THE YIELD CURVE AND THE STOCK MARKET: MIND THE LONG RUN

Gonçalo Faria 1 Fabio Verona 2

1 Católica Porto Business School and CEGE
2 Bank of Finland (Research Unit) and cef.up

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The views expressed are ours and do not necessarily reflect the views of the Bank of Finland
Summary of the paper

What we do

We extract cycles from the slope of the yield curve (the term spread) and study their role for predicting the equity premium using linear models.

Key findings

When properly extracted, the trend of the term spread is a strong and robust out-of-sample equity premium predictor, both from a statistical and an economic point of view. It outperforms several variables recently proposed as good equity premium predictors.

Our results support recent findings in the asset pricing literature that it is the low-frequency components of macroeconomic variables — rather than their business cycle or higher frequencies components — that shape the dynamics of equity markets.
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Our results support recent findings in the asset pricing literature that it is the low-frequency components of macroeconomic variables – rather than their business cycle or higher frequencies components – that shape the dynamics of equity markets.
Forecasting the equity premium (EP)

\[
\log (1+S&P500) - \log (1+R_{\text{free}})
\]
Forecasting the equity premium (EP)
Literature review – before Goyal and Welch (RFS 2008)

Source: Henkel, Spencer Martin and Nardari (JFE 2011, figure 2)
Forecasting the equity premium (EP)

Literature review – before Goyal and Welch (RFS 2008)

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Abstract

“Our article comprehensively reexamines the performance of variables that have been suggested by the academic literature to be good predictors of the equity premium. We find that by and large, these models have predicted poorly both in-sample (IS) and out-of-sample (OOS) for 30 years now; these models seem unstable, as diagnosed by their out-of-sample predictions and other statistics; and these models would not have helped an investor ... to profitably time the market.”

Conclusion

“... our article suggests only that the profession has yet to find some variable that has meaningful and robust empirical equity premium forecasting power, both IS and OOS.”
Develop and test new predictors

- Bollerslev, Tauchen and Zhou (RFS 2009): variance risk premium
- Cooper and Priestley (RFS 2009 / RoF 2013): output gap / world business cycle
- Kelly and Pruitt (JF 2013): a single factor extracted from the cross-section of book-to-market ratios
- Rapach, Strauss and Zhou (JF 2013): lagged US market returns as a predictor for stock returns of other countries
- Neely, Rapach, Tu and Zhou (MS 2014): technical indicators
- Li, Ng and Swaminathan (JFE 2013): aggregate implied cost of capital
- Huang, Jiang, Tu and Zhou (RFS 2015): investor sentiment
- Moller and Rangvid (JFE 2015): growth rate of macro variables
- Rapach, Ringgenberg and Zhou (JFE 2016): short interest aggregate position indicator

- Our paper: three frequency components of the term spread extracted using wavelet filtering methods
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- Our paper: three frequency components of the term spread extracted using wavelet filtering methods
**Forecasting the equity premium (EP)**

**Literature review – after Goyal and Welch (RFS 2008)**

- **Improve the forecasting strategy**
  - Ludvigson and Ng (JFE 2007): factor analysis approach
  - Rapach, Strauss and Zhou (RFS 2010): combination forecast
  - Ferreira and Santa-Clara (JFE 2011): sum-of-the-part method
  - Dangl and Halling (JFE 2012): regressions with time-varying coefficients
  - Pettenuzzo, Timmermann and Valkanov (JFE 2014): impose economic constraints on the forecast
  - Bollerslev, Todorov and Xu (JFE 2015): separate the predictor (the variance risk premium) into a jump and a diffusion component
  - Faria and Verona (JEF 2018): sum-of-the-part method in the time-frequency domain
  - Bandi *et al.* (JoEconometrics, forthcoming) and Faria and Verona (BoF wp 2018): time-frequency forecast of the EP
The term spread as EP predictor

