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This paper analyzes the basic risk-return relation of differnet volatility components in the cross-section of stock returns. Using option portfolio returns that have a constant exposure to either jump or diffusive risk, I decompose the total volatility risk into four components: market volatility risk, idiosyncratic volatility risk, market jump risk, and idiosyncratic jump risk. The analysis shows, that this decomposition helps in explaining contemporaneous and future returns. While all four components are at play when stocks earn contemporaneously negative returns, idiosyncratic jump and volatility risks are most impotent to explain the cross-sectional variation in positive returns. In addition, stocks that have higher idiosyncratic jump risk earn higher subsequent returns. This relation is robust to various stock characteristics and cannot be explained by the low beta anomaly.


Keywords: Options, Stock Returns, Idiosyncratic Risk, Volatility Risk, Jump Risk

JEL: G12, G13

# The Pricing of Market and Idiosyncratic Jump and Volatility Risks 

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This paper analyzes the basic risk-return relation of differnet volatility components in the cross-section of stock returns. Using option portfolio returns that have a constant exposure to either jump or diffusive risk, I decompose the total volatility risk into four components: market volatility risk, idiosyncratic volatility risk, market jump risk, and idiosyncratic jump risk. The analysis shows, that this decomposition helps in explaining contemporaneous and future returns. While all four components are at play when stocks earn contemporaneously negative returns, idiosyncratic jump and volatility risks are most impotent to explain the cross-sectional variation in positive returns. In addition, stocks that have higher idiosyncratic jump risk earn higher subsequent returns. This relation is robust to various stock characteristics and cannot be explained by the low beta anomaly.


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## 1 Introduction

Higher expected risks demand higher expected returns. This is the corner stone in every asset pricing model. While the classical CAPM of Sharpe (1964) and Lintner (1965) prices the covariation of stock returns with market returns only, recent studies (e.g, see Coval and Shumway (2001), Ang et al. (2006) or Adrian and Rosenberg (2008)) show that the covariation of stock returns with aggregate market volatility is an additional priced risk factor. Yet, aggregate market volatility may stem from diffusive movements of the market index and/or from sudden jumps in the index level. Looking at these different components of aggregate volatility, recent studies have shown that both components are important drivers of the market's equity premium (e.g., see Bates (1991), Duffie et al. (2000) or Santa-Clara and Yan (2010)). In addition, Cremers et al. (2015) highlight the importance of these risks for pricing the cross-section of stock returns. They show that the covariation of stock returns with both, diffusive and jump, market risks are independently priced in the crosssection of stock returns. While Cremers et al. (2015) among others concentrate on the covariation of cross-sectional returns with aggregate risk measures, a second strand of the literature focuses on the pricing implication of individual stock's total volatility. For example, Ang et al. (2006) show that realized idiosyncratic total volatility carries a negative price of risk and Bollerslev et al. (2017) argue that stock price jump risk is priced in the cross-section of stock returns. However, none of these studies looks at the risk return relation of these different volatility components at the same time.

Goal of this paper is to assess the risk return relation by analyzing the different components of a stock's total volatility simultaneously in a model-free fashion. Every stock price is subject to a continuous and a discontinuous movement e.g., the total volatility risk of a stock is driven by diffusive and jump risks. ${ }^{1}$ Both of these risks

[^1]may be driven by the market or are purely idiosyncratic. This makes a stock prone to four potential risks: market volatility risk, idiosyncratic volatility risk, market jump risk, and idiosyncratic jump risk. Martin (2017) and Martin and Wagner (2018) show that both the level of market's and the level of individual stocks' risk-neutral total volatility is an essential driver for expected returns. Thus, I expect the four different types of total volatility risks to have important, and potentially different pricing implications as well.

I make use of the cross-section of stock options for measuring a stock's jump and volatility risks. Stock options have the desirable feature of incorporating market participants' expectation into their prices. Thus, they allow to measure expected risks conditional on the current information set, without making use of historic data. This has the advantage that risk measures relying on options may be more accurate and adapt faster to innovations in these risks than measures estimated from historical returns. Building on, but extending the approach of Cremers et al. (2015), I construct option portfolios that have a constant exposure to either changes in the stochastic volatility or changes in the jump probabilities, while hedging the other risk. This generates option portfolios with returns solely driven by either volatility or jump risks. The constant risk exposure is important, since it allows comparing returns on the portfolios over time and across different stocks. More precise, the option portfolio returns are proportional to changes in the market and idiosyncratic volatility (jump) risk premium and the sensitivity of the portfolio values towards these risks. Keeping the sensitivities constant leaves the remaining variation in returns to changes in the risk prima. Constructing the two option portfolios for the market as well as for single stocks, allows to decompose the volatility (jump) risk of a stock into a component stemming from the market and an idiosyncratic component left unexplained.

The empirical analysis shows that both option portfolios are a good proxy ous and volatility risk synonymously if not explicitly stated otherwise. Similar, I use discontinuous risk and jump risk synonymously.
for either volatility or jump risks. On the market level, the highest returns on the jump risk portfolio are followed by the most extreme 5 minute returns in the sample. Similarly, the largest returns on the volatility risk portfolios are associated with the highest increases in realized volatility. Market participants are willing to pay a premium to hedge against volatility and jump risks on both, the market and single stock level. The daily median return on the volatility (jump) risk portfolio is -9 basis points ( -58 basis points) on the market and -1 basis points $(-56)$ on single stock level. Contemporaneously, I find volatility and jump risk to be negatively related to returns on the market. An increase in both risks lowers contemporaneous returns, an observation often described in the literature (e.g., see Pindyck (1984) or French et al. (1987)). On the single stock level, I find the market's risk components to be significantly negatively related to contemporaneous returns only. This is due to a diverse effect of the idiosyncratic risks. While all four risk components explain negative returns contemporaneously, most important in explaining positive returns are the idiosyncratic risks. A one standard deviation increase in the idiosyncratic jump (volatility) component increases positive returns by 130 (102) basis points, while the effect of the markets volatility component is insignificant and a one standard deviation increase in the market jump component increases positive returns by 3 basis points, only.

Cross-sectional portfolio sorts imply a positive price of idiosyncratic jump risk. Stocks that have a lower idiosyncratic jump risk component are the ones investors are willing to pay the largest premium to hedge these risks. The increase in expected riskneutral jumps is larger than the corresponding increase under the physical measure, for these stocks. Thus, investors dislike jumps and demand a premium to hold these stocks. This intuition is directly supported by the results of the portfolio sorts. The next month returns and alphas of value weighted portfolios are monotonically deceasing, giving rise to a statistically significant difference of the low-minus-high portfolio of $0.57 \%$ and an alpha of $0.67 \%$. On the contrary there is no significant
relation between the other risk measures and subsequent returns. Cross-sectional predictive regressions further show that the findings are robust to various stock characteristics and cannot be explained by the low beta anomaly or risk-neutral higher moments.

Central to the analysis is an appropriate measure for volatility and jump risk, at the market and the single stock level. As mentioned above, I rely on but extend the methodology of Cremers et al. (2015). Different from other measures as the implied tail measure of Bollerslev and Todorov (2011) or Bollerslev et al. (2015), Cremers et al. (2015) construct option portfolios that either proxy for volatility or jump risk using straddles with different times to maturity. These portfolios have the striking advantage that their construction relies on at-the-money options only, rather than deep out-of-the-money options as other tail measures. This is essential for an analysis on a single stock level, since mostly at-the-money equity options are liquidly traded on single stocks. Options that are no more that $10 \%$ in- or out-of-themoney account for $63,76 \%$ of the overall pooled trading volume. Thus, concentrating on at-the-money options allows to estimate the jump and volatility risk measures for a considerably large cross-section of stocks. Yet, when following the exact method of Cremers et al. (2015) the analysis would suffer from a drawback, when it comes to calculating the option portfolios on a single stock level. While Cremers et al. (2015) ensure the delta and the vega (gamma) of the portfolio to be always zero, they require the gamma (vega) to be positive, only. This results in a high time series variation of the gamma (vega) of the hedge portfolio and would induce a large dispersion in cross-sectional gamma (vega). However, the returns of the option portfolios are proportional to changes in the risk premium and the gamma (vega) of the portfolio. If gamma and vega are not constant, any difference between two returns of the jump (volatility) risk portfolios of differnt stocks might be either due to different changes in the risk prima itself (jump/volatility) or due to different exposures to these risks, while changes in the risk prima are the same. This makes
the measure as proposed by Cremers et al. (2015) inappropriate, when the main aim is to compare the cross-sectional differences in the returns of the option portfolios. That is why I keep the gamma and vega constant over time and across stocks using a numerical optimization to reduce any data noise. Thus, the constructed portfolios ensure that their gamma (vega) is always constant and any differences in portfolio returns should be directly related to differences in the risk prima.

The remainder of the paper is structured as follows. In the subsequent Section 2, I discuss the literature which is closest to my research. In Section 3 I describe the data and the methods to calculate the proxy for jump and volatility risk. Section 4 contains my main results and last, Section 5 concludes.

## 2 Literature Review

There is a considerably wide consensus that changes in the market volatility should command a negative risk premium (e.g., Campbell (1993), Campbell (1996) and Campbell et al. (2018)). Since an increase in the market volatility goes along with a deterioration of the investment opportunity set, any asset covarying positively with market volatility can be used as hedge and thus is expected to yield lower returns. Therefore, Ang et al. (2006) analyze changes in the VIX and find that these carry a significant negative premium for the cross-section of stock returns. In a similar spirit, Adrian and Rosenberg (2008) differentiate between short term and long term market volatility and find a negative premium too. In a more general setting Bansal et al. (2013) empirically show that changes in macroeconomic volatility are priced.

Next to market volatility, there is a considerably large discussion about the pricing of idiosyncratic volatility in the cross-section of stock returns. Initiated by Ang et al. (2006), who find the level of idiosyncratic volatility to be negatively priced, many follow-up studies find either a negative, a positive or no price of risk
(e.g., see Ang et al. (2009), Fu (2009) or Bali and Cakici (2008)). While there is no clear consensus about the explanation for these findings (for a broad discussion see Hou and Loh (2016) or Branger et al. (2018)), all these studies look at realized total volatility and thereby do not differentiate between volatility stemming from price jumps and diffusive price movements. Thus, I add to these discussions by splitting these two components up by analyzing wether idiosyncratic diffusive or jump risks drive the findings of Ang et al. (2006).

There is also evidence that price jumps in the market carry a risk premium. Chang et al. (2013) show that market skewness is priced in the cross-section of stock returns. They show that stocks which return co-varies positively with market skewness earn lower returns. Using high frequency data, Bollerslev et al. (2016) measure betas for continuous and discontinuous returns. They show that betas associated with jumps earn a significant risk premium in the cross-section of stock returns, while they find no such support for the continuous betas. Cremers et al. (2015) analyze the pricing of market jump and volatility risk in the cross-section of stock returns separately. To do so, the authors use options on S\&P 500 futures contracts and construct calendar-spread portfolios using two market-neutral straddles with different maturities. In order to measure jumps, the authors construct the option portfolio in such a way, that it is delta-vega-neutral, but gamma positive. This makes the portfolio returns insensitive to changes in the volatility, but sensitive to large changes in the price of the underlying. In the same spirit they construct the volatility factor to be delta-gamma-neutral, but vega positive. Again, this ensures that the returns of the portfolio are insensitive to small and large changes in the price of the underlying, but sensitive to changes in the expected volatility of the underlying. The authors conclude that both these market factors are priced in the cross-section of stock returns. However, they look at market risk only.

