On the Stock Market Variance-Return or Price Relations: A Tale of Two Variances

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ABSTRACT

Stock market variance-return or price relations are sometimes negative and sometimes positive. We explain these puzzling findings using a model with two variances, "bad" and "good". In the model, conditional equity premium depends positively on bad variance and negatively on good variance. Market prices, which correlate negatively with discount rates, decrease with bad variance and increase with good variance. Because market variance is the sum of bad and good variances, its relation to conditional equity premium or market prices can be negative or positive, depending on relative importance of two variances. Our empirical results support model's main implications.

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I. Introduction

In this paper, we address two puzzling asset pricing phenomena. When perceived stock market variance increases, leading asset pricing models (e.g., Campbell and Cochrane (1999) and Bansal and Yaron (2004)) predict that (1) scaled stock market prices, e.g., the price-dividend or earnings ratio, fall because (2) the required equity premium rises. Neither implication is supported by data, however. Numerous empirical studies have investigated the stock market variance-return relation in the past four decades and found mixed evidence, ranging from positive to insignificant or negative. Whitelaw (1994), Ghysels, Guerin, and Marcellino (2014), and others show that the relation is sometimes positive and sometimes negative. Surprisingly, few extant studies have investigated the stock market variance-price relation.[1] Schwert (1989) is an important exception. He notes that *relations between stock volatility with either dividend or earnings yield are sometimes positive and sometimes negative* (pp. 1134) over the 1859 to 1987 period. As Figure 1 shows, this intriguing finding is robust in the recent sample. The relation between the stock market variance and the price-earnings ratio is positive during the dotcom bubble period and is negative during the subprime mortgage crisis. To the best of our knowledge, no existing asset pricing theories have tried to explain the two puzzles simultaneously.

Our explanation is that stock market variance have two components that affect the conditional equity premium and prices in opposite ways. To formally illustrate this conjecture, we develop a stylized model that features two different risks: Disembodied technological (DT) shocks and investment-specific technological (IST) shocks. DT shocks are the main driver of economic fluctuations in classical real business cycle models, e.g.,Cooley (1995). Relatively recent studies emphasize that IST shocks also play a crucial role in explaining business cycles and economic growth. Specifically, Justiniano, Primiceri, and Tambalotti (2010, 2011) and others show that a positive IST shock increases output but reduces current consumption because households respond to the improved investment opportunities by investing more in physical capital so that they will have even more consumption in future. In contrast, a positive DT shock increases both output and current consumption.

We incorporate these stylized facts in a variant of the Bansal and Yaron (2004) long-run risk model. The risk price is positive for DT shocks because they correlate positively with consumption growth. The risk price is negative for IST shocks under certain parameterizations because of their negative correlation with changes in current consumption. Because both DT and IST shocks correlate positively with output or dividends, stock market returns load positively on DT-related consumption risk and negatively on IST-related consumption risk. As a result, the conditional equity premium depends positively on the variance of DT shocks and negatively on the variance of

[1] Shiller (1981) finds that future dividends account for little variation in stock market prices. The excess volatility puzzle leads to two competing asset pricing paradigms. Risk-based asset pricing theories argue that the time-varying conditional equity premium is the main driver of stock market prices, while time-varying investor sentiment is advocated in behavioral asset pricing theories. It is difficult to obtain direct empirical evidence on behavioral models because sentiment is hard to quantify. *Surprisingly*, we also know very little about whether the dividend yield actually correlates with the risks advocated by risk-based asset pricing models. Shiller's puzzle remains unanswered.

IST shocks. DT variance is “bad” because an increase in DT variance raises the conditional equity premium and hence lowers stock market prices. IST variance is “good” because an increase in IST variance raises stock market prices by lowering the conditional equity premium.

Stock market variance, which is the sum of both good and bad variances, correlates positively with the stock market price when it comprises predominantly good variance, as during the dotcom bubble period. On the other hand, stock market variance correlates negatively with the price when it comprises predominantly bad variance, as during the subprime mortgage crisis period. As Figure 2 shows, our model implies that the conditional stock market variance is a V-shaped function of the price-dividend ratio, and thus explains the stock market variance-price puzzle. Similarly, there is a negative (positive) stock market variance-return relation when market variance comprises predominantly good (bad) variance. To illustrate this point visually, Figure 3 shows that consistent with the present-value relation, the conditional equity premium decreases monotonically with the price-dividend ratio in our model. Figures 2 and 3 together imply that the relation between the conditional equity premium and market variance can be positive, negative, or insignificant in finite samples. Our model thus explains the stock market variance-return puzzle.

Good variance is a novel addition to risk-based asset pricing theories, and our model predicts that we can measure it empirically in two indirect ways. First, stocks with more loadings on good variance have higher prices, and the value-weighted average stock variance correlates closely with good variance. Second, in our model the conditional variance of long-term Treasury bonds is proportional to good variance because IST (DT) shocks have persistent (temporary) effects on consumption growth. Interestingly, we find that in the US data both measures correlate closely with the variance of IST shock proxies proposed in existing empirical studies, e.g., Papanikolaou (2011) and Kogan and Papanikolaou (2013, 2014). These results provide empirical support to our interpretation that good variance is related to IST shocks. Moreover, all three good variance measures lend strong empirical support to our model's main implications.

First, while the relation between stock market variance and price is unstable in the univariate regression, it becomes negative when in conjunction with good variance that correlates positively with the market price. The two variances account for about 60% variation of some scaled market price measures. Second, while the stock market variance-return relation is unstable in the univariate regression, it becomes positive when in conjunction with good variance that correlates negatively with the conditional equity premium. Third, stock market variance and good variance jointly forecast stock market returns in sample and out of sample even when we control for commonly used market return predictors. Fourth, both market variance and good variance affect the risk-free rate through the precautionary saving effect. Consistent with model calibration, the risk-free rate correlates negatively with market variance and positively with good variance in multivariate regressions. This implication provides a potential explanation for high (low) risk-free rates during the dotcom bubble (subprime mortgage crisis) period. Last but not the least, expected excess returns on individual stocks are linear functions of market variance and good variance, and loadings on these variances explain the cross-section of expected excess stock returns.

Using a general equilibrium model, Papanikolaou (2011) shows IST shocks have a negative risk price because a positive IST shock reduces current consumption. Kogan and Papanikolaou (2013, 2014) use these implications to explain the cross-section of expected stock returns. By contrast, Garlappi and Song (2017) argue that under some alternative assumptions a positive IST shock increases current consumption and thus has a positive risk price. The empirical evidence is also mixed. Papanikolaou (2011) and Kogan and Papanikolaou (2013, 2014), and Dissanayake, Watanabe, and Watanabe (2017) find IST shocks have a negative risk price in the post-1960 sample, while Garlappi and Song (2016) document a positive risk price using the long sample starting from 1930. We contribute to this growing literature by allowing for heteroscedastic DT and IST shocks and focus on their effects on the conditional equity premium.

Cochrane (2011) highlights that *discount-rate variation is the central organizing question of current asset-pricing research*. We shed new light on this literature in three important ways. First, while stock market variance is the sole determinant of the conditional equity premium in leading asset pricing models, we advocate for a two-factor model of the conditional equity premium. Second, Goyal and Welch (2008) have cast doubts on market return predictability because commonly used predictors have weak out of sample forecasting power. By contrast, our theoretically motivated direct risk measures, stock market variance and good variance, jointly forecast market returns in sample and out of sample. Last, using variances of cross-sectional risk factors to forecast market returns and using loadings on these variances to explain the cross-section of stock returns, we establish an explicit link between time-series and cross-sectional stock return predictability.

Guo (2004) argues that stock market variance is a U-shaped function of the stock market price using a limited stock market participation model. In Guo's model, shareholder's liquidity condition is the main driver of financial market dynamics because of occasionally binding constraints on risk sharing between shareholders and non-shareholders. While positive (negative) shocks to shareholders' liquidity conditions increase (decrease) stock market prices, both types of shocks increase stock market variance. For example, the subprime mortgage crisis arguably originated from negative liquidity shocks that raise stock market variance and depress stock market prices (e.g., Brunnermeier and Pedersen (2009) and He and Krishnamurthy (2013)). Similarly, the dotcom bubble may be the ramification of positive liquidity shocks. Justiniano et al. (2011) find that financial market frictions are an important source of IST shocks through their effects on the cost of capital. Therefore, good and bad variances may be associated with positive and negative liquidity shocks, respectively. This interpretation is also consistent with Ludvigson, Ma, and Ng (2017)'s empirical findings that financial market uncertainty has a causal effect on real economy. Nevertheless, Guo (2004) does not provide a formal decomposition of stock market variance into good and bad variances and the associated asset pricing implications.[2]

Segal, Shaliastovich, and Yaron (2015) also consider a variant of the long-run risk model in which good (bad) variance correlates positively (negatively) with the stock market price.[3] Their economic

[2]Kogan, Papanikolaou, and Stoffman (2018) propose a general equilibrium model that features both IST shocks and limited stock market participation.

[3]Their model setup is similar to that adopted in Bekaert and Engstrom (2017), who focus mainly on the relation

interpretation of good and bad variances is distinct from ours, however. Segal et al. (2015) argue that good (bad) variance is associated with positive (negative) economic growth shocks. Their economic mechanism is also different. The conditional equity premium depends positively on both good and bad variances in the Segal et al. (2015) model. As a result, there is a positive stock market variance-return relation and the direct impact of both good and bad variances on the stock market price is negative. The relation between good variance and the stock market price is positive because Segal et al. (2015) impose a positive relation between good variance and expected long-run economic growth. That is, Segal et al. (2015) use the interplay between cash flows and variances to explain the unstable stock market variance-price relation, while we emphasize the interplay between the conditional equity premium or discount rates and variances.

The rest of the paper is organized as follows. We develop the theoretical model in Section II. We present simulation results to illustrate the model's main implications in Section III. We discuss the data in Section IV. We provide empirical evidence of the model's main implications in Section V. We offer some concluding remarks in Section VI.

II. The Model

A. Preference and Aggregate Consumption Dynamics

The representative agent has the Epstein and Zin (1989) recursive utility function:

$$U_t = \left[(1-\delta)C_t^{\frac{1-\gamma}{\theta}} + \delta\big(\mathbb{E}_t[U_t^{1-\gamma}]\big)^{\frac{1}{\theta}}\right]^{\frac{\theta}{1-\gamma}},$$

where $0 < \delta < 1$ is the time discount factor, $\gamma > 0$ is the relative risk aversion, ψ is the elasticity of intertemporal substitution or EIS, and $\theta = \frac{1-\gamma}{1-\frac{1}{\psi}}$.

Aggregate consumption dynamics are as follows

$$\begin{aligned}
\Delta c_{t+1} &= \mu_c + x_t + \sigma_{g,t}\eta_{t+1} - \psi_x \sigma_{x,t} e_{t+1}, \\
x_{t+1} &= \rho x_t + \varphi_e \sigma_{x,t} e_{t+1}, \\
\sigma^2_{g,t+1} &= \sigma^2_g + v_g(\sigma^2_{g,t} - \sigma^2_g) + \sigma_1 z_{1,t+1}, \\
\sigma^2_{x,t+1} &= \sigma^2_x + v_x(\sigma^2_{x,t} - \sigma^2_x) + \sigma_2 z_{1,t+1} + \sigma_3 z_{2,t+1}.
\end{aligned} \tag{1}$$

Δc_{t+1} is the log consumption growth rate with the unconditional mean μ_c. x_t is the expected log consumption growth rate or a measure of long-run consumption growth that has zero mean and follows a persistent AR(1) process. Empirical studies, e.g., Greenwood, Hercowitz, and Krusell (1997) and Fisher (2006), argue that IST shocks are an important determinant of long-run economic growth. We interpret e_{t+1}, the innovation in x_{t+1}, as the IST shock and conduct empirical tests of

between options prices and skewness of consumption growth. Zhou and Zhu (2015) consider a variant of the long-run risk model with a short-run variance and a long-run variance. Their motivation and empirical implications are different from those of our model, however.

the model implications using proxies of IST shocks. A positive IST shock increases expected consumption growth or $\varphi_e > 0$. In our model, a positive IST shock also reduces the contemporaneous consumption, i.e., $\psi_x > 0$. This is because a positive IST shock improves investment opportunities and prompts householders to save and invest more in physical capital by reducing current consumption in exchange for even more future consumption.

The positive relation between IST shocks and long-run consumption growth and the negative relation between IST shocks and contemporaneous consumption growth are consistent with the theoretical implications and empirical findings in Justiniano et al. (2010, 2011) and Papanikolaou (2011). Khan and Tsoukalas (2011), Furlanetto and Seneca (2014), and Garlappi and Song (2017), however, argue that under some alternative assumptions a positive IST shock increases both the current and future consumption. In that case, ψ_x is negative. We shed new light on this debate by investigating empirical whether IST shocks are positively or negatively price in the stock market.

Following Papanikolaou (2011) and Kogan and Papanikolaou (2013, 2014), we interpret η_{t+1} as a DT shock that affects only the contemporaneous consumption. Of course, it may capture other shocks that have only short-term effects on consumption. $\sigma_{g,t}$ and $\sigma_{x,t}$ are the conditional variances of DT shocks and IST shocks, respectively. Both $\sigma_{g,t}$ and $\sigma_{x,t}$ follow persistent AR(1) processes with the unconditional means σ_g^2 and σ_x^2 and with homoscedastic shocks $z_{1,t+1}$ and $z_{2,t+1}$, respectively. The term $\sigma_2 z_{1,t+1}$ captures the potential correlation between $\sigma_{g,t}$ and $\sigma_{x,t}$. The shocks, η_{t+1}, e_{t+1}, $z_{1,t+1}$, and $z_{2,t+1}$ have i.i.d. standard normal distributions.

B. *Pricing kernel*

Using a log-linear approximation of Campbell and Shiller (1988), we can write the log return on the claim to aggregate consumption as

$$\begin{aligned} r_{a,t+1} &= \ln \frac{P_{t+1}+C_{t+1}}{P_t} = \ln \frac{P_{t+1}+C_{t+1}}{C_{t+1}} - \ln \frac{P_t}{C_t} + \ln \frac{C_{t+1}}{C_t} \\ &= k_0 + k_1 z_{t+1} - z_t + \Delta c_{t+1}, \end{aligned} \tag{2}$$

where $z_t = \ln \frac{P_t}{C_t}$, $\bar{z} = \mathbb{E}[z_t]$, $k_1 = \dfrac{e^{\bar{z}}}{e^{\bar{z}}+1} < 1$, and $k_0 = \ln(e^{\bar{z}}+1) - \dfrac{\bar{z}e^{\bar{z}}}{e^{\bar{z}}+1}$. From Epstein and Zin (1989), the log pricing kernel is

$$m_{t+1} = \ln M_{t+1} = \theta \ln \delta - \frac{\theta}{\psi}\Delta c_{t+1} + (\theta - 1) r_{a,t+1}. \tag{3}$$

The Euler equation for return on any asset, $R_{i,t+1}$, is $\mathbb{E}_t[M_{t+1}R_{i,t+1}] = 1$. Log-linearizing the Euler equation, we have

$$\mathbb{E}_t\big[\exp\big(\theta \ln \delta - \frac{\theta}{\psi}\Delta c_{t+1} + (\theta-1) r_{a,t+1} + r_{i,t+1}\big)\big] = 1. \tag{4}$$

The Euler equation holds for $r_{a,t+1}$:

$$\mathbb{E}_t\big[\exp\big(\theta\ln\delta-\frac{\theta}{\psi}\Delta c_{t+1}+\theta r_{a,t+1}\big)\big]=1. \tag{5}$$

We conjecture that the log price-consumption ratio is a linear function of state variables:

$$z_t=A_0+A_1\sigma^2_{g,t}+A_2\sigma^2_{x,t}+A_3x_t. \tag{6}$$

where A_0, A_1, A_2, A_3 are coefficients to be determined. Appendix A.A shows that these coefficients are functions of model's parameters:

$$\begin{aligned}
A_0 &= \frac{1}{1-k_1}\Big[\ln\delta+k_0+(1-\frac{1}{\psi})\mu_c+\frac{1}{2}\theta k_1^2(A_1\sigma_1+A_2\sigma_2)^2+\frac{1}{2}\theta k_1^2A_2^2\sigma_3^2 \\
&\qquad +k_1A_1\sigma_g^2(1-v_g)+k_1A_2\sigma_x^2(1-v_x)\Big], \\
A_1 &= \frac{(1-\gamma)^2}{2\theta(1-k_1v_g)}, \\
A_2 &= \frac{[\theta k_1A_3\varphi_e+(\gamma-1)\psi_x]^2}{2\theta(1-k_1v_x)}, \\
A_3 &= \frac{1-\frac{1}{\psi}}{1-k_1\rho}.
\end{aligned}$$

Combining equations (2), (3), and (6), we rewrite the log pricing kernel as

$$\begin{aligned}
m_{t+1} &= c_2+[A_3(\theta-1)(\rho k_1-1)-\gamma]x_t+(\theta-1)(k_1v_g-1)A_1\sigma^2_{g,t} \\
&\quad +(\theta-1)(k_1v_x-1)A_2\sigma^2_{x,t}+k_1(\theta-1)(A_1\sigma_1+A_2\sigma_2)z_{1,t+1} \\
&\quad +k_1(\theta-1)A_2\sigma_3z_{2,t+1}+[(\theta-1)k_1A_3\varphi_e+\gamma\psi_x]\sigma_{x,t}e_{t+1}-\gamma\sigma_{g,t}\eta_{t+1},
\end{aligned} \tag{7}$$

where $c_1=k_0+(k_1-1)A_0+k_1A_1\sigma_g^2(1-v_g)+k_1A_2\sigma_x^2(1-v_x)+\mu_c$, and $c_2=\theta\ln\delta-\frac{\theta}{\psi}\mu_c+(\theta-1)c_1$.

The shock to the log pricing kernel is

$$\begin{aligned}
m_{t+1}-\mathbb{E}_t[m_{t+1}] &= k_1(\theta-1)(A_1\sigma_1+A_2\sigma_2)z_{1,t+1}+k_1(\theta-1)A_2\sigma_3z_{2,t+1} \\
&\quad +[\gamma\psi_x+k_1\varphi_e\frac{\frac{1}{\psi}-\gamma}{1-k_1\rho}]\sigma_{x,t}e_{t+1}-\gamma\sigma_{g,t}\eta_{t+1}.
\end{aligned} \tag{8}$$

The risk price of IST shocks, $-\gamma\psi_x-k_1\varphi_e\frac{\frac{1}{\psi}-\gamma}{1-k_1\rho}$, has two components. The first component $-\gamma\psi_x$ reflects the concern about current consumption. It is negative because a positive IST shock lowers the current consumption. The second component $-k_1\varphi_e\frac{\frac{1}{\psi}-\gamma}{1-k_1\rho}$ captures the concern about future consumption because a positive IST shock increases expected future consumption. Its sign depends on the relative risk aversion γ and the elasticity of intertemporal substitution ψ. It is positive if $\gamma>\frac{1}{\psi}$ or households prefer early resolution of uncertainty and is negative if $\gamma<\frac{1}{\psi}$ or households prefer late resolution of uncertainty. We follow Bansal and Yaron (2004) and assume $\gamma>\frac{1}{\psi}$; the price of IST shocks is negative in our calibration because the first component dominates the second component in magnitude. The price of DT shocks, γ, is unambiguously positive.

C. Stock Market Returns

The dividend growth rate of the stock market portfolio is

$$\Delta d_{t+1} = \mu_d + \phi x_t + \pi_\eta \sigma_{g,t} \eta_{t+1} + \pi_e \sigma_{x,t} e_{t+1}. \tag{9}$$

The log dividend growth rate, Δd_{t+1}, depends positively on x_t. It also depends positively on DT and IST shocks because they both improve productivity and thus increase output and dividends. For example, Kogan and Papanikolaou (2014) argue that a positive DT shock and a positive IST shock increase the profitability of consumption-goods producers and investment goods producers, respectively. Below, we outline the main results for stock market returns, and provide detailed derivations in Appendix A.C.

Log-linearizing the stock market return, we have

$$\begin{aligned} r_{m,t+1} &= \ln \frac{P_{m,t+1} + D_{t+1}}{P_{m,t}} = \ln \frac{P_{m,t+1} + D_{t+1}}{D_{t+1}} - \ln \frac{P_{m,t}}{D_t} + \ln \frac{D_{t+1}}{D_t} \\ &= k_{0,m} + k_{1,m} z_{m,t+1} - z_{m,t} + \Delta d_{t+1}, \end{aligned} \tag{10}$$

where $z_{m,t} = \ln \frac{P_{m,t}}{D_t}$, $\bar{z}_m = \mathbb{E}[z_{m,t}]$ and

$$k_{1,m} = \frac{e^{\bar{z}_m}}{e^{\bar{z}_m} + 1} < 1, \; k_{0,m} = \ln(e^{\bar{z}_m} + 1) - \frac{\bar{z}_m e^{\bar{z}_m}}{e^{\bar{z}_m} + 1}.$$

The log stock market price-dividend ratio is a linear function of state variables:

$$z_{m,t} = A_{0,m} + A_{1,m} \sigma_{g,t}^2 + A_{2,m} \sigma_{x,t}^2 + A_{3,m} x_t, \tag{11}$$

where $A_{0,m}, A_{1,m}, A_{2,m}, A_{3,m}$ are coefficients to be determined:

$$\begin{aligned} A_{0,m} &= \frac{1}{1-k_{1,m}} \Big[c_2 + k_{0,m} + k_{1,m} A_{1,m} \sigma_g^2 (1 - v_g) + k_{1,m} A_{2,m} \sigma_x^2 (1 - v_x) + \mu_d + \\ &\qquad + \frac{1}{2} [k_1 (\theta - 1)(A_1 \sigma_1 + A_2 \sigma_2) + k_{1,m} (A_{1,m} \sigma_1 + A_{2,m} \sigma_2)]^2 \\ &\qquad + \frac{1}{2} [(\theta - 1) k_1 A_2 + k_{1,m} A_{2,m}]^2 \sigma_3^2 \Big], \\ A_{1,m} &= \frac{(\gamma - \frac{1}{\psi})(1 - \gamma) + (\pi_\eta - \gamma)^2}{2(1 - k_{1,m} v_g)}, \\ A_{2,m} &= \frac{1}{1 - k_{1,m} v_x} \big[(\theta - 1)(k_1 v_x - 1) A_2 + \frac{1}{2} ((\theta - 1) k_1 A_3 \varphi_e + \gamma \psi_x + k_{1,m} A_{3,m} \varphi_e + \pi_e)^2 \big], \\ A_{3,m} &= \frac{\phi - \frac{1}{\psi}}{1 - k_{1,m} \rho}. \end{aligned}$$

Combining equations (10) and (11), we rewrite the log market return as

$$\begin{aligned} r_{m,t+1} &= c_3 + (k_{1,m}v_g - 1)A_{1,m}\sigma^2_{g,t} + (k_{1,m}v_x - 1)A_{2,m}\sigma^2_{x,t} + (k_{1,m}A_{3,m}\rho - A_{3,m} + \phi)x_t \\ &\quad +(k_{1,m}A_{1,m}\sigma_1 + k_{1,m}A_{2,m}\sigma_2)z_{1,t+1} + k_{1,m}A_{2,m}\sigma_3 z_{2,t+1} \\ &\quad +(k_{1,m}A_{3,m}\varphi_e + \pi_e)\sigma_{x,t}e_{t+1} + \pi_\eta\sigma_{g,t}\eta_{t+1}, \end{aligned} \tag{12}$$

where $c_3 = k_{0,m} + (k_{1,m} - 1)A_{0,m} + k_{1,m}A_{1,m}\sigma^2_g(1 - v_g) + k_{1,m}A_{2,m}\sigma^2_x(1 - v_x) + \mu_d$.