- The term spread (TMS): difference between the long-term government bond yield and the 3-months T-bill
- Within the large set of equity premium (EP) predictors considered in the literature, the TMS has received a lot of attention
  - The term structure “predicts” the EP in the US (Campbell JFE 1987)
  - Closely linked with the business cycle
    - The slope of the yield curve is a reliable predictor of future real economic activity (several papers from the NY FED)
    - The TMS tracks changes in the EP in response to business cycles (Fama and French JFE 1989)
  - Continuously monitored by market participants and policymakers
  - Easy to compute from publicly available data
Data

- Monthly data, U.S., January 1973 to December 2017

- Sources
  - TMS: New York FED website
  - EP: log return on the S&P500 index (including dividends) minus the log return on a one-month Treasury bill
    - CRSP for the S&P500 index
    - St. Louis FED for the one-month Treasury bill
**Data**

We start the analysis in 1973 because ...

1. ... the beginning of the sample coincides with the collapse of the Bretton Woods system, which led to a different way of conducting monetary policy.

2. ... of the issue of model instability, which typically becomes more apparent in longer samples and can make finding a good forecasting model more difficult. In fact, unless structural breaks (like different monetary policy regimes) are properly modelled, past data can be of limited use in constructing useful forecasting models to be used at the end of the sample.
Forecasting the equity premium (EP)

The predictors

- The time series of the TMS: $TMS_{TS}$
- The high-frequency component of the TMS: $TMS_{HF}$
  - Captures fluctuations less than 16 months
- The business-cycle frequency component of the TMS: $TMS_{BCF}$
  - Captures fluctuations between 16 and 128 months
- The low-frequency component of the TMS: $TMS_{LF}$
  - Captures fluctuations more than 128 months

- The frequency components of the TMS extracted with the
  - one-sided Hodrick and Prescott filter
  - Christiano and Fitzgerald (IER 2003) band-pass filter


**Forecasting the equity premium (EP)**

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Wavelet decomposition of the TMS

$TMS_{TS}$
Wavelet decomposition of the TMS

$TMS_{TS}$ & $TMS_{HF}$
Wavelet decomposition of the TMS

\[ TMS_{TS} \& TMS_{HF} \& TMS_{BCF} \]
Wavelet decomposition of the TMS

$TMS_{TS}$ & $TMS_{HF}$ & $TMS_{BCF}$ & $TMS_{LF}$
Wavelet decomposition of a time series
Maximal Overlap Discrete Wavelet Transform MultiResolution Analysis

- Gallegati et al. (OBES 2011), “The US wage Phillips curve across frequencies and over time”
- Gallegati and Ramsey (JEF 2013), “Bond vs stock market’s Q: testing for stability across frequencies and over time”
- Barunik and Vacha (QF 2015), “Realized wavelet-based estimation of integrated variance and jumps in the presence of noise”
- Faria and Verona (JEF 2018), “Forecasting stock market returns by summing the frequency-decomposed parts”
Wavelet decomposition of a time series

It is a sequence of high-pass and low-pass filters

\[ TMS_t \]
Wavelet decomposition of a time series

It is a sequence of high-pass and low-pass filters

\[ TMS_t = TMS_{D1}^{t} + TMS_{S1}^{t} \]

- \( TMS_{D1}^{t} \) for \( 2m \sim 4m \)
- \( TMS_{S1}^{t} \) for \( >4m \)
Wavelet decomposition of a time series

It is a sequence of high-pass and low-pass filters

\[ TMS_t = TMS_{D_1}^{2m \sim 4m} + TMS_{S_1}^{>4m} \]
**Wavelet Decomposition of a Time Series**

It is a sequence of high-pass and low-pass filters

\[
TMS_t = TMS_{t,D1}^{D1} + TMS_{t,S1}^{S1} \\
\underline{2m \sim 4m} + \underline{>4m}
\]

\[\downarrow\]

\[
TMS_{t,D2}^{D2} + TMS_{t,S2}^{S2} \\
\underline{4m \sim 8m} + \underline{>8m}
\]
Wavelet decomposition of a time series

