Uncertainty, Liquidity and Financial Cycles

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Annual Growth Rate of SP 500 and Fixed Capital Formation Ratio

- Growth Rate of SP 500
- Fixed Capital Formation Ratio

Recession
Motivation

- **Obvious Phenomenon: Inconsistence between macroeconomic fluctuations and financial cycles**
  - After the Great Recession, a slow economic recovery together with financial booms
  - Macroeconomy and financial markets often blossom together.
- **Very FEW Theoretical Explanation**
Main Question: What are the roles of **Uncertainty**?

My Answer: An endogenous **Liquidity Allocation Mechanism** between real economy and financial system

A simple theory based on a tractable continuous-time DSGE model of heterogenous agents

Numerical Analysis of Global Dynamics
Key Assumptions

- Slower Adjustment of Macroeconomy than Financial Market.
  - Physical capital has less liquidity than its corresponding equity.

- Financial Frictions: “Skin in the Game”
  - Entrepreneurs have to take partial risk of their own investment.

- This leads to
  - entrepreneur faces a leverage constraint that depends on his capital structure
  - investment depends on not only equity price but also the capital structure
Main Insights

- Net Worth of an entrepreneur
  - determines his ability of risk tolerance
  - affects corporate investment

- Recession Stage:
  - Risk-averse entrepreneurs who have low net worth, will disinvest and hold more financial assets.
  - More funds flows into financial system from real economy.
  - There is a financial booms but a slow economic recovery.

- Booms Stage:
  - Entrepreneurs with high net worth have high investment demand.
  - This leads to high equity prices.
Bolton, Wang, Yang (Forthcoming in JF)
Brunnermeier, Eisenbach and Sannikov (2013)
Brunnermeier and Sannikov (AER 2014, 2017)
Di Tella (JPE, 2017)
He and Kondor (Econometrica, 2016)
Kiyotakky and Moore (2012)
The Model: Environment

- Infinite identical risk-averse entrepreneurs that totally measured by 1.
- Infinite identical risk-neutral investors that totally measured by 1.
- ONLY entrepreneurs can run physical capital.
- Entrepreneurs raise funds by issuing equity and debt.
- Investors can buy equity and risk-free bonds.
The Economic Structure
The Model: Entrepreneur's Balance Sheet

<table>
<thead>
<tr>
<th>Assets</th>
<th>Liabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>Capital: ( qK )</td>
<td>(Outside) Equity: ( (1 - \chi) qK )</td>
</tr>
<tr>
<td>(Others’) Equity: ( qH )</td>
<td>Bond: ( (v - 1) W )</td>
</tr>
<tr>
<td></td>
<td>Net Worth: ( W )</td>
</tr>
</tbody>
</table>
The Model: Key Assumption

- Less liquidity of physical capital because of transaction cost: \( \Psi(\kappa) < \kappa \)
  - This implies that capital and equity are imperfect substitutive.
  - The market price of physical capital is denoted by \( p \).
  - The evolving process of capital is given by
    \[
    dK = [\Phi(\iota) + \Psi(\kappa) - \delta]Kdt + \sigma KdZ
    \]
- Entrepreneurs face equity issuance constraint: \( \chi \geq \bar{\chi} \).
- This leads to
  - a leverage constraint for entrepreneurs from the equity issuance constraint
  - the optimal investment choice depends on equity price, \( q \), and the capital structure
The Model: Entrepreneurs

\[
\max_{\{C, \chi, \iota, \kappa, \nu, \zeta\}} E \int e^{-\rho t} \frac{C^{1-\gamma}}{1-\gamma} dt
\]

s.t.

\[
\chi \geq \bar{\chi}
\]

\[
qH \geq 0
\]

\[
\nu W \equiv \chi qK + qH
\]

\[
dW = \nu WdR + (1 - \nu) Wr dt - C dt
\]

\[
\frac{d(qH)}{qH} = (\zeta - \delta + \mu^q + \sigma \sigma^q) dt + (\sigma + \sigma^q) dZ
\]

\[
\frac{d(qK)}{qK} = [\Phi(\iota) + \Psi(\kappa) - \delta + \mu^q + \sigma \sigma^q] dt + (\sigma + \sigma^q) dZ
\]

where

\[
dR = \left(\frac{A - \iota - \rho \kappa}{q}\right) dt + [\Phi(\iota) + \Psi(\kappa) - \delta + \mu^q + \sigma \sigma^q] dt + (\sigma + \sigma^q) dZ
\]
The Model: Entrepreneurs

- **Optimal Investment Ratio:**

\[
\left[ \frac{(1 - \gamma) \varphi(w) - w \varphi'(w)}{\nu w \varphi'(w)} + 1 \right] \Phi'(\iota) = \frac{1}{q}
\]