The conditional variance of the log market return in equation (12) is

$$\sigma^2_{m,t} = c_4 + (k_{1,m}A_{3,m}\varphi_e + \pi_e)^2\sigma^2_{x,t} + \pi^2_\eta\sigma^2_{g,t}, \tag{13}$$

where $c_4 = k^2_{1,m}(A_{1,m}\sigma_1 + A_{2,m}\sigma_2)^2 + k^2_{1,m}A^2_{2,m}\sigma^2_3$. Equation (13) shows that market variance is a linear function of DT variance and IST variance. Because market variance and IST variance are more reliably available in data than DT variance, we use equation (13) to substitute DT variance out by market variance in our model's main implications.

Combining equations (11) and (13) we have

$$z_{m,t} = A_{0,m} - \frac{A_{1,m}}{\pi^2_\eta}c_4 + a\sigma^2_{m,t} + b\sigma^2_{x,t} + A_{3,m}x_t, \tag{14}$$

where $a = \frac{A_{1,m}}{\pi^2_\eta}$ and $b = A_{2,m} - \frac{A_{1,m}}{\pi^2_\eta}(k_{1,m}A_{3,m}\varphi_e + \pi_e)^2$. In equation (14), the log stock market price-dividend ratio is a linear function of stock market variance, IST variance, and the expected consumption growth rate. The coefficient on market variance, $\frac{A_{1,m}}{\pi^2_\eta}$, has the same sign as the coefficient on DT variance in equation (11). Intuitively, because market variance is a linear function of DT and IST variances, it is a proxy for DT variance when we control for its correlation with IST variance.

Using the Euler equation $\mathbb{E}_t[M_{t+1}R_{m,t+1}] = 1$, we derive the conditional equity premium

$$\begin{aligned} \mathbb{E}_t[r_{m,t+1} - r^f_t] &= -c_5 - \frac{1}{2}\sigma^2_{m,t} + \gamma\pi_\eta\sigma^2_{g,t} \\ &\quad -[(\theta - 1)k_1A_3\varphi_e + \gamma\psi_x](k_{1,m}A_{3,m}\varphi_e + \pi_e)\sigma^2_{x,t}. \end{aligned} \tag{15}$$

In equation (15), $-\frac{1}{2}\sigma^2_{m,t}$ is the Jensen's inequality adjustment term. The coefficient $\gamma\pi_\eta$ is positive or the conditional equity premium depends positively on $\sigma^2_{g,t}$. As we mentioned above, $-[(\theta - 1)k_1A_3\varphi_e + \gamma\psi_x]$ is the risk price of IST shocks in equation (8). $(k_{1,m}A_{3,m}\varphi_e + \pi_e)$ is positive if $A_{3,m} > 0$ or $\phi > \frac{1}{\psi}$. If the risk price of IST shocks is negative and $\phi > \frac{1}{\psi}$, the conditional equity premium depends negatively on $\sigma^2_{x,t}$. That is, under certain parameterizations, variances of DT shocks and IST shocks have opposite effects on the conditional equity premium. Specifically, an increase in $\sigma^2_{g,t}$ ($\sigma^2_{x,t}$) increases (decreases) the conditional equity premium.

The present-value relation implies a mechanical link between the stock price and the discount rate. In particular, the coefficient on bad variance, $A_{1,m}$, in equation (11) is negative and decrease with $\gamma\pi_\eta$, the coefficient on $\sigma_{g,t}^2$ in equation (15), when γ is relatively large.[4] Intuitively, an increase in $\sigma_{g,t}^2$ raises the conditional equity premium and thus lowers the stock market price. In a similar vein, the coefficient $A_{2,m}$ in equation (11) is positive if the coefficient on $\sigma_{x,t}^2$ in equation (15), $-[(\theta-1)k_1A_3\varphi_e+\gamma\psi_x](k_{1,m}A_{3,m}\varphi_e+\pi_e)$, is negative and large in magnitude. That is, an increase in $\sigma_{x,t}^2$ lowers the conditional equity premium and thus raises the stock market price. To highlight their different effects on stock market prices, we dubs $\sigma_{g,t}^2$ bad variance and $\sigma_{x,t}^2$ good variance.

Combining equation (13) and equation (15), we can rewrite the conditional equity premium as a linear function of market variance and good variance

$$\mathbb{E}_t[r_{m,t+1}-r_t^f]=c_6+\alpha\sigma_{m,t}^2+\beta\sigma_{x,t}^2, \tag{16}$$

where

$$\begin{aligned}
c_5 &= k_1k_{1,m}(\theta-1)(A_1\sigma_1+A_2\sigma_2)(A_{1,m}\sigma_1+A_{2,m}\sigma_2)+(\theta-1)k_1k_{1,m}A_{2,m}A_2\sigma_3^2,\\
c_6 &= -c_5-\frac{\gamma}{\pi_\eta}c_4,\\
\alpha &= -\frac{1}{2}+\frac{\gamma}{\pi_\eta},\\
\beta &= -[(\theta-1)k_1A_3\varphi_e+\gamma\psi_x](k_{1,m}A_{3,m}\varphi_e+\pi_e)-\frac{\gamma}{\pi_\eta}(k_{1,m}A_{3,m}\varphi_e+\pi_e)^2.
\end{aligned}$$

The coefficient on market variance is identical to the coefficient on bad variance in equation (15) except for the Jensen's inequality adjustment term $-\frac{1}{2}\sigma_{m,t}^2$. β is negative if the coefficient on good variance in equation (15) is negative.

D. *The Risk-Free Rate*

Using the Euler equation $\mathbb{E}_t[M_{t+1}R_t^f]=1$, we have the risk-free rate

$$\begin{aligned}
r_t^f &= -\mathbb{E}_t[m_{t+1}]-\frac{1}{2}\mathrm{Var}_t[m_{t+1}]\\
&= c_7+\frac{1}{\psi}x_t+c\sigma_{g,t}^2+d\sigma_{x,t}^2\\
&= c_7-\frac{cc_4}{\pi_\eta^2}+\frac{1}{\psi}x_t+\frac{c}{\pi_\eta^2}\sigma_{m,t}^2+[d-\frac{c}{\pi_\eta^2}(k_{1,m}A_{3,m}\varphi_e+\pi_e)^2]\sigma_{x,t}^2,
\end{aligned} \tag{17}$$

where

$$\begin{aligned}
c_7 &= -c_2-\frac{1}{2}k_1^2(\theta-1)^2[(A_1\sigma_1+A_2\sigma_2)^2+A_2^2\sigma_3^2],\\
c &= -\frac{1}{2}[\gamma+\frac{\gamma}{\psi}-\frac{1}{\psi}],\\
d &= -[(\theta-1)(k_1v_x-1)A_2+\frac{1}{2}((\theta-1)k_1A_3\varphi_e+\gamma\psi_x)^2].
\end{aligned}$$

[4] For example, for $\psi=1.5$ as in the calibration, $A_{1,m}$ is negative and decrease with $\gamma\pi_\eta$ when $\pi_\eta\le 4$ and $\gamma\ge 3$.

Note in the last line of equation (17), we use equation (13) to substitute out $\sigma^2_{g,t}$ by $\sigma^2_{m,t}$. The risk-free rate depends on good and bad variances because of the precautionary saving effect. In our model, good variance and bad variance have different effects on the risk-free rate.

E. *Individual Stock or Portfolio Returns*

In this subsection, we present the main results for individual stock or portfolio returns and provide detailed deviations in Appendix A.D. In our model, a stock differs from the market portfolio in two ways. First, it has different loadings on systemic risks. Second, it has idiosyncratic risk. Specifically, the dividend growth rate of stock p is

$$\Delta d_{p,t+1} = \mu_d + \phi_p x_t + \pi_{\eta,p}\sigma_{g,t}\eta_{t+1} + \pi_{e,p}\sigma_{x,t}e_{t+1} + \pi_p z_{p,t+1}, \tag{18}$$

where $z_{p,t+1}$ is an i.i.d. homoscedastic idiosyncratic shock.

Using the log-linear approximation for the portfolio return, we have

$$r_{p,t+1} = \ln\frac{P_{p,t+1} + D_{p,t+1}}{P_{p,t}} = k_{0,p} + k_{1,p}z_{t+1} - z_{p,t} + \Delta d_{p,t+1}, \tag{19}$$

where $z_{p,t} = \ln\frac{P_{p,t}}{D_{p,t}}$, $\bar{z}_p = \mathbb{E}[z_{p,t}]$ and

$$k_{1,p} = \frac{e^{\bar{z}_p}}{e^{\bar{z}_p}+1} < 1,\ \ k_{0,p} = \ln(e^{\bar{z}_p}+1) - \frac{\bar{z}_p e^{\bar{z}_p}}{e^{\bar{z}_p}+1}.$$

The log price-dividend ratio is

$$z_{p,t} = A_{0,p} + A_{1,p}\sigma^2_{g,t} + A_{2,p}\sigma^2_{x,t} + A_{3,p}x_t, \tag{20}$$

where $A_{0,p}, A_{1,p}, A_{2,p}, A_{3,p}$ are constants to be determined:

$$\begin{aligned}
A_{0,p} &= \frac{1}{1-k_{1,p}}\Big[c_2 + k_{0,p} + k_{1,m}A_{1,p}\sigma^2_g(1-v_g) + k_{1,p}A_{2,p}\sigma^2_x(1-v_x) + \mu_d + \\
&\qquad +\frac{1}{2}[k_1(\theta-1)(A_1\sigma_1 + A_2\sigma_2) + k_{1,p}(A_{1,p}\sigma_1 + A_{2,p}\sigma_2)]^2 \\
&\qquad +\frac{1}{2}[(\theta-1)k_1A_2 + k_{1,p}A_{2,p}]^2\sigma_3^2 + \frac{1}{2}\pi^2\Big], \\
A_{1,p} &= \frac{(\gamma-\frac{1}{\psi})(1-\gamma) + (\pi_{\eta,p}-\gamma)^2}{2(1-k_{1,p}v_g)}, \\
A_{2,p} &= \frac{1}{1-k_{1,p}v_x}\big[(\theta-1)(k_1v_x-1)A_2 + \frac{1}{2}((\theta-1)k_1A_3\varphi_e + \gamma\psi_x + k_{1,p}A_{3,p}\varphi_e + \pi_{e,p})^2\big], \\
A_{3,p} &= \frac{\phi_p - \frac{1}{\psi}}{1-k_{1,p}\rho}.
\end{aligned}$$

The conditional variance of the stock return is

$$\sigma_{p,t}^2 = c_{4,p} + (k_{1,p}A_{3,p}\varphi_e + \pi_{e,p})^2\sigma_{x,t}^2 + \pi_{\eta,p}^2\sigma_{g,t}^2, \tag{21}$$

where $c_{4,p} = k_{1,p}^2(A_{1,p}\sigma_1 + A_{2,p}\sigma_2)^2 + k_{1,p}^2A_{2,p}^2\sigma_3^2 + \pi_p^2$.

The stock risk premium is

$$\begin{aligned}\mathbb{E}_t[r_{p,t+1} - r_t^f] &= -\frac{1}{2}\sigma_{p,t}^2 - Cov_t[m_{t+1}, r_{p,t+1}] \\ &= -c_{5,p} - \frac{1}{2}c_{4,p} - \frac{1}{2}\sigma_{p,t}^2 + \gamma\pi_{\eta,p}\sigma_{g,t}^2 \\ &\quad -[(\theta-1)k_1A_3\varphi_e + \gamma\psi_x](k_{1,p}A_{3,p}\varphi_e + \pi_{e,p})\sigma_{x,t}^2, \end{aligned} \tag{22}$$

where $c_{5,p} = k_1k_{1,p}(\theta-1)(A_1\sigma_1+A_2\sigma_2)(A_{1,p}\sigma_1+A_{2,p}\sigma_2)+(\theta-1)k_1k_{1,p}A_{2,p}A_2\sigma_3^2$. In equation (22), the stock risk premium increases with $\pi_{\eta,p}$, the loading of stock p on DT shocks. If $[(\theta-1)k_1A_3\varphi_e+\gamma\psi_x]$ is positive, the risk premium decreases with $\pi_{e,p}$, the stock loading on IST shocks. In a similar vein, in equation (20), the coefficient $A_{1,p}$ decreases with $\pi_{\eta,p}$ when $\pi_{\eta,p} < \gamma$, indicating that stocks with high loadings on DT shocks have low prices. The coefficient $A_{2,p}$ increases with $\pi_{e,p}$ if $[(\theta-1)k_1A_3\varphi_e + \gamma\psi_x]$ is positive and $\phi_p > \frac{1}{\psi}$, indicating that stocks with high loadings on IST shocks have high prices.

Combining equations (21) and (22), we can rewrite the stock risk premium as a linear function of good and bad variances:

$$\begin{aligned}\mathbb{E}_t[r_{p,t+1} - r_t^f] &= -c_{5,p} - \frac{1}{2}c_{4,p} - \frac{1}{2}(k_{1,p}A_{3,p}\varphi_e + \pi_{e,p})^2\sigma_{x,t}^2 + [\gamma\pi_{\eta,p} - \frac{1}{2}\pi_{\eta,p}^2]\sigma_{g,t}^2 \\ &\quad -[(\theta-1)k_1A_3\varphi_e + \gamma\psi_x](k_{1,p}A_{3,p}\varphi_e + \pi_{e,p})\sigma_{x,t}^2. \end{aligned} \tag{23}$$

Combining equations (13) and (23), we can rewrite the risk premium as a linear function of market variance and good variance:

$$\mathbb{E}_t[r_{p,t+1} - r_t^f] = c_{6,p} + \alpha_p\sigma_{m,t}^2 + \beta_p\sigma_{x,t}^2, \tag{24}$$

where

$$\begin{aligned} c_{6,p} &= -c_{5,p} - \frac{1}{2}c_{4,p} - \frac{\gamma\pi_{\eta,p} - \frac{1}{2}\pi_{\eta,p}^2}{\pi_\eta^2}c_4, \\ \alpha_p &= \frac{\gamma\pi_{\eta,p} - \frac{1}{2}\pi_{\eta,p}^2}{\pi_\eta^2}, \\ \beta_p &= -[(\theta-1)k_1A_3\varphi_e + \gamma\psi_x](k_{1,p}A_{3,p}\varphi_e + \pi_{e,p}) - \frac{\gamma\pi_{\eta,p} - \frac{1}{2}\pi_{\eta,p}^2}{\pi_\eta^2}(k_{1,m}A_{3,m}\varphi_e + \pi_e)^2 \\ &\quad -\frac{1}{2}(k_{1,p}A_{3,p}\varphi_e + \pi_{e,p})^2. \end{aligned}$$

The coefficient on market variance in equation (24) has the same sign as the coefficient on bad variance in equation (23).

F. *Long-Term Real Treasury Bonds and Good Variance*

In our model, the long-term real Treasury bond is affected by IST shocks but not DT shocks. As a result, its conditional variance is a linear function of good variance. We illustrate this point using a perpetual bond that pays \$1 every period. Using the approximation method in Campbell and Shiller (1988), we have the log bond return

$$r_{b,t+1} = \ln \frac{P_{b,t+1} + 1}{P_{b,t}} = k_{0,b} + k_{1,b} z_{b,t+1} - z_{b,t}, \tag{25}$$

where $P_{b,t}$ is the bond price, $z_{b,t} = \ln P_{b,t}$, $\bar{z}_b = \mathbb{E}[z_{b,t}]$, $k_{1,b} = \dfrac{e^{\bar{z}_b}}{e^{\bar{z}_b}+1} < 1$, and $k_{0,b} = \ln(e^{\bar{z}_b}+1) - \dfrac{\bar{z}_b e^{\bar{z}_b}}{e^{\bar{z}_b}+1}$. The log bond price is a linear function of state variables:

$$z_{b,t} = A_{0,b} + A_{1,b}\sigma^2_{g,t} + A_{2,b}\sigma^2_{x,t} + A_{3,b}x_t, \tag{26}$$

where $A_{0,b}, A_{1,b}, A_{2,b}, A_{3,b}$ are constants to be determined:

$$\begin{aligned}
A_{0,b} &= \frac{1}{1-k_{1,p}}\Big[c_2 + k_{0,p} + k_{1,m}A_{1,p}\sigma_g^2(1-v_g) + k_{1,p}A_{2,p}\sigma_x^2(1-v_x) + \\
&\qquad + \frac{1}{2}[k_1(\theta-1)(A_1\sigma_1 + A_2\sigma_2) + k_{1,p}(A_{1,p}\sigma_1 + A_{2,p}\sigma_2)]^2 \\
&\qquad + \frac{1}{2}[(\theta-1)k_1A_2 + k_{1,p}A_{2,p}]^2\sigma_3^2\Big], \\
A_{1,b} &= \frac{(\theta-1)(k_1v_g - 1)A_1 + \frac{1}{2}\gamma^2}{1-k_{1,p}v_g} = \frac{(\gamma - \frac{1}{\psi})(1-\gamma) + \gamma^2}{2(1-k_{1,p}v_g)}, \\
A_{2,b} &= \frac{1}{1-k_{1,p}v_x}\big[(\theta-1)(k_1v_x - 1)A_2 + \frac{1}{2}((\theta-1)k_1A_3\varphi_e + \gamma\psi_x + k_{1,p}A_{3,p}\varphi_e)^2\big] \\
&= \frac{1}{1-k_{1,p}v_x}\big[\frac{1-\theta}{2\theta}(\theta k_1A_3\varphi_e + (\gamma-1)\psi_x)^2 + \frac{1}{2}((\theta-1)k_1A_3\varphi_e + \gamma\psi_x + k_{1,p}A_{3,p}\varphi_e)^2\big], \\
A_{3,b} &= \frac{A_3(\theta-1)(\rho k_1 - 1) - \gamma}{1-k_{1,p}\rho} = \frac{-\frac{1}{\psi}}{1-k_{1,p}\rho}.
\end{aligned}$$

Combining equations (25) and (26), we have

$$\begin{aligned}
r_{b,t+1} &= c_{3,b} + (k_{1,b}v_g - 1)A_{1,b}\sigma^2_{g,t} + (k_{1,b}v_x - 1)A_{2,b}\sigma^2_{x,t} + (k_{1,b}A_{3,b}\rho - A_{3,b})x_t \\
&\quad + (k_{1,b}A_{1,b}\sigma_1 + k_{1,b}A_{2,b}\sigma_2)z_{1,t+1} + k_{1,b}A_{2,b}\sigma_3 z_{2,t+1} + k_{1,b}A_{3,b}\varphi_e\sigma_{x,t}e_{t+1},
\end{aligned}$$

where $c_{3,b} = k_{0,b} + (k_{1,b}-1)A_{0,b} + k_{1,b}A_{1,b}\sigma_g^2(1-v_g) + k_{1,b}A_{2,b}\sigma_x^2(1-v_x)$. Therefore, the conditional bond variance is a linear function of good variance:

$$Var_t[r_{b,t+1}] = c_{4,b} + (k_{1,b}A_{3,b}\varphi_e)^2\sigma_{x,t}^2, \tag{27}$$

where $c_{4,b} = k_{1,b}^2(A_{1,b}\sigma_1 + A_{2,b}\sigma_2)^2 + k_{1,b}^2 A_{2,b}^2 \sigma_3^2$.

G. *Model's Main Implications*

Stock market variance has two time-varying components, good (IST) variance and bad (DT) variance; and the two variances have opposite effects on the conditional equity premium and the stock market price under certain parameterizations. This feature has several novel implications for understanding dynamics of the stock market price and variance.

First, the stock market variance-price relation is unstable because in equation (11) the stock market price-dividend ratio depends negatively on bad variance, i.e., $A_{1,m} < 0$, and positively on good variance, i.e., $A_{2,m} > 0$. The relation is negative when stock market variance comprises predominantly of bad variance, and is positive when good variance is the dominant component. The stock-market variance-price relation is sometimes positive, sometimes negative, and sometimes insignificant. However, equation (14) shows that the *partial* relation between the stock market price-dividend ratio and variance is negative, i.e., $a = \frac{A_{1,m}}{\pi_\eta^2} < 0$, when we control for the effect of good variance on the stock market price. In addition, the partial relation between the stock market price-dividend ratio and good variance is positive when we control for market variance.

Second,the stock market variance-return relation is unstable because in equation (15) the conditional equity premium correlates positively with bad variance, i.e., $\gamma\pi_\eta > 0$, and negatively with good variance, i.e., $-[(\theta - 1)k_1 A_3\varphi_e + \gamma\psi_x](k_{1,m}A_{3,m}\varphi_e + \pi_e) < 0$. The relation is positive when stock market variance comprises predominantly of bad variance, and is negative when good variance is the dominant component. The stock-market variance-return relation is sometimes positive, sometimes negative, and sometimes insignificant. However, equation (16) shows that the *partial* relation between the conditional equity premium and market variance is positive, i.e., $\alpha = -\frac{1}{2} + \frac{\gamma}{\pi_\eta} > 0$, when we control for the effect of good variance on the conditional equity premium. In addition, the partial relation between the conditional equity premium and good variance is negative. Moreover, equation (16) shows that stock market variance and good variance jointly forecast excess stock market returns because they capture dynamics of the conditional equity premium.

Third, equation (17) shows that both $\sigma_{m,t}^2$ and $\sigma_{x,t}^2$ are important determinants of the risk-free rate.

Fourth, the model suggests that we can measure good variance in two ways. First, equation (27) shows that conditional variance of long-term real Treasury bonds is a linear function of good variance. Second, for a stock with high $\pi_{e,p}$, the loading on IST shocks, it has high price (equation (20)) and its variance has a close correlation with $\sigma_{x,t}^2$ (equation (21)). Therefore, a price- or value-weighted average stock variance is a proxy for $\sigma_{x,t}^2$.

Last, taking the unconditional expectation of equation (23), we have

$$\mathbb{E}[r_{p,t+1} - r_t^f] = c_{6,p} + \alpha_p E[\sigma_{m,t}^2] + \beta_p E[\sigma_{x,t}^2]. \tag{28}$$

Equation (28) shows that loadings α_p and β_p help explain the cross-section of stock returns. In the next section, we illustrate these implications using simulated data.

III. Model Simulation

A. *Calibration*

Table I reports the parameter values that we choose for the model at the monthly frequency. For comparison, most parameter values are identical to those adopted in Bansal, Kiku, and Yaron (2012) (BKY thereafter) when applicable with following noticeable exceptions. First, ψ_x is a new parameter in our model. It equals 0.0389 or a one standard deviation increase in IST shocks reduces the contemporaneous consumption by $\mathbb{E}[\psi_x \sigma_{x,t}] = 0.0389 * 0.006 * \sqrt{12} = 0.08\%$ per year. The effect is similar to the point estimate of about 0.10% in a year reported in Figure 3 of Justiniano et al. (2010).[5] This parameter value is sufficient to generate a negative risk price for IST shocks, $-\gamma\psi_x - k_1\varphi_e \frac{\frac{1}{\psi}-\gamma}{1-k_1\rho}$, which decreases with ψ_x.

Second, π_e, another new parameter capturing the effect of IST shocks on the dividend growth rate, equals 3. One standard deviation increase in IST shocks increases the contemporaneous dividend by $\mathbb{E}[\pi_e \sigma_{x,t}] = 3 * 0.006 * \sqrt{12} = 6.24\%$ per year. When we take into account that dividend is levered, this parameter value is consistent with the point estimate of about 2.2% increase in output in a year following one standard deviation increase in IST shocks reported in Figure 3 of Justiniano et al. (2010).

Third, IST shocks have opposite effects on consumption and dividend and thus dampen the positive correlation between these two variables caused by DT shocks. As a result, we do not need an idiosyncratic shock in aggregate dividend assumed by BKY to match the correlation between consumption and dividend in data.

Fourth, the unconditional volatility is 0.006 for IST shocks, σ_x, and is 0.0015 for DT shocks, σ_g. This calibration is consistent with the empirical finding by Justiniano et al. (2011) that IST shocks are more volatile than DT shocks.

