Worker Reallocation Over the Business Cycle:
Evidence from Canada

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Abstract

This paper studies the cyclical variability of job finding, separations, and employer-to-employer flows in Canada for 1978-2016, the longest such time series available. Our analysis is based on direct administrative records of job separations. Our measures provide a much cleaner record of gross worker flows than standard household surveys. They are not subject to time-aggregation bias or to the measurement error problems that plague standard household surveys on employment dynamics. Employer-to-employer flows are strongly procyclical and are the dominant component of both job finding and separation. We document several additional facts regarding the role of job-to-job flows in labor market fluidity, the near-constancy of the ratio of hires coming from employment versus unemployment, and the roles of “ins” vs “outs” in the Canadian labor market.

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JEL Classification: E24, E32.

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1 Introduction

As new data has become available, there has been a renewed discussion of the cyclical behavior of gross worker flows. Early studies captured net flows instead of gross flows. However, gross flows are large relative to net flows. Since gross worker flows are a reallocation of resources, their behavior has a direct bearing on the extent to which there are cleansing or sullying effects from recessions (Caballero and Hammour, 2005; Foster et al., 2016). Cleansing effects arise if less productive job matches are destroyed during recessions making room for more productive matches (Hall, 1991; Davis and Haltiwanger, 1992; Mortensen and Pissarides, 1994). However, if reallocation is procyclical there may be a sullying effect. Instead of increasing the average quality of job matches, recessions can cause lower quality matches to last longer since fewer job-to-job transitions occur (Shleifer, 1986; Stadler, 1990; Aghion and Saint-Paul, 1998; Barlevy, 2002).

Worker flows are also needed to estimate search and matching functions, which are a key input into macro-labor models. They provide a means to distinguish between the different search and matching models. Moreover, as these flows are the result of decisions to work or search for work, they may provide insight into labor supply decisions and labor market frictions (Krusell et al., 2017). A growing literature studies employer-to-employer flows, which are vital in developing appropriate macro-labor models (Nagypal, 2004; Krause and Lubik, 2006; Kiyotaki and Lagos, 2007; Menzio and Shi, 2011), and understanding wage dynamics (Karahan et al., 2017).

We have four main results. First, we provide evidence on the cyclical behavior of gross worker flows, including employer-to-employer flows, based on direct administrative data on job separations for the Canadian labor market. Our time-series on employer-to-employer flows is the longest that we are aware of in the literature. We find that the rate of job finding is strongly procyclical. We also find that employer-to-employer flows are procyclical. Our work builds heavily on a previous analysis of the Canadian labor market by Picot et al. (1998).

Second, we compare the flows into new jobs from unemployment versus workers coming from other jobs, and find that these flows are roughly proportional. This supports common assumptions in models of random, on-the-job search. In particular, this pattern arises in simple models where employed and unemployed job search are modeled as having the same search technology,  

\footnotesize{\textsuperscript{1}Also see Caballero et al. (1996), Gomes et al. (2001), and Hornstein et al. (2003).  
\textsuperscript{2}Also see Van Zandweghe (2010) and Mukoyama (2014).}
and fixed differential search intensity.

Third, we decompose a measure of labor market fluidity into employer-to-employer flows, employment inflows, and employment outflows. Our results show that about 60% of variation in labor market fluidity can be accounted for by employer-to-employer transitions, with the remainder almost equally split between employment inflows and outflows.

Fourth, we revisit the ins and outs debate for Canada. The question is how much time-variation in the unemployment rate can be explained by flows into unemployment versus flows out of unemployment. Shimer (2012) and Elsby et al. (2013) argue that flows out (i.e., the hiring margin) play a dominant role, and this has motivated models in which the layoff rate is constant. For Canada, we find that outflows can explain about 60-70% of the variation in the unemployment rate, with inflows explaining about 20-40%, depending on the model and decomposition method used. We find a somewhat larger role for “ins” relative to previous work for Canada by Elsby et al. (2013), partly because of the greater detail of the data we have access to.³

Employer-to-employer flows account for a huge fraction of total worker flows, and plausibly play a fundamental role in the functioning of the labor market. Several studies have found that these flows exhibit procyclicality (Fallick and Fleischman, 2004; Mazumder, 2007; Bjelland et al., 2011). Yet, the empirical evidence on employer-to-employer flows is limited for a number of reasons. First, measuring employer-to-employer flows accurately requires either direct data on when a worker changes jobs or else high frequency data. However, standard data sets such as the CPS only measure labor market status – such as employed or unemployment at a monthly frequency. This introduces time aggregation bias, as multiple transitions cannot be captured between measurement periods. Moreover, the leading US data sources have short time series or important measurement issues.⁴

For example, the Current Population Survey (CPS) only permits estimation of employer-to-employer flows after the 1994 redesign, when the CPS began to ask returning employed respon-

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³Fujita and Ramey (2009), Elsby et al. (2009), and Shimer (2012) decompose the variance in the unemployment rate in the US to analyze whether unemployment inflows or outflows contribute more to cyclical unemployment rate fluctuations. Campolieti (2011) performs this decomposition for Canada. Petrongolo and Pissarides (2008) decompose unemployment variation for several European countries, and Elsby et al. (2013) perform the decomposition for OECD countries including Canada.

⁴Employer-to-employer flows have essentially been included in search models as early as Burdett (1978) in the form of on-the-job-search. However, there does not appear to be any work estimating these flows until Blanchard and Diamond (1989). Blanchard and Diamond constructed a rough estimate of employer-to-employer flows based on the strong assumption that these flows were 40% of job quits, a statistic they took from Akerlof et al. (1988).
dents whether they still worked for the same employer from the previous period (Fallick and Fleischman, 2004).

The Longitudinal Employer Household Dynamics (LEHD) dataset provides direct quarterly matched worker-firm data from the unemployment insurance system that allows one to measure worker flows, but this begins only in 2000 for all states.\(^5\) Bjelland et al. (2011) exploit worker histories to estimate job-to-job flows by identifying workers whose employers changed between periods. However, time aggregation bias is a serious issue in the LEHD as it reports data quarterly. This makes it impossible to determine whether an employment change should be classified as a job-to-job transition or a transition with an intervening nonemployment spell.

The Job Openings and Labor Turnover Survey (JOLTS) provides separation and hiring data from a survey of about 16,000 business establishments that could be used to estimate job-to-job flows if combined with other data. However the data only goes back to the end of 2000.\(^6\)

The SIPP has asked about the identity of the employer from its inception of 1983, which Mazumder (2007) uses to estimate employer-to-employer flows. However, as is well-known, the SIPP is susceptible to important recall and response bias because respondents are only interviewed every four months. In particular, the SIPP exhibits seam bias: respondents are more likely to record a change between interview periods than within interview periods. This potentially biases the cyclicality of worker flows since the timing of the transitions is distorted.\(^7\)

While a long time series of such flows is available for Germany, we believe our estimates provide a useful complement to the international evidence because of the similarity between the Canadian and US labor markets.\(^8\) The US and Canada have comparable levels of labor fluidity, with both countries exhibiting worker reallocation rates of about 40% over the period 2000 to 2007 (Bassanini and Garnero, 2013). The two countries are also similar in their collective bargaining structure (Flanagan, 1999; OECD, 1994, 2017). Moreover, Canada’s business cycles are strongly correlated with the US at around 0.9 (Artis and Zhang, 1997, 1999).\(^9\)

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\(^5\)Data for some states begins as early as 1990. The aggregate data is publicly available, however richer data with granular firm and employee characteristics are confidential.

