Financial Intermediation, Capital Accumulation and Crisis Recovery

Hans Gersbach (ETH Zurich)
Jean-Charles Rochet (Univ. Zurich)
Martin Scheffel (KIT)

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1 Introduction

2 Model Setup

3 Intra-temporal Equilibrium

4 Analysis of Steady States and Transition Dynamics

5 Short-run Dynamics and Sensitivity of Bank Leverage

6 Speeding up Recovery

7 Quantitative Analysis

8 Extensions and Conclusion
Motivation/Contribution

- Conceptual: Integration of banks into two-sector neoclassical growth model → existence and type of steady states?

- Issues to be investigated:
  1. Role of bank leverage as amplifier and automatic stabilizer.
  2. Optimal crisis recovery with bank recapitalization and dividend payout restrictions.
  3. Explaining typical business cycle patterns such as procyclical leverage, bank lending and countercyclical bond issuance.
Relation to the Literature (1)


- Difference:
  - Dual role of bank leverage and quantitative analysis.
  - Set-up with two sectors (bank and bond finance) and smooth consumption / savings decisions.
  - Coupled accumulation rules for household capital and bank capital (like Rampini and Viswanathan).
• New DSGE models with an explicit banking sector examine the impact of financial frictions on
  • Efficiency of monetary policy: Gertler and Kiyotaki (2010), Gertler and Karadi (2011),
  • Role of bank capital in propagating shocks: Meh and Moran (2010), Angeloni and Faia (2013), Rampini and Viswanathan (2014),
  • Bank leverage cycles and crises: Adrian and Boyarchenko (2012), Brunnermeier and Sannikov (2014).
Relation to the Literature (3)


- Stylized facts: During recessions and banking crises,
  - volume of loans decreases but volume of bonds increases (Kashyap, Stein and Wilcox (1993), De Fiore and Uhlig (2012)),
  - bank leverage is pro-cyclical (Adrian and Shin (2014)).

Both bank loans and bonds are qualitatively important in the financing of firms.

We proceed as follows:

1. Introduction
2. Model Setup
3. Intra-temporal Equilibrium
4. Analysis of Steady States and Transition Dynamics
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Model Set-up

Firm $M$

$F^M(K^M_t, L^M_t)$

$K^M_t$ $r^M_t K^M_t$

$K^I_t$ $w^I_t K^I_t$

$K^I_t$

$D_t$

$E_t$

$K^I_t$

$D_t$

$w^I_t L^I_t$

$L^I_t$

$K^I_t$ $r^I_t K^I_t$

$K^I_t$

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$w^M_t L^M_t$

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$K^M_t$

$D_t$

$w^M_t L^M_t$

$L^M_t$

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$K^M_t$

$D_t$

$w^I_t L^I_t$

$L^I_t$

$K^I_t$ $w^I_t K^I_t$

$K^I_t$
Sequence of Events

- investors own $\Omega_t$
- bankers own $E_t$
- investors choose bonds and deposits
- bankers choose bonds and loans
- factor markets clear
- production takes place
- factors are paid
- capital depreciates
- consumption/saving decisions
- investors own $\Omega_{t+1}$
- bankers own $E_{t+1}$
• Incentive compatibility condition for deposit contracts:
\[
(1 + r_t^M)(K_t^I - E_t) \leq K_t^I(1 + r_t^I - \theta).
\]

• As bankers maximize \( \theta K_t^I \), this condition will always be binding in equilibrium when \( E_t \) is not too large.

• ⇒ Bank leverage:
\[
\lambda_t = \frac{K_t^F}{E_t} = \frac{1 + r_t^M}{r_t^M - r_t^I + \theta}.
\]

• Remark: As \( r_t^I > r_t^M \) in equilibrium when financial frictions matter, bankers are always better off by leveraging.
Equilibrium Definition

Definition

A sequential markets equilibrium is a sequence of factor prices and allocations 
\[ \{w^M_t, w^I_t, r^M_t, r^I_t, \Omega_t, E_t, K^M_t, K^I_t, C^H_t, C^B_t\}_{t=0}^{\infty} \] such that

1. given \( \Omega_0 \) and \( \{r^M_t\}_{t=0}^{\infty} \), the allocation \( \{C^H_t, \Omega_t\}_{t=0}^{\infty} \) solves the investor’s problem (1),

2. given \( E_0 \) and \( \{r^M_t, r^I_t\}_{t=0}^{\infty} \), the allocation \( \{C^B_t, E_t\}_{t=0}^{\infty} \) solves the banker’s problem (2),

3. for each \( t \geq 0 \), given \( \{w^M_t, w^I_t, r^M_t, r^I_t\} \), the firm allocation \( \{K^M_t, K^I_t, L^M_t, L^I_t\} \) solves the firms’ problems,

4. factor and output markets clear,

5. leverage constraint is binding (if financial frictions matter) or non-binding (with \( r^I_t = r^M_t \)).
Three Cases

(A): Immobile labor.

