The Time Variation in Risk Appetite and Uncertainty

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Abstract

We develop new measures of time-varying risk aversion and economic uncertainty that can be calculated from observable financial information at high frequencies. To do so, we formulate a dynamic no-arbitrage asset pricing model for the two main risky asset classes, equities and corporate bonds. The joint dynamics among asset-specific cash flows, macroeconomic fundamentals and risk aversion feature time-varying heteroskedasticity and non-Gaussianity. We use returns but also realized volatility and option prices to help distinguish time variation in economic uncertainty (the amount of risk) from time variation in risk aversion (the price of risk). We find that variance risk premiums on equity are very informative about risk aversion, whereas credit spreads and corporate bond volatility are highly correlated with economic uncertainty. Our model-implied risk premiums outperform standard instrument sets for predicting excess returns on equity and corporate bonds. A financial proxy to our economic uncertainty predicts output growth negatively and significantly, even in the presence of the VIX.

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I Introduction

Changes in risk appetite are increasingly viewed as an important determinant of asset price dynamics. The behavioral finance literature (see, e.g., Lemmon and Portnaiguina, 2006; and Baker and Wurgler, 2006) has developed "sentiment indices" to discuss financial market anomalies whereas in the "structural" dynamic asset pricing literature, habit models featuring time-varying risk aversion have become prominent (see Campbell and Cochrane (1999) and a large number of related articles). Reduced-form asset pricing models, aiming to simultaneously explaining stock return dynamics and option prices, have also concluded that time-varying prices of risk are important drivers (see, e.g., Bakshi and Wu, 2010; Broadie, Chernov, and Johannes, 2007; Pan, 2002). Risk aversion has also featured prominently in recent monetary economics papers that suggest a potential link between loose monetary policy and the risk appetite of market participants, spurring a literature on what structural economic factors drive risk aversion changes (see, e.g., Rajan, 2006; Adrian and Shin, 2009; Bekaert, Hoerova, and Lo Duca, 2013). A growing literature has suggested that aggregate risk aversion may be linked to financial constraints, time-varying leverage or risk preferences of financial intermediaries (see, e.g., He and Krishnamurthy, 2013; Adrian and Shin, 2013). In international finance, Miranda-Agrippino and Rey (2015) and Rey (2015) suggest that global risk aversion is a key transmission mechanism for US monetary policy to be exported to countries worldwide and is a major source of asset return comovements across countries (see also Xu, 2017). Finally, several papers on sovereign bonds (e.g. Bernoth and Erdogan, 2012) have stressed the importance of global risk aversion in explaining their dynamics and contagion across countries.

Given the real-world importance of understanding asset pricing dynamics, financial institutions have developed a wide variety of "sentiment" or "risk aversion" indices (see Coudert and Gex, 2008, for a survey), with (sometimes tenuous) links to the asset pricing literature. Our goal is to develop a measure of aggregate time-varying risk aversion that is both consistent with the structural asset pricing literature and relatively easy to estimate and compute, so that it can be compared to existing indices and tracked over time. To do so, we build on a dynamic asset pricing model related to the habit models of Campbell and Cochrane (1999), Menzly, Santos, and Veronesi (2004) and Wachter (2006). Analogous to Bekaert, Engstrom, and Xing (2009) and Bekaert, Engstrom, and Grenadier (2010), we allow for a stochastic risk aversion component that is not perfectly correlated with fundamentals. Essentially, our risk aversion measure constitutes a second factor in the pricing kernel that is not driven exclusively by macroeconomic fundamentals.

Our approach is agnostic regarding the economic sources of the non-fundamental component of risk aversion. It may truly reflect sentiment induced by news (see the recent experimental evidence in Cohn, Engelmann, Fehr, and Maréchal, 2015), or even reflect mood swings induced by the weather (Kamstra, Kramer, and Levi, 2003). It could also reflect those unmodeled institutional factors (such as endogenous risk constraints faced by financial institutions) that end up affecting the aggregate risk aversion implied by the aggregate pricing kernel.

In implementing our approach, we confront several challenges. First, we must separately identify both the price of risk (risk aversion) and the amount of risk (economic uncertainty). After all, a large class of successful models (see e.g. Bansal, Kiku, Shaliastovich, and Yaron, 2014) relies on time variation in economic uncertainty as the main mechanism to generate variation in financial risk premiums. Moreover, uncertainty shocks play an increasingly prominent role in dynamic macro-models (see Bloom, 2009; Christiano, Motto, and Rostagno, 2014), perhaps contributing to a recent cottage industry of creating indices of economic uncertainty (see e.g. Baker, Bloom, and Davis, 2016; Jurado, Ludvigson, and Ng, 2015). Accounting for uncertainty in fundamentals empirically is challenging because the empirical macro literature suggests that macro shocks feature non-Gaussian distributions with time-varying second and higher order moments (Hamilton, 1990; Fagiolo, Napoletano and Roventini, 2008; Gambetti, Pappa and Canova, 2008; Adrian, Boyarchenko, and Giannone, 2018). To accommodate these non-linearities in a tractable fashion, we use the Bad Environment-Good Environment (BEGE, henceforth) framework developed in Bekaert and Engstrom (2017). Shocks are modeled as the sum of two variables with de-meaned gamma distributions, whose shape parameters vary through time. The model delivers conditional non-Gaussian shocks, with changes in "good" or "bad" volatility also changing the conditional distribution of the process. By realistically modeling economic uncertainty, our model delivers an economic uncertainty index, essentially the conditional variance of industrial production, as a by-product.

Second, to develop the risk aversion measure in an internally consistent manner, we must solve for asset prices as a function of preferences and cash flow dynamics. Focusing on the two main risky asset classes, corporate bonds and equities, we also use the BEGE-class of models to capture their cash flow dynamics (dividends or earnings for equities; default rates for corporate bonds).[1] Despite the fact that the model accommodates non-Gaussianities, our formulation admits (quasi) closed-form solutions for asset prices within the affine class. Summing up, the model has three channels for generating time-variation in risk premiums and volatility for asset returns: (1) macroeconomic

[1] In contrast, a number of articles develop time-varying risk aversion measures motivated by models that really assume "constant" prices of risk and hence are inherently inconsistent (see, for example, Bollerslev, Gibson, and Zhou, 2011), or fail to fully model the link between fundamentals and asset prices (see e.g. Bekaert and Hoerova, 2016).

fundamental uncertainty, (2) stochastic risk aversion and (3) cash flow uncertainty.

Third, to identify the model parameters and stochastic risk aversion, it is paramount to go beyond information in returns and historical risk premiums—which are known to be noisy. We use both realized variances and option-implied variances to help estimate the model and thus identify the risk aversion process. A large empirical literature (see e.g. Andersen, Bollerslev, Diebold and Labys, 2003) shows that realized variances can be measured fairly precisely and provide accurate forecasts of future return variances. Moreover, conditional return variances are an exact function of the relevant state variables (including risk aversion) in our pricing framework (see Joslin, Le, Singleton (2013) for a similar observation in a term structure model). Therefore, realized variances greatly facilitate identifying the risk aversion process. There is also a large literature on inferring risk and risk preferences from option prices, which we discuss in more detail in Section II. Option-implied volatility, such as the famous VIX index in the equity market, reflects both the physical return distribution, including the probability of crashes, and risk aversion. The risk aversion of rational agents creates a demand for insurance against potential losses, making (out-of-money) put options relatively more expensive than call options. Such expensive put options are the source of the consistent presence of a positive variance risk premium (often empirically measured as the difference between the VIX index-squared and the physical conditional return variance) (see Bekaert and Hoerova, 2016; Bakshi and Madan (2006) for formal arguments). Option data should also be informative about conditional risk premiums, which are difficult to observe from the data. Martin (2017) uses option-implied variances to provide bounds on equity premiums, and several articles (see Bollerslev and Todorov, 2011; Liu, Pan, and Wang, 2004; Santa-Clara and Yan, 2010) suggest that compensation for rare events ("jumps") accounts for a large fraction of equity risk premiums. We therefore use both realized and option -implied variances to estimate the model and to infer the risk aversion process.

The remainder of the paper is organized as follows. Sections II and III present the model and estimation strategy in detail. Section IV briefly outlines the data we use. Section V presents the estimation results and extracts risk aversion and uncertainty from asset prices. It also discusses the links between the risk aversion estimates and various financial variables. We find significant time variation in the volatilities and higher-order moments of the fundamentals, especially in real activity. The time variation in uncertainty is dominated by strongly countercyclical "bad" volatility. Moreover, we find that macroeconomic uncertainty is highly correlated with uncertainty about risky asset cash flows, both for the equity and corporate bond markets. Nonetheless, we do find evidence of independent time variation in the volatility of corporate bond loss rates.

The extracted risk aversion process loads most significantly on equity risk neutral variances (with a positive sign) and realized variances (with a negative sign), supporting the variance premium as a good proxy for aggregate risk aversion. This finding is consistent with recent work in the consumption-based asset pricing literature, showing the variance premium to be very informative in identifying equilibrium models featuring complex data generating processes for the fundamentals (see Drechsler and Yaron, 2010; Bollerslev, Tauchen, and Zhou, 2009; Bekaert and Engstrom, 2017). The risk aversion process is much more rapidly mean reverting than would be implied by habit models, which is consistent with the results in Martin (2017). Economic uncertainty is highly correlated with corporate bond volatility and, especially, with credit spreads.

In Section VI, we show that our model-implied risk premiums significantly predict equity returns, whereas our economic uncertainty index predicts output negatively and significantly. We then also link our measures of risk appetite and uncertainty to alternative indices, including ones produced by practitioners. Finally, we illustrate the use of the indices at high frequencies by examining their behavior around the Bear Stearns and Lehman Brothers bankruptcies. Concluding remarks are in Section VII.

II Modeling Risk Appetite and Uncertainty

In this section, we first define our concept of risk aversion in general terms in Section II.A. We then build a dynamic model with stochastic risk aversion and macroeconomic factors affecting the cash flows processes of two main risky asset classes, corporate bonds and equity. The state variables are described in Section II.B and the pricing kernel in Section II.C.

II.A General Strategy

An ideal measure of risk aversion would be model-free and does not confound time variation in economic uncertainty with time variation in risk aversion. There are many attempts in the literature to approximate this ideal, but invariably various modeling and statistical assumptions are necessary to tie down risk aversion. For example, in the options literature, a number of articles (Aıt-Sahalia and Lo, 2000; Engle and Rosenberg, 2002; Jackwerth, 2000; Bakshi, Kapadia and Madan, 2003; Britten-Jones and Neuberger, 2000; Bliss and Panigirtzoglou, 2004) appear at first glance to infer risk aversion from equity options prices in a model-free fashion, but it is generally the case that the utility function is assumed to be of a particular form and/or to depend only on stock prices.[2]

Another strand of the literature relies on general properties of pricing kernels. A

[2]This is also true for the recent debate about the recoverability of physical probabilities from option prices, which claims to identify risk aversion as well (Ross, 2015; Carr and Wu, 2016).

strictly positive pricing kernel or stochastic discount factor, M_{t+1}, under no-arbitrage conditions, implies that for all gross returns, R,

$$E_t\left[M_{t+1}R_{t+1}\right] = 1 \tag{1}$$

It is then straightforward to derive that any asset's expected excess return can be written as an asset specific risk exposure ("beta", or β_t) times a price of risk (or λ_t), which applies to all assets (see also Coudert and Gex, 2008):

$$E_t\left[R_{t+1}\right] - R_t^f = \beta_t\lambda_t \tag{2}$$

where R_t^f is the risk free rate, $\beta_t = -\frac{Colp_t(R_{t+1},M_{t+1})}{Var_t(M_{t+1})}$, and $\lambda_t = \frac{Var_t(M_{t+1})}{E_t(M_{t+1})}$.

Unfortunately, this price of risk is not equal to time-varying risk aversion, and in particular may confound economic uncertainty with risk aversion. In a simple power utility framework, it is easy to show that the price of risk is linked to both the coefficient of relative risk aversion and the volatility of consumption growth, the latter being a reasonable measure of economic uncertainty. Importantly, we use the terms "risk aversion" and "risk appetite" as each other's inverse. Gai and Vause (2006) and Pflueger, Siriwardane, and Sunderam (2018) however use the term "risk appetite" to indicate the price of risk, that is the product of "risk aversion" and "the amount of risk."

Our approach is to start from a fairly general utility function defined over both consumption ("fundamentals") and a "non-fundamental" factor. Our measure of risk aversion is then the coefficient of relative risk aversion implied by the utility function. We specify a fairly general consumption process accommodating time variation in economic uncertainty and use the utility framework to price assets, given general processes for their cash flows. Therefore, while certainly not model free, our risk aversion process is consistent with a wide set of economic models that respect no-arbitrage conditions. Moreover, we can use any risky asset for which we can model cash flows to help identify risk aversion. The identification of the risk aversion process takes into account that economic uncertainty varies through time and controls for non-Gaussianities in cash flow processes.

Consider a period utility function in the HARA class:

$$U\left(\frac{C}{Q}\right) = \frac{\left(\frac{C}{Q}\right)^{1-\gamma}}{1-\gamma} \tag{3}$$

where C is consumption and Q is a process that will be shown to drive time-variation in risk aversion. Essentially, when Q is high, consumption delivers less utility and marginal utility increases. For the general HARA class of utility functions,

$$Q = \left(\frac{a}{\gamma} - \frac{b}{C}\right)^{-1} = f(C) \tag{4}$$

where a and γ are positive parameters, and b is an exogenous benchmark parameter or process. Note that γ (the curvature parameter) is not equal to risk aversion in this framework. In principle, all parameters (a, γ, b) could have time subscripts, but we only allow time-variation in b. Note that the Q process depends on consumption, but we do not allow b to depend on consumption. This excludes internal habit models, for example.

The coefficient of relative risk aversion for this class of models is given by

$$RRA = -\frac{CU''(C)}{U'(C)} = aQ \tag{5}$$

and is thus proportional to Q. Note that $\frac{dQ}{dC} = -b\left(\frac{a}{\gamma}C - b\right)^{-2} < 0$; in good times when consumption increases, risk aversion decreases.

For pricing assets, we need to derive the log pricing kernel which is the intertemporal marginal rate of substitution in a dynamic economy. We assume an infinitely lived agent, facing a constant discount factor of β, and the HARA period utility function given above. The pricing kernel is then given by

$$m_{t+1} = \ln(\beta) + \ln\left[\frac{U'(C_{t+1})}{U'(C_t)}\right] = \ln(\beta) - \gamma\Delta c_{t+1} + \gamma\Delta q_{t+1} \tag{6}$$

where we use t to indicate time, lower case letters to indicate logs of uppercase variables, and Δ to represent the difference operator.

To get more intuition for this framework, note that the Campbell and Cochrane (1999) (CC henceforth) utility function is a special case. CC use an external habit model, with utility being a power function over $C_t - H_t$, where H_t is the habit stock. Of course, we can also write

$$C_t - H_t = \frac{C_t}{Q_t} \tag{7}$$

with $Q_t = \frac{C_t}{C_t - H_t}$. So the CC utility function is a special case of our framework with $a = \gamma$ and $b = H$. As C_t gets closer to the habit stock, risk aversion increases. Q_t is thus the inverse of the surplus ratio in the CC article. CC also model q_t exogenously but restrict the correlation between q_t and Δc_t to be perfect. The "moody investor" economy in Bekaert, Engstrom, and Grenadier (2010) is also a special case. In that model, q_t is also exogenously modeled, but has its own shock; that is, there are preference shocks not correlated with fundamentals. Another special case is the model in Brandt and

Wang (2003), but their risk aversion process specifically depends on inflation in addition to consumption growth. In fact, DSGE models in macro-economics routinely feature preference shocks (see e.g. Besley and Coate, 2003). For example, in the famous New Keynesian Smets and Wouters (2007) model, there is a persistent preference shock not directly related to fundamentals, which could reflect pure "animal spirits."

The experimental literature (see e.g. Cohn, Engelmann, Fehr, and Maréchal, 2015) shows that the subjective willingness to take risk is indeed lower during a recession, which is simulated by "priming" people with a stock market crash (versus boom), and that this risk aversion is rooted in emotions of fear. Even if this is the dominant source of preference variation, it is unlikely that the aggregate component of this type of counter-cyclical risk aversion is perfectly correlated with aggregate consumption growth. Another channel that may cause changes in aggregate risk aversion is shifts in the wealth distribution. If we think of risk aversion of the representative agent as the wealth weighted average of risk aversion in the economy, it is conceivable that in bad times, the wealth of the richer people goes down proportionally more than that of poorer people (because more of their wealth is tied up in risky asset classes). This in turn would then increase aggregate risk aversion.

Therefore, our approach specifies a stochastic process for q (risk aversion), which is partly but not fully driven by fundamentals (consumption growth) and features an independent shock.

II.B Economic Environment: State Variables

II.B.1 Macroeconomic Factors

In canonical asset pricing models agents have utility over consumption, but it is well known that consumption growth and asset returns show very little correlation. Moreover, consumption data are only available at the quarterly frequency. Because the use of options data is key to our identification strategy and these data are only available since 1986, it is important to use macro-economic data that are available at the monthly frequency. We therefore chose to use industrial production, which is available at the monthly frequency, as our main macroeconomic factor. In the macro-economic literature, much attention has been devoted recently to the measurement of "real" uncertainty (see e.g. Jurado, Ludvigson and Ng, 2015) and its effects on the real economy (see e.g. Bloom, 2009). We add to this literature by using a novel econometric framework to extract two macro risk factors from industrial production: "good" uncertainty, denoted by p_t, and "bad" uncertainty, denoted by n_t.

Specifically, the change in log industrial production index, θ_t, has time-varying conditional moments governed by two state variables: p_t and n_t. The conditional mean is modeled as a persistent process to accommodate a time-varying long-run mean of output growth:

$$\theta_{t+1} = \overline{\theta} + \rho_\theta(\theta_t - \overline{\theta}) + m_p(p_t - \overline{p}) + m_n(n_t - \overline{n}) + u^\theta_{t+1}, \tag{8}$$

where the growth shock is decomposed into two independent centered gamma shocks,

$$u^\theta_{t+1} \quad = \quad \sigma_{\theta p}\omega_{p,t+1} - \sigma_{\theta n}\omega_{n,t+1}. \tag{9}$$

The shocks follow centered gamma distributions with time-varying shape parameters,

$$\omega_{p,t+1} \sim \widetilde{\Gamma}\left(p_t, 1\right) \tag{10}$$
$$\omega_{n,t+1} \sim \widetilde{\Gamma}\left(n_t, 1\right), \tag{11}$$

where $\widetilde{\Gamma}\left(x, 1\right)$ denotes a centered gamma distribution with shape parameter x and a unit scale parameter. The shape factors, p_t and n_t, follow autoregressive processes,

$$p_{t+1} = \overline{p} + \rho_p(p_t - \overline{p}) + \sigma_{pp}\omega_{p,t+1} \tag{12}$$
$$n_{t+1} = \overline{n} + \rho_n(n_t - \overline{n}) + \sigma_{nn}\omega_{n,t+1}, \tag{13}$$

where ρ_x denotes the autoregressive term of process x_{t+1}, σ_{xx} the sensitivity to shock $\omega_{x,t+1}$, and $\overline{x}$ the long-run mean. We denote the macroeconomic state variables as, $\boldsymbol{Y_t^{mac}} = \begin{bmatrix}\theta_t & p_t & n_t\end{bmatrix}'$, and the set of unknown parameters are $\overline{\theta}, \rho_\theta, m_p, m_n, \overline{n}, \sigma_{\theta p}, \sigma_{\theta n}, \rho_p$, σ_{pp}, ρ_n, and σ_{nn}.

In this model, the conditional mean has an autoregressive component, but macro risks also affect expected growth. This can both accommodate cyclical effects (lower conditional means in bad times), or the uncertainty effect described in Bloom (2009). The shocks reflect the BEGE framework of Bekaert and Engstrom (2017), implying that the conditional higher moments of output growth are linear functions of the bad and good uncertainties. For example, the conditional variance and the conditional unscaled skewness are as follows,

$$\begin{aligned} \text{Conditional Variance:} \quad & E_t\left[\left(u^\theta_{t+1}\right)^2\right] = \sigma^2_{\theta p}p_t + \sigma^2_{\theta n}n_t, \\ \text{Conditional Unscaled Skewness:} \quad & E_t\left[\left(u^\theta_{t+1}\right)^3\right] = 2\sigma^3_{\theta p}p_t - 2\sigma^3_{\theta n}n_t. \end{aligned}$$

This reveals the sense in which p_t represents "good" and n_t "bad" volatility: p_t (n_t) increases (decreases) the skewness of industrial production growth.

The industrial production process is a key determinant of the consumption growth process, but we model consumption growth jointly with the cash flow processes for equities imposing the economic restriction that those processes are cointegrated.

II.B.2 Cash Flows and Cash Flow Uncertainty

To model the cash flows for equities and corporate bonds, we focus attention on two variables that exhibit strong cyclical movements, namely earnings (see e.g. Longstaff and Piazzesi, 2004) and corporate defaults (see e.g. Gilchrist and Zakrajšek, 2012).

Corporate Bond Loss Rate To model corporate bonds, we must model the possibility of defaults. Suppose a portfolio of one-period nominal bonds has a promised payoff of $C \equiv exp(c)$ at $(t+1)$, but will in fact only pay an unknown fraction $F_{t+1} \leq 1$ of that amount. Therefore, the nominal payoffs for a one-period zero-coupon defaultable corporate bond at period $t+1$ is $C \times F_{t+1} = exp(c + ln(F_{t+1})) = exp(c - l_{t+1})$. Thus, l_{t+1} is defined as $-ln(F_{t+1}) = -ln(1 - L_{t+1})$ where L_{t+1} (i.e., $1 - F_{t+1}$) is the aggregate corporate loss rate, which can be computed as the default rate times one minus the recovery rate. We provide more detail on the pricing of defaultable bonds in the asset pricing section (Section II.C).

The dynamic system of the aggregate corporate bond log loss rate, l_t, is modeled as follows:

$$l_{t+1} = l_0 + \rho_{ll} l_t + m_{lp} p_t + m_{ln} n_t + \sigma_{lp}\omega_{p,t+1} + \sigma_{ln}\omega_{n,t+1} + u^l_{t+1} \tag{14}$$

$$u^l_{t+1} = \sigma_{llp}\omega_{lp,t+1} - \sigma_{lln}\omega_{ln,t+1} \tag{15}$$

$$\omega_{lp,t+1} \sim \tilde{\Gamma}(lp_t, 1), \tag{16}$$

$$\omega_{ln,t+1} \sim \tilde{\Gamma}(\overline{ln}, 1), \tag{17}$$

where the variance equation is,

$$lp_{t+1} = \overline{lp} + \rho_{lp}(lp_t - \overline{lp}) + \sigma_{lplp}\omega_{lp,t+1}, \tag{18}$$

$$\overline{ln} > 0. \tag{19}$$

The conditional mean depends on an autoregressive term and the good and bad uncertainty state variables p_t and n_t. The loss rate total disturbance is governed by three independent heteroskedastic shocks: the good and bad environment macro shocks $\{\omega_{p,t+1}, \omega_{n,t+1}\}$ and the (orthogonal) loss rate shock $u_{l,t+1}$. The loss rate shock follows a typical BEGE process, but we only allow ω_{lp}'s shape parameter to be time-varying.[3]

[3]This final model is not chosen at random. We experimented with 5 other models (i.e., with and

This dynamic system allows macro-economic uncertainty to affect both the conditional mean and conditional variance of the loss rate process. However, it also allows the loss rate to have an autonomous autoregressive component in its conditional mean (making l_t a state variable) and accommodates heteroskedasticity not spanned by macro-economic uncertainty. This "financial" cash flow uncertainty has a time-varying component, denoted by lp_t, and a constant component denoted by $\overline{ln}$. If σ_{llp} and σ_{lplp} are positive, as we would expect, the loss rate and its volatility are positively correlated; that is, in bad times with a high incidence of defaults, there is also more uncertainty about the loss rate, and because the gamma distribution is positively skewed, the (unscaled) skewness of the process increases. We would also expect the sensitivities to the good (bad) economic environment shocks, σ_{lp} (σ_{ln}) to be negative (positive): intuitively, defaults should decrease (increase) in relatively good (bad) times.

The conditional variance of the loss rate is $\sigma_{lp}^2 p_t + \sigma_{ln}^2 n_t + \sigma_{llp}^2 lp_t + \sigma_{lln}^2 \overline{ln}_t$, and its conditional unscaled skewness is $2\left(\sigma_{lp}^3 p_t + \sigma_{ln}^3 n_t + \sigma_{llp}^3 lp_t - \sigma_{lln}^3 \overline{ln}_t\right)$. We denote the financial state variables as, $\boldsymbol{Y_t^{fin}} = \begin{bmatrix} l_t & lp_t \end{bmatrix}'$. The set of unknown parameters are l_0, ρ_{ll}, m_{lp}, m_{ln}, σ_{lp}, σ_{ln}, σ_{llp}, σ_{lln}, $\overline{lp}$, ρ_{lp}, σ_{lplp}, and $\overline{ln}$.

Log Earnings Growth Log earnings growth, g_t, is defined as the change in log real earnings of the aggregate stock market. It is modeled as follows:

$$\begin{aligned} g_{t+1} &= g_0 + \rho_{gg} g_t + \boldsymbol{\rho'_{g,mac}} \boldsymbol{Y_t^{mac}} + \boldsymbol{\rho'_{g,fin}} \boldsymbol{Y_t^{fin}} \\ &\quad + \sigma_{gp}\omega_{p,t+1} + \sigma_{gn}\omega_{n,t+1} + \sigma_{glp}\omega_{lp,t+1} + \sigma_{gln}\omega_{ln,t+1} + u_{t+1}^g \qquad (20) \\ u_{t+1}^g &= \sigma_{gg}\omega_{g,t+1} \qquad (21) \\ \omega_{g,t+1} &\sim N(0,1). \qquad (22) \end{aligned}$$

The conditional mean is governed by an autoregressive component and the three macro factors; the time variation in the conditional variance comes from the good and bad uncertainty factors, and the loss rate uncertainty factor. The earnings shock is assumed to be Gaussian and homoskedastic, implying that the time variation in the conditional variance of earnings growth is spanned by macro-economic uncertainty and the financial uncertainty present in default rates. In principle, the earnings growth rate shock can also be modeled using a heteroskedastic gamma shock. However, we fail to reject the null that the residuals series, after controlling for the heteroskedastic fundamental shocks, are Gaussian and homoskedastic. The set of unknown parameters is

without cyclical mean, AR(1), relaxing heteroskedasticity assumptions on the loss rate BEGE shocks, using a single gamma shock structure or BEGE.), and the model chosen outperforms other models on standard model selection criteria. Details on alternative models are available upon request.

$\{g_0, \rho_{gg}, \boldsymbol{\rho'_{g,mac}}, \boldsymbol{\rho'_{g,fin}}, \sigma_{gp}, \sigma_{gn}, \sigma_{glp}, \sigma_{gln}, \sigma_{gg}\}$.

Log Consumption-Earnings Ratio We model consumption as stochastically cointegrated with earnings so that the consumption-earnings ratio becomes a relevant state variable. Define $\kappa_t \equiv \ln\left(\frac{C_t}{E_t}\right)$ which is assumed to follow:

$$\begin{aligned}\kappa_{t+1} &= \kappa_0 + \rho_{\kappa\kappa}\kappa_t + \boldsymbol{\rho'_{\kappa,mac}}\boldsymbol{Y_t^{mac}} + \boldsymbol{\rho'_{\kappa,fin}}\boldsymbol{Y_t^{fin}} \\ &\quad + \sigma_{\kappa p}\omega_{p,t+1} + \sigma_{\kappa n}\omega_{n,t+1} + \sigma_{\kappa lp}\omega_{lp,t+1} + \sigma_{\kappa ln}\omega_{ln,t+1} + u_{t+1}^{\kappa} \qquad (23)\\ u_{t+1}^{\kappa} &= \sigma_{\kappa\kappa}\omega_{\kappa,t+1} \qquad (24)\\ \omega_{\kappa,t+1} &\sim N(0,1). \qquad (25)\end{aligned}$$

Similarly to earnings growth, there is an autonomous conditional mean component but the heteroskedasticity of κ_t is spanned by other state variables. As with log earnings growth, we fail to reject Gaussianity and homoskedasticity of u_{t+1}^{κ}. The set of unknown parameters is $\{\kappa_0, \rho_{\kappa\kappa}, \boldsymbol{\rho'_{\kappa,mac}}, \boldsymbol{\rho'_{\kappa,fin}}, \sigma_{\kappa p}, \sigma_{\kappa n}, \sigma_{\kappa lp}, \sigma_{\kappa ln}, \sigma_{\kappa\kappa}\}$.