\(^6\)Davis et al. (2012) construct synthetic JOLTS data that extends the time-series back to 1990.

\(^7\)Moreover, use of the SIPP requires adjustments as data is unavailable for the year 2000, and households in poverty are oversampled for the years 1990, 1996, and 2001 (Mazumder, 2007).

\(^8\)Bachmann (2005) finds that employer-to-employer flows are procyclical in West Germany (1975-2001). Other international evidence includes Gomes (2012), who finds evidence of procyclicality within the UK (1993-2010), and Bobbio and Bunzel (2018), who find procyclicality for Denmark (1985-2003).

\(^9\)Germany differs from the US and Canada in these regards. Germany’s average worker reallocation rate about 30%. Collective bargaining in Germany is usually at the industry-level with strong coordination. In comparison bargaining
We describe our data sources in Section 2. In Section 3 we discuss our methodology. In Section 4 we give basic statistics on gross job finding and separations from the ROE data, and transitions under a three-state model from LFS data. In Section 5 we present our main results. We conclude in Section 6.

2 Data

Since 1976, all Canadian employers have been required to issue a Record of Employment (ROE) when a worker separates from a full-time job. Workers employed less than 20 hours a week have also been included in the ROE since 1997, though this seems to have had little impact on the statistics. The self-employed do not receive a ROE so we exclude them from our analysis of the ROE data.

We use administrative data on the number of ROE’s issued each year over the period 1978-2016 to measure the separation rate. This measure of separations is not subject to time aggregation bias since all separations are recorded, even if the subsequent unemployment spell is very short. Furthermore, the data distinguishes between different types of separations, allowing us to identify the flows of quits, layoffs, or other separations.

We measure total insurable employment excluding the self-employed, using the Canadian Employment Insurance Statistics. The insurable employment stock is about 80% of the total employment stock estimated by the Canadian Labour Force Survey, on average. While the ROE extends as far back as 1976, the extracts available to us begin in 1987. Thus, we extend the data back to

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10We thank Roger Hubley and Lesle Wesa for providing us with administrative details regarding the ROE. Over time, the minimum coverage requirement for the ROE has changed somewhat. The ROE was initiated in 1976. Over 1972 - 1978, the minimum weekly earnings requirement was 20% of maximum weekly insurable earnings. For 1979 - 1980, the minimum requirement was set at 20 hours of work a week or 20% of maximum weekly insurable earnings. Over 1981 - 1986, 15 hours a week and 20% of maximum weekly insurable earnings became the minimum. Over 1987 - 1996, 15 hours a week or 20% of maximum weekly insurable earnings was the minimum. Persons not reaching at least one of these requirements would not receive an ROE. Effective January 1, 1997, the minimum requirement was abolished and every hour of work became insurable. All persons with a job separation from paid employment should receive an ROE.

11This series is referred to by Statistics Canada as “insured employment”, and is available in Statistics Canada’s CANSIM database in Table 276-0011. Insured employment is defined as all “employees”, excluding the self-employed, as defined in the Canadian Labour Force Survey plus the members of the armed forces, who are not included in the Labor Force Survey. Our measure of job finding therefore differs from the measure in Picot et al. (1998). They calculate job finding from the number of “person-jobs” per year measure recorded in the Longitudinal Worker File. This procedure double-counts the employment of workers who change jobs during the year, generating a substantially more volatile measure of employment.

In addition to the ROE, we also use Canadian Labour Force Survey (LFS) public-use microdata. This is a monthly survey similar to the Current Population Survey, focusing on working-age individuals. From the LFS we take variables on labor force state, duration of unemployment, duration of joblessness, class of a worker’s main job, and job tenure length. Using the weighted labor force status we estimate the stock of employed (including self-employed individuals), unemployed, and out of the labor force individuals.

We use Shimer’s (2012) method to estimate gross flows using these stocks. This method also requires measures of short-term stocks, i.e. individuals who have recently entered a given state. We estimate the stock of short-term unemployment as the weighted number of individuals with a duration of unemployment that is four weeks or less. Short-term employment is estimated as the weighted number of individuals with a job tenure that is one month or less. All of these estimated stocks are seasonally adjusted using the Census Bureau’s X-12-ARIMA program. These data are then used to estimate gross labor flows using a two-state model and a three-state model.

We also use aggregated seasonally unadjusted employment and unemployment stock data that is averaged over the calendar year from Statistics Canada’s CANSIM database based on the LFS, as these will be based on the full sample. These data are used to compute rates. For comparisons against the US we use separations and employment stock data from the Job Openings and Labor Turnover Survey (JOLTS).

3 Methodology

Our main goals are to estimate the unemployment to employment rate, the employer-to-employer flow rate, and a measure of labor market fluidity. The direct data on total separation flows from the ROE, in conjunction with data on the stocks for labor force states, allow us to provide a complete view of the Canadian labor market.

To estimate the unemployment to employment rate we perform a decomposition of worker flows using a three-state model with employment, unemployment, and labor force inactivity

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12 We estimate short-term non-employment similarly, using the weighted number of individuals with a duration of joblessness that is one month or less.
Based on the following equations. Let \( f \) denote flow levels and \( p \) denote flow probabilities with superscripts indicating the labor force states (e.g., \( p_{t}^{EU} \) is the probability of making a transition from employment to unemployment). Then we have:

\[
U_{t+1} = U_{t} + E_{t}p_{t}^{EU} + O_{t}p_{t}^{OU} - U_{t}(p_{t}^{UE} + p_{t}^{UO}) \tag{1}
\]

\[
E_{t+1} = E_{t} + U_{t}p_{t}^{UE} + O_{t}p_{t}^{OE} - E_{t}(p_{t}^{EU} + p_{t}^{EO}) \tag{2}
\]

\[
U_{t+1}^{s} = E_{t}p_{t}^{EU} + O_{t}p_{t}^{OU} \tag{3}
\]

\[
N_{t+1}^{s} = E_{t}(p_{t}^{EU} + p_{t}^{EO}) \tag{4}
\]

\[
f_{t}^{EU} = E_{t}p_{t}^{EU} \tag{5}
\]

Where \( U_{t} \) is unemployment, \( E_{t} \) is employment, \( O_{t} \) is labor force inactivity, \( U_{t}^{s} \) is short-term unemployment, and \( N_{t}^{s} \) is short-term non-employment, ie., the sum of unemployment and inactivity.\(^{13}\)

With these five equations, we only need one more condition to solve the system. We assume that the probability of a transition from labor force inactivity to employment, \( p^{OE} \) is constant.\(^{14}\) Given this assumption, we can bound \( p^{OE} \) by noting that \( p^{UO} \) and \( p^{UE} \) should be non-negative. This yields our baseline assumption that \( p^{OE} = 0.006 \), the approximate midpoint of the boundary values. We set \( p^{OE} \) at the boundary values in Appendix B and show it makes little difference. Moreover, since the solutions for the flow rates are linear in their parameters, changing our assumption for \( p^{OE} \) affects the levels of \( p^{UE} \) and \( p^{UO} \), but not their cyclicality.\(^{15}\) The full solution for this model is shown in Appendix A. Note that our normalizing assumption is only needed to estimate \( p^{UO} \) and \( p^{UE} \). Since we can directly estimate \( p^{EU} \) from the data, we can solve for \( p^{OU} \) and \( p^{EO} \).