I am not the first to analyze the impact of individual jump risk on stock
returns. Conrad et al. (2013) analyze the effect of risk-neutral skewness on a single stock level on subsequent stock returns. They find that stocks with an high exante skewness yield lower returns. Bollerslev et al. (2017) analyze the normalized difference between realized semi-variances, which they measure with high frequency data. They argue that their measure proxies for jumps in the stock prices, since it isolates the discontinuous part of the stock price dynamics while hedging the continuous component. They find that their measure indicates a highly significant premium for jump risk, which is robust to various stock characteristics. Kapadia and Zekhnini (2017) analyze cross-sectional stock returns and find a large fraction of the average return on a stock to be driven by idiosyncratic jump events. To do so, they measure realized idiosyncratic jumps as idiosyncratic returns which are larger than three standard deviations of the stocks return distribution. The authors argue that on average the total annual return on a stock is gained only on four to five days on which idiosyncratic jumps are detected. While this analysis only holds ex-post, they use the implied tail measure of Bollerslev and Todorov (2011) to proxy for expected jumps in the stock price. Thereby, the authors conclude that they do find evidence for a priced jump risk premium in the cross-section of returns. However, all these papers either focus on total jumps in the stock price or on idiosyncratic jumps only, rather than comparing it to the market jump risk. Bégin et al. (2017) estimate a parametric model, which incorporates both, diffusive and jump risk. They separate each risk into market driven and purely idiosyncratic and find that idiosyncratic jumps are largely responsible for the equity premium. However, while their findings are very interesting, their parametrization imposes an rather high model risk. Any estimated price of risk might be due to a misclassification of the model. Thus, I add to these findings by analyzing the impact of jump and volatility risk on stock returns separately for both, the idiosyncratic and the market component of these risks and du so in a model-free fashion.

## 3 Data and Methodology

In this section I discuss my measure of jump and volatility risk first and then elaborate on the data used.

### 3.1 Measures of Jump and Volatility Risks

Assume changes in the stock price are due to a continuous and a discontinuous component, where both are driven to a certain degree from the markets continuous and discontinuous components. In such a case the price process of any asset stems from four components and can be described as:

$$
\begin{equation*}
\frac{d S_{t}^{i}}{S_{t^{-}}^{i}}=\alpha_{t}^{i} d t+\beta_{V O L}^{i} \sqrt{V_{t}^{M}} d W_{t}^{M}+\sqrt{V_{t}^{\epsilon^{i}}} d W_{t}^{\epsilon^{i}}+\beta_{J U M P}^{i} k_{t}^{M} d q_{t}^{M}+k_{t}^{\epsilon^{i}} d q_{t}^{\epsilon^{i}} \tag{1}
\end{equation*}
$$

where $\alpha_{t}^{i}$ is some drift. $W_{t}^{M}$ and $W_{t}^{\epsilon^{i}}$ are the market and idiosyncratic Brownian motion and orthogonal to each other. $V_{t}^{M}$ and $V_{t}^{\epsilon^{i}}$ are the variances of the diffusive components and are stochastic themselves. The last two terms are due to jumps. $q^{M}$ and $q^{\epsilon^{i}}$ are Poisson counters with orthogonal, instantaneous intensities $\lambda_{t}^{M}$ and $\lambda_{t}^{\epsilon^{i}}$ and jump sizes $k_{t}^{M}$ and $k_{t}^{\epsilon^{i}}$. In this specification, the overall continuous volatility of a stock is partly explainable by the continuous volatility of the market and a rest which is idiosyncratic. In the same spirit the total discontinuous price movement of a stock is partly explainable by market jumps, while the rest remains idiosyncratic. Thus, the price movement of any asset is driven by four components. These four components might constitute separately priced risk factors in the cross-section of stock returns, which implies that the drift $\alpha_{t}^{i}$ is directly related to the pricing of these risks.

Central for my analysis are measures for jump and volatility risk for each stock as well as for the market. In order to run a meaningful analysis on the cross-section,
these measures should be applicable for a rather large fraction of the cross-section. There are a couple of studies using high frequency data to measure the realized continuous and discontinuous component (e.g., see Bollerslev et al. (2016), Bollerslev et al. (2017) or Guo et al. (2017)). However, as Cremers et al. (2015) argue, when looking at realized, rather than expected jumps it might be that jumps do not materialize even if the jump probability is high. Thus, a naturale attempt to extract these expected measures is to make use of stock options, since these incorporate markets expectations about future jump probabilities and changes in the stochastic volatility. There are different attempts to do so. Some papers use a model free approach to gain insights from the risk neutral distribution (e.g., see Bakshi et al. (2003), Du and Kapadia (2012), or Martin (2017)). Andersen et al. (2015) show that under rather mild assumptions, the VIX measures the risk-neutral expected realized variance stemming from diffusive movements and jumps. Thus, measures relying on model-free risk-neutral volatilities generally capture both, the continuous and the discontinuous part. In a similar manner Bollerslev and Todorov (2011) and Bollerslev et al. (2015) estimate in a semi-parametric fashion the left and right tail risk variation for the market. Their approach heavily relies on out-of-the-money options.

In the cross-section, out-of-the-money options are rarely traded, as indicated by Table 1. Panel A of Table 1 summarizes the percentage of traded option contracts for different moneyness and maturity buckets of the option sample used in the later analysis. Overall, most contracts are traded for options that mature within the next 30 days and are up to $10 \%$ out-of-the-money ( $8.79 \%$ ). The second highest fraction accounts to short term options which are up to $10 \%$ in-the-money ( $7.42 \%$ ). The same holds true when looking at the percentage of trading volume in Panel B. Here, short term options that are up to $10 \%$ out-of-the-money (in-the-money) account for $17.66 \%(12.25 \%)$ of the total volume. For both, number and volume, the percentage sharply decreases in maturity (aggregated over all monyness buckets: 21.92-1.03\%
and $36.59-0.86 \%$, respectively). A simular picture can be seen when looking at the moneyness and aggregating all maturities. While options that are no more than $10 \%$ in- or out-of-the-money account combined for $50.31 \%$ ( $63.76 \%$ ) of all contracts (total trading volume), deep in/out-of-the-money options that are more than $40 \%$ in- or out-of-the-money account combined for $4.35 \%$ ( $2.6 \%$ ) of all contracts (total trading volume), only. Due to the lack of available deep out-of-the-money equity options, methods as in Bollerslev and Todorov (2011) and Bollerslev et al. (2015) are not applicable to obtain a rather large cross-section of these jump and volatility measures.

Cremers et al. (2015) use a different approach and rely on returns of option portfolios of at-the-money option straddles. As Coval and Shumway (2001) argue, due to a high vega, straddle returns strongly react to changes in the expected volatility but are insensitive to small changes in the underlying. However, next to a high vega, straddles are also gamma positive. This makes the portfolio sensitive to large realized and expected movements in the underlying and thus sensitive to jumps in the stock price. Since the vega of an option is increasing in the maturity, while the gamma is decreasing in the maturity, Cremers et al. (2015) use two straddles with different maturity to construct an option portfolio that is delta-gamma neutral, but vega positive, and a portfolio that is delta-vega neutral but gamma positive. As they argue, these portfolios proxy for expected changes in the continuous and discontinuous component of the market in Equation (1). While this approach is intuitively convenient, their option portfolios show a high time variation in the vega and gamma, respectively. In Table 1 of their paper, Cremers et al. (2015) report a mean vega (gamma) of 180.01 (0.0124) with a standard deviation of 88.155 (0.0089).

This results in difficulties of estimating the direct effect of jump and volatility risk in the cross-section of returns. In the appendix I show that the instantaneous excess returns of the market and single stock volatility risk portfolios are given by:

$$
\begin{align*}
\frac{d O_{t}^{M}}{O_{t^{-}}^{M}}-r d t & =\frac{1}{O_{t}^{M}} \frac{\partial O_{t}^{M}}{\partial V_{t}^{M}}\left(d V_{t}^{M}-\mathbb{E}^{\mathbb{Q}}\left[d V_{t}^{M}\right]\right)  \tag{2}\\
\frac{d O_{t}^{i}}{O_{t^{-}}^{i}}-r d t & =\frac{1}{O_{t}^{i}} \frac{\partial O_{t}^{i}}{\partial V_{t}^{M}}\left(d V_{t}^{M}-\mathbb{E}^{\mathbb{Q}}\left[d V_{t}^{M}\right]\right)+\frac{1}{O_{t}^{i}} \frac{\partial O_{t}^{i}}{\partial V_{t}^{\epsilon^{i}}}\left(d V_{t}^{\epsilon^{i}}-\mathbb{E}^{\mathbb{Q}}\left[d V_{t}^{\epsilon^{i}}\right]\right) \tag{3}
\end{align*}
$$

where $\frac{\partial O_{t}^{M}}{\partial V_{t}^{M}}$ is the vega of the market portfolio and $\frac{\partial O_{t}^{i}}{\partial V_{t}^{M}}$ and $\frac{\partial O_{t}^{i}}{\partial V_{t}^{\epsilon^{i}}}$ are the sensitivities of the portfolio value with respect to changes in market and idiosyncratic volatility, where $\frac{\partial O_{t}^{i}}{\partial V_{t}}=\frac{\partial O_{t}^{i}}{\partial V_{t}^{M}}+\frac{\partial O_{t}^{i}}{\partial V_{t}^{\epsilon^{i}}}$ is the total vega of the single stock portfolio. If realized volatility is an martingale under the physical probability measure, then $d V_{t}^{M}-\mathbb{E}^{\mathbb{Q}}\left[d V_{t}^{M}\right]$ states the changes in the market variance risk premium stemming from the continuous component in Equation (1). Similar, $d V_{t}^{\epsilon^{i}}-\mathbb{E}^{\mathbb{Q}}\left[d V_{t}^{\epsilon^{i}}\right]$ states the changes in the idiosyncratic variance risk premium. Thus, the option portfolio returns will be proportional to changes in the risk prima scaled by the sensitivities towards these risks. Using the exact approach of Cremers et al. (2015) makes a direct comparison of two option portfolio returns therefore impossible. A higher return might either signal a lager change in a risk premium or a higher gamma (vega) given the same change in the risk premium.

