The Impact of Equity Tail Risk on Bond Risk Premia: Evidence of Flight-to-Safety in the U.S. Term Structure

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Flight to Safety (FTS): market stress event with (↑) expected returns for stocks and (↓) expected returns for Treasuries.

We study FTS in bond pricing by examining the effects of equity tail risk on the U.S. yield curve dynamics. To do so, we rely on:

- **Equity Left Tail Factor** for the downside tail risk of the stock market

- **Gaussian ATSM**\(^1\) in which bond yields are driven both by factors of bond-market origin and by the **equity left tail factor**

We pick a risk measure able to predict equity returns and we examine its role in a term structure model for U.S. interest rates.

We find that equity tail risk is priced within the ATSM and short-term Treasuries are more strongly affected by FTS than are long-term ones.

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\(^1\) Affine Term Structure Model
Introduction: Empirical Application

U.S. Treasury Bond Term Premia

Vertical gray bars indicate elevated equity tail risk (≥ 85%-ile)
Equity Tail Risk

For the U.S., U.K. and Euro-zone stock market index, we estimate the 3-Factor Double Exponential Model by Andersen et al. (2015):

\[
\frac{dX_t}{X_t} = (r_t - \delta_t)dt + \sqrt{V_{1,t}} \, dW_{1,t}^Q + \sqrt{V_{2,t}} \, dW_{2,t}^Q + \int_{\mathbb{R}^2} (e^x - 1)\tilde{\mu}_Q^Q(dt, dx, dy)
\]

\[
dV_{1,t} = \kappa_1(\bar{V}_1 - V_{1,t})dt + \sigma_1 \sqrt{V_{1,t}} \, dB_{1,t}^Q + \mu_1 \int_{\mathbb{R}^2} x^2 1_{\{x<0\}} \mu(dt, dx, dy)
\]

\[
dV_{2,t} = \kappa_2(\bar{V}_2 - V_{2,t})dt + \sigma_2 \sqrt{V_{2,t}} \, dB_{2,t}^Q
\]

\[
dU_t = -\kappa_u U_t dt + \mu_u \int_{\mathbb{R}^2} [(1 - \rho_u)x^2 1_{\{x<0\}} + \rho_u y^2] \mu(dt, dx, dy)
\]

For each stock market index, we obtain the “pure tail” factor \(\tilde{U}\) as the residual of the regression of \(U\) on the spot variance \(V = V_1 + V_2\).

We define the **Equity Left Tail Factor** as the market capitalization weighted average of the \(\tilde{U}\) factor of the three stock market indices:

\[
\tilde{U}_t^{\text{Equity}} = \sum_{i=1}^{3} w_i^t \tilde{U}_t^i
\]
Term Structure Modeling (I)

- We let the U.S. Term Structure be driven by the following factors:
  \[ X_t = \begin{bmatrix} \tilde{U}_t^{\text{Equity}}, PC1_t, PC2_t, PC3_t, PC4_t, PC5_t \end{bmatrix}' \]

- The price of the zero-coupon Treasury bond with maturity \( n \) is:
  \[ P^{(n)}_t = E_t \left[ M_{t+1} P^{(n-1)}_{t+1} \right] \]

- The pricing kernel, \( M_{t+1} \), is exponentially affine in the factors:
  \[ M_{t+1} = \exp \left( -r_t - \frac{1}{2} \lambda_t' \lambda_t - \lambda_t' \Sigma^{-1/2} v_{t+1} \right) \]

- The market prices of risk, \( \lambda_t \), are affine in the factors:
  \[ \lambda_t = \Sigma^{-1/2} (\lambda_0 + \lambda_1 X_t) \]
The data generating process for log excess returns is:

\[
rx_{t+1}^{(n-1)} = \beta^{(n-1)'} (\lambda_0 + \lambda_1 X_t) - \frac{1}{2} (\beta^{(n-1)'} \Sigma \beta^{(n-1)} + \sigma^2) + \beta^{(n-1)'} v_{t+1} + e^{(n-1)}_{t+1}
\]

