

Valuing Private Equity Investments Strip by Strip

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AFA

1. **Private equity valuation has not taken into account the multivariate nature of risk nor the temporal composition of risk**
 - Standard approaches:
 - PME
 - Korteweg and Nagel (2016)
 - Limitations: only one aggregate source of risk
 - If a bad assumption in equities (CAPM) likely also the case in PE
 - Especially in “alternative” categories like Real Estate Funds

Measurement Problems in Assessment of Risk and Return in Private Equity

1. Asset Pricing in Private Equity has struggled with cross-section and term structure of risk
2. **We draw from Asset Pricing Literature emphasizing term structure and multifactor models**
 - Multifactor models: Fama and French (2016, 2018) and Hou, Xue, and Zhang (2015, 2017, 2018)
 - Term Structure: van Binsbergen, Brandt, and Koijen (2012), van Binsbergen and Koijen (2017)
 - Term structure of risk potentially upward or downward sloping, depending on factor

Measurement Problems in Assessment of Risk and Return in Private Equity

1. Asset Pricing in Private Equity has struggled with cross-section and term structure of risk
2. We draw from other Asset Pricing Literature emphasizing multifactor models
3. **Problem: Observe cashflows, not returns**
 - Use insights from asset pricing to price cash flows from PE funds strip by strip
 - Estimate risk exposures of cash flows by imposing cross-equation restrictions on the PE funds cash flow exposures
 - Estimate multi-factor exposure and allow term structure of risk to vary by factor

1. **Price public assets.** Model delivers term structure fit and valuation of cash flow.
2. Estimate Private Equity **fund exposure** to capital market assets
3. Produce budget-feasible **replicating portfolios** matching risk exposure
4. Use asset pricing model to understand private equity **risk/return characteristics**

1. Price Public Assets

Asset pricing model fits capital market assets

Asset Pricing Model

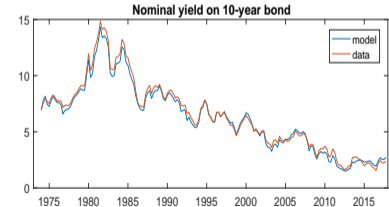
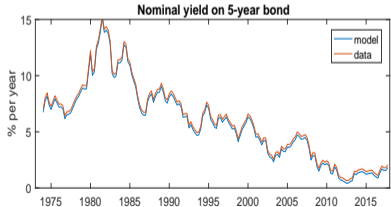
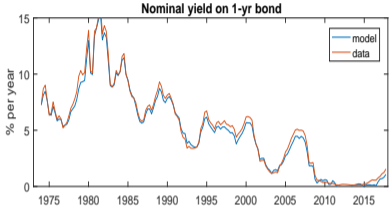
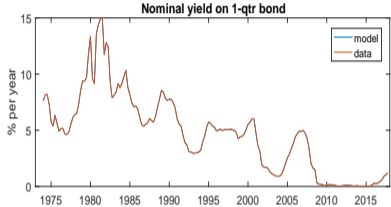
- State variables follow Gaussian first-order VAR:

$$\mathbf{z}_t = \Psi \mathbf{z}_{t-1} + \Sigma^{\frac{1}{2}} \varepsilon_t, \quad \varepsilon_t \sim i.i.d. \mathcal{N}(0, I)$$

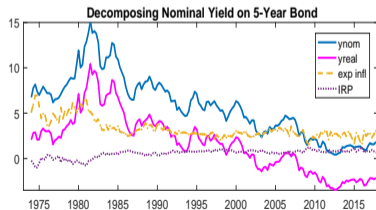
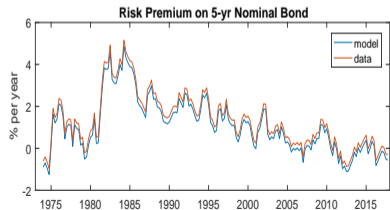
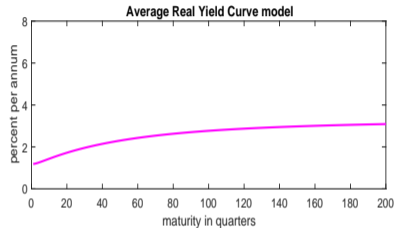
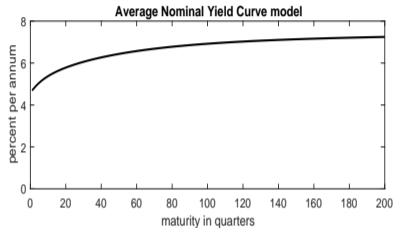
- Bond variables: CP factor, nominal short rate, realized inflation, 5-year - 1-month Treasury spread
- Stock variables: log price-dividend, log real dividend growth for: CRSP, NAREIT, listed infra
 - Approach can be extended to include other cross-sectional equity risk factors such as: size, value, investment, profitability, etc.
- SDF:

$$m_{t+1}^{\$} = -y_t^{\$(1)} - \frac{1}{2} \Lambda_t' \Lambda_t - \Lambda_t' \varepsilon_{t+1},$$

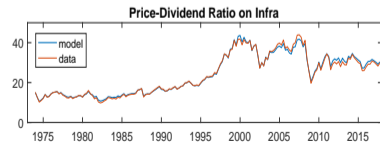
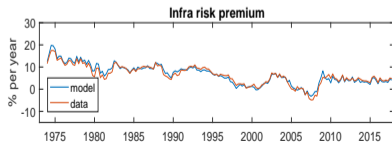
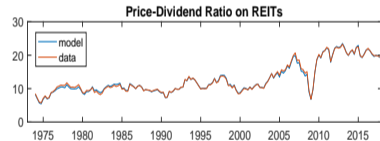
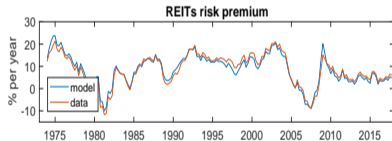
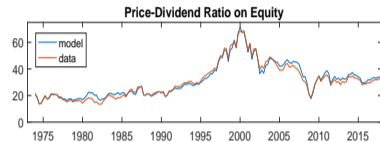
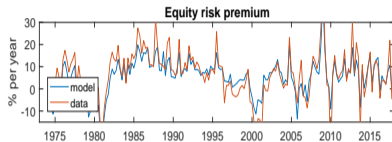
Model Matches Time-Series of Bond Yields



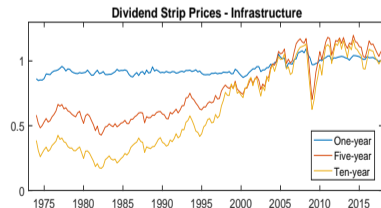
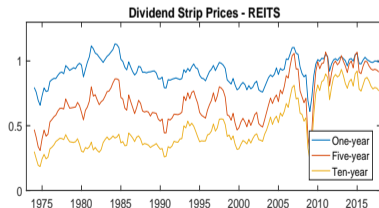
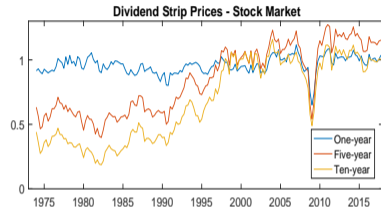
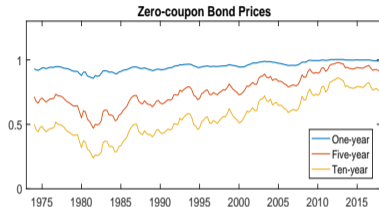
Also Matches underlying Components of Bond Yield: Real + Nominal