We can estimate employer-to-employer flows as the difference between total job finding and

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\(^{13}\)These flows will not correct for time-aggregation bias. However, the bias is likely to be relatively small when using monthly data. Using two-state models, where we can explicitly correct for time-aggregation bias, we can directly measure this bias as the difference between the corrected unemployment inflows and short-term unemployment. This suggests that not correcting for time-aggregation bias understates unemployment inflows by about 16%.

\(^{14}\)Jones (1993) finds the average hazard rate for this flow is 3.7% using restricted LFS data, however this leads to negative unemployment to employment transitions in our model.

\(^{15}\)Following Shimer (2012), we assume that all transitions follow a Poisson process such that the continuous rate \( \tilde{p}_{t} \) can be computed from the discrete rate \( p_{t} \) according to the formula \( \tilde{p}_{t} = -\log(1 - p_{t}) \). We apply this transformation to all rates we compute and only report the transformed rates, although the difference is essentially negligible for our three-state results. Unlike Shimer we do not make a distinction between ‘rates’ and ‘probabilities’ and use the terms interchangeably to refer to the continuous rates.
employment inflows. Since employment inflows capture all new entrants into employment, any job finding flows in excess of this must be from employer-to-employer transitions, as shown in Figure 1.\textsuperscript{16} We can calculate employment inflows as the sum of flows from unemployment and inactivity to employment. To estimate total job finding we use the following identity,

\[ E_t = E_{t-1} - S_{t-1} + H_{t-1}, \]  

(6)

where \( E_t \) is the number of people in insurable employment in year \( t \), \( S_{t-1} \) is the number of separations (measured by the number of ROE’s issued) and \( H_{t-1} \) is the number of jobs found.\textsuperscript{17} One issue is that the LFS data includes self-employment flows. These need to be netted out. We estimate new-entrants into self-employment as the number of individuals whose main job is considered self-employment and whose self-employment tenure is one month or less.

We can obtain a measure of labor market fluidity by summing together our job finding and separation measures. Following Davis and Haltiwanger (1990), we call this measure ‘worker reallocation’. However, worker reallocation flows double-count employer-to-employer transitions. Following Kiyotaki and Lagos (2007) we subtract employer-to-employer flows from worker reallocation. We refer to this measure as ‘total worker flows’, and use it as our measure of labor market fluidity.

We detrend all rates with a Hodrick-Prescott (HP) filter, except where explicitly noted. Following Shimer (2012) we use the parameter of \( \lambda = 10^4 \) for the annual ROE data and the parameter of \( \lambda = 10^6 \) for the monthly Labour Force Survey data, which removes secular trends.

4 Basic Statistics

4.1 Canada vs. US

Business cycle fluctuations in the Canadian and US unemployment rates have mirrored one another closely over recent decades. Figure 2a plots seasonally adjusted monthly Canadian and US

\textsuperscript{16}While our framework provides a relatively simple way to estimate employer-to-employer flows, it is subject to some limitations. Davis et al. (2006) warn that without longitudinal data, estimates will include spurious employer-to-employer flows. For example, with the data we have it is impossible to distinguish whether an employer-to-employer transition has occurred, or if an individual has taken on multiple jobs.

\textsuperscript{17}This procedure for estimating the number of people hired implicitly assumes that each worker has only a single job.
unemployment over the period 1978-2016. We use data that have not been detrended for this figure. The business cycles in Canada and the US have been remarkably similar over this period. Figure 2b shows how the Canadian ROE separation rates compare to US separation rates from the JOLTS using data that have not been detrended. The Canadian and US series track each other closely and exhibit similar magnitudes.

4.2 Job Finding and Separation Rates

The level of gross flows is shown in Figure 3a. A striking feature of this figure is how closely job finding and separations comove. This is because of the large role of employer-to-employer flows, which are crucial to understanding employment dynamics. We elaborate on this later.

Table 1a presents some basic statistics on the dynamics of the different types of gross worker flows from the ROE. The average instantaneous job finding probability out of unemployment is 0.71, while the separation probability out of insurable employment is 0.053. Of this, about 45% is

\footnote{Our US results are similar to what Davis et al. (2006) find using the JOLTS.}
Figure 2: Canada vs US

Notes: Panel 2a plots seasonally adjusted monthly Canadian and US unemployment rates over 1978-2016 from the Labour Force Survey and Current Population Survey, respectively. These data are not detrended. Panel 2b plots imputed monthly rates of separations for Canada and the US over 1978-2016 based on annual data from the Record of Employment for Canada, and annualized data from the Job Openings and Labor Turnover Survey for the US. These data are not detrended.

Figure 3: Gross Flows

Notes: Panel 3a plots imputed job finding and separation flows (thousands of workers) for Canada over 1978-2016 based on annual data from the Record of Employment. These data are not detrended. Panel 3b plots monthly rates of quits, layoffs and other separations for Canada over 1978-2016 based on annual data from the Record of Employment. The data are detrended using an HP filter with $\lambda = 10^4$. 
layoffs (0.022), about 20% is quits (0.012) and about 35% is other types of separations (0.017). The standard deviation of job finding is extremely large at 0.21.

The job finding probability is highly negatively correlated with the unemployment rate: the correlation is $-0.87$. The separation probability is also moderately negatively correlated with the unemployment rate: the correlation is about $-0.3$. As we discuss above, this arises from a combination of a highly positively correlated layoff rate ($0.70$), highly negatively correlated quits ($-0.81$) and almost acyclical separations of other types ($-0.14$).

We present the probabilities of layoffs, quits and other types of separations individually in Figure 3b. This figure shows that procyclical total separations are a combination of strongly procyclical quits and mildly countercyclical layoffs.$^{19}$

### 4.3 Three-State Model Results

Flow rates between states in the three-state model are presented in Table 1b. The average UE rate is 15%, while the average EU rate is 0.6%.$^{20}$ Both the UE and EU rates exhibit a high degree of

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19 The importance of heterogeneity in determining the cyclicity of gross worker flows has recently been emphasized by Darby et al. (1986), Davis (2005), Davis et al. (2006) and Elsby et al. (2009).

20 Our results depart, in some cases substantially, from Jones (1993) who uses the panel component of the restricted-use LFS data to estimate worker flows. For example, Jones estimates an average of 1.3 million transitions between dif-
Worker flow levels are shown in Figure 4. This figure illustrates importance of employer-to-employer flows for understanding worker dynamics as employer-to-employer flows dwarf the other flows, with only inactivity-unemployment flows coming close. Furthermore, employer-to-employer flows show a marked upward trend, in stark contrast to the almost flat trends exhibited by other flows in and out of employment.