(B): Flexible labor ($w^I_t = w^M_t$).

(C): Some labor mobile and some labor immobile.
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## Comparative Statics

<table>
<thead>
<tr>
<th>Shocks</th>
<th>Bank leverage</th>
<th>Loans</th>
<th>Bonds</th>
<th>Output</th>
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</thead>
<tbody>
<tr>
<td>TFP↓</td>
<td>−</td>
<td>−</td>
<td>+</td>
<td>−</td>
</tr>
<tr>
<td>Ω↓</td>
<td>−</td>
<td>−</td>
<td>−</td>
<td>−</td>
</tr>
<tr>
<td>E↓</td>
<td>+</td>
<td>−</td>
<td>+</td>
<td>−</td>
</tr>
</tbody>
</table>
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Laws of Motion for Log-utilities

- Log-utilities imply

\[ \Omega_{t+1} = \beta_H (1 + r_t^M - \delta) \Omega_t \]
\[ E_{t+1} = \beta_B R_t^B E_t, \]

where \( R_t^B \) is the (net) return on equity factor in period \( t \) given by

\[ R_t^B := \begin{cases} 
\theta \lambda(r_t^M, r_t^I) - \delta & \text{if } E_t < \bar{E}(K_t), \\
1 + r_t^M - \delta & \text{if } E_t \geq \bar{E}(K_t).
\end{cases} \]

- Bankers benefit from capital return differences between sector I and M and from leverage:

\[ R_t^B = 1 + r_t^M - \delta + \lambda(r_t^M, r_t^I)(r_t^I - r_t^M). \]

- Assumption: \( \beta_B < \beta_H \) (\( \iff \rho_B > \rho_H \)).
Existence of Steady States

Proposition

Suppose $\rho_B > \rho_H$. Then, the system has a unique and globally stable state $(\hat{E}, \hat{\Omega})$. Financial frictions always bind in the long run.

Remarks:

- Steady state can be explicitly (iteratively) calculated for log utilities.
- Interesting consequence of permanent shock: An increase of $\theta$ increases the banker’s utility if $\rho_B$ is close to $\rho_H$. 
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Amplification, Persistence and Stabilization

- Temporary negative shock to TFP: \((K^M \uparrow, K^I \downarrow)\), \(Y \downarrow\) but \(\lambda \downarrow, Y \down\downarrow\): amplification and persistence.

- Temporary negative shock to \(E\): \((K^M \uparrow, K^I \down\downarrow)\), \(Y \down\down\down\) but automatic stabilization, \(\lambda \uparrow, Y \uparrow\).
Bond and Loan Financing over the Business Cycle

Empirical literature: De Fiore and Uhlig (2012), Contessi et al. (2013)

- Bank lending is procyclical,
- Bond issuing reacts little to booms and busts, and may even be countercyclical.

This feature can be derived when a downturn is associated with
- a temporary negative aggregate productivity shock,
- a negative shock to bank equity,
- a negative trust shock,
- or any combination of these shocks.
Proposition (Dividend Payout Restrictions and Capital Injections)

Suppose there is a shock that leads to a temporary decline in bank equity capital in period 0, with \(1 - \delta_1^E > \beta_H (1 - \delta)\). Then, there exists a feasible sequence of transfer payments from investors to banks, \(\{T_{rt}\}_{t=0}^\infty\), and an associated sequence of dividend payout restrictions, \(\{d_t\}_{t=0}^\infty\), with the following properties:

(i) Total capital \(K_t\) and total output \(Y_t\) exceed their respective laissez-faire values in all periods.

(ii) Lifetime-utility of bankers is constant by construction and lifetime-utility of workers increases. The impact on lifetime-utility of investors is ambiguous.
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Quantitative Analysis

- Calibration to US economy (1991 Q1 – 2017 Q4) with shock process involving $A_t, \delta_t^E, \theta_t$ captured by VAR(1) process.