Log Dividend Payout Ratio The log dividend payout ratio, η_t, is expressed as the log ratio of dividends to earnings. Recent evidence in Kostakis, Magdalinos, and Stamatogiannis (2015) shows that the monthly dividend payout ratio is stationary. We model η_t analogously to κ_t and g_t, again assuming a Gaussian and homoskedastic residual shock (which can be justified in the data):

$$\begin{aligned}\eta_{t+1} &= \eta_0 + \rho_{\eta\eta}\eta_t + \boldsymbol{\rho'_{\eta,mac}}\boldsymbol{Y_t^{mac}} + \boldsymbol{\rho'_{\eta,fin}}\boldsymbol{Y_t^{fin}} \\ &\quad + \sigma_{\eta p}\omega_{p,t+1} + \sigma_{\eta n}\omega_{n,t+1} + \sigma_{\eta lp}\omega_{lp,t+1} + \sigma_{\eta ln}\omega_{ln,t+1} + u_{t+1}^{\eta} \qquad (26)\\ u_{t+1}^{\eta} &= \sigma_{\eta\eta}\omega_{\eta,t+1} \qquad (27)\\ \omega_{\eta,t+1} &\sim N(0,1). \qquad (28)\end{aligned}$$

The set of unknown parameters is $\{\eta_0, \rho_{\eta\eta}, \boldsymbol{\rho'_{\eta,mac}}, \boldsymbol{\rho'_{\eta,fin}}, \sigma_{\eta p}, \sigma_{\eta n}, \sigma_{\eta lp}, \sigma_{\eta ln}, \sigma_{\eta\eta}\}$. Using η_{t+1} and g_{t+1}, dividend growth Δd_{t+1}, is given by $\eta_{t+1} - \eta_t + g_{t+1}$.

II.B.3 Pricing Kernel State Variables

In the model we introduced above, the real pricing kernel depends on consumption growth and changes in risk aversion. To price nominal cash flows (or to price default free nominal bonds), we also need an inflation process. We discuss the modeling of these variables here.

Consumption Growth By definition, log real consumption growth, $\Delta c_{t+1} = \ln\left(\frac{C_{t+1}}{C_t}\right) = g_{t+1} + \Delta\kappa_{t+1}$. Therefore, consumption growth is spanned by the previously defined state variables and shocks.

Risk Aversion The state variable capturing risk aversion, $q_t \equiv \ln\left(\frac{C_t}{C_t - H_t}\right)$ is, by definition, nonnegative. We impose the following structure,

$$q_{t+1} = q_0 + \rho_{qq}q_t + \rho_{qp}p_t + \rho_{qn}n_t + \sigma_{qp}\omega_{p,t+1} + \sigma_{qn}\omega_{n,t+1} + u^q_{t+1} \tag{29}$$

$$u^q_{t+1} = \sigma_{qq}\omega_{q,t+1} \tag{30}$$

$$\omega_{q,t+1} \sim \widetilde{\Gamma}(q_t, 1). \tag{31}$$

The risk aversion disturbance is comprised of three parts, exposure to the good uncertainty shock, exposure to the bad uncertainty shock, and an orthogonal preference shock. Thus, given the distributional assumptions on these shocks, the model-implied conditional variance is $\sigma^2_{qp}p_t + \sigma^2_{qn}n_t + \sigma^2_{qq}q_t$, and the conditional unscaled skewness $2\left(\sigma^3_{qp}p_t + \sigma^3_{qn}n_t + \sigma^3_{qq}q_t\right)$. With $\sigma_{qq} = 0$ and certain restrictions on σ_{qp} and σ_{qn}, the model would imply a perfect correlation between the conditional correlation between risk aversion and real activity, as in the Campbell-Cochrane (1999) model.

We model the pure preference shock also with a demeaned gamma distributed shock, so that its variance and (unscaled) skewness are proportional to its own level. Controlling for current business conditions, when risk aversion is high, so is its conditional variability and unscaled skewness. This seems like a plausible assumption. For example, option-implied volatilities, which are partially driven by risk aversion, are much more volatile in stress times. The higher moments of risk aversion are perfectly spanned by macroeconomic uncertainty on the one hand and pure sentiment (q_t) on the other hand. Note that our identifying assumption is that q_t itself does not affect the macro variables and $u_{q,t+1}$ represents a pure preference shock. The conditional mean is modeled as before: an autonomous autoregressive component and dependence on p_t and n_t. The set of unknown parameters describing the risk aversion process is $\{q_0, \rho_{qq}, \rho_{qp}, \rho_{qn}, \sigma_{qp}, \sigma_{qn}, \sigma_{qq}\}$.

Inflation To price nominal cash flows, we must specify an inflation process. The conditional mean of inflation depends on an autoregressive term and the three macro factors $\boldsymbol{Y^{mac}_t}$. The conditional variance and higher moments of inflation are proportional to the good and bad uncertainty factors $\{p_t, n_t\}$. The inflation innovation u^π_{t+1} is assumed to be Gaussian and homoskedastic. There is no feedback from inflation to the macro variables:

$$\pi_{t+1} = \pi_0 + \rho_{\pi\pi}\pi_t + \boldsymbol{\rho'_{\pi y}}\boldsymbol{Y^{mac}_t} + \sigma_{\pi p}\omega_{p,t+1} + \sigma_{\pi n}\omega_{n,t+1} + u^\pi_{t+1} \tag{32}$$

$$
\begin{aligned}
u^{\pi}_{t+1} &= \sigma_{\pi\pi}\omega_{\pi,t+1} && (33)\\
\omega_{\pi,t+1} &\sim N(0,1). && (34)
\end{aligned}
$$

The set of unknown parameters is $\{\pi_0, \rho_{\pi\pi}, \boldsymbol{\rho'_{\pi y}}, \sigma_{\pi p}, \sigma_{\pi n}, \sigma_{\pi\pi}\}$.

II.B.4 Matrix Representation

The dynamics of all state variables introduced above can be written compactly in matrix notation. We define the macro factors $\boldsymbol{Y^{mac}_t} = \begin{bmatrix}\theta_t & p_t & n_t\end{bmatrix}'$ and other state variables $\boldsymbol{Y^{other}_t} = \begin{bmatrix}\pi_t & l_t & g_t & \kappa_t & \eta_t & lp_t & q_t\end{bmatrix}'$. Among the ten state variables, the industrial production growth θ_t, the inflation rate π_t, the loss rate l_t, earnings growth g_t, the log consumption-earnings ratio κ_t and the log divided payout ratio η_t are observable, while the other four state variables, $\{p_t, n_t, lp_t, q_t\}$ are latent. There are eight independent centered gamma and Gaussian shocks in this economy. The system can be formally described as follows (technical details are relegated to the Appendix):

$$\boldsymbol{Y_{t+1}} = \boldsymbol{\mu} + \boldsymbol{A}\boldsymbol{Y_t} + \boldsymbol{\Sigma}\boldsymbol{\omega_{t+1}}, \tag{35}$$

where constant matrices, $\boldsymbol{\mu}$ (10×1), $\boldsymbol{A}$ (10×10) and $\boldsymbol{\Sigma}$ (10×9), are implicitly defined, $\boldsymbol{Y_t} = \begin{bmatrix}\boldsymbol{Y^{mac\prime}_t} & \boldsymbol{Y^{other\prime}_t}\end{bmatrix}'$ (10×1) is a vector comprised of the state variable levels, and $\boldsymbol{\omega_{t+1}} = \begin{bmatrix}\omega_{p,t+1} & \omega_{n,t+1} & \omega_{\pi,t+1} & \omega_{lp,t+1} & \omega_{ln,t+1} & \omega_{g,t+1} & \omega_{\kappa,t+1} & \omega_{\eta,t+1} & \omega_{q,t+1}\end{bmatrix}'$ (9×1) is a vector comprised of all the independent shocks in the economy.

Note that, among the nine shocks, five shocks are gamma distributed—the good uncertainty shock ($\omega_{p,t+1}$), the bad uncertainty shock ($\omega_{n,t+1}$), the right-tail loss rate shock ($\omega_{lp,t+1}$), the left-tail loss rate shock ($\omega_{ln,t+1}$), and the risk aversion shock ($\omega_{q,t+1}$). The remaining four shocks are standard homoskedastic Gaussian shocks (i.e., $N(0,1)$). Importantly, given our preference structure, the state variables driving the time variation in the higher order moments of these shocks are the only ones driving the time variation in asset risk premiums and their higher order moments. Economically, we therefore rely on time variation in risk aversion—as in the classic Campbell-Cochrane model and its variants (see e.g. Bekaert, Engstrom, and Grenadier, 2010; Wachter, 2006)—and time variation in economic uncertainty—as in the Bansal-Yaron (2004) model—to explain risk premiums.[4] The model's implications for conditional asset return variances are critical in identifying the dynamics of risk aversion (see also Joslin, Le, and Singleton, 2013).

Our specific structure admits conditional non-Gaussianity yet generates affine pricing solutions. The model is tractable because the moment generating functions of gamma

[4]Previous research by Bekaert, Engstrom, and Xing (2009) and Bekaert and Engstrom (2017) also combines time variation in economic uncertainty with changes in risk aversion.

and Gaussian distributed variables can be derived in closed form, delivering exponentiated affine functions of the state variables. In particular,

$$E_t\left[\exp(\boldsymbol{\nu}'\boldsymbol{Y_{t+1}})\right] = \exp\left[\boldsymbol{\nu}'\boldsymbol{S_0} + \frac{1}{2}\boldsymbol{\nu}'\boldsymbol{S_1}\boldsymbol{\Sigma}^{\boldsymbol{other}}\boldsymbol{S_1'}\boldsymbol{\nu} + \boldsymbol{f_S(\nu)}\boldsymbol{Y_t} + S_2(\boldsymbol{\nu})\overline{ln}\right], \qquad (36)$$

where $\boldsymbol{S_0}$ (10×1) is a vector of drifts; $\boldsymbol{S_1}$ (10×4) is a selection matrix of 0s and 1s which picks out the Jensen's inequality terms of the four Gaussian shocks; $\boldsymbol{\Sigma}^{\boldsymbol{other}}$ (4×4) represents the covariance of the Gaussian shocks. The matrix $\boldsymbol{f_S(\nu)}$ (the scalar $S_2(\boldsymbol{\nu})$) is a non-linear function of $\boldsymbol{\nu}$, involving the feedback matrix and the scale parameters of the gamma-distributed variables, see Appendix A.1 for more details.

II.C Asset Pricing

In this section, we present the model solutions. First, we formally define the real and nominal pricing kernel as a function of the previously defined state variables. Assuming complete markets, this kernel prices any cash flow pattern spanned by our state variable dynamics. Second, asset prices for two risky assets—defaultable corporate bonds and equities—are derived. The solution of the model shows that asset prices are (quasi) affine functions of the state variables, which is crucial in developing the estimation procedure in this paper. In particular, we derive approximate expressions for endogenous returns to use in estimating the model parameters, and deriving a risk appetite index.

II.C.1 The pricing kernel

Taking the ratio of marginal utilities at time $t+1$ and t, we obtain the intertemporal marginal rate of substitution which constitutes the real pricing kernel denoted by M_{t+1}. As Equation (6) indicates, it has the same form as the pricing kernel in the Campbell and Cochrane model, however, the kernel state variables and kernel shocks are quite different. Unlike the CC model, changes in the log surplus consumption ratio (the inverse of risk aversion) are not perfectly correlated with the consumption growth shock, and consumption growth is heteroskedastic. The real pricing kernel in our model follows an affine process as well:

$$m_{t+1} \quad = \quad m_0 + \boldsymbol{m_2'}\boldsymbol{Y_t} + \boldsymbol{m_1'}\boldsymbol{\Sigma}\boldsymbol{\omega_{t+1}}, \qquad (37)$$

where m_0, $\boldsymbol{m_1}$ ($10{\times}1$), $\boldsymbol{m_2}$ ($10{\times}1$) are constant scalar or matrices that are implicitly defined using Equations (18)–(23) and (27)-(29). To price nominal assets, we define the nominal pricing kernel, $\widetilde{m}_{t+1}$, which is a simple transformation of the log real pricing kernel, m_{t+1},

$$\begin{aligned} \widetilde{m}_{t+1} &= m_{t+1} - \pi_{t+1}, && (38) \\ &= \widetilde{m}_0 + \boldsymbol{\widetilde{m}_2'} \boldsymbol{Y_t} + \boldsymbol{\widetilde{m}_1'} \boldsymbol{\Sigma \omega_{t+1}}, && (39) \end{aligned}$$

where $\widetilde{m}_0$, $\boldsymbol{\widetilde{m}_1}$ (10×1) and $\boldsymbol{\widetilde{m}_2}$ (10×1) are implicitly defined. The nominal risk free rate, $\widetilde{rf}_t$, is defined as $-\ln\{E_t[\exp(\widetilde{m}_{t+1})]\}$ which can be expressed as an affine function of the state vector.

II.C.2 Asset prices

In this section, we further discuss the pricing of the two risky assets—corporate bonds and equities. The Appendix contains detailed proofs and derivations.

Defaultable Nominal Bonds Above, we assume that a one period nominal bond faces a fractional (logarithmic) loss of l_t. Given the structure assumed for l_t and Equation (34), the log price-coupon ratio of the one-period defaultable bond portfolio is

$$\begin{aligned} pc_t^1 &= \ln\{E_t[\exp(\widetilde{m}_{t+1} - l_{t+1})]\} && (40) \\ &= b_0^1 + \boldsymbol{b_1^{1\prime}} \boldsymbol{Y_t}, && (41) \end{aligned}$$

where b_0^1 and $\boldsymbol{b_1^{1\prime}}$ are implicitly defined. Consider next a portfolio of multi-period zero-coupon defaultable bonds with a promised terminal payment of C at period $(t+N)$. As for the N-period bond, the actual payment will be less than or equal to the promised payment, and the ex-post nominal payoff can be expressed as $\exp(c - l_{t+N})$. We ignore the possibility of early default or prepayment. Then, the price-coupon ratio of this bond at period $(t+N-1)$, one period before maturity, PC_{t+N-1}^1, is $\exp(b_0^1 + b_1^{1\prime} Y_{t+N-1})$. Given the Euler equation and the law of iterated expectations, it then follows by induction that all earlier dated zero-coupon nominally defaultable corporate bond prices are similarly affine in the state variables, in particular:

$$\begin{aligned} pc_t^N &= \ln\left\{E_t[\widetilde{M}_{t+1} PC_{t+1}^{N-1}]\right\}, \\ &= b_0^N + \boldsymbol{b_1^{N\prime}} \boldsymbol{Y_t}. && (42) \end{aligned}$$

The assumed zero-coupon structure of the payments before maturity implies that the unexpected returns to this portfolio are exactly linearly spanned by the shocks to $\boldsymbol{Y_t}$.

Equities Equity is a claim to the dividend stream; let P_t denote the ex-dividend price of the claim, then, the price-dividend ratio, PD_t, is given by:

$$\begin{aligned} PD_t &= E_t\left[M_{t+1}\frac{(P_{t+1}+D_{t+1})}{D_t}\right] && (43)\\ &= \sum_{n=1}^{\infty}\underbrace{E_t\left[\exp\left(\sum_{j=1}^{n} m_{t+j}+\Delta d_{t+j}\right)\right]}_{\equiv F_t^n}, && (44)\end{aligned}$$

When $n=1$, $F_t^1 = E_t[\exp(m_{t+1}+\Delta d_{t+1})]$ can be expressed as an exact exponential affine function of the state vector. Recursively, the n-th summation term yields the following identity:

$$\begin{aligned} F_t^n &= E_t\left[\exp\left(\sum_{j=1}^{n} m_{t+j}+\Delta d_{t+j}\right)\right] && (45)\\ &= E_t\left[\exp(m_{t+1}+\Delta d_{t+1})F_{t+1}^{n-1}\right]. && (46)\end{aligned}$$

Therefore, by induction, any summation term with $n>1$ can also be expressed as an exponential affine function of the state vector. Therefore, the price-dividend ratio is the sum of an infinite number of exponential affine functions of the state vector, with the coefficients following simple difference equations.

II.C.3 Asset Returns

Given that the log price-coupon ratio of a defaultable corporate bond can be expressed as an exact affine function of the state variables, it immediately implies that the log nominal return (before maturity), $\widetilde{r}_{t+1}^{cb} = pc_{t+1} - pc_t$, can be represented in closed-form. For equities, the log nominal equity return is derived as follows, $\widetilde{r}_{t+1}^{eq} = \ln\left(\frac{PD_{t+1}+1}{PD_t}\frac{D_{t+1}}{D_t}\exp(\pi_{t+1})\right)$. It is therefore a non-linear but known function of the state variables. We approximate this function by a linear function (See the Appendix for details). Note that this procedure is very different from the very popular Campbell-Shiller (1988) model which approximates returns with a linear expression. Because they approximate the return expression and then price future cash flows with approximate expected returns, their procedure accumulates pricing errors. We approximate a known quasi-affine pricing function in deriving a return expression.

To account for the approximation error, we allow for two asset-specific homoskedastic shocks that are orthogonal to the state variable innovations. As a result, log nominal asset returns approximately satisfy the following factor model,

$$\widetilde{r}^i_{t+1} = \widetilde{\xi}^i_0 + \widetilde{\boldsymbol{\xi}}^{\boldsymbol{i}\prime}_{\boldsymbol{1}} \boldsymbol{Y_t} + \widetilde{\boldsymbol{r}}^{\boldsymbol{i}\prime} \boldsymbol{\Sigma} \boldsymbol{\omega_{t+1}} + \varepsilon^i_{t+1}, \tag{47}$$

where $\widetilde{r}^i_{t+1}$ is the log nominal asset return i from t to $t+1$, $\forall i = \{eq, cb\}$; $\widetilde{\boldsymbol{\xi}}^{\boldsymbol{i}}_{\boldsymbol{1}}$ (10×1) is the loading vector on the state vector; $\widetilde{\boldsymbol{r}}^{\boldsymbol{i}}$ (10×1) is the loading vector on the state variable shocks, and ε^i_{t+1} is a homoskedastic error term with unconditional volatility σ_i.

Rather than exploiting the model restrictions on prices, we exploit the restrictions the economy imposes on asset returns, physical variances and risk-neutral variances. Given Equation (47) and the pricing kernel, the model implies that one period expected log excess returns are given by:

$$\begin{aligned} RP^i_t \equiv E_t(\widetilde{r}^i_{t+1}) - \widetilde{rf}_t \quad = \quad & \left\{ \sigma_p(\widetilde{\boldsymbol{r}}^{\boldsymbol{i}}) + \ln\left[\frac{1 - \sigma_p(\widetilde{\boldsymbol{m}}_{\boldsymbol{1}} + \widetilde{\boldsymbol{r}}^{\boldsymbol{i}})}{1 - \sigma_p(\widetilde{\boldsymbol{m}}_{\boldsymbol{1}})}\right] \right\} p_t \\ + \quad & \left\{ \sigma_n(\widetilde{\boldsymbol{r}}^{\boldsymbol{i}}) + \ln\left[\frac{1 - \sigma_n(\widetilde{\boldsymbol{m}}_{\boldsymbol{1}} + \widetilde{\boldsymbol{r}}^{\boldsymbol{i}})}{1 - \sigma_n(\widetilde{\boldsymbol{m}}_{\boldsymbol{1}})}\right] \right\} n_t \\ + \quad & \left\{ \sigma_{lp}(\widetilde{\boldsymbol{r}}^{\boldsymbol{i}}) + \ln\left[\frac{1 - \sigma_{lp}(\widetilde{\boldsymbol{m}}_{\boldsymbol{1}} + \widetilde{\boldsymbol{r}}^{\boldsymbol{i}})}{1 - \sigma_{lp}(\widetilde{\boldsymbol{m}}_{\boldsymbol{1}})}\right] \right\} lp_t \\ + \quad & \left\{ \sigma_{ln}(\widetilde{\boldsymbol{r}}^{\boldsymbol{i}}) + \ln\left[\frac{1 - \sigma_{ln}(\widetilde{\boldsymbol{m}}_{\boldsymbol{1}} + \widetilde{\boldsymbol{r}}^{\boldsymbol{i}})}{1 - \sigma_{ln}(\widetilde{\boldsymbol{m}}_{\boldsymbol{1}})}\right] \right\} \overline{ln} \\ + \quad & \left\{ \sigma_q(\widetilde{\boldsymbol{r}}^{\boldsymbol{i}}) + \ln\left[\frac{1 - \sigma_q(\widetilde{\boldsymbol{m}}_{\boldsymbol{1}} + \widetilde{\boldsymbol{r}}^{\boldsymbol{i}})}{1 - \sigma_q(\widetilde{\boldsymbol{m}}_{\boldsymbol{1}})}\right] \right\} q_t \\ - \quad & \boldsymbol{m}'_{\boldsymbol{1}} \boldsymbol{S_1} \boldsymbol{\Sigma}^{\boldsymbol{other}} \boldsymbol{S}'_{\boldsymbol{1}} \widetilde{\boldsymbol{r}}^{\boldsymbol{i}} - \frac{1}{2}\left[\widetilde{\boldsymbol{r}}^{\boldsymbol{i}\prime} \boldsymbol{S_1} \boldsymbol{\Sigma}^{\boldsymbol{other}} \boldsymbol{S}'_{\boldsymbol{1}} \widetilde{\boldsymbol{r}}^{\boldsymbol{i}} + \sigma^2_i\right]. \end{aligned} \tag{48}$$

Here (as before), $\widetilde{\boldsymbol{m}}_{\boldsymbol{1}}$ and $\widetilde{\boldsymbol{r}}^{\boldsymbol{i}}$ are vectors containing the sensitivities of the log nominal pricing kernel and the log nominal asset returns to the state variable shocks, respectively. The symbols $\sigma_p(\boldsymbol{x})$, $\sigma_n(\boldsymbol{x})$, $\sigma_{lp}(\boldsymbol{x})$, $\sigma_{ln}(\boldsymbol{x})$ and $\sigma_q(\boldsymbol{x})$ represent linear functions of state variables' sensitivities to the good uncertainty shock ($\omega_{p,t+1}$), the bad uncertainty shock ($\omega_{n,t+1}$), the right-tail loss rate shock ($\omega_{lp,t+1}$), the left-tail loss rate shock ($\omega_{ln,t+1}$) and the risk aversion shock ($\omega_{q,t+1}$). For instance, because $\widetilde{\boldsymbol{m}}_{\boldsymbol{1}} = \begin{bmatrix} 0 & 0 & 0 & -1 & 0 & -\gamma & -\gamma & 0 & 0 & \gamma \end{bmatrix}'$ and $\boldsymbol{\Sigma_{\bullet 9}} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \sigma_{qq} \end{bmatrix}'$.[5]

The signs of state variable coefficients are also intuitive. Below we discuss the risk aversion coefficient. $\sigma_q(\widetilde{\boldsymbol{m}}_{\boldsymbol{1}}) = \widetilde{\boldsymbol{m}}_{\boldsymbol{1}}{}' \boldsymbol{\Sigma_{\bullet 9}} = \gamma\sigma_{qq} > 0$, where $\gamma > 0$ follows from the concavity of the utility function and $\sigma_{qq} > 0$ implies positive skewness of risk aversion in Equation (29). It immediately implies that an asset with a negative return sensitivity to the risk aversion shock exhibits a higher risk premium when risk aversion is high. That is,

[5]Matrix $\boldsymbol{\Sigma_{\bullet j}}$ is the j-th column of the shock coefficient matrix in the state variable process, or $\boldsymbol{\Sigma}$ in Equation (35).

for such an asset, $\sigma_q(\widetilde{\boldsymbol{r}}^{\boldsymbol{i}}) < 0$; then, it can be easily shown that $\sigma_q(\widetilde{\boldsymbol{r}}^{\boldsymbol{i}}) + \ln\left[\frac{1-\sigma_q(\widetilde{\boldsymbol{m}}_{\boldsymbol{1}}+\widetilde{\boldsymbol{r}}^{\boldsymbol{i}})}{1-\sigma_q(\widetilde{\boldsymbol{m}}_{\boldsymbol{1}})}\right] \approx \sigma_q(\widetilde{\boldsymbol{r}}^{\boldsymbol{i}}) - \frac{\sigma_q(\widetilde{\boldsymbol{r}}^{\boldsymbol{i}})}{1-\sigma_q(\widetilde{\boldsymbol{m}}_{\boldsymbol{1}})} > 0$. Under realistic parameter values, expected excess returns thus vary through time and are affine in p_t, n_t, lp_t (macroeconomic and cash flow uncertainties) and q_t (aggregate risk aversion).

The physical conditional return variance is obtained given the return loadings of Equation (47):

$$\begin{aligned} VAR_t^i \equiv VAR_t(\widetilde{r}_{t+1}^i) \quad &= \quad \left(\sigma_p(\widetilde{\boldsymbol{r}}^{\boldsymbol{i}})\right)^2 p_t + \left(\sigma_n(\widetilde{\boldsymbol{r}}^{\boldsymbol{i}})\right)^2 n_t + \left(\sigma_{lp}(\widetilde{\boldsymbol{r}}^{\boldsymbol{i}})\right)^2 lp_t + \left(\sigma_q(\widetilde{\boldsymbol{r}}^{\boldsymbol{i}})\right)^2 q_t \\ &+ \quad \underbrace{\left(\sigma_{ln}(\widetilde{\boldsymbol{r}}^{\boldsymbol{i}})\right)^2 \overline{ln} + \widetilde{\boldsymbol{r}}^{\boldsymbol{i}\prime} \boldsymbol{S_1} \boldsymbol{\Sigma}^{\boldsymbol{other}} \boldsymbol{S_1'} \widetilde{\boldsymbol{r}}^{\boldsymbol{i}} + \sigma_i^2}_{constant}, \end{aligned} \tag{49}$$

where $\boldsymbol{S_1}$ is defined in the appendix. The expected variance under the physical measure is time-varying and affine in p_t, n_t, lp_t and q_t.

The one-period risk-neutral conditional return variance is:

$$\begin{aligned} VAR_t^{i,Q} \equiv VAR_t^Q(\widetilde{r}_{t+1}^i) \quad &= \quad \left(\frac{\sigma_p(\widetilde{\boldsymbol{r}}^{\boldsymbol{i}})}{1-\sigma_p(\widetilde{\boldsymbol{m}}_{\boldsymbol{1}})}\right)^2 p_t + \left(\frac{\sigma_n(\widetilde{\boldsymbol{r}}^{\boldsymbol{i}})}{1-\sigma_n(\widetilde{\boldsymbol{m}}_{\boldsymbol{1}})}\right)^2 n_t + \left(\frac{\sigma_{lp}(\widetilde{\boldsymbol{r}}^{\boldsymbol{i}})}{1-\sigma_{lp}(\widetilde{\boldsymbol{m}}_{\boldsymbol{1}})}\right)^2 lp_t \\ &+ \quad \left(\frac{\sigma_q(\widetilde{\boldsymbol{r}}^{\boldsymbol{i}})}{1-\sigma_q(\widetilde{\boldsymbol{m}}_{\boldsymbol{1}})}\right)^2 q_t \\ &+ \quad \left(\frac{\sigma_{ln}(\widetilde{\boldsymbol{r}}^{\boldsymbol{i}})}{1-\sigma_{ln}(\widetilde{\boldsymbol{m}}_{\boldsymbol{1}})}\right)^2 \overline{ln} + \widetilde{\boldsymbol{r}}^{\boldsymbol{i}\prime} \boldsymbol{S_1} \boldsymbol{\Sigma}^{\boldsymbol{other}} \boldsymbol{S_1'} \widetilde{\boldsymbol{r}}^{\boldsymbol{i}} + \sigma_i^2. \end{aligned} \tag{50}$$

Note that the functions in Equation (50) are affine transformations from the ones in Equation (49), using the "$\sigma(m)$" functions. Under normal circumstances, we would expect that the relative importance of "bad" uncertainty, the loss rate's uncertainty and risk aversion increases under the risk neutral measure relative to the importance of "good" uncertainty. In Equation (50), this intuition can be formally established as $\sigma_n(m)$, $\sigma_{lp}(m)$, $\sigma_q(m)$ are positive and $\sigma_p(m)$ is negative. For example, as derived above, $\sigma_q(\widetilde{m}_1) = \gamma\sigma_{qq}$ is strictly positive. Therefore, because σ_{qq} is likely a small number, as long as γ is not very large, the risk neutral variance should load more heavily on q_t than does the physical variance.

III The Identification and Estimation of Risk Aversion and Uncertainty

In what follows, we first describe our general estimation philosophy which is focused on retrieving a risk aversion process that can be traced at high frequencies, and then outline the estimation methodology in detail, which consists of two steps. The first step is the identification of macro-economic and cash flow uncertainties; the second step is the actual estimation of the remainder of the model parameters and the identification of risk aversion.