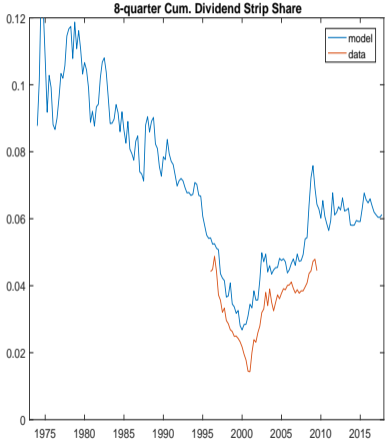
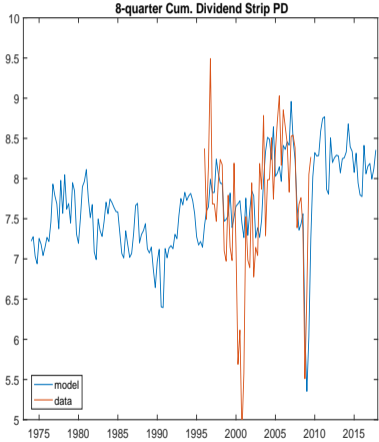
Fits Equity Risk Premia as well as Stock Price Levels



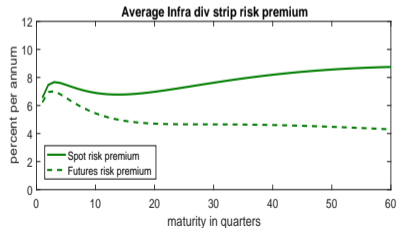
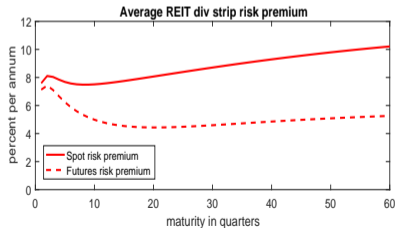
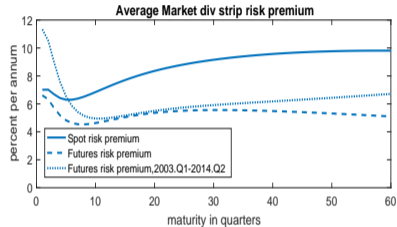
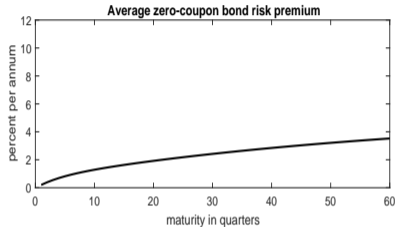
Outcome of Model: Bond + Dividend Strip Prices



Imputed Dividend Strip Model Matches Data when Available



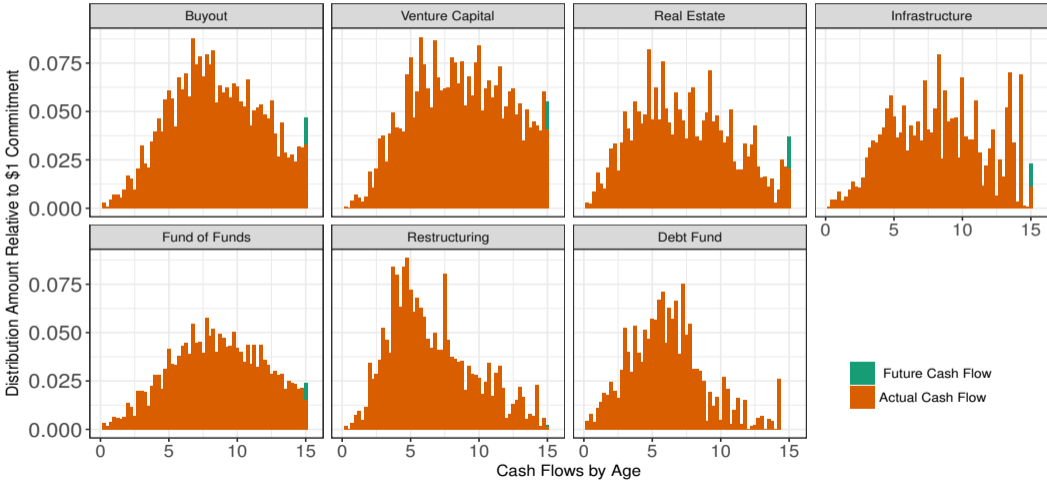
Rich Patterns in Temporal Pricing of Risk



2. Private Equity Fund Exposures

Estimate fund exposure to multi-factor model

Want to Understand Cash-Flow Profiles of Private Equity Funds

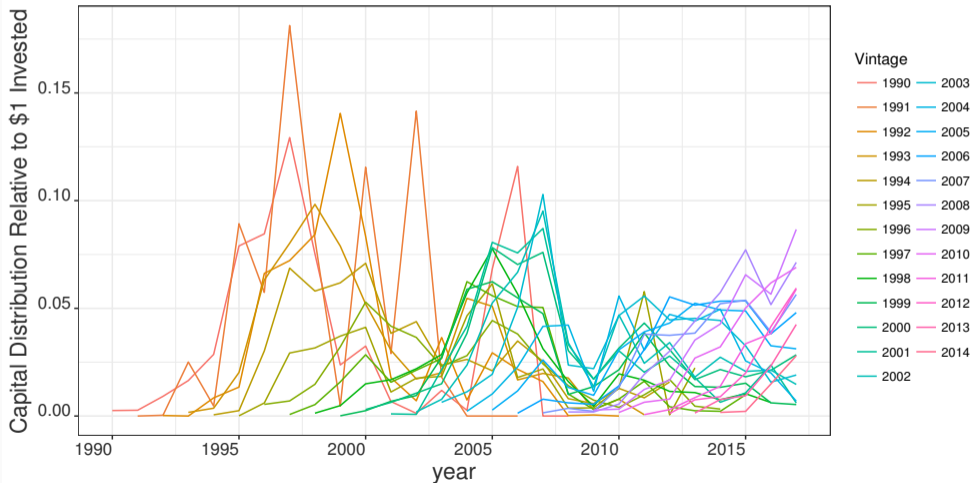


Cash-Flow Variation Across Horizon and Vintage — Buyout

Venture Capital

Real Estate

Infrastructure



Estimate Factor Exposure Strip-by-Strip

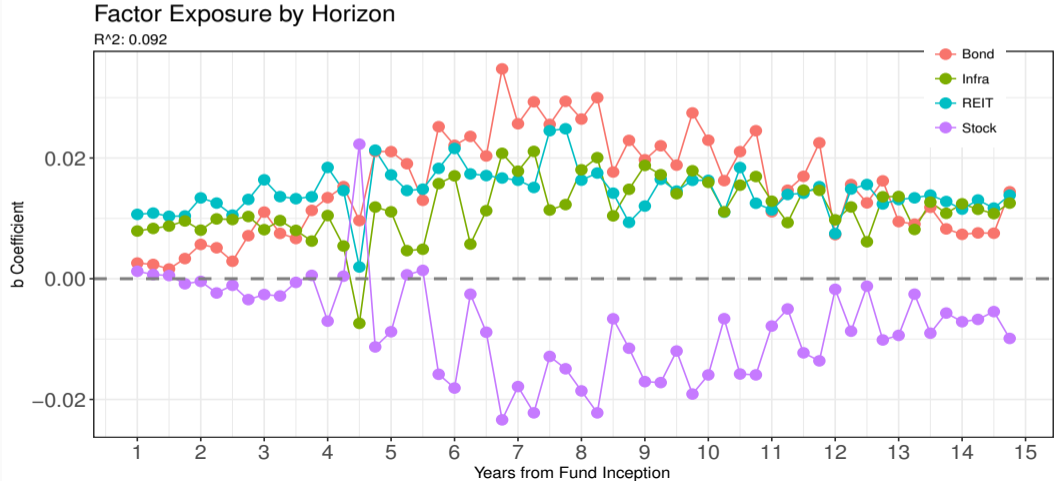
- Four-factor model fitting PE fund cash flows against four traded factors (bonds, stocks, real estate, infra)

$$\begin{aligned} X_{t+h}^i &= \beta_{t,h}^i Y_{t+h} + e_{t+h}^i \\ &= a_{1,t}^c b_{1,h}^c + a_{2,t}^c b_{2,h}^c Y_{t+h}^m + a_{3,t}^c b_{3,h}^c Y_{t+h}^{\text{reit}} + a_{4,t}^c b_{4,h}^c Y_{t+h}^{\text{infra}} + v_{t+h}^{i \in c}. \end{aligned}$$

- Fit for each cash-flow horizon strip
- Estimate vintage (a_1, a_2, a_3, a_4) and age (b_1, b_2, b_3, b_4) effects
- Cross-equation restrictions: coefficients same for all funds in categories (i.e., Real Estate). Proportional shifts in vintage and age effects across factor exposures.