I extend the measure of Cremers et al. (2015) by constructing an option portfolio consisting of straddles that is delta-vega neutral and has a constant gamma of 0.01 to measure jump risk. In the same fashion I construct an option portfolio from straddles that is delta-gamma neutral and has a constant vega of 100 in order to measure volatility risk. ${ }^{2}$ Specifically, on every trading day I pick those two option pairs (call and put with same maturity) that are closest to being at-the-money and that have two different maturities between 7 to 90 days. If multiple option pairs are equally close to the money, I pick the ones with shortest and longest maturities. The

[^2]vega of an option is increasing in maturity and gammas is deceasing in maturity, so I require the two options of the short maturity pair to be short (long) and the two options with larger maturities to be long (short) in order to construct the volatility (jump) portfolio. The construction of the option portfolios requires to solve a linear equation system with four unknowns, the absolute number of contracts for each option, and three equations, the portfolio delta, gamma and vega. This gives one degree of freedom, which I use to minimize the relative weight each option constitutes to the portfolio value. I do so, in order to minimize any potential data noise associated with the options when calculating returns. In contrast, Cremers et al. (2015) calculate two market neutral straddles before neutralizing the last greek (vega or gamma). However, this might result in extreme relative portfolio weights and the portfolio return might be driven by few options, only. This becomes critical, when looking at equity option where the data quality might be less good and bid-ask spreads larger than for the market. Thus, on every day I run the following optimization:
\[

$$
\begin{equation*}
\arg \min _{\omega} \sum_{i=1}^{4}\left(\frac{\omega^{i} O^{i}}{a b s(\omega)^{\top} O}\right)^{2} \tag{4}
\end{equation*}
$$

\]

s.t.

$$
\begin{aligned}
\omega^{\top} \Delta & =0 \\
\omega^{\top} \mathcal{V} & =0 \\
\omega^{\top} \Gamma & =0.01
\end{aligned}
$$

in order to construct my jump factor. $\omega$ is a vector, containing the number of Options invested in the portfolio and $\omega^{i}$ is one element in $\omega . O$ is the vector of the single option values with elements $O^{i} . \Delta, \Gamma$ and $\mathcal{V}$ are vectors of the corresponding option greeks, so that the constrains ensure that the delta and vega of the portfolio is
always zero, while the gamma is always equal to 0.01 . I follow the same approach to calculate my volatility factor, but replace the last two constrains in Equation (4) with $\omega^{\top} \mathcal{V}=100$ and $\omega^{\top} \Gamma=0$. This ensures that the option portfolios will load positively on the different risks and the objective function in Equation (4) reduces potential data noise only. While the greeks are hold constant, the value of the option portfolio might differ. That is the portfolio meith become a zero cost portfolio. In order to keep the return always well defined, I calculate the return relatively to the absolute amount invested into the short and long position. Therefore, the objective function is defined relatively to the absolute amount invested in the options.

I hold the option portfolio for one trading day and measure its return over that day. I pick new option pairs the next day. This gives me a continuous time series for my jump and volatility factor for both, the market and on a single stock level.

### 3.2 Data

I merge stock price data from CRSP with stock options data from OptionMetrics. Thereby I look at more than 20 years of daily data, spanning a sample period from 01/1996 until 04/2016. I use daily bid/ask prices, implied volatilities, trading volumes, and open interests of American stock-options as well of SPX options and the zero yield curve from Ivy DB US provided by OptionMetrics. From CRSP I obtain daily and monthly stock data, such as split-adjusted returns, prices, dividend amounts, dividend frequency and trading volume. Further, to calculate the book-to-market ratio I include the book-value on an annual basis from Compustat in my analysis. Last, I obtain daily Fama-French factors from Kenneth French's data library.

For my stock-option data set I employ filters similar to Goyal and Saretto (2009). That is I exclude all options with zero open interest and exclude options with time-to-maturity of less than 5 days. I calculate option prices as the mid bid-ask price
and delete any option violating arbitrage bounds. The stock-options have American exercise style. However, to construct my factors I rely on standard Black-Scholes greeks. Thus, given the American option prices I calculate synthetic European option prices to make my analysis robust with respect to the early exercise premium. I do so, in the same way as OptionMetrics calculates implied volatilities with a binomialtree following Cox et al. (1979). Specifically, on every day and for every option in my data set I use the quoted implied volatility to span a CRR-tree with 1,000 time steps. Thereby, I explicitly account for expected dividends and reprice the American options using that tree, first. I exclude all options, where I could not match the American option price with the observed one. That is if the mid bid-ask price deviates more than $1 \%$ from the calculated price. For the remaining options I calculated European option prices using the same CCR-trees.

In order to construct my jump and my volatility factor, I pick two traded option pairs which are closest to being at-the-money on every trading day and have different maturities beween 7 to 90 days. I calculate option sensitivities according to the Black-Scholes model and run the optimization in Equation (4). I exclude a factor for a stock on an observation day if the optimization failed. That is if the constrains in Equation (4) are violated. Having the weights I, calculated the portfolio return over the next trading day relatively to the absolute wealth invested into the long and short position and interpolate implied volatilities if an option is not traded on that day. Specifically, I apply a Gaussian kernel smoother, where I smooth over moneyness, log maturity and a put-call identifier with a constant bandwidth. ${ }^{3}$ If too few observations are available to interpolate, I drop the jump and volatility measure on that day for the given stock.

[^3]
## 4 Results

In this section I discuss my main results. I start with analyzing the empirical performance of my jump and volatility measure first and analyze the pricing of the different risks afterwards.

### 4.1 Market Jump and Volatility Risks

Before analyzing the pricing of jump and volatility risk, I analyze the measures itself first to test if they indeed proxy for jump and volatility risk. Therefore, Table 2 shows summary statistics for the measures on the market. Panel A displays the mean returns of the jump and volatility portfolio. Both are on average negative, indicating that market participants are on average willing to pay a premium to hedge against these risks. Comparing both, the returns for the volatility factor show less skewness (1.0056) and kurtosis (12.2926) as compared to the returns of the jump portfolio (3.1873 and 27.4118, respectively). Also the daily mean (median) returns of $-0.04 \%(-0.09 \%)$ of the volatility portfolio are closer to zero than the ones for the jump portfolio ( $-0.17 \%$ and $-0.58 \%$ ). This leads to a lower annualized Sharp Ratio for the jump portfolio of -0.7591 as compared to the volatility portfolio $(-0.4125)$.

The last row of Panel A in Table 2 displays changes in the risk-neutral expected variance, measured by VIX ${ }^{2}$. Andersen et al. (2015) show that under rather mild assumptions, the square of the VIX as computed by the CBOE is a jump robust measure for the risk-neutral expected total realized variance of the S\&P 500. Thus, changes in the VIX ${ }^{2}$ should stem from changes in the continuous part and/or changes in the discontinuous part of the total volatility. That is why these changes should be directly related to the measures of jump and volatility risk. The table highlights that changes in VIX ${ }^{2}$ show higher (lower) standard deviation, skewness and kurtosis ( $0.1356,1.9314$ and 16.0358 , respectively) than the returns of the volatility (jump)
portfolio. While on average changes in VIX ${ }^{2}$ are slightly positive ( $0.73 \%$ ), the median changes are negative $(-0.72 \%)$.

Panel B of Table 2 reports the pairwise correlation between the two measures and changes in the $\mathrm{VIX}^{2}$. The jump and volatility risks proxy are positively correlated with $12.41 \%$. In addition, both measures show a high correlation with changes in VIX ${ }^{2}$. The volatility portfolio is correlated with changes in VIX ${ }^{2}$ by $56.16 \%$ and returns on the jump risk portfolio show an correlation with $\Delta \mathrm{VIX}^{2}$ of $40.48 \%$. This indicates that both measures carry important information for the variation in total volatility.

To assess the time-series behaviour of the jump and volatility risk measurers Figure 1 plots the time series of returns of the markets jump and volatility risk portfolio. Again, the third plot of the figure displays the changes in the VIX ${ }^{2}$. Several observations emerge when comparing the three plots in the figure. First, while on average the returns of both portfolios are relatively small, at certain points in time these spike to rather large values. When comparing the portfolio returns with changes in the VIX ${ }^{2}$, it becomes obvious that when the VIX increases by a large amount, often at least one of the two portfolios shows a spike as well. For example, in 2001 the $\mathrm{VIX}^{2}$ and the return of the volatility portfolio jointly spike, while the jump portfolio is rather unaffected. In contrast, in 2009 the VIX and the jump portfolio spike jointly, but the volatility portfolio is rather unaffected, and in late 2007 all three measures spike jointly.

The spikes in the two portfolios take place when changes in the jump probability (stochastic volatility) occur, as supported by a deeper look into the figure. The vertical lines in Figure 1 indicate the $0.5 \%$ largest positive (dashed line) and negative (dotted line) 5 minute returns. Almost in all cases a vertical line occurs on the same day or few days after the return of the jump portfolio spiked up. This is a clear indication, that the portfolio proxies rather good expected jumps. Especially
during late 2008 when the return of the jump portfolio seems to be the most volatile also the most extreme 5 minute returns are observed. In contrast, this pattern is less pronounced when looking at the returns of the volatility portfolio. While some spikes go indeed along with an extreme 5 minute return, many others do not. Especially during late 2008 the returns on the volatility portfolio do not seem to be larger than during other periods. This picture changes when looking at Figure 2. In this figure the vertical lines indicate the $1 \%$ largest increases in daily realized volatility measured as the sum of squared 5 minutes $\log$ returns. Almost all spikes in the volatility factor go along with an increase in realized volatility. Again, this pattern is weaker when looking at the returns of the jump portfolio. All in all, these findings clearly indicate that the portfolios are a good measure for jump and volatility risk and that both risks are rather independent of each other.

To quantify the importance of the measures further, Table 3 reports results of additional regression analysis. For the full sample period I regress either the change in $\mathrm{VIX}^{2}$ or the return on the $\mathrm{S} \& \mathrm{P} 500$ on my two portfolio returns:

$$
\begin{equation*}
x_{t}^{M}=\alpha^{M}+\beta_{\mathrm{VOL}}^{M} \mathrm{VOL}_{t}^{M}+\beta_{\mathrm{JUMP}}^{M} \mathrm{JUMP}_{t}^{M}+\epsilon_{t}^{M} \tag{5}
\end{equation*}
$$

The first column in Table 3 shows that $\Delta$ VIX $^{2}$ loads positively on both, the return of the volatility and the jump risk portfolio. Both betas are highly significant on the $1 \%$ level and the adjusted $\mathrm{R}^{2}$ is $42.92 \%$, which supports the previous argumentation that changes in the VIX are directly related to expected changes in either the continuous or discontinuous part of the volatility. The second column reports results for regressing the market return on the jump and volatility risk measure. Again, the adjusted $\mathrm{R}^{2}$ is rather high (13.10\%). Both betas are negative and highly significant, indicating that the jump and the volatility factor capture an important facet for describing returns. Since a straddle loads on positive and negative returns, the jump risk portfolio does not differentiate between positive and negative jumps in the underlying. That is, a spike in the jump risk measure dose not revel the ex-
pected direction of the price jump, but only its increased probability. This is also supported by Figure 1, since spikes in the jump measure go along with positive and negative 5 minute returns. In order to gain a better picture on this relation the last two columns in Table 1 show results of regressing only positive (negative) market returns on the measures. While the over all picture stays unchanged for negative returns, for positive $\mathrm{S} \& \mathrm{P} 500$ returns the beta of the jump risk portfolio gets positive and statistically highly significant, while the beta of the volatility risk portfolio becomes insignificant. Thus, as intuitively expected, jumps have a rather diverse influence on returns. All in all, I conclude that both factors proxy rather good volatility and jump risk.