It is a sequence of high-pass and low-pass filters

\[ TMS_t = \underbrace{TMS_t^{D1}}_{2m \sim 4m} + \underbrace{TMS_t^{S1}}_{>4m} \]

\[ \downarrow \]

\[ \underbrace{TMS_t^{D2}}_{4m \sim 8m} + \underbrace{TMS_t^{S2}}_{>8m} \]
Wavelet decomposition of a time series

It is a sequence of high-pass and low-pass filters

\[
TMS_t = \underbrace{TMS_{D1}^t}_{2m \sim 4m} + \underbrace{TMS_{S1}^t}_{>4m} \\
\downarrow \\
\underbrace{TMS_{D2}^t}_{4m \sim 8m} + \underbrace{TMS_{S2}^t}_{>8m}
\]
Wavelet decomposition of a time series

It is a sequence of high-pass and low-pass filters

\[ TMS_t = TMS^{D_1}_t + TMS^{S_1}_t \]

\[ \underline{2m \sim 4m} \]

\[ \downarrow \]

\[ TMS^{D_2}_t + TMS^{S_2}_t \]

\[ \underline{4m \sim 8m} \]

\[ \downarrow \]

\[ TMS^{D_3}_t + TMS^{S_3}_t \]

\[ \underline{8m \sim 16m} \]

\[ \underline{>16m} \]
Wavelet decomposition of a time series
It is a sequence of high-pass and low-pass filters

\[ TMS_t = TMS_t^{D_1} + TMS_t^{S_1} \]

\[ TMS_t^{D_2} + TMS_t^{S_2} \]

\[ TMS_t^{D_3} + TMS_t^{S_3} \]
Wavelet decomposition of a time series

It is a sequence of high-pass and low-pass filters

\[ TMS_t = TMS_{t1}^{D_1} + TMS_{t1}^{S_1} \]

\[ \downarrow \]

\[ TMS_{t2}^{D_2} + TMS_{t2}^{S_2} \]

\[ \downarrow \]

\[ TMS_{t3}^{D_3} + TMS_{t3}^{S_3} \]

\[ TMS_t = TMS_{t1}^{D_1} + TMS_{t2}^{D_2} + TMS_{t3}^{D_3} + TMS_{t3}^{S_3} \]
Using the MODWT MRA decomposition (monthly data), we first extract (Haar wavelet filter, reflecting boundary conditions) 7 time series components from the time series of the TMS:

- $D_1$: 2 ~ 4 months
- $D_2$: 4 ~ 8 months
- $D_3$: 8 ~ 16 months
- $D_4$: 16 ~ 32 months
- $D_5$: 32 ~ 64 months
- $D_6$: 64 ~ 128 months
- $S_6$: >128 months

We recompute the time series components at each iteration of the OOS forecasting process to make sure that we only use current and past information when making the forecast.
Wavelet decomposition of the TMS

Step 1: MODWT MRA decomposition of the TMS

\[
TMS_t = TMS_{t}^{D_1} + TMS_{t}^{D_2} + TMS_{t}^{D_3} + \cdots
\]

\[
\underbrace{TMS_{t}^{D_1}}_{2m \sim 4m} \underbrace{TMS_{t}^{D_2}}_{4m \sim 8m} \underbrace{TMS_{t}^{D_3}}_{8m \sim 16m} + \cdots
\]

\[
\underbrace{TMS_{t}^{D_4}}_{16m \sim 32m} \underbrace{TMS_{t}^{D_5}}_{32m \sim 64m} \underbrace{TMS_{t}^{D_6}}_{64m \sim 128m} + \underbrace{TMS_{t}^{S_6}}_{>128m}
\]
We then aggregate the time series components of the TMS and get:

- the high-frequency component (fluctuations less than 16 months)

\[ TMS_{HF,t} = \sum_{i=1}^{3} TMS_{Di}^t \]