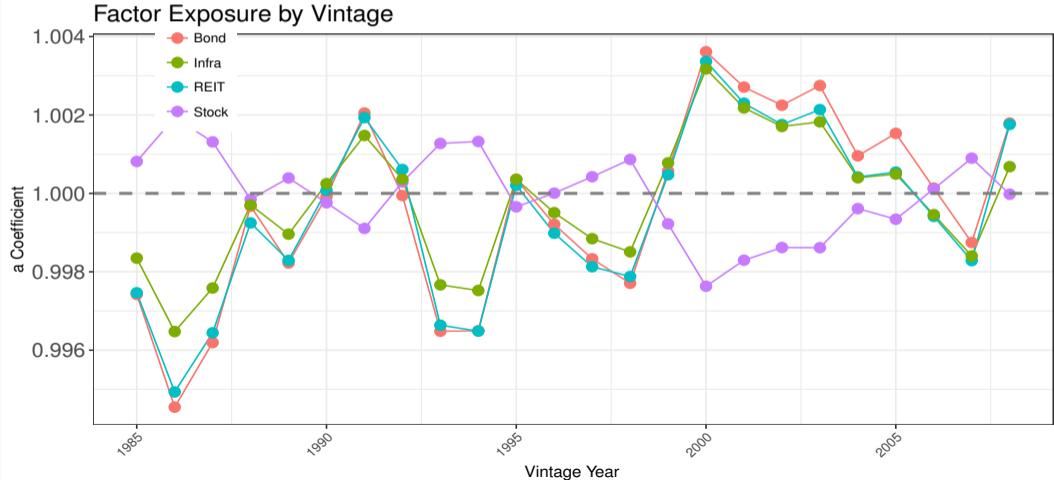
Factor Exposure in PE Funds by Horizon – Buyout

Venture Capital Real Estate Infrastructure



Factor Exposure in PE Funds by Vintage – Buyout

Venture Capital Real Estate Infrastructure



3. Replicating Portfolios

Construct budget-feasible portfolios replicating Private Equity Funds

We connect the Model + Fund Exposures = Replicating Portfolio

- Define scaled long-positions in each factor that are budget feasible, where

$$\mathbf{q}_{t,h}^{i,+} = \frac{\beta_{t,h}^{i,+}}{\sum_{h=0}^H \beta_{t,h}^{i,+} \mathbf{P}_{t,h}} \Rightarrow \sum_{h=0}^H \mathbf{q}_{t,h}^{i,+} \mathbf{P}_{t,h} = \$1.$$

where $\mathbf{P}_{t,h}$ comes from asset pricing model

- Null: present discounted value of fund cash distributions is 1:

$$\mathbb{E}_t \left[\sum_{h=0}^H M_{t+h}^h X_{t+h}^i \right] = \mathbb{E}_t \left[\sum_{h=0}^H M_{t+h}^h \mathbf{q}_{t,h}^{i,+} \mathbf{Y}_{t+h} \right] = \sum_{h=0}^H \mathbf{q}_{t,h}^{i,+} \mathbf{P}_{t,h} = 1$$

- Also use model to find expected return; $\mathbf{q}_{t,h}^{i,+} \mathbf{P}_{t,h}$ are portfolio weights:

$$\mathbb{E}_t [R^i] = \sum_{h=0}^H \mathbf{q}_{t,h}^{i,+} \mathbf{P}_{t,h} \mathbb{E}_t [R_{t,h}]$$

4. Private Equity Fund Characteristics

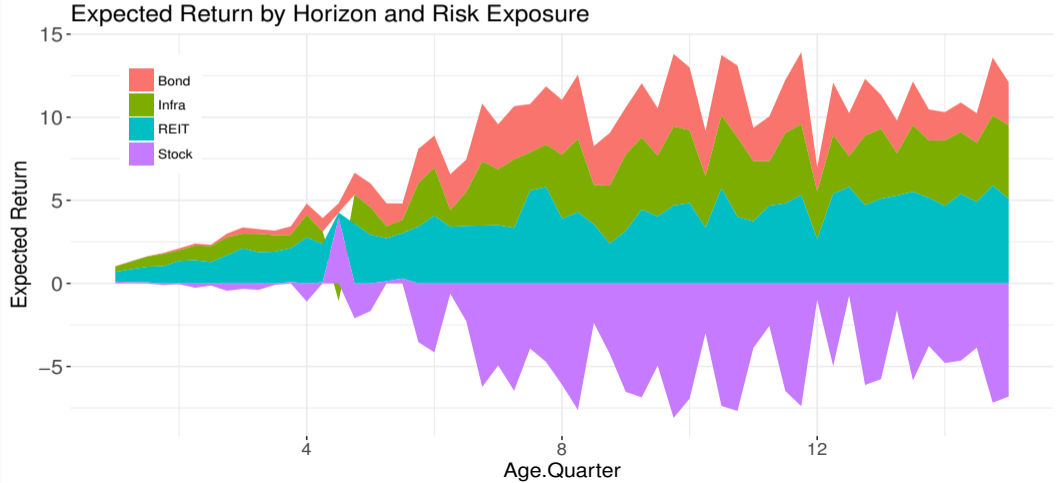
Use fitted model to understand risk and characteristics of private equity funds