### 4.2 Market and Idiosyncratic Jump and Volatility Risks

After having assessed that the returns on the option portfolios proxy rather good market jump and volatility risks, I analyze these measures for the cross-section of equity options. Table 4 reports summary statistics and Panel A displays mean, standard deviation, median, kurtosis, skewness and the annualized Sharp Ratio for the pooled sample, that is for all cross-sectional observations on all days. Comparing the numbers to the findings in Table 2, the results on single stock level seem a bit different than for the market. While the average daily return of the jump portfolio stays negative $(-0.27 \%)$, the mean daily return of the volatility portfolio is positive ( $0.08 \%$ ) now. Also the skewness of the volatility (jump) portfolio is higher on single stock level than for the market, 4.7852 (4.0309). Therefore, the median daily return of both portfolios is slightly negative ( $-0.01 \%$ and $-0.56 \%$ ), indicating that market participants are also willing to pay a premium to hedge against these risks on single stock level. Interestingly, the median return of the jump risk portfolio on single stock level $-0.56 \%$ is close to the one of the market $-0.58 \%$, suggesting a similar premium. Since the average daily return of both portfolios is smaller, compared to
the portfolios for the market, the Sharp Ratio is much smaller as well. In order to asses the quality of the jump and volatility risk proxy I calculate a $\mathrm{VIX}^{2^{i}}$ on a single sock level and thereby follow the CBOE approach. ${ }^{4}$ Again, following the argumentation in Andersen et al. (2015), any change in the square of the single stock VIX $^{i}$ should be related to the continuous or discontinuous component in Equation (1). Overall, the changes in $\mathrm{VIX}^{2^{i}}$ are more extreme than the ones for the market VIX $^{2^{M}}$, whit high skewness and extreme kurtosis.

Panel B of Table 4 displays pairwise correlations between the stocks jump and volatility risks, the changes in $\mathrm{VIX}^{2^{i}}$ and the markets jump and volatility risks. I calculate the correlation for each stocks time-series separately and report crosssectional averages and standard errors. Overall the picture is similar as for the market. Yet, the average correlation between the returns of the jump and volatility risks portfolios is $-32.36 \%$ and highly statistically significant. This suggests that different from the market jump and volatility risk is complementary on single stock level. Thus, the jump intensity should not be positively linked to the volatility level for single stocks. On the other hand, both portfolio returns show a rather high correlation with the $\Delta \mathrm{VIX}^{2^{2}}$. While overall a bit lower than for the market, the average correlation between the returns from the volatility risk portfolio and $\Delta \mathrm{VIX}^{2^{i}}$ $(15.54 \%)$ is a bit lower than the correlation between $\mathrm{JUMP}^{i}$ and $\Delta \mathrm{VIX}^{2^{i}}(19.60 \%)$, both being statistically different from zero at the $1 \%$ confidence level. The returns on the individual jump risk portfolio show on average a higher correlation with the markets jump risk portfolio (11.23\%) than with the markets volatility risk portfolio ( $5.26 \%$ ). While this finding makes intuitively sense, since both jump portfolios proxy for the same kind of risk, the correlation of $\mathrm{VOL}^{i}$ with the market risks proxies is generally lower ( $7.75 \%$ with $\mathrm{VOL}^{M}$ and $3.57 \%$ with $\mathrm{JUMP}^{M}$ ). The discontinuous

[^4]jump risks of the single stocks seem to coincide more with the continuous volatility risk of the market than this is the case for the jump risks of single stocks and the volatility risk of market. This suggests that during times of highly expected market jumps, also the continuous volatility of single stocks is expected to increase.

The two measures on single stock level proxy the total risk stemming from volatility or jumps, that is they do not differentiate between idiosyncratic and market risk. In order to measure purely the fraction coming from market and idiosyncratic risks, I run for every time-series in my cross-section a full sample regression. Thereby, I make use of the linear relation in Equation (3) to orthogonalize these components:

$$
\begin{equation*}
X_{t}^{i}=\alpha_{X}^{i}+\beta_{X}^{i} X_{t}^{M}+\epsilon_{X, t}^{i} \tag{6}
\end{equation*}
$$

where $X$ is either the return on the VOL or JUMP factor. I then define $\mathrm{JUMP}_{t}^{i, M}=\beta_{\mathrm{JUMP}}^{i} \mathrm{JUMP}_{t}^{M}$ and $\mathrm{JUMP}_{t}^{i, \epsilon}=\alpha_{\mathrm{JUMP}}^{i}+\epsilon_{\mathrm{JUMP}, t}^{i}\left(\mathrm{VOL}_{t}^{i, M}=\beta_{\mathrm{VOL}}^{i} \mathrm{VOL}_{t}^{M}\right.$ and $\operatorname{VOL}_{t}^{i, \epsilon}=\alpha_{\mathrm{VOL}}^{i}+\epsilon_{\mathrm{VOL}, t}^{i}$. Panel D in Table 4 reports summary statistic of these measures for the pooled sample. Due to robustness I include a time-series only if it has at least one year of data observable to run the regression in Equation (6). Once again, all median daily returns are negative, indicating that market participants are willing to pay a premium to hedge against all these risks, no matter if they stem from the market or are idiosyncratic only. The returns of the idiosyncratic jump risk portfolio are on average ( $-0.14 \%$ ), while the idiosyncratic volatility risk portfolio has extremely positive returns on average ( $11.05 \%$ ). The high average return is mainly driven by a few extremely high return realizations of the idiosyncratic volatility risk portfolio. ${ }^{5}$

[^5]In order to further assess the relation between the different continuous and discontinuous components on stock returns, Table 5 reports cross-sectional averages of regression coefficients stemming from time-series regressions similar to Equation 5. Average cross-sectional standard errors are given in parenthesis. The table highlights that changes in VIX $^{2^{i}}$ load on all four components, where the betas on the continuous and discontinuous components are on average statistically different from zero at the $1 \%$ level. Thus, the innovations in idiosyncratic and market jump and volatility risks are able to explain a considerably large fraction of the variation in $\Delta \mathrm{VIX}^{2^{i}}$ with an average adjusted $\mathrm{R}^{2}$ of $10.87 \%$. On the other hand, stock returns load on average only statistically significant on the market measures, where the average adjusted $R^{2}$ is $8.62 \%$. Since the jump risk portfolio loads on negative and positive expected jumps I regress positive and negative stock returns on these measures separably, to get a better picture of how these effect returns. While the betas of all four components are negative and become on average statistically significant on the $1 \%$ confidence level for negative returns, positive returns do not load on market volatility significantly. The other components are significant at the $1 \%$ level, increasing the adjusted $\mathrm{R}^{2}$ to $31.19 \%$ for positive returns and $37.97 \%$ for negative returns. The idiosyncratic components are economically the most important for explaining positive returns. A increase of one average standard deviation in the idiosyncratic jump (volatility) component increases positive returns by 130 (102) basis points, while the effect of the markets volatility component is insignificant and a one standard deviation increase in the market jump component increases positive returns by 3 basis points, only.

### 4.3 The Pricing of Jump and Volatility Risk

The previous Section shows that there is a clear link between the different continuous and discontinuous idiosyncratic and market risks and stock returns. However, the analysis has been contemporaneous and for every time-series separately so far. To see
if these four components are priced in the cross-section of stock returns, I analyze if they can predict cross-sectional returns. Thereby, I look at the characteristics of each stock. If they are priced, the alpha in Equation (1) should carry a compensation for these risks. That is, a stock with a higher volatility risk component stemming from the market should earn significantly different expected returns than a stock with a low market volatility risk component as long as the market's continuous volatility component carries any price of risk.

To do so, I run a portfolio sort analysis. That is on the end of every month in the sample I estimate $\mathrm{VOL}_{t}^{i, M}, \mathrm{VOL}_{t}^{i, \epsilon}, \mathrm{JUMP}_{t}^{i, M}$ and $\mathrm{JUMP}_{t}^{i, \epsilon}$ following Equation (6), first. Thereby I use the daily observations of the last month (from $t-1$ until $t$ ). I allow no more than two missing observations during the estimation period to include a stock's risk measures. Having estimated the sensitivities in Equation (6), I calculated the cumulative returns of the risks components over the last month (from $t-1$ until $t$ ). This aggregates changes in the risk prima over the previous month and these cumulate returns proxy for the expected risk of these factors. Finally, to test if they carry a price of risk, I sort stocks into quintile portfolios and calculate value weighted contemporaneous (from $t-1$ until $t$ ) and next month (from $t$ until $t+1$ ) returns as well as next month Fama-French three factor alpha.

Table 6 reports returns and alpha as well as Newey-West adjusted standard errors of the single sorts. In Panel A stocks are sorted according to their level of the market volatility risk component, Panel B reports sorts of the idiosyncratic volatility risk components, Panel C of the market jump risk components and Panel D sorts according to the idiosyncratic jump risk components. The contemporaneous returns of all portfolios are monotonically increasing in the portfolio rank. That is stocks where the increases in physical (expected) risk is higher than the increase in risk-neutral expected risk tend to earn higher returns contemporaneously than stocks where the increase in risk-neutral expected risk was higher. The returns of
the low-minus-high portfolios are all except for the market jump risk components significantly different from zero. When looking at expected returns, only idiosyncratic jumps are able to generate a significant spread of $0.54 \%$ ( $0.67 \%$ ) in returns (alpha) for the low-minus-high portfolio. Low $\mathrm{JUMP}_{t}^{i, \epsilon}$ stocks have an higher increase in risk-neutral expected idiosyncratic jump risk than in physical (expected) idiosyncratic jump risk. Thus, investors dislike the jump risk of these stocks more compared to high $\mathrm{JUMP}_{t}^{i, \epsilon}$ stocks and are willing to pay a premium to hedge these risks. Consequently, they will require a compensation by higher expected returns for holding low $\mathrm{JUMP}_{t}^{i, \epsilon}$ stocks compared to high $\mathrm{JUMP}_{t}^{i, \epsilon}$ stocks. This implies a positive price of idiosyncratic jump risk. On the other hand, the results of the other risk components suggest that these are not priced in the cross-section, since there is no significant difference in next month returns or alpha of the 1-5 portfolios.

In order to ensure the robustness of these results I run cross-sectional predictive regressions. Compared to portfolio sorts, these have the advantage of being able to control for multiple characteristics simultaneously. On every month I estimate $\operatorname{VOL}_{t}^{i, M}, \operatorname{VOL}_{t}^{i, \epsilon}, \mathrm{JUMP}_{t}^{i, M}$ and $\mathrm{JUMP}_{t}^{i, \epsilon}$ following the same approach as for the sorts and using the daily returns of the past month. Then I regress the next month returns on these risk components and additional controls:

$$
\begin{align*}
r_{t+1}^{i}= & \alpha_{t}+\beta_{t}^{V O L^{M}} \mathrm{VOL}_{t}^{i, M}+\beta_{t}^{V O L^{\epsilon}} \mathrm{VOL}_{t}^{i, \epsilon}  \tag{7}\\
& +\beta_{t}^{J U M P^{M}} \mathrm{JUMP}_{t}^{i, M}+\beta_{t}^{J U M P^{\epsilon}} \mathrm{JUMP}_{t}^{i, \epsilon}+\beta_{t}^{\text {cont }} X_{t}^{i}+\epsilon_{t}^{i}
\end{align*}
$$

Table 7 reports time-series averages as well as Newey-West adjusted standard errors of the estimated coefficients. As a control I include the firms size, as it might explain the cross-sectional variation according to Fama and French (1992). To rule out that the results are driven by liquidity issues, I further add the illiquidity measure (ILLIQ) of Amihud (2002) to the regression analysis. Frazzini and Pedersen (2014) show, that a strategy going long low beta stocks and short high beta stocks, earns on average a significant positive subsequent return. Thus I also control for the market
beta, where calculate the beta following Frazzini and Pedersen (2014). Conrad et al. (2013) show that risk-neutral higher moments (e.g., volatility and skewness) imply a negative price of risk in the cross-section of stock returns. Thus I include these measures as a control. In addition, Bollerslev et al. (2017) argue that the normalized difference between realized good and bad (total) volatility can be used as a measure for jumps, which has pricing implications in the cross-section of stock returns. Thus, I calculate their measure of signaled relative jumps (SRJ) using risk neutral moments to make use of the forward looking information. That is I calculated the differences between the good volatility, which comes from the right side of the risk neutral distribution following Bakshi et al. (2003), and bad volatility and normalize it by total volatility.

