsymbol{\eta}_{nt}$ $(E_t \boldsymbol{\eta}_{n,t+1} = \Pi_n \boldsymbol{\eta}_{nt})$
  - $\lambda_0 W_n Y_{nt}^*$: contemporaneous spatial effect
  - $\gamma_0 Y_{n,t-1}$: dynamic effect
  - $\rho_0 W_n Y_{n,t-1}$: spatial-past time effect
  - $\delta Q_n^* Y_{nt}^*$: additional expected spatial-future time effect
  - $\delta L_n^* \Pi_n \boldsymbol{\eta}_{nt}$: expected future exogenous effect

3. Econometric model

- An econometric model based on Eq. (4)?

  $\Rightarrow$ Econometric model

$$Y_{nt} = \left(\lambda_0 W_n + \delta Q_n^*\right) Y_{nt} + \left(\gamma_0 I_n + \rho_0 W_n\right) Y_{n,t-1} + \underbrace{\left(I_n + \delta L_n^* \Pi_n\right) X_{nt} \beta_0}_{\text{regression func}} + \underbrace{\mathbf{c}_{n0} + \alpha_{t,0} l_n}_{\text{ind./time eff.}} + \underbrace{\mathcal{E}_{nt}}_{\text{disturbance}} \quad (5)$$

- Main parameters: $\theta_0 = \left(\lambda_0, \gamma_0, \rho_0, \beta_0', \sigma_{\epsilon,0}^2\right)'$ where $Var\left(\mathcal{E}_{nt}\right) = \sigma_{\epsilon,0}^2 I_n$.
- Estimation framework: large n and T, increasing domain asymptotics
  - spatial filter: $R_n = I_n - \lambda_0 W_n - \delta Q_n^*$
  - difficulty: how to get Q_n^* and L_n^*?

4. Estimation

4.1 Quasi-maximum likelihood estimation

- The concentrated log-likelihood function with nT observations:

$$\ln L_{nT,c}(\theta) = c - \frac{nT}{2}\ln\sigma_\epsilon^2 + T\ln|R_n(\theta_1)| - \frac{1}{2\sigma_\epsilon^2}\sum_{t=1}^{T}\tilde{\mathcal{E}}'_{nt}(\theta) J_n \tilde{\mathcal{E}}_{nt}(\theta) \quad (6)$$

where

$$\tilde{\mathcal{E}}_{nt}(\theta) = R_n(\theta_1)\tilde{Y}_{nt} - (\gamma I_n + \rho W_n)\tilde{Y}^{(-)}_{n,t-1} - (I_n + \delta L^*_n(\theta_1)\Pi_n)\tilde{X}_{nt}\beta$$

with $\tilde{Y}_{nt} = Y_{nt} - \bar{Y}_{nT}$, $\tilde{Y}^{(-)}_{n,t-1} = Y_{n,t-1} - \bar{Y}_{nT,-1}$, and $\tilde{X}_{nt} = X_{nt} - \bar{X}_{nT}$, and $J_n = I_n - \frac{1}{n}l_n l'_n$.

  - $R_n(\theta_1)$ and $L^*_n(\theta_1)$: additional parts evaluated at $\theta_1 = (\lambda, \gamma, \rho)$

- The QMLE, $\hat{\theta}_{ml,nT} \equiv \arg\max_{\theta\in\Theta} \ln L_{nT,c}(\theta)$.

4. Estimation

4.1 Quasi-maximum likelihood estimation

- Estimation procedure:
  - outer loop: search over different parameter value θ.
    - demanding part: evaluating $\ln |R_n(\theta_1)|$
  - inner loop: For θ, we compute $R_n(\theta_1)$ (i.e., $Q_n^*(\theta_1)$) and $L_n^*(\theta_1)$ (=main components of value functions). $\Rightarrow$ compute $\ln L_{nT,c}(\theta)$.
    - we do not need to compute all components in V_i's.

Theorem (Consistency)

Under some regularity conditions, $\hat{\theta}_{ml,nT} \xrightarrow{p} \theta_0$.