- the business-cycle frequency component (fluctuations between 16 and 128 months)

\[ TMS_{BCF,t} = \sum_{i=4}^{6} TMS_{Di}^t \]

- the low-frequency component (fluctuations more than 128 months)

\[ TMS_{LF,t} = TMS_{S6}^t \]
Wavelet decomposition of the TMS

$TMS_{TS} \& TMS_{HF} \& TMS_{BCF} \& TMS_{LF}$
In-sample (IS) analysis

- For each predictor variable $x_t$, the predictive regression is
  \[ r_{t:t+h} = \alpha + \beta x_t + \epsilon_{t:t+h} \quad \forall t = 1, ..., T - h , \]
  where $r_{t:t+h} = \left( \frac{1}{h} \right) (r_{t+1} + \cdots + r_{t+h})$

- Five forecasting horizons
  - one-month ($h = 1$) ahead
  - one-quarter ($h = 3$) ahead
  - one-semester ($h = 6$) ahead
  - one-year ($h = 12$) ahead
  - two-years ($h = 24$) ahead
The $h$-step ahead OOS forecasts of stock market returns are generated using a sequence of expanding windows.

We use an initial sample (1973M01 to 1989M12) to make the first $h$-step ahead OOS forecast.

The sample is then increased by one observation and a new $h$-step ahead OOS forecast is produced.

The full OOS period thus spans from January 1990 to December 2017.
**Out-of-sample (OOS) forecasts**

- OLS estimation of $r_{t:t+h} = \alpha + \beta x_t + \varepsilon_{t:t+h}$, using data until month $t$

- $h$-step ahead OOS forecast: $E_t r_{t+h} = \hat{r}_{t:t+h} = \hat{\alpha} + \hat{\beta} x_t$

- $h$-step ahead forecast error: $e_{t:t+h} = r_{t:t+h} - \hat{r}_{t:t+h}$

- Squared forecast error: $e_{t:t+h}^2 = (r_{t:t+h} - \hat{r}_{t:t+h})^2$

- Mean squared forecast error (MSFE): $\frac{1}{T-t_0} \sum_{t=t_0}^{T-h} e_{t:t+h}^2$
Out-of-sample (OOS) forecasts

- The forecast performance is evaluated using the Campbell and Thompson (RFS 2008) OOS $R^2(R^2_{OS})$

- The $R^2_{OS}$ measures the proportional reduction in the MSFE for the predictor relative to the historical mean (HM):

$$R^2_{OS} = 100 \left[ 1 - \frac{\sum_{t=t_0}^{T-h} (r_{t:t+h} - \hat{r}_{t:t+h})^2}{\sum_{t=t_0}^{T-h} (r_{t:t+h} - \bar{r}_t)^2} \right]$$

- A positive (negative) $R^2_{OS}$ indicates that the predictor outperforms (underperforms) the HM
### $R^2_{OS}$: 1990-2017

<table>
<thead>
<tr>
<th>Predictor</th>
<th>$h=1$</th>
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<td>$TMS_{TS}$</td>
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*, ** and *** denote statistical significance at the 10%, 5% and 1% level, respectively, computed using the Clark and West (2007) statistic.
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<tr>
<td>( TMS_{HP-CY} )</td>
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<td>3.89***</td>
<td>8.10***</td>
<td>15.5***</td>
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$R^2_{OS}$: ALTERNATIVE PREDICTORS (1990-2014)

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<td>EBP</td>
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<td>Yield gap</td>
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<td>TI - MA(2,12)</td>
<td>1.20*</td>
<td>0.76</td>
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<td>6.52***</td>
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INTERPRETATION OF THE RESULTS

1. Economic source of predictability
2. Asset allocation
3. Dynamics and correlations
4. Model – to be done (?)
Interpretation of the results (#1)
Economic source of predictability

- The predictive power of any predictor of stock returns may result from either the discount rate channel or the cash flow channel, or both.