Overall, column (1) to (3) indicate that market jump and volatility risk as well as idiosyncratic volatility risk have no cross-sectional predictive power and stay insignificant. On the contrary, the beta of idiosyncratic jump risk is negative and highly statistically significant, as indicated in column (4) and (5). Non of the betas of other controls has any statistical significance. This is especially remarkable. By construction my measure of jump risk cannot differentiate between upward or downward jumps, while the sign of SRJ should clearly indicates the expected jump direction. Similar, a positive risk-neutral skewness might be due to fat tails of the physical distribution as argued by Bakshi et al. (2003). Still, my measure of jump risks seem to carry superior information, when relating it to stock returns. Also the insignificant loading on the risk-neutral volatility measured by VIX ${ }^{i}$ suggests splitting up total volatility in a diffusive and a jump component adds valuable information in understanding the pricing of stock return dynamics.

All in all, I conclude that idiosyncratic jump risk is a priced risk factors in the cross-section of stock returns. This relation seems to be very robust. No matter of the controls added to the predictive regressions in Table 7, the average loading on
idiosyncratic jump risks stays in a rather narrow corridor. While the positive price of risk might seem to contradict Ang et al. (2006), two things are worth mentioning. First, Ang et al. (2006) focus in their study exclusively on realized idiosyncratic volatility and expected returns, while I analyse a risk prima and expected returns. Second, while these authors analyse the absolute level of the volatility, I concentrate on the changes in the risks prima. Overall, the findings indicate the importance of idiosyncratic jumps. These should constituted price risk factors in stock and stock option pricing.

## 5 Conclusion

This Paper analyzes the pricing of different volatility components in the cross-section of stock returns. Using returns on stock options I construct option portfolios that load on either diffusive or on jump risk. Since the Portfolios relies on at-the-money options, it is largely applicable for the cross-section of equity options. This allows to decompose the total diffusive (jump) risks of a stock into a part stemming from the market and a part being idiosyncratic only. The analysis indicates that these diffusive and jump measures are a rather good risk proxy. It holds, whenever the return on the market jump risk portfolio spikes, the $\mathrm{S} \& \mathrm{P} 500$ realizes a jump on that day or shortly after that day. Similar, whenever the return on the volatility portfolio spikes, the realized volatility shows the highest changes. Both measures embed important information to explain changes in the total volatility, measured by the VIX.

On the single stock level, the same observations hold. Both kind of risks - diffusive and jump risk - help in explaining the changes in the total volatility, measured by a VIX $^{i}$ on single stock level. While idiosyncratic diffusive and jump risk are the most important components in order to explain positive returns contemporaneously.

Moreover, both market risk components are negatively related to contemporaneous stock returns. On the other hand, both market risk components are not able to cross-sectionally predict future returns. In addition, idiosyncratic diffusive risk is not priced but idiosyncratic jump risk carries a positive price of risk. This relation seems very robust with respect to various stock characteristics, risk-neutral higher moments and the low beta anomaly. My results shead light on the relation between stocks volatility risks and expected returns and have important implications for asset and option pricing.

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## A Appendix

In the following I show, that the returns of the VOL portfolios are proportional to the variance risk in a stochastic volatility model. Assume that the dynamics of a stock are given by the following process:

$$
\begin{align*}
d S_{t}^{i} / S_{t}^{i} & =\alpha_{t}^{i} d t+\beta_{V O L}^{i} \sqrt{V_{t}^{M}} d W_{t}^{M}+\sqrt{V_{t}^{\epsilon^{i}}} d W_{t}^{\epsilon^{i}}  \tag{8}\\
d V_{t}^{M} & =\kappa^{M}\left(\bar{V}^{M}-V_{t}^{M}\right) d t+\sigma d W_{t}^{V, M}  \tag{9}\\
d V_{t}^{\epsilon^{i}} & =\kappa^{\epsilon^{i}}\left(\bar{V}^{\epsilon^{i}}-V_{t}^{\epsilon^{i}}\right) d t+\sigma d W_{t}^{V, \epsilon^{i}} \tag{10}
\end{align*}
$$

where $V_{t}^{M}$ is the market variance and $V_{t}^{\epsilon^{i}}$ is the idiosyncratic variance. Thus, $d W_{t}^{M}$ and $d W_{t}^{\epsilon^{i}}$ are two Brownian motions and orthogonal to each other. Using Ito's Lemma the dynamics of any option on the underlying are given by:

$$
\begin{align*}
d U_{t}^{i}= & \frac{\partial U}{\partial t} d t+\frac{\partial U}{\partial S_{t}} d S_{t}+\frac{\partial U}{\partial V_{t}^{M}} d V_{t}^{M}+\frac{\partial U}{\partial V_{t}^{\epsilon^{i}}} d V_{t}^{\epsilon^{i}} \\
& +\frac{1}{2} \frac{\partial^{2} U}{\partial S_{t}^{2}}\left(d S_{t}\right)^{2}+\frac{1}{2} \frac{\partial^{2} U}{\partial V_{t}^{M^{2}}}\left(V_{t}^{M}\right)^{2}+\frac{1}{2} \frac{\partial^{2} U}{\partial V_{t}^{\epsilon^{i^{2}}}}\left(d V_{t}^{\epsilon^{i}}\right)^{2} \\
& +\frac{\partial^{2} U}{\partial S_{t} \partial V_{t}^{M}}\left(d V_{t}^{M}\right)\left(d S_{t}\right)+\frac{\partial^{2} U}{\partial S_{t} \partial V_{t}^{\epsilon^{i}}}\left(d S_{t}\right)\left(d V_{t}^{\epsilon^{i}}\right) \tag{11}
\end{align*}
$$

Further assume that both, market and idiosyncratic, variance risks are priced and carry an individual market price of risk $\lambda^{*}$. Thus, the PDE is given by:

$$
\begin{align*}
0= & \frac{\partial U}{\partial t} d t+\frac{\partial U}{\partial S_{t}}(r-\delta) S_{t}+\frac{\partial U}{\partial V_{t}^{M}}\left(\kappa^{m}\left(\bar{V}^{M}-V_{t}^{M}\right)-\lambda^{M}\right)+\frac{\partial U}{\partial V_{t}^{\epsilon^{i}}}\left(\kappa^{\epsilon}\left(\bar{V}^{\epsilon}-V_{t}^{\epsilon}\right)-\lambda^{\epsilon}\right) \\
& +\frac{1}{2} \frac{\partial^{2} U}{\partial S_{t}^{2}}\left(\beta^{2} V_{t}^{M}+V_{t}^{\epsilon^{i}}\right) S_{t}^{2}+\frac{1}{2} \frac{\partial^{2} U}{\partial V_{t}^{M^{2}}} \sigma^{M^{2}}+\frac{1}{2} \frac{\partial^{2} U}{\partial V_{t}^{\epsilon^{2}}} \sigma^{\epsilon 2} \\
& +\frac{\partial^{2} U}{\partial S_{t} \partial V_{t}^{M}} \rho^{M} \sigma^{M} S_{t}+\frac{\partial^{2} U}{\partial S_{t} \partial V_{t}^{\epsilon^{i}}} \rho^{\epsilon} \sigma^{\epsilon} S_{t}-r U \tag{12}
\end{align*}
$$

Inserting the PDE into Equation (11), one can rewrite option returns as:

$$
\begin{align*}
\frac{d U_{t}^{i}}{U_{t}^{i}}= & r d t+\frac{1}{U_{t}^{i}} \frac{\partial U}{\partial S_{t}} d S_{t}+\frac{1}{U_{t}^{i}} \frac{\partial U}{\partial V_{t}^{M}} d V_{t}^{M}+\frac{1}{U_{t}^{i}} \frac{\partial U}{\partial V_{t}^{\epsilon^{i}}} d V_{t}^{\epsilon^{i}} \\
& -\frac{1}{U_{t}^{i}} \frac{\partial U}{\partial S_{t}}(r-\delta) S_{t} d t-\frac{1}{U_{t}^{i}} \frac{\partial U}{\partial V_{t}^{M}}\left(\kappa^{m}\left(\bar{V}^{M}-V_{t}^{M}\right)-\lambda^{M}\right) d t \\
& -\frac{1}{U_{t}^{i}} \frac{\partial U}{\partial V_{t}^{\epsilon^{i}}}\left(\kappa^{\epsilon}\left(\bar{V}^{\epsilon}-V_{t}^{\epsilon}\right)-\lambda^{\epsilon}\right) d t \\
= & r d t+\frac{1}{U_{t}^{i}} \frac{\partial U}{\partial S_{t}}\left(d S_{t}-(r-\delta) d t\right)+\frac{1}{U_{t}^{i}} \frac{\partial U}{\partial V_{t}^{M}}\left(d V_{t}^{M}-\left(\kappa^{m}\left(\bar{V}^{M}-V_{t}^{M}\right)-\lambda^{M}\right) d t\right) \\
& +\frac{1}{U_{t}^{i}} \frac{\partial U}{\partial V_{t}^{\epsilon^{i}}}\left(d V_{t}^{\epsilon^{i}}-\left(\kappa^{\epsilon}\left(\bar{V}^{\epsilon}-V_{t}^{\epsilon}\right)-\lambda^{\epsilon}\right) d t\right) \\
= & r d t+\frac{1}{U_{t}^{i}} \frac{\partial U}{\partial S_{t}}\left(d S_{t}-\mathbb{E}^{\mathbb{Q}}\left[d S_{t}\right]\right) \\
& +\frac{1}{U_{t}^{i}} \frac{\partial U}{\partial V_{t}^{M}}\left(d V_{t}^{M}-\mathbb{E}^{\mathbb{Q}}\left[d V_{t}^{M}\right]\right)+\frac{1}{U_{t}^{i}} \frac{\partial U}{\partial V_{t}^{\epsilon^{i}}}\left(d V_{t}^{\epsilon^{i}}-\mathbb{E}^{\mathbb{Q}}\left[d V_{t}^{\epsilon^{i}}\right]\right) \tag{13}
\end{align*}
$$

Since the option portfolios $(O)$ are constructed in such a way that they have a zero delta but positive vega, the excess returns are proportional in the two volatility components:

$$
\frac{d O_{t}^{i}}{O_{t}^{i}}-r d t=\frac{1}{O_{t}^{i}} \frac{\partial O}{\partial V_{t}^{M}}\left(d V_{t}^{M}-\mathbb{E}^{\mathbb{Q}}\left[d V_{t}^{M}\right]\right)+\frac{1}{O_{t}^{i}} \frac{\partial O}{\partial V_{t}^{\epsilon^{i}}}\left(d V_{t}^{\epsilon^{i}}-\mathbb{E}^{\mathbb{Q}}\left[d V_{t}^{\epsilon^{i}}\right]\right)(1
$$