III.A General Estimation Philosophy

While there are 10 state variables in the model, there are only four latent state variables that drive risk premiums and conditional physical and risk neutral variances in the model as described in Equations (48)–(50). Three of these state variables, good uncertainty, p_t, bad uncertainty, n_t and cash flow uncertainty, lp_t, describe economic uncertainty. We want to ensure that these variables are identified from macro-economic and cash flow information alone and are not contaminated by asset prices. We therefore pre-estimate these variables. This constitutes the first step in the estimation methodology.

Given the dynamics of these variables, there are a variety of ways that we can retrieve risk aversion from the model and data on corporate bonds and equities. However, an important goal of the paper is to make risk aversion observable, even at high frequencies. Under the null of the model, asset prices, risk premiums and variances are an exact function of the state variables, including risk aversion. It thus follows that (market-wide) risk aversion should be spanned by a judiciously chosen set of asset prices and risk variables. Given our desire to generate a high frequency risk aversion index, we select these instruments to be observable at high frequencies and to reflect risk and return information for our two asset classes. In particular, we assume

$$q_t = \boldsymbol{\chi}' \boldsymbol{z_t}, \tag{51}$$

where $\boldsymbol{z_t}$ is a vector of 6 observed asset prices and ones. The instruments include (1) term spread (the difference between the 10-year and 3-month Treasury bond yield), (2) the credit spread (the difference between Moody's Baa yield and the 10-year Treasury bond yield), (3) a "detrended" dividend yield or earnings yield, (4) the realized equity return variance, (5) the risk-neutral equity return variance, and (6) the realized corporate bond return variance.

The term spread may reflect information about the macro-economy (see e.g. Harvey, 1988) and was also included in the risk appetite index of Bekaert and Hoerova (2016). The credit spread and price yields contain direct price information from the corporate bond and equity market respectively and thus reflect partially information about risk premiums. Ideally, we would include information on both risk-neutral and physical variances for both equities and corporate bonds, but we do not have data on the risk neutral corporate bond return variance. We use the realized variance for both markets, rather than an estimate of the physical conditional variance, because realized variances are effectively observed, whereas conditional variances must be estimated. Given a loading vector $\boldsymbol{\chi}$, the risk aversion process can be computed daily from observable data.

So far, the methodology is reminiscent of the FAVAR literature (see Bernanke, Boivin, and Eliasz, 2005) and Stock and Watson (2002), where unobserved macro-factors are identified using large date sets of observable macro-data using a spanning assumption. However, in contrast to the above literature and all "principal component" type analysis, we exploit the restrictions the economy imposes on risk premiums, and physical and risk neutral variances to estimate the loadings of the time-varying risk aversion process. That is, our risk aversion estimate is forced to satisfy the (dynamic) properties of risk aversion implied by the above model: it is an element of the pricing kernel, which must, in turn, correctly price asset returns and be consistent with observed measures of return volatility under both the physical and risk-neutral measures. To do so, we adopt a GMM procedure detailed in Section III.C.[6]

III.B Identifying Economic and Cash Flow Uncertainties

Given that there is no feedback from risk aversion to the three uncertainty state variables, we can pre-estimate the uncertainty factors without using financial asset prices.

First, we use the monthly log real growth rate of industrial production to measure θ_t. In the system for θ_t, described in Equations (8)–(13), there are three state variables, which we collect in $\boldsymbol{Y_t^{mac}}$,

$$\widehat{\boldsymbol{Y^{mac}}}_{\boldsymbol{t}} = \begin{bmatrix} \theta_t & \widehat{p}_t & \widehat{n}_t \end{bmatrix}'.^7$$

We denote the filtered shocks,

$$\widehat{\boldsymbol{\omega^{mac}}}_{\boldsymbol{t}} = \begin{bmatrix} \widehat{\omega}_{p,t} & \widehat{\omega}_{n,t} \end{bmatrix}'.$$

The system is estimated using Bates (2006)'s approximate MLE procedure (see the Appendix for details).

Second, we must determine the latent cash flow uncertainty factor lp_t, which determines the time variation in the conditional variance of the log corporate bond default rate. The dynamics of the variables are described in Equations (14)–(18). We use Bates' approximate MLE to estimate the model parameters. Unlike the BEGE structure for real shocks, for the idiosyncratic loss rate shock, only the right-tail shock is heteroskedastic. We denote the estimated right-tail loss rate shape parameter as $\widehat{lp}_t$, and the loss rate

[6] Imposing the model restrictions and no arbitrage through a positive pricing kernel also differentiates the estimation from the approach taken in Bekaert and Hoerova (2016).

[7] In the remainder of the paper, a hat superscript is used to indicate estimated variables or matrices.

shocks as $\widehat{\omega}_{lp,t+1}$ and $\widehat{\omega}_{ln,t+1}$.

III.C Identifying Risk Aversion

To identify the risk aversion process and the parameters in the spanning condition (Equation (51) above), we exploit the restrictions the model imposes on return risk premiums (equities and corporate bonds), physical variances (equities and corporate bonds) and risk neutral variances (for equities only). The estimation is a GMM system in which we use the same instruments as the ones used to span risk aversion ($\boldsymbol{z_t}$). Apart from the $\boldsymbol{\chi}$ parameters, we must also identify the parameters in the kernel (β, the discount factor, and γ, the curvature parameter), and the scale parameter of the preference shock, σ_{qq}. Note that the level of risk aversion is also driven by the q_t process, so that γ and β are not well identified. We impose $\gamma = 2$ and $\beta = 0.999$. The GMM system thus has 8 unknown parameters,

$$\boldsymbol{\Theta} = \left[\chi_0, \chi_{tsprd}, \chi_{csprd}, \chi_{CF5yr}, \chi_{rvareq}, \chi_{qvareq}, \chi_{rvarcb}, \sigma_{qq}\right],$$

where the notation is obvious, and $CF5yr$ refers to either a detrended dividend or earnings yield ("DY5yr" or "EY5yr"), described later. Before the moment conditions can be evaluated, we must identify the state variables and their shocks, the pricing kernel, and the return shocks. The estimation process is therefore intricate and consists of six steps: for each candidate $\widehat{\boldsymbol{\Theta}} = [\widehat{\boldsymbol{\chi}}', \widehat{\sigma}_{qq}]$ vector,

1. <u>Identify the implied risk aversion series given the loading choices, $\widehat{q}_t = \widehat{\boldsymbol{\chi}}'\boldsymbol{z_t}$.</u> We impose a lower boundary of 10^{-8} on q_t during the estimation. This is consistent with theory, as q_t is motivated from a habit model ($q_t = \ln(Q_t) = \ln\left(\frac{C_t}{C_t - H_t}\right) > 0$). It is also consistent with the distributional assumption for q_t as the positive shape parameter of the ω_q shock.[8]

2. <u>Identify the state variable levels ($\boldsymbol{Y_t}$) and shocks ($\boldsymbol{\Sigma\omega_{t+1}}$).</u>
   The parameters of the state variable processes, $\{\theta_t, p_t, n_t, l_t, lp_t\}$, are pre-determined according to Section III.B. For the remaining cash flow state variables $\{\pi_t, g_t, \kappa_t, \eta_t\}$, we estimate the parameters in each iteration using simple projections. To identify the risk aversion-specific shock in the risk aversion process, we first project $\widehat{q}_{t+1}$ on $\widehat{q}_t$, $\widehat{p}_t$, $\widehat{n}_t$, $\widehat{\omega}_{p,t+1}$ and $\widehat{\omega}_{n,t+1}$ to obtain the residual term $\widehat{u}^q_{t+1}$, and then divide it by $\widehat{\sigma}_{qq}$ to obtain the preference shock $\widehat{\omega}_{q,t+1}$ (see Equations (27)–(29)). We later use the implied residual variance and unscaled skewness calculated using the distributional properties of gamma shocks as two moment conditions in

[8]However, for the best model, the minimum q is 0.32 and the boundary is non-binding.

the GMM system. Now, given the choice of $\widehat{\boldsymbol{\chi}}$, a full set of state variables levels, $\widehat{\boldsymbol{Y}}_{\boldsymbol{t}} = \left[\widehat{\boldsymbol{Y}}^{\boldsymbol{mac\prime}}_{\boldsymbol{t}} \quad \pi_t \quad l_t \quad g_t \quad \kappa_t \quad \eta_t \quad \widehat{lp}_t \quad \widehat{q}_t\right]'$, and the eight independent shocks, $\widehat{\boldsymbol{\omega}}_{\boldsymbol{t+1}}$ including $\widehat{\omega}^q_{t+1}$, can be identified.[9]

3. Identify the nominal pricing kernel.
Consumption growth in this model is (endogenously) implied by two state variables, real log earnings growth and (changes in) the log consumption-earnings ratio. Given consumption growth (i.e., $g_t + \Delta\kappa_t$), the risk aversion process $\widehat{q}_t$, γ and β, the monthly nominal kernel is obtained:

$$\widehat{\widetilde{m}}_{t+1} = \ln(\beta) - \gamma\Delta c_{t+1} + \gamma\left(\widehat{q}_{t+1} - \widehat{q}_t\right) - \pi_{t+1}.$$

Constant matrices related to the log nominal kernel—$\widetilde{m}_0, \widetilde{\boldsymbol{m}}_{\boldsymbol{1}}, \widetilde{\boldsymbol{m}}_{\boldsymbol{2}}$ (as in the affine representation of the kernel; see Equation (39))—are implicitly identified.

4. Estimate the return loadings.
In this step, we obtain the loadings of nominal asset returns on the state variable shocks, controlling for time-varying conditional means. Note that there are 8 state variables $\{\theta_t, p_t, n_t, \pi_t, g_t, \kappa_t, lp_t, q_t\}$ affecting the pricing kernel. The remaining state variables, $\{l_t, \eta_t\}$, correspond to cash flow state variables in the corporate bond and equity markets. We estimate the loadings by simple projections, assuming the asset-specific approximation shock is homoskedastic:

$$\widetilde{r}^i_{t+1} = \xi^i_0 + \boldsymbol{\xi}^{\boldsymbol{i\prime}}_{\boldsymbol{1}}\widehat{\boldsymbol{Y}}_{\boldsymbol{t}} + \widetilde{\boldsymbol{r}}^{\boldsymbol{i\prime}}\widehat{\boldsymbol{\Sigma}}\widehat{\boldsymbol{\omega}}_{\boldsymbol{t+1}} + \varepsilon^i_{t+1}, \tag{52}$$

where $\widetilde{r}^i_{t+1}$ is the log nominal return for asset i, $\widehat{\boldsymbol{\Sigma}}$ and $\widehat{\boldsymbol{\omega}}_{\boldsymbol{t+1}}$ are identified previously, and ε^i_{t+1} has mean 0 and variance σ^2_i. To obtain asset moments, $\widetilde{\boldsymbol{r}}^{\boldsymbol{i\prime}}$ is the crucial shock loading vector, but we also need $\widehat{\sigma}_i$.

5. Obtain the model-implied endogenous moments.
We derive three moments for the asset returns: 1) the expected excess return implied by the model (using the pricing kernel), RP^i; 2) the physical (conditional expected) return variance, VAR^i, which only depends on the return definition in Equation (52) and 3) the risk neutral conditional variance, $VAR^{i,Q}$, which also uses the pricing kernel. The expressions for these variables are derived in Equations (48)–(50) where p_t, n_t, lp_t,q_t, $\widetilde{\boldsymbol{r}}^{\boldsymbol{i}}$, $\boldsymbol{\Sigma}^{\boldsymbol{other}}$ and σ_i have been estimated in previous steps.

[9] The parameters obtained from this substep are $\pi_0, \rho_{\pi\pi}, \rho_{\pi\theta}, \rho_{\pi p}, \rho_{\pi n}, \sigma_{\pi p}, \sigma_{\pi n}, \sigma_{\pi l}, \sigma_{\pi\pi}, l_0, \rho_{ll}, \rho_{lp}, \rho_{ln}, \sigma_{lp}, \sigma_{ln}, \sigma_{ll}, g_0, \rho_{gg}, \rho_{g\theta}, \rho_{gp}, \rho_{gn}, \sigma_{gp}, \sigma_{gn}, \sigma_{gl}, \sigma_{gg}, \kappa_0, \rho_{\kappa\kappa}, \rho_{\kappa\theta}, \rho_{\kappa p}, \rho_{\kappa n}, \sigma_{\kappa p}, \sigma_{\kappa n}, \sigma_{\kappa l}, \sigma_{\kappa\kappa}, \eta_0, \rho_{\eta\eta}, \rho_{\eta\theta}, \rho_{\eta p}, \rho_{\eta n}, \sigma_{\eta p}, \sigma_{\eta n}, \sigma_{\eta l}, \sigma_{\eta\eta}, v_0, \rho_{vv}, \rho_{vl}, q_0, \rho_{qq}, \rho_{qp}, \rho_{qn}, \sigma_{qp}$ and σ_{qn}.

6. Obtain the moment conditions $\boldsymbol{\varepsilon}(\Theta; \Psi_t)$. Given data on asset returns and options, we use the derived moments to define 7 error terms that can be used to create GMM orthogonality conditions. There are three types of errors we use in the system. First, neither risk premiums nor physical conditional variances are observed in the data, but we use the restriction that the observed returns/realized variances minus their expectations under the null of the model ought to have a conditional mean of zero:

$$\boldsymbol{\varepsilon_1}(\boldsymbol{\Theta}; \Psi_t) = \begin{bmatrix} \left(\widetilde{r}^{eq}_{t+1} - \widetilde{rf}_t\right) - \widehat{RP}^{eq}_t \\ RVAR^{eq}_{t+1} - \widehat{VAR}^{eq}_t \\ \left(\widetilde{r}^{cb}_{t+1} - \widetilde{rf}_t\right) - \widehat{RP}^{cb}_t \\ RVAR^{cb}_{t+1} - \widehat{VAR}^{cb}_t \end{bmatrix}, \tag{53}$$

where $\widetilde{r}^{i}_{t+1}$ is the realized nominal return from t to $t+1$, rf_t is the risk free rate, and $RVAR^{i}_{t+1}$ is the realized nominal variance from t to $t+1$ defined as the sum of the squares of the log high-frequency returns from t to $t+1$ (see the Data section for details). Here Ψ_t denotes the information set at time t. The risk neutral variance can be measured from options data (see e.g. Bakshi and Madan, 2000; Bakshi, Kapadia, and Madan, 2003), and so we use the error:

$$\varepsilon_2(\boldsymbol{\Theta}; \Psi_t) = \left[QVAR^{eq}_t - \widehat{VAR}^{eq,Q}_t\right], \tag{54}$$

where $QVAR^{eq}_t$ is the ex-ante risk-neutral variance of r^{eq}_{t+1} calculated from the data. We assume that $\varepsilon_2(\boldsymbol{\Theta}; \Psi_t)$ reflects model and measurement error, orthogonal to Ψ_t. Finally, we also construct two moment conditions to identify σ_{qq}, exploiting the model dynamics for u^q_{t+1} (i.e., the shock to the risk aversion process as in Equation (29)):

$$\boldsymbol{\varepsilon_3}(\boldsymbol{\Theta}; \Psi_t) = \begin{bmatrix} (\widehat{u}^q_{t+1})^2 - (\widehat{\sigma}_{qq})^2 \widehat{q}_t \\ (\widehat{u}^q_{t+1})^3 - 2(\widehat{\sigma}_{qq})^3 \widehat{q}_t \end{bmatrix} \tag{55}$$

Let $\boldsymbol{\varepsilon_{1,2}}(\boldsymbol{\Theta}; \Psi_t) = \left[\boldsymbol{\varepsilon_1}(\boldsymbol{\Theta}; \Psi_t)' \quad \varepsilon_2(\boldsymbol{\Theta}; \Psi_t)\right]$. Under our assumptions these errors are mean zero given the information set, Ψ_t. We can therefore use them to create the usual GMM moment conditions. Given our previously defined set of instruments,

$\boldsymbol{z_t}$ (7×1, including a vector of 1's), we define the moment conditions as:

$$E\left[\boldsymbol{g_t}(\boldsymbol{\Theta}; \Psi_t, \boldsymbol{z_t})\right] \equiv E\begin{bmatrix} \underbrace{\boldsymbol{\varepsilon_{1,2}}(\boldsymbol{\Theta}; \Psi_t)}_{5\times 1} \otimes \underbrace{\boldsymbol{z_t}}_{7\times 1} \\ \underbrace{\boldsymbol{\varepsilon_3}(\boldsymbol{\Theta}; \Psi_t)}_{2\times 1} \end{bmatrix} = \underbrace{\boldsymbol{0}}_{37\times 1}. \tag{56}$$

Note that to keep the set of moment conditions manageable, we only use two moment conditions for the identification of σ_{qq}. Denote $\boldsymbol{g_t}(\boldsymbol{\Theta}; \Psi_t, \boldsymbol{z_t})$ (37×1) as the vector of errors at time t, and $\boldsymbol{g_T}(\boldsymbol{\Theta}; \Psi, \boldsymbol{z})$ (37×1) the sample mean of $\boldsymbol{g_t}(\boldsymbol{\Theta}; \Psi_t, \boldsymbol{z_t})$ from $t = 1$ to $t = T$. Then, the GMM objective function is,

$$J(\boldsymbol{\Theta}; \Psi, \boldsymbol{z}) \equiv T\boldsymbol{g'_T}(\boldsymbol{\Theta}; \Psi, \boldsymbol{z})\boldsymbol{W}\boldsymbol{g_T}(\boldsymbol{\Theta}; \Psi, \boldsymbol{z}),$$

where $\boldsymbol{W}$ is the weighting matrix. We use the standard GMM procedure, first using an identity weighting matrix, yielding a first stage set of parameters $\widehat{\boldsymbol{\Theta}}_{\boldsymbol{1}}$. We then compute the usual optimal weighting matrix as the inverse of the spectral density at frequency zero of the orthogonality conditions, $\widehat{\boldsymbol{S}}_1$, using 5 Newey-West (1987) lags:

$$\widehat{\boldsymbol{S}}_1 = \sum_{j=-5}^{j=5} \frac{5-|j|}{5} \widehat{E}[\boldsymbol{g_t}(\widehat{\boldsymbol{\Theta}}_{\boldsymbol{1}}; \Psi_t, \boldsymbol{z_t})\boldsymbol{g_{t-j}}(\widehat{\boldsymbol{\Theta}}_{\boldsymbol{1}}; \Psi_{t-1}, \boldsymbol{z_{t-1}})']. \tag{57}$$

Then, the inverse of $\widehat{\boldsymbol{S}}$ is shrunk towards the identity matrix with a shrinkage parameter of 0.1 in obtaining the second-step weight matrix $\boldsymbol{W_2}$:

$$\boldsymbol{W_2} = 0.9\widehat{\boldsymbol{S}}^{-1} + 0.1\boldsymbol{I_{37\times 37}}, \tag{58}$$

where $\boldsymbol{I_{37\times 37}}$ is a identity matrix of dimension 37×37. This gives rise to a second-round $\widehat{\boldsymbol{\Theta}}_{\boldsymbol{2}}$ estimator. To ensure that poor first round estimates do not affect the estimation, we conduct one more iteration with shrinkage, compute $\widehat{\boldsymbol{S}}_2(\widehat{\boldsymbol{\Theta}}_{\boldsymbol{2}})$, and produce a third-round GMM estimator, $\widehat{\boldsymbol{\Theta}}_{\boldsymbol{3}}$. Lastly, the asymptotic distribution for the third-step GMM estimation parameter is, $\sqrt{T}(\widehat{\boldsymbol{\Theta}}_{\boldsymbol{3}} - \boldsymbol{\Theta_0}) \underset{d}{\rightarrow} N(0, \boldsymbol{Avar}(\widehat{\boldsymbol{\Theta}}_{\boldsymbol{3}}))$, where $\widehat{\boldsymbol{Avar}(\widehat{\boldsymbol{\Theta}}_{\boldsymbol{3}})} = (\boldsymbol{G'_T}(\widehat{\boldsymbol{\Theta}}_{\boldsymbol{3}})\widehat{\boldsymbol{S}}_{\boldsymbol{2}}^{\boldsymbol{-1}}\boldsymbol{G_T}(\widehat{\boldsymbol{\Theta}}_{\boldsymbol{3}}))^{-1}$ and where $\boldsymbol{G_T}$ denotes the gradient of $\boldsymbol{g_T}$.

Because the estimation involves several steps and is quite non-linear in the parameters, we increase the chance of finding the true global optimum by starting from 24,000 different starting values for $\widehat{\boldsymbol{\chi}}$ drawn randomly from a large set of possible starting values for each parameter. The global optimum is defined as the parameter estimates generating the lowest minimum objective function value.

IV Data

Because we combine macro and cash flow data to estimate the dynamics of the state variables, with financial data in the GMM estimation, we use the longest data available for the various estimations of the state variable dynamics. The estimation of the macroeconomic uncertainty state variables uses the period from January 1947 to February 2015, and the estimation of the loss rate uncertainty state variable uses data from January 1982 to February 2015. For the GMM estimation, the sample spans the period from June 1986 to February 2015 (T=345 months). All estimations are conducted at the monthly frequency.

IV.A State variables

Our output variable—delivering three state variables (θ_t, p_t and n_t)—is the change in log real industrial production where the monthly real industrial production index is obtained from the Federal Reserve Bank at St. Louis. Inflation (π), is defined as the change in the log of the consumer price index (CPI) obtained from the Bureau of Labor Statistics (BLS).

The fifth state variable, the log corporate bond loss rate (l), as indicated before requires data on default rates and recovery rates for the US corporate bond market. We obtain data on 3-month trailing all-corporate bond default rates and monthly recovery rates spanning November 1980 to February 2015 from the Federal Reserve Board. We use 6 month moving averages of these raw data to compute the log loss rate representative for each month.

The sixth state variable, real earnings growth (g), is defined as the change in log real earnings per capita. Real earnings is the product of real earnings per share and the number of shares outstanding during the same month. The seventh state variable, the log consumption-earnings ratio (κ), uses real consumption and real earnings. Real monthly consumption is defined as the sum of seasonally-adjusted real personal consumption expenditures on nondurable goods and services; as widely recognized in the literature, the consumption deflator is different from the CPI and is computed using monthly data in this paper. The eighth state variable, the log dividend payout ratio (η), uses the log ratio of real dividends and real earnings. Therefore, consumption growth (dividend growth) is implicitly defined given g and κ (g and η).

The source for the consumption data is the U.S. Bureau of Economic Analysis (BEA). The source for the dividends and earnings data is Robert Shiller's website. We use the 12-month trailing dividends and earnings, i.e., $E_t^{12} = E_{t-12} + ...E_{t-1}$ where E_t denotes the monthly earnings. There are no true monthly earnings data because almost all firms report earnings results only quarterly. According to Shiller's website, the monthly dividend and earnings data provided are inferred from the S&P four-quarter totals, which are available since 1926. Calculating 12-month trailing values of earnings and dividends is common practice to control for the strong seasonality in the data. Total market shares are obtained from CRSP. To obtain per capita units, we divide real consumption and real earnings by the population numbers provided by BEA.

IV.B Financial Variables

Daily equity returns are the continuously compounded value-weighted nominal market returns with dividends from CRSP. The monthly return (r^{eq}) is the sum of daily returns within the same month. To create excess returns, we subtract the one-month Treasury bill rate, also from CRSP. We use the square of the month-end VIX index (divided by 120000) as the one-period risk-neutral conditional variance of equity returns ($QVAR^{eq}$) which is obtained from the Chicago Board Options Exchange (CBOE) and is only available from the end of January 1990. We use the VXO index prior to 1990, also from CBOE, going back to June 1986. We construct the monthly one-period physical conditional variance of equity returns ($PVAR^{eq}$) in two steps. First, we calculate the monthly realized variance as the sum of the squared daily equity returns within the same month; then, we project the monthly realized variance onto the lagged risk-neutral variance and the lagged realized variance to obtain the monthly $PVAR^{eq}$, as in Bekaert, Hoerova, and Lo Duca (2013).

The daily corporate bond market return is the continuously compounded log change in daily Dow Jones corporate bond total return index (source: Global Financial Data). The monthly return (r^{cb}) is the sum of daily returns within the same month. The conditional variance under the physical measure ($PVAR^{cb}$) is the projection of monthly realized variance onto the lagged realized variance and the lagged credit spread (defined as the difference between the month-end Baa yield and the 10-year zero-coupon Treasury yield).

In some of our work below, we also use data on speculative corporate bonds, which may be particularly informative about the economic environment (source: FRED, "ICE BofAML US High Yield Total Return Index"). We obtain monthly realized speculative corporate bond return variances using the methodology described above for overall corporate returns. Because the daily index only starts in February 1990, we use an empirical model to fill in the missing data from June 1986 to January 1990.[10]

In attempting to span risk aversion, we use several observed financial variables. The

[10]Th exact model is described in the Online Appendix.

term spread is the difference between the 10-year Treasury yield and the 3-month Treasury yield, where the yield data is obtained from the Federal Reserve Bank of St. Louis. The credit spread is the difference between Moody's Baa yield and the 10-year Treasury bond yield. The detrended dividend yield is calculated as the difference between the raw dividend yield and an moving average term that takes the 5 year average of monthly dividend yields, starting one year before, or $DY5yr_t = DY_t - \sum_{i=1}^{60} DY_{t-12-i}$ where DY_t denotes the dividend yield level at time t (the ratio of 12-month trailing dividends and the equity market price). We create an analogous detrended earnings yield variable using earnings data.

V Estimation Results

In this section, we describe the estimation of the state variable processes, and the actual risk aversion process.

V.A State Variable Dynamics

V.A.1 Macro-economic factors

While we entertained a number of alternative model specifications for industrial production growth, the model described in Section II.B was best in terms of the standard BIC criterion. The parameter estimates are reported in Table 1. Industrial production growth features slight positive auto-correlation and high realizations of "bad" volatility decrease its conditional mean significantly. The p_t process is extremely persistent (almost a unit root) and quasi Gaussian, forcing us to fix its unconditional mean at 500 (for such values, skewness and kurtosis are effectively zero). The n_t process has a much lower mean featuring an unconditional skewness coefficient of 0.50 ($\frac{2}{\sqrt{16.14}}$) and excess kurtosis of 0.37 ($\frac{6}{16.14}$). It is also less persistent than the p_t process.

We graph the conditional mean and the p_t and n_t process in Figure 1 together with NBER recessions. The strong countercyclicality of the n_t process and the procyclicality of the conditional mean of "technology" or output growth are apparent from the graph. We also confirmed it by running a regression of the three processes (conditional mean, p_t, and n_t) on a constant and a NBER dummy. The NBER dummy obtains a highly statistically significant positive (negative) coefficient for the n_t (conditional mean) equation. The coefficient is in fact positive in the p_t equation as well, but not statistically significant. In fact, the n_t regression features an adjusted R^2 of almost 45%.

In Figure 2, we plot the conditional variance of industrial production and its conditional skewness. Clearly, macro-economic uncertainty is highly countercyclical, and thus exposure to such uncertainty may render asset prices countercyclical as well. Interestingly, the scaled skewness coefficient is procyclical. This arises from the fact that, while unscaled skewness is countercyclical, the countercyclicality of the variance in the denominator dominates.

V.A.2 Cash flow dynamics

The loss rate process is persistent with the autocorrelation coefficient close to 0.83. The p_t-process does not significantly affect the loss rate process, neither through the conditional mean or through shock exposures. However, the $\omega_{n,t}$ shock has a statistically significant effect on the loss rate process; moreover n_t affects the loss rate's conditional mean with a statistically significant positive coefficient. The time-varying part of the conditional variance, lp_t, is persistent with an autoregressive coefficient of 0.86. This process also exhibits substantial excess kurtosis (unconditional kurtosis = 1.15) and positive skewness (unconditional skewness = 0.90). The gamma shock generating negative skewness, which has a time-invariant shape parameter, is nearly Gaussian, with the shape parameter exceeding 100.

In Figure 3, we first plot the loss rate process l and its conditional mean. The loss rate clearly spikes around recessions, from an overall average of 0.6% to 2.1% on average in recessions (the maximum value is 5.6% during February 2009). The conditional mean of the loss rate inherits the countercyclicality of the loss rate itself, given the loss rate's high persistence and it positive dependence on n_t. Our model fits the positive skewness of the loss rate process through the positively skewed u^l shocks and the positive dependence on ω_n.

Then, we show the conditional higher-order moments of the loss rate process, including the lp_t process in the second half of Figure 3. While lp_t is overall countrycyclical, it appears to peak a few months after recessions. The conditional variance in the second panel ($Var_t(l_{t+1})$) also appears countercyclical, which is the combined result of a countercyclical lp_t process and a strongly countercyclical n_t process (σ_{ln} being positive). In fact, a regression of lp_t on a constant and a NBER dummy, yields a NBER coefficient of 6.78 with a t statistic of 3.03, but the t statistic increases to 8.87 when regressing total variance on the NBER dummy.