- First step: Time-invariant parameters to match steady state to long-run stylized facts.

- Second step: Estimation of joint stochastic process.

### Parameters and Calibration Targets

#### Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$z^M$</th>
<th>$z^I$</th>
<th>$\delta^H$</th>
<th>$\delta^B$</th>
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<tr>
<td>$\alpha$</td>
<td>0.3484</td>
<td>1.0168</td>
<td>0.0146</td>
<td>0.0146</td>
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<tr>
<td>$\beta^H$</td>
<td>0.9871</td>
<td>0.0967</td>
<td>1.0000</td>
<td>0.5885</td>
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#### Calibration Targets

<table>
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<tr>
<th>Target</th>
<th>$\bar{s}$</th>
<th>$\bar{K}/Y$</th>
<th>$\bar{\lambda}$</th>
<th>$\bar{r}^B$</th>
<th>$\bar{K}^I/K^M$</th>
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</thead>
<tbody>
<tr>
<td>$\bar{s}$</td>
<td>0.1801</td>
<td>12.3763</td>
<td>10.7808</td>
<td>0.0276</td>
<td>0.6667</td>
</tr>
</tbody>
</table>
## Correlation of Shocks and Leverage

<table>
<thead>
<tr>
<th></th>
<th>( \Delta \ln(A) )</th>
<th>( \Delta \delta^E )</th>
<th>( \Delta \ln(\theta) )</th>
<th>( \Delta \ln(Y) )</th>
<th>( \Delta \ln(\lambda) )</th>
<th>( \Delta \ln(K^I) )</th>
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</thead>
<tbody>
<tr>
<td>( \Delta \ln(A) )</td>
<td>+1.0000</td>
<td>-0.2746</td>
<td>+0.0295</td>
<td>+0.6591</td>
<td>-0.0939</td>
<td>+0.1688</td>
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<tr>
<td></td>
<td>(0.0042)</td>
<td>(0.7630)</td>
<td>(0.0000)</td>
<td>(0.3360)</td>
<td>(0.0822)</td>
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<tr>
<td>( \Delta \delta^E )</td>
<td>+1.0000</td>
<td>+0.7946</td>
<td>-0.4947</td>
<td>+0.0436</td>
<td>+0.2797</td>
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<tr>
<td></td>
<td>(0.0000)</td>
<td>(0.0000)</td>
<td>(0.6556)</td>
<td>(0.0035)</td>
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<tr>
<td>( \Delta \ln(\theta) )</td>
<td>+1.0000</td>
<td></td>
<td>-0.2642</td>
<td>-0.2491</td>
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<tr>
<td></td>
<td>(0.0060)</td>
<td>(0.0097)</td>
<td>(0.2503)</td>
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<tr>
<td>( \Delta \ln(Y) )</td>
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<td>+1.0000</td>
<td>+0.2073</td>
<td>+0.4888</td>
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<td>(0.0321)</td>
<td>(0.0000)</td>
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<td>( \Delta \ln(\lambda) )</td>
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<td>(0.0000)</td>
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</table>

**Note:** \( \Delta x \) refers to the deviation of \( x \) from its HP-trend with smoothing parameter 1600. The numbers are temporary cross-correlations and the associated p-values are in parentheses.
Great Recession – Shock Sequences

Note: $\Delta x$ refers to the deviation of $x$ from its HP-trend with smoothing parameter 1600. The deviations from trend are further normalized by their respective 2008Q1 value.
## Welfare and Output Costs of the Great Recession

<table>
<thead>
<tr>
<th>shocks to ...</th>
<th>output cost</th>
<th>investor</th>
<th>worker</th>
<th>banker</th>
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</thead>
<tbody>
<tr>
<td>( (A, \delta^E, \theta) )-shock ( \lambda_{\text{reg}} = \infty )</td>
<td>+0.5323</td>
<td>+0.5640</td>
<td>+0.3408</td>
<td>+3.3963</td>
</tr>
<tr>
<td>( (A, \delta^E) )-shock</td>
<td>+0.5235</td>
<td>+0.5408</td>
<td>+0.3349</td>
<td>+3.9666</td>
</tr>
<tr>
<td>( (A) )-shock</td>
<td>+0.2678</td>
<td>+0.0976</td>
<td>+0.1434</td>
<td>+0.1402</td>
</tr>
<tr>
<td>( (A, \delta^E, \theta) )-shock ( \lambda_{\text{reg}} = 1.01\hat{\lambda} )</td>
<td>+0.6257</td>
<td>+0.7367</td>
<td>+0.4152</td>
<td>+4.4756</td>
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<tr>
<td>( (A, \delta^E) )-shock</td>
<td>+0.9119</td>
<td>+1.2565</td>
<td>+0.6530</td>
<td>+8.361</td>
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<tr>
<td>( (A) )-shock</td>
<td>+0.2678</td>
<td>+0.0976</td>
<td>+0.1434</td>
<td>+0.1402</td>
</tr>
</tbody>
</table>