Since both risk charnels are orthogonal, $\frac{O}{V_{t}}=\frac{O}{V_{t}^{M}}+\frac{O}{V_{t}^{\epsilon}}$. In the empirical analysis $\frac{O}{V_{t}}$ is set to equal 100. Thus, all option portfolios have the overall same exposure to volatility risk. However, to portion stemming from the market or being idiosyncratic might differ.
Liquidity of Option Contracts

|  |  |  |  |  |  |  | Mo | yness |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  | $\geq 0.5$ | $\geq 0.4$ | $\geq 0.3$ | $\geq 0.2$ | $\geq 0.1$ | $\geq 0$ | $>0$ | $>0.1$ | $>0.2$ | $>0.3$ | > 0.4 | $>0.5$ |  |
|  |  |  |  |  |  | nel A | ercen | age of | ontrac |  |  |  |  |  |
|  | $\leq 30$ | 0.01 | 0.03 | 0.14 | 0.55 | 2.32 | 8.79 | 7.42 | 1.76 | 0.50 | 0.19 | 0.08 | 0.11 | 21.92 |
|  | $\leq 60$ | 0.04 | 0.11 | 0.36 | 1.11 | 3.40 | 7.60 | 5.27 | 1.88 | 0.69 | 0.30 | 0.14 | 0.20 | 21.10 |
|  | $\leq 90$ | 0.06 | 0.12 | 0.30 | 0.70 | 1.58 | 2.60 | 1.86 | 0.83 | 0.38 | 0.20 | 0.10 | 0.18 | 8.90 |
|  | $\leq 120$ | 0.08 | 0.15 | 0.34 | 0.76 | 1.58 | 2.43 | 1.69 | 0.79 | 0.38 | 0.21 | 0.12 | 0.22 | 8.74 |
|  | $\leq 150$ | 0.10 | 0.16 | 0.37 | 0.78 | 1.57 | 2.25 | 1.47 | 0.69 | 0.34 | 0.19 | 0.11 | 0.21 | 8.23 |
| \# | $\leq 180$ | 0.10 | 0.16 | 0.37 | 0.76 | 1.48 | 1.98 | 1.24 | 0.59 | 0.29 | 0.17 | 0.10 | 0.20 | 7.44 |
| * | $\leq 210$ | 0.07 | 0.13 | 0.28 | 0.58 | 1.10 | 1.48 | 0.94 | 0.45 | 0.23 | 0.13 | 0.08 | 0.16 | 5.62 |
| $\sum$ | $\leq 240$ | 0.06 | 0.11 | 0.23 | 0.49 | 0.91 | 1.16 | 0.70 | 0.32 | 0.16 | 0.09 | 0.05 | 0.11 | 4.39 |
|  | $\leq 270$ | 0.04 | 0.05 | 0.09 | 0.15 | 0.24 | 0.28 | 0.19 | 0.10 | 0.06 | 0.04 | 0.02 | 0.06 | 1.32 |
|  | $\leq 300$ | 0.04 | 0.04 | 0.07 | 0.12 | 0.17 | 0.19 | 0.14 | 0.08 | 0.05 | 0.03 | 0.02 | 0.06 | 1.01 |
|  | $\leq 330$ | 0.04 | 0.05 | 0.08 | 0.12 | 0.17 | 0.19 | 0.14 | 0.08 | 0.05 | 0.03 | 0.02 | 0.07 | 1.04 |
|  | $\leq 360$ | 0.05 | 0.05 | 0.08 | 0.13 | 0.17 | 0.18 | 0.13 | 0.08 | 0.05 | 0.03 | 0.02 | 0.07 | 1.03 |
|  | $\sum$ | 0.68 | 1.16 | 2.70 | 6.27 | 14.69 | 29.12 | 21.19 | 7.65 | 3.17 | 1.60 | 0.87 | 1.64 | 90.75 |
|  |  |  |  |  |  | Panel B | Perce | tage of | Volum |  |  |  |  |  |
|  | $\leq 30$ | 0.01 | 0.03 | 0.14 | 0.66 | 3.40 | 17.66 | 12.25 | 1.66 | 0.47 | 0.16 | 0.07 | 0.10 | 36.59 |
|  | $\leq 60$ | 0.03 | 0.09 | 0.29 | 1.00 | 3.73 | 11.05 | 5.78 | 1.29 | 0.45 | 0.19 | 0.09 | 0.16 | 24.17 |
|  | $\leq 90$ | 0.04 | 0.07 | 0.20 | 0.52 | 1.39 | 2.89 | 1.50 | 0.45 | 0.19 | 0.11 | 0.05 | 0.14 | 7.55 |
|  | $\leq 120$ | 0.05 | 0.09 | 0.22 | 0.54 | 1.32 | 2.43 | 1.22 | 0.40 | 0.17 | 0.09 | 0.05 | 0.12 | 6.72 |
|  | $\leq 150$ | 0.06 | 0.09 | 0.22 | 0.51 | 1.18 | 1.93 | 0.92 | 0.31 | 0.14 | 0.08 | 0.04 | 0.09 | 5.58 |
| 7 | $\leq 180$ | 0.05 | 0.09 | 0.20 | 0.46 | 0.98 | 1.49 | 0.72 | 0.25 | 0.11 | 0.06 | 0.04 | 0.10 | 4.57 |
| $\stackrel{\sim}{6}$ | $\leq 210$ | 0.04 | 0.06 | 0.14 | 0.32 | 0.65 | 0.97 | 0.49 | 0.18 | 0.10 | 0.05 | 0.03 | 0.06 | 3.08 |
| $\Sigma$ | $\leq 240$ | 0.03 | 0.06 | 0.12 | 0.26 | 0.52 | 0.72 | 0.36 | 0.13 | 0.06 | 0.04 | 0.02 | 0.05 | 2.37 |
|  | $\leq 270$ | 0.03 | 0.03 | 0.06 | 0.13 | 0.22 | 0.29 | 0.15 | 0.06 | 0.03 | 0.03 | 0.02 | 0.05 | 1.11 |
|  | $\leq 300$ | 0.03 | 0.03 | 0.05 | 0.10 | 0.15 | 0.20 | 0.11 | 0.05 | 0.04 | 0.02 | 0.01 | 0.04 | 0.82 |
|  | $\leq 330$ | 0.03 | 0.03 | 0.06 | 0.10 | 0.16 | 0.20 | 0.11 | 0.04 | 0.03 | 0.02 | 0.02 | 0.04 | 0.85 |
|  | $\leq 360$ | 0.03 | 0.03 | 0.06 | 0.11 | 0.17 | 0.20 | 0.11 | 0.05 | 0.03 | 0.02 | 0.01 | 0.05 | 0.86 |
|  | $\sum$ | 0.43 | 0.71 | 1.77 | 4.70 | 13.88 | 40.04 | 23.72 | 4.86 | 1.82 | 0.87 | 0.46 | 1.00 | 94.26 |

Table 1: The table shows the liquidity of option contracts for different moneyness and maturity buckets. Percentage is given relative to all traded contracts during the sample period from 1996/01 to 2016/04. Maturity is given in days. Moneyness is quoted in absolute terms and defined as $\frac{K}{S}-1$ for puts and $\frac{S}{K}-1$ for calls.

Summary Statistics for Market Volatility and Jump Risk Factors

| Panel A: Descriptive Statistics |  |  |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Mean | SD | Median | Skewness | Kurtosis | Sharpe Ratio |
| VOL $^{M}$ | -0.0004 | 0.0145 | -0.0009 | 1.0056 | 12.2926 | -0.4125 |
| JUMP $^{M}$ | -0.0017 | 0.0360 | -0.0058 | 3.1873 | 27.4118 | -0.7591 |
| $\Delta$ VIX $^{2^{M}}$ | 0.0073 | 0.1356 | -0.0072 | 1.9314 | 16.0358 | - |


| Panel B: Pairwise Correlations |  |  |  |
| :--- | :---: | :---: | :---: |
|  | $\mathrm{VOL}^{M}$ | $\mathrm{JUMP}^{M}$ | $\Delta \mathrm{VIX}^{2^{M}}$ |
| $\mathrm{VOL}^{M}$ | 1 |  |  |
| $\mathrm{JUMP}^{M}$ | 0.1241 | 1 | 1 |
| $\Delta \mathrm{VIX}^{2^{M}}$ | 0.5616 | 0.4048 |  |

Table 2: The table shows summary statistics of the jump and volatility risk proxy over the sample period from 1996/01 to 2016/04. Mean, standard deviation, median, skewness and kurtosis are given for daily returns. The Sharp Ratio is annualized.

## Contemporaneous Market Regressions

|  | $\Delta \mathrm{VIX}_{t}^{2^{M}}$ | $r_{t}^{M}$ | $r_{t}^{M^{+}}$ | $r_{t}^{M^{-}}$ |
| :---: | :---: | :---: | :---: | :---: |
| Intercept | $\begin{aligned} & 0.0040{ }^{* * *} \\ & (0.0007) \end{aligned}$ | $\begin{aligned} & \frac{\imath}{0.0001}(0.0002) \end{aligned}$ | $\begin{aligned} & \frac{l}{0.00990^{* * *}} \\ & (0.0002) \end{aligned}$ | $\underset{(0.0002)}{-0.0079^{* * *}}$ |
| VOL | ${\underset{(0.0484)}{2.358)^{* * *}}{ }^{2} .}^{2}$ | $\underset{(0.0112)}{-0.2637^{* * *}}$ | $-\underset{(0.0113)}{0.0171}$ | $\underset{(0.0108)}{-0.1064^{* * *}}$ |
| JUMP | ${\underset{(0.0195)}{0.6221 * *}}^{10 * *}$ | $-(0.0045)$ | ${\underset{(0.0054)}{0.1471 * * *}}^{(0)}$ | $-\underset{(0.0037)}{-0.1117^{* * *}}$ |
| adj. $\mathrm{R}^{2}$ | 0.4292 | 0.1310 | 0.2226 | 0.3479 |

Table 3: The table shows intercept, betas and adjusted $\mathrm{R}^{2}$ for different regressions: $x_{t}^{M}=\alpha^{M}+\beta_{\mathrm{VOL}}^{M} \mathrm{VOL}_{t}^{M}+\beta_{\mathrm{JUMP}}^{M} \mathrm{JUMP}_{t}^{M}+\epsilon_{t}^{M}$ over the full sample period from $01 / 1996$ until 04/2016. The first column shows results for regressing changes in the VIX $^{2}$ on the measures. In the second column, returns of the S\&P 500 are regressed on the factors. In the last two columns only positive (negative) returns of the S\&P 500 are regressed on the two factors. Standard errors are stated in parenthesis. *, ** and ${ }^{* * *}$ indicate statistical significance on the $90 \%, 95 \%$ and $99 \%$ confidence level.

