Valuing Private Equity Investments Strip by Strip

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1. Private equity valuation has not taken into account the multivariate nature of risk nor the temporal composition of risk

- Standard approaches:
  - PME
  - Korteweg and Nagel (2016)

- Limitations: only one aggregate source of risk
  - If a bad assumption in equities (CAPM) likely also the case in PE
  - Especially in “alternative” categories like Real Estate Funds
1. Asset Pricing in Private Equity has struggled with cross-section and term structure of risk

2. We draw from Asset Pricing Literature emphasizing term structure and multifactor models
   - Term structure of risk potentially upward or downward sloping, depending on factor
1. Asset Pricing in Private Equity has struggled with cross-section and term structure of risk

2. We draw from other Asset Pricing Literature emphasizing multifactor models

3. **Problem: Observe cashflows, not returns**
   - Use insights from asset pricing to price cash flows from PE funds strip by strip
   - Estimate risk exposures of cash flows by imposing cross-equation restrictions on the PE funds cash flow exposures
   - Estimate multi-factor exposure and allow term structure of risk to vary by factor
1. **Price public assets.** Model delivers term structure fit and valuation of cash flow.

2. Estimate Private Equity **fund exposure** to capital market assets

3. Produce budget-feasible **replicating portfolios** matching risk exposure

4. Use asset pricing model to understand private equity **risk/return characteristics**
1. Price Public Assets

Asset pricing model fits capital market assets
Asset Pricing Model

- State variables follow Gaussian first-order VAR:
  \[ z_t = \Psi z_{t-1} + \Sigma^{1/2} \varepsilon_t, \quad \varepsilon_t \sim i.i.d. \mathcal{N}(0, I) \]

- Bond variables: CP factor, nominal short rate, realized inflation, 5-year - 1-month Treasury spread

- Stock variables: log price-dividend, log real dividend growth for: CRSP, NAREIT, listed infra
  - Approach can be extended to include other cross-sectional equity risk factors such as: size, value, investment, profitability, etc.

- SDF:
  \[ m_{t+1}^s = -y_t^s(1) - \frac{1}{2} \Lambda_t' \Lambda_t - \Lambda_t' \varepsilon_{t+1}, \]
Model Matches Time-Series of Bond Yields

- **Nominal yield on 1-qtr bond**
- **Nominal yield on 1-yr bond**
- **Nominal yield on 5-year bond**
- **Nominal yield on 10-year bond**
Also Matches underlying Components of Bond Yield: Real + Nominal
Fits Equity Risk Premia as well as Stock Price Levels

Equity risk premium

REITs risk premium

Infra risk premium

Price-Dividend Ratio on Equity

Price-Dividend Ratio on REITs

Price-Dividend Ratio on Infra
Imputed Dividend Strip Model Matches Data when Available
Rich Patterns in Temporal Pricing of Risk

Average zero-coupon bond risk premium

Average Market div strip risk premium

Average REIT div strip risk premium

Average Infra div strip risk premium
2. Private Equity Fund Exposures

Estimate fund exposure to multi-factor model
Want to Understand Cash-Flow Profiles of Private Equity Funds
Cash-Flow Variation Across Horizon and Vintage — Buyout

Venture Capital  Real Estate  Infrastructure

[Graph showing capital distribution relative to $1 invested across different years and vintages for Venture Capital, Real Estate, and Infrastructure.]
Estimate Factor Exposure Strip-by-Strip

- Four-factor model fitting PE fund cash flows against four traded factors (bonds, stocks, real estate, infra)

\[
X_{t+h}^i = \beta_{t,h}^i Y_{t+h} + e_{t+h}^i \\
= a_{1,t}^c b_{1,h}^c + a_{2,t}^c b_{2,h}^c Y_{t+h}^m + a_{3,t}^c b_{3,h}^c Y_{t+h}^{reit} + a_{4,t}^c b_{4,h}^c Y_{t+h}^{infra} + v_{i \in c}^i.
\]

- Fit for each cash-flow horizon strip

- Estimate vintage \((a_1, a_2, a_3, a_4)\) and age \((b_1, b_2, b_3, b_4)\) effects

- Cross-equation restrictions: coefficients same for all funds in categories (i.e., Real Estate). Proportional shifts in vintage and age effects across factor exposures.
Factor Exposure in PE Funds by Horizon — Buyout

Factor Exposure by Horizon

\[ R^2: 0.092 \]

- Bond
- Infra
- REIT
- Stock

Years from Fund Inception

b Coefficient

-0.02
0.00
0.02

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15
Factor Exposure in PE Funds by Vintage – Buyout

Factor Exposure by Vintage

- Bond
- Infra
- REIT
- Stock

Vintage Year


Coefficient

0.996
0.998
1.000
1.002
1.004

15
3. Replicating Portfolios

Construct budget-feasible portfolios replicating Private Equity Funds
We connect the Model + Fund Exposures = Replicating Portfolio

- Define scaled long-positions in each factor that are budget feasible, where

\[
q_{t,h}^{i,+} = \frac{\beta_{t,h}^{i,+}}{\sum_{h=0}^{H} \beta_{t,h}^{i,+} p_{t,h}} \Rightarrow \sum_{h=0}^{H} q_{t,h}^{i,+} p_{t,h} = 1.
\]

where \( p_{t,h} \) comes from asset pricing model

- Null: present discounted value of fund cash distributions is 1:

\[
\mathbb{E}_t \left[ \sum_{h=0}^{H} M_{t+h}^h x_{t+h}^i \right] = \mathbb{E}_t \left[ \sum_{h=0}^{H} M_{t+h}^h q_{t,h}^{i,+} y_{t+h} \right] = \sum_{h=0}^{H} q_{t,h}^{i,+} p_{t,h} = 1
\]