Note: Simulation results for \( (A, \delta^E, \theta) \)-shocks – Great Recession – \( (A, \delta^E) \)-shocks, and \( (A) \)-shocks for different regulatory regimes: laissez faire refers to \( \lambda_{\text{reg}} = \infty \), weak regulation refers to \( \lambda_{\text{reg}} = 1.05\hat{\lambda} \), and strong regulation refers to \( \lambda_{\text{reg}} = 1.01\hat{\lambda} \). Output costs are denominated in percent of the present discounted value of output. Welfare costs are denominated in percent of consumption equivalent units.
### Accelerating Recoveries

Welfare and Output Costs of the Great Recession: *Balanced Bailout* vs *Laissez-Faire*

<table>
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<tr>
<th>shocks to . . .</th>
<th>output cost</th>
<th>welfare cost</th>
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</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>investor</td>
</tr>
<tr>
<td><em>laissez faire</em> ($\zeta = 0.00$)</td>
<td>+0.5323</td>
<td>+0.5640</td>
</tr>
<tr>
<td><em>balanced bailout</em> ($\zeta = 0.33$)</td>
<td>+0.5254</td>
<td>+0.6059</td>
</tr>
<tr>
<td>$\lambda = 10.7808$</td>
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</tr>
<tr>
<td><em>balanced bailout</em> ($\zeta = 0.66$)</td>
<td>+0.5220</td>
<td>+0.6492</td>
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Note: Simulation results for $(\Lambda, \delta^E, \theta)$-shocks – Great Recession – for different policy regimes. The policy regimes are convex combinations between the laissez-faire path of bank equity capital and the steady state value of bank equity capital, where parameter $\zeta$ is the weight given to laissez-faire. Output costs are denominated in percent of the present discounted value of output. Welfare costs are denominated in percent of consumption-equivalent units.
Policy Implications

- Automatic stabilization of leverage is quantitatively important → countercyclical capital requirements are important.

- Balanced bailout speeds up recovery.

- Unbalanced bailout strongly accelerates recovery. → debt-financed bank recapitalization and dividend payment restriction
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Extensions

• Extensions:
  - Anticipated bank equity shocks
  - Costs of intermediation
  - Saving workers
  - General utility and production function

• Application: Resilience of economies relying more heavily on bank loans (Eurozone and much of Asia) compared to the ones relying more on corporate bonds (USA).

• Challenge: Model with completely flexible labor.
Conclusion

- Parsimonious model of capital accumulation and growth in which both bank credit and bonds play an essential role.

- Useful for dual role of bank leverage, explaining facts and designing policy for crisis management and prevention.

- Many possible avenues for further research
Backup
Macroeconomic Environment

- Time $t \in \{0, 1, 2, \ldots\}$
- Four types of competitive agents (represented by continua in $[0, 1]$):
  - Workers (each one supplies one unit of labor)
  - Entrepreneurs (manage non-financial firms)
  - Investors (own capital $\Omega_t$)
  - Bankers (manage banks, own capital $E_i$)
- Competitive markets
  \[ \Rightarrow \text{Representative agents acting competitively} \]
- Two goods: physical good and labor
- Physical good
  - produced by capital $K_t$ and labor $L_t$
  - consumed or invested in future periods
- Capital depreciates at rate $\delta$. 
Production Technologies

- At the end of each period, agents decide how much to consume and how much to save.