- Following Cochrane (RFS 2008 and JF 2011), we use:
  - the dividend-price ratio (DP) as the proxy for the discount rate channel
  - the aggregate dividend growth (DG) as the proxy for the cash flow channel

- The classical Campbell and Schiller (RFS 1988) log linearization of stock return leads to:

\[
R_{t+1} = \kappa + DG_{t+1} - \rho DP_{t+1} + DP_t ,
\]

where \( R_{t+1} \) is the one-month ahead stock market return, and \( \kappa \) and \( \rho \) are positive log-linearization constants.
Interpretation of the results (#1)

Economic source of predictability

Consequently, if $TMS_{LF}$ has predictive power of the next period market return, then it must predict either $DP_{t+1}$ or $DG_{t+1}$, or both.

We estimate two bivariate predictive regressions:

$$Y_{t+1} = \rho + \delta TMS_{LF,t} + \psi DP_t + \vartheta_{t+1}, \; Y = DP, DG,$$

<table>
<thead>
<tr>
<th>$Y_{t+1}$</th>
<th>$\delta$</th>
<th>$\psi$</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$DP$</td>
<td>-0.68**</td>
<td>0.99***</td>
<td>98.9</td>
</tr>
<tr>
<td>$DG$</td>
<td>-0.04</td>
<td>0.05</td>
<td>0.41</td>
</tr>
</tbody>
</table>

** and *** denote statistical significance at the 5% and 1% level, respectively, accordingly to wild bootstrapped $p$-values.

The EP predictability power of the $TMS_{LF}$ comes exclusively from the discount rate channel (consistent with Fama and French JFE 1989, and Cochrane RFS 2008 and JF 2011).
Interpretation of the results (#1)

Economic source of predictability

\[ Y_{t+1} = \rho + \delta TMS_{XX,t} + \psi DP_t + \nu_{t+1}, \quad Y = DP, DG \]

<table>
<thead>
<tr>
<th>( X_t )</th>
<th>( Y_{t+1} )</th>
<th>( \delta )</th>
<th>( \psi )</th>
<th>( R^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( TMS_{TS} )</td>
<td>( DP )</td>
<td>-0.24*</td>
<td>0.99***</td>
<td>98.9</td>
</tr>
<tr>
<td>( DG )</td>
<td>-0.08***</td>
<td>0.04</td>
<td>3.8</td>
<td></td>
</tr>
<tr>
<td>( TMS_{HF} )</td>
<td>( DP )</td>
<td>-0.30</td>
<td>0.99***</td>
<td>98.9</td>
</tr>
<tr>
<td>( DG )</td>
<td>-0.02</td>
<td>0.07</td>
<td>0.30</td>
<td></td>
</tr>
<tr>
<td>( TMS_{BCF} )</td>
<td>( DP )</td>
<td>-0.24</td>
<td>0.99***</td>
<td>98.9</td>
</tr>
<tr>
<td>( DG )</td>
<td>-0.15***</td>
<td>0.09</td>
<td>6.1</td>
<td></td>
</tr>
<tr>
<td>( TMS_{LF} )</td>
<td>( DP )</td>
<td>-0.68**</td>
<td>0.99***</td>
<td>98.9</td>
</tr>
<tr>
<td>( DG )</td>
<td>-0.04</td>
<td>0.05</td>
<td>0.41</td>
<td></td>
</tr>
</tbody>
</table>

*, ** and *** denote statistical significance at the 10%, 5% and 1% level, respectively, accordingly to wild bootstrapped \( p \)-values.
We consider a mean-variance investor who dynamically allocates her wealth between equities and risk-free bills.

The rebalancing frequency of the portfolio is assumed to be equal to the forecasting horizon $h$ (non-overlapping return forecasts).