Much of the literature (e.g. Elsby et al., 2009; Fujita and Ramey, 2009; Shimer, 2012) has used a two-state model. How much difference does this make? It affects the level of the flows a bit

cyclicality (-0.67 and 0.66, respectively) that may explain why they can account for so much of the variation in the steady-state unemployment rate.\footnote{The large flow rates between unemployment and inactivity may seem surprising. However, in terms of levels, the lion’s share of unemployment inflows actually comes from inactivity. As a reminder, this is unrelated to our normalizing assumption, which is only needed to estimate UE and UO flows.}

Worker flow levels are shown in Figure 4. The figure plots imputed annual employer-to-employer flows and three-state worker flows for Canada over 1978-2016. The employer-to-employer flows are estimated using data from the Record of Employment and Labour Force Survey, while the three-state worker flows use data solely from the Labour Force Survey. These data are not detrended.

Our results suggests that on average during a similar time period just over 790,000 transitions are made. The direction of this difference is consistent with studies of response errors in US survey data. Poterba and Summers (1986) argue that response errors in labor force states cause spurious transitions to be recorded. This could cause Jones’ estimates to be inflated. Bowers and Horvath (1984) suggest that durations for the newly unemployed are overstated, which could cause our estimates to be understated.

\footnote{We show details of the two-state model and its results in Appendix C.}
(0.38 vs 0.38 for unemployment outflows, 0.028 vs 0.034 for inflows), but has almost no effect on the cyclicality. The level of unemployment outflows in the two-state model has a correlation of 0.99 with unemployment outflow levels from the 3-state model \((UE + UO)\); and the correlation is 0.99 for the unemployment inflow levels.

5 Main Results

5.1 Cyclicality of Employer-to-Employer Transitions

We show the basic statistics for employer-to-employer flow rates in the last column of Table 1b. The average monthly rate is 3%, which is substantially larger than the other flows originating from employment. It is comforting that the average rate we find is similar to those found for the US by Fallick and Fleischman (2004) and Nagypal (2004) who estimate monthly rates of 2.6% and 2.75% respectively for the US using CPS data.\(^{23}\) Figure 5 shows that employer-to-employer flows exhibit a strong negative correlation with the unemployment rate (-0.54).

5.2 Proportionality of EE and UE Flows

We compare our three-state measures against our direct measures of layoffs and job finding calculated using the ROE in Figures 6a and 6b. Our measure of the EU rate closely tracks layoffs. In contrast, there is almost no correlation between the ROE measure of separations and the EU rate. This reflects the massive role of employer-to-employer transitions (which are procyclical and dwarf the countercyclicality of layoffs) in total separations.

Similarly, the UE rate closely tracks the ROE measure of job finding, with a correlation of 0.64. This reflects the fact that UE and EE rates are close to proportional. What does this tell us about labor market search models?

Consider a simple model of random, on-the-job search based off of Petrongolo and Pissarides (2001).\(^{24}\) Let the meeting technology for both employed and unemployed job searchers be described by \(m(sE_t + U_t, V_t)\), where \(E_t\) denotes the employment stock, \(U_t\) denotes the unemployment stock, \(V_t\) is the level of vacancies, and \(s\) is the search intensity of employed searchers relative

\(^{23}\)Our results are substantially above those of Bjelland et al. (2011), who estimate average quarterly rates of 3.9% for the US using the Longitudinal Employer Household Dynamics database. This is not unexpected as Bjelland et al. (2011) focus on stable employer-to-employer transitions, and thus do not address transitions between short-term jobs.

\(^{24}\)We ignore search from outside the labor force.
Figure 5: Employer-to-Employer Transition Rate

Notes: The figure plots imputed monthly probabilities of employer-to-employer transitions, alongside the unemployment rate, for Canada over 1978-2016. The employer-to-employer flows are estimated using data from the Record of Employment and Labour Force Survey, while the unemployment rate is drawn from the Labour Force Survey. All data are detrended using an HP Filter with $\lambda = 10^6$.

It is simple to extend the model to allow unemployed searchers to receive more or less than their ‘fair’ proportion of meetings, as in Burgess (1993).
Notes: Panel 6a compares layoffs with a three-state measure of employment to unemployment transitions for Canada over 1978-2016. Layoffs is based on annual Record of Employment data, detrended using an HP filter with $\lambda = 10^4$. The EU measure is the quarterly average of monthly series constructed using Labour Force Survey data, detrended with $\lambda = 10^6$.

Panel 6b compares job finding with a three-state measure of unemployment to employment transitions for Canada over 1978-2016. Job finding is based on annual Record of Employment data, detrended using an HP Filter with $\lambda = 10^4$. The UE measure is the quarterly average of monthly series constructed using Labour Force Survey data, detrended with $\lambda = 10^6$. 

Figure 6: Layoffs vs EU and Job Finding vs UE
\[ EE = sA_E \frac{1}{sE_t + U_t} m(sE_t + U_t, V_t) \]  

(8)

The ratio of UE flows to EE flows will be:

\[ \frac{UE}{EE} = \frac{A_U}{sA_E} \]  

(9)

Thus, the proportionality result we find will be generated if the match efficiencies remain stable over time, or if they change proportionally. Studies of match efficiency for unemployed searchers have found that it exhibits procyclicality (Barlevy, 2011; Veracierto, 2011; Barnichon and Figura, 2015). Hall and Schulhofer-Wohl (2015) estimate match efficiency among heterogeneous job searchers, including employed searchers. They find an aggregate measure of match efficiency that includes employer-to-employer flows shows the same movement as an aggregate measure that excludes those flows.\(^{26}\) This suggests that the match efficiencies for employed and unemployed searchers comove.

In addition to the assumption that the ratio of aggregate matching efficiencies are constant, these results require two further assumptions. First, the structure and parameters of the meeting technology are identical. That is, the search behavior of the employed and unemployed can be modeled as being the same. Second, the total number of meetings between job searchers and vacancies is distributed based on their contribution to total search effort.

These two assumptions are common in random, on-the-job search models that endogenize the offer arrival rate (ie. the meeting technology) (Burgess, 1993; Pissarides, 1994, 2000), including state-of-the-art models such as Lise and Robin (2017) that allow for heterogeneity among job searchers and firms. The assumption of a constant ratio between match efficiencies depends on the structural details of the model. For example, Lise and Robin allow for the possibility that this ratio will be constant depending on the implementation of the model.\(^{27}\)

\(^{26}\)Hall and Schulhofer-Wohl (2015) estimate job finding probabilities for different classes of searchers using a logit function with CPS data. Then using vacancy data from the JOLTS, they estimate detrended match efficiencies as the residual of a regression of job finding probability on a measure of labor market tightness.