Summary Statistics for Individual Volatility and Jump Risk Factors

| Panel A: Descriptive Statistics |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Mean | SD | Median | Skewness | Kurtosis | Sharpe Ratio |
| $\mathrm{VOL}^{i}$ | 0.0008 | 0.0305 | -0.0001 | 4.7852 | 406.35 | 0.0000 |
| $\mathrm{JUMP}^{i}$ | -0.0029 | 0.0498 | -0.0056 | 4.0309 | 138.25 | -0.0001 |
| $\Delta \mathrm{VIX}^{2}$ | 0.1031 | 14.7282 | -0.0143 | 367.95 | 146125.52 | - |
| Panel B: Pairwise Correlations |  |  |  |  |  |  |
|  | $\mathrm{VOL}^{i}$ | $\mathrm{JUMP}^{i}$ | $\Delta \mathrm{VIX}^{2}$ | VOL ${ }^{M}$ | $\mathrm{JUMP}^{M}$ |  |
| $\mathrm{VOL}^{i}$ | 1 |  |  |  |  |  |
| $\mathrm{JUMP}^{i}$ | $\underset{(0.0032)}{-0.3236^{* * *}}$ | 1 |  |  |  |  |
| $\Delta \mathrm{VIX}^{2 i}$ | ${\underset{(0.0024)}{(0.1554)}}^{(0.00 *}$ | $\underset{(0.0018)}{0.1960 * * *}$ | 1 |  |  |  |
| VOL ${ }^{M}$ | $\underset{(0.0018)}{\left(0.00775^{* * *}\right.}$ | $\underbrace{0.035)^{* * *}}_{(0.0015)}$ | $\underset{(0.0017)}{0.0922^{* * *}}$ | 1 |  |  |
| $\mathrm{JUMP}^{M}$ | $\begin{aligned} & 0.0522^{* * *} \\ & (0.0013) \end{aligned}$ | $\begin{aligned} & 0.1123^{* * *} \\ & (0.018) \end{aligned}$ | $\begin{aligned} & 0.0578^{* * *} \\ & { }_{(0.0016)} \\ & \hline \end{aligned}$ | 0.0003 | 1 |  |
| Panel C: Descriptive Statistics of Decomposed Measures |  |  |  |  |  |  |
|  | Mean | SD | Median | Skewness | Kurtosis | Sharp Ratio |
| VOL ${ }^{i, M}$ | -0.0075 | 5.1704 | -0.0040 | -0.0379 | 774.2500 | -0.0228 |
| VOL ${ }^{i, \epsilon}$ | 0.1105 | 6.0085 | $-0.0007$ | 0.4764 | 472.2220 | 0.2907 |
| JUMP ${ }^{\text {i,M }}$ | -0.0004 | 0.0066 | -0.0006 | 1.3226 | 25.4584 | -1.0402 |
| JUMP ${ }^{\text {i, }}$, | -0.0014 | 0.0421 | -0.0045 | 4.5905 | 13.1580 | -0.5425 |

Table 4: The table shows summary statistics of the jump and volatility risk proxy over the sample period from 1996/01 to 2016/04. Mean, standard deviation, median, skewness and kurtosis are given for daily returns. The Sharp Ratio is annualized. Panel A displays the quantities for the pooled sample. Panel B reports cross-sectional means. Cross-sectional standard errors are given in brackets, where appropriate. *, ${ }^{* *}$ and ${ }^{* * *}$ indicate statistical significance on the $90 \%, 95 \%$ and $99 \%$ confidence level. Panel C displays summary statistics of the pooled sample of orthogonal market and idiosyncratic measures by the full sample regression: $X_{t}^{i}=\alpha^{i}+\beta^{i} X_{t}^{M}+\epsilon_{t}^{i}$. $\mathrm{VOL}^{i, M}=\beta^{i} \mathrm{VOL}_{t}^{M}$ and $\mathrm{VOL}^{i, \epsilon}=\alpha^{i}+\epsilon_{t}^{i}$.

Contemporaneous Time-Series Regressions

|  | $\Delta \mathrm{VIX}_{t}^{2^{i}}$ | $r_{t}^{i}$ | $r_{t}^{i^{+}}$ | $r_{t}^{i^{-}}$ |
| :---: | :---: | :---: | :---: | :---: |
| Intercept | $\underset{(0.0067)}{0.0148^{* * *}}$ | $\underset{(0.0013)}{\stackrel{\imath}{0.0007}}$ | $\underset{(0.0012)}{0.0250^{* * *}}$ | $\underset{(0.0010)}{-0.0221^{* * *}}$ |
| $\mathrm{VOL}^{i, M}$ | ${\underset{(0.1388)}{0.164)^{* * *}}}^{0}$ | ${\underset{(0.0279)}{0.0553^{* * *}}}^{(0.0}$ | $\begin{gathered} -0.0239 \\ (0.0272) \end{gathered}$ | $-\underset{(0.0246)}{-0.0060^{* * *}}$ |
| $\mathrm{VOL}^{i, \epsilon}$ | ${\underset{(0.0027)}{0.0130 * *}}^{0}$ | $\begin{array}{r} -0.0005 \\ (0.0004) \end{array}$ | ${\underset{(0.0004)}{0.001)^{* * *}}}^{0}$ | $-\underset{(0.0004)}{-0.0022^{* * *}}$ |
| JUMP ${ }^{i, M}$ | ${\underset{(4.7951)}{1.5870 * *}}^{0 * *}$ | $-\underset{(1.0824)}{-0.4716^{* * *}}$ | ${\underset{(1.2251)}{0.4147^{* * *}}}^{(2)}$ | $-(0.7554)$ |
| $\mathrm{JUMP}^{i, \epsilon}$ | ${\underset{(0.1469)}{0.8455^{* * *}}}^{(2)}$ | $\underset{(0.0280)}{0.0158}$ | ${\underset{(0.0257)}{0.2887^{* * *}}}^{(2)}$ | $-(0.0223)$ |
| adj. $\mathrm{R}^{2}$ | 0.1081 | 0.0862 | 0.3119 | 0.3797 |

Table 5: For every stock in the sample period from 1996/01 to 2016/04, having more than one year of observations, I regress changes in $\Delta \mathrm{VIX}_{t}^{2^{2}}$ and the return on the four volatility and jump measures: $X_{t}^{i}=\alpha^{i}+\beta_{\mathrm{VOL}}^{i, M} \mathrm{VOL}_{t}^{i, M}+\beta_{\mathrm{VOL}}^{i, \epsilon} \mathrm{VOL}_{t}^{i, \epsilon}+$ $\beta_{\mathrm{JUMP}}^{i, M} \mathrm{JUMP}_{t}^{i, M}+\beta_{\mathrm{JUMP}}^{i, \epsilon} \mathrm{JUMP}_{t}^{i, \epsilon}+\epsilon_{t}^{i}$. The table reports the regression coefficients and adjusted $\mathrm{R}^{2}$ as cross-section averages. Cross-sectional average standard errors are given in parentheses. ${ }^{*, * *}$ and ${ }^{* * *}$ indicate average statistical significance at the $90 \%, 95 \%$ and $99 \%$ confidence level.

Cross-Sectional Single Sorts

|  | 1 | 2 | 3 | 4 | 5 | 1-5 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Panel A: Sort on VOL ${ }^{i, M}$ |  |  |  |  |  |  |
| $\mathrm{VOL}^{i, M}$ | $\underset{(0.0001)}{-0.0294^{* * *}}$ | $\underset{(0.0001)}{-0.0091^{* * *}}$ | $\begin{aligned} & 0.0001 \\ & (0.0001) \end{aligned}$ | ${\underset{(0.0001)}{0.009)^{* * *}}}^{2}$ | $\underset{(0.0002)}{0.0310^{* * *}}$ | $\underset{(0.0002)}{-0.0606^{* * *}}$ |
| $r_{t}$ | ${\underset{(0.0036)}{0.0085^{* * *}}}^{(2)}$ | ${\underset{(0.0037)}{0.0104^{* * *}}}^{(2)}$ | ${\underset{(0.0038)}{0.0118}}^{* * *}$ | ${\underset{(0.0038)}{0.014)^{* * *}}}^{0}$ | ${\underset{(0.0040)}{0.0139)^{* * *}}}^{()^{*}}$ | ${\underset{(0.0028)}{-0.0060}}^{\text {a** }}$ |
| $r_{t+1}$ | ${\underset{(0.0034)}{0.0069^{* *}}}^{2 *}$ | $\begin{aligned} & 0.0043 \\ & (0.0038) \end{aligned}$ | $\underset{(0.0039)}{0.0070^{*}}$ | $\underset{(0.0040)}{0.0084^{* *}}$ | $\underset{(0.0042)}{0.0053}$ | $\begin{aligned} & 0.0007 \\ & (0.0023) \end{aligned}$ |
| $\alpha_{t+1}$ | $\begin{array}{r} 0.0014 \\ (0.0012) \\ \hline \end{array}$ | $\begin{array}{r} -0.0016 \\ (0.0015) \\ \hline \end{array}$ | $\begin{array}{r} 0.0012 \\ (0.0018) \\ \hline \end{array}$ | $\begin{array}{r} 0.0031 \\ (0.0019) \\ \hline \end{array}$ | $\begin{array}{r} -0.0011 \\ (0.0027) \\ \hline \end{array}$ | $\begin{array}{r} 0.0017 \\ (0.0027) \\ \hline \end{array}$ |
| Panel B: Sort on VOL ${ }^{i, \epsilon}$ |  |  |  |  |  |  |
| $\mathrm{VOL}^{i, \epsilon}$ | $\underbrace{-0.0963^{* * *}}_{(0.0001)}$ | $\underset{(0.0001)}{-0.0300^{* * *}}$ | $\begin{aligned} & 0.0049^{* * *} \\ & (0.0001) \end{aligned}$ | $\underset{(0.0001)}{0.0427^{* * *}}$ | $\underset{(0.0002)_{* * * *}^{0.1313 * *}}{ }$ | $-\underset{(0.0002)}{-0.2278^{* * *}}$ |
| $r_{t}$ | $\underbrace{0.0083^{* *}}_{(0.0037)}$ | ${\underset{(0.0033)}{0.0098 * *}}^{(0)}$ | ${\underset{(0.0036)}{0.0116^{* * *}}}^{(0)}$ | $\underset{(0.0039)}{0.0133^{* * *}}$ | $\underset{(0.0062)}{0.0188^{* * *}}$ | $-\underset{(0.0060)}{-0.0111^{*}}$ |
| $r_{t+1}$ | $\underset{(0.0044)}{0.0079^{*}}$ | $\begin{aligned} & 0.0060 \\ & (0.0040) \end{aligned}$ | $\underset{(0.0035)}{0.0067^{*}}$ | $\underset{(0.0037)}{0.0041}$ | $\begin{aligned} & 0.0041 \\ & (0.0038) \end{aligned}$ | $\underset{(0.0029)}{0.0030}$ |
| $\alpha_{t+1}$ | $\begin{array}{r} 0.0019 \\ (0.0019) \\ \hline \end{array}$ | $\begin{aligned} & 0.0001 \\ & (0.0015) \\ & \hline \end{aligned}$ | $\begin{gathered} 0.0011 \\ (0.0014) \\ \hline \end{gathered}$ | $\begin{array}{r} -0.0015 \\ (0.0015) \\ \hline \end{array}$ | $\begin{array}{r} -0.0016 \\ (0.0025) \\ \hline \end{array}$ | $\begin{aligned} & 0.0027 \\ & (0.0029) \end{aligned}$ |
| Panel C: Sort on JUMP ${ }^{i, M}$ |  |  |  |  |  |  |
| $\mathrm{JUMP}^{i, M}$ | $\begin{gathered} -0.0637^{* * *} \\ (0.0002) \end{gathered}$ | $-\underset{(0.0001)}{-0.0239^{* * *}}$ | $\underset{(0.0001)}{-0.0057^{* * *}}$ | $\begin{aligned} & 0.0131^{* * *} \\ & (0.0001) \end{aligned}$ | $\begin{aligned} & 0.0558^{* * *} \\ & (0.0003) \end{aligned}$ | $\underset{(0.0004)}{-0.1200^{* * *}}$ |
| $r_{t}$ | ${\underset{(0.0040)}{0.0098^{* * *}}}^{0}$ | $\underset{(0.0036)}{0.0114 * *}$ | ${\underset{(0.0036)}{0.0135^{* * *}}}^{2}$ | ${\underset{(0.0035)}{0.0116}}^{* * *}$ | $\underset{(0.0038)}{0.0132^{* * *}}$ | $\begin{array}{r} -0.0039 \\ (0.0027) \end{array}$ |
| $r_{t+1}$ | $\underset{(0.0052)}{0.0045}$ | ${\underset{(0.0033)}{0.0080}}^{0 * *}$ | $\underset{(0.0035)}{0.0056}$ | ${\underset{(0.0035)}{0.008)^{* * *}}}^{2}$ | $\underset{(0.0040)}{0.0051}$ | $\underset{(0.0036)}{-0.0014}$ |
| $\alpha_{t+1}$ | $\begin{array}{r} -0.0025 \\ (0.0030) \\ \hline \end{array}$ | $\underbrace{0.003)^{* * *}}_{(0.0011)}$ | $\begin{aligned} & 0.0003 \\ & (0.0016) \end{aligned}$ | ${\underset{(0.0012)}{0.003)^{* * *}}}^{( }$ | $\begin{array}{r} -0.0008 \\ (0.0020) \\ \hline \end{array}$ | $\begin{array}{r} -0.0025 \\ (0.0039) \\ \hline \end{array}$ |
| Panel D: Sort on JUMP ${ }^{\text {i }, \epsilon}$ |  |  |  |  |  |  |
| $\mathrm{JUMP}^{i, \epsilon}$ | $-\underset{(0.0002)}{0.2404}{ }^{* * *}$ | $\underset{(0.0002)}{-0.1358^{* * *}}$ | $-\underset{(0.0002)}{-0.0654^{* * *}}$ | $\underset{(0.0002)}{0.0156^{* * *}}$ | ${\underset{(0.0003)}{0.1993 * * *}}_{\mathbf{N}^{* * *}}$ | $-\underset{(0.0003)}{-0.4413^{* * *}}$ |
| $r_{t}$ | ${\underset{(0.0031)}{0.0064^{* *}}}^{*}$ | ${\underset{(0.0028)}{0.007)^{* * *}}}^{*}$ | ${\underset{(0.0035)}{0.0096}}^{* * *}$ | ${\underset{(0.0038)}{0.013)^{* * *}}}^{(2)}$ | ${\underset{(0.0050)}{0.0171^{* * *}}}^{\left(y^{*}\right.}$ | $-\underset{(0.0039)}{-0.0114^{* * *}}$ |
| $r_{t+1}$ | $\underset{(0.0042)}{0.0091^{* *}}$ | $\underset{(0.0034)}{0.0077^{* *}}$ | $\underset{(0.0036)}{0.0071^{* *}}$ | $\underset{(0.0040)}{0.0059}$ | $\underset{(0.0047)}{0.0028}$ | ${\underset{(0.0033)}{0.005)^{* *}}}^{* * * *}$ |
| $\alpha_{t+1}$ | $\begin{aligned} & 0.0032 \\ & (0.0023) \\ & \hline \end{aligned}$ | $\begin{aligned} & 0.0024^{*} \\ & (0.0015) \end{aligned}$ | $\begin{aligned} & 0.0014 \\ & (0.0014) \end{aligned}$ | $\begin{aligned} & 0.0000 \\ & (0.0016) \end{aligned}$ | $\underset{(0.0025)}{-0.0043^{*}}$ | ${\underset{(0.0035)}{0.0067^{* * *}}}_{\substack{* * \\ \hline}}$ |