The asset allocation decision is made at the end of month $t$, and the optimal share allocated to equities during period $t+h$ is given by

$$w_t = \frac{1}{\gamma} \frac{\hat{R}_{t+h}}{\hat{\sigma}^2_{t+h}}$$

where

- $\gamma = 3$ is the investor’s risk aversion coefficient
- $\hat{R}_{t+h}$ is the predicted excess stock return at time $t$ for period $t+h$
- $\hat{\sigma}^2_{t+h}$ is the forecast of the variance of the excess return
INTERPRETATION OF THE RESULTS (♯2)

ASSET ALLOCATION: EQUITY WEIGHT (BLUE LINE)

High (low) TMS (black line) predicts high (low) returns, because it predicts low (high) discount rates. This implies an increased (decreased) appetite for risk-taking, triggering an increased (decreased) future equity exposure.
High (low) $TMS_{LF}$ predicts high (low) returns, because it predicts low (high) discount rates. This implies an increased (decreased) appetite for risk-taking, triggering an increased (decreased) future equity exposure.
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Asset allocation: equity weight (blue line) and $TMS_{LF}$ (black line)

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Interpretation of the results (#3)

Dynamics and correlations

EP frequency components and forecast based on the $TMS_{LF}$
To wrap up

‘... our article suggests only that the profession has yet to find some variable that has meaningful and robust empirical equity premium forecasting power, both IS and OOS.’

Goyal and Welch (RFS 2008, page 1505)

In this paper we show that the low-frequency component of the term spread – extracted using wavelet methods – can be such a variable

- Good in-sample fit
- Remarkable OOS forecasting performance: the outperformance exists for one-month horizon, increases with the forecasting horizon and is consistently stable throughout an OOS period comprising 28 years of monthly data
- It also forecasts well in expansions and outperforms several variables that have recently been proposed as good EP predictors
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Frequency decomposition
Fourier
Frequency decomposition
MODWT
The Baxter and King (REStat 99) band-pass filter is a combination of a moving average in the time domain with a Fourier decomposition in the frequency domain optimized by minimizing the distance between the Fourier transform and an ideal filter

- It applies a kind of optimal Fourier filtering on a sliding window (in the time domain), keeping the size of the window constant
- Similar to the so-called short-time Fourier transform (also known as Gabor or windowed Fourier transform)

The MODWT automatically adjusts the size of the window according to the frequency
Wavelet / BP / HP Filter

A. Wavelet and band-pass filters

B. Wavelet and Hodrick-Prescott filters
### Summary statistics

<table>
<thead>
<tr>
<th>Variable</th>
<th>Mean</th>
<th>Median</th>
<th>1(^{st}) perc.</th>
<th>99(^{th}) perc.</th>
<th>St.dev.</th>
<th>AR(1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>EP (%)</td>
<td>0.43</td>
<td>0.85</td>
<td>-11.7</td>
<td>10.5</td>
<td>4.40</td>
<td>0.05</td>
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<tr>
<td>TMS(_{TS}) (%)</td>
<td>1.58</td>
<td>1.75</td>
<td>-2.38</td>
<td>3.67</td>
<td>1.35</td>
<td>0.95</td>
</tr>
<tr>
<td>TMS(_{HF}) (%)</td>
<td>0.00</td>
<td>-0.01</td>
<td>-1.03</td>
<td>1.67</td>
<td>0.42</td>
<td>0.60</td>
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<tr>
<td>TMS(_{BCF}) (%)</td>
<td>0.00</td>
<td>0.09</td>
<td>-2.00</td>
<td>1.53</td>
<td>0.95</td>
<td>0.99</td>
</tr>
<tr>
<td>TMS(_{LF}) (%)</td>
<td>1.58</td>
<td>1.76</td>
<td>0.56</td>
<td>2.37</td>
<td>0.52</td>
<td>1.00</td>
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</table>
### Correlations