\(^{27}\)In Lise and Robin (2017) the acceptance function is based on match surplus (the difference between the value of a match and the outside option). Match surplus varies over time only through aggregate productivity shocks that affect both match values and the outside option. If aggregate productivity shocks affect match values and the outside option in the same way the model will generate strict proportionality of UE and EE flows. However, in their implementation of the model, only match value is a function of aggregate shocks, while the outside option is not. How closely UE and EE comove depends, therefore, on how much the ratio of acceptance probabilities of employed versus unemployed workers varies over the business cycle.
5.3 Role of EE Flows in Labor Market Fluidity

We have described gross worker flows, but how do these various flows contribute to labor market fluidity? We can approach this using total worker flows \((wf)\) as our measure of labor market fluidity, which we base off of Kiyotaki and Lagos (2007).\(^{28}\) This can be decomposed into three components: employer-to-employer transitions \((EE)\), employment inflows \((NE)\), and employment outflows \((EN)\).\(^{29}\)

\[
wf = EE + NE + EN
\]  

(10)

The latter two flows are simply job finding less employer-to-employer flows, and separations less employer-to-employer flows, all of which we detrend using the HP filter with \(\lambda = 10^4\).\(^{30}\)

As these components are additively separable, we can easily decompose the variance of total worker flows by following the logic of Fujita and Ramey (2009).

\[
\text{Var}(wf) = \text{Var}(EE) + \text{Var}(NE) + \text{Var}(EN) + 2\text{Cov}(EE, NE) + 2\text{Cov}(EE, EN) + 2\text{Cov}(NE, EN)
\]  

(11)

\[
= \text{Cov}(EE, wf) + \text{Cov}(NE, wf) + \text{Cov}(EN, wf)
\]

Since the variance of total worker flows will be the sum of the covariances between it and its components, we can determine the contribution of each component by dividing both sides of the equation by the variance of \(wf\). For example, the contribution from \(EE\) flows will be,

\[
\beta_{EE} = \frac{\text{Cov}(EE, wf)}{\text{Var}(wf)}
\]  

(12)

This allows us to rewrite Equation 11 as follows.\(^{31,32}\)

---

\(^{28}\)Kiyotaki and Lagos (2007) note that worker turnover = job finding + separations, and that \(wf\) = worker turnover – \(EE\), from which our Equation 10 can be inferred.

\(^{29}\)Haltiwanger et al. (2015) also decompose worker flows into employer-to-employer flows, employment inflows, and unemployment outflows. Our paper performs this decomposition for gross worker flows, while they decompose net flows.

\(^{30}\)We construct these measures using the ROE data instead our three-state flows from the LFS. This is because our measure of employer-to-employer flows excludes self-employment.

\(^{31}\)\(\beta_{EE}\) can be easily computed as the slope coefficient from a simple regression of \(EE\) on \(wf\).

\(^{32}\)As Fujita and Ramey note, these terms are defined identically to the betas used in finance.
Table 2: Total Worker Flows Decomposition

<table>
<thead>
<tr>
<th></th>
<th>Employment Inflows</th>
<th>Employment Outflows</th>
</tr>
</thead>
<tbody>
<tr>
<td>EE</td>
<td>0.62</td>
<td>0.20</td>
</tr>
<tr>
<td>NE</td>
<td>0.18</td>
<td></td>
</tr>
</tbody>
</table>

Notes: The table presents decompositions of the variance of total worker flows for Canada over 1978-2016. These flows are estimated using data from the Record of Employment and Labour Force Survey, and are detrended using an HP filter with $\lambda = 10^4$.

\[ 1 = \beta^{EE} + \beta^{NE} + \beta^{EN} \] (13)

Table 2 shows the contributions from each component. We find that about 60% of the cyclical variation in total worker flows can be accounted for by employer-to-employer transitions. Employment inflows and outflows each explain about another 20%. The dominant role of employer-to-employer transitions, coupled with its procyclicality, suggest that recessions have a sullying effect on the Canadian Labor Market. The reduction in worker reallocation during recessions potentially leads to lower quality job matches.

5.4 The Ins and Outs of Unemployment in Canada

Variance decompositions have also been used to study whether unemployment inflows or outflows contribute more to the variation in the unemployment rate. A recent generation of search models has included only endogenous job creation, as opposed to endogenous separation, motivated by the notion that endogenous separations are acyclical (Hall, 2006; Shimer, 2012). However, contemporary papers that have decomposed the unemployment rate have found a more nuanced picture.\textsuperscript{33}

Shimer (2012) decomposes the variance in unemployment rate fluctuations by computing hypothetical steady-state unemployment rates where only one type of transition is allowed to vary, with the others held at their average value. He derives an expression for steady-state unemployment under a three-state setting by assuming that employment, unemployment, and labor force inactivity are all at their steady-state levels,

\textsuperscript{33}Earlier papers in the literature framed the argument in terms of unemployment incidence (inflows) versus unemployment duration (outflows). Sider (1985) and Baker (1992) argue that outflows play a more prominent role in understanding unemployment dynamics. Darby et al. (1985, 1986), Blanchard and Diamond (1990), Davis and Halliwanger (1990, 1992), Burgess (1992), and Burgess and Turon (2005) support the position that inflows are the dominant factor. Cross-national studies such as Petrongolo and Pissarides (2008) and Elsby et al. (2013) find that the relative importance of outflows versus inflows varies between nations.
Table 3: Unemployment Rate Decomposition

<table>
<thead>
<tr>
<th></th>
<th>UE</th>
<th>EU</th>
<th>UO</th>
<th>OU</th>
<th>EO</th>
<th>OE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Canada</td>
<td>0.69</td>
<td>0.26</td>
<td>0.054</td>
<td>0.022</td>
<td>0.066</td>
<td>-</td>
</tr>
<tr>
<td>US Shimer (2012)</td>
<td>0.49</td>
<td>0.22</td>
<td>0.17</td>
<td>0.12</td>
<td>-0.05</td>
<td>0.08</td>
</tr>
</tbody>
</table>

Notes: The table presents various decompositions of the variability in the steady-state unemployment rate for Canada over 1978-2016, and for the US over different periods. The Canadian data are detrended using an HP filter with $\lambda = 10^6$. Canadian rates are detrended using an HP Filter with $\lambda = 10^6$. Canadian data are drawn from the Labour Force Survey. The US results are drawn from Shimer (2012), covering the period 1967-2010. The US results are drawn from data that have been detrended using the HP Filter.

$$u_t^* = \frac{p_t^{EO} p_t^{OU} + p_t^{OE} p_t^{EU} + p_t^{OU} p_t^{EU}}{(p_t^{EO} p_t^{OU} + p_t^{OE} p_t^{EU} + p_t^{OU} p_t^{EU}) + (p_t^{EO} p_t^{OU} + p_t^{OE} p_t^{EU} + p_t^{OU} p_t^{EU})}$$ (14)

These hypothetical steady-states are then regressed on the actual next-period unemployment rate. The slope coefficient thus shows the proportion of the total variation in the actual unemployment rate that can be accounted for by the covariance between the actual unemployment rate and the hypothetical rates. Full details can be found in Appendix D. This is not an exact decomposition, so the contributions will not sum to one. However, this method gives a quantitative measure of the comovement between unemployment rate variation, and the variation in worker flows when using a two-state model or a three-state model.