We decompose the total conditional variance of the loss rate in its contributions coming from lp_t, p_t and n_t in Figure 4. The dominant sources of variation are lp_t (accounting for 29% of the total variance on average) and n_t (accounting for 40%). The relative importance of lp_t drops slightly in recessions while that of n_t increases. It tends to peak when the economy starts recovering, reaching as high as 93%. The p_t process has a negligible effect on the loss rate variance. Clearly, the loss rate variance has substantial independent variation not spanned by macro-economic uncertainty.

With the loss rate process estimated, the dynamics of the other cash flow state variables (earnings growth, the consumption earnings ratio and the payout ratio) follow straightforwardly. We simply use linear projections of the variables onto the previously identified state variables and shocks. The results are contained in Table 3. Earnings growth is less persistent than the two ratio variables, but loads positively and significantly on industrial production growth. The n_t state variable has a positive effect on the conditional mean of the consumption-earnings and dividend-earnings ratio, indicating that in recessions these ratios are expected to be larger than in normal times. This makes economic sense as consumption and dividends are likely smoothed over the cycle whereas earnings are particularly cycle sensitive (see also Longstaff and Piazessi, 2004). Yet, the cyclicality of earnings growth does not show through a significant effect of n_t but rather appears through its positive dependence on industrial production growth directly and its negative dependence on the loss rate. Again, the ratio variables load significantly, but positively on the loss rate. The same intuition explains why the ratio variables load positively on ω_n shocks and earnings growth loads negatively on this shock. The ω_p and ω_{lp} shocks do not have a significant effect on these state variables. The ω_{ln} shock, which has a negative effect on loss rates, somewhat surprisingly affects the ratio variables positively and significantly.

The projections implicitly define the residuals shocks for the cash flow variables, which were shown to be homoskedastic. Table 3 indicates that they still feature substantial and significant variability. We do not impose any correlation structure on these shocks, and Table 4 shows that they are quite correlated. Essentially, because earnings growth is quite variable, the ratio variables are positively correlated with one another and negatively correlated with earnings growth. When pricing assets with the model, this correlation structure must be accounted for (see below). The correlations with the other state variable shocks and between these state variable shocks (ω_p, ω_n, ω_{lp}, ω_{ln}) ought to be zero in theory and the table shows that they are economically indeed close to zero.

V.B Risk Aversion

Recall that we assume risk aversion to be spanned by 6 financial instruments. In Table 5, we report some properties of these financial instruments. We show both dividend and earnings yields, but we only use one of the yield variables in the estimation. First, all of them are highly persistent. This is the main reason we use a stochastically detrended dividend (AR(1)=0.982) or earnings yield (AR(1)=0.984) series rather than the actual dividend or earnings yield series. The dividend yield shows a secular decline over part of the sample that induces much autocorrelation. This decline is likely due to American tax policy and therefore not likely informative about risk aversion (see e.g. Boudoukh et.al, 2007). Second, the various instruments are positively correlated but the correlations never exceed 85% so that we should not worry about multi-collinearity. Perhaps surprisingly, the term spread is also positively correlated with the other instruments, even though it is generally believed that high term spreads indicate good times, whereas the yield and variance instruments would tend to be high in bad times. Third, 4 of the instruments show significant positive skewness. This is critical as we have assumed that the risk aversion dynamics are positively skewed through its gamma distributed shock (see Equation (31)), and we need the linear spanning model to be consistent with the assumed dynamics for risk aversion. The term spread, and earnings yields are significantly negatively skewed so that a negative weight on one of them could also induce positive skewness in risk aversion, but their skewness coefficients are much smaller in magnitude.

Table 6 reports the reduced form estimates in the spanning relation. The system estimates 8 parameters with 37 moment conditions. The test of the over-identifying restrictions fails to reject at the 5% level but we investigate the fit of the model along various dimensions in more detail later. The significant determinants of the risk aversion process are the credit spread, realized equity return and corporate bond return variances and the equity return risk neutral variance. The positive coefficient on the risk neutral and the negative coefficient on the physical realized equity return variances is consistent with the idea that the variance risk premium may be quite informative about risk aversion in financial markets (see also Bekaert and Hoerova, 2016). The implied risk aversion process shows a 0.40 correlation with the NBER indicator and is thus highly counter-cyclical. We also estimated the system using the detrended dividend yield instead of the earnings yield, which produces very similar results. The two q_t processes are 98% correlated.

In Table 7, we estimate the dynamic properties of the risk aversion process according to Equation (29). All the parameters are estimated by OLS, except for the σ_{qq} parameter, which is delivered by the GMM estimation (see Section III.C). The process shows moderate persistence (an autocorrelation coefficient of 0.67) but the conditional mean surprisingly shows a significant positive loading on p_t, which accounts for 77% of the variation in the conditional mean. Risk aversion shocks do not load significantly on the macro-economic uncertainty shocks and therefore most of their variation is driven by the risk aversion specific shock. It appears that economic models that impose a very tight link between aggregate fundamentals and risk aversion, such as pure habit models (Campbell and Cochrane, 1999) are missing important variation in actual risk aversion. In addition, risk aversion is much less persistent than the risk aversion implied by these models; the autocorrelation coefficient of the surplus ratio process in the CC model is

0.99 at the monthly level; the first-order autocorrelation coefficient of q_t derived in this paper is 0.67.

While the test of the over-identifying restrictions fails to reject, Table 8 examines in more detail how well the estimated dynamic system fits critical asset price moments in the data. The moments are reported in monthly units; for example, the monthly equity premium produced by the model is 78.5 basis points. All model moments are within two standard errors of the data moments and most are within one standard error of the data moment.[11] The model over-estimates the equity premium but is still close to within one standard error of the data moment. The corporate bond risk premium is only 4 basis points higher than data moment. The model implied variance moments are all quite close to their empirical counterparts. Finally, the table also reports the model-implied variance and unscaled skewness of the risk aversion innovation, $\sigma_{qq}^2 q_t$ and $2\sigma_{qq}^3 q_t$ (respectively).

Of all the asset return moments examined here, the only observed one is the risk neutral variance (the VIX index). Because we have filtered state variables, we can verify how well this process fits the actual observed risk neutral variance at each point in time. Figure 5 graphs the empirical and model implied risk neutral variance. While the model fails to match the distinct spikes of the VIX in several crisis periods, the fit is remarkably good, with the correlation between the two series being 90.06%.

VI Risk Aversion, Uncertainty and Asset Prices

In this section, we first characterize the link between risk aversion and macroeconomic uncertainty, on the one hand and asset prices, on the other hand. We compare the time variation in risk aversion and macroeconomic uncertainty and document how our measures correlate with extant measures of uncertainty and risk aversion.

VI.A Risk Aversion, Macro-Economic Uncertainty and the First and Second Moments of Asset Returns

Figure 6 graphs the risk aversion process, which in our model is:

$$ra_t^{BEX} = \gamma \exp(q_t). \tag{59}$$

[11] Bootstrapped standard errors for the five asset price moments (equity risk premium, equity physical variance, equity risk-neutral variance, corporate bond risk premium, and corporate bond physical variance) use different block sizes to accommodate different serial auto-correlations, to ensure that the sampled blocks are approximately i.i.d.. In particular, Politis and Romano (1995) (and later discussed in Politis and White, 2004) suggest looking for the smallest integer after which the correlogram appears negligible, where the significance of the autocorrelation estimates is tested using the Ljung-Box Q Test (Ljung and Box, 1978).

The weak countercyclicality of the process is immediately apparent with risk aversion spiking in all three recessions, but also showing distinct peaks in other periods. The highest risk aversion of 6.31 is reached at the end of February in 2009, at the height of the Great Recession. However, the risk aversion process also peaks in the October 1987 crash, the August 1998 crisis (Russia default and LTCM collapse), after the TMT bull market ended in August 2002 and in August 2011 (Euro area debt crisis).

How important is risk aversion for asset prices? In this article's model, the priced state variables for risk premiums and variances are those entering the conditional covariance between asset returns and the pricing kernel and therefore are limited to risk aversion q_t, the macro-economic uncertainty state variables, p_t and n_t and the loss rate variability lp_t. In Table 9, we report the loadings of risk premiums and variances on the 4 state variables. To help interpret these coefficients, we scaled the projection coefficients by the standard deviation of the state variables so that they can be interpreted as the response to a one standard deviation move in the state variable. For the equity premium, by far the most important state variable is q_t which has an effect more than 10 times larger than that of n_t. The effects of p_t and lp_t are trivially small. The economic effect of a one standard deviation change in q_t is large representing 52 basis points at the monthly level (this is a bit lower than the average monthly equity premium). For the corporate bond premium, n_t and q_t are again the most important state variables. A one standard deviation increase in n_t increases the monthly corporate bond risk premium by 5 basis points, representing more than 10% of the average monthly premium. The effect of q_t is about twice as large n_t.

The coefficients for variances are somewhat harder to interpret, but n_t and q_t remain the most important state variables with the former (latter) more important for corporate bond (equity) variances. The one variable for which q_t is only the third most important variable is the corporate bond physical variance, which reacts more strongly to n_t, and lp_t. Recall that lp_t measures the idiosyncratic component in corporate loss rates but that loss rates load very significantly on our business cycle variable.

Because the relationship between asset prices and state variables is affine, we also compute a variance decomposition. That is, we compute, for $x \in \{p, n, v, q\}$, coefficient on $x_t \times \frac{Cov(x_t, Mom_t)}{Var(Mom_t)}$ where Mom represents an asset price moment like the equity risk premium, or corporate bond physical variance. These variance proportions add up to one. In the model, 96% of the equity risk premium's variance is driven by risk aversion; only 72.5% of the corporate bond risk premium is driven by risk aversion, while more than 27% is accounted for by "bad" macro-economic uncertainty. The physical equity variance is predominantly driven by risk aversion (69%) while 89% of the corporate bond return's physical variance is driven by bad macroeconomic uncertainty.

Nevertheless, macro-economic uncertainty also accounts for 31% of the variance of the physical equity variance. It would be logical that the risk neutral variance would load more on risk aversion and less on macroeconomic uncertainty than the physical variance and this is indeed the case, with risk aversion accounting for 85% of the variance of the risk neutral variance.

Bekaert, Hoerova and Lo Duca (2013) argue that the variance risk premium houses much information about risk aversion. Is this true in our model? To answer this question, we compute the model-implied variance risk premium as the difference between the risk neutral variance and the physical variance. A projection on the 4 state variables reveals that 96.8% of the variance of the variance risk premium is accounted for by risk aversion. Conversely, regressing risk aversion on the variance premium, the coefficient is 160.40 with a t-stat of 93.37, and the R^2 is 96.2%. Through the lens of our model, the variance premium is clearly a good proxy for risk aversion.

Finally, because the state variables perfectly explain conditional first and second moments of asset returns in the model, they should help predict realized returns and variances in the data. We test this in Table 10. We regress realized monthly excess returns and variances in both the equity and corporate bond markets on 1) the 4 state variables, or 2) the model-implied conditional moment (either the conditional risk premium or the conditional variance). Not imposing the model restrictions on how the state variables combine to the model implied conditional moment, slightly decreases the adjusted R^2, but for equity returns the R^2 decreases strongly from 5.5% to 0.1%. In this case, the coefficient on the model implied moment is about 0.61 and not significantly different from 1, but it is significantly different from zero. For corporate bond risk premiums, the coefficient is only borderline significantly different from zero. Not surprisingly, the R^2s are higher for the realized equity and corporate bond variances hovering around 20–25%. The coefficients on the model-implied conditional moments are higher than 1 in this case. When investigating the coefficients of individual state variables, risk aversion significantly predicts equity returns and both realized variances. Bad economic uncertainty predicts the realized variances and equity returns, the earlier with a negative sign. The latter result is reminiscent of a result in Bekaert and Hoerova (2014). They show that the variance risk premium predicts stock returns with a positive sign whereas uncertainty, as measured by an estimate of the conditional variance in the stock market, predicts stock returns with a negative sign. As n_t is highly correlated with the stock market physical variance and q_t with the variance risk premium, this result is therefore no surprise. The loss rate variance has an intuitive positive coefficient in the variance and corporate bond return regressions but is not statistically significantly different form zero. However, it predicts equity returns with a negative sign, with the coefficient significant at the 10%

level. The last line reports correlations of the model-implied moments with the NBER recession indicator, showing all of them to be significantly countercyclical.

Given the vast literature on return predictability, it is informative to contrast the predictive power of our model implied premiums with the predictive power of the usual instruments used in the literature. We do this exercise out of sample as the literature has shown huge biases due to in sample over-fitting (Goyal and Welch, 2008). Our model premium candidates are either derived from a projection of excess returns on the 4 state variables (Model 2) or the actual model-implied risk premium (Model 1). We consider three empirical models, depending on the instruments used: 1) earning yield, 2) earnings yield, term spread and credit spread, 3) physical uncertainty and variance risk premium estimate. For equity (corporate bond) returns, we use the physical uncertainty derived from equity (corporate bond) returns. We then generate out-of-sample predictions for the risk premiums according to the various empirical models by starting the sample after five years of data and then running rolling samples to generate predictions from the five-year point to one month before the end of the sample. For the model implied risk premiums, Model 1 uses whatever the model predicts the premium to be. For Model 2, the projections are also conducted in a rolling fashion, but note that the construction of the state variables uses information from the full sample. With those competing risk premium estimates in hand, we then run simple horse races over the full sample by estimating:

$$\widetilde{r}_{t+1} - rf_t = a \ \mathrm{Mod}(t,i) + (1-a) \ \mathrm{Emp\ Mod}(t,j) + e_{t+1}, \ \text{ for } i = 1,2, j = 1,2,3. \quad (60)$$

The results are reported in Table 11. Using the implied risk premiums form the model (Model 1) clearly outperforms the empirical model for equity returns with the "a"-coefficients being well over 0.50, varying between 0.73 and 0.85. For corporate bond returns, the "a"-coefficient varies between 0.45 and 0.56, with all "a"-coefficients highly statistically different from zero. Thus, while the data suggest that the model implied risk premiums are very useful in forecasting excess corporate bond returns out of sample (despite the rather poor results in Table 10), the empirical instruments yield information about risk premiums that is equally important.

Model 2 fares less well, with the coefficients only being above 0.5 for the equity return regressions when the model is pitted against empirical models 2 and 3. Yet, the "a"-coefficients remain statistically significant different from zero in all cases except for corporate bond returns when pitted against the first instrument set. The "a"-coefficients vary between 0.36 and 0.56 in the other 5 cases. We conclude that our model seems to capture the predictable variation better, or at least as well, than the fitted values extracted from standard instruments used in the literature. While the model premium is not strictly out of sample, the model imposes numerous restrictions relative to the empirical models.

VI.B Interpreting Economic Uncertainty

An advantage of the risk aversion process we estimated is that because of its dependence on financial instruments it can be computed at even a daily level. Unfortunately, our filtered macro-economic uncertainty variables were extracted from industrial production which is only available at the monthly level. Here we consider whether we can use the financial instruments to approximate macro uncertainty. First, total macro-economic uncertainty, the conditional variance of industrial production growth, is a function of both p_t and n_t, $\sigma^2_{\theta p} p_t + \sigma^2_{\theta n} n_t$ where the coefficient estimates of σ_p and σ_n are provided in Table 1. In Table 12, we show the coefficients from a regression of uncertainty on the financial instruments used to span risk aversion and two additional instruments, the detrended dividend yield and realized variances of speculative bond returns. The R^2 is almost 50% and uncertainty loads significantly on all instruments except for the realized equity and speculative bond return variances. Unlike the risk aversion process, uncertainty loads very strongly on both credit spreads and the physical corporate bond variance. The term spread also has a significant negative effect on uncertainty (and no effect on risk aversion). This makes sense as flattening yield curves are associated with future economic downturns. The table also reports regressions from the two components in macro-economic uncertainty, bad and good uncertainty, onto the instruments. Clearly, the variation in macro-economic uncertainty is dominated by the bad component and the coefficients for the bad component projection coefficients are very similar to those of total uncertainty. We also report the results from a variance decomposition applied to the fitted values of the regression. The credit spread explains almost 63% of the explained uncertainty variation. The dividend and earnings yield variables likely offset one another partially with one contributing a positive, the other a negative amount to the total variation but jointly the equity yield variables still explain close to 20%. Finally, the risk neutral equity variance and the physical corporate bond return variance contribute about 13-14% of the explained variation of uncertainty.

From this analysis, we create an uncertainty index representing the part of total uncertainty that is explained by the financial instruments:

$$unc_t^{BEX} = \boldsymbol{\chi}^{\boldsymbol{unc}\prime} \boldsymbol{z_t}. \tag{61}$$

In Figure 7, we graph the financial instrument proxies to uncertainty and risk aversion for comparison. The correlation between actual uncertainty and risk aversion is 51%; when we use the proxy the correlation increases to 70%. Obviously, most of the time crisis periods feature both high uncertainty and high risk aversion. There are exceptions however. For example, the October 1987 crash happened during a time of relatively low economic uncertainty. It also appears that at the end of the 90s, macro-uncertainty seems to secularly increase, consistent with the Great Moderation ending around that time (see also Baele et al., 2015).

Bloom (2009) has argued that uncertainty precedes bad economic outcomes. We regress future real industrial production growth at various horizons on our uncertainty index—its financial proxy and the actual one—and the risk aversion process. In addition, we use the VIX as suggested in Bloom (2009). The results are in Table 13. We use Hodrick (1992) standard errors to accommodate the overlap in the data. Panel A shows univariate results. All indices predict growth with a negative sign at the one month, one quarter and one year horizons. Our financial instrument uncertainty index generates the highest R^2 by far. This suggests that it is indeed macro uncertainty predicting output growth, with the VIX having much lower predictive power in univariate regressions. The actual macroeconomic uncertainty (Column "unc^{true}") exhibits substantially more predictive power than the VIX (Column "$QVAR$"), but still substantially less than the combination of financial instruments most correlated with it (Column "unc^{BEX}"). This is likely due to the important role played by the credit spread in unc^{BEX}; with the credit spread known to predict future economic activity (see De Santis, 2018, and the references therein).

This result is confirmed in multivariate regressions. In Panel B, we use our risk aversion index (ra^{BEX}), "financial instruments" uncertainty index (unc^{BEX}), and the VIX simultaneously. The "financial instruments" uncertainty index is statistically significant for all horizons, whereas risk aversion (with a positive sign) and the VIX squared (with a negative sign) are significant at the quarterly horizon and beyond. Thus, our uncertainty index dominates the VIX index in terms of its predictive power for real activity. In Panel C, the "financial instruments" uncertainty index is replaced by the actual economic uncertainty. The latter variable is statistically significant at all three horizons. However, the predictive power of the multivariate model drops substantially at all horizons being only 1/4 to 2/3's of that present in Panel B (by 24% $\sim$ 64%). Not surprisingly, Panel D shows that the "financial instruments" uncertainty proxy also dominates actual economic uncertainty (unc^{true}), when both are present. The actual economic uncertainty is insignificant at all horizons.

In ongoing work, we also try to create a high frequency proxy to macro uncertainty, that uses both past information in macroeconomic uncertainty and financial instruments.

Because macro-uncertainty is highly persistent, past uncertainty plays a dominant role in such an index.

Finally, we also examine the correlation between lp_t, the idiosyncratic variance component of corporate bond loss rates, with financial instruments, but the R2 in such a regression is only 9% (see the Online Appendix for detailed results).

VI.C Correlations with Extant Measures

In this section, we examine the correlation of our risk aversion and uncertainty indices with existing measures. For risk aversion, we consider three categories: risk aversion indices based on "fundamental" habit models, sentiment indices and commercially available risk aversions indices. For uncertainty, we focus on the recent uncertainty index developed by Jurado, Ludvigson and Ng (2015) and the Baker, Bloom and Davis (2016) political uncertainty measure.

Recall that in an external habit model such as Campbell and Cochrane (1999), the curvature of the utility function is a negative affine function of the log "consumption surplus ratio," which in turns follows a heteroskedastic autoregressive process with shocks perfectly correlated with consumption growth. We follow Wachter (2006) and create a "fundamental" risk aversion process from consumption data and CC's parameter estimates, which we denote by RA_{CC}. Table 14 shows that it is only weakly correlated with our risk aversion measure but the correlation is still significantly different from zero. Clearly, the asset pricing literature should start accepting that risk aversion shows higher frequency movements inconsistent with the focus on low frequency changes tightly linked to consumption growth as in the extant habit models. Work by Bekaert, Engstrom and Grenadier (2010) and Martin (2017) also suggests the existence of more variable risk aversion in financial markets.[12]

The behavioral finance literature suggests that the sentiment of retail investors may drive asset prices and cause non-fundamental price swings. As a well-known representative of this work, we use the sentiment index from Baker and Wurgler (2006). The index is based on the first principal component of six (standardized) sentiment proxies including: the closed-end fund discount, the NYSE share turnover, the number and the average first-day returns of IPOs, the share of equity issues in total equity and debt issues, and the dividend premium (the log-difference of the average market-to-book ratios of payers and nonpayers). We denote their index by $Sent_{BW}$. High values mean positive sentiment so we expect a negative correlation with our risk aversion indicator, and indeed

[12]When computing the same correlations, with our alternative risk aversion index (using the dividend rather than the earnings yield as the equity yield instrument), all the correlations in Table 14 are quite similar (but mostly slightly higher in absolute magnitude).

the correlation is significantly negative but still relatively small at -0.17.

Because the Baker-Wurgler index relies on financial data, it may not directly reflect the sentiment of retail investors. Lemmon and Portnaiguina (2006) and Qiu and Welch (2006) therefore suggest using a consumer sentiment index such as the Michigan Consumer Sentiment Index (MCSI). The correlation with this index is also negative, as expected, and larger in absolute magnitude at -0.28. Hence, our risk aversion index correlates more with a pure consumer sentiment index than with $Sent_{BW}$, derived from financial variables.

Finally, many financial services companies create their own risk appetite indices. As a well-known example, we obtain data on the Credit Suisse First Boston Risk Appetite Index (RAI). The indicator draws on the correlation between risk appetite and the relative performance of safe assets (proxied by seven to ten-year government bonds) and risky assets (equities and emerging market bonds). The underlying assumption is that an increasing risk preference shifts the demand from less risky investments to assets associated with higher risks, thus pushing their prices up relative to low-risk assets (and vice versa). The indicator is based on a cross-sectional linear regression of excess returns of 64 international stock and bond indices on their risk, approximated by historic volatility. The slope of the regression line represents the risk appetite index. The index shows a -0.48 correlation with our index and is thus highly correlated with our concept of risk aversion.

Our uncertainty measure only uses industrial production data. Jurado, Ludvigson and Ng (2015) use the weighted sum of the conditional volatilities of 132 financial and macroeconomic series, with the bulk of them being macroeconomic. They have three versions of the measure depending on the forecasting horizon, but we focus on the one month horizon, which is most consistent with our model. The correlation with our uncertainty index is highly significant and substantial at 81%.

Macroeconomic uncertainty may be correlated with political uncertainty, which has recently been proposed as a source of asset market risk premiums (Pastor and Veronesi, 2013). Baker, Bloom and Davis (2016) create a policy uncertainty measure, based on newspaper coverage frequency, which we denote by UC_{BBD}. The index shows a highly significant correlation of 0.34 with our uncertainty index. One advantage of UC_{BBD} relative to the Jurado, Ludvigson, and Ng (2015) measure is that it can also be computed at the daily frequency. However, our financial proxy to uncertainty can also be computed at the daily frequency.

VI.D Daily Risk Aversion and Uncertainty Indices

Monthly indices may hide important variation within the month in uncertainty and risk aversion. To demonstrate this, Figure 8 shows how the indices behaved around two critical events in the recent global financial crisis: the Bear Stearns collapse and bail out and the Lehman Brothers bankruptcy. In general, Bear Stearns' woes generated less effect on our measures than did Lehman Brothers, as expected. To be more specific, Figure 8 plots 2-month intervals of our daily risk aversion index (top plots) and our daily financial instrument proxy to economic uncertainty index (bottom plots) around the two events. By the end of February and March 2008, the risk aversion index reached 3.0 and 2.7, respectively; the difference is small, considering the substantive time variation in the full sample. However, our daily risk aversion index climbed mildly to around 3.8 on the day of Bear Stearns bailout (March 14, 2008, Friday). The uncertainty index also kept increasing until that day, in a mild way. Uncertainty and risk aversion drop steeply afterwards. During August and September 2008, both risk aversion and uncertainty gradually increase, with risk aversion rising to around 3.7 on the day of Lehman Brothers' bankruptcy—which is the same value reached during Bear Stearns' collapse. However, as the magnification of the Lehman Brothers bankruptcy became clear to financial market participants, both risk aversion and uncertainty continues to rise, with risk aversion rising to 13.225 on October 10th which corresponds to the coordinated global action by central banks to lower interest rate.[13]

VII Conclusion

We formulate a no arbitrage model where fundamentals such as industrial production, consumption earnings ratios, corporate loss rates, etc. follow dynamic processes that admit time-variation in both conditional variances and the shape of the shock distribution. The agent in the economy takes this time-varying uncertainty into account when pricing equity and corporate bonds, but also faces preference shocks imperfectly correlated with fundamentals. The state variables in the economy that drive risk premiums and higher order moments of asset prices involve risk aversion, good and bad economic uncertainty and the conditional variance of loss rates on corporate bonds. We use equity and corporate bond returns, physical equity and corporate bond return variances and the risk neutral equity variance to estimate the model parameters and simultaneously

[13]On October 8th, the Federal Reserve and the central banks of the EU, Canada, UK, Sweden and Switzerland cut their rates by half a point. China's central bank cut its rate by .27 of a point. This was done to lower LIBOR, thus lowering the cost of bank borrowing. Overnight bank lending rates dropped in response, indicating a potential turning point in the crisis. (Source: Guardian, "Global rate cuts helps ease overnight interbank rates," October 8, 2008)

derive a risk aversion spanning process . Risk aversion is a function of 6 financial instruments, namely the term spread, credit spread, a detrended earnings yield, realized and risk-neutral equity return variance, and realized corporate bond return variance.

We find that risk aversion loads significantly and positively on the risk neutral equity variance and the realized corporate bond variance, and negatively on the realized equity return variance. Risk aversion is much less persistent than the risk aversion process implied by standard habit models. It is the main driver of the equity premium and the equity return risk neutral variance. It also accounts for 68% of the conditional variance of equity returns with the remainder accounted for by bad macro uncertainty. For corporate bonds, bad economic uncertainty plays a relatively more important role. It accounts for 27% of the risk premium variation and 89% of the corporate bond physical variance. Hence, different asset markets reflect differential information about risk appetite versus economic uncertainty. Our model-implied risk premiums often beat standard predictors of equity and corporate bond returns in an out-of-sample horse race, but the performance for corporate bonds is weaker than for equities.

While our risk aversion measure is correlated with some existing risk appetite and sentiment indices, the simplest approximation may be the variance risk premium in equity markets which is 96% correlated with our risk appetite index.

Because the spanning instruments are financial data, we can track the risk aversion index at higher frequencies. Similarly, we obtain a financial proxy to economic uncertainty (the conditional variance of industrial production growth) which can be obtained at the daily frequency as well. This measure is 81% correlated with the well-known Jurado, Ludvigson and Ng (2015) measure, extracted from macro data. The financial proxy to economic uncertainty predicts output growth negatively and significantly and is a much stronger predictor of output growth than is the VIX. We plan to make both our risk aversion and uncertainty indices available on our websites and update them regularly, which could potentially be useful for both academic researchers and practitioners.

Our work has important implications for the dynamic asset pricing literature. Clearly, to provide a unified framework explaining asset return dynamics in different asset classes, both changes in risk aversion and economic uncertainty must be accommodated. In addition, aggregate risk aversion must contain a relatively non-persistent variable component.

Finally, we only used risky asset classes to create the risk appetite index, omitting Treasury bonds, arguably the second most important asset class. In principle, given a process for inflation our model should also price Treasury bonds. In fact, Cremers, Fleckenstein and Ghandi (2017) claim that an implied volatility measure computed from Treasury bonds predicts the level and volatility of macro-economic indicators better than stock market implied indicators do. However, for our purposes, the problem with considering Treasuries as determining general risk aversion is that they are often viewed as the benchmark "safe" assets and are subject to occasional flights-to safety (see Baele et al, 2017). This makes it ex-ante unlikely that a simple pricing model such as ours can jointly price the three assets classes. We therefore defer incorporating Treasury bonds to future work.