Fifth, the volatility of volatility (σ_1 and σ_3) is 0.000006, compared with 0.0000028 in BKY. Because good variance and bad variance have opposite effects on the equity premium, using BKY's volatility of volatility calibration generates a somewhat lower equity premium, although it does not affect our main results qualitatively.

Sixth, because IST shocks affect the dividend growth process directly, we adopt a smaller value for π_η (2.2 in our model versus 2.6 in BKY) and a smaller value for ϕ (2.2 in our model versus 2.5

[5] Justiniano et al. (2010) report the effect using quarterly data and we convert it into annual data by multiplying it by 2.

in BKY) so that the volatility of the dividend growth rate in simulated data does not exceed that in actual data.

Last, BKY consider only one variance process, and we assume that good variance and bad variance follow different stochastic processes. For simplicity, we assume that the two variances are uncorrelated by setting the parameter σ_2 to zero.

B. Aggregate Quantities and Asset Prices

In Table II, we report the summary statistics of the consumption growth rate, the dividend growth rate, stock market returns, the stock market price-dividend ratio, and the risk-free rate in annual frequency. The column under the title "Data" reproduces the BKY estimation from the actual data spanning the 1930 to 2008 period with 79 annual observations. For each simulation, we generate 1,948 monthly observations, discard the first 1,000 observations, and convert the remaining 948 observations into 79 annual observations. We conduct 10,000 simulations and report the distribution of the summary statistics in columns under the title "Model". The column "Pop" reports the summary statistics from the simulation of 100,000 annual observations.

Beeler and Campbell (2012) emphasize that consumption follows a mean-reverting process in the data. The third- to fifth-order autocorrelations of consumption growth are negative. While the first-order autocorrelation coefficient is 0.45, it may be partly accounted for by the time-aggregation bias pointed out by Working (1960) that the annual autocorrelation with i.i.d. growth rates would be 0.25. Overall, as we reproduce in Table II, Beeler and Campbell (2012) report that the variance ratio of 6-year consumption growth to 1-year consumption growth is 0.84. Because a positive IST shock decreases concurrent consumption growth and increases future consumption growth, our model does imply negative autocorrelations in consumption growth. Taken into account of the positive time-aggregation bias in the first-order autocorrelation, we show in Table II that key statistics of consumption data are within the 95% interval of simulated data.

The model also does a good job in matching main properties of dividends, stock market returns and prices, and the risk-free rate. Their summary statistics from the data are within the 95 percent interval of simulated data except that as in BKY, the standard deviations of the risk-free rate and the log price-dividend ratio are somewhat smaller in simulated data than in the actual data.

C. Stock Market Variance-Price Relation

This subsection illustrates the model's implication for the relation between stock market variance and the log stock market price-dividend ratio. For comparison with empirical findings that are based on the quarterly sample spanning the 1963Q1 to 2016Q4 period, we use 216 quarterly observations in each simulated sample. Specifically, we generate a monthly sample of 1,648 observations, discard the first 1,000 observations, and convert the remaining into 216 quarterly observations. We generate 10,000 simulated samples and report their distributions in Table III. The column "Pop" reports the results obtained from 100,000 simulated quarterly observations.

In Panel A of Table III, we report the ordinary least squares (OLS) estimation results of regressing the log stock market price-dividend ratio on a constant and concurrent conditional stock market variance. Leading asset pricing models stipulate a negative variance-price relation, and we sort the coefficient on stock market variance and its t-value from high to low. The R^2 is sorted from low to high. The simulation results illustrate that the univariate stock market variance-price relation is unstable in our model. The coefficient is positive in over 30% of simulated samples, while the median coefficient is negative. In addition, the median t-value and the median R^2 are -0.587 and 12.87%, respectively, indicating that on average the variance-price relation is weak.

In our model, the stock market variance-price relation is sometimes negative because the stock market price-dividend ratio depends negatively on bad variance. The relation is sometimes positive because as we show in Panel B of Table III, the stock market price-dividend ratio correlates positively with good variance. That is, because stock market variance is the sum of bad and good variances, the relation between stock market variance and price depends on the relative importance of good and bad variances. When stock market variance comprises predominantly of bad variance, the stock market price is relatively low and decreases with stock market variance. When good variance is the dominant component, the stock market price is relatively high and increases with stock market variance. We illustrate these points formally in Figure 2, in which conditional market variance is a V-shaped function of the stock market price-dividend ratio.

In Equation (14), the stock market price-dividend ratio is a linear function of stock market variance, good variance, and expected dividend growth. In particular, the coefficient on stock market variance is negative when we control for good variance that has a positive effect on the stock market price. To illustrate this point, in Panel C of Table III, we report the OLS estimation results of regressing the stock market price-dividend ratio on stock market variance and good variance. The coefficient on stock market variance is always negative and the coefficient on good variance is always positive. In addition, the R^2 is close to 100%, indicating that expected dividend growth has negligible explanatory power for stock market prices in our calibration. This result also suggests time-varying conditional equity premium, which is a linear function of stock market variance good variance, accounts for most stock market price variation in the model. Specifically, Figure 3 shows that conditional equity premium decreases monotonically with the stock market price-dividend ratio. Therefore, as we discuss in the next subsection, the unstable stock market variance-price relation reflects the unstable stock market variance-return relation.

D. Stock Market Variance-Return Relation

Our model implies an unstable relation between conditional stock market variance and conditional equity premium. Conditional equity premium depends positively (negatively) on bad (good) variance. When stock market variance comprises primarily bad (good) variance, stock market prices are low (high) and the variance-return relation is positive (negative). We can illustrate these results using two figures. Figure 3 shows that conditional equity premium decreases monotonically with the stock market price-dividend ratio, while stock market variance is a V-shaped function of the price-dividend ratio in Figure 2. Therefore, our model suggests that the stock market variance-return relation is positive (negative) when stock market prices are low (high).

In the empirical analysis, we often use realized equity premium as a proxy for conditional equity premium. Following this specification, we use the expected market variance $\sigma^2_{m,t}$ based on information at time t to forecast the time $t+1$ excess market return $r_{m,t+1}$ using simulated data, and report the OLS regression results in Panel A of Table IV. The coefficient on conditional market variance, VMKT, is negative in over 30% of simulated samples but has a positive median, indicating an unstable stock market variance-return relation. In panel B, we report the OLS regression results of forecasting one-quarter-ahead excess stock market returns using conditional good variance, VG. The coefficient on VG is positive in about 30% of simulated samples, while its median is negative. In Panel C, we include both stock market variance and good variance as the predictive variables. The coefficient on market variance (good variance) is positive (negative) in most simulated samples. In addition, the coefficients, t-values, and R^2 are substantially larger in magnitude than their univariate regression counterparts reported in panels A and B. The difference reflects an omitted variables problem. In simulated data, the median correlation coefficient between market variance and good variance is 87%, although they have opposite effects on future market returns. As a result, in the univariate regressions, the estimated coefficient on the market variance (good variance) is biased downward (upward) toward zero.

There is a strong correlation between the stock market price-dividend ratio and conditional equity premium in our calibration. The results in Table III essentially illustrate the stock market variance-return relation using ex-ante equity premium measure. Noticeably, ex-ante equity premium measure allows us to estimate the stock market variance-return relation more precisely than does ex-post equity premium measure used in Table IV. That is, using scaled stock market prices provides a more powerful test of the stock market risk-return tradeoff than using realized excess market returns. The reason is that, as Elton (1999) points out, realized excess stock market return is a poor proxy for the conditional equity premium because the latter accounts for a small fraction of variation in the former.

E. Uncertainties and the Risk-Free Rate

In Table V, we illustrate the relation between the risk-free rate and variances stipulated in equation (17). Panel A reveals a negative relation between the risk-free rate and stock market variance. Panel B shows that the simple relation between the risk-free rate and good variance is unstable. When we use both variances as the explanatory variables, stock market variance and good variance are significantly negative and positive, respectively. Moreover, the median R^2 is 96%, indicating that uncertainties account for most variation in the risk-free rate in our model.

F. Value-Weighted Average Stock Variance

To illustrate the implications for the cross-section of stock returns, we construct 125 portfolios that have different loadings on systematic risks in equation (18). Specifically, ϕ_p takes one of five possible values [1.4, 1.8, 2.2, 2.6, 3.0], $\pi_{\eta,p}$ takes one of five possible values [1.4, 1.8, 2.2, 2.6, 3.0], and $\pi_{e,p}$ takes one of five possible values [1,9, 2.3, 2.7, 3.1, 3.5]. We assume π_p, the volatility of the idiosyncratic risk is 0.005 for all portfolios. The average values of ϕ_p, $\pi_{\eta,p}$, and $\pi_{e,p}$ equal those of the market portfolio.

Stocks with larger loadings on $\pi_{e,p}$ have higher price-dividend ratio because of their larger loadings on good variance in equation (20). Therefore, a value-weighted average stock variance or VWASV have a stronger correlation with good variance than with bad variance. For illustration, we use squared price-dividend ratio as the weight in simulated data. Of 10,000 simulated samples, the median coefficient of correlation between VWASV and good variance is 95%, compared with only 38% for the correlation between VWASV and bad variance.[6] More importantly, Panel D of Tables III, IV, and V show that the explanatory power of VWASV for the log price-dividend ratio, the equity premium, and the risk-free rate is almost identical to those of good variance. This implication is important because it provides a robustness check on empirical proxies of good variance.

G. The Cross-Section of Stock Returns

Equation (24) shows that loadings on market variance and good variance help explain the cross-section of stock returns. To illustrate this implication, we run the Fama and MacBeth (1973) regression using the 125 portfolios discussed in the preceding subsection. In the first stage, for each portfolio, we run a time-series forecasting regression of its excess returns on conditional market variance and good variance as in equation (24). In the second stage, we run the cross-sectional regression of portfolio returns on their loadings on market variance $\hat{\alpha_p}$ and good variance $\hat{\beta_p}$. The estimated prices of loadings $\hat{\alpha_p}$ and $\hat{\beta_p}$ are positive because they equal unconditional means of market variance and good variance, respectively. We illustrate these points in Table VI. In panel A, the risk prices of loadings on stock market variance, VMKT, and good variance, VG, are both positive in most simulated samples. The median R^2 is 78%, suggesting that market variance and good variance account for a significant portion of variation in the cross-section of stock returns in our model. Note that the intercept term in the second stage regression is also positive in most simulated sample because it reflects the risk premium associated with homoscedastic shocks to good and bad variances. Panel B shows that results are almost identical when we use VWASV as a proxy for good variance.

IV. Data

We briefly discuss the main variables used in the empirical analysis and provide details of data construction in Appendix B. We use quarterly data spanning the 1963Q1 to 2016Q4 period unless otherwise indicated. Daily and monthly stock return data are from the Center of Research in Security Prices (CRSP), annual accounting data are from Compustat, and analysts earnings

[6] Equal-weighted average stock variance has a weak correlation (around 50%) with both good and bad variances.

forecast data are from I/B/E/S. We obtain the Fama-French 5 factor portfolio returns from Ken French at Dartmouth College, the aggregate earnings-price ratio data from Robert Shiller at Yale University, and industry classification data from Dimitris Papanikolaou at Northwestern University.

We follow Boudoukh, Michaely, Richardson, and Roberts (2007) to construct the dividend-price ratio and the net payout-price ratio. We employ two methods to calculate corporate dividend payments: (1) the CRSP stock market indices with and without the dividend distribution and (2) the CRSP dividend payments (CRSP item DIVAMT). We define corporate net payout as the difference between dividend payments and equity issuance that we compute using the monthly change in the number of shares outstanding. We use several dividend reinvestment assumptions, including no reinvestment, the risk-free rate, and the market rate at the end of each month. We get similar results from all these alternative methods. For brevity, we use the dividend payments inferred from CRSP dividend payments data and assume zero-reinvestment to construct the dividend-price ratio and the net payout-price ratio.

We construct three sets of proxies for good variance that are related to IST shocks. First, Papanikolaou (2011) shows that the spread in equity returns between investment-goods producers and consumption-goods producers (IMC) correlates strongly with standard IST shock measures such as the relative price of new equipment. The advantage of IMC is that it is available at a higher, i.e., daily frequency, and we can construct its conditional variance more precisely using realized variance. In addition, Kogan and Papanikolaou (2013, 2014) argue that stocks with higher investment-capital ratios, Tobin's Q, price-earnings ratios, book-to-market ratios, market betas, idiosyncratic volatilities, and IMC betas are more sensitive to IST shocks. The high-minus-low spreads in equity returns on portfolios sorted by these characteristics are also proxies for IST shocks. Kogan and Papanikolaou (2013) document a strong comovement among the IST proxies. As a robustness check, We construct the average and the first principle component of the eight IST proxies as two additional IST measures. We use the realized variances of the ten IST measures as proxies for good variance. Second, our model suggests that VWASV is a proxy for good variance. Last, in our model, variance of real Treasury bonds is a linear function of good variance. We obtain options-implied nominal Treasury bond volatility, TYVIX, from Chicago Board Options Exchange (CBOE). Because inflation is stable over the 2003Q1 to 2016Q4 period over which TYVIX is available to us, we use TYVIX as an additional measure of good variance.

To construct the daily IMC spread, we use industry classification data to sort stocks into two portfolios, investment-goods producers and consumption-goods producers. We calculate the daily value-weighted portfolio returns, and IMC is the difference in returns between the two portfolios. To construct daily high-minus-low portfolio spreads, we first sort stocks into two portfolios using the median NYSE market cap as the breaking point. Within each size portfolio, we sort stocks equally into three portfolios by one of the aforementioned seven characteristics. If the characteristic uses accounting data that have release delays, we form the portfolios at the end of June of year $t+1$ and hold the portfolios for a year. Otherwise, we form the portfolios at the end of December of year t and hold the portfolios for a year. We construct daily portfolio returns using value weight.[7] We then construct a high-minus-low hedging portfolio for each characteristic. For example, we construct the return differences between high and low Tobin's Q portfolios for both small and big stocks and use their simple average as a proxy for IST shocks.

We construct quarter t realized variance of each daily IST measure as a proxy for good variance:

$$RV_t = \sum_{i=1}^{N_t} r_{i,t}^2 + 2\sum_{i=1}^{N_t} r_{i,t} r_{i+1,t}, \tag{29}$$

where $r_{i,t}$ is the ith day excess return and N_t is the number of daily returns in quarter t. The second term in equation (29) is the correction of serial correlation in returns. For the first principle component of the eight IST proxies, we do not include the second term because it generates negative realized variance in some quarters. Kogan and Papanikolaou (2013) document a strong comovement among the IST proxies. We document a strong comovement among their variances (untabulated). Because of their strong comovement, we also use the average and the first principle component of the ten standardized IST-based good variance measures as additional good variance measures.

To construct VWASV, we first construct quarterly realized variance of individual stocks using equation (29) and then aggregate them using the value weight. Because options-implied variance is a better measure of conditional variance than is realized variance, we use value-weighted options-implied variance instead of VWASV after 1996. Consistent with the model implication, we document a strong relation between VWASV and IST-based good variance measures. The coefficient of correlation of VWASV with the 12 IST-based good variance measures ranges from 59% to 79% over the 1963Q1 to 2016Q4 period, with an average of 69%. Our model also suggests that bond variance is a proxy for good variance. Consistent with this conjecture, we find a strong relation between TYVIX and the 12 IST-based good variance measures, with an average correlation coefficient of 65% over the 2003Q1 to 2016Q4 period. Similarly, TYVIX is closely correlated to VWASV, with a correlation coefficient of 78%. For brevity, these results are not tabulated.

We use equation (29) to construct realized stock market variance as a proxy for conditional stock market variance. We use options-implied market variance VOX or VIX obtained from CBOE after 1986.

Following Pastor, Sinha, and Swaminathan (2008), we use the implied cost of capital (ICC) as a proxy for conditional equity premium to test the stock market variance-return relation. To ensure that our results are not sensitive to any particular ICC measure, we use common stocks traded on NYSE, AMEX, and Nasdaq to construct five commonly used ICC measures proposed by Pastor et al. (2008), Gebhardt, Lee, and Swaminathan (2001), Easton (2004), Ohlson and Juettner-Nauroth (2005), and Gordon and Gordon (1997). We obtain the Li, Ng, and Swaminathan (2013) ICC measure from David Ng at Cornell University. I/B/E/S publishes monthly consensus forecasts on the third Thursday of each month. We impose a minimum reporting lag of three months to make sure that earnings forecasts are made based on publicly available accounting information.

[7] We obtain similar results using monthly rebalanced portfolios or independently sorted portfolios.

Table VII provides summary statistics of main variables used in the empirical analysis. Panel A reports log price ratios. PD is the price-dividend ratio. PPO is the price-payout ratio. PE is the price-earnings ratio. Panel B reports the implied cost of capital measures. PSS, GLS, Easton, OJ, GG are ICC measures proposed by Pastor et al. (2008), Gebhardt et al. (2001), Easton (2004), Ohlson and Juettner-Nauroth (2005), and Gordon and Gordon (1997), respectively. AICC is the average of these five ICC measures. LNS is the ICC measure used in Li et al. (2013). Panel C reports empirical measures of good variance and stock market variance. We have eight proxies for IST shocks. VIMC is quarterly realized variance of IMC. VIK, VTobinQ, VPE, VIMCIV, Vβ_{IMC}, VIMC, Vβ_{MKT}, and VHML are quarterly realized variances of hedging portfolios formed by characteristics IK, Tobin's Q, PE ratio, IMC idiosyncratic volatilities, IMC beta, Market Beta, and book-to-market equity ratio, respectively. We also calculate first principle component and the average of the eight IST measures, and VFPC and VAVE are their realized variances, respectively. FPCV and AVGV are the first principle component and the average of these IST-based good variance measures. VWASV is the value-weighted average stock variance. EWASV is the equity-weighted average stock variance. TYVIX is the options-implied bond variance. VMKT is stock market variance. Panel D reports asset returns. IK, TobinQ, PE, IMCIV, β_{IMC}, β_{MKT}, and HML are quarterly returns on hedging portfolios formed by characteristics IK, Tobin's Q, PE ratio, IMC idiosyncratic volatilities, IMC beta, Market Beta, and book-to-market equity ratio, respectively. AVE is the average of returns on the seven hedging portfolio returns. CMA, RMW, and SMB are the conservative-minus-aggressive, robust-minus-weak, and small-minus-big factors, respectively. ERET is the excess stock market return, and RF is the real risk-free rate.

V. Empirical Results

In this section, we investigate the model's main implications using actual data.

A. *Forecasting Excess Stock Market Returns*

Panel A of Table VIII reports the univariate regression results of forecasting one-quarter-ahead excess stock market returns with stock market variance and various measures of good variance. Over the 1963Q1 to 2016Q4 period, stock market variance, VMKT, correlates positively and significantly with future excess stock market returns at the 5% level. By contrast, the correlation is negative for the IST-based good variance measures except for Vβ_{MKT}, although it is statistically insignificant in most cases. The correlation is negative albeit statistically insignificant for the value-weighted average stock variance (VWASV) and bond variance (TYVIX).

In panel B of Table VIII, we include both stock market variance and a good variance measure as forecasting variables. Consistent with our model's prediction, we find that the two variances have much stronger forecasting power for excess stock market returns in bivariate regressions than in univariate regressions. The coefficient on VMKT is always significantly positive, and the coefficient on good variance is always significantly negative. More importantly, the coefficients and t-values are substantially larger in magnitude than their univariate counterparts reported in panel A for both stock market variance and good variance. In addition, the R^2 is much higher in bivariate regressions than in corresponding univariate regressions. The difference reflects the omitted variables problem. The coefficient of correlation between VMKT and good variance measures is positive, ranging between 30% to 70%, while VMKT and good variance have opposite effects on conditional equity premium. If we omit good variance (VMKT) in the forecast regression, the coefficient on VMKT (good variance) is downward (upward) biased toward zero.[8]

For comparison, we include the equal-weighted average stock variance, EWASV, as a predictor in Table VIII. Its predictive power for excess stock market returns is much weaker than that of VWASV. Specifically, the effect of EWASV on conditional equity premium is statistically insignificant at the 10% level in both univariate and bivariate regressions. By contrast, VWASV is statistically significant at the 1% level in the bivariate regression. These results are consistent with the model's prediction that VWASV is a good proxy for good variance.

As a robustness check, we also investigate the out-of-sample predictive power of stock market variance and good variance in panel C of Table VIII. For TYVIX, we use the 2003Q1 to 2009Q4 period for the initial in-sample estimation and make the out-of-sample forecast for the 2010Q1 to 2016Q4 period using an expanding sample. For the other good variance measures, we use the 1963Q1 to 1989Q4 period for initial in-sample estimation and make the out-of-sample forecast for the 1990Q1 to 2016Q4 period using an expanding sample. We use two standard measures to gauge the out-of-sample performance. MSER is the mean squared forecasting errors ratio of the forecasting model to a benchmark model in which conditional equity premium equals average equity premium in historical data. ENC_NEW is the encompassing test proposed by Clark and McCracken (2001). 8 out of 12 IST-based good variance measures have smaller mean squared forecasting errors than does the benchmark model. The encompassing test shows that the out-of-sample predictive power is statistically significant at the 5% level for all IST-based good variance measures. Results are similar for VWASV and TYVIX.

As expected, VWASV has market return predictive power similar to that of IST-based good variance measures. For example, it drives out IST-based good variance measures except for VHML in the multivariate regressions of forecasting excess stock market returns. In addition, the predictive power of TYVIX is similar to that of VWASV: TYVIX becomes statistically insignificant when we control for VWASV in the forecasting regression. These results are not reported here but are available upon request. Because IST-based good variance measures have similar predictive for excess stock market returns, for brevity, in the remainder of the paper we use their first principle component, FPCV, and their average, AVGV as IST-based proxies for good variance. Because TYVIX is available only for a short sample period, we use VWASV as the alternative good variance measure in the remainder of the paper.

[8] The multicollinearity problem cannot explain our findings because it inflates standard errors and does not increases R^2. As a further robustness check, we orthogonalize market variance by good variance and vice versa, and find that the orthogonalized market variance or good variance has significant predictive power for excess market returns (Untabulated).

To summarize, consistent with the model implication, we find that conditional equity premium depends positively on stock market variance and negatively on good variance. The two variances jointly have significant forecasting power for excess stock market returns.

B. ICC as a Measure of the Conditional Equity premium

In the proceeding subsection, we investigate stock market variance-return relation using realized excess stock market return as a proxy for conditional equity premium. As a robustness check, we follow Pastor et al. (2008) and use ICC as a proxy for conditional equity premium. Panel A of Table IX investigates the relation between ICC and market variance. Consistent with Pastor et al. (2008)'s finding, the relation is positive and statistically significant at the 10% level using their ICC measure, PSS, over the extended sample period. We find similar results using the Li et al. (2013) ICC measure, LNS, which is very similar to PSS. For the other ICC measures, the relation is positive albeit insignificant.

In Panel B of Table IX, we add FPCV, a measure of good variance, as an additional explanatory variable. All ICC measures correlate positively and significantly with market variance at least at the 5% level. Their correlation with FPCV is significantly negative at least at the 5% level. The adjusted R^2 is also substantially higher than its counterpart reported in Panel A. The results are qualitatively similar when we use AVGV and VWASV as proxies for good variance in Panels C and D, respectively. Therefore, the relatively weak relation between ICC and stock market variance documented in panel A reflects the omitted variables problem: Both stock market variance and good variance are significant determinants of the implied cost of capital.