Steady-state decompositions are only valid if the correlation between the steady-state and actual unemployment rate are high. Following Shimer (2012), we look at the correlation between the steady-state unemployment rate and the actual unemployment rate in the following period. We use quarterly averaged data as Shimer does, resulting in a correlation of 0.93.34

Table 3 shows the coefficients from Shimer’s regression-based decomposition method. We find that the UE rate accounts for about 70% of unemployment rate variation. The EU rate accounts for another 25%, with flow in and out of the labor force collectively contributing about 15%.35

Table 3 also shows Shimer’s results for the US over 1967-2010. He finds that UE rates account for about 50% of unemployment rate variation, while the EU rate accounts for about 25%, and about 35% for flows in and out of the labor force. Thus, our results suggest that the UE rate plays

---

34This is consistent with Campolieti (2011). Moreover, the correlation we find for Canada is lower than the correlation Shimer (2012) finds for the US (0.99). This matches Elsby et al. (2013), who find that unemployment dynamics are ‘faster’ in the US than Canada.

35Our three-state flows are not corrected for time aggregation bias. Under a two-state setting, using flows that have not been corrected for time aggregation bias in a regression-based decomposition method decreases the outflow contribution from 0.69 to 0.60, and increases the inflow contribution from 0.22 to 0.30.
a larger role in Canada, while flows in and out of the labor force have only a minor role.

5.5 Comparison to Literature

Within the literature, variance decompositions of the unemployment rate have differed along two dimensions: whether the decomposition takes place under a three-state or two-state model, and the method used for the decomposition. In addition to Shimer’s regression-based decomposition method, Elsby et al. (2009) and Fujita and Ramey (2009) derive a log-differentiation decomposition method. This method decomposes the steady-state unemployment rate into unemployment inflow and outflow components. The advantage of their method is that the contributions approximately sum to one. However, it is not obvious how to extend the log-differentiation method to a three-state setting, which is why we only present the regression-based decomposition above. Details of the log-differentiation decomposition method can be found in Appendix E.

In order to compare against the existing literature, including results for Canada, we show the decomposition results using Shimer’s (2012) two-state model in Table 4. Under the log-differentiation method, we find that unemployment outflows account for about 60% of the variation in steady-state unemployment rate, while the inflow rate accounts for the remaining 40%. The regression-based method attributes a noticeably smaller role for unemployment inflows: just over 20%, while about 70% of unemployment rate variation is explained by outflows.

Our two-state unemployment rate decompositions for Canada are similar to earlier estimates for the US when holding the method constant. Shimer (2012) finds an outflow contribution of 85% and an inflow contribution of 15% for the US using his regression-based method. Fujita and Ramey (2009) attribute about 60% of variation to unemployment outflows and 40% to inflows for the US, using a log-differentiation decomposition. Notably, these are almost identical to our

---

36A third dimension exists that considers whether the decompositions take place in steady-state or out of steady-state. Elsby et al. (2013) suggest the difference is negligible for Canada. We only consider steady-state decompositions.
37Smith (2011) has made progress in extending this decomposition model to three-states, but the role of flows in and out of the labor force are still difficult to interpret under her extension.
38The correlation between the steady-state and actual unemployment rate is 0.95 for the two-state model, using monthly data. The difference in correlations between the two and three-state settings appears to exist because deriving a steady-state formula for the unemployment rate requires stronger assumptions in the three-state model than the two-state model. The three-state model assumes that employment, unemployment, and inactivity are all at their steady-state values. The two-state model only assumes that unemployment and the sum total of employment and inactivity are at their steady-state. Thus it allows the proportion of employment and inactivity to vary.
39Shimer uses duration-based flow data from the CPS over 1976-2010, detrended with an HP filter with the non-standard parameter of $\lambda = 10^5$.
40Fujita and Ramey perform this analysis on gross flows data from the CPS over 1976-2005 detrended with an HP
Table 4: Two-State Unemployment Rate Decomposition

<table>
<thead>
<tr>
<th></th>
<th>Canada</th>
<th>US</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Outflows</td>
<td>Inflows</td>
</tr>
<tr>
<td>Log-Differentiation Decomposition</td>
<td>0.59</td>
<td>0.41</td>
</tr>
<tr>
<td>Regression-based Decomposition</td>
<td>0.69</td>
<td>0.22</td>
</tr>
</tbody>
</table>

Notes: The table presents various decompositions of the variability in the steady-state unemployment rate for Canada over 1978-2016, and for the US over different periods. The Canadian data are detrended using an HP filter with $\lambda = 10^6$. Canadian rates are detrended using an HP Filter with $\lambda = 10^6$. Canadian data are drawn from the Labour Force Survey. The US results are drawn from Shimer (2012) for the regression-based method, which covers the period 1976-2010, and Fujita and Ramey (2009) for the log-differentiation method, which covers 1976-2005. All US results are drawn from data that have been detrended using the HP Filter.

Results for Canada when using their decomposition method.

Using a log-differentiation decomposition, we find a somewhat larger role for unemployment inflows than previous studies that look at Canada, such as Elsby et al. (2013) and Campolieti (2011). The differences between our results and Elsby et al. arise from limitations in their data, as well as small methodological differences. Similar small definitional changes are likely to explain the gap between our results and Campolieti as well.

6 Conclusion

We have four main results. First, using Record of Employment data and the Labour Force Survey, we provide a complete decomposition of worker flows for Canada, including employer-to-employer transitions. Employer-to-employer flows are large and strongly procyclical. Second, we find that unemployment to employment transitions, and gross job finding flows tend to be proportional, which supports common assumptions used in random search models. Third, we decompose the variation of a measure labor market fluidity into employer-to-employer transitions, employment inflows, and employment outflows. Employer-to-employer transitions account for about 60% of the variation. Fourth, we find that unemployment outflows contribute more to unemployment rate variation than inflows. However, the contribution from inflows is also substantial, in line with the previous literature.

\[ \text{filter with a standard parameter of } \lambda = 1600. \]

\footnote{Elsby et al. use OECD data derived from the LFS beginning in 1976 and ending in 2009. Campolieti uses LFS data beginning in 1976 and ending in 2008.}
A Three-State Model Solution

As Equation 5 implies, we can estimate the level of employment to unemployment transitions probability ($f_{EU}^t$) directly from the data. This allows us to determine the probability of this transition by dividing by the employment stock, $E_t$

From there we can solve for the probability of employment to inactivity transitions ($p_{EO}^t$) using Equation 4, and the probability of inactivity to unemployment transitions ($p_{OU}^t$) using Equation 3.

\[ p_{EO}^t = \frac{N_{s+1}^t}{E_t^t} - p_{EU}^t \]  
\[ (15) \]
\[ p_{OU}^t = \frac{U_{s+1}^t - E_t^t p_{EU}^t}{O_t^t} \]  
\[ (16) \]

Where $N_{s+1}^t$ denotes the next month’s level of individuals who have been jobless for one month or less given previous employment, $U_{s+1}^t$ denotes the next month’s level of individuals who have been unemployed for four weeks or less, and $O_t^t$ is the stock of out-of-labor force individuals.