- Total capital $K_t = E_t + \Omega_t$ allocated between two sectors:
  - $j = M$ (firms obtaining market finance) and $j = I$ (firms needing intermediated finance)

- Cobb-Douglas technologies:
  \[
  Y^j_t = z^j A(K^j_t)\alpha (L^j_t)^{1-\alpha}
  \]

- $z^j$ specific productivity in each sector: allows to calibrate the relative size of two sectors
  $K_t = K^M_t + K^I_t$
Financial Frictions (1)

- **Sector M (large/mature firms)**
  - Uninformed lending through financial markets
  - $K^M_t$ supplied by households only

- **Sector I (small/young firms)**
  - Moral hazard problem of entrepreneurs
  - Monitoring technology of banks (in basic version: costless)
  - $K^I_t$ denotes bank capital supplied by bankers and households
  - Access to capital markets only through informed bank lending
Financial Frictions (2)

- Banking technology
  - Moral hazard at bank managers’ level
  - Bankers cannot pledge a fraction $\theta$ of their banks’ assets
  - Non-pledgeable part is thus $\theta K_t^I$
  - Can be explained by
    - moral hazard à la Holmström and Tirole (1997)
    - asset diversion (Gertler and Karadi (2011))
    - non-alienability of human capital
      (Hart and Moore (1994), Diamond and Rajan (2000))
Labor and Capital

- Competitive firms maximize profits, given interest rates $r^j_t$ and wages $w^j_t$.
- Segmented labor markets, fixed labor supply ($L^M_t = 1, L^I_t = 1$)
- Segmented capital markets:
  - $I$-firms only financed by banks (loan rate $r^I_t$);
  - $M$-firms financed by markets (interest rate $r^M_t$).
- In equilibrium:
  - positive spread between loan and bond rates $r^I_t > r^M_t$
Preferences

- Bankers and investors (households) choose their saving and consumption levels to maximize

$$\sum_{t=0}^{\infty} (\beta^k)^t \ln(C^k_t), k = B, H. \quad \beta^B \equiv \frac{1}{1 + \rho^B} < \beta^H \equiv \frac{1}{1 + \rho^H}. $$

s.t. Budget Constraints.

- Investors are indifferent between bonds and deposits.
- Banks issue deposits to leverage their equity.
- Workers supply labor and own no assets. For implicit solutions: focus on case in which they consume all of their income.
- Entrepreneurs are competitive and make zero profit.
Intertemporal Budget Constraints

- **Bankers**

\[ C_t^B + E_{t+1} = \theta K_t^{I} - \delta E_t = \left( \theta \frac{1 + r_t^M}{r_t^M - r_t^I + \theta} - \delta \right) E_t \]

\[ C_t^B, E_{t+1} \geq 0, \quad E_0 \text{ is given} \]

- **Households**

\[ C_t^H + \Omega_{t+1} = r_t^M K_t^M + r_t^D D_t + (1 - \delta) \Omega_t = r_t^M \Omega_t + (1 - \delta) \Omega_t \]

\[ K_t^M + D_t = \Omega_t \]

\[ C_t^H, D_t, K_t^M, \Omega_{t+1} \geq 0, \quad \Omega_0 \text{ is given} \]
Investors

\[
\max_{\{C_t^H, \Omega_{t+1}\}_{t=0}^{\infty}} \left\{ \sum_{t=0}^{\infty} \beta_t^H \ln(C_t^H) \right\}
\]

s.t. \[ C_t^H + \Omega_{t+1} = r_t^M K_t^M + r_t^D D_t + (1 - \delta) \Omega_t \]

\[ K_t^M + D_t = \Omega_t \]

\[ C_t^H, D_t, K_t^M, \Omega_{t+1} \geq 0 \]

\[ \Omega_0 \] given

- \( C_t^H \) denotes investors’ consumption.
- \( D_t \) denotes the (aggregate) amount of deposits.
- \( \beta_H = \frac{1}{1 + \rho_H} \) (0 < \( \beta_H < 1 \)) denotes the discount factor and \( \rho_H \) the discount rate.
Bankers

Maximize over \( \{C_t^B, E_{t+1}\}_{t=0}^{\infty} \) \( \sum_{t=0}^{\infty} \beta_t^B \ln(C_t^B) \) \( s.t. \) \( C_t^B + E_{t+1} = \theta K_t^I - \delta E_t = \left( \theta \frac{1 + r_t^M}{r_t^M - r_t^I + \theta} - \delta \right) E_t \)

\( C_t^B, E_{t+1} \geq 0 \)