- Also use model to find expected return; \( q_{t,h}^{i,+} p_{t,h} \) are portfolio weights:

\[
\mathbb{E}_t \left[ r_t^i \right] = \sum_{h=0}^{H} q_{t,h}^{i,+} p_{t,h} \mathbb{E}_t \left[ r_{t,h} \right]
\]
4. Private Equity Fund Characteristics

Use fitted model to understand risk and characteristics of private equity funds
Private Equity Fund Expected Return — Buyout

Expected Return by Horizon and Risk Exposure

- Bond
- Infra
- REIT
- Stock

Age.Quarter
Expected Return

Venture Capital  Real Estate  Infrastructure
PE Expected Return — Buyout

Expected Return by Vintage and Risk Exposure

- Bond
- Infra
- REIT
- Stock

Vintage

Expected Return


-0.01 0.00 0.01 0.02
Histogram of Fund-Level Profit Relative to Replicating Portfolio

Avg Profit is: -0.003  Fraction above 10% is: 0.05
Takeaways

1. Develop methodology to value and understand risk/return characteristics when only cash flows, not returns, are available.

2. Find PE funds take asset-specific specific exposure.

3. On average, PE funds do not have risk-adjusted outperformance.

4. Future work: alternate ways to deal with call timing, consider more cross-sectional return factors.
Cash-Flow Variation Across Horizon and Vintage — Venture Capital
Cash-Flow Variation Across Horizon and Vintage — Real Estate

Capital Distribution Relative to $1 Invested

Vintage

- 2000
- 2001
- 2002
- 2003
- 2004
- 2005
- 2006
- 2007
- 2008
- 2009
- 2010
- 2011
- 2012
- 2013
- 2014

Year


Capital Distribution Relative to $1 Invested vs. Year for Different Vintages
Cash-Flow Variation Across Horizon and Vintage — Infrastructure
Factor Exposure in PE Funds by Horizon – Venture Capital

Factor Exposure by Horizon

R^2: 0.19

Years from Fund Inception

b Coefficient

Bond
Infra
REIT
Stock

R^2: 0.19
Factor Exposure in PE Funds by Horizon — Real Estate

Factor Exposure by Horizon

R^2: 0.144

- Bond
- Infra
- REIT
- Stock

Years from Fund Inception

b Coefficient

R^2: 0.144
Factor Exposure in PE Funds by Vintage – Real Estate

![Factor Exposure by Vintage](image)

- Bond
- Infra
- REIT
- Stock

<table>
<thead>
<tr>
<th>Vintage Year</th>
<th>Bond Coefficient</th>
<th>Infra Coefficient</th>
<th>REIT Coefficient</th>
<th>Stock Coefficient</th>
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<td>0.998</td>
<td>1.000</td>
<td>1.002</td>
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<td>2000</td>
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<td>2005</td>
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</tbody>
</table>
Factor Exposure in PE Funds by Horizon — Infrastructure

Factor Exposure by Horizon

R^2: 0.097

Years from Fund Inception

b Coefficient

- Bond
- Infra
- REIT
- Stock

Factor Exposure by Horizon

b Coefficient

- Bond
- Infra
- REIT
- Stock

Years from Fund Inception

R^2: 0.097
Factor Exposure in PE Funds by Vintage – Infrastructure

Factor Exposure by Vintage

- Bond
- Infra
- REIT
- Stock

Vintage Year

a Coefficient

Private Equity Fund Expected Return — Real Estate

Expected Return by Horizon and Risk Exposure

- Bond
- Infra
- REIT
- Stock

Expected Return vs Age of Quarter
Expected Return by Horizon and Risk Exposure

- Bond
- Infra
- REIT
- Stock
Expected Return by Vintage and Risk Exposure
Real Estate Fund Expected Return – Infrastructure

Expected Return by Vintage and Risk Exposure

- Bond
- Infra
- REIT
- Stock
Private Equity Fund Risk-Adjusted Profits — Venture Capital

Histogram of Fund-Level Profit Relative to Replicating Portfolio
Avg Profit is: -0.009  Fraction above 10% is: 0.026

Fund Count
Losses
Excess Profits
Avg Profit is: -0.009  Fraction above 10% is: 0.026
Histogram of Fund–Level Profit Relative to Replicating Portfolio

Avg Profit is: $-0.003$  
Fraction above 10% is: $0.048$
Private Equity Fund Risk-Adjusted Profits – Infrastructure

Histogram of Fund–Level Profit Relative to Replicating Portfolio

Avg Profit is: −0.003  Fraction above 10% is: 0.05

Lostes
Excess Profits

Profit Relative to $1 Committed to Replicating Portfolio

Fund Count

0.0
0.1
0.2

−0.1
0.0
0.1
0.2

Fund Count
Private Equity Fund Risk-Adjusted Profits — Venture Capital

Average Profit by Vintage Year

Profit (Relative to $1 Committed)

-0.04 0.00 0.04

2001 2004 2007 2010

Vintage
Average Profit by Vintage Year

Profit (Relative to $1 Committed)
Average Profit by Vintage Year