Private Equity Fund Expected Return – Buyout

Venture Capital

Real Estate

Infrastructure

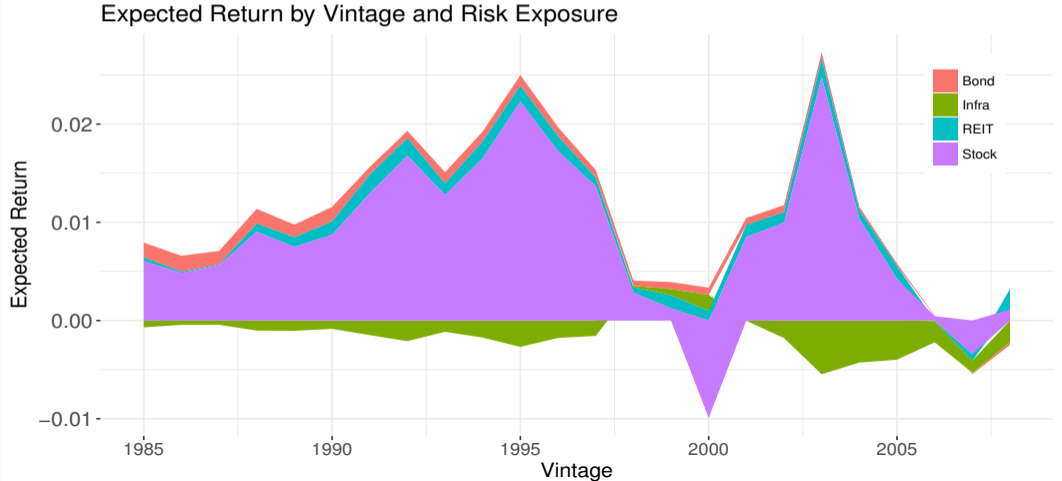


PE Expected Return – Buyout

Venture Capital

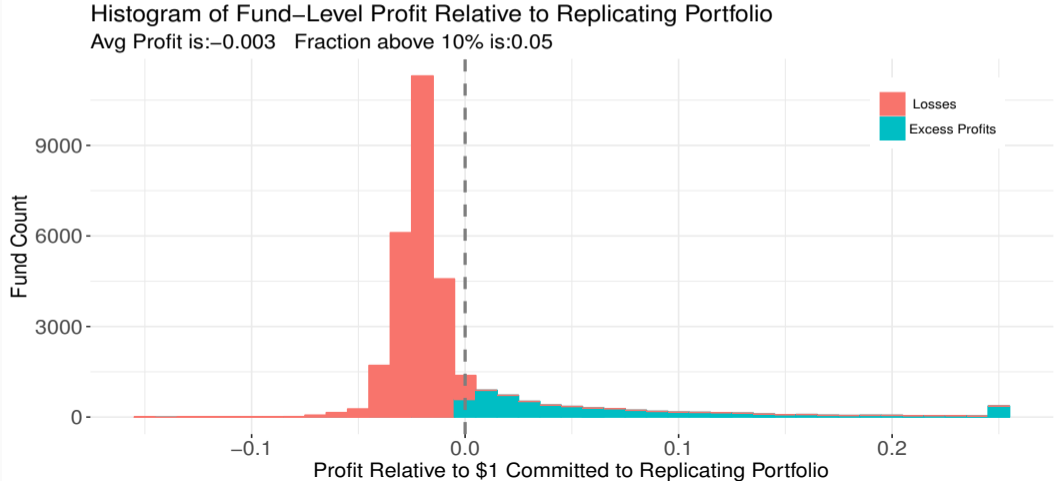
Real Estate

Infrastructure



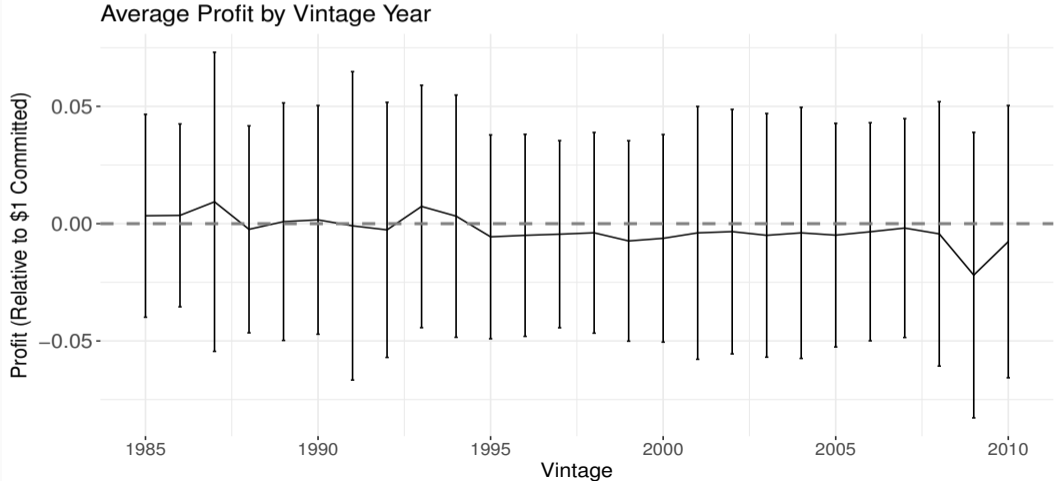
PE Fund Risk-Adjusted Profits – Buyout

Venture Capital Real Estate Infrastructure



PE Fund Risk-Adjusted Profits – Buyout

Venture Capital Real Estate Infrastructure

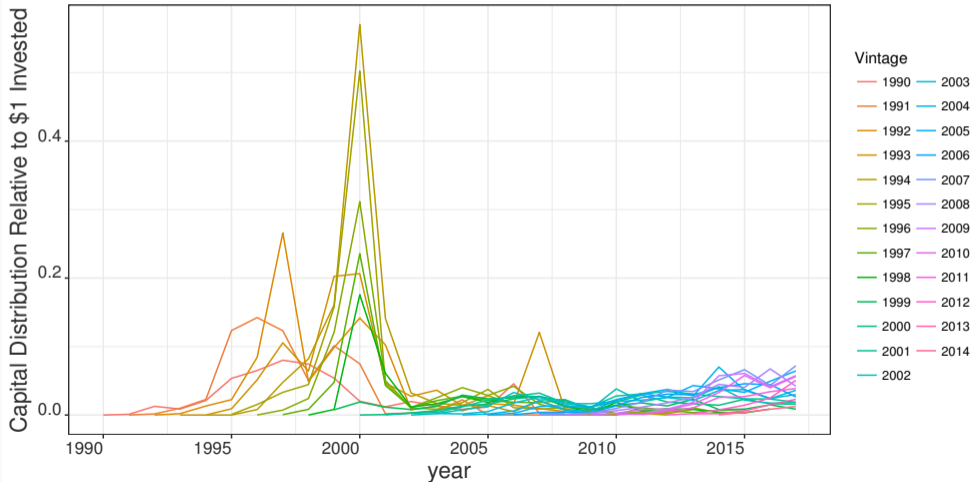


Takeaways

1. Develop methodology to value and understand risk/return characteristics when only cash flows, not returns, are available
2. Find PE funds take asset-specific specific exposure
3. On average, PE funds do not have risk-adjusted outperformance
4. Future work: alternate ways to deal with call timing, consider more cross-sectional return factors

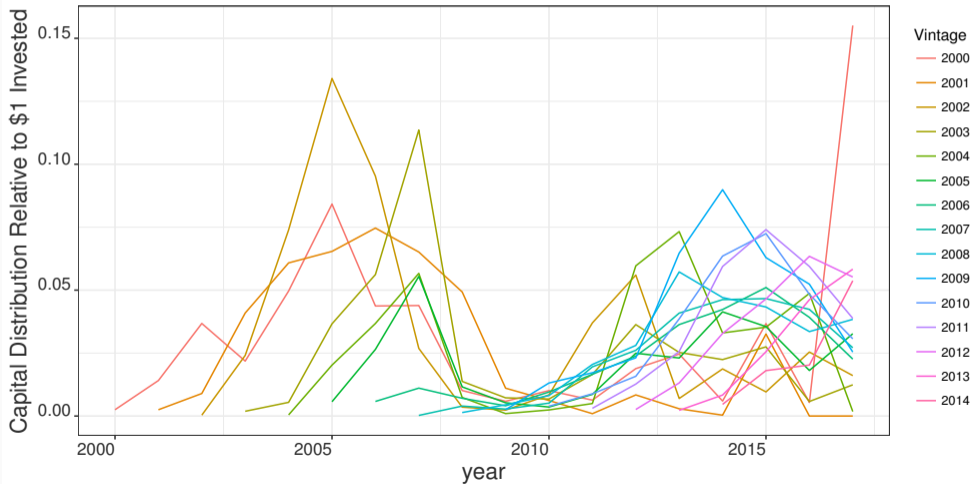
Cash-Flow Variation Across Horizon and Vintage — Venture Capital