Once a value is assumed for the inactivity to employment flow rate ($p_{OE}^t$), we can solve for the unemployment to employment probability ($p_{UE}^t$) by combining and rearranging Equations 2 and 4:

\[ E_{t+1} = E_t - N_{t+1}^s + U_t p_{UE}^t + O_t p_{OE}^t \]  
\[ (17) \]
\[ p_{UE}^t = \frac{1}{U_t} \left( E_{t+1} - E_t + N_{t+1}^s - O_t p_{OE}^t \right) \]  
\[ (18) \]

Likewise, we can solve for the unemployment to out-of-the-labor force probability ($p_{UO}^t$) by combining and rearranging Equations 1 through 4:

\[ U_{t+1} + E_{t+1} + N_{t+1}^s - U_{t+1}^s = U_t - U_t (p_{UE}^t + p_{UO}^t) + E_t + U_t p_{UE}^t + O_t p_{OE}^t \]  
\[ (19) \]
\[ p_{UO}^t = \frac{1}{U_t} \left( U_t - U_{t+1} + E_t - E_{t+1} - N_{t+1}^s + U_{t+1}^s + O_t p_{OE}^t \right) \]  
\[ (20) \]

Where $U_t$ denotes the stock of the unemployed individuals.
We show these flow rates assuming that the flow rate from out-of-the-labor force to employment is constant at 0.006 in Figure A.1.

B Three-State Model Assumptions

We assume that the inactivity to employment flow rate is 0.006. We use this assumption as most flow rates result in some negative unemployment to employment or unemployment to inactivity flows. Using numerical analysis, we find that all flows are non-negative when the inactivity to employment flow rate is between about 0.002 and 0.0105. Thus, we select 0.006 as the approximate midpoint between these boundaries. In Figure B.2 we show the flow rates if the boundaries are used.

C Shimer Two-State Model

We denote the probability of these two-state transitions as $p_{tie}^u$ and $p_{tie}^e$. Note that we use uppercase superscripts to denote three-state models, and lowercase superscripts to denote two-state models.

Assume that workers are homogeneous with identical unemployment inflow and outflow probabilities. The continuous unemployment outflow probability can be calculated as:

$$\tilde{p}_t^{ue} = -\ln \left( \frac{U_{t+1} - U_{t+1}^s}{U_t} \right)$$

Calculating the continuous inflow rate to unemployment, $\tilde{p}_t^{eu}$ is slightly more involved. Again following Shimer, we derive an implicit function of the inflow rate to unemployment.

$$U_{t+1} = \frac{(1 - e^{-\tilde{p}_t^{ue} - \tilde{p}_t^{eu}})\tilde{p}_t^{eu}}{\tilde{p}_t^{ue} + \tilde{p}_t^{eu}} (E_t + U_t) + e^{-\tilde{p}_t^{ue} - \tilde{p}_t^{eu}} U_t$$

where $E_t$ is the average employment stock over period $t$. We assume that the working-age non-institutionalized population does not change within a period.

When comparing results from the two-state model and three-state model, we calculate the three-state unemployment inflow rate as the sum of EU and OU levels divided by the sum of employment and inactivity, with the continuous transformation applied to the result. The three-state outflow rate is computed under a similar procedure.
Figure A.1: Complete Decomposition of Worker Flows Between States

Notes: The figure plots imputed flow rates between employment, unemployment, and out-of-the-labor-force for Canada over 1978-2016 based on monthly data from the Labour Force Survey. All data are detrended using an HP Filter with $\lambda = 10^6$. 

23
Figure B.2: Three-State Flow Bounds

Notes: The figure plots imputed flow rates between unemployment and inactivity, and unemployment and employment under different inactivity to employment flow rates for Canada over 1978-2016 based on monthly data from the Labour Force Survey.

Table C.1: Two-State Model Statistics

<table>
<thead>
<tr>
<th></th>
<th>Unemployment Outflows</th>
<th>Unemployment Inflows</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>0.38</td>
<td>0.034</td>
</tr>
<tr>
<td>Std. Dev.</td>
<td>0.052</td>
<td>0.0027</td>
</tr>
<tr>
<td>Unemployment Rate Corr.</td>
<td>-0.82</td>
<td>0.45</td>
</tr>
</tbody>
</table>

Notes: The table presents average monthly statistics for worker flows for Canada over 1978-2016, calculated using a two-state Shimer model from Labour Force Survey data. All series are detrended using an HP Filter with $\lambda = 10^6$. 
If we use a two-state measure of job finding instead of a three-state measure to compute employer-to-employer flows, the level of the employer-to-employer rate rises slightly (3.2%), though the standard deviation and cyclicality remain substantially the same. Further, the employer-to-employer rate we estimate is not sensitive to our normalizing assumption for the three-state model, as job finding under the three-state model is $U_E + U_O$, and the three-state model computes essentially the same job finding flows regardless of our normalizing assumption. The normalizing assumption simply changes the proportion of job finding flows that come from UE versus UO.

## D Regression-based Decomposition

Shimer (2012) proposes the following decomposition. Suppose $p_{tu}$ is the unemployment outflow rate, $p_{ue}$ is the inflow rate, and $u_t$ is the unemployment rate, $\bar{p}^{ue}$ is the average inflow rate and $\bar{p}^{eu}$ is the average outflow rate. The steady-state unemployment can be expressed as:

$$u^* = \frac{p_{tu}}{p_{tu} + p_{ue}} \quad (23)$$

The “outflow” component of steady-state unemployment is $\frac{\bar{p}^{eu}}{p_{tu} + p_{ue}}$, and the “inflow” component is $\frac{\bar{p}^{eu}}{p_{tu} + p_{ue}}$. The fraction of the variation in the unemployment rate due to outflows is the covariance between $\frac{\bar{p}^{eu}}{p_{tu} + p_{ue}}$ and $u_{t+1}$ divided by the variance of $u_{t+1}$, which is conveniently the coefficient of a regression of $\frac{\bar{p}^{eu}}{p_{tu} + p_{ue}}$ on $u_{t+1}$. The fraction of the variation due to outflows is the covariance of $\frac{\bar{p}^{eu}}{p_{tu} + p_{ue}}$ with $u_{t+1}$ divided by the variance of $u_{t+1}$. As Shimer (2012) discusses, this is not an exact decomposition so the two parts need not sum to one.\(^{42}\)

This method can also be applied to three-state models, using the conditions that employment, unemployment, and inactivity are all at steady state.

$$E_t(p_t^{EU} + p_t^{EO}) = U_t p_t^{UE} + O_t p_t^{OE}$$

$$U_t(p_t^{UE} + p_t^{UO}) = E_t p_t^{EU} + O_t p_t^{OU} \quad (24)$$

$$O_t(p_t^{OE} + p_t^{OU}) = E_t p_t^{EO} + U_t p_t^{UO}$$

Manipulating these equations, the steady-state unemployment rate can be expressed as:

\(^{42}\)Nevertheless, in Shimer’s (2012) application, the two parts sum almost exactly to one.
\[
\bar{u}_t^* = \frac{p_t \bar{p}_t \bar{E}_t \bar{O}_t + p_t \bar{P}_t \bar{E}_t + p_t \bar{O}_t \bar{E}_t}{(p_t \bar{P}_t \bar{O}_t + p_t \bar{P}_t \bar{E}_t + p_t \bar{O}_t \bar{E}_t) + (p_t \bar{P}_t \bar{O}_t + p_t \bar{P}_t \bar{E}_t + p_t \bar{O}_t \bar{E}_t)}
\] (25)

By constructing hypothetical steady-states where only one transition is allowed to vary, while the others are held constant at their mean value, we can determine how much that transition contributes to variation in the unemployment rate. E.g. the EU component of steady-state unemployment rate is

\[
\bar{p}_t \bar{E}_t \bar{O}_t \bar{E}_t \quad \text{and} \quad \bar{p}_t \bar{P}_t \bar{E}_t \bar{O}_t \bar{E}_t
\]