- \( w \equiv \frac{W}{qK} \): the capital ratio; \( \varphi(w) \equiv J(W, qK) / (qK)^{1-\gamma} \)

- **Relative Price between capital and equity valued by entrepreneurs:**

\[
\frac{(1 - \gamma) \varphi(w) - w \varphi'(w)}{\nu w \varphi'(w)} + 1 = \frac{qKJ_{qK}'}{\nu WJ_W'}
\]
Trade-Off between Producing Capital and Purchasing Capital:

\[ \Phi'(l) K = \Psi'(\kappa) K / p \]

Capital Formation by Producing  
Capital Formation by Purchasing

Asset Pricing of Inside Equity:

\[ A - \iota - p\kappa - \frac{q}{p} + \Phi(l) + \Psi(\kappa) - \delta + \mu q + \sigma \sigma^q - r + \frac{\lambda_2}{V - \gamma \phi'} \]

\[ E(dR)/dt \]

\[ \text{Liquidity Premium} \]

\[ = \left[ \gamma + (1 - \nu) \frac{\varphi''}{\varphi'} \right] (\sigma + \sigma^q)^2 \]

\( \lambda_2 \) : the Lagrangian multiplier of equity issuance constraint \( \chi \geq \bar{\chi} \)
BS(2014, 2017): physical capital $\leftrightarrow$ equity: **relative price is 1**

$$\Phi'(t) K = K/q$$

- Capital Formation by Producing = New Capital by Purchasing

**Non-Arbitrage Condition:** one more unit of investment ratio

$$\Phi'(t) K = \Psi'(\kappa) K/p$$

- Capital Formation by Producing = Capital Formation by Purchasing
The Model: Investors

\[
\max_{\{c \geq 0, \nu\}} E_0 \left[ \int_0^\infty e^{-rt} C dt \right]
\]

s.t.

\[
dW = \nu W dR + r (1 - \nu) W dt - C dt
\]

Asset Pricing of (Outside) Equity:

\[
\frac{A - \nu}{q} + \Phi(\nu) + \Psi(\kappa) - \delta + \mu^q + \sigma \sigma^q = r
\]

\[
E(dR/dt)
\]
Markov Equilibrium: 2nd-Order ODEs

$$\mu^g q = q' (\eta) \mu^\eta \eta + \frac{1}{2} q'' (\eta) (\sigma^\eta \eta)^2$$

$$\left\{ \rho - (1 - \gamma) \left[ \Phi (i) - \delta + \mu^q + \sigma^q \sigma - \frac{\gamma}{2} (\sigma + \sigma^q)^2 \right] \right\} \varphi$$

$$= \max_{\{c, i, v\}} \frac{(c \eta)^{1-\gamma}}{1 - \gamma} + \frac{1}{2} (v - 1)^2 (\sigma + \sigma^q)^2 \eta^2 \varphi''$$

$$+ \left\{ \nu \frac{A - i}{q} + (v - 1) \left[ \Phi (i) - \delta + \mu^q + \sigma^q \sigma \right] \right. \right.$$\left. \left. + (1 - v) r - c + (1 - v) \gamma (\sigma + \sigma^q)^2 \right\} \eta \varphi'$$

$$\eta \equiv \frac{\int_0^1 W^i di}{\int_0^1 q K^i di} : \text{entrepreneurs' wealth ratio over the aggregate wealth}$$
Markov Equilibrium: Parameterization

- Set Similar Parameters Values as Brunnermeier and Sannikov (2014):

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Meaning</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho$</td>
<td>time discount rate of entrepreneurs</td>
<td>6%</td>
</tr>
<tr>
<td>$r$</td>
<td>time discount rate of investors</td>
<td>5%</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>entrepreneur’s risk aversion</td>
<td>2</td>
</tr>
<tr>
<td>$A$</td>
<td>productivity level</td>
<td>12%</td>
</tr>
<tr>
<td>$\delta$</td>
<td>depreciation rate</td>
<td>3%</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>capital quality shock</td>
<td>2%</td>
</tr>
<tr>
<td>$\phi$</td>
<td>investment function</td>
<td>10</td>
</tr>
<tr>
<td>$\bar{\chi}$</td>
<td>equity issuance constraint</td>
<td>70%</td>
</tr>
</tbody>
</table>

- Investment function:

$$\Phi(t) \equiv \frac{1}{\phi} \left( \sqrt{1 + 2\phi t} - 1 \right)$$
Investment Ratio, Equity Price Growth and Equity Market Risk

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Global Dynamics with Different Extents of Equity Issuance Constraint

![Graphs showing dynamic changes in equity issuance](image)

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A tractable DSGE model of an endogenous liquidity allocation mechanism between real economy and financial markets.

The endogenous risks and liquidity allocation are helpful to understand the inconsistency between macroeconomic fluctuations and financial cycles.