Appendices

A The state variables

I.A Matrix representation of the state variables

In this section, we show the matrix representation of the system of ten state variables in this economy. The ten state variables, as introduced in Section 3, are as follows,

$$\boldsymbol{Y_t} = \left[\theta_t, p_t, n_t, \pi_t, l_t, g_t, \kappa_t, \eta_t, lp_t, q_t\right]',$$

where $\{p_t, n_t\}$ denote the upside uncertainty factor and the downside uncertainty factor, as latent variables extracted from the system of output growth (i.e., change in log real industrial production index); π_t represents the inflation rate; l_t represents the log of corporate loss rate; g_t represents the log change in real earnings; κ_t represents the log consumption-earnings ratio; η_t represents the log dividend payout ratio; lp_t represents the cash flow uncertainty factor, as the latent variable extracted from the system of corporate loss rate l_t; q_t represents the latent risk aversion of the economy. The state variables have the following matrix representation:

$$\boldsymbol{Y_{t+1}} = \boldsymbol{\mu} + \boldsymbol{A}\boldsymbol{Y_t} + \boldsymbol{\Sigma}\boldsymbol{\omega_{t+1}}, \tag{A.1}$$

where $\boldsymbol{\omega_{t+1}} = [\omega_{p,t+1}, \omega_{n,t+1}, \omega_{\pi,t+1}, \omega_{lp,t+1}, \omega_{ln,t+1}, \omega_{g,t+1}, \omega_{\kappa,t+1}, \omega_{\eta,t+1}, \omega_{q,t+1}]$ (9×1) is a vector comprised of eight independent shocks in the economy. Among the nine shocks, $\{\omega_{\pi,t+1}, \omega_{ln,t+1}, \omega_{g,t+1}, \omega_{\kappa,t+1}, \omega_{\eta,t+1}\}$ shocks are homoskedastic. The conditional variance, skewness and higher-order moments of the following four centered gamma shocks—$\omega_{p,t+1}$, $\omega_{n,t+1}$, $\omega_{lp,t+1}$, and $\omega_{q,t+1}$ —are assumed to be proportional to p_t, n_t, lp_t, and q_t respectively. The underlying distributions for the rest four shocks are assumed to be Gaussian with unit standard deviation.

The constant matrices are defined implicitly,

$$\boldsymbol{\mu} = \begin{bmatrix} (1-\rho_\theta)\bar{\theta} - m_p\bar{p} - m_n\bar{n} \equiv \theta_0 \\ (1-\rho_p)\bar{p} \equiv p_0 \\ (1-\rho_n)\bar{n} \equiv n_0 \\ \pi_0 \\ l_0 \\ g_0 \\ \kappa_0 \\ \eta_0 \\ v_0 \\ q_0 \end{bmatrix}, \tag{A.2}$$

$$\boldsymbol{A} = \begin{bmatrix} \rho_\theta & m_p & m_n & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \rho_p & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \rho_n & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \rho_{\pi\theta} & \rho_{\pi p} & \rho_{\pi n} & \rho_{\pi\pi} & \rho_{\pi l} & 0 & 0 & 0 & \rho_{\pi v} & 0 \\ 0 & \rho_{lp} & \rho_{ln} & 0 & \rho_{ll} & 0 & 0 & 0 & \rho_{lv} & 0 \\ \rho_{g\theta} & \rho_{gp} & \rho_{gn} & \rho_{gl} & 0 & \rho_{gg} & 0 & 0 & \rho_{gv} & 0 \\ \rho_{\kappa\theta} & \rho_{\kappa p} & \rho_{\kappa n} & \rho_{\kappa l} & 0 & 0 & \rho_{\kappa\kappa} & 0 & \rho_{\kappa v} & 0 \\ \rho_{\eta\theta} & \rho_{\eta p} & \rho_{\eta n} & \rho_{\eta l} & 0 & 0 & 0 & \rho_{\eta\eta} & \rho_{\eta v} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \rho_{vv} & 0 \\ 0 & \rho_{qp} & \rho_{qn} & 0 & 0 & 0 & 0 & 0 & 0 & \rho_{qq} \end{bmatrix}, \tag{A.3}$$

$$
\mathbf{\Sigma} = \begin{bmatrix} \sigma_{\theta p} & -\sigma_{\theta n} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \sigma_{pp} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \sigma_{nn} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \sigma_{\pi p} & \sigma_{\pi n} & \sigma_{\pi\pi} & \sigma_{\pi lp} & \sigma_{\pi ln} & 0 & 0 & 0 & 0 \\ \sigma_{lp} & \sigma_{ln} & 0 & \sigma_{llp} & -\sigma_{lln} & 0 & 0 & 0 & 0 \\ \sigma_{gp} & \sigma_{gn} & 0 & \sigma_{glp} & \sigma_{gln} & \sigma_{gg} & 0 & 0 & 0 \\ \sigma_{\kappa p} & \sigma_{\kappa n} & 0 & \sigma_{\kappa lp} & \sigma_{\kappa ln} & 0 & \sigma_{\kappa\kappa} & 0 & 0 \\ \sigma_{\eta p} & \sigma_{\eta n} & 0 & \sigma_{\eta lp} & \sigma_{\eta ln} & 0 & 0 & \sigma_{\eta\eta} & 0 \\ 0 & 0 & 0 & \sigma_{vl} & 0 & 0 & 0 & 0 & 0 \\ \sigma_{qp} & \sigma_{qn} & 0 & 0 & 0 & 0 & 0 & 0 & \sigma_{qq} \end{bmatrix}. \tag{A.4}
$$

Given the moment generating functions (mgf) of gamma and Gaussian distributions, we show that the model is affine, $\forall \boldsymbol{\nu} \in \mathbb{R}^{10}$,

$$
\begin{aligned}
M_Y(\boldsymbol{\nu}) := E_t\left[\exp(\boldsymbol{\nu}'\boldsymbol{Y_{t+1}})\right] &= \exp(\boldsymbol{\nu}'\boldsymbol{\mu} + \boldsymbol{\nu}'\boldsymbol{A}\boldsymbol{Y_t})E_t\left[\exp(\boldsymbol{\nu}'\mathbf{\Sigma}\boldsymbol{\omega_{t+1}})\right] \\
&= \exp\left[\boldsymbol{\nu}'\boldsymbol{S_0} + \frac{1}{2}\boldsymbol{\nu}'\boldsymbol{S_1}\mathbf{\Sigma}^{\boldsymbol{other}}\boldsymbol{S_1'}\boldsymbol{\nu} + \boldsymbol{f_S}(\boldsymbol{\nu})\boldsymbol{Y_t} + S_2(\boldsymbol{\nu})v_n\right], \qquad \text{(A.5)}
\end{aligned}
$$

where $\boldsymbol{S_0} = \boldsymbol{\mu}$ (10×1),

$$
\boldsymbol{S_1} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \tag{A.6}
$$

$$
\mathbf{\Sigma}^{\boldsymbol{other}} = \begin{bmatrix} \sigma^2_{\pi\pi} & \sigma_{\pi g} & \sigma_{\pi\kappa} & \sigma_{\pi\eta} \\ \sigma_{g\pi} & \sigma^2_{gg} & \sigma_{g\kappa} & \sigma_{g\eta} \\ \sigma_{\kappa\pi} & \sigma_{\kappa g} & \sigma^2_{\kappa\kappa} & \sigma_{\kappa\eta} \\ \sigma_{\eta\pi} & \sigma_{\eta g} & \sigma_{\eta\kappa} & \sigma^2_{\eta\eta} \end{bmatrix} \text{(cov-var matrix of } \{\omega_\pi, \omega_g, \omega_\kappa, \omega_\eta\}\text{)}, \tag{A.7}
$$

$$
\boldsymbol{f_S}(\boldsymbol{\nu}) = \boldsymbol{\nu}'\boldsymbol{A} + \begin{bmatrix} 0 \\ -\sigma_p(\boldsymbol{\nu}) - \ln(1-\sigma_p(\boldsymbol{\nu})) \\ -\sigma_n(\boldsymbol{\nu}) - \ln(1-\sigma_n(\boldsymbol{\nu})) \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -\sigma_{lp}(\boldsymbol{\nu}) - \ln(1-\sigma_{lp}(\boldsymbol{\nu})) \\ -\sigma_q(\boldsymbol{\nu}) - \ln(1-\sigma_q(\boldsymbol{\nu})) \end{bmatrix}', \tag{A.8}
$$

$$
\begin{aligned}
S_2(\boldsymbol{\nu}) &= -\sigma_{vn}(\boldsymbol{\nu}) - \ln(1-\sigma_{vn}(\boldsymbol{\nu})), && \text{(A.9)} \\
\sigma_p(\boldsymbol{\nu}) &= \boldsymbol{\nu}'\mathbf{\Sigma}_{\bullet\mathbf{1}}, && \text{(A.10)} \\
\sigma_n(\boldsymbol{\nu}) &= \boldsymbol{\nu}'\mathbf{\Sigma}_{\bullet\mathbf{2}}, && \text{(A.11)} \\
\sigma_{lp}(\boldsymbol{\nu}) &= \boldsymbol{\nu}'\mathbf{\Sigma}_{\bullet\mathbf{4}}, && \text{(A.12)} \\
\sigma_{vn}(\boldsymbol{\nu}) &= \boldsymbol{\nu}'\mathbf{\Sigma}_{\bullet\mathbf{5}}, && \text{(A.13)} \\
\sigma_q(\boldsymbol{\nu}) &= \boldsymbol{\nu}'\mathbf{\Sigma}_{\bullet\mathbf{9}}, && \text{(A.14)}
\end{aligned}
$$

where $\boldsymbol{M_{\bullet j}}$ denotes the j-th column of the matrix M.

I.B Consumption growth

Consumption growth in this economy is endogenous defined and can be expressed in an affine function:

$$\begin{aligned}
\Delta c_{t+1} &= g_{t+1} + \Delta\kappa_{t+1} && \text{(A.15)}\\
&= c_0 + \boldsymbol{c_2'Y_t} + \boldsymbol{c_1'\Sigma\omega_{t+1}}, && \text{(A.16)}\\
& && \text{(A.17)}
\end{aligned}$$

where $c_0 = g_0 + \kappa_0$, $\boldsymbol{c_1} = \begin{bmatrix}0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0\end{bmatrix}'$, and

$$\boldsymbol{c_2} = \begin{bmatrix} \rho_{g\theta} + \rho_{\kappa\theta} \\ \rho_{gp} + \rho_{\kappa p} \\ \rho_{gn} + \rho_{\kappa n} \\ 0 \\ 0 \\ \rho_{gg} \\ \rho_{\kappa\kappa} - 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}. \qquad \text{(A.18)}$$

B Asset Pricing

In this section, we solve the model analytically. First, given consumption growth and changes in risk aversion, the log of real pricing kernel of the economy is derived as an affine function of the state variables. Next, we show that asset prices of claims on cash flows from three different asset markets can be expressed in (quasi) affine equations. The model is solved using the non-arbitrage condition. The goal of this section is to derive the analytical solutions for the expected excess returns, the physical variance of asset returns and the risk-neutral variance of asset returns in closed forms. The implied moments are crucial for the estimation procedure.

II.A The real pricing kernel

The log real pricing kernel for this economy is given by,

$$\begin{aligned}
m_{t+1} &= \ln(\beta) - \gamma\Delta c_{t+1} + \gamma\Delta q_{t+1} && \text{(B.1)}\\
&= m_0 + \boldsymbol{m_2'Y_t} + \boldsymbol{m_1'\Sigma\omega_{t+1}}, && \text{(B.2)}
\end{aligned}$$

where $m_0 = \ln(\beta) + \gamma(q_0 - g_0 - \kappa_0)$, $\boldsymbol{m_1} = \begin{bmatrix}0 & 0 & 0 & 0 & 0 & -\gamma & -\gamma & 0 & 0 & \gamma\end{bmatrix}'$, and

$$\boldsymbol{m_2} = \begin{bmatrix} \gamma(-\rho_{g\theta} - \rho_{\kappa\theta}) \\ \gamma(\rho_{qp} - \rho_{gp} - \rho_{\kappa p}) \\ \gamma(\rho_{qn} - \rho_{gn} - \rho_{\kappa n}) \\ 0 \\ 0 \\ -\gamma\rho_{gg} \\ -\gamma(\rho_{\kappa\kappa} - 1) \\ 0 \\ 0 \\ \gamma(\rho_{qq} - 1) \end{bmatrix}. \qquad \text{(B.3)}$$

As a result, the moment generating function of the real pricing kernel is, $\forall\nu \in \mathbb{R}$,

$$E_t\left[\exp(\nu m_{t+1})\right] = \exp\left[\nu m_0 + \nu\boldsymbol{m_2'Y_t}\right]$$

$$\cdot \quad \exp\left\{\left[-\nu\sigma_p(\boldsymbol{m_1}) - \ln\left(1-\nu\sigma_p(\boldsymbol{m_1})\right)\right] p_t + \left[-\nu\sigma_n(\boldsymbol{m_1}) - \ln\left(1-\nu\sigma_n(\boldsymbol{m_1})\right)\right] n_t\right\}$$
$$\cdot \quad \exp\left\{\left[-\nu\sigma_{lp}(\boldsymbol{m_1}) - \ln\left(1-\nu\sigma_{lp}(\boldsymbol{m_1})\right)\right] lp_t + \left[-\nu\sigma_q(\boldsymbol{m_1}) - \ln\left(1-\nu\sigma_q(\boldsymbol{m_1})\right)\right] q_t\right\}$$
$$\cdot \quad \exp\left\{\left[-\nu\sigma_{vn}(\boldsymbol{m_1}) - \ln\left(1-\nu\sigma_{vn}(\boldsymbol{m_1})\right)\right] v_n + \frac{1}{2}\nu^2\left[\boldsymbol{m_1'S_1\Sigma^{other}S_1'm_1}\right]\right\}, \quad \text{(B.4)}$$

where m_0, $\boldsymbol{m_1}$, $\boldsymbol{m_2}$, $\boldsymbol{S_1}$, and $\boldsymbol{\Sigma^{other}}$ are constant matrices defined earlier, and

$$\begin{aligned}
\sigma_p(\boldsymbol{m_1}) &= \boldsymbol{m_1'\Sigma_{\bullet 1}}, & \text{(B.5)}\\
\sigma_n(\boldsymbol{m_1}) &= \boldsymbol{m_1'\Sigma_{\bullet 2}}, & \text{(B.6)}\\
\sigma_{lp}(\boldsymbol{m_1}) &= \boldsymbol{m_1'\Sigma_{\bullet 4}}, & \text{(B.7)}\\
\sigma_{vn}(\boldsymbol{m_1}) &= \boldsymbol{m_1'\Sigma_{\bullet 5}}, & \text{(B.8)}\\
\sigma_q(\boldsymbol{m_1}) &= \boldsymbol{m_1'\Sigma_{\bullet 9}}. & \text{(B.9)}
\end{aligned}$$

Accordingly, the model-implied short rate rf_t is,

$$\begin{aligned}
rf_t &= -\ln\left\{E_t\left[\exp(m_{t+1})\right]\right\} & \text{(B.10)}\\
&= -m_0 - \boldsymbol{m_2'Y_t} & \text{(B.11)}\\
&+ \left[\sigma_p(\boldsymbol{m_1}) + \ln\left(1-\sigma_p(\boldsymbol{m_1})\right)\right] p_t + \left[\sigma_n(\boldsymbol{m_1}) + \ln\left(1-\sigma_n(\boldsymbol{m_1})\right)\right] n_t & \text{(B.12)}\\
&+ \left[\sigma_{lp}(\boldsymbol{m_1}) + \ln\left(1-\sigma_{lp}(\boldsymbol{m_1})\right)\right] lp_t + \left[\sigma_q(\boldsymbol{m_1}) + \ln\left(1-\sigma_q(\boldsymbol{m_1})\right)\right] q_t & \text{(B.13)}\\
&+ \left[\sigma_{vn}(\boldsymbol{m_1}) + \ln\left(1-\sigma_{vn}(\boldsymbol{m_1})\right)\right] v_n - \frac{1}{2}\left[\boldsymbol{m_1'S_1\Sigma^{other}S_1'm_1}\right], & \text{(B.14)}\\
&= rf_0 + \boldsymbol{rf_2'Y_t}. & \text{(B.15)}
\end{aligned}$$

To price nominal assets, we define the nominal pricing kernel, $\widetilde{m}_{t+1}$, which is a simple transformation of the log real pricing kernel, m_{t+1},

$$\begin{aligned}
\widetilde{m}_{t+1} &= m_{t+1} - \pi_{t+1}, & \text{(B.16)}\\
&= \widetilde{m}_0 + \boldsymbol{\widetilde{m}_2'Y_t} + \boldsymbol{\widetilde{m}_1'\Sigma\omega_{t+1}}, & \text{(B.17)}
\end{aligned}$$

where $\widetilde{m}_0 = m_0 - \pi_0$, $\boldsymbol{\widetilde{m}_1} = \boldsymbol{m_1} - \begin{bmatrix}0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0\end{bmatrix}'$, and

$$\boldsymbol{\widetilde{m}_2} = \boldsymbol{m_2} - \begin{bmatrix}\rho_{\pi\theta}\\ \rho_{\pi p}\\ \rho_{\pi n}\\ \rho_{\pi\pi}\\ 0\\ 0\\ 0\\ 0\\ 0\\ 0\end{bmatrix}. \quad \text{(B.18)}$$

The nominal risk free rate $\widetilde{rf}_t$ is defined as $-\ln\left\{E_t\left[\exp(\widetilde{m}_{t+1})\right]\right\}$.

II.B Valuation ratio

It is a crucial step in this paper to show that asset prices are (quasi) affine functions of the state variables. It is especially not obvious for equity price-dividend ratio, of which we provide proofs below. First, we rewrite the real dividend growth in a general matrix expression:

$$\begin{aligned}
\Delta d_{t+1} &= g_{t+1} + \Delta\eta_{t+1}\\
&= h_0 + \boldsymbol{h_2'Y_t} + \boldsymbol{h_1'\Sigma\omega_{t+1}}, & \text{(B.19)}
\end{aligned}$$

where $h_0 = g_0 + \eta_0$, $\boldsymbol{h_1} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \end{bmatrix}'$, and

$$\boldsymbol{h_2} = \begin{bmatrix} \rho_{g\theta} + \rho_{\eta\theta} \\ \rho_{gp} + \rho_{\eta p} \\ \rho_{gn} + \rho_{\eta n} \\ 0 \\ 0 \\ \rho_{gg} \\ 0 \\ \rho_{\eta\eta} - 1 \\ 0 \\ 0 \end{bmatrix}. \tag{B.20}$$

The price-dividend ratio, $PD_t = E_t \left[M_{t+1} \left(\frac{P_{t+1} + D_{t+1}}{D_t} \right) \right]$, can be rewritten as,

$$PD_t = \sum_{n=1}^{\infty} E_t \left[\exp \left(\sum_{j=1}^{n} m_{t+j} + \Delta d_{t+j} \right) \right]. \tag{B.21}$$

Let F_t^n denote the n-th term in the summation:

$$F_t^n = E_t \left[\exp \left(\sum_{j=1}^{n} m_{t+j} + \Delta d_{t+j} \right) \right], \tag{B.22}$$

and $F_t^n D_t$ is the price of zero-coupon equity that matures in n periods.

To show that equity price is an approximate affine function of the state variables, we first prove that $F_t^n (\forall n \geq 1)$ is exactly affine using induction. First, when $n = 1$,

$$\begin{aligned} F_t^1 &= E_t \left[\exp \left(m_{t+1} + \Delta d_{t+1} \right) \right] \\ &= E_t \left\{ \exp \left[(m_0 + h_0) + \boldsymbol{(m_2' + h_2') Y_t} + \boldsymbol{(m_1' + h_1') \Sigma \omega_{t+1}} \right] \right\} \\ &= \exp \left[(m_0 + h_0) + \boldsymbol{(m_2' + h_2') Y_t} \right] \\ &\cdot \exp \left\{ \left[-\sigma_p(\boldsymbol{m_1} + \boldsymbol{h_1}) - \ln \left(1 - \sigma_p(\boldsymbol{m_1} + \boldsymbol{h_1}) \right) \right] p_t + \left[-\sigma_n(\boldsymbol{m_1} + \boldsymbol{h_1}) - \ln \left(1 - \sigma_n(\boldsymbol{m_1} + \boldsymbol{h_1}) \right) \right] n_t \right\} \\ &\cdot \exp \left\{ \left[-\sigma_{lp}(\boldsymbol{m_1} + \boldsymbol{h_1}) - \ln \left(1 - \sigma_{lp}(\boldsymbol{m_1} + \boldsymbol{h_1}) \right) \right] lp_t + \left[-\sigma_q(\boldsymbol{m_1} + \boldsymbol{h_1}) - \ln \left(1 - \sigma_q(\boldsymbol{m_1} + \boldsymbol{h_1}) \right) \right] q_t \right\} \\ &\cdot \exp \left\{ \left[-\sigma_{vn}(\boldsymbol{m_1} + \boldsymbol{h_1}) - \ln \left(1 - \sigma_{vn}(\boldsymbol{m_1} + \boldsymbol{h_1}) \right) \right] v_n + \frac{1}{2} \left[\boldsymbol{(m_1' + h_1') S_1 \Sigma^{other} S_1' (m_1 + h_1)} \right] \right\} \\ &= \exp \left(e_0^1 + \boldsymbol{e_1^{1\prime} Y_t} \right), \end{aligned} \tag{B.23}$$

where m_0, $\boldsymbol{m_1}$, $\boldsymbol{m_2}$, h_0, $\boldsymbol{h_1}$, $\boldsymbol{h_2}$, $\boldsymbol{S_1}$, and $\boldsymbol{\Sigma^{other}}$ are constant matrices defined earlier, and

$$\sigma_p(\boldsymbol{m_1} + \boldsymbol{h_1}) = (\boldsymbol{m_1'} + \boldsymbol{h_1'}) \boldsymbol{\Sigma_{\bullet 1}}, \tag{B.24}$$
$$\sigma_n(\boldsymbol{m_1} + \boldsymbol{h_1}) = (\boldsymbol{m_1'} + \boldsymbol{h_1'}) \boldsymbol{\Sigma_{\bullet 2}}, \tag{B.25}$$
$$\sigma_{lp}(\boldsymbol{m_1} + \boldsymbol{h_1}) = (\boldsymbol{m_1'} + \boldsymbol{h_1'}) \boldsymbol{\Sigma_{\bullet 4}}, \tag{B.26}$$
$$\sigma_{vn}(\boldsymbol{m_1} + \boldsymbol{h_1}) = (\boldsymbol{m_1'} + \boldsymbol{h_1'}) \boldsymbol{\Sigma_{\bullet 5}}, \tag{B.27}$$
$$\sigma_q(\boldsymbol{m_1} + \boldsymbol{h_1}) = (\boldsymbol{m_1'} + \boldsymbol{h_1'}) \boldsymbol{\Sigma_{\bullet 9}}, \tag{B.28}$$

and $e_0^1 = m_0 + h_0 + [-\sigma_{vn}(\boldsymbol{m_1} + \boldsymbol{h_1}) - \ln(1 - \sigma_{vn}(\boldsymbol{m_1} + \boldsymbol{h_1}))]\, v_n + \frac{1}{2}\left[(\boldsymbol{m_1'} + \boldsymbol{h_1'})\boldsymbol{S_1}\boldsymbol{\Sigma^{other}}\boldsymbol{S_1'}(\boldsymbol{m_1} + \boldsymbol{h_1})\right]$, and

$$\boldsymbol{e_1^1} = \boldsymbol{m_2} + \boldsymbol{h_2} + \begin{bmatrix} 0 \\ -\sigma_p \boldsymbol{m_1} + \boldsymbol{h_1}) - \ln(1 - \sigma_p(\boldsymbol{m_1} + \boldsymbol{h_1})) \\ -\sigma_n(\boldsymbol{m_1} + \boldsymbol{h_1}) - \ln(1 - \sigma_n(\boldsymbol{m_1} + \boldsymbol{h_1})) \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -\sigma_{lp}(\boldsymbol{m_1} + \boldsymbol{h_1}) - \ln(1 - \sigma_{lp}(\boldsymbol{m_1} + \boldsymbol{h_1})) \\ -\sigma_q(\boldsymbol{m_1} + \boldsymbol{h_1}) - \ln(1 - \sigma_q(\boldsymbol{m_1} + \boldsymbol{h_1})) \end{bmatrix}. \tag{B.29}$$

Now, suppose that the $(n-1)$-th term $F_t^{n-1} = \exp\left(e_0^{n-1} + \boldsymbol{e_1^{n-1\prime}}\boldsymbol{Y_t}\right)$, then

$$\begin{aligned} F_t^n &= E_t\left[\exp\left(\sum_{j=1}^{n} m_{t+j} + \Delta d_{t+j}\right)\right] \\ &= E_t\left\{E_{t+1}\left[\exp(m_{t+1} + \Delta d_{t+1})\exp\left(\sum_{j=1}^{n-1} m_{t+j+1} + \Delta d_{t+j+1}\right)\right]\right\} \\ &= E_t\left\{\exp(m_{t+1} + \Delta d_{t+1})\underbrace{E_{t+1}\left[\exp\left(\sum_{j=1}^{n-1} m_{t+j+1} + \Delta d_{t+j+1}\right)\right]}_{F_{t+1}^{n-1}}\right\} \\ &= E_t\left[\exp(m_{t+1} + \Delta d_{t+1})\exp\left(e_0^{n-1} + \boldsymbol{e_1^{n-1\prime}}\boldsymbol{Y_{t+1}}\right)\right] \\ &= \exp\left(e_0^n + \boldsymbol{e_1^{n\prime}}\boldsymbol{Y_t}\right), \end{aligned} \tag{B.30}$$

where e_0^n and $\boldsymbol{e_1^{n\prime}}$ are defined implicitly.

Hence, the price-dividend ratio is approximately affine:

$$\begin{aligned} PD_t &= \sum_{n=1}^{\infty} E_t\left[\exp\left(\sum_{j=1}^{n} m_{t+j} + \Delta d_{t+j}\right)\right] \\ &= \sum_{n=1}^{\infty} F_t^n \\ &= \sum_{n=1}^{\infty} \exp\left(e_0^n + \boldsymbol{e_1^{n\prime}}\boldsymbol{Y_t}\right). \end{aligned} \tag{B.31}$$

■

II.C Log nominal equity return

We apply first-order Taylor approximations to the log nominal equity return, and obtain a linear system,

$$\begin{aligned} \tilde{r}_{t+1}^{eq} &= \ln\left(\frac{P_{t+1} + D_{t+1}}{P_t}\Pi_{t+1}\right) \\ &= \ln\left(\frac{PD_{t+1} + 1}{PD_t}\right)\ln\left(\frac{D_{t+1}}{D_t}\right)\ln(\Pi_{t+1}) \end{aligned}$$

$$= \Delta d_{t+1} + \pi_{t+1} + \ln\left[\frac{1+\sum_{n=1}^{\infty}\exp\left(e_0^n + \boldsymbol{e_1^{n\prime}Y_{t+1}}\right)}{\sum_{n=1}^{\infty}\exp\left(e_0^n + \boldsymbol{e_1^{n\prime}Y_t}\right)}\right]$$
$$\approx \Delta d_{t+1} + \pi_{t+1} + \text{const.} + \frac{\sum_{n=1}^{\infty}\exp\left(e_0^n + \boldsymbol{e_1^{n\prime}\bar{Y}}\right)\boldsymbol{e_1^{n\prime}}}{1+\sum_{n=1}^{\infty}\exp\left(e_0^n + \boldsymbol{e_1^{n\prime}\bar{Y}}\right)}\boldsymbol{Y_{t+1}} - \frac{\sum_{n=1}^{\infty}\exp\left(e_0^n + \boldsymbol{e_1^{n\prime}\bar{Y}}\right)\boldsymbol{e_1^{n\prime}}}{\sum_{n=1}^{\infty}\exp\left(e_0^n + \boldsymbol{e_1^{n\prime}\bar{Y}}\right)}\boldsymbol{Y_t}$$
$$= \widetilde{\xi}_0^{eq} + \boldsymbol{\widetilde{\xi}_1^{eq\prime}Y_t} + \boldsymbol{\widetilde{r}^{eq\prime}\Sigma\omega_{t+1}}, \tag{B.32}$$

where $\widetilde{r}_{t+1}^{eq}$ is the log *nominal* return of asset i from t to $t+1$, $\widetilde{\xi}_0^{eq}$ is constant, $\boldsymbol{\widetilde{\xi}_1^{eq}}$ is a vector of state vector coefficients, and $\boldsymbol{\widetilde{r}^{eq}}$ is a vector of shock coefficients. Thus, this step involves linear approximation.

More generally, to acknowledge the errors that are potentially caused by the linear approximations (the Taylor approximation in log price-dividend ratio in the return equation), we write down the return innovations for asset i with an idiosyncratic shock:

$$\widetilde{r}_{t+1}^i - E_t\left(\widetilde{r}_{t+1}^i\right) = \boldsymbol{\widetilde{r}^{i\prime}\Sigma\omega_{t+1}} + \varepsilon_{t+1}^i, \tag{B.33}$$

where $E_t\left(\widetilde{r}_{t+1}^i\right)$ is the expected return, $\boldsymbol{\widetilde{r}^i}$ (10×1) is the asset i return loadings on selected state variable innovations (the choice of which depends on the asset classes), and ε_{t+1}^i is the Gaussian noise uncorrelated with the state variable shocks but may be cross-correlated (with other asset-specific shocks). The Gaussian shock ε_{t+1}^i has an unconditional variance σ_i^2.