\( E_0 \) given

- \( C_t^B \) denotes the bankers’ consumption.
- \( \beta_B = \frac{1}{1 + \rho_B} \) denotes the bankers’ discount factor and \( \rho_B \) the discount rate.
• Profit-maximization of firms yields

\begin{align*}
  r_j^t &= \alpha A z_j^j (K_t^j)^{\alpha-1}, \quad j \in \{M, I\} \\
  w_j^t &= (1 - \alpha) A z_j^j (K_t^j)^{\alpha}, \quad j \in \{M, I\}
\end{align*}

\begin{align*}
  r_j^t &= \alpha A z_j^j (K_t^j)^{\alpha-1}, \quad j \in \{M, I\} \\
  w_j^t &= (1 - \alpha) A z_j^j (K_t^j)^{\alpha}, \quad j \in \{M, I\}
\end{align*}
Irrelevant Financial Frictions

- No funds channeled from households to the banking technology

\[ K_t^M = (E_t - K_t^I) + \Omega_t \]

- \( r_t^I = r_t^M \)

- Equilibrium values

\[ K_t^M = \frac{K_t}{1 + z}, \quad K_t^I = \frac{zK_t}{1 + z} \]

- Net earnings of bankers amount to \( K_t^I(1 + r_t^I) + (E_t - K_t^I)(1 + r_t^M) = E_t(1 + r_t^M) \)

- Incentive compatibility constraint requires that \( E_t(1 + r_t^M) \geq \theta K_t^I \)
  or equivalently

\[ E_t \geq \frac{\theta z}{1 + \alpha z^M \left( \frac{K_t}{1 + z} \right)^{\alpha - 1}} \frac{K_t}{1 + z} \equiv \bar{E}(K) \]
Binding Financial Frictions

• Allocation

\[ K_t^I = \lambda_t E_t \]
\[ K_t^M = K_t - \lambda_t E_t = \Omega_t + E_t - \lambda_t E_t \]
\[ \lambda_t = \frac{1 + \alpha z^M (\Omega_t + E_t - \lambda_t E_t)^{\alpha - 1}}{\alpha z^M (\Omega_t + E_t - \lambda_t E_t)^{\alpha - 1} - \alpha z^I (\lambda_t E_t)^{\alpha - 1} + \theta} \]

• Equilibrium leverage \( \lambda \) satisfies

\[ \varphi(\lambda) = \alpha z^M (\Omega + E - \lambda E)^{\alpha - 1} \left( 1 - \frac{1}{\lambda} \right) - \frac{1}{\lambda} + \theta - \alpha z^I (\lambda E)^{\alpha - 1} = 0 \] (5)

• If \( E < \bar{E}(K) \), the intermediate value theorem and strict monotonicity of \( \varphi(\lambda) \) delivers the existence and uniqueness of \( \lambda^*_t \) that solves (5).

• \( r^I_t > r^M_t \)
Existence and Uniqueness

Proposition (Intra-temporal Equilibrium)

For all pairs \((E_t, K_t)\) with \(0 < E_t < K_t\), there exists a unique equilibrium.

(i) If \(E_t \geq \bar{E}(K_t)\), we obtain \(K^M_t = \frac{K_t}{1+z}, K^I_t = z \frac{K_t}{1+z}\) and financial frictions do not matter.

(ii) If \(E_t < \bar{E}(K_t)\), financial constraints bind and leverage \(\lambda_t\) is determined by

\[
\theta \lambda_t = 1 + r^M_t(\lambda_t) + \lambda_t(r^I_t(\lambda_t) - r^M_t(\lambda_t)).
\]
Comparative Statics (1/2)

Corollary

Suppose that financial frictions matter, i.e. $E_t < \bar{E}(K_t)$. Then,

(i) $\lambda_t$ increases in $z^I$, $\Omega_t$,

(ii) $\lambda_t$ decreases in $z^M$, $E_t$ and $\theta$.

• Suppose both total factor productivity parameters $z^M$ and $z^I$ are affected by the same relative shock

$$\epsilon := \frac{\Delta z^M}{z^M} = \frac{\Delta z^I}{z^I}.$$  

• Then, the effect on leverage is as follows:

Corollary

Suppose financial frictions are binding. Then $\frac{\partial \lambda}{\partial \epsilon} > 0$ where $\epsilon$ is a proportional change of $z^M$ and $z^I$. 
Comparative Statics (2/2)

- Impact of higher $E_t$ and thereby higher $K_t$

Corollary

Suppose that financial frictions matter. Then, an increase in bank equity $E_t$ (and a corresponding increase of $K_t$) raises $K^I_t$.