(E)

**Log-Differentiation Decomposition**

Begin with Shimer’s steady-state unemployment rate equation under a two-state model:

\[
\bar{u}_t^* = \frac{p_t \bar{p}_t \bar{E}_t \bar{O}_t}{p_t \bar{P}_t \bar{O}_t + p_t \bar{P}_t \bar{E}_t + p_t \bar{O}_t \bar{E}_t}
\] (26)

Log differentiation of both sides yields:

\[
d\ln \bar{u}_t^* = d\ln p_t \bar{p}_t \bar{E}_t \bar{O}_t - d\ln (p_t \bar{P}_t \bar{O}_t + p_t \bar{P}_t \bar{E}_t + p_t \bar{O}_t \bar{E}_t)
\] (27)

\[
d\ln \bar{u}_t^* = d\ln p_t \bar{p}_t \bar{E}_t \bar{O}_t - \frac{1}{p_t \bar{p}_t \bar{E}_t \bar{O}_t + p_t \bar{P}_t \bar{O}_t + p_t \bar{P}_t \bar{E}_t + p_t \bar{O}_t \bar{E}_t} \cdot d(p_t \bar{P}_t \bar{O}_t + p_t \bar{P}_t \bar{E}_t + p_t \bar{O}_t \bar{E}_t)
\] (28)

\[
d\ln \bar{u}_t^* = d\ln p_t \bar{p}_t \bar{E}_t \bar{O}_t - \frac{1}{p_t \bar{p}_t \bar{E}_t \bar{O}_t + p_t \bar{P}_t \bar{O}_t + p_t \bar{P}_t \bar{E}_t + p_t \bar{O}_t \bar{E}_t} \cdot d(p_t \bar{P}_t \bar{O}_t + p_t \bar{P}_t \bar{E}_t + p_t \bar{O}_t \bar{E}_t)
\] (29)

\[
d\ln \bar{u}_t^* = d\ln p_t \bar{p}_t \bar{E}_t \bar{O}_t - \frac{1}{p_t \bar{p}_t \bar{E}_t \bar{O}_t + p_t \bar{P}_t \bar{O}_t + p_t \bar{P}_t \bar{E}_t + p_t \bar{O}_t \bar{E}_t} \cdot d(p_t \bar{P}_t \bar{O}_t + p_t \bar{P}_t \bar{E}_t + p_t \bar{O}_t \bar{E}_t)
\] (30)

\[
d\ln \bar{u}_t^* = d\ln p_t \bar{p}_t \bar{E}_t \bar{O}_t - \frac{1}{p_t \bar{p}_t \bar{E}_t \bar{O}_t + p_t \bar{P}_t \bar{O}_t + p_t \bar{P}_t \bar{E}_t + p_t \bar{O}_t \bar{E}_t} \cdot d(p_t \bar{P}_t \bar{O}_t + p_t \bar{P}_t \bar{E}_t + p_t \bar{O}_t \bar{E}_t)
\] (31)

Fujita and Ramey (2009) then write this as a generic equation.
du_t^* = du_t^{cu} + du_t^{ue} \quad (33)

Taking the variance of the generic equation is akin to taking the variance of the sum of correlated variables. Thus the variance of the generic equation will equal the sum of the covariances.

\[
\text{Var}(du_t^*) = \text{Cov}(du_t^*, du_t^{cu}) + \text{Cov}(du_t^*, du_t^{ue}) \quad (34)
\]

Define:

\[
\beta^{cu} = \frac{\text{Cov}(du_t^*, du_t^{cu})}{\text{Var}(du_t^*)} \quad (35)
\]

As Fujita and Ramey (2009) notes, this is equivalent to the betas in finance.

The decomposition can then be written as:

\[
1 = \beta^{cu} + \beta^{ue} \quad (36)
\]

\(\beta^{cu}\) can then be estimated as the coefficients of simple linear regressions of \((1 - u_t^*)\Delta \ln p_t^{cu}\) on \(\Delta \ln u_t^*,\) with a similar procedure being used to estimate \(\beta^{ue}.\)

F Differences between our study and Elsby, Hobijn, and Şahin (2013)

We summarize the key differences between our analysis and Elsby et al. (2013) [EHS] in Table F.2, and show the cumulative effect of incorporating details of the EHS analysis. Rows 1-4 are differences caused by data limitations, while rows 5-8 are methodological differences. The data limitations are caused by their use of OECD harmonized data. The OECD data only have annual data; hence, EHS have to approximate the model solution to allow for time aggregation. The OECD data also define short-term unemployment as people who have become unemployed in the last 0-3 weeks rather than 0-4 weeks, as one would wish to use in a monthly model. In contrast, we are able to use monthly data with short-term unemployed defined as people who became unemployed in the last 0-4 weeks (we calculate this directly from the LFS). The largest methodological difference is that we include all unemployed people in calculating the fraction of short-term unemployed, so we are assuming those who do not report a duration have durations greater than 27...
1 month (whereas Elsby et al. assume they are drawn equally from all durations). Furthermore, EHS perform their decomposition using the actual unemployment rate, while we decompose the steady-state unemployment rate.

Table F.2: Differences between Elsby et al. (2013) and Our Analysis

<table>
<thead>
<tr>
<th></th>
<th>Outflow Contribution</th>
<th>Inflow Contribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) Baseline</td>
<td>0.59</td>
<td>0.41</td>
</tr>
<tr>
<td>(2) Data Averaged Annually(^a)</td>
<td>0.61</td>
<td>0.38</td>
</tr>
<tr>
<td>(3) EHS Annual Formula Used(^b)</td>
<td>0.66</td>
<td>0.32</td>
</tr>
<tr>
<td>(4) (U^s): 0-3 Weeks</td>
<td>0.69</td>
<td>0.30</td>
</tr>
<tr>
<td>(5) (U^s) Incidence Definition(^c)</td>
<td>0.73</td>
<td>0.25</td>
</tr>
<tr>
<td>(6) No Detrending</td>
<td>0.75</td>
<td>0.24</td>
</tr>
<tr>
<td>(7) Time Period Ends in 2009</td>
<td>0.75</td>
<td>0.23</td>
</tr>
<tr>
<td>(8) Actual vs Steady-State Decomposition</td>
<td>0.78</td>
<td>0.25</td>
</tr>
<tr>
<td>(9) EHS results</td>
<td>0.80</td>
<td>0.24</td>
</tr>
</tbody>
</table>

Notes: The table presents the differences between the analysis of Elsby et al. (2013) and our analysis, as well as the cumulative effect of adopting elements of the analysis of EHS. A gap remains between our results and EHS because we do not account for all differences, such as differences in the numerical method used to solve for unemployment inflow rates.

\(^a\) The data are averaged annually, but remain on a monthly basis.

\(^b\) The use of annual data means EHS have to approximate the model solution to allow for time aggregation.

\(^c\) EHS define short-term unemployment incidence as the proportion of short-term unemployed over the number of unemployed who report a duration, instead of dividing by the total number of unemployed.
References