4. Estimation

4.1 Quasi-maximum likelihood estimation

Theorem (Asymptotic normality)

$$\sqrt{nT}\left(\hat{\theta}_{ml,nT}-\theta_0\right)+\sqrt{\frac{n}{T}}\Sigma_{\theta_0,nT}^{-1}\underbrace{a_{n,1}(\theta_0)}_{\text{from estimating } \mathbf{c}_{n0}}$$

$$+\sqrt{\frac{T}{n}}\Sigma_{\theta_0,nT}^{-1}\underbrace{a_{n,2}(\theta_0)}_{\text{from estimating } \alpha_{t,0}}+O_p\left(\max\left(\sqrt{\frac{n}{T^3}},\sqrt{\frac{T}{n^3}},\sqrt{\frac{1}{T}}\right)\right)$$

$$\xrightarrow{d} N\left(0,\Sigma_{\theta_0}^{-1}\Omega_{\theta_0}\Sigma_{\theta_0}^{-1}\right).$$

$\Omega_{\theta_0,nT}=E\left(\frac{1}{nT}\frac{\partial \ln L_{nT,c}^{(u)}(\theta_0)}{\partial\theta}\frac{\partial \ln L_{nT,c}^{(u)}(\theta_0)}{\partial\theta'}\right)$, $\Omega_{\theta_0}=\lim_{T\to\infty}\Omega_{\theta_0,nT}$,

$\Sigma_{\theta_0,nT}=-E\left(\frac{1}{nT}\frac{\partial^2 \ln L_{nT,c}(\theta_0)}{\partial\theta\partial\theta'}\right)$ and $\Sigma_{\theta_0}=\lim_{T\to\infty}\Sigma_{\theta_0,nT}$.

4. Estimation

4.1 Quasi-maximum likelihood estimation

- Bias correction of $\hat{\theta}_{ml,nT}$:

$$\hat{\theta}^{c}_{ml,nT} = \hat{\theta}_{ml,nT} - \frac{1}{T}\left[-\Sigma^{-1}_{\theta,nT} a_{n,1}(\theta)\right]|_{\theta=\hat{\theta}_{ml,nT}} - \frac{1}{n}\left[-\Sigma^{-1}_{\theta,nT} a_{n,2}(\theta)\right]|_{\theta=\hat{\theta}_{ml,nT}}.$$

Corollary

Regularity conditions and $\frac{n}{T^3} \to 0$ *and* $\frac{T}{n^3} \to 0$,

$$\sqrt{nT}\left(\hat{\theta}^{c}_{ml,nT} - \theta_0\right) \xrightarrow{d} N\left(0, \Sigma^{-1}_{\theta_0}\Omega_{\theta_0}\Sigma^{-1}_{\theta_0}\right).$$

5. Simulations

- Overall performance of $\hat{\theta}_{ml,nT}$ and $\hat{\theta}^{c}_{ml,nT}$/ comparison with $\hat{\theta}^{S}_{ml,nT}$ and $\hat{\theta}^{S,c}_{ml,nT}$ (= the QMLEs from the conventional SDPD model (Lee & Yu (2010))
  - $\hat{\theta}^{c}_{ml,nT}$ performs better than $\hat{\theta}_{ml,nT}$.
  - crucial misspecification errors of $\hat{\theta}^{S}_{ml,nT}$ and $\hat{\theta}^{S,c}_{ml,nT}$

6. Application

- Public safety spending among counties in NC
- Two types of optimal reaction functions
  (i) conventional SDPD model: myopic agent model ($\delta = 0$)
  (ii) our model: forward-looking agent model
    - $\delta = 0.9704$ ($\Leftarrow$ average annual long-run interest rates in the sampling periods)

6. Application

- Selected model via the sample log-likelihood

	Myopic	Forward-looking
Total revenue	0.1023*** [0.0054]	0.1239*** [0.0066]
Neighbor's total revenue	-0.052*** [0.0158]	-0.0667*** [0.0191]
λ	0.0142 [0.0657]	0.0058 [0.0845]
γ	0.3937*** [0.0251]	0.5081*** [0.065]
ρ	0.0705 [0.0784]	0.1726* [0.0984]
Sample log-likelihood	-2712.9	-2712.5

- * indicates 10% level of significance and *** indicates 1% level of significance.
- Dollar amounts are real per capita values adjusted by the GDPD.
- Sample log-likelihood is a good measure to capture the true model ($\Leftarrow$ simulation study)

7. Conclusion

- Summary
  - Spatial dynamic panel data model with the forward-looking agent assumption
  - QML estimation method, asymptotic properties, bias correction
  - application: policy interdependence of counties' public safety spending in NC