- Impact of higher $\Omega_t$ and thereby higher $K_t$

Corollary

Suppose that financial frictions matter. Then, an increase in household wealth $\Omega_t$ (and a corresponding increase of $K_t$) raises $K^M_t$. 
Intuition why Bank Leverage is Pro-cyclical

- In Adrian and Shin (2008) and Adrian and Boyarchenko (2013), banks are confronted with VaR constraints: the higher the risk the lower the leverage. Then leverage is pro-cyclical because risk is anti-cyclical.
- In our model, leverage is given by the "skin in the game" constraint for bankers:

\[
\lambda = \frac{1 + ar^M}{\theta - a(r^l - r^M)} \quad \text{increases in TFP } a.
\]
Pro-cyclicality of Bank Lending

Figure: Total growth of US banks’ assets.
Source: Adrian and Boyarchenko (2013). NBER recessions in grey.
Comparative Statics

- Second row (financial crisis): both bank loans and bond issuance decreases.
- Third row (banking crisis without capital injections): bank leverage increases, bank credit decreases, bond issuance increases.
Lemma

The necessary conditions for the solution of the investor’s problem imply

\[
C^H_t = (1 - \beta_H)(1 + r^M_t - \delta)\Omega_t,
\]

\[
\Omega_{t+1} = \beta_H(1 + r^M_t - \delta)\Omega_t.
\]

• We make the following assumption:

Assumption

Bankers are more impatient than investors, i.e. \( \beta_B < \beta_H \) or \( \rho_B > \rho_H \).
Existence of Steady States

• There is no steady state when financial frictions are irrelevant.
• Otherwise, if \( \hat{E} > 0 \), the laws of motion would imply
  \[
  \frac{\hat{E}}{\hat{\Omega}} = \frac{\beta_B}{\beta_H} \frac{\hat{E}}{\hat{\Omega}} < \frac{\hat{E}}{\hat{\Omega}}.
  \]

• Note that the case \( \hat{E} = 0 \) will be excluded, based on the analysis of the transitional dynamics.
• Therefore, we obtain

**Proposition**

Suppose \( \rho_B > \rho_H \). Then, the system has a unique and globally stable state \((\hat{E}, \hat{\Omega})\) described by equations (11) to (16). Financial frictions always bind in the long run.
Phase Diagram (1/4)

- Suppose first that financial frictions matter.
- The laws of motion reads

\[
E_{t+1} = \beta B E_t [\theta \lambda(E_t, \Omega_t) - \delta], \\
\Omega_{t+1} = \beta H \Omega_t [1 - \delta + \alpha z^M (E_t + \Omega_t - \lambda(E_t, \Omega_t) E_t)^{\alpha^{-1}}].
\]

- We define \( \Omega^1(E) \) and \( \Omega^2(E) \) such that

\[
E_{t+1} = E_t \iff \Omega^1(E_t) = \Omega_t, \\
\Omega_{t+1} = \Omega_t \iff \Omega^2(E_t) = \Omega_t.
\]

- We obtain

Lemma

\[
\Omega^2(E_t) > \Omega^1(E_t) \iff E_t < \hat{E}_t
\]
We also obtain

**Corollary**

*When financial frictions are binding,*

(i) \( \Omega^1(E_t) < \Omega_t \Leftrightarrow E_t < E_{t+1} \),

(ii) \( \Omega_t < \Omega^2(E_t) \Leftrightarrow \Omega_{t+1} > \Omega_t \).

- We define \( E^i \) for \( i = 1, 2 \) implicitly by

\[
E^i = \bar{E}(E^i + \Omega^i(E^i)).
\]

- By continuity,
  - at \((E^1, \Omega^1(E^1))\), \( r^M = \hat{r}^M = \delta + \rho_B \),
  - at \((E^2, \Omega^2(E^2))\), \( r^M = \delta + \rho_H \).