II.D Model-implied moments

In this section, we derive three model-implied asset conditional moments— expected excess returns, physical and risk-neutral conditional variances of nominal asset returns. The moments are crucial in creating the moment conditions during the third step of model estimation.

II.D.1 One-period expected excess return

We impose the no-arbitrage condition, $1 = E_t[\exp(\widetilde{m}_{t+1} + \widetilde{r}_{t+1}^i)]$ ($\forall i \in$\{equity, treasury bond, corporate bond\}), and obtain the expected excess returns. Expand the law of one price (LOOP) equation:

$$\begin{aligned}
1 &= E_t[\exp(\widetilde{m}_{t+1} + \widetilde{r}_{t+1}^i)] \\
&= \exp\left[E_t(\widetilde{m}_{t+1}) + E_t(\widetilde{r}_{t+1}^i)\right] \\
&\cdot \exp\left\{\left[-\sigma_p(\boldsymbol{\widetilde{m}_1} + \boldsymbol{\widetilde{r}^i}) - \ln\left(1 - \sigma_p(\boldsymbol{\widetilde{m}_1} + \boldsymbol{\widetilde{r}^i})\right)\right]p_t + \left[-\sigma_n(\boldsymbol{\widetilde{m}_1} + \boldsymbol{\widetilde{r}^i}) - \ln\left(1 - \sigma_n(\boldsymbol{\widetilde{m}_1} + \boldsymbol{\widetilde{r}^i})\right)\right]n_t\right\} \\
&\cdot \exp\left\{\left[-\sigma_{lp}(\boldsymbol{\widetilde{m}_1} + \boldsymbol{\widetilde{r}^i}) - \ln\left(1 - \sigma_{lp}(\boldsymbol{\widetilde{m}_1} + \boldsymbol{\widetilde{r}^i})\right)\right]lp_t + \left[-\sigma_q(\boldsymbol{\widetilde{m}_1} + \boldsymbol{\widetilde{r}^i}) - \ln\left(1 - \sigma_q(\boldsymbol{\widetilde{m}_1} + \boldsymbol{\widetilde{r}^i})\right)\right]q_t\right\} \\
&\cdot \exp\left\{\left[-\sigma_{vn}(\boldsymbol{\widetilde{m}_1} + \boldsymbol{\widetilde{r}^i}) - \ln\left(1 - \sigma_{vn}(\boldsymbol{\widetilde{m}_1} + \boldsymbol{\widetilde{r}^i})\right)\right]v_n + \frac{1}{2}\left[(\boldsymbol{\widetilde{m}_1'} + \boldsymbol{\widetilde{r}^{i\prime}})\boldsymbol{S_1\Sigma^{other}S_1'}(\boldsymbol{\widetilde{m}_1} + \boldsymbol{\widetilde{r}^i}) + \sigma_i^2\right]\right\}
\end{aligned} \tag{B.34}$$

where $\boldsymbol{\widetilde{m}_1}$, $\boldsymbol{\widetilde{r}^i}$, σ_i, $\boldsymbol{S_1}$, and $\boldsymbol{\Sigma^{other}}$ are constant matrices defined earlier, and

$$\begin{aligned}
\sigma_p(\boldsymbol{\widetilde{m}_1} + \boldsymbol{\widetilde{r}^i}) &= (\boldsymbol{\widetilde{m}_1'} + \boldsymbol{\widetilde{r}^{i\prime}})\boldsymbol{\Sigma_{\bullet 1}}, \\
\sigma_n(\boldsymbol{\widetilde{m}_1} + \boldsymbol{\widetilde{r}^i}) &= (\boldsymbol{\widetilde{m}_1'} + \boldsymbol{\widetilde{r}^{i\prime}})\boldsymbol{\Sigma_{\bullet 2}}, \\
\sigma_{lp}(\boldsymbol{\widetilde{m}_1} + \boldsymbol{\widetilde{r}^i}) &= (\boldsymbol{\widetilde{m}_1'} + \boldsymbol{\widetilde{r}^{i\prime}})\boldsymbol{\Sigma_{\bullet 4}}, \\
\sigma_{vn}(\boldsymbol{\widetilde{m}_1} + \boldsymbol{\widetilde{r}^i}) &= (\boldsymbol{\widetilde{m}_1'} + \boldsymbol{\widetilde{r}^{i\prime}})\boldsymbol{\Sigma_{\bullet 5}}, \\
\sigma_q(\boldsymbol{\widetilde{m}_1} + \boldsymbol{\widetilde{r}^i}) &= (\boldsymbol{\widetilde{m}_1'} + \boldsymbol{\widetilde{r}^{i\prime}})\boldsymbol{\Sigma_{\bullet 9}}.
\end{aligned} \tag{B.35}$$

Given the nominal risk free rate derived earlier using real pricing kernel and inflation, the nominal excess return is,

$$\begin{aligned}
E_t(\widetilde{r}_{t+1}^i) - \widetilde{rf}_t &= \left\{\sigma_p(\boldsymbol{\widetilde{r}^i}) + \ln\left[\frac{1 - \sigma_p(\boldsymbol{\widetilde{m}_1} + \boldsymbol{\widetilde{r}^i})}{1 - \sigma_p(\boldsymbol{\widetilde{m}_1})}\right]\right\}p_t \\
&+ \left\{\sigma_n(\boldsymbol{\widetilde{r}^i}) + \ln\left[\frac{1 - \sigma_n(\boldsymbol{\widetilde{m}_1} + \boldsymbol{\widetilde{r}^i})}{1 - \sigma_n(\boldsymbol{\widetilde{m}_1})}\right]\right\}n_t
\end{aligned}$$

$$
\begin{aligned}
&+ && \left\{\sigma_{lp}(\widetilde{\boldsymbol{r}}^{\boldsymbol{i}}) + \ln\left[\frac{1-\sigma_{lp}(\widetilde{\boldsymbol{m}}_{\boldsymbol{1}}+\widetilde{\boldsymbol{r}}^{\boldsymbol{i}})}{1-\sigma_{lp}(\widetilde{\boldsymbol{m}}_{\boldsymbol{1}})}\right]\right\} lp_t \\
&+ && \left\{\sigma_{q}(\widetilde{\boldsymbol{r}}^{\boldsymbol{i}}) + \ln\left[\frac{1-\sigma_{q}(\widetilde{\boldsymbol{m}}_{\boldsymbol{1}}+\widetilde{\boldsymbol{r}}^{\boldsymbol{i}})}{1-\sigma_{q}(\widetilde{\boldsymbol{m}}_{\boldsymbol{1}})}\right]\right\} q_t \\
&+ && \left\{\sigma_{vn}(\widetilde{\boldsymbol{r}}^{\boldsymbol{i}}) + \ln\left[\frac{1-\sigma_{vn}(\widetilde{\boldsymbol{m}}_{\boldsymbol{1}}+\widetilde{\boldsymbol{r}}^{\boldsymbol{i}})}{1-\sigma_{vn}(\widetilde{\boldsymbol{m}}_{\boldsymbol{1}})}\right]\right\} v_n - \widetilde{\boldsymbol{m}}_{\boldsymbol{1}}' \boldsymbol{S}_{\boldsymbol{1}} \boldsymbol{\Sigma}^{\boldsymbol{other}} \boldsymbol{S}_{\boldsymbol{1}}' \widetilde{\boldsymbol{r}}^{\boldsymbol{i}} - \frac{1}{2}\left[\widetilde{\boldsymbol{r}}^{\boldsymbol{i}\prime} \boldsymbol{S}_{\boldsymbol{1}} \boldsymbol{\Sigma}^{\boldsymbol{other}} \boldsymbol{S}_{\boldsymbol{1}}' \widetilde{\boldsymbol{r}}^{\boldsymbol{i}} + \sigma_i^2\right]
\end{aligned}
\tag{B.36}
$$

where

$$\sigma_p(\widetilde{\boldsymbol{r}}^{\boldsymbol{i}}) = \widetilde{\boldsymbol{r}}^{\boldsymbol{i}\prime}\boldsymbol{\Sigma}_{\bullet\boldsymbol{1}}, \tag{B.37}$$

$$\sigma_n(\widetilde{\boldsymbol{r}}^{\boldsymbol{i}}) = \widetilde{\boldsymbol{r}}^{\boldsymbol{i}\prime}\boldsymbol{\Sigma}_{\bullet\boldsymbol{2}}, \tag{B.38}$$

$$\sigma_{lp}(\widetilde{\boldsymbol{r}}^{\boldsymbol{i}}) = \widetilde{\boldsymbol{r}}^{\boldsymbol{i}\prime}\boldsymbol{\Sigma}_{\bullet\boldsymbol{4}}, \tag{B.39}$$

$$\sigma_{vn}(\widetilde{\boldsymbol{r}}^{\boldsymbol{i}}) = \widetilde{\boldsymbol{r}}^{\boldsymbol{i}\prime}\boldsymbol{\Sigma}_{\bullet\boldsymbol{5}}, \tag{B.40}$$

$$\sigma_q(\widetilde{\boldsymbol{r}}^{\boldsymbol{i}}) = \widetilde{\boldsymbol{r}}^{\boldsymbol{i}\prime}\boldsymbol{\Sigma}_{\bullet\boldsymbol{9}}, \tag{B.41}$$

$$\sigma_p(\widetilde{\boldsymbol{m}}_{\boldsymbol{1}}+\widetilde{\boldsymbol{r}}^{\boldsymbol{i}}) = (\widetilde{\boldsymbol{m}}_{\boldsymbol{1}}' + \widetilde{\boldsymbol{r}}^{\boldsymbol{i}\prime})\boldsymbol{\Sigma}_{\bullet\boldsymbol{1}}, \tag{B.42}$$

$$\sigma_n(\widetilde{\boldsymbol{m}}_{\boldsymbol{1}}+\widetilde{\boldsymbol{r}}^{\boldsymbol{i}}) = (\widetilde{\boldsymbol{m}}_{\boldsymbol{1}}' + \widetilde{\boldsymbol{r}}^{\boldsymbol{i}\prime})\boldsymbol{\Sigma}_{\bullet\boldsymbol{2}}, \tag{B.43}$$

$$\sigma_{lp}(\widetilde{\boldsymbol{m}}_{\boldsymbol{1}}+\widetilde{\boldsymbol{r}}^{\boldsymbol{i}}) = (\widetilde{\boldsymbol{m}}_{\boldsymbol{1}}' + \widetilde{\boldsymbol{r}}^{\boldsymbol{i}\prime})\boldsymbol{\Sigma}_{\bullet\boldsymbol{4}}, \tag{B.44}$$

$$\sigma_{vn}(\widetilde{\boldsymbol{m}}_{\boldsymbol{1}}+\widetilde{\boldsymbol{r}}^{\boldsymbol{i}}) = (\widetilde{\boldsymbol{m}}_{\boldsymbol{1}}' + \widetilde{\boldsymbol{r}}^{\boldsymbol{i}\prime})\boldsymbol{\Sigma}_{\bullet\boldsymbol{5}}, \tag{B.45}$$

$$\sigma_q(\widetilde{\boldsymbol{m}}_{\boldsymbol{1}}+\widetilde{\boldsymbol{r}}^{\boldsymbol{i}}) = (\widetilde{\boldsymbol{m}}_{\boldsymbol{1}}' + \widetilde{\boldsymbol{r}}^{\boldsymbol{i}\prime})\boldsymbol{\Sigma}_{\bullet\boldsymbol{9}}. \tag{B.46}$$

II.D.2 One-period physical conditional return variance

The physical variance is easily obtained given the loadings:

$$
\begin{aligned}
VAR_t(\widetilde{r}^i_{t+1}) &= \left(\sigma_p(\widetilde{\boldsymbol{r}}^{\boldsymbol{i}})\right)^2 p_t + \left(\sigma_n(\widetilde{\boldsymbol{r}}^{\boldsymbol{i}})\right)^2 n_t + \left(\sigma_{lp}(\widetilde{\boldsymbol{r}}^{\boldsymbol{i}})\right)^2 lp_t + \left(\sigma_q(\widetilde{\boldsymbol{r}}^{\boldsymbol{i}})\right)^2 q_t \\
&+ \left(\sigma_{vn}\widetilde{\boldsymbol{r}}^{\boldsymbol{i}})\right)^2 v_n + \widetilde{\boldsymbol{r}}^{\boldsymbol{i}\prime} \boldsymbol{S}_{\boldsymbol{1}} \boldsymbol{\Sigma}^{\boldsymbol{other}} \boldsymbol{S}_{\boldsymbol{1}}' \widetilde{\boldsymbol{r}}^{\boldsymbol{i}} + \sigma_i^2.
\end{aligned}
\tag{B.47}
$$

II.D.3 One-period risk-neutral conditional return variance

To obtain the risk-neutral variance of the asset returns, we use the moment generating function under the risk-neutral measure:

$$
\begin{aligned}
mgf_t^Q(\widetilde{r}^i_{t+1};\nu) &= \frac{E_t\left[\exp\left(\widetilde{m}_{t+1} + \nu\widetilde{r}^i_{t+1}\right)\right]}{E_t\left[\exp\left(\widetilde{m}_{t+1}\right)\right]} \\
&= \exp\left\{E_t(\widetilde{m}_{t+1}) + \nu E_t(\widetilde{r}^i_{t+1})\right\} \\
&\cdot \exp\left\{\left[-\sigma_p(\widetilde{\boldsymbol{m}}_{\boldsymbol{1}}+\nu\widetilde{\boldsymbol{r}}^{\boldsymbol{i}}) - \ln\left(1-\sigma_p(\widetilde{\boldsymbol{m}}_{\boldsymbol{1}}+\nu\widetilde{\boldsymbol{r}}^{\boldsymbol{i}})\right)\right] p_t\right\} \\
&\cdot \exp\left\{\left[-\sigma_n(\widetilde{\boldsymbol{m}}_{\boldsymbol{1}}+\nu\widetilde{\boldsymbol{r}}^{\boldsymbol{i}}) - \ln\left(1-\sigma_n(\widetilde{\boldsymbol{m}}_{\boldsymbol{1}}+\nu\widetilde{\boldsymbol{r}}^{\boldsymbol{i}})\right)\right] n_t\right\} \\
&\cdot \exp\left\{\left[-\sigma_{lp}(\widetilde{\boldsymbol{m}}_{\boldsymbol{1}}+\nu\widetilde{\boldsymbol{r}}^{\boldsymbol{i}}) - \ln\left(1-\sigma_{lp}(\widetilde{\boldsymbol{m}}_{\boldsymbol{1}}+\nu\widetilde{\boldsymbol{r}}^{\boldsymbol{i}})\right)\right] lp_t\right\} \\
&\cdot \exp\left\{\left[-\sigma_q(\widetilde{\boldsymbol{m}}_{\boldsymbol{1}}+\nu\widetilde{\boldsymbol{r}}^{\boldsymbol{i}}) - \ln\left(1-\sigma_q(\widetilde{\boldsymbol{m}}_{\boldsymbol{1}}+\nu\widetilde{\boldsymbol{r}}^{\boldsymbol{i}})\right)\right] q_t\right\} \\
&\cdot \exp\left\{\left[-\sigma_{vn}(\widetilde{\boldsymbol{m}}_{\boldsymbol{1}}+\nu\widetilde{\boldsymbol{r}}^{\boldsymbol{i}}) - \ln\left(1-\sigma_{vn}(\widetilde{\boldsymbol{m}}_{\boldsymbol{1}}+\nu\widetilde{\boldsymbol{r}}^{\boldsymbol{i}})\right)\right] v_n + \frac{1}{2}\left[(\widetilde{\boldsymbol{m}}_{\boldsymbol{1}}' + \nu\widetilde{\boldsymbol{r}}^{\boldsymbol{i}\prime}) \boldsymbol{S}_{\boldsymbol{1}} \boldsymbol{\Sigma}^{\boldsymbol{other}} \boldsymbol{S}_{\boldsymbol{1}}' (\widetilde{\boldsymbol{m}}_{\boldsymbol{1}}+\nu\widetilde{\boldsymbol{r}}^{\boldsymbol{i}}) + \nu\right.\right. \\
&/ \exp\left\{E_t(\widetilde{m}_{t+1})\right\} \\
&/ \exp\left\{\left[-\sigma_p(\widetilde{\boldsymbol{m}}_{\boldsymbol{1}}) - \ln\left(1-\sigma_p(\widetilde{\boldsymbol{m}}_{\boldsymbol{1}})\right)\right] p_t + \left[-\sigma_n(\widetilde{\boldsymbol{m}}_{\boldsymbol{1}}) - \ln\left(1-\sigma_n(\widetilde{\boldsymbol{m}}_{\boldsymbol{1}})\right)\right] n_t\right\} \\
&/ \exp\left\{\left[-\sigma_{lp}(\widetilde{\boldsymbol{m}}_{\boldsymbol{1}}) - \ln\left(1-\sigma_{lp}(\widetilde{\boldsymbol{m}}_{\boldsymbol{1}})\right)\right] lp_t + \left[-\sigma_q(\widetilde{\boldsymbol{m}}_{\boldsymbol{1}}) - \ln\left(1-\sigma_q(\widetilde{\boldsymbol{m}}_{\boldsymbol{1}})\right)\right] q_t\right\} \\
&/ \exp\left\{\left[-\sigma_{vn}(\widetilde{\boldsymbol{m}}_{\boldsymbol{1}}) - \ln\left(1-\sigma_{vn}(\widetilde{\boldsymbol{m}}_{\boldsymbol{1}})\right)\right] v_n + \frac{1}{2}\left[\widetilde{\boldsymbol{m}}_{\boldsymbol{1}}' \boldsymbol{S}_{\boldsymbol{1}} \boldsymbol{\Sigma}^{\boldsymbol{other}} \boldsymbol{S}_{\boldsymbol{1}}' \widetilde{\boldsymbol{m}}_{\boldsymbol{1}}\right]\right\} \\
&= \exp\left\{\nu E_t(\widetilde{r}^i_{t+1})\right\}
\end{aligned}
$$

$$
\begin{aligned}
&\cdot \quad \exp\left\{\left[-\sigma_p(\nu\widetilde{\boldsymbol{r}}^{\boldsymbol{i}}) - \ln\left(\frac{1-\sigma_p(\widetilde{\boldsymbol{m}}_{\boldsymbol{1}}+\nu\widetilde{\boldsymbol{r}}^{\boldsymbol{i}})}{1-\sigma_p(\widetilde{\boldsymbol{m}}_{\boldsymbol{1}})}\right)\right] p_t\right\}\\
&\cdot \quad \exp\left\{\left[-\sigma_n(\nu\widetilde{\boldsymbol{r}}^{\boldsymbol{i}}) - \ln\left(\frac{1-\sigma_n(\widetilde{\boldsymbol{m}}_{\boldsymbol{1}}+\nu\widetilde{\boldsymbol{r}}^{\boldsymbol{i}})}{1-\sigma_n(\widetilde{\boldsymbol{m}}_{\boldsymbol{1}})}\right)\right] n_t\right\}\\
&\cdot \quad \exp\left\{\left[-\sigma_{lp}(\nu\widetilde{\boldsymbol{r}}^{\boldsymbol{i}}) - \ln\left(\frac{1-\sigma_{lp}(\widetilde{\boldsymbol{m}}_{\boldsymbol{1}}+\nu\widetilde{\boldsymbol{r}}^{\boldsymbol{i}})}{1-\sigma_{lp}(\widetilde{\boldsymbol{m}}_{\boldsymbol{1}})}\right)\right] lp_t\right\}\\
&\cdot \quad \exp\left\{\left[-\sigma_q(\nu\widetilde{\boldsymbol{r}}^{\boldsymbol{i}}) - \ln\left(\frac{1-\sigma_q(\widetilde{\boldsymbol{m}}_{\boldsymbol{1}}+\nu\widetilde{\boldsymbol{r}}^{\boldsymbol{i}})}{1-\sigma_q(\widetilde{\boldsymbol{m}}_{\boldsymbol{1}})}\right)\right] q_t\right\}\\
&\cdot \quad A(\nu),
\end{aligned}
$$

where

$$
\begin{aligned}
A(\nu) &= \exp\left\{\left[-\sigma_{vn}(\nu\widetilde{\boldsymbol{r}}^{\boldsymbol{i}}) - \ln\left(\frac{1-\sigma_{vn}(\widetilde{\boldsymbol{m}}_{\boldsymbol{1}}+\nu\widetilde{\boldsymbol{r}}^{\boldsymbol{i}})}{1-\sigma_{vn}(\widetilde{\boldsymbol{m}}_{\boldsymbol{1}})}\right)\right] v_n\right\}\\
&+ \exp\left\{\frac{1}{2}\left[(\widetilde{\boldsymbol{m}}'_{\boldsymbol{1}}+\nu\widetilde{\boldsymbol{r}}^{\boldsymbol{i}\prime})\boldsymbol{S}_{\boldsymbol{1}}\boldsymbol{\Sigma}^{\boldsymbol{other}}\boldsymbol{S}'_{\boldsymbol{1}}(\widetilde{\boldsymbol{m}}_{\boldsymbol{1}}+\nu\widetilde{\boldsymbol{r}}^{\boldsymbol{i}}) - \widetilde{\boldsymbol{m}}'_{\boldsymbol{1}}\boldsymbol{S}_{\boldsymbol{1}}\boldsymbol{\Sigma}^{\boldsymbol{other}}\boldsymbol{S}'_{\boldsymbol{1}}\widetilde{\boldsymbol{m}}_{\boldsymbol{1}} + \nu^2\sigma_i^2\right]\right\} \qquad \text{(B.49)}
\end{aligned}
$$

, and

$$
\begin{aligned}
\sigma_p(\widetilde{\boldsymbol{m}}'_{\boldsymbol{1}}+\nu\widetilde{\boldsymbol{r}}^{\boldsymbol{i}\prime}) &= (\widetilde{\boldsymbol{m}}'_{\boldsymbol{1}}+\nu\widetilde{\boldsymbol{r}}^{\boldsymbol{i}\prime})\boldsymbol{\Sigma}_{\bullet\boldsymbol{1}}, & \text{(B.50)}\\
\sigma_n(\widetilde{\boldsymbol{m}}'_{\boldsymbol{1}}+\nu\widetilde{\boldsymbol{r}}^{\boldsymbol{i}\prime}) &= (\widetilde{\boldsymbol{m}}'_{\boldsymbol{1}}+\nu\widetilde{\boldsymbol{r}}^{\boldsymbol{i}\prime})\boldsymbol{\Sigma}_{\bullet\boldsymbol{2}}, & \text{(B.51)}\\
\sigma_{lp}(\widetilde{\boldsymbol{m}}'_{\boldsymbol{1}}+\nu\widetilde{\boldsymbol{r}}^{\boldsymbol{i}\prime}) &= (\widetilde{\boldsymbol{m}}'_{\boldsymbol{1}}+\nu\widetilde{\boldsymbol{r}}^{\boldsymbol{i}\prime})\boldsymbol{\Sigma}_{\bullet\boldsymbol{4}}, & \text{(B.52)}\\
\sigma_{vn}(\widetilde{\boldsymbol{m}}'_{\boldsymbol{1}}+\nu\widetilde{\boldsymbol{r}}^{\boldsymbol{i}\prime}) &= (\widetilde{\boldsymbol{m}}'_{\boldsymbol{1}}+\nu\widetilde{\boldsymbol{r}}^{\boldsymbol{i}\prime})\boldsymbol{\Sigma}_{\bullet\boldsymbol{5}}, & \text{(B.53)}\\
\sigma_q(\widetilde{\boldsymbol{m}}'_{\boldsymbol{1}}+\nu\widetilde{\boldsymbol{r}}^{\boldsymbol{i}\prime}) &= (\widetilde{\boldsymbol{m}}'_{\boldsymbol{1}}+\nu\widetilde{\boldsymbol{r}}^{\boldsymbol{i}\prime})\boldsymbol{\Sigma}_{\bullet\boldsymbol{9}}. & \text{(B.54)}
\end{aligned}
$$

The first-order moment is the first-order derivate at $\nu = 0$:

$$
\begin{aligned}
E_t^Q(\widetilde{r}^i_{t+1}) &= \frac{\partial mgf_t^Q(\widetilde{r}^i_{t+1};\nu)}{\partial\nu}|_{\nu=0}\\
&= E_t(\widetilde{r}^i_{t+1}) + \frac{\sigma_p(\widetilde{\boldsymbol{m}}_{\boldsymbol{1}})\sigma_p(\widetilde{\boldsymbol{r}}^{\boldsymbol{i}})}{1-\sigma_p(\widetilde{\boldsymbol{m}}_{\boldsymbol{1}})}p_t + \frac{\sigma_n(\widetilde{\boldsymbol{m}}_{\boldsymbol{1}})\sigma_n(\widetilde{\boldsymbol{r}}^{\boldsymbol{i}})}{1-\sigma_n(\widetilde{\boldsymbol{m}}_{\boldsymbol{1}})}n_t + \frac{\sigma_{lp}(\widetilde{\boldsymbol{m}}_{\boldsymbol{1}})\sigma_{lp}(\widetilde{\boldsymbol{r}}^{\boldsymbol{i}})}{1-\sigma_{lp}(\widetilde{\boldsymbol{m}}_{\boldsymbol{1}})}lp_t + \frac{\sigma_q(\widetilde{\boldsymbol{m}}_{\boldsymbol{1}})\sigma_q(\widetilde{\boldsymbol{r}}^{\boldsymbol{i}})}{1-\sigma_q(\widetilde{\boldsymbol{m}}_{\boldsymbol{1}})}q_t\\
&+ \frac{\sigma_{vn}(\widetilde{\boldsymbol{m}}_{\boldsymbol{1}})\sigma_{vn}(\widetilde{\boldsymbol{r}}^{\boldsymbol{i}})}{1-\sigma_{vn}(\widetilde{\boldsymbol{m}}_{\boldsymbol{1}})}v_n + \widetilde{\boldsymbol{m}}'_{\boldsymbol{1}}\boldsymbol{S}_{\boldsymbol{1}}\boldsymbol{\Sigma}^{\boldsymbol{other}}\boldsymbol{S}'_{\boldsymbol{1}}\widetilde{\boldsymbol{r}}^{\boldsymbol{i}}. \qquad \text{(B.55)}
\end{aligned}
$$

Note the similarity between $E_t(\widetilde{r}^i_{t+1}) - E_t^Q(\widetilde{r}^i_{t+1})$ from this equation and the equity premium derived before using the no-arbitrage condition. The second-order moment is derived,

$$
\begin{aligned}
VAR_t^Q(\widetilde{r}^i_{t+1}) &= E_t^Q\left((\widetilde{r}^i_{t+1})^2\right) - \left(E_t^Q(\widetilde{r}^i_{t+1})\right)^2\\
&= \frac{\partial^2 mgf_t^Q(\widetilde{r}^i_{t+1};\nu)}{\partial\nu^2}|_{\nu=0} - \left(\frac{\partial mgf_t^Q(\widetilde{r}^i_{t+1};\nu)}{\partial\nu}|_{\nu=0}\right)^2\\
&= \left(\frac{\sigma_p(\widetilde{\boldsymbol{r}}^{\boldsymbol{i}})}{1-\sigma_p(\widetilde{\boldsymbol{m}}_{\boldsymbol{1}})}\right)^2 p_t + \left(\frac{\sigma_n(\widetilde{\boldsymbol{r}}^{\boldsymbol{i}})}{1-\sigma_n(\widetilde{\boldsymbol{m}}_{\boldsymbol{1}})}\right)^2 n_t + \left(\frac{\sigma_{lp}(\widetilde{\boldsymbol{r}}^{\boldsymbol{i}})}{1-\sigma_{lp}(\widetilde{\boldsymbol{m}}_{\boldsymbol{1}})}\right)^2 lp_t + \left(\frac{\sigma_q(\widetilde{\boldsymbol{r}}^{\boldsymbol{i}})}{1-\sigma_q(\widetilde{\boldsymbol{m}}_{\boldsymbol{1}})}\right)^2 q_t\\
&+ \left(\frac{\sigma_{vn}(\widetilde{\boldsymbol{r}}^{\boldsymbol{i}})}{1-\sigma_{vn}(\widetilde{\boldsymbol{m}}_{\boldsymbol{1}})}\right)^2 v_n + \widetilde{\boldsymbol{r}}^{\boldsymbol{i}\prime}\boldsymbol{S}_{\boldsymbol{1}}\boldsymbol{\Sigma}^{\boldsymbol{other}}\boldsymbol{S}'_{\boldsymbol{1}}\widetilde{\boldsymbol{r}}^{\boldsymbol{i}} + \sigma_i^2. \qquad \text{(B.56)}
\end{aligned}
$$

C Variables and parameters

Table C.1: Variables. (In order of first appearance)

Symbol	
θ_t	change in log real industrial production index or growth
p_t	positive uncertainty factor
n_t	negative uncertainty factor
$\omega_{p,t}$	"good environment" shock
$\omega_{n,t}$	"bad environment" shock
$\boldsymbol{Y_t^{mac}}$	technology factors consisting of $\{\theta_t, p_t, n_t\}$
ω_t^{mac}	technology shocks consisting of $\{\omega_{p,t}, \omega_{n,t}\}$
π_t	change in log historical consumer price index
u_t^{π}	independent state variable shock of π
$\omega_{\pi,t}$	inflation shock
l_t	log corporate bond loss rate
u_t^l	independent state variable shock of l
$\omega_{l,t}$	loss rate shock
g_t	change in log earnings
u_t^g	independent state variable shock of e
$\omega_{g,t}$	earnings shock
κ_t	log consumption-earnings ratio
u_t^{κ}	independent state variable shock of κ
$\omega_{\kappa,t}$	log consumption-earnings ratio shock
η_t	log dividend payout ratio
u_t^{η}	independent state variable shock of η
$\omega_{\eta,t}$	log dividend payout ratio shock
lp_t	loss rate shock shape parameter
q_t	risk aversion
u_t^q	independent state variable shock of q
$\omega_{q,t}$	risk aversion shock
Y_t	a vector of 10 state variables
ω_t	a vector of 8 independent shocks
Δc_t	change in log consumption
m_t	log real pricing kernel
$\widetilde{m}_t$	log nominal pricing kernel
y_t^1	nominal short rate
PC_t	price-to-coupon ratio of one period defaultable bond
PD_t	price-dividend ratio
r_t^i	log asset return for assets i
$E_{t-1}\left(r_t^i\right)$	expected return for assets i
u_t^i	asset-specific shock of assets i
VAR_{t-1}^i	model-implied one-period physical conditional return variance of assets i
$VAR_{t-1}^{i,Q}$	model-implied one-period risk-neutral conditional return variance of assets i
z_t	a vector of observable asset prices / instruments
$PVAR_t^i$	empirical one-period physical conditional return variance of assets i for t+1
$QVAR_t^i$	empirical one-period risk-neutral conditional return variance of assets i for t+1

Table C.2: Parameters.