Back



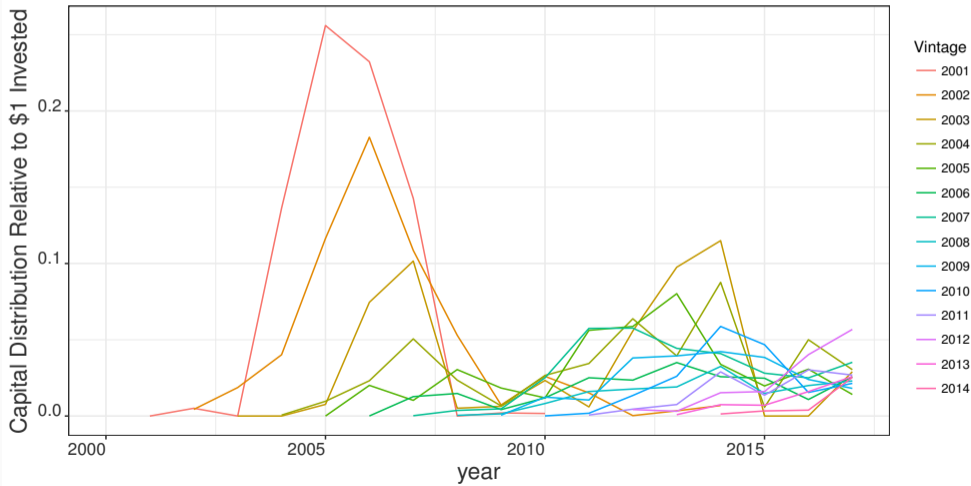
Cash-Flow Variation Across Horizon and Vintage — Real Estate

Back



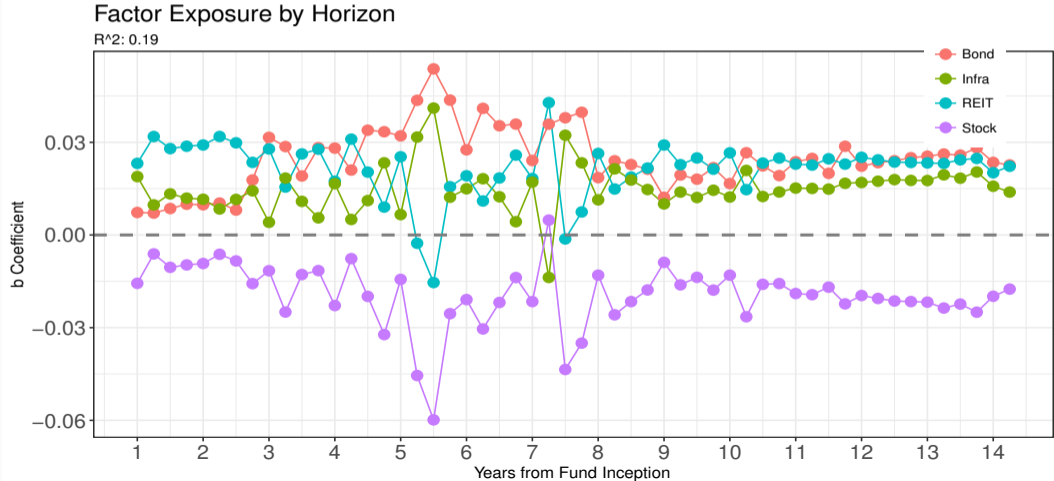
Cash-Flow Variation Across Horizon and Vintage — Infrastructure

Back



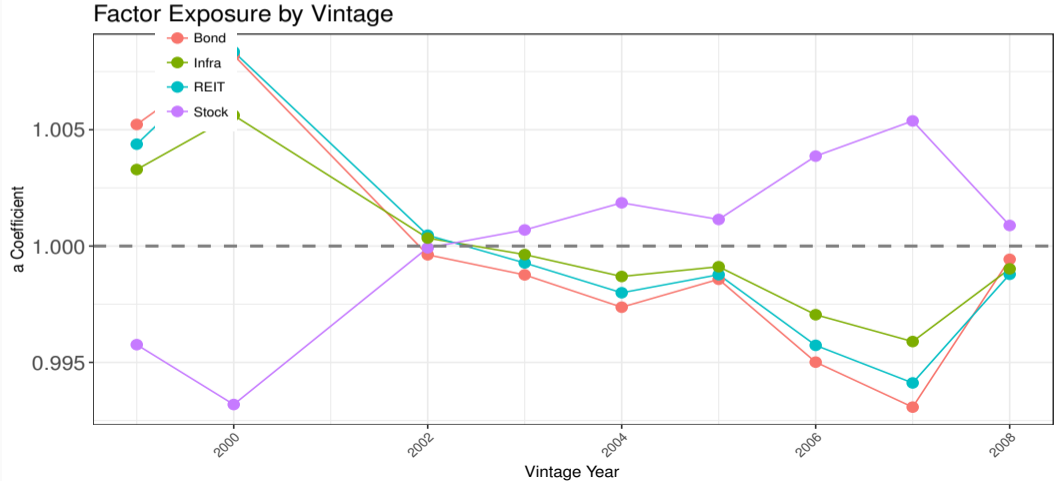
Factor Exposure in PE Funds by Horizon – Venture Capital

Back



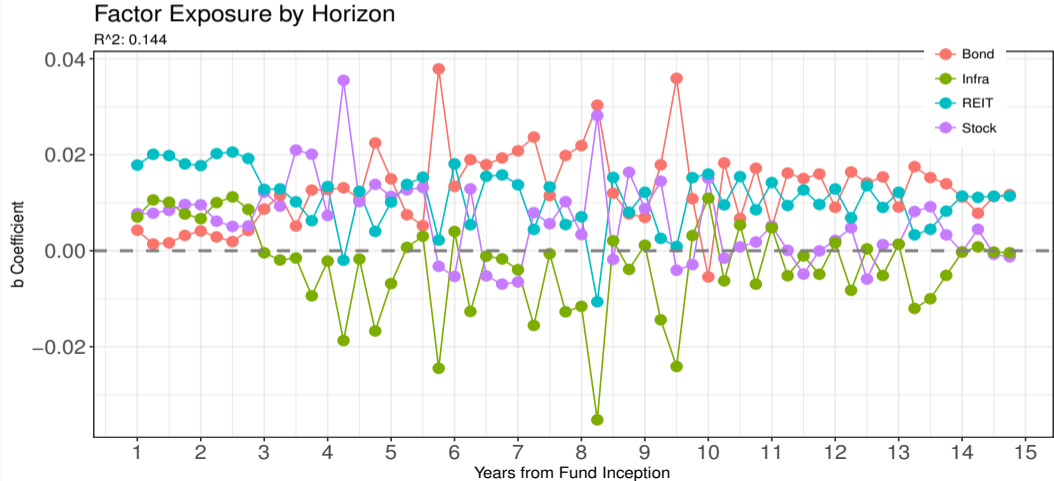
Factor Exposure in PE Funds by Vintage – Venture Capital

Back



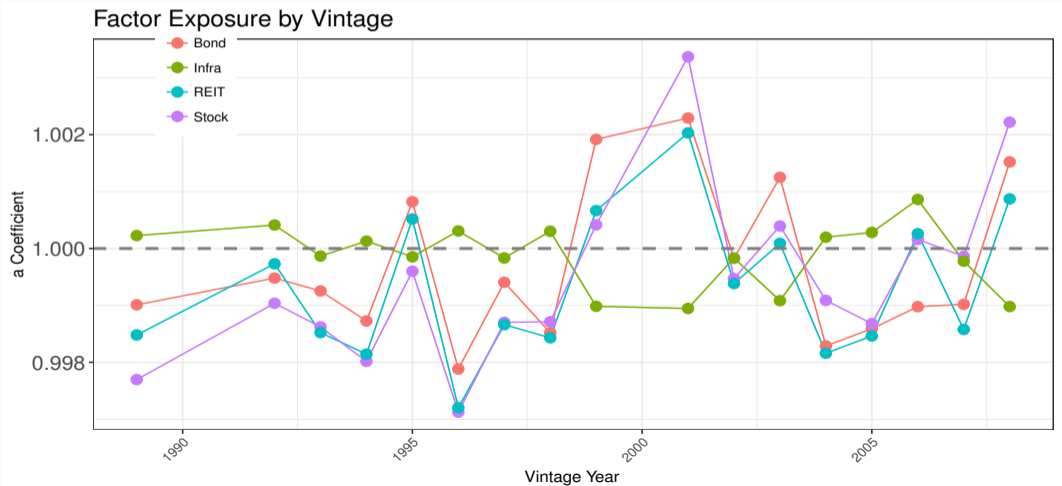
Factor Exposure in PE Funds by Horizon — Real Estate