If ICC is a measure of conditional equity premium, it may forecast excess stock market returns. Consistent with this conjecture, Li et al. (2013) show that their ICC measure does have significant predictive power for excess stock market returns. We replicate their main finding in panel A of Table X that LNS correlates positively and significantly with the one-quarter-ahead excess stock market return at the 5% level. The other ICC measures also correlate positively with future excess stock market returns; however, the relation is statistically insignificant at the 5% level. To investigate whether the forecasting power of ICC for excess stock market returns reflects its correlation with stock market variance and good variance, we decompose ICC into two components by regressing it on stock market variance and good variance, as in Table IX. We use FPCV as the good variance measure in Panel A of Table X. The fitted component of ICC measures correlates positively and significantly with future stock market returns, while the residual component has negligible predictive power. Results are similar when we use AVGV and VWASV as good variance measures in Panels B and C, respectively.

To summarize, consistent with the model's implication, stock market variance and good variance are important determinants of the conditional equity premium.

C. Stock Market Variance and Prices

We investigate the relation between the scaled stock market price and variances in Table XI. Equation (20) shows that the log price-dividend ratio depends on the expected long-run growth rate, x_t, in addition to market variance and good variance. Similarly, Campbell and Shiller (1988) show that the log price-dividend ratio approximately equals $\sum_{i=0}^{\infty} k_{1,m}^{i}[\Delta d_{t+1+i} + r_{m,t+1+i}]$. The expected cash flows are unobservable. Following Sadka (2007) and Guo and Jiang (2011), we use realized real earnings growth in the following 20 quarters as a proxy for the expected cash flow growth: FEG=$\sum_{i=0}^{20} k_{1,m}^{i}[\Delta e_{t+1+i}]$, where $k_{1,m} = 0.996^3$ for quarterly data. Panel A shows that the results are mixed for the relation between stock market variance and scaled stock market prices. It is positive for the price-dividend ratio and the price-payout ratio and is negative for the price-earnings ratio. Nevertheless, the relation is statistically insignificant in all cases.

In panel B of Table XI, we add FPCV, a measure of good variance, as an additional explanatory variable. The scaled stock market prices correlate negatively with stock market variance, and the correlation is statistically significant at the 1% level for the price-payout ratio, at the 5% level for the price-earnings ratio, and at the 10% level for the price-dividend ratio. In addition, the three scaled stock market prices correlate positively and significantly with FPCV at the 1% level. By contrast, the coefficient on FEG is statistically insignificant at the 10% level in all cases. This result, which confirms Shiller's finding of a disconnect between stock market prices and fundamentals, is an important feature of our model. Overall, the adjusted R^2 ranges from 19% for the price-earnings ratio to 51% for the price-payout ratio, indicating that stock market variance and good variance account for a significant portion of variation in the stock market price. We find similar results using AVGV and VWASV as proxies for good variance in Panels C and D, respectively.

In our model, the price-dividend ratio correlates with stock market variance and good variance because these variances are the determinants of conditional equity premium. To investigate this implication, we decompose the scaled stock market price into two components by regressing it on stock market variance and good variance. In Panel A of Table X, we use FPCV as the proxy for good variance. For all three stock market price measures, the fitted component correlates negatively and significantly with one-quarter-ahead excess stock market returns at the 1% level, while the predictive power is negligible for the residual component. Panels B and C show that results are similar when we use AVGV and VWASV, respectively, as proxies for good variance.

D. Market Variance and the Risk-Free Rate

We investigate the relation between the risk-free rate and variances in Table XII. Panel A reports the univariate regression results. While the risk-free rate correlates negatively with market variance, its correlations with three good variance measures are positive. Nevertheless, the relation is statistically insignificant in all cases. Panel B reports the estimation results of the bivariate regression. The negative relation between stock market variance and the risk-free rate becomes statistically significant at the 1% level when in conjunction with VWASV and at the 10% level when in conjunction with FPCV and AVGV. Similarly, the relation between the risk-free rate and good variance is significantly positive at the 1% level for VWASV and at the 10% for FPCV and AVGV. These findings are consistent with the simulation results reported in Table V.

E. Forecasting Anomalies

In our model, loadings on stock market variance and good variance help explain the cross-section of stock returns. Stocks that are more sensitive to IST shocks have more negative loadings on good variance and thus lower expected returns. Stocks that are more sensitive to DT shocks have more positive loadings on bad variance and thus higher expected returns. To investigate this implication, we form portfolios on the investment capital ratio, Tobin's Q, the price-earnings ratio, idiosyncratic volatility, IMC beta, and market beta. We construct hedging portfolios that are long (short) in stocks are least (most) sensitive to IST shocks. For example, we buy low investment capital ratio stocks and short high investment capital ratio stocks for the hedging portfolio formed on the investment capital ratio. We expect that these long-short portfolios have a positive loadings on good variance. We also consider the four hedging risk factors in the Fama and French (2015) five-factor model, HML, CMA, RMW, and SMB. HML longs (shorts) stocks with high (low) book-to-market equity ratios; CMA longs (shorts) stocks with low (high) total asset growth; RMW longs (shorts) stocks with high (low) profitability; and SMB longs (shorts) stocks with small (big) market capitalization. Because stocks with lower book-to-market equities ratios and higher investment are more sensitive IST shocks and have more negative loadings on good variance, HML and CMA should have positive loadings on good variance. Extant studies, e.g., Kogan and Papanikolaou (2013, 2014) and Fama and French (2015), have shown that these long-short portfolios have significantly positive alpha except that the alpha has diminished for SMB in the past three decades. Kogan and Papanikolaou (2013, 2014) argue that the strong comovement among portfolios formed on the investment capital ratio, Tobin's Q, the price-earnings ratio, idiosyncratic volatility, IMC beta, market beta, and the book-to-market equity ratio reflects their loadings on IST shocks. To investigate this conjecture, we calculate the average of returns on the long-short portfolios formed on these characteristics, AVE, as a measure of the comovement.

In Table XIII, we report the OLS regression results of forecasting long-short portfolio returns using stock market variance and good variance. We use FPCV as a proxy for good variance in panel A. As expected, the coefficient on good variance is positive in all cases and is statistically significant at least at the 10% in most cases. The coefficient on stock market variance is negative in all cases except for SMB, and is statistically significant at least at the 10% level except for CMA and SMB. Again, we find similar results using AVGV and VWASV as proxies for good variance in panels B and C, respectively. To summarize, stocks with different sensitivity to IST shocks have different loadings on stock market variance and good variance. In the next subsection, we investigate whether loadings on stock market variance and good variance help explain the cross-section of expected stock returns.

F. Explaining the Cross-Section of Expected Stock Returns

We use the Fama and MacBeth (1973) cross-sectional regression to test whether loadings on stock market variance and loadings on good variance account for the cross-section of expected stock returns. Specifically, we first regress excess returns on each test portfolio on lagged stock market variance and lagged good variance, and use the estimated loadings in the second-stage cross-sectional regressions. Because both stock market variance and good variance are persistent and have measurement errors, we include two lags of market variance and two lags of good variance in the first-stage regression, and the loadings are the sum of the coefficients on two lags of stock market variance or two lags of good variance.[9]

We use four sets of test portfolios. We first sort stocks equally into five portfolios by market capitalization, and then within size portfolio, we sort stocks equally into five portfolios by each of the seven characteristics: the investment capital ratio, Tobin's Q, the price-earnings ratio, idiosyncratic volatility, IMC beta, the book-to-market equity ratio, and market beta. We use equal or value weights to construct 175 portfolios. We obtain from Kenneth French at Dartmouth College the 32 triple-sorted portfolios formed on market capitalization, operation profit, and total asset growth. Last, we obtain from Kenneth French at Dartmouth College the 32 triple-sorted portfolios formed on market capitalization, book-to-market equity ratios, and total asset growth.

In panel A of Table XIV, we report the Fama and MacBeth (1973) regression results for the 175 value-weighted double-sorted portfolios. The risk price of loadings on good variance is positive and significant at the 1% level for all three good variance measures. In addition, the risk price of loadings on stock market variance is significantly positive at the 1% level. The cross-sectional R-squared ranges from 68% to 73%, which are comparable to the median R-squared in simulated data reported in TableVI. Figure 6 shows that the expected portfolio returns line up with the average portfolio returns along the 45-degree line. Panel B shows that results are similar for the 175 equal-weighted doubt-sorted portfolios. In Panel C, we report the results for the 32 triple-sorted portfolios formed on market capitalization, operation profit, and total asset growth. The risk price is significantly positive at the 1% level for loadings on good variance and at the 10% level for loadings on stock market variance. Panel D reports that results are qualitatively similar for the 32 triple-sorted portfolios formed on market capitalization, book-to-market equity ratios, and total asset growth. Therefore, consistent with the model's implication, loadings on stock market variance and loadings on good variance explain the cross-section of expected stock returns.

VI. Conclusion

The price-dividend ratio is a function of expected future discount rates and expected future dividend growth rates. Time-varying equity premium has become the central organizing question in rational-expectations asset pricing paradigm since Shiller (1981)'s finding that the dividend component accounts for little variation in stock market prices. The price-dividend ratio plays a pivotal

[9]Results are qualitatively similar when we include one lag of stock market variance and one lag of good variance.

role in modern asset pricing models of time-varying equity premium because of the mechanical link between the two variables. No extant empirical studies, however, have attempted to address *directly* the most fundamental question in asset pricing that Shiller raised over three decades ago: What are economic origins of stock market price fluctuations? In this paper, we try to fill the gap by investigating empirically the relation between the stock market price and systematic risks stipulated in asset pricing models.

Leading asset pricing models suggest that stock market variance is the main driver of variation in the stock market price. By contrast with this view, we overwhelmingly reject the null hypothesis of a negative relation between the stock market price-dividend ratio and variance. The relation is sometimes positive and sometimes negative. Our findings echo extensive empirical evidence of an unstable stock market variance-return relation.

We provide a theoretical explanation for the findings. Stock market variance has two components, good variance and bad variance. The equity premium depends positively on bad variance but negatively on good variance. Because stock market variance is the sum of the two variances, its relation with conditional equity premium can be positive, negative, or insignificant, depending on the relative importance of its two components. The unstable stock market variance-return relation implies an unstable relation between stock market variance-price relation, as we document in data. Nevertheless, our model suggests that stock market variance and good variance jointly can explain variation in stock market prices or the conditional equity premium, and our empirical evidence strongly supports this conjecture.

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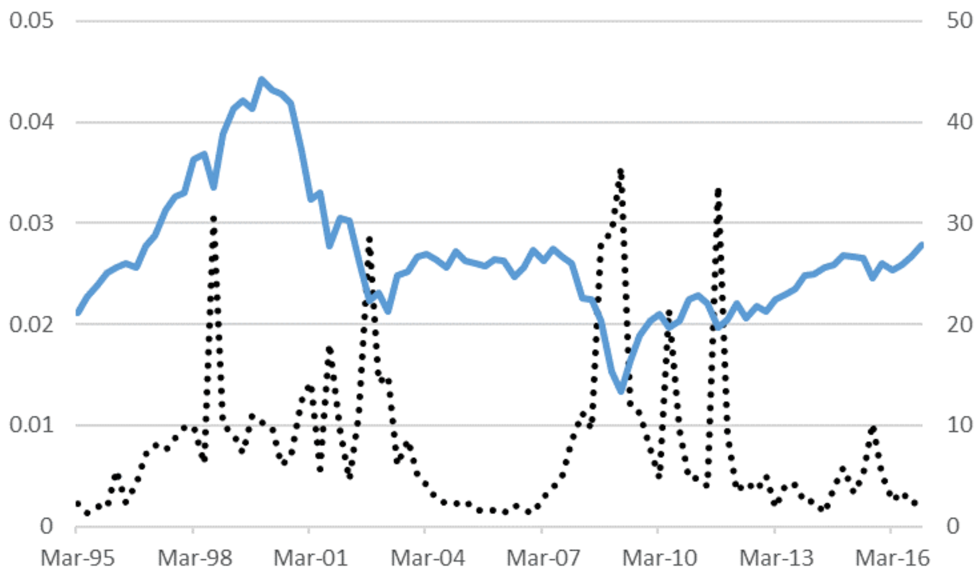

Figure 1. Stock Market Variance (Dashed Line) and the Price-Earnings Ratio (Solid Line)

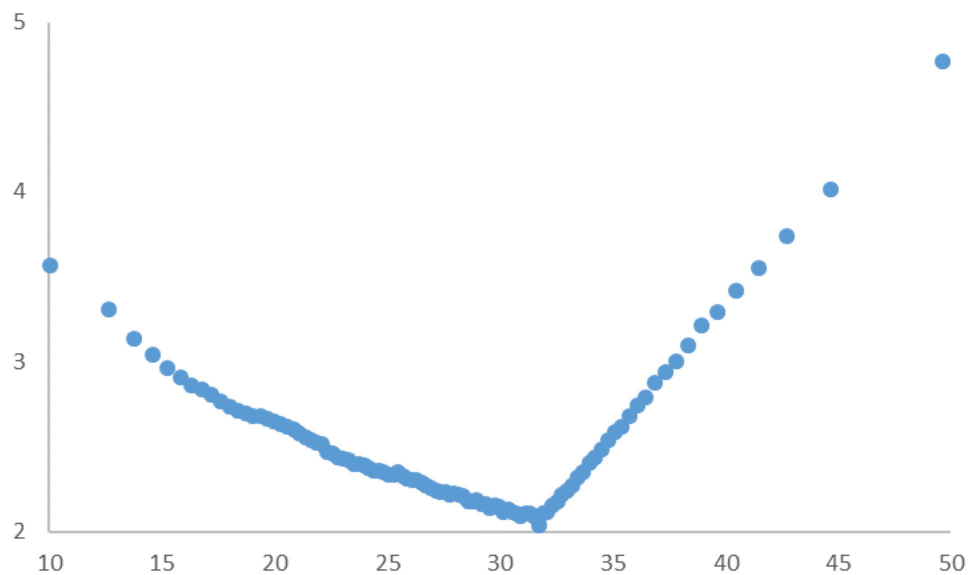

Figure 2. Relation between Conditional Stock Market Variance (in Percentage, Vertical Axis) and Price-Dividend Ratio (Horizontal Axis) in Simulated Data

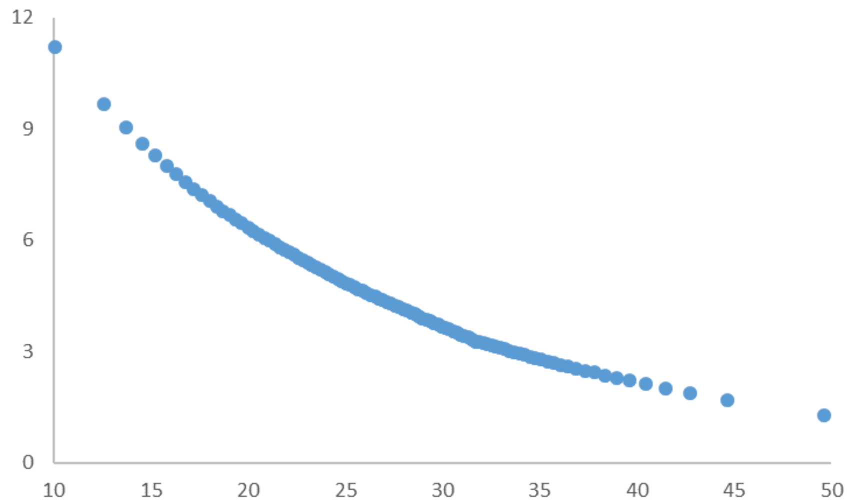

Figure 3. Relation between Conditional Equity Premium (in Percentage, Vertical Axis) and Price-Dividend Ratio (Horizontal Axis) in Simulated Data

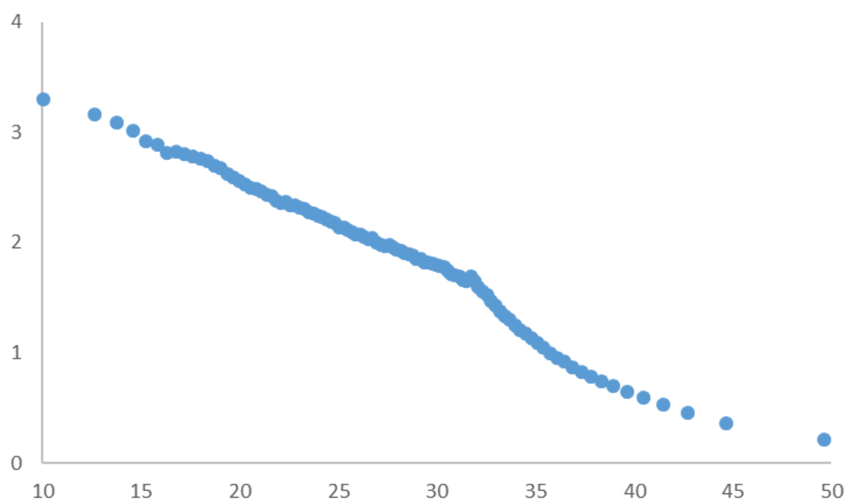

Figure 4. Relation between Equity Premium-Variance Ratio (Vertical Axis) and Price-Dividend Ratio (Horizontal Axis) in Simulated Data

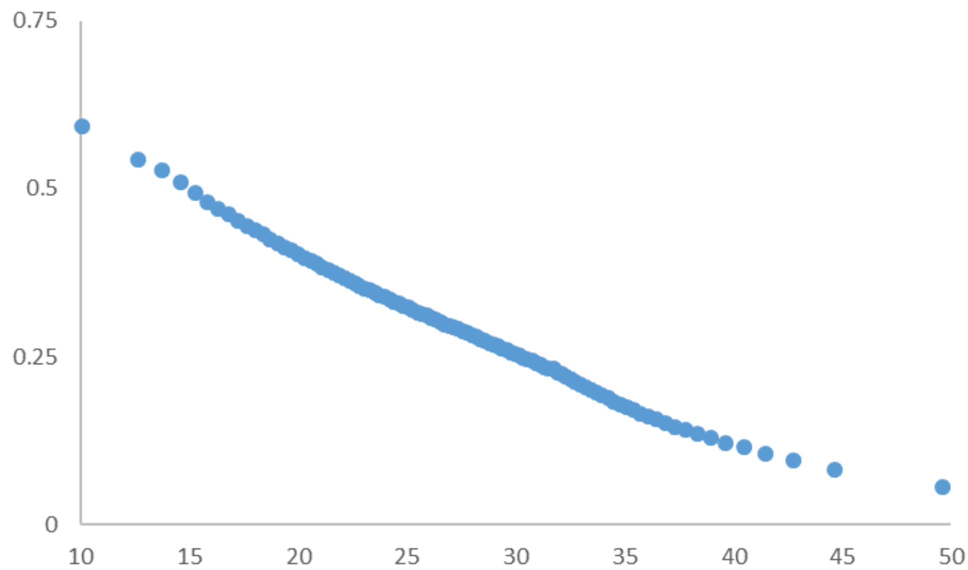

Figure 5. Relation between Sharpe Ratio (Vertical Axis) and Price-Dividend Ratio (Horizontal Axis) in Simulated Data

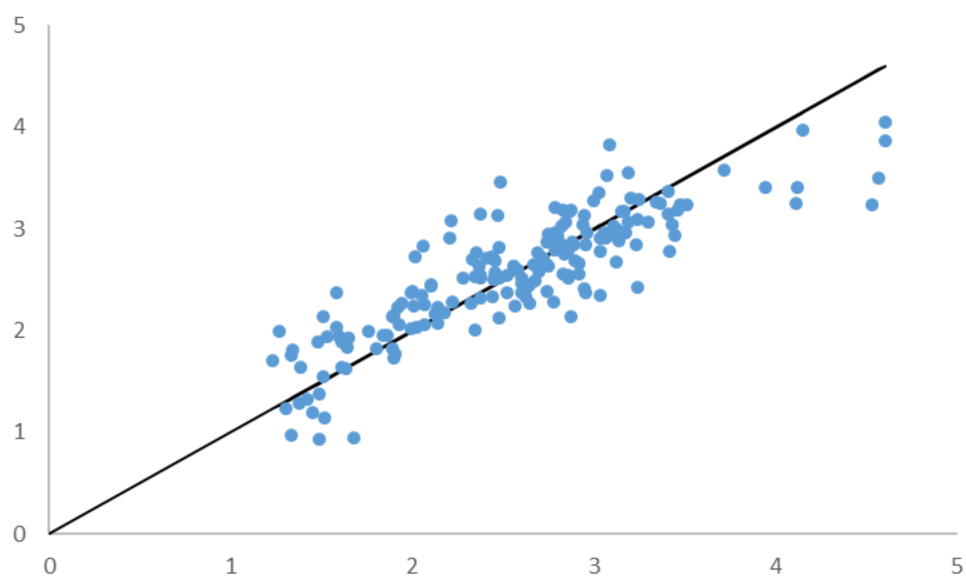

Figure 6. Scatter Plot of Expected Portfolio Returns (in Percentage, Vertical Axis) v.s. Average Portfolio Returns (in Percentage, Horizontal Axis)

Table I Configuration of Model Parameters

Preferences	δ	γ	ψ								
	0.9989	10	1.5								
Consumption	μ_c	ρ	φ_e	ψ_x	σ_g	σ_x	v_g	v_x	σ_1	σ_2	σ_3
	0.0015	0.975	0.001	0.0389	0.0015	0.006	0.999	0.999	0.000006	0	0.000006
Dividends	μ_d	ϕ	π_e	π_η	π_p						
	0.0015	2.2	3	2.2	0.005						

Note: The table reports the parameter values used in the model.

Table II Consumption, Dividend, and Asset Returns

	Data	Model					
Moment	Estimate	Median	2.5%	5%	95%	97.5%	Pop
$E[\Delta c]$	1.93	1.80	0.99	1.15	2.43	2.59	1.80
$\sigma(\Delta c)$	2.16	3.21	1.84	1.99	5.20	5.60	3.56
$AC1(\Delta c)$	0.45	-0.01	-0.26	-0.22	0.19	0.23	0.00
$AC2(\Delta c)$	0.16	-0.01	-0.26	-0.22	0.20	0.24	0.00
$AC3(\Delta c)$	-0.10	-0.01	-0.26	-0.22	0.19	0.23	0.00
$AC4(\Delta c)$	-0.24	-0.01	-0.26	-0.22	0.19	0.23	0.00
$AC5(\Delta c)$	-0.02	-0.01	-0.25	-0.22	0.19	0.23	0.00
$VR6(\Delta c)$	0.84	0.89	0.45	0.51	1.48	1.62	1.00
$E[\Delta d]$	1.15	1.79	-1.32	-0.74	4.40	4.96	1.80
$\sigma(\Delta d)$	11.05	12.92	8.25	8.80	18.69	19.88	13.84
$AC1(\Delta d)$	0.21	-0.01	-0.25	-0.20	0.18	0.22	0.00
$VR6(\Delta d)$	0.59	0.91	0.48	0.53	1.46	1.59	1.02
$Corr(\Delta c, \Delta d)$	0.55	0.54	0.17	0.23	0.80	0.83	0.54
$E[R]$	7.66	7.11	3.77	4.32	10.55	11.37	7.29
$\sigma(R)$	20.28	16.16	11.35	11.99	22.36	23.79	17.13
$AC1(R)$	0.02	-0.02	-0.24	-0.21	0.17	0.21	0.00
$E[p-d]$	3.36	3.31	2.76	2.86	3.56	3.61	3.27
$\sigma(p-d)$	0.45	0.17	0.09	0.10	0.31	0.34	0.29
$AC1(p-d)$	0.87	0.89	0.71	0.75	0.96	0.96	0.97
$E[R^f]$	0.57	1.32	-0.28	0.03	1.83	1.87	1.14
$\sigma(R^f)$	2.86	0.46	0.22	0.25	0.83	0.91	0.79
$AC1(R^f)$	0.65	0.94	0.78	0.82	0.98	0.98	0.98

Note: The table reports key statistics of the consumption growth rate, Δc; the dividend growth rate, Δd; the stock market return, R; the log price-dividend ratio, $p-d$; and the risk-free rate, R^f. E is the mean; σ is the standard deviation; ACi is the ith-order autocorrelation coefficient; $VR6$ is the variance ratio of six-year growth rate to six times one-year growth rate; and $Corr$ is the correlation coefficient. The column under the name "Data" reproduces annual estimates from the 1930 to 2008 period reported in Bansal et al. (2012) and Beeler and Campbell (2012). The column under the name "Model" reports the distribution of annual estimates from 10,000 simulated samples of 79 years each. "Pop" reports annual estimates from a long simulated sample of 100,000 years.