Symbol	
$\bar{\theta}$	unconditional mean of growth
m_p	sensitivity of output growth on lagged upside uncertainty
m_n	sensitivity of output growth on lagged downside uncertainty
$\bar{p}$	unconditional mean of positive uncertainty factor
$\bar{n}$	unconditional mean of negative uncertainty factor

ρ_p	autocorrelation coefficient of positive uncertainty factor
ρ_n	autocorrelation coefficient of negative uncertainty factor
$\sigma_{\theta p}$	scale parameter of growth to "good environment" shock
$\sigma_{\theta n}$	scale parameter of growth to "bad environment" shock
σ_{pp}	scale parameter of positive uncertainty factor to "good environment" shock
σ_{nn}	scale parameter of negative uncertainty factor to "bad environment" shock
j_0 *	constant in Variable j process
ρ_{jj} *	autocorrelation coefficient of Variable j
ρ_{jyp} *	sensitivity coefficient of Variable j to positive uncertainty factor
ρ_{jyn} *	sensitivity coefficient of Variable j to negative uncertainty factor
$\rho_{jy\theta}$ *	sensitivity coefficient of Variable j to output growth factor
ρ_{jy} *	$[\rho_{jp}, \rho_{jn}, \rho_{jx}]$
σ_{jp} *	scale parameter of Variable j to "good environment" shock
σ_{jn} *	scale parameter of Variable j to "bad environment" shock
σ_{jj} **	unconditional volatility of u_t^j
σ_{jj} ***	scale parameter of the state variable gamma shock u_t^j
σ_{vl}	scale parameter of the lp_t to the loss shock
μ	constant vector in the state variable system (10 $\times$ 1)
A	autocorrelation vector in the state variable system (10 $\times$ 10)
Σ	scale / volatility parameter matrix of the 8 shocks (10 $\times$ 8)
c_0	constant in the consumption growth process
c_1	sensitivity vector of consumption growth to state variable shocks
c_2	sensitivity vector of consumption growth to state variable levels
m_0	constant in the real pricing kernel process
m_1	sensitivity vector of real pricing kernel to state variable shocks
m_2	sensitivity vector of real pricing kernel to state variable levels
r^i	return loadings on state variable shocks
σ_i	unconditional volatility of u_t^i
χ	risk aversion loadings on observed asset prices

* for all $j \in \{\pi, l, g, \kappa, \eta, v, q\}$:
** for all $j \in \{\pi, g, \kappa, \eta\}$:
*** for all $j \in \{l, q\}$:

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Table 1: The Dynamics of the Macro Factors

This tables reports parameter estimates from the model below using the monthly log growth rate of U.S. industrial production from January 1947 to February 2015. This system involves latent processes (good shape parameter governing positive skewness, p_t, and bad shape parameter governing negative skewness, n_t) and is estimated using the MLE-filtration methodology described in Bates (2006). Bold (italic) coefficients have <5% (10%) p-values.

$$\begin{aligned}
\theta_{t+1} &= \overline{\theta} + \rho_\theta(\theta_t - \overline{\theta}) + m_p(p_t - 500) + m_n(n_t - \overline{n}) + u^\theta_{t+1} \\
p_{t+1} &= 500 + \rho_p(p_t - 500) + \sigma_{pp}\omega_{p,t+1} \\
n_{t+1} &= \overline{n} + \rho_n(n_t - \overline{n}) + \sigma_{nn}\omega_{n,t+1}
\end{aligned}$$

, where

$$\begin{aligned}
u^\theta_{t+1} &= \sigma_{\theta p}\omega_{p,t+1} - \sigma_{\theta n}\omega_{n,t+1} \\
\omega_{p,t+1} &\sim \tilde{\Gamma}(p_t, 1) \\
\omega_{n,t+1} &\sim \tilde{\Gamma}(n_t, 1) \\
\sigma_{pp} &> 0 \\
\sigma_{nn} &> 0.
\end{aligned}$$

Standard errors are displayed in parentheses. Note that Row " $\omega_{n,t}$ loading" in Column "θ_t" corresponds to the parameter estimate of $\sigma_{\theta n}$, and the actual loading of θ_t on $\omega_{n,t}$ is $-\sigma_{\theta n}$; Row " $\omega_{n,t}$ loading" in Column "p_t" ("n_t") is $+\sigma_{pp}$ ($+\sigma_{nn}$).

	θ_t	p_t	n_t
mean	0.0000	500 (fix)	**16.1421**
	(0.0005)		(2.1453)
AR	**0.1310**	**0.9997**	**0.9108**
	(0.0309)	(0.0192)	(0.0135)
m_p	0.0000		
	(0.0003)		
m_n	**-0.0002**		
	(0.0000)		
$\omega_{p,t}$ loading	**0.0001**	**0.5528**	
	(0.0000)	(0.0707)	
$\omega_{n,t}$ loading	**0.0017**		**2.1775**
	(0.0001)		(0.1503)
LL	2861.308		
BIC	-5648.85		
AIC	-5700.62		

Table 2: The Dynamics of the Corporate Loss Rate

This table reports parameter estimates for the corporate loss rate model using monthly data from June 1984 to February 2015. The actual nominal payoff of a one-period zero-coupon defaultable (risky) corporate bond at period $t+1$ is $C \times F_{t+1}$ where $F_{t+1} < 1$ is an unknown fraction of the total promised payment C. The nominal payment can be rewritten as, $CF_{t+1} = exp(c + ln(F_{t+1})) = exp(c - l_{t+1})$. Thus, l_{t+1} is $-ln(F_{t+1}) = -ln(1 - L_{t+1})$ where L_{t+1} is corporate loss rate. The empirical proxy for L_t is obtained using the following equality, $DEF_t \times (1 - RECOV_t)$. Both the default rate DEF_t and recovery rate $RECOV_t$ pertain to the overall corporate bond market and are empirically constructed as 6-month trailing measures from 3-month trailing all-corporate bond default rate and monthly recovery rate, respectively; both raw series are obtained from the Federal Reserve Board; $(1 - RECOV_t)$ is the monthly loss-given-default rate. The dynamic process of l_{t+1} is as follows:

$$\begin{aligned}
l_{t+1} &= l_0 + \rho_{ll} l_t + m_{lp} p_t + m_{ln} n_t + \sigma_{lp}\omega_{p,t+1} + \sigma_{ln}\omega_{n,t+1} + u^l_{t+1} \\
u^l_{t+1} &= \sigma_{llp}\omega_{lp,t+1} - \sigma_{lln}\omega_{ln,t+1} \\
\omega_{lp,t+1} &\sim \tilde{\Gamma}(lp_t, 1), \\
\omega_{ln,t+1} &\sim \tilde{\Gamma}(\overline{ln}, 1),
\end{aligned}$$

where the variance equation is,

$$\begin{aligned}
lp_{t+1} &= \overline{lp} + \rho_{lp}(lp_t - \overline{lp}) + \sigma_{lplp}\omega_{lp,t+1}, \\
\overline{ln} &> 0.
\end{aligned}$$

The mean equation is estimated by projection, the variance equation by Bates' approximate MLE-filtration. Standard errors are displayed in parentheses. Bold (italic) coefficients have <5% (10%) p-values.

	Mean:		
l_0	ρ_{ll}	m_{lp}	m_{ln}
-0.0009	**0.8306**	1.95E-06	**0.0001**
(0.0017)	(0.0241)	3.57E-06	(0.0000)
	Shock Sensitivities:		
σ_{lp}	σ_{ln}	σ_{llp}	σ_{lln}
0.0000	**0.0005**	**0.0006**	*0.0001*
(0.0000)	(0.0001)	(0.0001)	(0.0001)
	Shape Parameter Dynamics:		
$\overline{lp}$	ρ_{lp}	σ_{lplp}	$\overline{ln}$
5.2153	**0.8556**	**1.8615**	**103.58**
(0.2566)	(0.0126)	(0.1809)	(1.2566)
	Model Specifications:		
LL	AIC	BIC	
1814.51	-3617.01	-3593.58	

Table 3: Cash Flow Dynamics

This table shows the estimation results of cash flow dynamics. The dynamic processes of cash flow variables are expressed in Equation (20) for the log earnings growth, g_{t+1}, Equation (23) for the log consumption-earnings ratio, κ_{t+1}, Equation (26) for the log dividend-earnings ratio, η_{t+1}, and Equation (32) for the inflation rate, π_{t+1}. These coefficients are estimated using simple linear projections. Bold (italic) coefficients have <5% (10%) p-values. Robust errors are shown in parentheses. The adjusted R^2 of the conditional mean part (with information set t) is reported in the last row. The sample period is 1986/06 to 2015/02 (345 months).

	earnings growth	log CE	log DE	inflation
Coefficients on:	g_{t+1}	κ_{t+1}	η_{t+1}	π_{t+1}
drift	0.0207	**0.1451**	**-0.0966**	**0.0031**
	(0.0277)	(0.0459)	(0.0340)	(0.0014)
AR	**0.6589**	**0.9303**	**0.9102**	**0.3973**
	(0.0433)	(0.0095)	(0.0109)	(0.0500)
θ_t	**1.5181**	-0.2731	0.1765	**-0.0718**
	(0.6933)	(0.8565)	(0.8880)	(0.0315)
p_t	-0.0001	0.0000	0.0000	0.0000
	(0.0001)	(0.0001)	(0.0001)	(0.0000)
n_t	0.0005	**0.0031**	**0.0037**	**-0.0001**
	(0.0007)	(0.0008)	(0.0008)	(0.0000)
l_t	*-1.2318*	**2.3791**	**2.1250**	
	(0.7376)	(0.9352)	(0.9596)	
lp_t	*0.0007*	*-0.0008*	*-0.0008*	
	(0.0004)	(0.0004)	(0.0004)	
$\omega_{p,t+1}$ loading	-0.0001	0.0001	0.0001	0.0000
	(0.0001)	(0.0001)	(0.0001)	(0.0000)
$\omega_{n,t+1}$ loading	**-0.0033**	**0.0066**	**0.0068**	0.0001
	(0.0011)	(0.0013)	(0.0014)	(0.0001)
$\omega_{lp,t+1}$ loading	-0.0005	0.0008	0.0008	
	(0.0010)	(0.0013)	(0.0013)	
$\omega_{ln,t+1}$ loading	*-0.0002*	**0.0004**	**0.0004**	
	(0.0001)	(0.0001)	(0.0002)	
Gaussian shock volatility	**0.0462**	**0.0558**	**0.0574**	**0.0023**
	(0.0018)	(0.0021)	(0.0022)	(0.0001)
Adjusted R^2 (conditional mean)	56.76%	98.34%	98.06%	20.85%

Table 4: Shock Correlation Matrix

The table provides a correlation matrix of the shock structure of the economy. The shocks are summarized as follows:

$\omega_{p,t+1}$:	good uncertainty shock	$\widetilde{\Gamma}(p_t, 1)$;
$\omega_{n,t+1}$:	bad uncertainty shock	$\widetilde{\Gamma}(n_t, 1)$;
$\omega_{lp,t+1}$:	cash flow good uncertainty shock	$\widetilde{\Gamma}(lp_t, 1)$;
$\omega_{ln,t+1}$:	cash flow bad uncertainty shock	$\widetilde{\Gamma}(v_n, 1)$;
$\omega_{g,t+1}$:	log earnings growth-specific shock	N(0,1);
$\omega_{\kappa,t+1}$:	log C/E-specific shock	N(0,1);
$\omega_{\eta,t+1}$:	log D/E-specific shock	N(0,1);
$\omega_{q,t+1}$:	risk aversion-specific shock	$\widetilde{\Gamma}(q_t, 1)$.

Bold (italic) coefficients have <5% (10%) p-values. The sample period is 1986/06 to 2015/02 (345 months).

	ω_p	ω_n	ω_π	ω_{lp}	ω_{ln}	ω_g	ω_κ	ω_η	ω_q
ω_p	1	**-0.1129**	0.0000	-0.0011	-0.0007	0.0000	0.0000	0.0000	0.0000
ω_n		1	0.0000	*-0.0912*	-0.0828	0.0000	0.0000	0.0000	0.0000
ω_π			1	*0.0943*	-0.0442	**0.1060**	-0.0120	-0.0536	0.0360
ω_{lp}				1	**-0.1877**	0.0000	0.0000	0.0000	0.0296
ω_{ln}					1	0.0000	0.0000	0.0000	**-0.1789**
ω_g						1	**-0.6765**	**-0.6589**	0.0613
ω_κ							1	**0.9863**	-0.0413
ω_η								1	-0.0351
ω_q									1

Table 5: Financial Instruments Spanning Risk Aversion

This table presents summary statistics of the 6 financial instruments that are used to span our risk aversion measure: "tsprd" is the difference between 10-year treasury yield and 3-month Treasury yield; "csprd" is the difference between Moody's Baa yield and the 10-year zero-coupon Treasury yield; "EY5yr" ("DY5yr") is the detrended earnings (dividend) yield where the moving average takes the 5 year average of monthly earnings yield, starting one year before; "rvareq" and "rvarcb" are realized variances of log equity returns and log corporate bond returns, calculated from daily returns; "qvareq" is the risk-neutral conditional variance of log equity returns; for the early years (before 1990), we use VXO and authors' calculations. Bold (italic) coefficients have <5% (10%) p-values. Block bootstrapped errors are shown in parentheses. The sample period is from 1986/06 to 2015/02 (345 months).

	tsprd	csprd	DY5yr	EY5yr	rvareq	qvareq	rvarcb
			Correlation Matrix				
tsprd	1	**0.3524**	**0.2595**	**0.2526**	**0.1269**	**0.1244**	**0.2952**
csprd		1	**0.4990**	**0.5083**	**0.4786**	**0.5988**	**0.5330**
DY5yr			**1.0000**	**0.8966**	**0.1678**	**0.1650**	**0.3101**
EY5yr				1	**0.1399**	**0.1564**	**0.3359**
rvareq					1	**0.8431**	**0.5943**
qvareq						1	**0.5376**
rvarcb							1
			Summary Statistics				
Mean	**0.0179**	**0.0231**	**-0.0030**	**-0.0074**	**0.0029**	**0.0040**	**0.0002**
Boot.SE	(0.0006)	(0.0004)	(0.0003)	(0.0008)	(0.0003)	(0.0002)	(0.0000)
S.D.	**0.0116**	**0.0075**	**0.0061**	**0.0149**	**0.0059**	**0.0037**	**0.0003**
Boot.SE	(0.0003)	(0.0005)	(0.0003)	(0.0007)	(0.0014)	(0.0005)	(0.0000)
Skewness	**-0.2322**	**1.7891**	0.0959	**-0.3495**	**8.1198**	**3.7225**	**4.2227**
Boot.SE	(0.0810)	(0.2515)	(0.1882)	(0.1502)	(1.5951)	(0.5123)	(0.6872)
AR(1)	**0.9668**	**0.9640**	**0.9822**	**0.9843**	**0.4312**	**0.7462**	**0.5775**
SE	(0.0137)	(0.0143)	(0.0083)	(0.0068)	(0.0488)	(0.0360)	(0.0441)

Table 6: Reduced-Form Risk Aversion Parameters

This table presents the GMM estimation results for risk aversion, $q_t = \boldsymbol{\chi' z_t}$, using equity market and corporate bond market asset prices. The utility curvature parameter, γ, is fixed at 2 in the estimation. The first-step GMM weighting matrix is an identity matrix; the second-step weighting matrix builds on the Newey-West spectral density function with 5-month lags, and then is shrunk towards an identity matrix where the shrinkage parameter is 0.1. The GMM system also consistently estimates σ_{qq}. Therefore, the system has 8 unknown parameters. The p-value of Hansen's overidentification test (J test) is calculated from the asymptotic χ^2 distribution with the degree of freedom being 29 (37-8). Bold (italic) coefficients have <5% (10%) p-values. Efficient standard errors are shown in parentheses. The sample period is 1986/06 to 2015/02 (345 months).

	q_t
Efficient GMM Estimators	
constant	**0.059**
	(0.011)
χ_{tsprd}	0.222
	(0.511)
χ_{csprd}	**1.948**
	(0.564)
χ_{EY5yr}	0.126
	(0.265)
χ_{rvareq}	**-20.195**
	(0.460)
χ_{qvareq}	**61.196**
	(1.551)
χ_{rvarcb}	**105.721**
	(11.500)
Correlation with the NBER Indicator	
$\rho(q_t, NBER_t)$	**0.401**
	(0.045)
Model Specifications	
Hansen's J	41.2904
p-value	0.0649

Table 7: Structural Risk Aversion Parameters

This table presents the model-implied risk aversion process parameters. In the first panel, parameter estimates are obtained either from simple projection or from the GMM procedure introduced earlier (see Table 6). The second and third panels report the variance decomposition (VARC) results of the conditional mean and shock structure of $\widehat{q}_{t+1}$, respectively. In the second panel, VARC of a linear variable x is as follows, $VARC = \beta_x \frac{cov(\widehat{y},x)}{var(\widehat{y})})$ where $\widehat{y} = \widehat{E}_t(\widehat{q}_{t+1})$. VARC in the third panel is calculated using the residual, $\widehat{y} = \widehat{q}_{t+1} - \widehat{E}_t(\widehat{q}_{t+1})$. Therefore, the sum of VARC in explaining the conditional mean and the shock structure is 100% each. Bold (italic) coefficients have <5% (10%) p-values. Robust and efficient standard errors are shown in parentheses. The sample period is 1986/06 to 2015/02 (345 months).

$$\begin{aligned}
\widehat{q}_{t+1} &= q_0 + \rho_{qq}\widehat{q}_t + \rho_{qp}\widehat{p}_t + \rho_{qn}\widehat{n}_t + \sigma_{qp}\widehat{\omega}_{p,t+1} + \sigma_{qn}\widehat{\omega}_{n,t+1} + u^q_{t+1}, \\
u^q_{t+1} &= \sigma_{qq}\omega_{q,t+1}, \\
\omega_{q,t+1} &= \widetilde{\Gamma}(q_t, 1).
\end{aligned}$$

Structual Risk Aversion Parameters, q_{t+1}							
	◦ Projection						◦ GMM
	Constant	p_t	n_t	q_t	$\omega_{p,t+1}$	$\omega_{n,t+1}$	$\omega_{q,t+1}$
Est	-0.0886	**0.0003**	**0.0037**	**0.6712**	0.0004	-0.0004	**0.1557**
(SE)	(0.0566)	(0.0001)	(0.0007)	(0.0387)	(0.0002)	(0.0023)	(0.0019)
Conditional Mean Variance Decomposition (65% of Total Variance)							
		p_t	n_t	q_t			
VARC		1.06%	18.74%	80.20%			
Shock Structure Variance Decomposition (35% of Total Variance)							
					$\omega_{p,t+1}$	$\omega_{n,t+1}$	$\omega_{q,t+1}$
VARC					0.77%	0.02%	99.22%

Table 8: Fit of Moments

This table evaluates the fit of conditional moments of equity and corporate bond returns. Column "Model" reports the averages of the relevant model-implied conditional moments. The "Empirical Averages" represent the sample averages of the excess returns (for "Mom 1" and "Mom 4"), the sample averages of empirical conditional variances (for "Mom 2", "Mom 3", and "Mom 5"). For "Mom 6" and "Mom 7", "Risk Aversion Innovation" is u^q_{t+1} in Equation (29). The variance and unscaled skewness rows compare the average model-implied conditional moments with the unconditional moments. Bold numbers denote a distance of less than 1.645 standard errors from the corresponding empirical point estimate. Block bootstrapped standard errors are shown in parentheses; we allow the block size to vary for different moments: block sizes=[0 6 15 1 10] for Mom 1 to Mom 5, respectively. Asymptotic standard errors (standard deviation divided by square root of the number of observations) are reported for Mom 6 and Mom 7. The sample period is 1986/06 to 2015/02 (345 months).

	Moment	Model	Empirical Average	Boot.SE/SE
Mom 1	Equity Risk Premium	**0.00785**	0.00530	(0.00246)
Mom 2	Equity Physical Variance	**0.00346**	0.00286	(0.00051)
Mom 3	Equity Risk-neutral Variance	**0.00412**	0.00397	(0.00049)
Mom 4	Corporate Bond Risk Premium	**0.00429**	0.00388	(0.00050)
Mom 5	Corporate Bond Physical Variance	**0.00023**	0.00024	(0.00003)
Mom 6	Risk Aversion Innovation Variance	**0.00767**	0.00929	(0.00195)
Mom 7	Risk Aversion Innovation Unscaled Skewness	**0.00239**	0.00216	(0.00111)

Table 9: Asset Prices and the State Variables

This table reports the decomposition of model-implied conditional moments by the underlying dynamic drivers. The closed form solutions of each conditional moments are shown in the main text. The asset conditional moments are explained by four premium state variables, $\{p_t, n_t, lp_t, q_t\}$. This table presents 1. scaled coefficients for interpretation purpose and 2. variance decomposition. 1. The coefficients are multiplied by standard deviations of the corresponding state variables of the same column and then multiplied by 10000. 2. VARC (as previously defined) is coefficient$*\frac{Cov(x_t,Mom_t)}{Var(Mom_t)}$ where $x \in \{p, n, v, q\}$ and Mom is from Mom 1 to Mom 5. The variance decomposition is reported in a bold italic font. The sum of the four VARCs in the same add up to 100% by design.

Model-implied coefficients of moments on state variables $\{p_t, n_t, lp_t, q_t\}$					
	Moment	p_t	n_t	lp_t	q_t
Mom 1	Equity Risk Premium	0.1473	4.0169	-0.0852	52.6093
	VARC	***0.014%***	***4.014%***	***-0.033%***	***96.004%***
Mom 2	Equity Physical Variance	0.0492	3.1165	0.0555	5.6090
	VARC	***-0.042%***	***31.249%***	***0.148%***	***68.645%***
Mom 3	Equity Risk-neutral Variance	0.0493	3.0708	0.0554	11.8273
	VARC	***0.000%***	***14.717%***	***0.086%***	***85.197%***
Mom 4	Corporate Bond Risk Premium	0.0625	4.7338	0.1157	9.8541
	VARC	***-0.025%***	***27.310%***	***0.187%***	***72.528%***
Mom 5	Corporate Bond Physical Variance	0.0004	0.1610	0.0408	0.0101
	VARC	***-0.076%***	***88.928%***	***7.921%***	***3.227%***

Table 10: Predicting Realized Excess Returns and Variances

This table reports the regression coefficients of realized future excess returns and realized variances of equity and corporate bond on model implications. In the first set of analysis, moment realizations are regressed on the four premium state variables, $\{p_t, n_t, lp_t, q_t\}$. The coefficients are multiplied by standard deviations of the corresponding state variables of the same column and then multiplied by 100. In the second set of analysis, moment realizations are regressed on "Model-Implied Moments" which are risk premiums (for realized excess returns) and physical variances (for realized variances). Bold (italic) coefficients have <5% (10%) p-values. Adjusted R^2s are reported. Standard errors are shown in parentheses. The sample period is 1986/06 to 2015/02 (345 months).

	$\widetilde{r}^{eq}_{t+1} - rf_t$		$RVAR^{eq}_{t+1}$		$\widetilde{r}^{cb}_{t+1} - rf_t$		$RVAR^{cb}_{t+1}$	
p_t	-0.2263		0.0146		-0.0821		-0.0007	
	(0.2649)		(0.0313)		(0.0944)		(0.0015)	
n_t	**-1.1843**		**0.1461**		0.0530		**0.0081**	
	(0.2974)		(0.0352)		(0.1059)		(0.0016)	
lp_t	*-0.4346*		0.0014		0.0674		0.0010	
	(0.2483)		(0.0294)		(0.0885)		(0.0014)	
q_t	**0.8623**		**0.1976**		0.0911		**0.0071**	
	(0.2916)		(0.0345)		(0.1039)		(0.0016)	
Model-Implied Moments		**0.6108**		**2.1525**		*0.1553*		**3.9400**
		(0.2460)		(0.2025)		(0.0907)		(0.4189)
Adjusted R^2	5.5%	0.1%	25.0%	24.8%	1.4%	0.9%	24.4%	20.6%
$\rho(NBER)$		0.4361		0.5798		0.5651		0.7041

Table 11: Out-Of-Sample Exercise

This table evaluates the ability of in-sample (see Section III.C) and out-of-sample risk premium estimates of equity returns and corporate bond returns to explain realized future excess returns. To be more specific, model-implied risk premium estimates (from the paper) and empirical risk premium estimates (from a wide set of empirical predictors that are widely-used in the stock return predictability literature) are both included to predict one-month future excess returns. **1. Model-implied risk premium estimates, "Mod":** "Mod (1)" indicates the in-sample (full-sample) estimates of model-implied risk premiums, the dynamics of which are determined by $\{p_t, n_t, lp_t, q_t\}$. "Mod (2)" indicates the out-of-sample estimates of model-implied risk premiums: Define a 60-month rolling window from $t-60$ to $t-1$, then project one-period ahead excess returns on the 4 premium state variables, and then use the coefficient estimates to obtain $E_t(\widetilde{r}_{t+1} - rf_t)$, repeatedly. **2. Empirical risk premium estimates, "Emp Mod":** We also consider three out-of-sample empirical risk premium estimates that use three instruments sets (subsets of $\boldsymbol{z_t}$), respectively: (1) 5-year detrended earnings-price ratio, (2) 5-year detrended earnings-price ratio + term spread + credit spread, (3) market return physical uncertainty plus variance risk premium estimate. The table then reports the optimal combination of Mod and Emp Mod estimates which minimize the sum of squared innovations. Least Square standard errors are shown in parentheses. Bold (italic) coefficients have <5% (10%) p-values (against zero). The (full) sample period is 1986/06 to 2015/02 (345 months).

Least-Square Estimate of a in $\widetilde{r}_{t+1} - rf_t = a \times \text{Mod}(t,i) + (1-a) \times \text{Emp Mod}(t,j) + e_{t+1}$, $i = 1,2; j = 1,2,3$				
	○ *Equity:*		○ *Corporate Bond:*	
	Mod (1)	Mod (2)	Mod (1)	Mod (2)
Emp Mod (1)	**0.7322**	**0.3987**	**0.4501**	0.1051
	(0.0958)	(0.0894)	(0.0740)	(0.0778)
Emp Mod (2)	**0.8483**	**0.5628**	**0.5509**	**0.3614**
	(0.0878)	(0.0848)	(0.0591)	(0.0720)
Emp Mod (3)	**0.7609**	**0.5619**	**0.5579**	**0.4507**
	(0.0758)	(0.0763)	(0.0579)	(0.0836)

Table 12: Projecting Macroeconomic Uncertainty on Financial Instruments

This table presents regression results of the estimated monthly macroeconomic uncertainty (from industrial production growth) on a set of monthly asset prices; some are used to span the time-varying risk aversion. "Total" is the total industrial production growth variance, which is a function of p_t and n_t, $\sigma^2_{\theta p} p_t + \sigma^2_{\theta n} n_t$. "$\times 10^3$" in the header means that the coefficients and their SEs reported are multiplied by 1000 for reporting convenience."VARC" reports the variance decomposition. Bold (italic) coefficients have <5% (10%) p-values. Robust and efficient standard errors are shown in parentheses. Adjusted R^2s are reported. The sample period is 1986/06 to 2015/02 (345 months).