Back



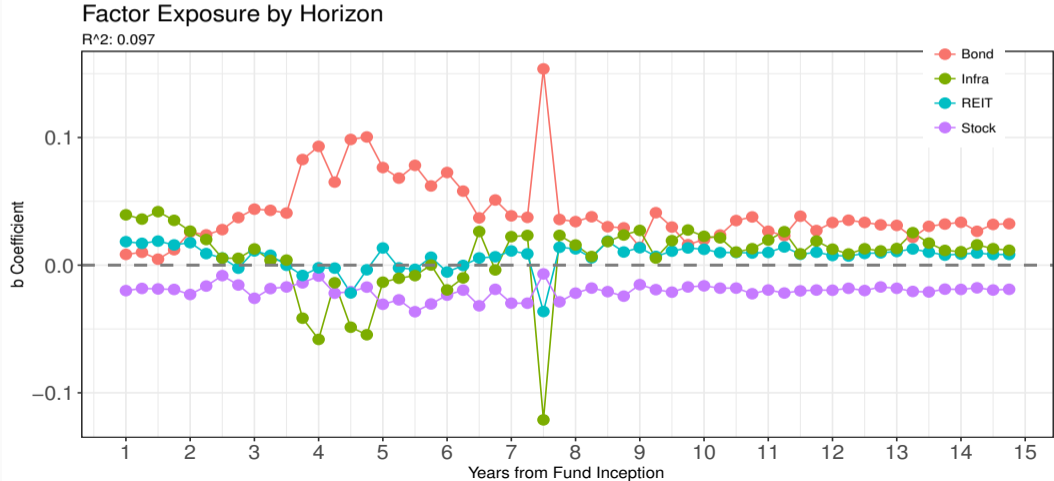
Factor Exposure in PE Funds by Vintage – Real Estate

Back



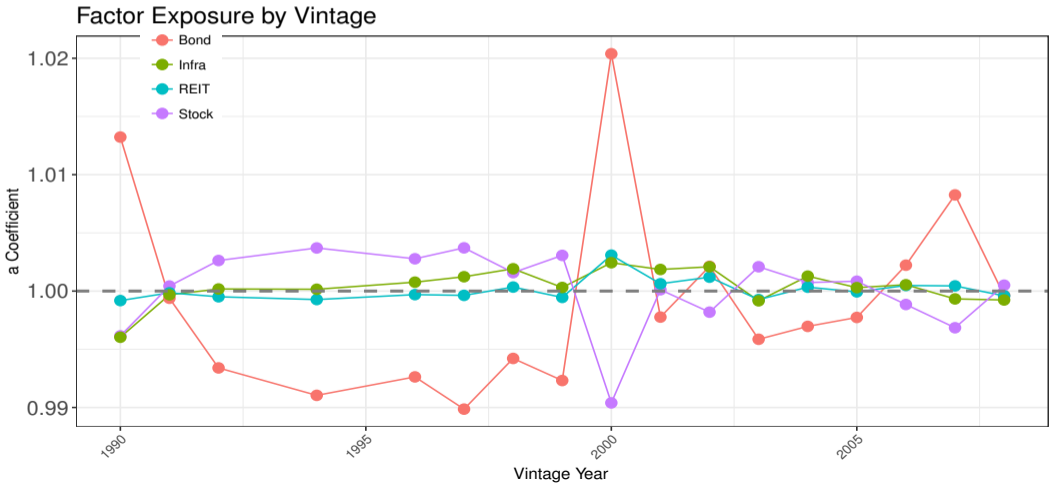
Factor Exposure in PE Funds by Horizon – Infrastructure

Back



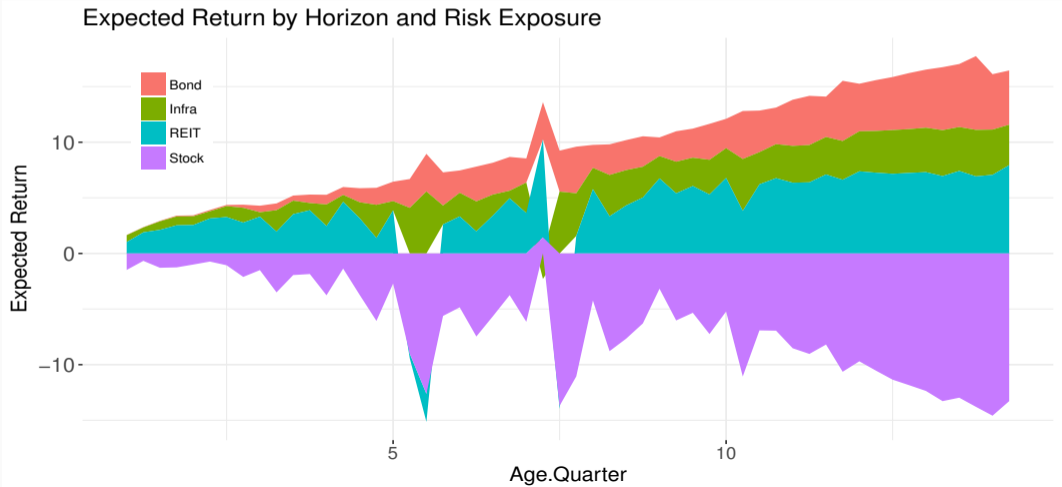
Factor Exposure in PE Funds by Vintage – Infrastructure

Back



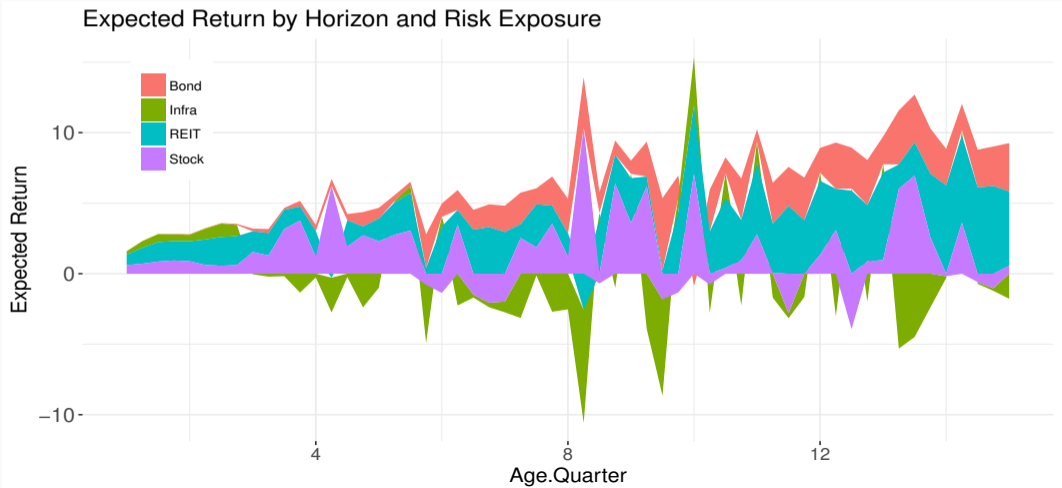
Private Equity Fund Expected Return – Venture Capital