Table 6: The table reports time-series averages of contemporaneous and next month returns for value weighted portfolios. For every month of the entire sample period from 1996/01 until 2016/04 the market and idiosyncratic components of VOL and JUMP are estimated using daily observations of the last month ( $\mathrm{t}-1$ to t ). Stocks are sorted into quintile portfolios. Newey-West adjusted standard errors are given in parentheses. ${ }^{*}$, ${ }^{* *}$ and ${ }^{* * *}$ indicate statistical significance at the $90 \%, 95 \%$ and $99 \%$ confidence level.

Cross-Sectional Predictive Regressions

|  | (1) | (2) | (3) | (4) | (5) |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Intercept | $\underset{(0.0083)}{0.021)^{* * *}}$ | $\underset{(0.0083)^{0.0}}{0.0207^{* * *}}$ | $\underset{(0.0083)}{0.020)^{* * *}}$ | ${\underset{(0.0082)}{0.018)^{* *}}}^{2}$ | ${\underset{(0.0081)}{0.0190^{* * *}}}^{2}$ |
| VOL ${ }^{i, M}$ | $-\underset{(1.4161)}{ }$ |  |  |  | $\begin{array}{r} -1.1519 \\ (1.4115) \end{array}$ |
| $\mathrm{VOL}^{i, \epsilon}$ |  | $\underset{(0.1731)}{0.1233}$ |  |  | $\underset{(0.1642)}{0.0058}$ |
| $\mathrm{JUMP}^{i, M}$ |  |  | $\underset{(0.5402)}{0.0304}$ |  | $\underset{(0.4858)}{-0.0503}$ |
| JUMP ${ }^{i, \epsilon}$ |  |  |  | $\underset{(0.1070)}{-0.2305^{* *}}$ | $\underset{(0.1238)}{-0.2941^{* * *}}$ |
| Size | $\underset{(0.0010)}{-0.0007}$ | $\begin{array}{r} -0.0009 \\ (0.0010) \end{array}$ | $\underset{(0.0010)}{-0.0008}$ | $-\underset{(0.0010)}{-0.0007}$ | $\underset{(0.0010)}{-0.0004}$ |
| ILLIQ | $\begin{aligned} & 0.0000 \\ & (0.1044) \end{aligned}$ | $\begin{aligned} & 0.0023 \\ & (0.1048) \end{aligned}$ | $\begin{aligned} & 0.0030 \\ & (0.1053) \end{aligned}$ | $\begin{aligned} & 0.0480 \\ & (0.1097) \end{aligned}$ | $\begin{aligned} & 0.0552 \\ & (0.1089) \end{aligned}$ |
| $\beta_{F P}$ | $\underset{(0.0039)}{-0.0002}$ | $\begin{aligned} & 0.0002 \\ & (0.0040) \end{aligned}$ | $\underset{(0.0039)}{-0.0005}$ | $\underset{(0.0039)}{-0.0009}$ | $-\underset{(0.0040)}{-0.0012}$ |
| VIX ${ }^{i}$ | $-\underset{(0.0125)}{-0.0107}$ | $-\underset{(0.0126)}{-0.0107}$ | $-\underset{(0.0125)}{-0.0091}$ | $-\underset{(0.0125)}{-0.0121}$ | $\underset{(0.0124)}{-0.0122}$ |
| $\mathrm{SRJ}_{R N}$ | $\begin{aligned} & 0.0010 \\ & (0.0024) \end{aligned}$ | $\begin{aligned} & 0.0011 \\ & (0.0022) \end{aligned}$ | $\begin{aligned} & 0.0012 \\ & (0.0023) \end{aligned}$ | $\begin{aligned} & 0.0012 \\ & (0.0023) \end{aligned}$ | $\begin{aligned} & 0.0009 \\ & (0.0024) \end{aligned}$ |
| $\mathrm{SKEW}_{R N}$ | $\begin{array}{r} -0.0014 \\ (0.0011) \\ \hline \end{array}$ | $\begin{array}{r} -0.0013 \\ (0.0011) \\ \hline \end{array}$ | $\begin{array}{r} -0.0013 \\ (0.0011) \\ \hline \end{array}$ | $\begin{array}{r} -0.0010 \\ (0.0011) \\ \hline \end{array}$ | $\begin{array}{r} -0.0014 \\ (0.0011) \\ \hline \end{array}$ |

Table 7: The table reports time-series averages of cross-sectional predictive regressions. For every month of the entire sample period from 1996/01 until 2016/04, the next month realized returns are regressed on the JUMP and VOL measures: $r_{t+1}^{i}=\alpha_{t}+\beta_{t}^{V O L^{M}} \mathrm{VOL}_{t}^{i, M}+\beta_{t}^{V O L^{\epsilon}} \mathrm{VOL}_{t}^{i, \epsilon}+\beta_{t}^{J U M P^{M}} \mathrm{JUMP}_{t}^{i, M}+\beta_{t}^{J U M P^{\epsilon}} \mathrm{JUMP}_{t}^{i, \epsilon}+\epsilon_{t}^{i}$. The market and idiosyncratic components of VOL and JUMP are estimated using daily observations of the last month ( $\mathrm{t}-1$ to t ). Newey-West adjusted standard errors are given in parentheses. ${ }^{*}$, ${ }^{* *}$ and ${ }^{* * *}$ indicate statistical significance at the $90 \%$, $95 \%$ and $99 \%$ confidence level.


Figure 1: The figure displays the changes in the VIX and daily returns of the jump and volatility factor on the market over the sample periods 01/1996-04/2016. Dashed vertical lines indicate the $0.5 \%$ largest positive 5 minute returns of S\&P 500 futures. Dotted lines show the $0.5 \%$ largest negative 5 minute returns. Solid vertical lines if one of the largest positive and negative 5 minute returns occurred on the same trading day.


Figure 2: The figure displays the changes in the VIX and daily returns of the jump and volatility factor on the market over the sample period 01/1996-04/2016. Vertical dashed lines indicate the $1 \%$ largest increases in realized volatility measured with 5 minutes returns on S\&P 500 futures.


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[^1]:    ${ }^{1}$ Throughout this paper I will use the term total volatility to describe the stock price movements stemming from diffusive movements and jumps. In addition, I will use the terms diffusive, continu-

[^2]:    ${ }^{2}$ I choose $\Gamma=0.01$ and $\mathcal{V}=100$, since it is close to the reported average in Cremers et al. (2015). As long as gamma and vega are kept constant, the exact level is from minor importance since it just scales the portfolio returns equally for the entire cross-section.

[^3]:    ${ }^{3}$ A very similar approach is done by OptionMetrics to estimate the implied volatility surface.

[^4]:    ${ }^{4}$ I follow the same approach for single stocks as the CBOE (2016) does for the S\&P500. This is done by the CBOE for selected stocks too (currently Amazon, Apple, Goldman Sachs, Google and IBM), as well as selected ETFs.

[^5]:    ${ }^{5}$ I avoid winsorizing the sample, since as seen in Figure 1 spikes in the portfolio returns might be due to changes in the risk prima. In unreported results I confirm that results are the same when excluding $1 \%$ of the most extreme realizations.