<table>
<thead>
<tr>
<th>Variable</th>
<th>$TMS_{TS}$</th>
<th>$TMS_{HF}$</th>
<th>$TMS_{BCF}$</th>
<th>$TMS_{LF}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$TMS_{TS}$</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>$TMS_{HF}$</td>
<td>0.47</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$TMS_{BCF}$</td>
<td>0.89</td>
<td>0.22</td>
<td>1</td>
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</tr>
<tr>
<td>$TMS_{LF}$</td>
<td>0.61</td>
<td>0.02</td>
<td>0.31</td>
<td>1</td>
</tr>
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</table>
## IS predictive regression results

<table>
<thead>
<tr>
<th>Predictor</th>
<th>$h=1$</th>
<th>$h=3$</th>
<th>$h=12$</th>
<th>$h=24$</th>
</tr>
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<tbody>
<tr>
<td></td>
<td>$\hat{\beta}$</td>
<td>$R^2$</td>
<td>$\hat{\beta}$</td>
<td>$R^2$</td>
</tr>
<tr>
<td>$TMS_{TS}$</td>
<td>0.33</td>
<td>0.56</td>
<td>0.30</td>
<td>1.35</td>
</tr>
<tr>
<td></td>
<td>[1.65]*</td>
<td></td>
<td>[1.72]*</td>
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</tr>
<tr>
<td>$TMS_{BCF}$</td>
<td>0.22</td>
<td>0.24</td>
<td>0.24</td>
<td>0.85</td>
</tr>
<tr>
<td></td>
<td>[1.16]</td>
<td></td>
<td>[1.42]</td>
<td></td>
</tr>
<tr>
<td>$TMS_{LF}$</td>
<td>0.32</td>
<td>0.53</td>
<td>0.32</td>
<td>1.51</td>
</tr>
<tr>
<td></td>
<td>[1.64]**</td>
<td></td>
<td>[1.87]*</td>
<td></td>
</tr>
<tr>
<td>$TMS_{BP-LF}$</td>
<td>0.29</td>
<td>0.43</td>
<td>0.29</td>
<td>1.22</td>
</tr>
<tr>
<td></td>
<td>[1.39]*</td>
<td></td>
<td>[1.55]*</td>
<td></td>
</tr>
<tr>
<td>$TMS_{HP-TR}$</td>
<td>0.52</td>
<td>1.35</td>
<td>0.52</td>
<td>3.93</td>
</tr>
<tr>
<td></td>
<td>[2.52]***</td>
<td></td>
<td>[2.81]**</td>
<td></td>
</tr>
</tbody>
</table>

*, ** and *** denote statistical significance at the 10%, 5% and 1% level, respectively, computed using the Clark and West (2007) statistic.
$R^2_{OS}$: 1990-2006 AND 2007-2017

<table>
<thead>
<tr>
<th>Sample period</th>
<th>Predictor</th>
<th>$R^2_{OS}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$h=1$</td>
</tr>
<tr>
<td>1990-2006</td>
<td>$TMS_{TS}$</td>
<td>-1.12</td>
</tr>
<tr>
<td></td>
<td>$TMS_{LF}$</td>
<td>1.66***</td>
</tr>
<tr>
<td>2007-2017</td>
<td>$TMS_{TS}$</td>
<td>-0.16</td>
</tr>
<tr>
<td></td>
<td>$TMS_{LF}$</td>
<td>2.67***</td>
</tr>
<tr>
<td>1990-2006</td>
<td>$TMS_{BP-LF}$</td>
<td>0.05</td>
</tr>
<tr>
<td></td>
<td>$TMS_{HP-TR}$</td>
<td>0.35</td>
</tr>
<tr>
<td>2007-2017</td>
<td>$TMS_{BP-LF}$</td>
<td>-0.10</td>
</tr>
<tr>
<td></td>
<td>$TMS_{HP-TR}$</td>
<td>2.48**</td>
</tr>
</tbody>
</table>