Table III Relation between Price-Dividend Ratio and Variances in Simulated Data

	Median	10%	30%	70%	90%	Pop	Scaler
Panel A: Stock Market Variance							
VMKT	-7.205	57.329	21.486	-36.702	-86.125	-0.333	1
	(-0.587)	(5.293)	(1.749)	(-2.942)	(-7.227)	(-39.200)	1
R^2	12.870	0.477	4.503	26.556	50.586	8.031	0.01
Panel B: Good Variance							
VG	4.509	-2.136	2.091	6.896	10.876	3.067	100
	(3.779)	(-1.456)	(1.560)	(6.377)	(10.789)	(41.977)	1
R^2	21.708	0.974	8.037	39.867	63.553	7.811	0.01
Panel C: Stock Market Variance and Good Variance							
VMKT	-2.268	-2.253	-2.263	-2.273	-2.283	-2.086	100
	(-4.879)	(-2.529)	(-3.675)	(-6.508)	(-10.063)	(770.050)	100
VG	2.569	2.553	2.563	2.575	2.585	19.483	1000
	(5.228)	(2.772)	(4.012)	(6.860)	(9.972)	(151.655)	100
R^2	99.981	99.937	99.969	99.989	99.995	99.991	0.01
Panel D: Stock Market Variance and Value-Weighted Average Stock Variance							
VMKT	-7.691	-5.554	-6.713	-8.875	-10.784	-4.471	100
	(-34.303)	(-18.645)	(-27.306)	(-43.263)	(-60.458)	(-394.924)	1
VWASV	8.759	6.185	7.570	10.212	12.521	4.105	100
	(34.031)	(18.545)	(26.902)	(42.974)	(60.211)	(322.695)	1
R^2	97.172	91.185	95.649	98.167	99.033	95.703	0.01

Note: The table reports the OLS estimation results of regressing the stock market price-dividend ratio on contemporaneous variances for simulated data. We generate a monthly sample of 1,648 observations, discard the first 1,000 observations, and convert the remaining into 216 quarterly observations. We generate 10,000 simulated samples and report their distributions. The column "Pop" reports the results obtained from 100,000 simulated quarterly observations. VMKT is stock market variance, VG is good variance, and VWASV is value-weighted average stock variance. t-values are reported in parentheses. The coefficient and the t-value of stock market variance are sorted from the highest to the lowest. All other statistics are sorted from the lowest to the highest. The column "Scaler" indicates the actual values of the statistics reported in a row are the reported values time the scaler in that row. For example, the scaler for R^2 is 0.01, indicating that it is reported in percentage.

Table IV Relation between Excess Stock market Returns and Variances in Simulated Data

	Median	10%	30%	70%	90%	Pop
Panel A: Stock Market Variance						
VMKT	0.577	-4.461	-1.322	2.551	5.845	0.859
	(0.167)	(-1.192)	(-0.389)	(0.730)	(1.555)	(7.989)
R^2	0.227	0.008	0.076	0.539	1.399	0.077
Panel B: Good Variance						
VG	-12.771	38.981	7.462	-33.867	-71.727	-3.855
	(-0.345)	(1.018)	(0.209)	(-0.895)	(-1.698)	(-3.785)
R^2	0.248	0.009	0.079	0.578	1.420	0.018
Panel C: Stock Market Variance and Good Variance						
VMKT	9.329	-1.970	4.673	14.828	24.606	4.133
	(1.112)	(-0.238)	(0.575)	(1.645)	(2.440)	(21.327)
VG	-104.825	17.694	-52.230	-163.844	-269.035	-36.383
	(-1.126)	(0.198)	(-0.592)	(-1.684)	(-2.480)	(-19.788)
R^2	1.133	0.205	0.608	1.849	3.207	0.504
Panel D: Stock Market Variance and Value-Weighted Average Stock Variance						
VMKT	31.642	-4.837	15.650	51.183	90.259	8.922
	(1.122)	(-0.184)	(0.590)	(1.662)	(2.482)	(20.176)
VWASV	-35.496	6.099	-17.440	-58.211	-102.337	-7.977
	(-1.096)	(0.196)	(-0.573)	(-1.637)	(-2.457)	(-18.791)
R^2	1.135	0.187	0.607	1.859	3.349	0.559

Note: The table reports the OLS estimation results of regressing one-quarter-ahead excess stock market returns on stock variances for simulated data. We generate a monthly sample of 1,648 observations, discard the first 1,000 observations, and convert the remaining into 216 quarterly observations. We generate 10,000 simulated samples and report their distributions. The column "Pop" reports the results obtained from 100,000 simulated quarterly observations. VMKT is stock market variance, VG is good variance, and VWASV is value-weighted average stock variance. t-values are reported in parentheses. R^2 is reported in percentage.

Table V Relation between the Risk-Free Rate and Variances in Simulated Data

	Median	10%	30%	70%	90%	Pop
Panel A: Stock Market Variance						
VMKT	-0.330	0.057	-0.156	-0.508	-0.803	-0.552
	(-4.391)	(0.750)	(-2.140)	(-7.300)	(-12.597)	(-95.703)
R^2	25.154	1.052	9.793	44.832	68.888	34.156
Panel B: Good Variance						
VG	0.022	4.727	1.758	-1.807	-4.892	-0.461
	(0.026)	(5.157)	(1.967)	(-2.013)	(-5.454)	(-7.964)
R^2	9.972	0.398	3.377	21.227	43.645	0.303
Panel C: Stock Market Variance and Good Variance						
VMKT	-1.552	-1.411	-1.502	-1.590	-1.630	-1.640
	(-45.723)	(-25.944)	(-36.013)	(-58.059)	(-82.125)	(-2421.912)
VG	14.457	13.059	13.946	14.847	15.297	12.442
	(40.015)	(23.298)	(31.841)	(50.455)	(69.519)	(1897.623)
R^2	94.372	86.416	91.804	96.249	97.877	99.015
Panel D: Stock Market Variance and Value-Weighted Average Stock Variance						
VMKT	-4.845	-3.568	-4.264	-5.551	-6.653	-3.179
	(-31.929)	(-17.946)	(-25.763)	(-39.390)	(-53.290)	(-425.995)
VG	5.189	3.657	4.477	6.061	7.408	2.634
	(29.978)	(16.815)	(24.004)	(36.890)	(49.436)	(314.120)
R^2	96.088	87.734	96.369	94.041	97.303	96.364

Note: The table reports the OLS estimation results of regressing the risk-free rate on contemporaneous stock variances for simulated data. We generate a monthly sample of 1,648 observations, discard the first 1,000 observations, and convert the remaining into 216 quarterly observations. We generate 10,000 simulated samples and report their distributions. The column "Pop" reports the results obtained from 100,000 simulated quarterly observations. VMKT is stock market variance, VG is good variance, and VWASV is value-weighted average stock variance. t-values are reported in parentheses. R^2 is reported in percentage.

Table VI Relation between Cross-Section of Expected Returns and Variances in Simulated Data

	Median	10%	30%	70%	90%	Pop	Scaler
Panel A: Stock Market Variance and Good Variance							
Const	0.332	-0.047	0.174	0.552	0.983	-0.111	0.01
	(1.922)	(-0.255)	(1.006)	(2.868)	(4.234)	(-1.100)	1
VMKT	1.199	-0.709	0.568	1.867	3.127	6.438	0.001
	(2.148)	(-1.072)	(1.054)	(2.952)	(3.898)	(30.043)	1
VG	0.057	-0.141	-0.013	0.125	0.244	0.387	0.001
	(1.084)	(-2.159)	(-0.239)	(2.193)	(3.366)	(15.071)	1
R^2	77.532	37.098	65.333	84.886	90.801	99.674	0.01
Panel B: Stock Market Variance and Value-Weighted Average Stock Variance							
Const	0.333	-0.046	0.177	0.552	0.989	-0.189	0.01
	(1.946)	(-0.241)	(1.021)	(2.909)	(4.262)	(-1.909)	1
VMKT	4.812	-2.744	2.332	7.517	12.563	26.422	0.001
	(2.169)	(-1.077)	(1.093)	(2.961)	(3.900)	(31.536)	1
VWASV	1.042	-1.100	0.337	1.781	3.125	5.731	0.001
	(1.777)	(-1.530)	(0.580)	(2.716)	(3.725)	(24.130)	1
R^2	77.484	36.683	65.312	84.915	90.864	99.666	0.01

Note: The table reports the Fama and MacBeth (1973) regression results for simulated data. We construct 125 portfolios that have different loadings on systematic risks in equation (18). Specifically, ϕ_p takes one of five possible values [1.4, 1.8, 2.2, 2.6, 3.0], $\pi_{\eta,p}$ takes one of five possible values [1.4, 1.8, 2.2, 2.6, 3.0], and $\pi_{e,p}$ takes one of five possible values [1,9, 2.3, 2.7, 3.1, 3.5]. We assume π_p, the volatility of the idiosyncratic risk is 0.005 for all portfolios. We run the Fama and MacBeth (1973) regression using the 125 portfolios. In the first stage, for each portfolio, we run a time-series forecasting regression of its returns on conditional stock market variance and good variance. In the second stage, we run the cross-sectional regression of portfolio returns on their loadings on stock market variance and good variance. The table reports the estimated risk prices of loadings on variances. t-values are reported in parentheses. VMKT is stock market variance, VG is good variance, and VWASV is value-weighted average stock variance. The column "Scaler" indicates the actual values of the statistics reported in a row are the reported values time the scaler in that row. For example, the scaler for R^2 is 0.01, indicating that it is reported in percentage. We generate a monthly sample of 1,648 observations, discard the first 1,000 observations, and convert the remaining into 216 quarterly observations. We generate 10,000 simulated samples and report their distributions. The column "Pop" reports the results obtained from 100,000 simulated quarterly observations.

Table VII Summary Statistics of Selected Variables

Variable	Mean	Std Err	Kurt	Skew	AR(1)	Sampling Period
Panel A: Stock Market Price						
PD	3.704	0.030	-0.577	-0.345	0.979	1963Q1-2016Q4
PPO	2.203	0.016	18.615	-3.742	0.940	1963Q1-2016Q4
PE	1.697	0.029	-0.502	0.385	0.982	1963Q1-2016Q4
Panel B: Implied Costs of Capital						
PSS	1.602	0.053	-0.873	0.527	0.909	1981Q1-2016Q4
GLS	1.128	0.052	-0.965	0.342	0.923	1982Q1-2016Q4
Easton	1.830	0.046	-0.896	-0.015	0.895	1981Q1-2016Q4
OJ	1.881	0.040	-0.809	-0.168	0.891	1981Q1-2016Q4
GG	0.711	0.056	-0.799	0.412	0.904	1981Q1-2016Q4
AICC	1.444	0.048	-0.906	0.265	0.910	1982Q1-2016Q4
LNS	1.806	0.059	-0.751	-0.019	0.866	1981Q1-2011Q4
Panel C: Stock Return Variances						
VIK	0.080	0.005	13.517	3.327	0.592	1963Q1-2016Q4
VTobinQ	0.136	0.009	8.730	2.711	0.630	1963Q1-2016Q4
VPE	0.095	0.005	3.634	1.895	0.545	1963Q1-2016Q4
$V\beta_{\mathrm{MKT}}$	0.144	0.010	15.822	3.532	0.632	1963Q1-2016Q4
$V\beta_{\mathrm{IMC}}$	0.192	0.017	17.898	3.952	0.559	1963Q1-2016Q4
VIMCIV	0.188	0.018	37.512	5.329	0.556	1963Q1-2016Q4
VIMC	0.128	0.015	50.097	6.107	0.693	1963Q1-2016Q4
VHML	0.109	0.010	43.494	5.847	0.633	1963Q1-2016Q4
VFPC	0.003	0.026	20.698	4.155	0.688	1963Q1-2016Q4
VAVE	0.042	0.004	30.959	5.009	0.559	1963Q1-2016Q4
FPCV	0.000	0.068	16.067	3.629	0.649	1963Q1-2016Q4
AVGV	0.000	0.058	15.388	3.556	0.652	1963Q1-2016Q4
VWASV	0.029	0.022	7.588	2.582	0.647	1963Q1-2016Q4
EWASV	0.082	0.051	12.780	2.959	0.745	1963Q1-2016Q4
TYVIX	0.001	0.008	8.090	2.511	0.696	2003Q1-2016Q4
VMKT	0.653	0.042	6.960	2.466	0.503	1963Q1-2016Q4

Variable	Mean	Std Err	Kurt	Skew	AR(1)	Sampling Period
Panel D: Asset Returns						
IK	0.714	0.299	2.110	0.356	0.057	1963Q1-2016Q4
TobinQ	0.888	0.413	1.869	-0.208	0.124	1963Q1-2016Q4
PE	0.879	0.306	1.327	-0.235	0.134	1963Q1-2016Q4
IMCIV	0.249	0.554	1.901	0.552	0.048	1963Q1-2016Q4
β_{MKT}	0.177	0.475	2.391	0.338	-0.006	1963Q1-2016Q4
β_{IMC}	0.177	0.475	2.391	0.338	-0.006	1963Q1-2016Q4
HML	1.108	0.390	1.703	0.439	0.121	1963Q1-2016Q4
AVE	0.587	0.341	3.125	-0.050	0.085	1963Q1-2016Q4
CMA	0.922	0.274	1.911	0.907	0.048	1963Q1-2016Q4
RMW	0.735	0.283	7.035	0.915	0.143	1963Q1-2016Q4
SMB	0.788	0.379	-0.080	0.142	-0.001	1963Q1-2016Q4
ERET	1.638	0.576	0.815	-0.505	0.062	1963Q1-2016Q4
RF	0.334	0.037	-0.364	0.268	0.865	1963Q1-2016Q4

Note: The table reports the quarterly summary statistics for the stock market price (panel A), the implied cost of capital (panel B), variances (panel C), and asset returns (panel D). In panel A, DP, POP, and EP are log dividend-price ratio, log net payout-price ratio, and log earning-price ratio, respectively. In panel B, PSS, GLS, Easton, OJ, GG, and LNS are the implied cost of capital measures constructed following Pastor et al. (2008), Gebhardt et al. (2001), Easton (2004), Ohlson and Juettner-Nauroth (2005), and Gordon and Gordon (1997), respectively. AICC is the average of these five ICC measures. LNS is the ICC measure used in Li et al. (2013). In panel C, VIK, VTobinQ, VPE, VIMCIV, Vβ_{IMC}, VIMC, Vβ_{MKT}, and VHML are realized variances of daily returns on portfolios formed on IK, Tobin's Q, PE ratio, idiosyncratic volatility, IMC beta, IMC spread, Market Beta, and book-to-market equity ratio, respectively. VFPC and VAVE are realized variances of the first principle component and average of these eight daily portfolio returns, respectively. FPCV and AVGV are the first principle component and average, respectively, of VIK, VTobinQ, VPE, VIMCIV, Vβ_{IMC}, VIMC, Vβ_{MKT}, VHML, VFPC, and VAVE. VWASV and EWASV are value-weighted and equal-weighted average stock variances, respectively. VMKT is stock market variance. In panel D, IK, TobinQ, PE, IMCIV, β_{IMC}, β_{MKT}, and HML are returns on portfolios formed by IK, Tobin's Q, PE ratio, idiosyncratic volatility, IMC beta, Market Beta, and book-to-market ratio, respectively. AVE is the average of these seven portfolio returns. CMA, RMW, and SMB are the Fama and French (2015) conservative-minus-aggressive, robust-minus-weak, and small-minus-big factors, respectively. ERET is the excess stock market return. RF is the risk-free rate. Mean and standard errors in panel B, C and D are reported in percentage. VPC1 is scaled by 10^{-4}, and PC1V and AVGV are scaled by 10^{-2}.

Table VIII Forecasting One-Quarter-Ahead Excess Stock Market Returns Using Stock Variances

	Panel A		Panel B			Panel C		
Variable	All Variance	R^2	Good Variance	Market Variance	R^2	MSER	ENC_NEW Statistics	5% BSCV
VMKT	2.799**	3.707						
	(2.054)							
VIK	-11.408*	0.641	-26.902***	4.338***	8.192	0.957	11.699	2.381
	(-1.831)		-(5.060)	(2.851)				
VTobinQ	-3.993	-0.069	-11.062**	3.776***	5.833	0.997	10.846	2.370
	(-1.013)		(-2.043)	(2.997)				
VPE	-11.042	0.477	-29.331***	4.526***	8.368	0.927	15.510	2.331
	(-1.112)		(-2.927)	(3.543)				
VIV	-1.535	-0.235	-4.807***	3.612***	5.208	1.171	2.667	2.525
	(-0.933)		(-3.467)	(2.614)				
$V\beta_{IMC}$	-4.414*	1.280	-9.446***	4.557***	9.650	0.931	12.380	2.379
	(-1.662)		(-2.725)	(4.904)				
VIMC	-3.020	0.136	-5.761**	3.381**	5.286	1.048	6.239	2.503
	(-1.583)		(-2.551)	(2.523)				
$V\beta_{MKT}$	0.773	-0.451	-8.748**	4.025***	4.877	1.006	5.426	2.379
	(0.220)		(-2.419)	(2.960)				
VHML	-8.357**	1.852	-19.934***	5.442***	12.781	0.823	31.010	2.484
	(-2.291)		(-6.038)	(6.088)				
VFPC	-1.634	0.076	-4.756***	4.165***	6.895	0.963	8.892	2.414
	(-1.233)		(-4.707)	(3.078)				
VAVE	-4.564	-0.377	-20.845***	3.586**	4.857	1.033	4.255	2.436
	(-0.523)		(-2.661)	(2.301)				
FPCV	-0.740	0.300	-2.247***	4.699***	8.448	0.917	12.985	2.380
	(-1.454)		(-4.389)	(4.295)				
AVGV	-0.898	0.347	-2.715***	4.765***	8.679	0.913	13.586	2.370
	(-1.481)		(-4.339)	(4.453)				
VWASV	-0.065	-0.440	-2.096***	8.979***	13.473	0.825	21.880	2.330
	(-0.168)		(-4.063)	(6.849)				
EWASV	0.078	-0.241	-0.211	3.897**	4.279	1.013	5.104	2.406
	(0.644)		(-1.515)	(2.474)				
TYVIX	-24.718	5.722	-53.546***	4.658***	17.143	0.771	8.849	2.629
	(-1.495)		(-2.798)	(5.699)				

Note: The table reports the OLS estimation results of forecasting one-quarter-ahead excess stock market returns using stock variances. VIK, VTobinQ, VPE, VIMCIV, Vβ_{IMC}, VIMC, Vβ_{MKT}, and VHML are realized variances of daily returns on portfolios formed on IK, Tobin's Q, PE ratio, idiosyncratic volatility, IMC beta, IMC spread, Market Beta, and book-to-market equity ratio, respectively. VFPC and VAVE are realized variances of the first principle component and average of these eight daily portfolio returns, respectively. FPCV and AVGV are the first principle component and average, respectively, of VIK, VTobinQ, VPE, VIMCIV, Vβ_{IMC}, VIMC, Vβ_{MKT}, VHML, VFPC, and VAVE. VWASV and EWASV are value-weighted and equal-weighted average stock variances, respectively. VMKT is stock market variance. TYVIX is the options-implied Treasury bond variance. TYVIX is available over the 2003Q1 to 2016Q4 period and the other variance measures are available over the 1963Q1 to 2016Q4 period. Panel A reports the univariate regression results. Panel B reports the bivariate regression results with stock market variance and a good variance measure as the forecasting variables. Panel C reports the out-of-sample forecast results. For TYVIX, we use the 2003Q1 to 2009Q4 period for the initial in-sample estimation and make the out-of-sample forecast recursively for the 2010Q1 to 2016Q4 period using an expanding sample. For the other good variance measures, we use the 1963Q1 to 1989Q4 period for initial in-sample estimation and make the out-of-sample forecast recursively for the 1990Q1 to 2016Q4 period using an expanding sample. We use two standard measures to gauge the out-of-sample performance. MSER is the mean squared forecasting errors ratio of the forecasting model to a benchmark model in which conditional equity premium equals average equity premium in historical data. ENC_NEW is the encompassing test proposed by Clark and McCracken (2001). t-values are reported in parentheses. ***, **, and * denote significance at the 1%, 5%, and 10% levels, respectively.

Table IX Implied Cost of Capital and Stock Market Variance

	PSS	GLS	Easton	OJ	GG	AICC	LNS
Panel A: Stock Market Variance							
VMKT	0.161*	0.137	0.075	0.110	0.145	0.125	0.225**
	(1.787)	(1.593)	(0.820)	(1.393)	(1.510)	(1.428)	(2.217)
R^2	5.060	3.149	0.283	1.815	3.239	2.529	4.694
Panel B: Stock Market Variance and First Principle Component of Good Variance Measures							
FPCV	-0.127**	-0.158***	-0.133**	-0.123**	-0.158**	-0.140**	-0.157**
	(-2.408)	(-2.626)	(-2.228)	(-2.366)	(-2.591)	(-2.476)	(-2.094)
VMKT	0.274**	0.282***	0.197*	0.222**	0.289**	0.254**	0.371***
	(2.546)	(2.605)	(1.726)	(2.216)	(2.549)	(2.330)	(2.684)
R^2	12.784	14.369	6.764	8.513	13.397	11.314	10.200
Panel C: Stock Market Variance and Average of Good Variance Measures							
AVGV	-0.148**	-0.184**	-0.154**	-0.144**	-0.184**	-0.163**	-0.184**
	(-2.279)	(-2.517)	(-2.117)	(-2.286)	(-2.463)	(-2.362)	(-2.012)
VMKT	0.277**	0.282***	0.196*	0.222**	0.289**	0.253**	0.371**
	(2.519)	(2.576)	(1.701)	(2.189)	(2.524)	(2.303)	(2.648)
R^2	12.453	14.041	6.472	8.311	13.055	10.999	9.980
Panel D: Stock Market Variance and Value-Weighted Average Stock Variance							
VWASV	-0.116***	-0.146***	-0.122***	-0.109***	-14.935***	-0.129***	-0.122*
	(-3.415)	(-4.332)	(-3.058)	(-3.129)	(-4.566)	(-3.707)	(-1.923)
VMKT	0.522***	0.596***	0.455***	0.451***	0.612***	0.529***	0.599***
	(3.885)	(4.398)	(3.090)	(3.408)	(4.472)	(3.872)	(2.812)
R^2	19.683	25.114	12.911	13.989	24.022	19.490	12.446

Note: The table reports the OLS estimation results of regressing the implied cost of capital on contemporaneous stock market variance and good variance measures. We de-trend the implied cost of capital by a linear time trend. PSS, GLS, Easton, OJ, GG, and LNS are the implied cost of capital measures constructed following Pastor et al. (2008), Gebhardt et al. (2001), Easton (2004), Ohlson and Juettner-Nauroth (2005), and Gordon and Gordon (1997), respectively. AICC is the average of these five ICC measures. LNS is the ICC measure used in Li et al. (2013). LNS is available over the 1981Q1 to 2011Q4 period, GLS and AICC are available over the 1982Q1 to 2016Q4 period, and the other ICC measures are available over the 1981Q1 to 2016Q4 period. VMKT is stock market variance. FPCV and AVGV are the first principle component and average, respectively, of 10 good variance measures: VIK, VTobinQ, VPE, VIMCIV, $V\beta_{\mathrm{IMC}}$, VIMC, $V\beta_{\mathrm{MKT}}$, VHML, VFPC, and VAVE. VWASV is the value-weighted average stock variance. *t*-values are reported in parentheses. ***, **, and * denote significance at the 1%, 5%, and 10% levels, respectively.