Back



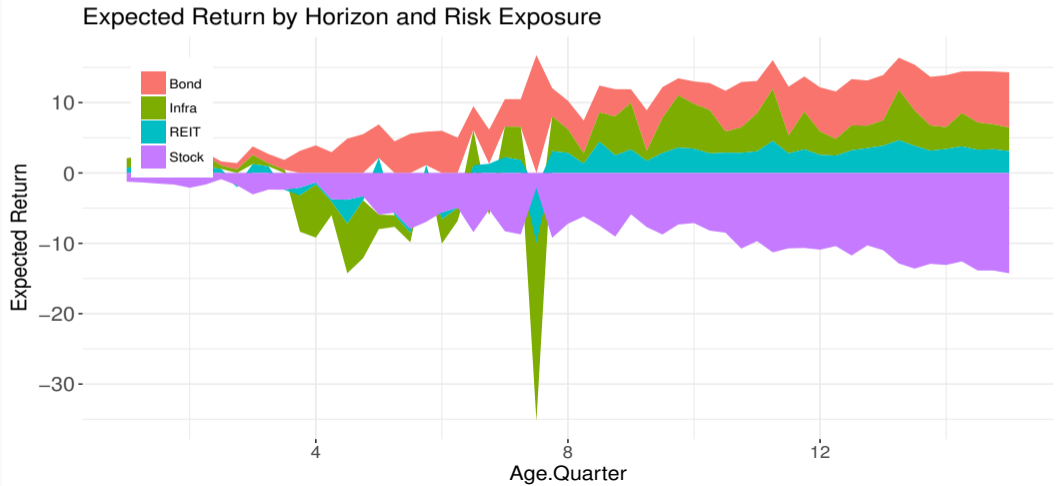
Private Equity Fund Expected Return – Real Estate

Back



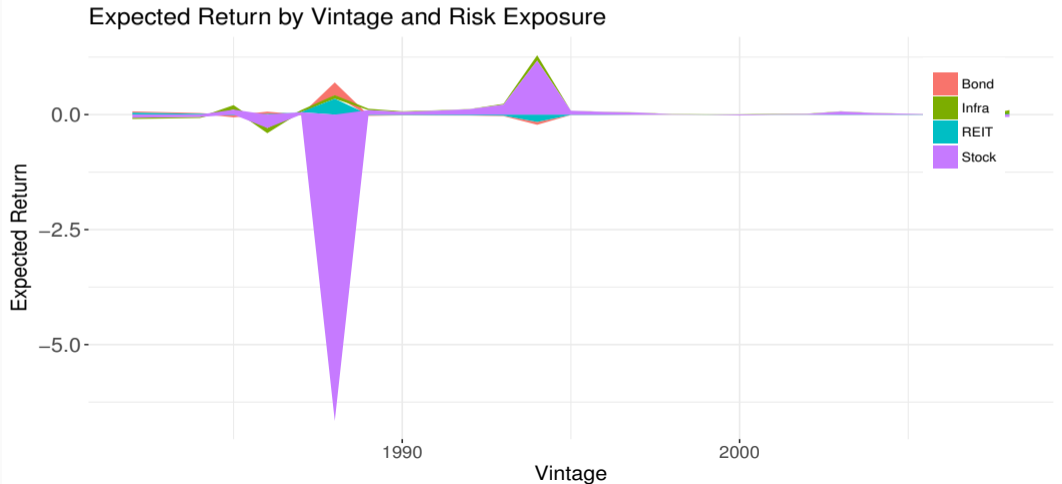
Private Equity Fund Expected Return – Infrastructure

Back



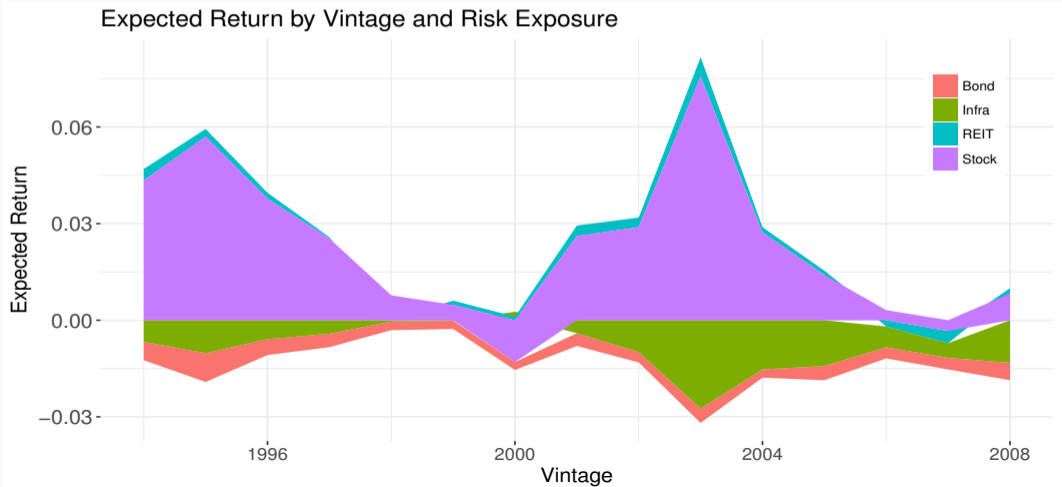
Real Estate Fund Expected Return – Venture Capital

Back



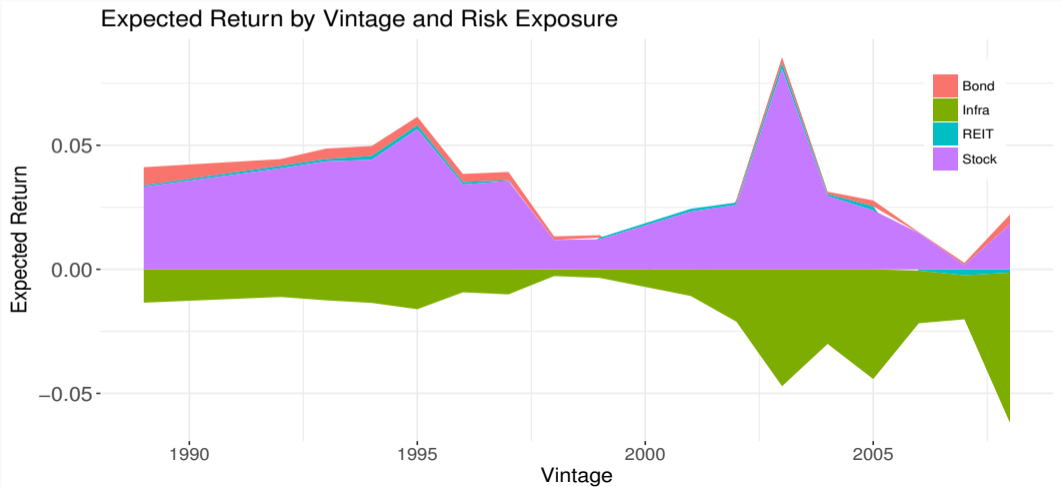
Real Estate Fund Expected Return – Real Estate

[Back](#)



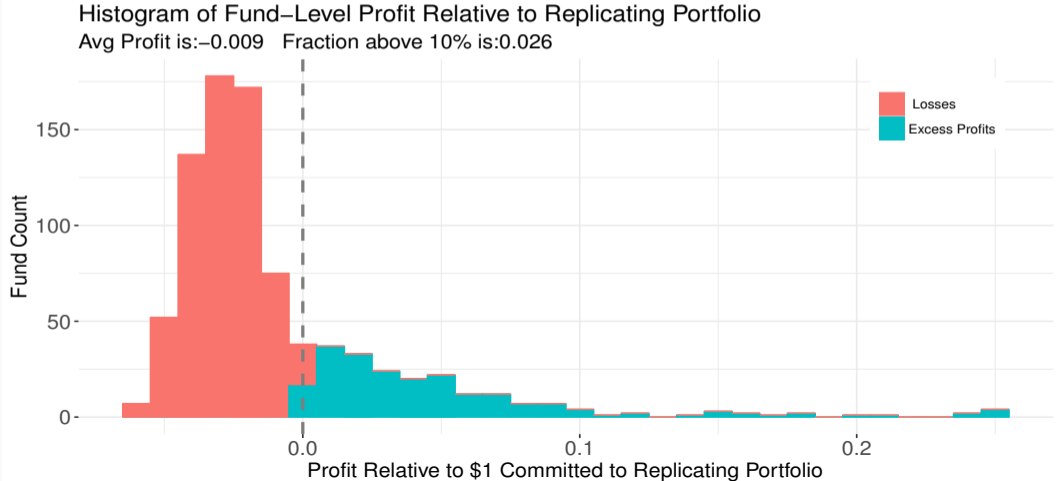
Real Estate Fund Expected Return – Infrastructure