Zero-coupon bond yields, risk-neutral yields and bond term premia are calculated as follows:

\[
y_t^{(n)} = -\frac{1}{n} \left[ a_n + b_n' X_t \right] + u_t^{(n)}
\]

\[
y_t^{(n) \text{ RN}} = \frac{1}{n} \sum_{i=0}^{n-1} E_t[r_{t+i}] = -\frac{1}{n} [a_{n \text{ RN}} + b_{n \text{ RN}'} X_t]
\]

\[
TP_t^{(n)} = y_t^{(n)} - y_t^{(n) \text{ RN}}
\]
Empirical Application: Pricing Factors of U.S. Treasuries

\( \tilde{U}^{\text{Equity}} \): market capitalization weighted average of the “pure tail” factor of U.S., U.K. and Euro-zone stock market indices.

\( PC_1 - PC_5 \): first 5 principal components extracted from Treasury yields of maturities \( n = 3, 6, \ldots, 120m \), orthogonal to \( \tilde{U}^{\text{Equity}} \).
Empirical Application: Factor Exposures and Prices of Risk

- **Test for unspanned factors**: the Wald statistic, under $H_0 : \beta_i = 0_{N \times 1}$, is defined as:
  \[ W_{\beta_i} = \hat{\beta}_i' \hat{\Sigma}_{\beta_i}^{-1} \hat{\beta}_i \overset{\alpha}{\sim} \chi^2(N) \]

- **Test for priced risk factors**: the Wald statistic, under $H_0 : \lambda'_i = 0_{1 \times (K+1)}$, is defined as:
  \[ W_{\lambda_i} = \hat{\lambda}'_i \hat{\Sigma}_{\lambda_i}^{-1} \hat{\lambda}_i \overset{\alpha}{\sim} \chi^2(K + 1) \]

- **Test for time-varying market prices of risk**: the Wald statistic, under $H_0 : \lambda'_{1i} = 0_{1 \times (K)}$, is defined as:
  \[ W_{\lambda_{1i}} = \hat{\lambda}'_{1i} \hat{\Sigma}_{\lambda_{1i}}^{-1} \hat{\lambda}_{1i} \overset{\alpha}{\sim} \chi^2(K) \]

<table>
<thead>
<tr>
<th>$p$-value</th>
<th>$\tilde{U}^{Equity}$</th>
<th>PC1</th>
<th>PC2</th>
<th>PC3</th>
<th>PC4</th>
<th>PC5</th>
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<td>0.021</td>
<td>0.002</td>
<td>0.349</td>
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Empirical Application: Yield Loadings

Yield loadings on $\tilde{U}^{\text{Equity}}$ are ($-$) across all maturities: Equity Tail Risk ($\uparrow$) $\Rightarrow$ Bond Prices ($\uparrow$) $\Rightarrow$ FTS
Expected return loadings on $\tilde{U}_{Equity}$ are (−) across all maturities: Equity Tail Risk ($\uparrow$) ⇒ Bond Risk Premia ($\downarrow$) ⇒ FTS
Empirical Application: Equity Tail Risk and Term Premia

Impact of Equity Tail Risk on U.S. Bond Term Premia

- Financial crisis (Dec-08)
- Eurozone debt crisis (Dec-11)
- Fed's Taper Tantrum (Dec-13)
- Trump wins elections (Nov-16)
Concluding Remarks

- We study Flight to Safety in the context of bond pricing with equity tail risk driving the U.S. Treasury yield curve.

- Equity left tail factor is extracted from options on international stock market indices and is used as a pricing factor in a Gaussian ATSM.

- Application to our dataset of U.S. zero-coupon yields and S&P 500, FTSE 100 and EURO STOXX 50 equity-index options shows:
  - Equity tail risk is significantly priced within the term structure model.
  - Consistent with the theory of FTS, bond prices increase and future excess returns shrink in response to a shock to the equity left tail factor.
  - The equity left tail factor has significant explanatory power for future returns on Treasuries with maturities up to four years.
  - The short end of the U.S. yield curve has strongly been affected by equity tail risk since the outburst of the recent financial crisis.