Table X Forecasting One-Quarter-Ahead Excess Stock Market Returns Using Implied Cost of Capital and Scaled Stock Market Prices

	Original Value	R^2	Fitted Value	R^2	Residual Value	R^2
Panel A: First Principle Component of Good Variance Measures						
PSS	1.348	0.348	16.242***	9.461	0.287*	-0.690
	(1.173)		(4.504)		(1.803)	
GLS	1.563	0.593	14.954***	10.179	1.069	-0.427
	(1.416)		(5.150)		(0.584)	
Easton	1.540	0.296	18.967***	9.481	0.222	-0.693
	(1.230)		(4.976)		(0.146)	
OJ	2.531*	1.385	19.119***	9.767	0.942	-0.474
	(1.851)		(5.095)		(0.568)	
GG	1.809*	1.378	14.793***	9.762	1.266	-0.259
	(1.703)		(5.070)		(0.710)	
AICC	1.476	0.297	16.703***	10.167	0.672	-0.607
	(1.217)		(5.126)		(0.390)	
LNS	2.332**	2.212	13.075***	10.297	0.915	-0.408
	(1.957)		(4.407)		(0.653)	
PD	-0.018	0.479	-0.085***	3.984	0.001	-0.468
	(-1.315)		(-3.876)		(0.049)	
PPO	-0.050***	1.535	-0.118***	3.658	-0.009	-0.425
	(-2.740)		(-3.820)		(-0.196)	
PE	-0.016	0.144	-0.126***	6.305	0.007	-0.370
	(-1.057)		(-4.297)		(0.426)	
Panel B: Average of Good Variance Measures						
PSS	1.348	0.348	16.617***	9.685	0.288	-0.689
	(1.173)		(4.618)		(0.160)	
GLS	1.563	0.593	15.322***	10.483	1.056	-0.433
	(1.416)		(5.287)		(0.578)	
Easton	1.540	0.296	19.641***	9.824	0.224	-0.693
	(1.230)		(5.081)		(0.148)	
OJ	2.531*	1.385	19.591***	10.064	0.933	-0.478
	(1.851)		(5.230)		(0.563)	
GG	1.809*	1.378	15.177***	10.054	1.256	-0.264
	(1.703)		(5.205)		(0.704)	
AICC	1.476	0.297	17.145***	10.464	0.667	-0.608
	(1.217)		(5.262)		(0.387)	
LNS	2.332**	2.212	13.364***	10.577	0.907	-0.413
	(1.957)		(4.551)		(0.648)	
PD	-0.018	0.479	-0.088***	4.191	0.001	-0.466
	(-1.315)		(-4.945)		(0.073)	
PPO	-0.050***	1.535	-0.122***	3.848	-0.009	-0.432
	(-2.740)		(-3.876)		(-0.178)	
PE	-0.016	0.144	-0.129***	6.573	0.007	-0.360
	(-1.057)		(-4.367)		(0.447)	

	Original Value	R^2	Fitted Value	R^2	Residual Value	R^2
Panel C: Value-Weighted Average Stock Variance						
PSS	1.348	0.348	16.684***	15.302	-1.212	-0.388
	(1.173)		(6.532)		(-0.704)	
GLS	1.563	0.593	13.819***	14.972	-0.533	-0.664
	(1.416)		(5.760)		(-0.309)	
Easton	1.540	0.296	17.934***	15.303	-0.938	-0.443
	(1.230)		(4.903)		(-0.649)	
OJ	2.531*	1.385	19.132***	15.631	-0.226	-0.697
	(1.851)		(5.845)		(-0.152)	
GG	1.809*	1.378	14.074***	15.632	-0.397	-0.670
	(1.703)		(5.768)		(-0.229)	
AICC	1.476	0.297	15.62***	14.968	-0.686	-0.614
	(1.217)		(5.845)		(-0.425)	
LNS	2.332**	2.212	14.304***	15.004	0.405	-0.741
	(1.957)		(6.358)		(0.302)	
PD	-0.018	0.479	-0.137***	8.562	0.007	-0.369
	(-1.315)		(-3.726)		(0.478)	
PPO	-0.050***	1.535	-0.158***	8.341	0.036	-0.116
	(-2.740)		(-3.706)		(0.651)	
PE	-0.016	0.144	-0.221***	12.322	0.008	-0.322
	(-1.057)		(-4.485)		(0.564)	

Note: The table reports the OLS estimation results of forecasting excess stock market returns with implied cost of capital measures and scaled stock market prices. We de-trend the implied cost of capital by a linear time trend. PSS, GLS, Easton, OJ, GG, and LNS are the implied cost of capital measures constructed following Pastor et al. (2008), Gebhardt et al. (2001), Easton (2004), Ohlson and Juettner-Nauroth (2005), and Gordon and Gordon (1997), respectively. AICC is the average of these five ICC measures. LNS is the ICC measure used in Li et al. (2013). LNS is available over the 1981Q1 to 2011Q4 period, GLS and AICC are available over the 1982Q1 to 2016Q4 period, and the other ICC measures are available over the 1981Q1 to 2016Q4 period. PD is the price-dividend ratio. PPO is the price-payout ratio. PE is the price-earnings ratio. PD, PPO, and PE are available over the 1963Q1 to 2016Q4 period. In the column under the name"Original Value," we use the raw data of implied cost of capital measures and the scaled stock market prices as the predictive variables. We also decompose implied cost of capital measures and the scaled stock market prices by regressing them on a constant, stock market variance, and a good variance measure. We use the fitted value as the forecasting variable in the column under the name "Fitted Value" and use the residual value as the forecasting variable in the column under the name "Residual Value." t-values are reported in parentheses. ***, **, and * denote significance at the 1%, 5%, and 10% levels, respectively.

Table XI Relation between Stock Variances and Scaled Stock Market Prices

	PD	PPO	PE
Panel A: Stock Market Variance			
VMKT	4.411	7.486	-4.015
	(0.377)	(1.181)	(-0.380)
FEG	0.058	0.025	0.045
	(0.292)	(0.412)	(0.245)
R^2	-0.328	1.660	-0.569
Panel B: Stock Market Variance and The First Component of Good Variance Measures			
FPCV	0.288***	0.207***	0.233***
	(8.289)	(5.493)	(6.683)
VMKT	-20.640*	-10.582**	-24.267***
	(-1.840)	(-2.043)	(-2.190)
FEG	0.030	0.005	0.022
	(0.192)	(0.169)	(0.148)
R^2	27.998	51.030	19.211
Panel C: Stock Market Variance and the Average of Good Variance Measures			
AVGV	0.342***	0.247***	0.276***
	(8.366)	(5.473)	(6.726)
VMKT	-20.671*	-10.655***	-24.260***
	(-1.837)	(-2.040)	(-2.185)
FEG	0.033	0.007	0.025
	(0.210)	(0.238)	(0.163)
R^2	27.905	51.148	19.080
Panel D: Stock Market Variance and Value-Weighted Average Stock Variance			
VWASV	18.072***	14.965***	13.184***
	(6.525)	(5.046)	(5.219)
VMKT	-48.463***	-36.296***	-42.586***
	(-4.523)	(-4.759)	(-3.857)
FEG	0.077	0.040	0.059
	(0.468)	(1.262)	(0.369)
R^2	24.571	59.108	13.461

Note: The table reports the OLS estimation results of regressing scaled stock market prices on contemporaneous stock market variance and good variance. PD is the price-dividend ratio. PPO is the price-payout ratio. PE is the price-earnings ratio. VMKT is stock market variance. FPCV is the first principle component of the 10 IST-based good variance measures. AVGV is the average of the 10 IST-based good variance measures. VWASV is the value-weighted average stock variance. FEG is the earnings growth in the following 10 years. Data span the 1963Q1 to 2016Q4 period. t-value is reported in parentheses. ***, **, and * denote significance at the 1%, 5%, and 10% levels, respectively.

Table XII Relation between Stock Variances and the Risk-Free Rate

	Panel A		Panel B		
	Market Variance	R^2	Good Variance	Market Variance	R^2
VMKT	-0.068	0.683			
	(-1.201)				
FPCV	0.000	0.294	0.001*	-0.133*	3.496
	(0.806)		(1.797)	(-1.723)	
AVGV	0.000	0.257	0.001*	-0.134*	3.464
	(0.789)		(1.769)	(-1.708)	
VWASV	0.010	-0.259	0.071***	-0.288***	7.625
	(0.477)		(3.087)	(-2.923)	

Note: The table reports the OLS estimation results of regressing the risk-free rate on contemporaneous stock market variance and good variance. PD is the price-dividend ratio. PPO is the price-payout ratio. PE is the price-earnings ratio. VMKT is stock market variance. FPCV is the first principle component of the 10 IST-based good variance measures. AVGV is the average of the 10 IST-based good variance measures. VWASV is the value-weighted average stock variance. Data span the 1963Q1 to 2016Q4 period. t-values are reported in parentheses. ***, **, and * denote significance at the 1%, 5%, and 10% levels, respectively.

Table XIII Forecasting One-Quarter-ahead Anomaly Returns

	Good Variance	Market Variance	R^2
Panel A: The First Principle Component of Good Variance Measures			
IK	0.442	-1.857**	4.193
	(1.098)	(-2.130)	
Tobin Q	0.794*	-1.975*	2.086
	(1.727)	(-1.833)	
PE	0.604*	-1.778***	3.423
	(1.757)	(-2.976)	
IMC IV	1.106*	-4.808***	9.099
	(1.681)	(-3.736)	
β_{IMC}	1.075	-3.643***	6.461
	(1.347)	(-2.837)	
β_{MKT}	0.826*	-4.490***	11.402
	(1.950)	(-4.638)	
HML	0.64	-2.271***	3.539
	(1.102)	(-3.101)	
AVE	0.786*	-2.965***	9.086
	(1.845)	(-3.874)	
CMA	0.674**	-0.715	1.202
	(2.272)	(-1.302)	
RMW	0.894**	-1.169**	2.896
	(2.272)	(-2.029)	
SMB	0.185	0.974	0.737
	(0.461)	(1.465)	

	Good Variance	Market Variance	R^2
Panel B: The Average of Good Variance Measures			
IK	0.551	-1.882**	4.276
	(1.164)	(-2.149)	
TobinQ	0.971*	-2.007*	2.175
	(1.768)	(-1.857)	
PE	0.734*	-1.799***	3.496
	(1.783)	(-2.996)	
IMC IV	1.332*	-4.837***	9.15
	(1.712)	(-3.741)	
β_{IMC}	1.286	-3.665***	6.501
	(1.366)	(-2.834)	
β_{MKT}	1.001**	-4.516***	11.453
	(1.978)	(-4.650)	
HML	0.768	-2.286***	2.568
	(1.100)	(-3.098)	
AVE	0.952*	-2.990***	9.175
	(1.884)	(-3.875)	
CMA	0.810**	-0.731	1.271
	(2.109)	(-1.320)	
RMW	1.060**	-1.180**	2.914
	(2.252)	(-2.025)	
SMB	0.2	0.986	0.723
	(0.413)	(1.474)	
Panel C: Value-Weighted Average Stock Variance			
IK	0.489*	-2.926**	5.519
	(1.672)	(-2.402)	
TobinQ	0.824*	-3.734**	3.899
	(1.911)	(-2.291)	
PE	0.865**	-3.819***	8.249
	(2.560)	(-3.696)	
IMC IV	1.169**	-7.321***	11.178
	(2.445)	(-4.016)	
β_{IMC}	1.089**	-5.946***	8.701
	(2.100)	(-2.950)	
β_{MKT}	0.931**	-6.537***	13.334
	(2.539)	(-4.832)	
HML	0.835*	-4.189***	6.201
	(1.824)	(-2.963)	
AVE	0.890***	-4.922***	12.578
	(2.605)	(-4.019)	
CMA	0.672***	-2.124**	3.813
	(2.764)	(-2.292)	
RMW	0.776**	-2.701**	5.334
	(2.133)	(-2.303)	
SMB	0.301	0.245	1.148
	(1.152)	(0.286)	

Note: The table reports the OLS estimation results of forecasting one-quarter-ahead anomaly returns. IK, TobinQ, PE, IMCIV, β_{IMC}, β_{MKT}, and HML are returns on long-short portfolios formed by investment-capital ratio, Tobin's Q, price-earnings ratio, idiosyncratic volatility, IMC beta, Market Beta, and book-to-market equity ratio, respectively. AVE is the average of these seven portfolio returns. CMA, RMW, and SMB are the Fama and French (2015) conservative-minus-aggressive, robust-minus-weak, and small-minus-big factors, respectively. We use three proxies for good variance. We use the first principle component and the average of the 10 IST-based good variance measures in panels A and B, respectively. We use the value-weighted average sock variance in panel C. Data span the 1963Q1 to 2016Q4 period. t-values are reported in parentheses. ***, **, and * denote significance at the 1%, 5%, and 10% levels, respectively.

Table XIV Stock Variances and the Cross-Section of Expected Portfolio Returns

	Constant	Good Variance	Market Variance	R^2
Panel A: 175 Value-weighted double-sorted Portfolios				
FPCV	0.027***	0.993***	0.002**	67.538
	(4.972)	(3.670)	(2.021)	
AVGV	0.027***	0.842***	0.002**	67.814
	(4.995)	(3.668)	(2.022)	
VWASV	0.030***	0.020***	0.003***	72.697
	(5.210)	(3.435)	(2.372)	
Panel B: 175 Equal-weighted double-sorted Portfolios				
FPCV	0.026***	1.575***	0.004***	76.360
	(4.466)	(5.609)	(3.568)	
AVGV	0.026***	1.328***	0.004***	76.754
	(4.546)	(5.604)	(3.548)	
VWASV	0.032***	0.029***	0.005***	81.950
	(5.310)	(5.402)	(3.759)	
Panel C: 32 Portfolios Sorted by Size, Profitability, and Asset Growth				
FPCV	0.011**	1.164***	0.003*	57.279
	(2.142)	(4.221)	(1.858)	
AVGV	0.012**	0.996***	0.003*	57.600
	(2.206)	(4.221)	(1.877)	
VWASV	0.018***	0.022***	0.003*	61.867
	(3.390)	(3.384)	(1.946)	
Panel D: 32 Portfolios sorted by Size, BM, and Asset Growth				
FPCV	0.003	1.084***	0.005**	51.925
	(0.569)	(3.616)	(2.536)	
AVGV	0.003	0.925***	0.005**	51.967
	(0.617)	(3.618)	(2.543)	
VWASV	0.011*	0.023***	0.005**	59.010
	(1.797)	(3.275)	(2.475)	

Note: The table reports the Fama and MacBeth (1973) cross-sectional regression results. We use four sets of test portfolios. In panels A and B, we first sort stocks equally into five portfolios by market capitalization, and then within each size portfolio we sort stocks equally into five portfolios by each of the seven characteristics: the investment-capital ratio, Tobin's Q, the price-earnings ratio, idiosyncratic volatility, IMC beta, the book-to-market equity ratio, and market beta. We use the value weighted 175 portfolios in panel A and the equal-weighted 175 portfolios in panel B. In panel C, we use the 32 triple-sorted portfolios formed on market capitalization, operation profit, and total asset growth obtained from Kenneth French at Dartmouth College. In panel C, we use the 32 triple-sorted portfolios formed on market capitalization, book-to-market equity ratios, and total asset growth obtained from Kenneth French at Dartmouth College. In the Fama and MacBeth regression, we first regress returns on each test portfolio on lagged stock market variance and lagged good variance, and use the estimated loadings in the second-stage cross-sectional regressions. We include two lags of stock market variance and two lags of good variance in the first-stage regression, and the loadings are the sum of the coefficients on two lags of stock market variance or two lags of good variance. VMKT is stock market variance. We use three proxies of good variance. FPCV is the first principle component of the 10 IST-based good variance measures. AVGV is the average of the 10 IST-based good variance measures. VWASV is the value-weighted average stock variance. The data span the 1963Q1 to 2016Q4 period. ***, **, and * denote significance at the 1%, 5%, and 10% levels, respectively.

Appendix A. Model Appendix

Appendix A. Consumption Dynamics

Aggregate consumption dynamics are as follows

$$\begin{aligned}
\Delta c_{t+1} &= \mu_c + x_t + \sigma_{g,t}\eta_{t+1} - \psi_x \sigma_{x,t} e_{t+1}, \\
x_{t+1} &= \rho x_t + \varphi_e \sigma_{x,t} e_{t+1}, \\
\sigma^2_{g,t+1} &= \sigma^2_g + v_g(\sigma^2_{g,t} - \sigma^2_g) + \sigma_1 z_{1,t+1}, \\
\sigma^2_{x,t+1} &= \sigma^2_x + v_x(\sigma^2_{x,t} - \sigma^2_x) + \sigma_2 z_{1,t+1} + \sigma_3 z_{2,t+1}.
\end{aligned}$$

The shocks $\eta_{t+1}, e_{t+1}, z_{1,t+1}, z_{2,t+1}, z_{3,t+1}$ are i.i.d. standard normal and uncorrelated.

Using the log-linear approximation of Campbell and Shiller (1988), we can write the log return on the claim to aggregate consumption as

$$\begin{aligned}
r_{a,t+1} &= \ln \frac{P_{t+1} + C_{t+1}}{P_t} = \ln \frac{P_{t+1} + C_{t+1}}{C_{t+1}} - \ln \frac{P_t}{C_t} + \ln \frac{C_{t+1}}{C_t} \\
&= k_0 + k_1 z_{t+1} - z_t + \Delta c_{t+1}, \qquad \text{(A1)}
\end{aligned}$$

where $z_t = \ln \frac{P_t}{C_t}$, $\bar{z} = \mathbb{E}[z_t]$, $k_1 = \dfrac{e^{\bar{z}}}{e^{\bar{z}} + 1} < 1$,$k_0 = \ln(e^{\bar{z}} + 1) - \dfrac{\bar{z}e^{\bar{z}}}{e^{\bar{z}} + 1}$. From Epstein and Zin (1989), the log pricing kernel is

$$m_{t+1} = \ln M_{t+1} = \theta \ln \delta - \frac{\theta}{\psi}\Delta c_{t+1} + (\theta - 1) r_{a,t+1}. \qquad \text{(A2)}$$

The Euler equation for return on any asset i is $\mathbb{E}_t[M_{t+1} R_{i,t+1}] = 1$, which can be rewritten as

$$\mathbb{E}_t \big[\exp \big(\theta \ln \delta - \frac{\theta}{\psi}\Delta c_{t+1} + (\theta - 1) r_{a,t+1} + r_{i,t+1} \big) \big] = 1. \qquad \text{(A3)}$$

Equation (A3) holds for the return on the claim to aggregate consumption $r_{a,t+1}$ or

$$\mathbb{E}_t \big[\exp \big(\theta \ln \delta - \frac{\theta}{\psi}\Delta c_{t+1} + \theta r_{a,t+1} \big) \big] = 1. \qquad \text{(A4)}$$

The log price-consumption ratio is a linear function of state variables:

$$z_t = A_0 + A_1 \sigma^2_{g,t} + A_2 \sigma^2_{x,t} + A_3 x_t, \qquad \text{(A5)}$$

where A_0, A_1, A_2, A_3 are constants to be determined below. Combining equation (A1) and equation (A5), we have

$$\begin{aligned}
r_{a,t+1} &= c_1 + (k_1 v_g - 1) A_1 \sigma^2_{g,t} + (k_1 v_x - 1) A_2 \sigma^2_{x,t} + (k_1 A_1 \sigma_1 + k_1 A_2 \sigma_2) z_{1,t+1} \\
&\quad + k_1 A_2 \sigma_3 z_{2,t+1} + (k_1 A_3 \rho - A_3 + 1) x_t + (k_1 A_3 \varphi_e - \psi_x) \sigma_{x,t} e_{t+1} + \sigma_{g,t} \eta_{t+1},
\end{aligned}$$

where $c_1 = k_0 + (k_1 - 1)A_0 + k_1 A_1 \sigma_g^2(1 - v_g) + k_1 A_2 \sigma_x^2 (1 - v_x) + \mu_c$. Note that

$$\begin{aligned}
& \theta \ln \delta - \frac{\theta}{\psi} \Delta c_{t+1} + \theta r_{a,t+1} \\
= \; & \theta \ln \delta + \theta c_1 - \frac{\theta}{\psi} \mu_c + [A_3 \theta(\rho k_1 - 1) + 1 - \gamma] x_t + \theta (k_1 v_g - 1) A_1 \sigma_{g,t}^2 \\
& + \theta(k_1 v_x - 1) A_2 \sigma_{x,t}^2 + \theta k_1 (A_1 \sigma_1 + A_2 \sigma_2) z_{1,t+1} + \theta k_1 A_2 \sigma_3 z_{2,t+1} \\
& + [\theta k_1 A_3 \varphi_e + (\gamma - 1)\psi_x] \sigma_{x,t} e_{t+1} + (1 - \gamma) \sigma_{g,t} \eta_{t+1}.
\end{aligned}$$

Using equation (A4) and the fact that $\ln(\mathbb{E}[X]) = \mathbb{E}[\ln(X)] - \frac{1}{2}\mathrm{Var}[\ln(X)]$ for log normal distributed variable X, we have

$$\begin{aligned}
& A_3 \theta(\rho k_1 - 1) + 1 - \gamma = 0, \\
& \theta(k_1 v_g - 1) A_1 + \frac{1}{2}(1 - \gamma)^2 = 0, \\
& \theta(k_1 v_x - 1) A_2 + \frac{1}{2}[\theta k_1 A_3 \varphi_e + (\gamma - 1)\psi_x]^2 = 0, \\
& \theta \ln \delta + \theta c_1 - \frac{\theta}{\psi} \mu_c + \frac{1}{2} \theta^2 k_1^2 (A_1 \sigma_1 + A_2 \sigma_2)^2 + \frac{1}{2} \theta^2 k_1^2 A_2^2 \sigma_3^2 = 0,
\end{aligned}$$

from which we get

$$\begin{aligned}
A_0 &= \frac{1}{1 - k_1} \Big[\ln \delta + k_0 + (1 - \frac{1}{\psi}) \mu_c + \frac{1}{2} \theta k_1^2 (A_1 \sigma_1 + A_2 \sigma_2)^2 + \frac{1}{2} \theta k_1^2 A_2^2 \sigma_3^2 \\
& \qquad + k_1 A_1 \sigma_g^2 (1 - v_g) + k_1 A_2 \sigma_x^2 (1 - v_x) \Big], \\
A_1 &= \frac{(1 - \gamma)^2}{2\theta(1 - k_1 v_g)}, \\
A_2 &= \frac{[\theta k_1 A_3 \varphi_e + (\gamma - 1)\psi_x]^2}{2\theta(1 - k_1 v_x)}, \\
A_3 &= \frac{1 - \frac{1}{\psi}}{1 - k_1 \rho}.
\end{aligned}$$

Appendix B. Pricing kernel

The log pricing kernel is

$$\begin{aligned}
m_{t+1} &= \theta \ln \delta - \frac{\theta}{\psi} \Delta c_{t+1} + (\theta - 1) r_{a,t+1} \\
&= c_2 + [A_3 (\theta - 1)(\rho k_1 - 1) - \gamma] x_t + (\theta - 1)(k_1 v_g - 1) A_1 \sigma_{g,t}^2 \\
& \quad + (\theta - 1)(k_1 v_x - 1) A_2 \sigma_{x,t}^2 + k_1 (\theta - 1)(A_1 \sigma_1 + A_2 \sigma_2) z_{1,t+1} \\
& \quad + (\theta - 1) k_1 A_2 \sigma_3 z_{2,t+1} + [(\theta - 1) k_1 A_3 \varphi_e + \gamma \psi_x] \sigma_{x,t} e_{t+1} \\
& \quad - \gamma \sigma_{g,t} \eta_{t+1}, \qquad \text{(A6)}
\end{aligned}$$

where $c_2 = \theta \ln \delta - \frac{\theta}{\psi}\mu_c + (\theta-1)c_1$. The shock to the pricing kernel is

$$
\begin{aligned}
m_{t+1} - \mathbb{E}_t[m_{t+1}] &= k_1(\theta-1)(A_1\sigma_1 + A_2\sigma_2)z_{1,t+1} + (\theta-1)k_1A_2\sigma_3 z_{2,t+1} \\
&\quad +[(\theta-1)k_1A_3\varphi_e + \gamma\psi_x]\sigma_{x,t}e_{t+1} - \gamma\sigma_{g,t}\eta_{t+1}.
\end{aligned}
$$