- With obvious notations, we obtain \( K^1 < \hat{K} < K^2 \).
• When financial frictions do not matter,

\[ r_t^M = r_t^I = \alpha z^M \left( \frac{K_t}{1 + z} \right)^{\alpha - 1} = \alpha z^I \left( \frac{zK_t}{1 + z} \right)^{\alpha - 1}, \]  

(8)

\[ E_{t+1} = \beta_B \left[ 1 + \alpha z^M \left( \frac{K_t}{1 + z} \right)^{\alpha - 1} - \delta \right] E_t, \]  

(9)

\[ \Omega_{t+1} = \beta_H \left[ 1 + \alpha z^M \left( \frac{K_t}{1 + z} \right)^{\alpha - 1} - \delta \right] \Omega_t. \]  

(10)

• We can easily derive the following:

**Corollary**

*When financial frictions do not matter,*

(i) \( K_t < K^1 \iff E_{t+1} > E_t, \)

(ii) \( K_t < K^2 \iff \Omega_{t+1} > \Omega_t. \)

• From these considerations, we can draw the phase diagram and derive convergence towards the steady state.
Existence of Steady States

\[
\hat{r}^M = \delta + \rho_H \\
\hat{r}^I = \hat{r}^M + \frac{\theta (\rho_B - \rho_H)}{1 + \delta + \rho_B} \\
\hat{K}^M = \left( \frac{\alpha z^M}{\hat{r}^M} \right)^{\frac{1}{1-\alpha}} \\
\hat{K}^I = \left( \frac{\alpha z^I}{\hat{r}^I} \right)^{\frac{1}{1-\alpha}} \\
\hat{E} = \left( \frac{\alpha z^I}{\hat{r}^I} \right)^{\frac{1}{1-\alpha}} \frac{\theta}{1 + \delta + \rho_B} \\
\hat{\Omega} = \hat{K} - \hat{E}
\]

Remark: Frictionless case: \(\hat{r}^M = \hat{r}^I = \delta + \rho_H\)
Phase Diagram
Impact of Financial Frictions

- They reduce the steady state capital stock in the intermediated sector (but not in the market sector).
- Spread between loan rates and bonds rate persists in the limit, due to the combination of financial frictions and the bankers’ impatience.
- Frictions reduce the speed of convergence towards steady state.
Permanent Shocks to Financial Frictions (1)

- A negative shock to financial frictions ($\theta \rightarrow \theta'$ with $\theta < \theta'$) may result from
  - worsening moral hazard in banking,
  - lowered trust in bankers.

Corollary

*An increase of the intensity of financial frictions, i.e. an increase of $\theta$,*

(i) lowers the steady state value $\hat{K}$,
(ii) increases bank equity $\hat{E}$ if bankers are not too impatient.
Moreover,

**Proposition**

Suppose that $\rho_B$ is sufficiently close to $\rho_H$ and that the economy is hit by a negative permanent shock to financial frictions ($\theta \rightarrow \theta'$). Then, the bankers’ intertemporal utility after the shock is higher than in the steady state associated with $\theta$. 
Exogenous technological progress leaves structure of economy (e.g. share of banking) unchanged.
Temporary Shocks to Financial Frictions

- Shock: $\theta \to \theta'$ where $\theta < \theta'$
  - Lowers output but boosts bank equity accumulation
  - Shock ends: $\theta' \to \theta$
  - Higher levels of bank equity may allow temporary higher investment in sector I, thereby boosting output.

Hypothesis

A temporary shock $\theta \to \theta'$ ($\theta < \theta'$) may cause a bust/boom cycle, i.e. aggregate output first declines, then turns into a boom before it returns to the steady state.

Remark: The same may occur when an negative shock to household wealth occurs (in particular when labor markets are not segmented).
Temporary Shocks to Productivity

**Hitting both sectors:** \( \epsilon = \frac{\Delta z^M}{z^M} = \frac{\Delta z^I}{z^I} < 0 \) (only at \( t = 0 \))

- The borrowing constraint on bankers is tightened;
- leverage decreases (cf. Corollary 1);
- at period 1, bank equity will decline;
- as a consequence of lower bank equity and leverage, more capital will be employed in sector \( M \), meaning that \( r^M \) will decline;
- at period 1, households’ wealth will decline;
- then, recovery occurs with capital starting its build-up.

**Hitting sector \( M \) only:** \( \Delta z^M < 0 \) (only at \( t = 0 \))

- Leverage increases (cf. Corollary 1);
- therefore, \( K^I_0 > \hat{K}^I \) and \( r^I_0 < \hat{r}^I \);
- lower returns in sector \( I \) implies lower returns in sector \( M \): \( r^M_0 < \hat{r}^M \);
- therefore, \( E_1 > \hat{E} \) and \( \Omega_1 < \hat{\Omega} \);
- shock hurts households, but benefits bankers;
- recovery is qualitatively different than for aggregate productivity shock.