Back



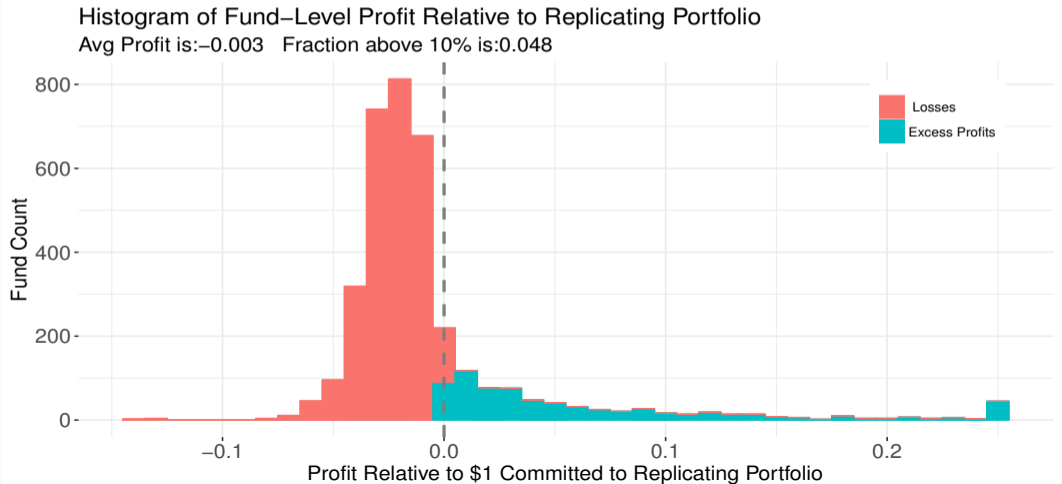
Private Equity Fund Risk-Adjusted Profits – Venture Capital

Back



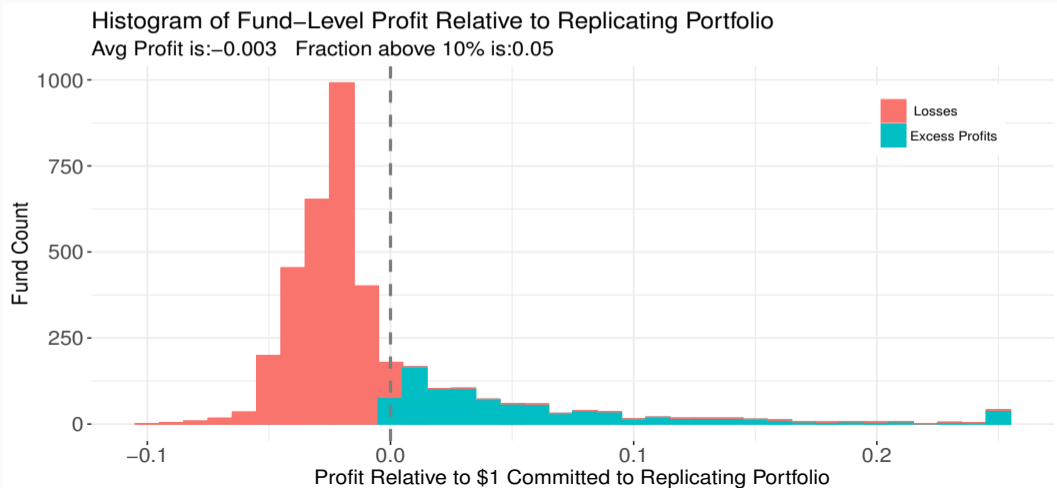
Private Equity Fund Risk-Adjusted Profits – Real Estate

Back



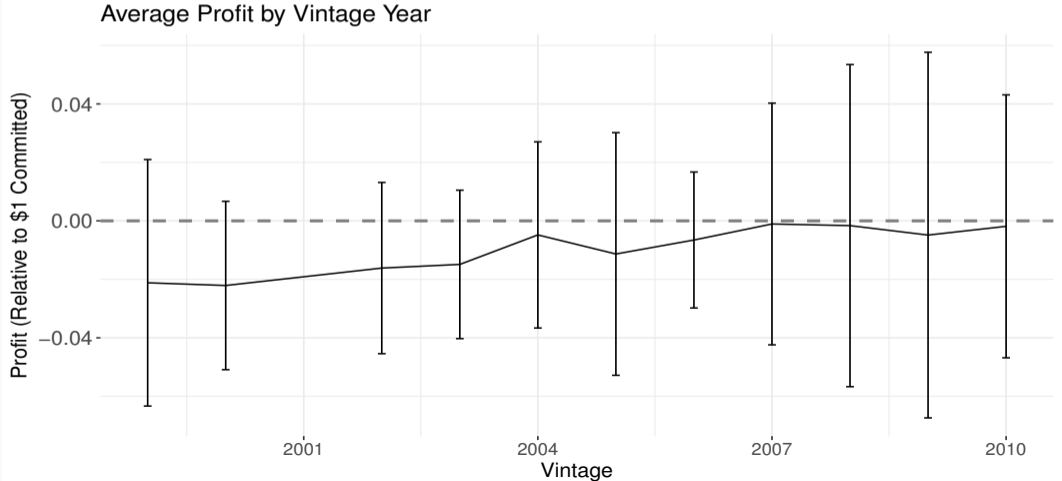
Private Equity Fund Risk-Adjusted Profits – Infrastructure

[Back](#)



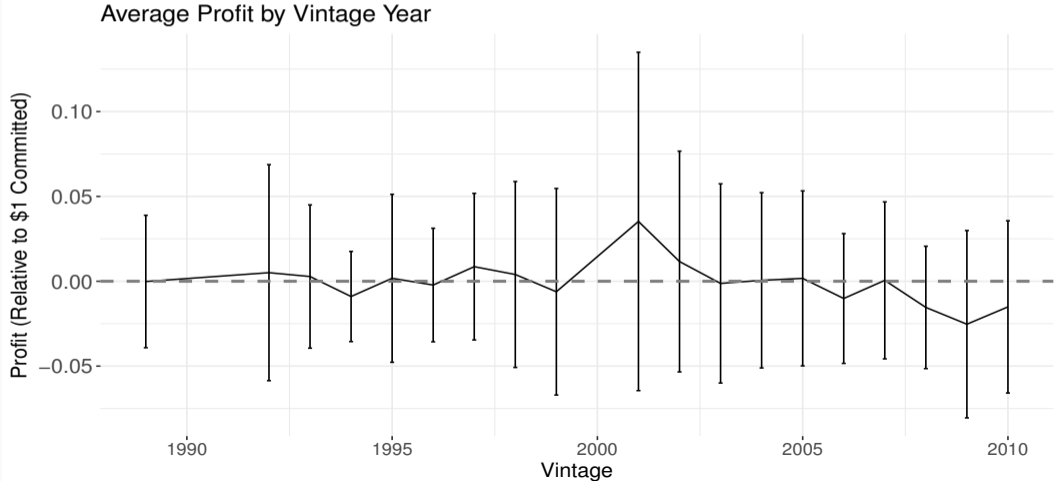
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Back



Private Equity Fund Risk-Adjusted Profits – Real Estate

Back



Private Equity Fund Risk-Adjusted Profits – Infrastructure

[Back](#)