	($\times 10^3$)		($\times 10^3$)		($\times 10^3$)	
	Total	VARC	Upside	VARC	Downside	VARC
constant	*-0.009*		**0.006**		**-0.015**	
	(0.005)		(0.000)		(0.005)	
χ_{tsprd}	**-0.577**	-2.33%	**-0.004**	2.70%	**-0.573**	-2.47%
	(0.112)		(0.002)		(0.112)	
χ_{csprd}	**2.024**	62.69%	**-0.016**	6.52%	**2.040**	62.32%
	(0.246)		(0.004)		(0.246)	
χ_{DY5yr}	**2.343**	41.57%	**-0.162**	139.79%	**2.505**	44.74%
	(0.456)		(0.007)		(0.456)	
χ_{EY5yr}	**-0.609**	-22.57%	**0.048**	-55.56%	**-0.657**	-24.28%
	(0.189)		(0.003)		(0.189)	
χ_{rvareq}	-0.257	-3.76%	-0.002	-0.03%	-0.255	-3.65%
	(0.620)		(0.010)		(0.621)	
χ_{qvareq}	*1.190*	13.25%	**0.066**	5.20%	*1.124*	12.20%
	(0.669)		(0.010)		(0.670)	
χ_{rvarcb}	**17.792**	13.67%	-0.056	0.37%	**17.848**	13.49%
	(5.927)		(0.092)		(5.935)	
$\chi_{rvarcbSPEC}$	-2.233	-2.51%	-0.108	1.01%	-2.125	-2.35%
	(5.564)		(0.087)		(5.571)	
R^2	50.20%		70.80%		50.60%	

Table 13: On the Predictive Power of Risk Aversion and Uncertainty for Future Output Growth

This table reports the coefficient estimates of the following predictive regression,

$$\theta_{t+k} = a_k + \boldsymbol{b_k'}\boldsymbol{x_t} + res_{t+k},$$

where θ_{t+k} represents future industrial production growth during period $t+1$ and $t+k$ ($\sum_{\tau=1}^{k}\theta_{t+\tau}$) and $\boldsymbol{x_t}$ represents a vector of current predictors. We consider (1) our GMM-implied risk aversion index, ra^{BEX}, (2) our financial instrument proxy of economic uncertainty, unc^{BEX}, (3) the risk-neutral conditional variance (the square of the month-end VIX (after 1990) / VXO (prior to 1990) index divided by 120000), $QVAR$, and (4) the true total macroeconomic uncertainty filtered from industrial production growth unc^{true}. The coefficients are scaled by the standard deviation of the predictor in the same column for reporting purposes. Hodrick (1992) standard errors are reported in parentheses, and adjusted R^2s are in %. Bold (italic) coefficients have <5% (10%) p-values.

	ra^{BEX}	unc^{BEX}	$QVAR$	unc^{true}	
		A. Univariate			
1m	**-0.002**	**-0.003**	**-0.002**	**-0.002**	
	(0.001)	(0.000)	(0.001)	(0.001)	
	7.2%	20.6%	6.5%	13.1%	
1q	**-0.005**	**-0.008**	**-0.005**	**-0.007**	
	(0.001)	(0.001)	(0.002)	(0.001)	
	14.9%	37.9%	15.3%	26.5%	
4q	**-0.006**	**-0.017**	**-0.008**	**-0.010**	
	(0.003)	(0.004)	(0.003)	(0.004)	
	2.5%	17.7%	3.7%	6.5%	
		B. Multivariate (1)			R^2
1m	0.001	**-0.003**	0.000		21.2%
	(0.001)	(0.001)	(0.001)		
1q	**0.004**	**-0.009**	**-0.003**		39.1%
	(0.001)	(0.002)	(0.001)		
4q	**0.021**	**-0.025**	**-0.011**		22.8%
	(0.004)	(0.005)	(0.002)		
		C. Multivariate (2)			R^2
1m	0.000		0.000	**-0.002**	14.1%
	(0.001)		(0.001)	(0.001)	
1q	0.001		**-0.003**	**-0.006**	29.6%
	(0.001)		(0.002)	(0.001)	
4q	**0.010**		**-0.012**	**-0.010**	8.1%
	(0.003)		(0.003)	(0.003)	
		D. Multivariate (3)			R^2
1m	0.001	**-0.003**	0.000	-0.001	21.5%
	(0.001)	(0.001)	(0.001)	(0.001)	
1q	**0.004**	**-0.008**	**-0.003**	-0.002	40.4%
	(0.001)	(0.001)	(0.001)	(0.001)	
4q	**0.021**	**-0.027**	**-0.011**	0.003	23.1%
	(0.004)	(0.006)	(0.002)	(0.003)	

Table 14: Alternative Risk Aversion and Uncertainty Measures.

This table report the correlation between our risk aversion and economic uncertainty indices and existing measures. For risk aversion (Panel A), we consider three categories. A.1) We follow Wachter (2006) to create a fundamental risk aversion process from inflation-adjusted (real) quarterly consumption growth ($\sum_{j=0}^{4} \Delta c_{t-j}$); A.2) we consider the well-known sentiment index by Baker and Wurgler (2006) from the behavior finance literature, and the Michigan Consumer Sentiment Index (that directly measures the consumer sentiment); A.3) we also consider an industry index, the Credit Suisse First Boston Risk Appetite Index. For economic uncertainty (Panel B), we consider B.1) the macroeconomic uncertainty index created by Jurado, Ludvigson, and Ng (2015), and B.2) the Economic Policy Uncertainty Index created by Baker, Bloom, and Davis (2016). Correlations are calculated using overlapping samples at the monthly frequency. Standard errors are shown in parentheses. Bold correlation coefficients have <5% p-values.

A. Correlations with Extant Risk Aversion Indices	
A.1) "Fundamental" Habit Model:	
Wachter (2006) / Campbell and Cochrane (1999)	**0.1307**
A.2) Sentiment Index:	(0.0534)
Baker and Wurgler (2006)	**-0.1652**
	(0.0531)
Michigan Consumer Sentiment Index	**-0.2752**
	(0.0518)
A.3) Industry Index	**-0.4830**
Credit Suisse First Boston Risk Appetite Index	(0.0471)
B. Correlations with Extant Uncertainty Indices	
B.1) Macroeconomic Uncertainty:	
Jurado, Ludvigson, and Ng (2015)	**0.8094**
	(0.0316)
B.2) Political Uncertainty:	
Baker, Bloom, and Davis (2016)	**0.3428**
	(0.0506)

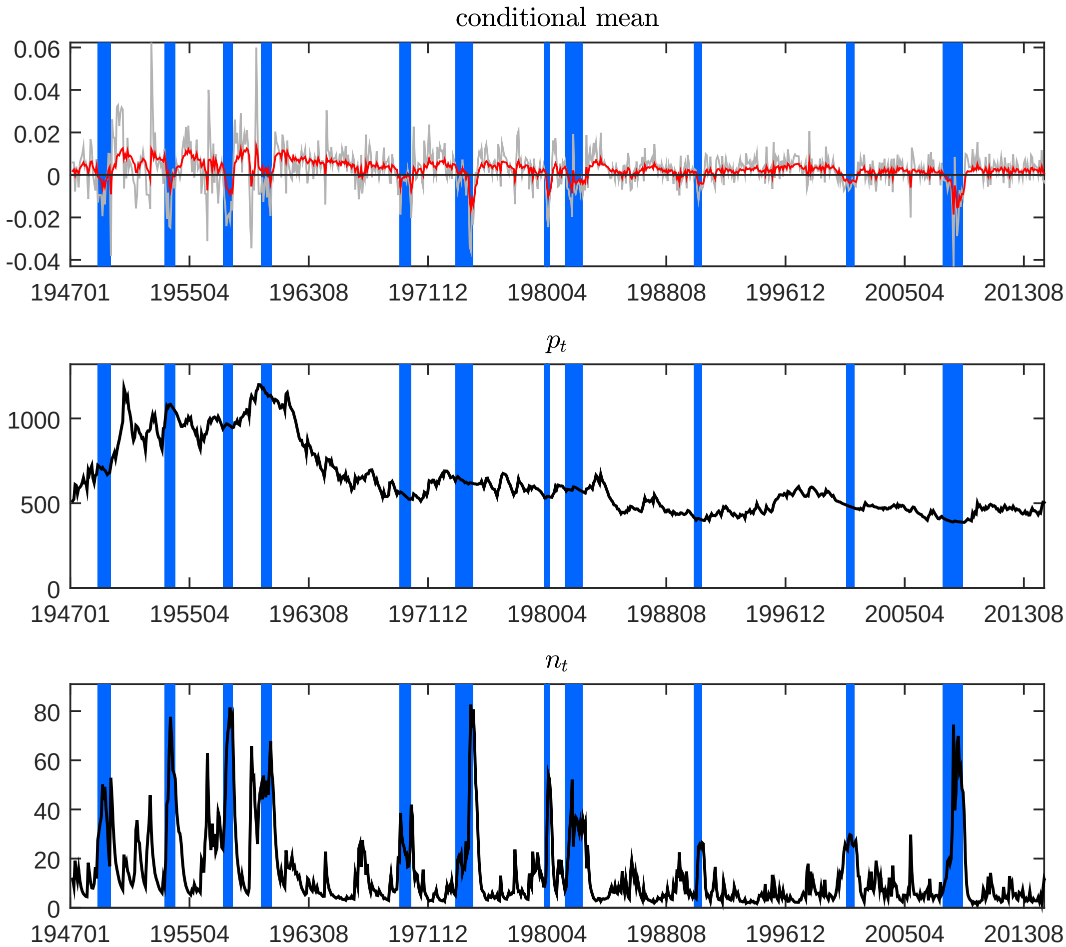

Figure 1: Filtered state variables extracted from industrial production growth. The estimation results are displayed in Table 1. The plot covers the estimation period from January 1947 to February 2015. The shaded regions are NBER recession months from the NBER website.

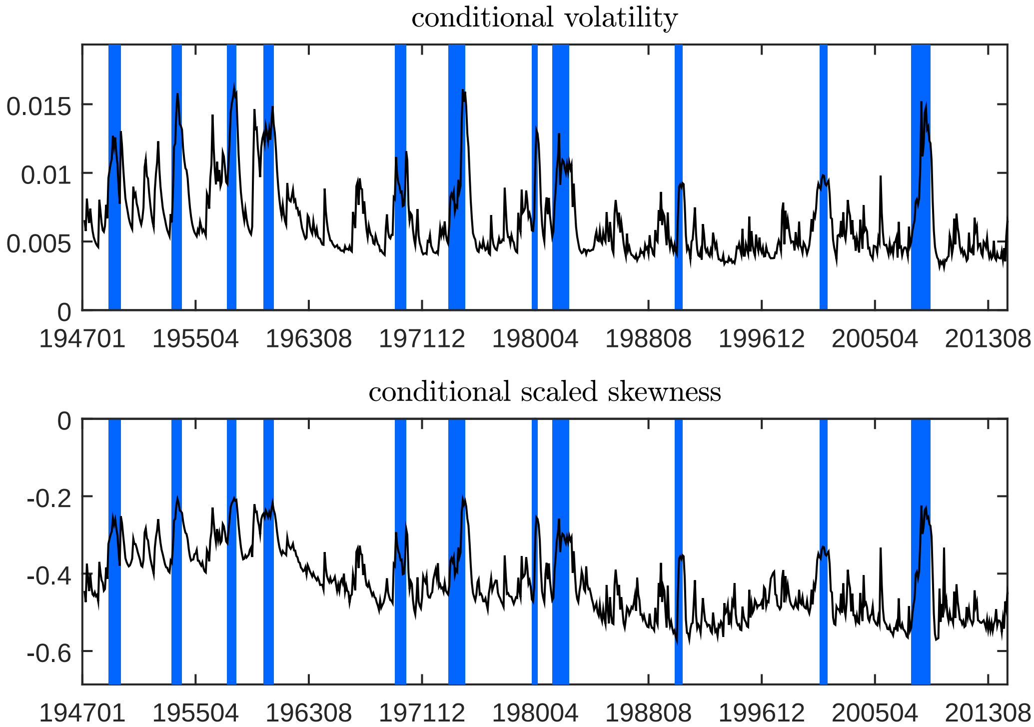

Figure 2: Model-implied conditional moments of industrial production growth. The shaded regions are NBER recession months from the NBER website.

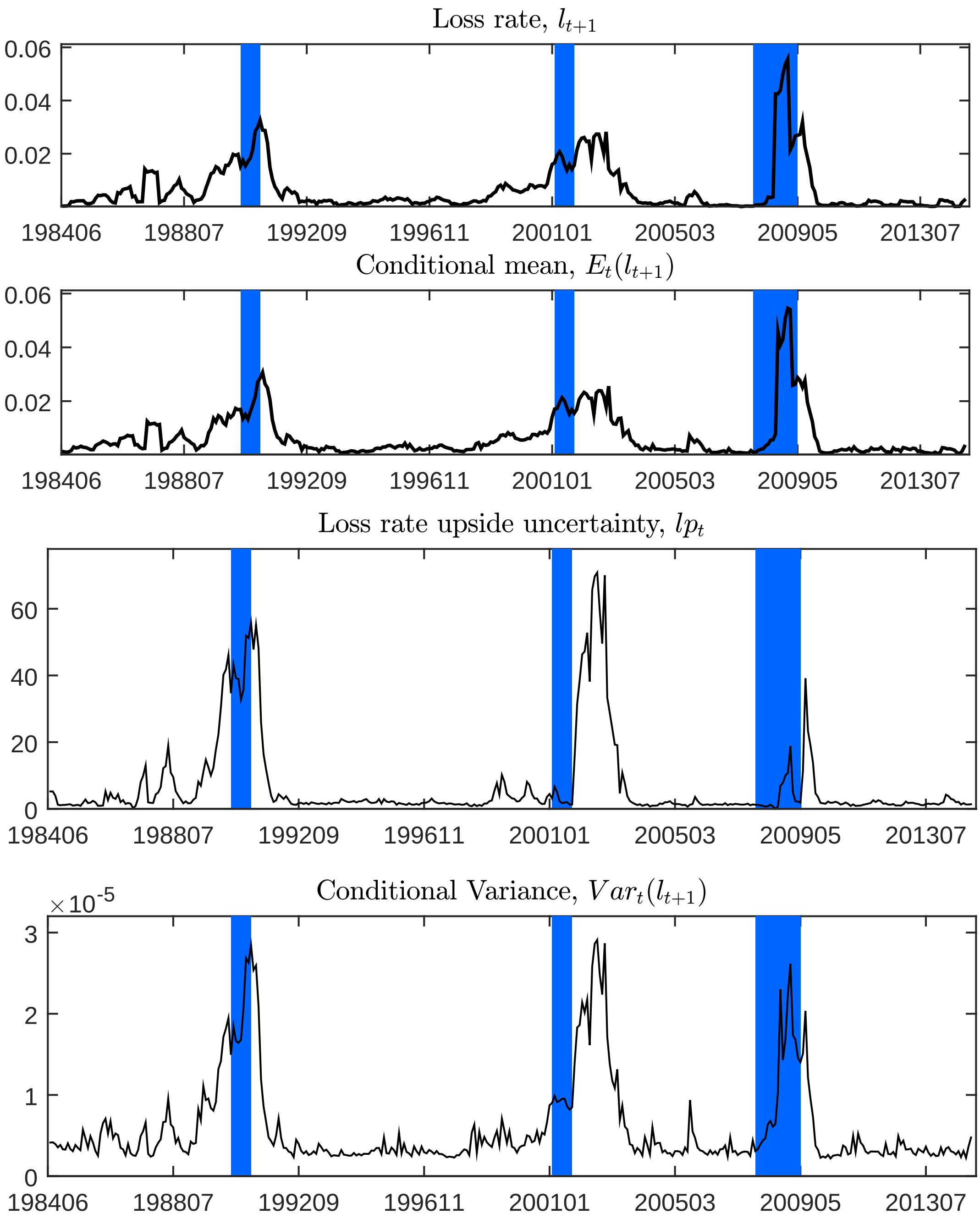

Figure 3: Loss rate and its conditional moments. The dynamics and estimation results are shown in Table 2. From top to bottom: loss rate l_{t+1}, conditional mean, the cash flow uncertainty state variable lp_t, and the total conditional variance. The plot covers the estimation period from June 1984 to February 2015. The shaded regions are NBER recession months from the NBER website.

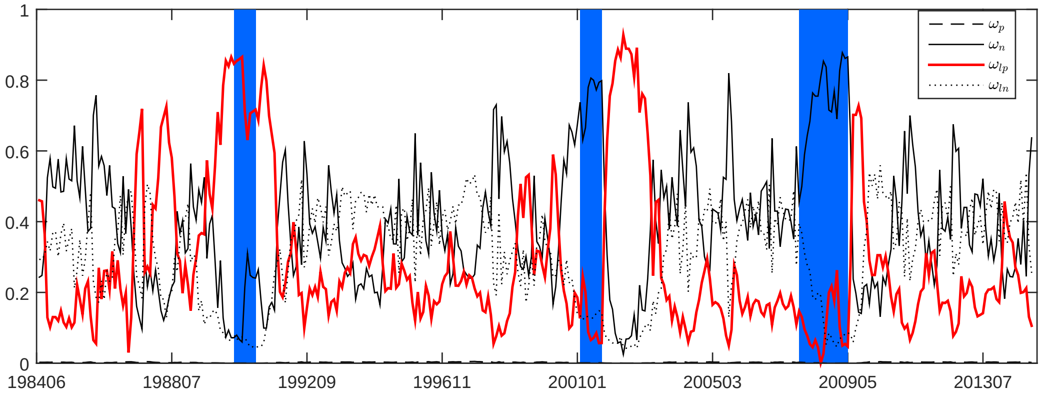

Figure 4: Conditional variance decomposition of the loss rate. (Conditional) Variances of four independent gamma shocks in the system contribute to the magnitude and the dynamics of total loss rate: real economic shocks, ω_p and ω_n, and pure cash flow shock, ω_{lp} and ω_{ln}. Among them, only ω_{ln} is homoskedastic, capturing the left-tail behavior of the pure loss rate movement that is not explained by the economic shocks. The shaded regions are NBER recession months from the NBER website.

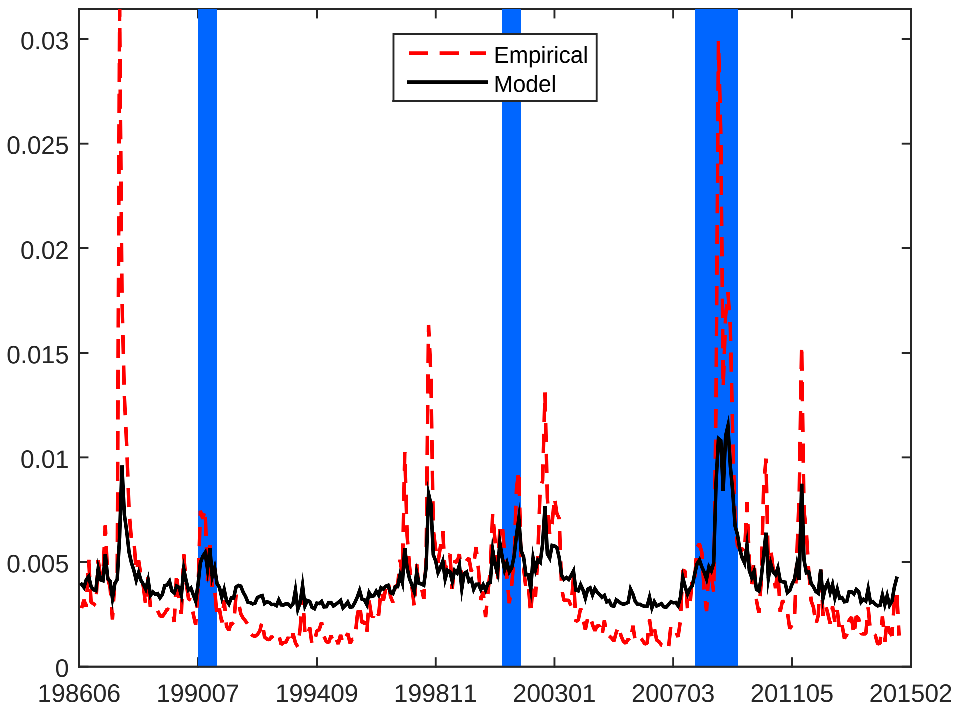

Figure 5: Model-implied and empirical risk-neutral conditional variances of equity market returns. The shaded regions are NBER recession months from the NBER website. The two series are 90.06% correlated.

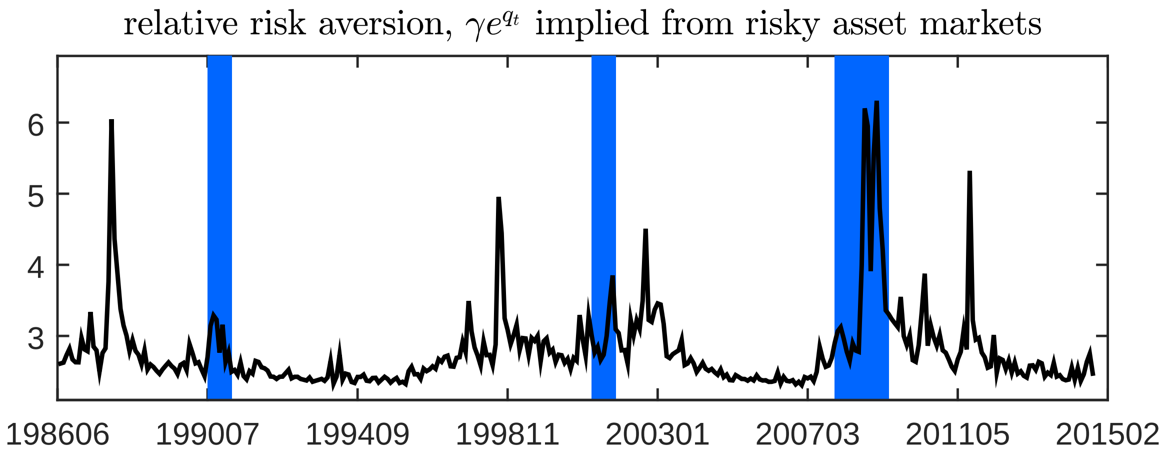

Figure 6: The risk aversion index ($\gamma \exp(q_t)$). The utility curvature parameter γ is 2. The shaded regions are NBER recession months from the NBER website.

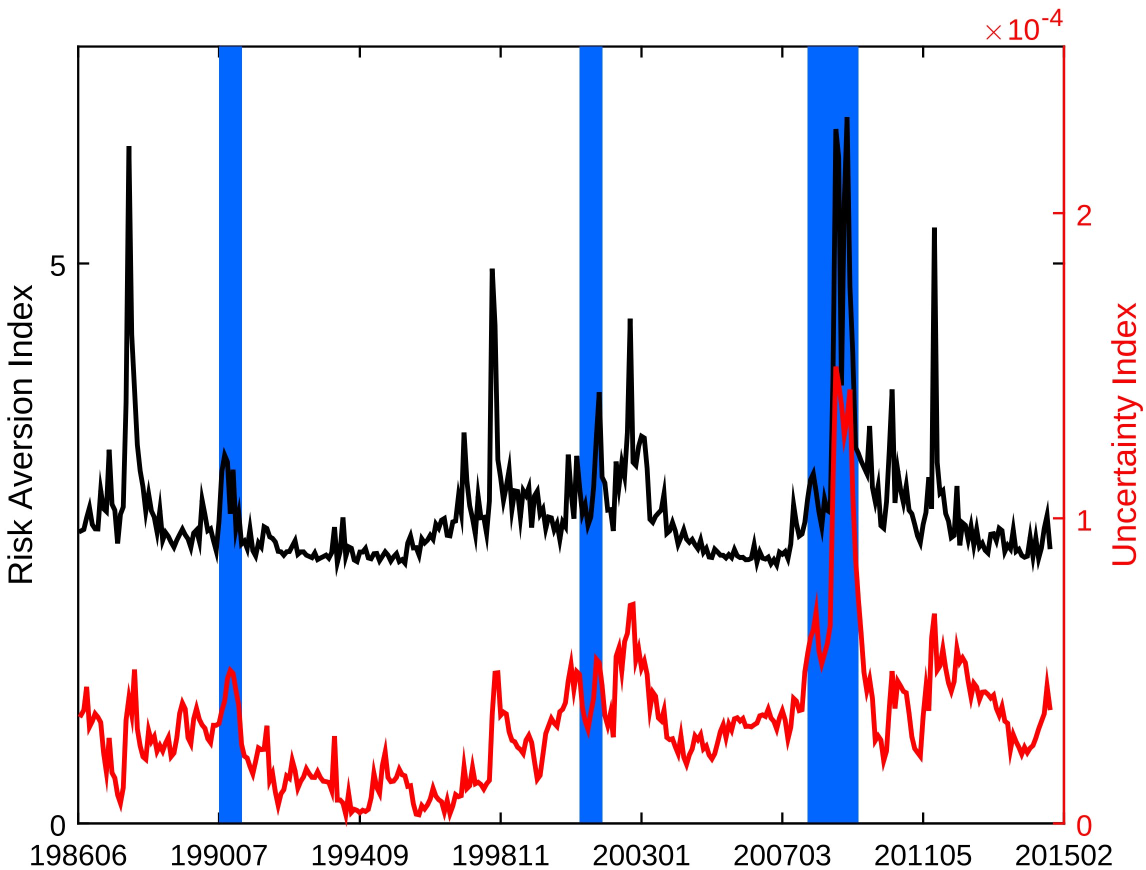

Figure 7: Risk aversion index (black/left y-axis) and Uncertainty index (red/right y-axis). The risk aversion index denoted as $ra^{BEX} = \gamma \exp(q_t)$ and the uncertainty index denoted as unc^{BEX} are linear functions of a set of financial instruments. Correlation between the two series is 70.43%. The shaded regions are NBER recession months from the NBER website.

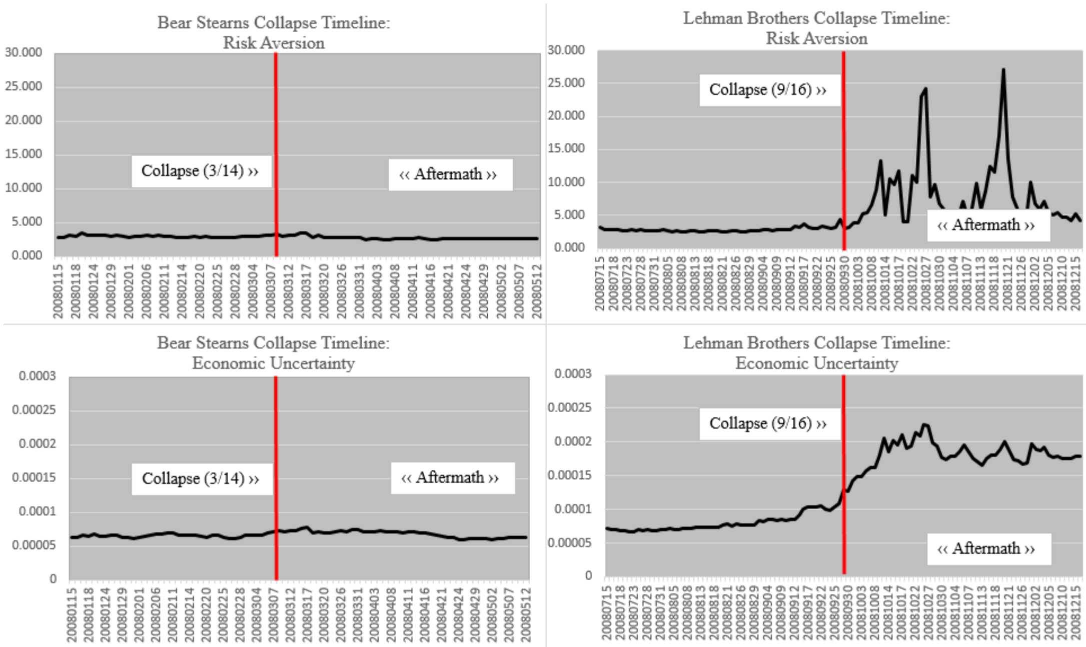

Figure 8: Risk aversion and economic uncertainty at daily frequencies around the Bear Stearns and Lehman Brothers Collapses in 2008.