*, ** and *** denote statistical significance at the 10%, 5% and 1% level, respectively, computed using the Clark and West (2007) statistic.
### $R^2_{OS}$: Bad, Normal and Good Growth Periods

<table>
<thead>
<tr>
<th>Predictor</th>
<th>Bad growth $R^2_{OS}$ (h=1)</th>
<th>Normal growth $R^2_{OS}$ (h=1)</th>
<th>Good growth $R^2_{OS}$ (h=1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$TMS_{TS}$</td>
<td>0.57</td>
<td>-2.61</td>
<td>-0.82</td>
</tr>
<tr>
<td>$TMS_{LF}$</td>
<td>2.87***</td>
<td>2.17**</td>
<td>1.16**</td>
</tr>
<tr>
<td>$TMS_{BP-LF}$</td>
<td>-0.10</td>
<td>-0.44</td>
<td>0.37</td>
</tr>
<tr>
<td>$TMS_{HP-TR}$</td>
<td>2.51***</td>
<td>0.12</td>
<td>0.62</td>
</tr>
</tbody>
</table>

** and *** denote statistical significance at the 5\% and 1\% level, respectively, computed using the Clark and West (2007) statistic.
Each line reports the cumulative difference in squared forecast errors of the HM forecast relative to the forecast based on different predictors.
As in Rapach, Ringgenberg and Zhou (JFE 2016), we

- use a ten-year moving window of past excess returns to estimate the variance forecast
- impose portfolio constraints by restricting the weights $w_t$ to lie between -0.5 and 1.5

The portfolio return at time $t+h$, $RP_{t+h}$, is then given by

$$RP_{t+h} = w_t R_{t+h} + RF_{s+h}$$
ECONOMIC ANALYSIS
CERTAINTY EQUIVALENT RETURN (CER)

- An investor who allocates using the previous equity allocation rule \((w_t)\) realizes an average utility or CER of

\[
CER = \overline{RP} - 0.5\gamma\sigma_{RP}^2
\]

where \(\overline{RP}\) and \(\sigma_{RP}^2\) are the mean and variance of the portfolio return.

- The CER is the risk-free rate of return that an investor would be willing to accept instead of holding the risky portfolio.

- We report the (annualized) CER gain (difference between the CER using the predictive model to forecast stock returns and the CER using the HM forecasting strategy), which can be interpreted as the (annual) management fee that an investor would be willing to pay in order to be exposed to a trading strategy based on the alternative forecasting model instead of being based on the HM.
## CER Gains: 1990-2017

<table>
<thead>
<tr>
<th>Predictor</th>
<th>CER gains</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$h=1$</td>
</tr>
<tr>
<td>$TMS_{TS}$</td>
<td>0.10</td>
</tr>
<tr>
<td>$TMS_{HF}$</td>
<td>-1.16</td>
</tr>
<tr>
<td>$TMS_{BCF}$</td>
<td>-2.25</td>
</tr>
<tr>
<td>$TMS_{LF}$</td>
<td>5.91</td>
</tr>
<tr>
<td>$TMS_{BP-HF}$</td>
<td>1.28</td>
</tr>
<tr>
<td>$TMS_{BP-BCF}$</td>
<td>-1.29</td>
</tr>
<tr>
<td>$TMS_{BP-LF}$</td>
<td>0.90</td>
</tr>
<tr>
<td>$TMS_{HP-CY}$</td>
<td>0.38</td>
</tr>
<tr>
<td>$TMS_{HP-TR}$</td>
<td>3.83</td>
</tr>
</tbody>
</table>
ECONOMIC ANALYSIS

EQUITY WEIGHTS ($w_t$) OVER THE OOS PERIOD ($h = 1$)
Economic analysis

Log cumulative wealth over the OOS period ($h = 1$)
Interest rates

$TMS_{TS} \ & \ LTY_{TS} \ & \ STY_{TS}$
INTEREST RATES

$TMS_{LF}$ & $LTY_{LF}$ & $STY_{LF}$