Appendix C. Equity premium, Conditional Stock Market Variance, and Risk-Free Rate

Using the log linear approximation for the stock market return, we have

$$
\begin{aligned}
r_{m,t+1} &= \ln \frac{P_{m,t+1} + D_{t+1}}{P_{m,t}} = \ln \frac{P_{m,t+1} + D_{t+1}}{D_{t+1}} - \ln \frac{P_{m,t}}{D_t} + \ln \frac{D_{t+1}}{D_t} \\
&= k_{0,m} + k_{1,m} z_{m,t+1} - z_{m,t} + \Delta d_{t+1}, \qquad \text{(A7)}
\end{aligned}
$$

where $z_{m,t} = \ln \frac{P_{m,t}}{D_t}$, $\bar{z}_m = \mathbb{E}[z_{m,t}]$, $k_{1,m} = \dfrac{e^{\bar{z}_m}}{e^{\bar{z}_m}+1} < 1$, and $k_{0,m} = \ln(e^{\bar{z}_m}+1) - \dfrac{\bar{z}_m e^{\bar{z}_m}}{e^{\bar{z}_m}+1}$. The market portfolio's dividend growth process is

$$
\Delta d_{t+1} = \mu_d + \phi x_t + \pi_\eta \sigma_{g,t}\eta_{t+1} + \pi_e \sigma_{x,t} e_{t+1}.
$$

Suppose that the log stock market price-dividend ratio is a linear function of state variables

$$
z_{m,t} = A_{0,m} + A_{1,m}\sigma_{g,t}^2 + A_{2,m}\sigma_{x,t}^2 + A_{3,m}x_t, \qquad \text{(A8)}
$$

where $A_{0,m}, A_{1,m}, A_{2,m}, A_{3,m}$ are constants to be determined below. Combining Equation (A7)and Equation (A8) we have

$$
\begin{aligned}
r_{m,t+1} &= k_{0,m} + k_{1,m}z_{m,t+1} - z_{m,t} + \Delta d_{t+1} \\
&= c_3 + (k_{1,m}v_g - 1)A_{1,m}\sigma_{g,t}^2 + (k_{1,m}v_x - 1)A_{2,m}\sigma_{x,t}^2 + (k_{1,m}A_{3,m}\rho - A_{3,m} + \phi)x_t \\
&\quad +(k_{1,m}A_{1,m}\sigma_1 + k_{1,m}A_{2,m}\sigma_2)z_{1,t+1} + k_{1,m}A_{2,m}\sigma_3 z_{2,t+1} \\
&\quad +(k_{1,m}A_{3,m}\varphi_e + \pi_e)\sigma_{x,t}e_{t+1} + \pi_\eta\sigma_{g,t}\eta_{t+1}, \qquad \text{(A9)}
\end{aligned}
$$

where $c_3 = k_{0,m} + (k_{1,m}-1)A_{0,m} + k_{1,m}A_{1,m}\sigma_g^2(1-v_g) + k_{1,m}A_{2,m}\sigma_x^2(1-v_x) + \mu_d$.

Combining Equation (A6) and Equation (A9) we have

$$
\begin{aligned}
& m_{t+1} + r_{m,t+1} \\
= \;& \theta \ln \delta - \frac{\theta}{\psi}\Delta c_{t+1} + (\theta-1) r_{a,t+1} + r_{m,t+1} \\
= \;& c_2 + c_3 + [A_3(\theta-1)(\rho k_1 - 1) - \gamma + k_{1,m}A_{3,m}\rho - A_{3,m} + \phi] x_t \\
& + [(\theta-1)(k_1 v_g - 1)A_1 + (k_{1,m}v_g - 1)A_{1,m}]\sigma_{g,t}^2 \\
& + [(\theta-1)(k_1 v_x - 1)A_2 + (k_{1,m}v_x - 1)A_{2,m}]\sigma_{x,t}^2 \\
& + [k_1(\theta-1)(A_1\sigma_1 + A_2\sigma_2) + k_{1,m}(A_{1,m}\sigma_1 + A_{2,m}\sigma_2)] z_{1,t+1} \\
& + [(\theta-1)k_1 A_2 + k_{1,m}A_{2,m}]\sigma_3 z_{2,t+1} + (\pi_\eta - \gamma)\sigma_{g,t}\eta_{t+1} \\
& + [(\theta-1)k_1 A_3 \varphi_e + \gamma \psi_x + k_{1,m}A_{3,m}\varphi_e + \pi_e]\sigma_{x,t} e_{t+1}.
\end{aligned}
$$

Using the Euler equation $\mathbb{E}_t[M_{t+1}R_{m,t+1}] = 1$ and the fact that $\ln(\mathbb{E}[X]) = \mathbb{E}[\ln(X)] - \frac{1}{2}\mathrm{Var}[\ln(X)]$ for log normal distributed variable X, we have

$$
\begin{aligned}
A_3(\theta-1)(\rho k_1 - 1) - \gamma + k_{1,m}A_{3,m}\rho - A_{3,m} + \phi &= 0, \\
(\theta-1)(k_1 v_g - 1)A_1 + (k_{1,m}v_g - 1)A_{1,m} + \frac{1}{2}(\pi_\eta - \gamma)^2 &= 0, \\
(\theta-1)(k_1 v_x - 1)A_2 + (k_{1,m}v_x - 1)A_{2,m} & \\
+ \frac{1}{2}((\theta-1)k_1 A_3 \varphi_e + \gamma\psi_x + k_{1,m}A_{3,m}\varphi_e + \pi_e)^2 &= 0, \\
c_2 + c_3 + \frac{1}{2}[k_1(\theta-1)(A_1\sigma_1 + A_2\sigma_2) + k_{1,m}(A_{1,m}\sigma_1 + A_{2,m}\sigma_2)]^2 & \\
+ \frac{1}{2}[(\theta-1)k_1 A_2 + k_{1,m}A_{2,m}]^2 \sigma_3^2 &= 0,
\end{aligned}
$$

from which we have

$$
\begin{aligned}
A_{0,m} &= \frac{1}{1-k_{1,m}}\Big[c_2 + k_{0,m} + k_{1,m}A_{1,m}\sigma_g^2(1-v_g) + k_{1,m}A_{2,m}\sigma_x^2(1-v_x) + \mu_d + \\
&\qquad + \frac{1}{2}[k_1(\theta-1)(A_1\sigma_1 + A_2\sigma_2) + k_{1,m}(A_{1,m}\sigma_1 + A_{2,m}\sigma_2)]^2 \\
&\qquad + \frac{1}{2}[(\theta-1)k_1 A_2 + k_{1,m}A_{2,m}]^2\sigma_3^2\Big], \\
A_{1,m} &= \frac{(\gamma - \frac{1}{\psi})(1-\gamma) + (\pi_\eta - \gamma)^2}{2(1 - k_{1,m}v_g)}, \\
A_{2,m} &= \frac{1}{1 - k_{1,m}v_x}\big[(\theta-1)(k_1 v_x - 1)A_2 + \frac{1}{2}((\theta-1)k_1 A_3\varphi_e + \gamma\psi_x + k_{1,m}A_{3,m}\varphi_e + \pi_e)^2\big], \\
A_{3,m} &= \frac{\phi - \frac{1}{\psi}}{1 - k_{1,m}\rho}.
\end{aligned}
$$

From Equation (A9), we can derive the conditional stock market variance

$$
\sigma_{m,t}^2 = c_4 + (k_{1,m}A_{3,m}\varphi_e + \pi_e)^2\sigma_{x,t}^2 + \pi_\eta^2\sigma_{g,t}^2, \tag{A10}
$$

where $c_4 = k_{1,m}^2(A_{1,m}\sigma_1 + A_{2,m}\sigma_2)^2 + k_{1,m}^2 A_{2,m}^2 \sigma_3^2$. Using Equation (A10), we can substitute $\sigma_{g,t}^2$ out from Equation (A8) by $\sigma_{m,t}^2$:

$$\begin{aligned} z_{m,t} &= A_{0,m} + A_{1,m}\sigma_{g,t}^2 + A_{2,m}\sigma_{x,t}^2 + A_{3,m}x_t \\ &= A_{0,m} - \frac{A_{1,m}}{\pi_\eta^2}c_4 + a\sigma_{m,t}^2 + b\sigma_{x,t}^2 + A_{3,m}x_t, \end{aligned} \tag{A11}$$

where $a = \frac{A_{1,m}}{\pi_\eta^2}$ and $b = A_{2,m} - \frac{A_{1,m}}{\pi_\eta^2}(k_{1,m}A_{3,m}\varphi_e + \pi_e)^2$.

Using Equation (A6) and Equation (A9) we have

$$Cov_t[m_{t+1}, r_{m,t+1}] = c_5 - \gamma\pi_\eta\sigma_{g,t}^2 + [(\theta-1)k_1A_3\varphi_e + \gamma\psi_x](k_{1,m}A_{3,m}\varphi_e + \pi_e)\sigma_{x,t}^2.$$

where $c_5 = k_1k_{1,m}(\theta-1)(A_1\sigma_1 + A_2\sigma_2)(A_{1,m}\sigma_1 + A_{2,m}\sigma_2) + (\theta-1)k_1k_{1,m}A_{2,m}A_2\sigma_3^2$ By the Euler equations $\mathbb{E}_t[M_{t+1}R_{m,t+1}] = 1$ and $\mathbb{E}_t[M_{t+1}R_t^f] = 1$ we have

$$\begin{aligned} \mathbb{E}_t[r_{m,t+1} - r_t^f] &= -\frac{1}{2}\sigma_{m,t}^2 - Cov_t[m_{t+1}, r_{t+1}] \\ &= -c_5 - \frac{1}{2}\sigma_{m,t}^2 + \gamma\pi_\eta\sigma_{g,t}^2 \\ &\quad -[(\theta-1)k_1A_3\varphi_e + \gamma\psi_x](k_{1,m}A_{3,m}\varphi_e + \pi_e)\sigma_{x,t}^2. \end{aligned} \tag{A12}$$

From (A10) and (A12) we have

$$\mathbb{E}_t[r_{m,t+1} - r_t^f] = c_6 + \alpha\sigma_{m,t}^2 + \beta\sigma_{x,t}^2,$$

where

$$\begin{aligned} c_6 &= -c_5 - \frac{\gamma}{\pi_\eta}c_4, \\ \alpha &= -\frac{1}{2} + \frac{\gamma}{\pi_\eta}, \\ \beta &= -[(\theta-1)k_1A_3\varphi_e + \gamma\psi_x](k_{1,m}A_{3,m}\varphi_e + \pi_e) - \frac{\gamma}{\pi_\eta}(k_{1,m}A_{3,m}\varphi_e + \pi_e)^2. \end{aligned}$$

By the Euler equation $\mathbb{E}_t[M_{t+1}R_t^f] = 1$ we have

$$\begin{aligned} r_t^f &= -\mathbb{E}_t[m_{t+1}] - \frac{1}{2}\mathrm{Var}_t[m_{t+1}] \\ &= c_7 - [A_3(\theta-1)(\rho k_1 - 1) - \gamma]x_t + c\sigma_{g,t}^2 + d\sigma_{x,t}^2 \\ &= c_7 - \frac{cc_4}{\pi_\eta^2} + \frac{1}{\psi}x_t + \frac{c}{\pi_\eta^2}\sigma_{m,t}^2 + [d - \frac{c}{\pi_\eta^2}(k_{1,m}A_{3,m}\varphi_e + \pi_e)^2]\sigma_{x,t}^2, \end{aligned}$$

where

$$\begin{aligned} c_7 &= -c_2 - \frac{1}{2}k_1^2(\theta-1)^2[(A_1\sigma_1 + A_2\sigma_2)^2 + A_2^2\sigma_3^2], \\ c &= -[(\theta-1)(k_1 v_g - 1)A_1 + \frac{1}{2}\gamma^2], \\ d &= -[(\theta-1)(k_1 v_x - 1)A_2 + \frac{1}{2}((\theta-1)k_1 A_3\varphi_e + \gamma\psi_x)^2]. \end{aligned}$$

Appendix D. Stock Portfolio Returns

Using the log linear approximation for the return on portfolio p, we have

$$r_{p,t+1} = \ln\frac{P_{p,t+1} + D_{p,t+1}}{P_{p,t}} = k_{0,p} + k_{1,p}z_{t+1} - z_{p,t} + \Delta d_{p,t+1}, \tag{A13}$$

where $z_{p,t} = \ln\frac{P_{p,t}}{D_{p,t}}$, $\bar{z}_p = \mathbb{E}[z_{p,t}]$, $k_{1,p} = \dfrac{e^{\bar{z}_p}}{e^{\bar{z}_p}+1} < 1$, and $k_{0,p} = \ln(e^{\bar{z}_p}+1) - \dfrac{\bar{z}_p e^{\bar{z}_p}}{e^{\bar{z}_p}+1}$.

The portfolio's dividend growth process is

$$\Delta d_{p,t+1} = \mu_d + \phi_p x_t + \pi_{\eta,p}\sigma_{g,t}\eta_{t+1} + \pi_{e,p}\sigma_{x,t}e_{t+1} + \pi z_{p,t+1}.$$

We suppose that the log price-dividend ratio has the following form

$$z_{p,t} = A_{0,p} + A_{1,p}\sigma_{g,t}^2 + A_{2,p}\sigma_{x,t}^2 + A_{3,p}x_t, \tag{A14}$$

where $A_{0,p}, A_{1,p}, A_{2,p}, A_{3,p}$ are constants to be determined below.

Combining Equation (A13) and Equation (A14), we have

$$\begin{aligned} r_{p,t+1} = \; & c_{3,p} + (k_{1,p}v_g - 1)A_{1,p}\sigma_{g,t}^2 + (k_{1,p}v_x - 1)A_{2,p}\sigma_{x,t}^2 \\ & + (k_{1,p}A_{3,p}\rho - A_{3,p} + \phi_p)x_t + (k_{1,p}A_{1,p}\sigma_1 + k_{1,p}A_{2,p}\sigma_2)z_{1,t+1} \\ & + k_{1,p}A_{2,p}\sigma_3 z_{2,t+1} + \pi z_{3,t+1} + (k_{1,p}A_{3,p}\varphi_e + \pi_{e,p})\sigma_{x,t}e_{t+1} \\ & + \pi_{\eta,p}\sigma_{g,t}\eta_{t+1}, \end{aligned} \tag{A15}$$

where $c_{3,p} = k_{0,p} + (k_{1,p}-1)A_{0,p} + k_{1,p}A_{1,p}\sigma_g^2(1-v_g) + k_{1,p}A_{2,p}\sigma_x^2(1-v_x) + \mu_d$. The conditional variance of the portfolio return is

$$\sigma_{p,t}^2 = c_{4,p} + (k_{1,p}A_{3,p}\varphi_e + \pi_{e,p})^2\sigma_{x,t}^2 + \pi_{\eta,p}^2\sigma_{g,t}^2, \tag{A16}$$

where $c_{4,p} = k_{1,p}^2(A_{1,p}\sigma_1 + A_{2,p}\sigma_2)^2 + k_{1,p}^2A_{2,p}^2\sigma_3^2 + \pi^2$.

The conditional covariance of the portfolio return with the log pricing kernel is

$$Cov_t[m_{t+1}, r_{p,t+1}] = c_{5,p} - \gamma\pi_\eta\sigma_{g,t}^2 + [(\theta-1)k_1A_3\varphi_e + \gamma\psi_x](k_{1,p}A_{3,p}\varphi_e + \pi_{e,p})\sigma_{x,t}^2.$$

where $c_{5,p} = k_1k_{1,p}(\theta-1)(A_1\sigma_1 + A_2\sigma_2)(A_{1,p}\sigma_1 + A_{2,p}\sigma_2) + (\theta-1)k_1k_{1,p}A_{2,p}A_2\sigma_3^2$.

By the Euler equations $\mathbb{E}_t[M_{t+1}R_{p,t+1}] = 1$ and $\mathbb{E}_t[M_{t+1}R_t^f] = 1$ we have

$$\begin{aligned}
\mathbb{E}_t[r_{p,t+1} - r_t^f] &= -\frac{1}{2}\sigma_{p,t}^2 - Cov_t[m_{t+1}, r_{p,t+1}] \\
&= -c_{5,p} - \frac{1}{2}c_{4,p} - \frac{1}{2}(k_{1,p}A_{3,p}\varphi_e + \pi_{e,p})^2\sigma_{x,t}^2 + [\gamma\pi_{\eta,p} - \frac{1}{2}\pi_{\eta,p}^2]\sigma_{g,t}^2 \\
&\quad -[(\theta-1)k_1A_3\varphi_e + \gamma\psi_x](k_{1,p}A_{3,p}\varphi_e + \pi_{e,p})\sigma_{x,t}^2. \qquad \text{(A17)}
\end{aligned}$$

Substituting Equation (A16) into Equation (A17), we have

$$\mathbb{E}_t[r_{p,t+1} - r_t^f] = c_{6,p} + \alpha_p\sigma_{m,t}^2 + \beta_p\sigma_{x,t}^2,$$

where

$$\begin{aligned}
c_{6,p} &= -c_{5,p} - \frac{1}{2}c_{4,p} - \frac{\gamma\pi_{\eta,p} - \frac{1}{2}\pi_{\eta,p}^2}{\pi_\eta^2}c_4, \\
\alpha_p &= \frac{\gamma\pi_{\eta,p} - \frac{1}{2}\pi_{\eta,p}^2}{\pi_\eta^2}, \\
\beta_p &= -[(\theta-1)k_1A_3\varphi_e + \gamma\psi_x](k_{1,p}A_{3,p}\varphi_e + \pi_{e,p}) - \frac{\gamma\pi_{\eta,p} - \frac{1}{2}\pi_{\eta,p}^2}{\pi_\eta^2}(k_{1,m}A_{3,m}\varphi_e + \pi_e)^2 \\
&\quad -\frac{1}{2}(k_{1,p}A_{3,p}\varphi_e + \pi_{e,p})^2.
\end{aligned}$$

Combining Equation (A6) and Equation (A15), we have

$$\begin{aligned}
m_{t+1} + r_{p,t+1} &= \theta\ln\delta - \frac{\theta}{\psi}\Delta c_{t+1} + (\theta-1)r_{a,t+1} + r_{p,t+1} \\
&= c_2 + c_{3,p} + [A_3(\theta-1)(\rho k_1 - 1) - \gamma + k_{1,p}A_{3,p}\rho - A_{3,p} + \phi_p]x_t \\
&\quad +[(\theta-1)(k_1v_g - 1)A_1 + (k_{1,p}v_g - 1)A_{1,p}]\sigma_{g,t}^2 \\
&\quad +[(\theta-1)(k_1v_x - 1)A_2 + (k_{1,p}v_x - 1)A_{2,p}]\sigma_{x,t}^2 \\
&\quad +[k_1(\theta-1)(A_1\sigma_1 + A_2\sigma_2) + k_{1,p}(A_{1,p}\sigma_1 + A_{2,p}\sigma_2)]z_{1,t+1} \\
&\quad +[(\theta-1)k_1A_2 + k_{1,p}A_{2,p}]\sigma_3 z_{2,t+1} \\
&\quad +[(\theta-1)k_1A_3\varphi_e + \gamma\psi_x + k_{1,p}A_{3,p}\varphi_e + \pi_{e,p}]\sigma_{x,t}e_{t+1} \\
&\quad +(\pi_{\eta,p} - \gamma)\sigma_{g,t}\eta_{t+1} + \pi z_{p,t+1}.
\end{aligned}$$

Using the Euler equation $\mathbb{E}_t[M_{t+1}R_{p,t+1}] = 1$ and the fact that $\ln(\mathbb{E}[X]) = \mathbb{E}[\ln(X)] - \frac{1}{2}\text{Var}[\ln(X)]$

for log normal distributed variable X, we have

$$\begin{aligned}
A_3(\theta-1)(\rho k_1-1)-\gamma+k_{1,p}A_{3,p}\rho-A_{3,p}+\phi_p &= 0,\\
(\theta-1)(k_1v_g-1)A_1+(k_{1,p}v_g-1)A_{1,p}+\frac{1}{2}(\pi_{\eta,p}-\gamma)^2 &= 0,\\
(\theta-1)(k_1v_x-1)A_2+(k_{1,p}v_x-1)A_{2,p}&\\
+\frac{1}{2}((\theta-1)k_1A_3\varphi_e+\gamma\psi_x+k_{1,p}A_{3,p}\varphi_e+\pi_{e,p})^2 &= 0,\\
c_2+c_{3,p}+\frac{1}{2}[k_1(\theta-1)(A_1\sigma_1+A_2\sigma_2)+k_{1,p}(A_{1,p}\sigma_1+A_{2,p}\sigma_2)]^2&\\
+\frac{1}{2}[(\theta-1)k_1A_2+k_{1,p}A_{2,p}]^2\sigma_3^2+\frac{1}{2}\pi^2 &= 0,
\end{aligned}$$

from which we get

$$\begin{aligned}
A_{0,p} &= \frac{1}{1-k_{1,p}}\Big[c_2+k_{0,p}+k_{1,p}A_{1,p}\sigma_g^2(1-v_g)+k_{1,p}A_{2,p}\sigma_x^2(1-v_x)+\mu_d+\\
&\quad +\frac{1}{2}[k_1(\theta-1)(A_1\sigma_1+A_2\sigma_2)+k_{1,p}(A_{1,p}\sigma_1+A_{2,p}\sigma_2)]^2\\
&\quad +\frac{1}{2}[(\theta-1)k_1A_2+k_{1,p}A_{2,p}]^2\sigma_3^2+\frac{1}{2}\pi^2\Big]\\
A_{1,p} &= \frac{(\gamma-\frac{1}{\psi})(1-\gamma)+(\pi_{\eta,p}-\gamma)^2}{2(1-k_{1,p}v_g)},\\
A_{2,p} &= \frac{1}{1-k_{1,p}v_x}\big[(\theta-1)(k_1v_x-1)A_2+\frac{1}{2}((\theta-1)k_1A_3\varphi_e+\gamma\psi_x+k_{1,p}A_{3,p}\varphi_e+\pi_{e,p})^2\big],\\
A_{3,p} &= \frac{\phi_p-\frac{1}{\psi}}{1-k_{1,p}\rho}.
\end{aligned}$$

Appendix E. Long-Term Treasury Bonds

Using the log linear approximation, we have

$$\begin{aligned}
r_{b,t+1} &= \ln\frac{P_{b,t+1}+1}{P_{b,t}}=\ln(P_{b,t+1}+1)-\ln P_{b,t}\\
&= k_{0,b}+k_{1,b}z_{b,t+1}-z_{b,t}
\end{aligned}$$

where $z_{b,t}=\ln P_{b,t}$, $\bar{z}_b=\mathbb{E}[z_{b,t}]$, $k_{1,b}=\frac{e^{\bar{z}_b}}{e^{\bar{z}_b}+1}<1$, and $k_{0,b}=\ln(e^{\bar{z}_b}+1)-\frac{\bar{z}_p e^{\bar{z}_b}}{e^{\bar{z}_b}+1}$. Suppose that the log bond price has the following form:

$$z_{b,t}=A_{0,b}+A_{1,b}\sigma_{g,t}^2+A_{2,b}\sigma_{x,t}^2+A_{3,b}x_t, \tag{A18}$$

where $A_{0,b}, A_{1,b}, A_{2,b}, A_{3,b}$ are constants to be determined.

Using Equation A18, we rewrite the bond return as

$$
\begin{aligned}
r_{b,t+1} &= k_{0,b} + k_{1,b} z_{b,t+1} - z_{b,t} \\
&= k_{0,b} + k_{1,b} A_{0,b} + k_{1,b} A_{1,b} \sigma^2_{g,t+1} + k_{1,b} A_{2,b} \sigma^2_{x,t+1} + k_{1,b} A_{3,b} x_{t+1} - z_{b,t} \\
&= c_{3,b} + k_{1,b} A_{1,b} v_g \sigma^2_{g,t} + k_{1,b} A_{1,b} \sigma_1 z_{1,t+1} + k_{1,b} A_{2,b} v_x \sigma^2_{x,t} + k_{1,b} A_{2,b} \sigma_2 z_{1,t+1} \\
&\quad + k_{1,p} A_{2,p} \sigma_3 z_{2,t+1} + k_{1,p} A_{3,p} \rho x_t + k_{1,p} A_{3,p} \varphi_e \sigma_{x,t} e_{t+1} - A_{1,b} \sigma^2_{g,t} - A_{2,b} \sigma^2_{x,t} - A_{3,b} x_t \\
&= c_{3,b} + (k_{1,b} v_g - 1) A_{1,b} \sigma^2_{g,t} + (k_{1,b} v_x - 1) A_{2,b} \sigma^2_{x,t} + (k_{1,b} A_{3,b} \rho - A_{3,b}) x_t \\
&\quad + (k_{1,b} A_{1,b} \sigma_1 + k_{1,b} A_{2,b} \sigma_2) z_{1,t+1} \\
&\quad + k_{1,b} A_{2,b} \sigma_3 z_{2,t+1} + k_{1,b} A_{3,b} \varphi_e \sigma_{x,t} e_{t+1},
\end{aligned}
$$

where $c_{3,b} = k_{0,b} + (k_{1,b} - 1) A_{0,b} + k_{1,b} A_{1,b} \sigma^2_g (1 - v_g) + k_{1,b} A_{2,b} \sigma^2_x (1 - v_x)$. The conditional bond variance is

$$
\mathrm{Var}_t[r_{b,t+1}] = c_{4,b} + (k_{1,p} A_{3,b} \varphi_e)^2 \sigma^2_{x,t}, \tag{A19}
$$

where $c_{4,b} = k^2_{1,b} (A_{1,b} \sigma_1 + A_{2,b} \sigma_2)^2 + k^2_{1,b} A^2_{2,b} \sigma^2_3$.

The conditional covariance of the bond return with the pricing kernel is

$$
Cov_t[m_{t+1}, r_{b,t+1}] = c_{5,b} + [(\theta - 1) k_1 A_3 \varphi_e + \gamma \psi_x] k_{1,b} A_{3,b} \varphi_e \sigma^2_{x,t},
$$

where $c_{5,b} = k_1 k_{1,b} (\theta - 1)(A_1 \sigma_1 + A_2 \sigma_2)(A_{1,b} \sigma_1 + A_{2,b} \sigma_2) + (\theta - 1) k_1 k_{1,b} A_{2,b} A_2 \sigma^2_3$. Using the Euler equations for bond and the risk free rate, we have

$$
\begin{aligned}
\mathbb{E}_t[r_{b,t+1} - r^f_t] &= -\frac{1}{2} Var_t[r_{b,t+1}] - Cov_t[m_{t+1}, r_{b,t+1}] \\
&= -\frac{1}{2} c_{4,b} - \frac{1}{2} (k_{1,b} A_{3,b} \varphi_e)^2 \sigma^2_{x,t} \\
&\quad - c_{5,b} - [(\theta - 1) k_1 A_3 \varphi_e + \gamma \psi_x](k_{1,b} A_{3,b} \varphi_e) \sigma^2_{x,t} \\
&= -c_{5,b} - \frac{1}{2} c_{4,b} - \frac{1}{2} (k_{1,b} A_{3,b} \varphi_e)^2 \sigma^2_{x,t} \\
&\quad - [(\theta - 1) k_1 A_3 \varphi_e + \gamma \psi_x](k_{1,b} A_{3,b} \varphi_e) \sigma^2_{x,t} \\
&= c_{6,b} + \beta_b \sigma^2_{x,t},
\end{aligned}
$$

where

$$
\begin{aligned}
c_{6,b} &= -c_{5,b} - \frac{1}{2} c_{4,b}, \\
\beta_b &= -[(\theta - 1) k_1 A_3 \varphi_e + \gamma \psi_x](k_{1,b} A_{3,b} \varphi_e) - \frac{1}{2} (k_{1,b} A_{3,b} \varphi_e)^2.
\end{aligned}
$$

Combining the pricing kernel and the bond return, we have

$$\begin{aligned}
m_{t+1} + r_{b,t+1} &= \theta \ln \delta - \frac{\theta}{\psi}\Delta c_{t+1} + (\theta - 1) r_{a,t+1} + r_{b,t+1} \\
&= c_2 + c_{3,b} + [A_3(\theta-1)(\rho k_1 - 1) - \gamma + k_{1,b}A_{3,b}\rho - A_{3,b} \\
&\quad +[(\theta-1)(k_1 v_g - 1)A_1 + (k_{1,b}v_g - 1)A_{1,b}]\sigma_{g,t}^2 \\
&\quad +[(\theta-1)(k_1 v_x - 1)A_2 + (k_{1,b}v_x - 1)A_{2,b}]\sigma_{x,t}^2 \\
&\quad +[k_1(\theta-1)(A_1\sigma_1 + A_2\sigma_2) + k_{1,b}(A_{1,b}\sigma_1 + A_{2,b}\sigma_2)]z_{1,t+1} \\
&\quad +[(\theta-1)k_1 A_2 + k_{1,b}A_{2,b}]\sigma_3 z_{2,t+1} \\
&\quad +[(\theta-1)k_1 A_3 \varphi_e + \gamma\psi_x + k_{1,b}A_{3,b}\varphi_e]\sigma_{x,t}e_{t+1} - \gamma\sigma_{g,t}\eta_{t+1}.
\end{aligned}$$

Using the Euler equation $\mathbb{E}_t[M_{t+1}R_{b,t+1}] = 1$ and the fact that $\ln(\mathbb{E}[X]) = \mathbb{E}[\ln(X)] - \frac{1}{2}\mathrm{Var}[\ln(X)]$ for log normal distributed variable X, we have

$$\begin{aligned}
A_3(\theta-1)(\rho k_1 - 1) - \gamma + k_{1,b}A_{3,b}\rho - A_{3,b} &= 0, \\
(\theta-1)(k_1 v_g - 1)A_1 + (k_{1,b}v_g - 1)A_{1,b} + \frac{1}{2}\gamma^2 &= 0, \\
(\theta-1)(k_1 v_x - 1)A_2 + (k_{1,b}v_x - 1)A_{2,b} + \frac{1}{2}((\theta-1)k_1 A_3\varphi_e + \gamma\psi_x + k_{1,b}A_{3,b}\varphi_e)^2 &= 0, \\
c_2 + c_{3,b} + \frac{1}{2}[k_1(\theta-1)(A_1\sigma_1 + A_2\sigma_2) + k_{1,b}(A_{1,b}\sigma_1 + A_{2,b}\sigma_2)]^2 & \\
+\frac{1}{2}[(\theta-1)k_1 A_2 + k_{1,b}A_{2,b}]^2\sigma_3^2 &= 0.
\end{aligned}$$

The solutions to the equations above are

$$\begin{aligned}
A_{0,b} &= \frac{1}{1-k_{1,b}}\Big[c_2 + k_{0,b} + k_{1,b}A_{1,b}\sigma_g^2(1-v_g) + k_{1,b}A_{2,b}\sigma_x^2(1-v_x) + \\
&\qquad +\frac{1}{2}[k_1(\theta-1)(A_1\sigma_1 + A_2\sigma_2) + k_{1,b}(A_{1,b}\sigma_1 + A_{2,b}\sigma_2)]^2 \\
&\qquad +\frac{1}{2}[(\theta-1)k_1 A_2 + k_{1,b}A_{2,b}]^2\sigma_3^2\Big], \\
A_{1,b} &= \frac{(\theta-1)(k_1 v_g - 1)A_1 + \frac{1}{2}\gamma^2}{1 - k_{1,b}v_g} = \frac{(\gamma - \frac{1}{\psi})(1-\gamma) + \gamma^2}{2(1-k_{1,b}v_g)}, \\
A_{2,b} &= \frac{1}{1-k_{1,b}v_x}\big[(\theta-1)(k_1 v_x - 1)A_2 + \frac{1}{2}((\theta-1)k_1 A_3\varphi_e + \gamma\psi_x + k_{1,b}A_{3,b}\varphi_e)^2\big] \\
&= \frac{1}{1-k_{1,b}v_x}\big[\frac{1-\theta}{2\theta}(\theta k_1 A_3\varphi_e + (\gamma-1)\psi_x)^2 + \frac{1}{2}((\theta-1)k_1 A_3\varphi_e + \gamma\psi_x + k_{1,b}A_{3,b}\varphi_e)^2\big], \\
A_{3,b} &= \frac{A_3(\theta-1)(\rho k_1 - 1) - \gamma}{1 - k_{1,b}\rho} = \frac{-\frac{1}{\psi}}{1-k_{1,b}\rho}.
\end{aligned}$$

Appendix B. Data Appendix

Appendix A. Daily and Monthly IST Factors

Accounting data are from Compustat Annual Fundamental files. Stock prices, stock returns, and shares outstanding of common stocks traded on NYSE, AMEX, and Nasdaq are from CRSP. Daily excess stock market returns and daily risk-free rates are from Ken French at Dartmouth College. We exclude Utility firms (SIC 4900-4949), financial firms (SIC 6000-6799), and firms that have negative or missing book values of equities. We follow Hou, Xue, and Zhang (2015) to construct book values of equities using Compustat annual data files. It equals (a) stockholders' book equities, plus (b) balance sheet deferred taxes and investment tax credit, and minus (c) book values of preferred stocks. We use the Compustat item *SEQ* as a measure of stockholders' book equities. If *SEQ* is not available, we use the sum of the book value of common equities *CEQ* and the par value of preferred stocks *PSTK*. If the sum of *CEQ* and *PSTK* is not available, we use the difference between the book value of total assets *AT* and the book value of total liabilities *LT*. Balance sheet deferred taxes and investment tax credit are measured by *TXDITC*. The book value of preferred stocks is redemption value *PSTKRV*, liquidation value *PSTKL*, or par value *PSTK* of preferred stocks, depending on the availability.

Papanikolaou (2011) argues that HML is closely related to IST shocks, and we obtain daily and monthly HML from Kenneth French at Dartmouth College. Following Papanikolaou (2011), we construct the daily investment-minus-consumption factor, IMC, as the difference in daily returns between the value-weighted portfolio of investment-goods producers and the value-weighted portfolio of consumption-goods producers. We thank Dimitris Papanikolaou at Kellogg School of Management of Northwestern University for providing the classification of investment-goods producers and consumption-goods producers used in Papanikolaou (2011).

Following Kogan and Papanikolaou (2013), we construct six additional proxies of IST shocks using portfolios formed by Tobin's Q, the investment-capital ratio (IK) the price-earnings ratio (PE), loadings on excess stock market returns (β_{MKT}), idiosyncratic volatility (IMCIV), and loadings on IMC (β_{IMC}). As in Kogan and Papanikolaou (2013), we exclude investment-goods producers from our sample. For portfolios that require accounting data, i.e., Tobin's Q, IK, and PE, we rank stocks using year t annual accounting data, and rebalance portfolios at the end of June, year $t+1$. For portfolios that require only stock return data,i.e., β_{MKT}, IMCIV, and β_{IMC}, we rank stocks using data available at the end of year t, and rebalance portfolios at the end of year t. We construct daily and monthly IST shock proxies using double sorts. We first sort stocks into two groups using the median NYSE market capitalization as the breakpoint. Within each size portfolio, we sort stocks into three portfolios by a firm characteristic, e.g., IK, using the NYSE $30th$ and $70th$ IK percentiles as the breakpoints. We construct the daily or monthly value-weighted portfolio returns and calculate the return difference between low and high IK, for example, portfolios. The IK factor is the average of the long-short portfolio returns of small and big stocks. We construct the other factors in the same way. Table B1 provides more details of IST factors.

We also construct five-by-five portfolios using each of the aforementioned six firm characteristics. We first sort all stocks into five size portfolios using the NYSE 20th, 40th, 60th, and 80th market capitalization percentiles as the breakpoints. Within each size portfolio, we sort stocks into five portfolios by a firm characteristic, e.g., IK, using the NYSE 20th, 40th, 60th, and 80th IK percentiles as breakpoints. We calculate monthly both equal-weighted and value-weighted returns for each portfolio. Monthly equal-weighed and value-weighted returns on the five-by-five portfolios formed on BM are obtained from Kenneth French at Dartmouth College.

Appendix B. Implied Cost of Capital

We construct five ICC measures. Analyst consensus (mean) earnings forecast data are from the I/B/E/S unadjusted summary file. Accounting data are from Compustat. The end-of-month stock price and shares outstanding data are from CRSP. The 10-year treasury yield and GDP growth rate are from the Federal Reserve Bank of St. Louis. We use WRDS's iclink to link I/B/E/S data and CRSP data and then merge them with Compustat data using the CRSP/Compustat Merged linking table. We impose following data requirements. First, firms must have common stocks traded on NYSE, AMEX, or NASDAQ. Second, a stock must have a valid SIC code that can be used to classify the stock into one of Fama-French 48 industries. The requirement allows us to construct the median payout ratio for each industry-size group. We use the historical SIC code from Compustat (Compustat item *SICH*). If *SICH* is unavailable, we use the SIC code from CRSP (CRSP item **SICCD**). Third, stocks must have non-missing CRSP price (CRSP item **PRC**) and shares outstanding (CRSP item **SHROUT**) that are used to calculate market capitalization. Fourth, we exclude observations with negative or missing I/B/E/S earnings forecast for the current fiscal year FE_{t+1} (I/B/E/S FPI=1). Fifth, I/B/E/S publishes monthly consensus forecasts on the third Thursday of each month. To ensure that earnings forecasts are made based on publicly available accounting information, we impose a minimum reporting lag of three months. Last, because of the low coverage in I/B/E/S data files in early years, our sample begins from January 1981.

Appendix B.1. Pastor, Sinha, and Swaminathan (2008) Measure

Pastor et al. (2008) define ICC as:

$$\mathrm{P}_t = \sum_{k=1}^{15} \frac{\mathrm{FE}_{t+k}(1-\mathrm{b}_{t+k})}{(1+\mathrm{r}_e)^k} + \frac{\mathrm{FE}_{t+16}}{\mathrm{r}_e(1+\mathrm{r}_e)^{15}},$$

where r_e is the implied cost of capital, b_{t+k} is the expected year $t+k$ plowback rate, FE_{t+k} is the analyst forecast of the $t+k$ year earnings per share, and P_t is the current month price per share. We calculate the implied cost of capital from the finite-horizon free cash flow valuation model using a three-stage procedure.

Stage 1: Earnings growth rate

We define earnings growth rate as

$$\mathrm{g}_{t+i} = \mathrm{g}_{t+i-1} \times \exp\left[\frac{\log(\frac{\mathrm{g}}{\mathrm{g}_{LT}})}{T-1}\right] \quad \text{for i = 4 to 16.}$$

We use I/B/E/S (FPI=0) item LTG as a measure of analyst long-term growth rate forecasts, g_{LT}. If LTG is missing, we use $(\mathrm{FE}_{t+2}/\mathrm{FE}_{t+1}) - 1$ instead. If consensus forecasts for year t+1 or t+2 are also missing, we use $(\mathrm{FE}_{t+1}/\mathrm{FE}_{t+0}) - 1$ as an alternative measure. If the analyst long-term growth rate forecast measure has a value below 2% (above 100%), we replace it with 2% (100%). We then measure earnings growth rate between year $t+4$ and year $t+16$ by assuming that firm earnings growth rates mean-revert to the steady-state growth rate by year $t+17$. We assume that the steady-state growth rate, g, equals the long-run nominal GDP growth rate, which is the expanding rolling average of the sum of annual real GDP growth rate and implicit price deflator growth rate. Our GDP data begins in 1930. The real GDP growth rate and implicit price deflator data are from the Federal Reserve Bank of St. Louis.

Stage 2: Expected Earnings Per Share

We calculate the expected earnings per share using the formula:

$$\mathrm{FE}_{t+i} = \mathrm{FE}_{t+i-1} \times (1 + g_{t+k}) \quad \text{for i = 4 to 16.}$$

We obtain FE_{t+2} from I/B/E/S. If it is missing, we assume that it equals $\mathrm{FE}_{t+1} \times (1 + g_{LT})$. After obtaining FE_{t+2}, we remove firms with missing or negative FE_{t+1} and FE_{t+2}. The forecast of three-year-ahead earnings is FE_{t+3}=$\mathrm{FE}_{t+2} \times (1 + g_{LT})$. We then use FE_{t+3} and the corresponding growth rate obtained from stage 1 to measure FE_{t+i} recursively.

Stage 3: Plowback rate

The plowback rate forecast for year $t+1$ and $t+2$ can be constructed using the most recent accounting data. We construct the forecast in the years after $t+2$ recursively using the formula:

$$b_{t+k} = b_{t+k-1} - \frac{b_{t+2} - b}{14} = b_{t+k-1} - \frac{b_{t+2} - \frac{g}{r_e}}{14} \quad \text{for k = 3 to 15.}$$

Plowback rate (PB_t) equals one minus net payout ratio NP_t. We measure NP_t in three ways. First, we define $\mathrm{NP}_t = \dfrac{\mathrm{D}_t + \mathrm{REP}_t - \mathrm{NE}_t}{\mathrm{NI}_t}$, where D_t is the common dividend (Compustat item *DVC*), REP_t is the share repurchase (Compustat item *PRSTKC*), NE_t is the net equity issuance (Compustat item *SSTK*), and NI_t is net income (Compustat item *IB*). Second, if IB is missing or has a negative value, we use the one-year ahead consensus earnings forecast made at the end of previous calendar year, FE_{t-1}, to measure NI_t or $\mathrm{NP}_t = \dfrac{\mathrm{D}_t + \mathrm{REP}_t - \mathrm{NE}_t}{\mathrm{FE}_{t-1}}$. Last, if NP_t is still unavailable or if the NP_t from the first two steps has a value above 1 or below -0.5, we use the median NP_t of the corresponding industry-size portfolio instead. To compute the median NP_t, we first sort firms into Fama-French 48 industries. Within each industry, we use firm market capitalization at the end of previous calendar year to sort firms equally into three portfolios. If the resulting NP_t from each industry-size portfolio has a value above 1 or below -0.5, we replace it with 1 or -0.5, respectively. Hence, the minimum (maximum) plowback rate is 0 (1.5). If a firm still does not have valid plowback rate after these procedures, we remove it from the sample.

We estimate the plowback rates for year $t+3$ to year $t+16$ recursively by assuming that the plowback rate mean-reverts linearly to a steady-state value at year t+17. The steady-state plowback rate is $b = g/r_e$, where the steady state growth rate g is obtained from stage 1 and r_e is the implied cost of capital that we are interested in. Therefore, the expanded free cash flow valuation model is

$$\begin{aligned} P_t &= \frac{\mathrm{FE}_{t+1}(1-\mathrm{PB}_t)}{(1+r_e)^1} + \frac{\mathrm{FE}_{t+2}(1-\mathrm{PB}_t)}{(1+r_e)^2} \\ &\quad + \sum_{k=3}^{15} \frac{\mathrm{FE}_{t+k}\left(1-\left(b_{t+k-1} - \frac{\mathrm{PB}_t - \frac{g}{r_e}}{14}\right)\right)}{(1+r_e)^k} + \frac{\mathrm{FE}_{t+16}}{r_e(1+r_e)^{15}}, \end{aligned}$$

and we can solve for r_e numerically.

Appendix B.2. Gebhardt et al. (2001) Measure

Gebhardt et al. (2001) use the following equation to solve for ICC:

$$\mathrm{P}_t = \mathrm{B}_t + \sum_{k=1}^{11} \frac{(\mathrm{FROE}_{t+k} - r_e)B_{t+k-1}}{(1+r_e)^k} + \frac{(\mathrm{FROE}_{t+12} - r_e)B_{t+11}}{r_e(1+r_e)^{11}}.$$

P_t is the stock price from CRSP monthly files. We use shares outstanding data from I/B/E/S to calculate the book equity value per share, B_t. If the shares outstanding value from I/B/E/S is missing, we construct an interpolated value using CRSP data: $d * \mathbf{SHROUT}_{m-1} + (1-d) * \mathbf{SHROUT}_m$, where d is the ratio of the number of days between previous month-end and current I/B/E/S statistical period to the total number of trading days in month m, and **SHROUT** is the number of monthly-end shares outstanding from CRSP. r_e is the implied cost of capital. FROE is the expected return on equity (ROE).

For years $t+1$ to $t+2$, $\mathrm{FROE}_{t+k} = \frac{\mathrm{FE}_{t+k}}{\mathrm{B}_{t+k-1}}$. We obtain FE_{t+1} and FE_{t+2} from I/B/E/S. For year $t+3$, we use the analyst long-term earnings growth rate forecast (LTG) from I/B/E/S (FPI=0) to calculate $\mathrm{FE}_{t+3} = \mathrm{FE}_{t+2} \times (1+\mathrm{LTG})$. If LTG is missing, we replace it with $(\mathrm{FE}_{t+2}/\mathrm{FE}_{t+1}) - 1$. If consensus forecasts in year $t+2$ is also missing, we use $(\mathrm{FE}_{t+1}/\mathrm{FE}_{t+0}) - 1$. We require non-negative and non-missing I/B/E/S consensus earnings forecasts. After year $t+3$, we estimate FROE by assuming that it linearly mean-reverts to the industry median ROE by year $t+11$.

$\text{ROE}_t = \frac{\text{E}_t}{\text{B}_t}$, where E_t is the actual EPS obtained from I/B/E/S unadjusted summary files. As in Gebhardt et al. (2001), we exclude firms with negative EPS when estimating the industry median ROE because profitable firms provide more accurate estimation over the industry's long-term equilibrium rate of return on equity than do unprofitable firms. We require a minimum of five years and a maximum of ten years rolling window to compute the industry median ROE, ROE_{int}. Hence, $\text{FROE}_{t+3+j} = \text{FROE}_{t+3} \times (1 + g_{int})^j$ where $g_{int} = (\frac{\text{ROE}_{int}}{\text{FROE}_{t+3}})^{\frac{1}{9}} - 1$.

The book equity value per share is obtained from clean surplus accounting $\text{B}_{t+j} = \text{B}_{t+j-1} + \text{FE}_{t+j} - \text{D}_{t+j}$ for $j = 1$ to 11. B_t is the book equity value per share measured as the ratio of most recent book equity value to the number of shares outstanding. FE_{t+k} is the year t forecast of EPS in year $t+k$. D_{t+k} is the year t forecast of dividend per shares in year $t+k$; it is the product of the most recent dividend payout ratio with FE_{t+k}. We use Compustat data to construct the dividend payout ratio as $\frac{DVC}{IB}$. For firms with negative or missing IB, we use $\frac{DVC}{(0.06 * AT)}$ as an alternative dividend payout ratio. Note that the historical average return on assets is 0.06 in the US data. We require firms to have a valid payout ratio. For firms with a payout ratio below zero or above one, we replace it with zero or one, respectively.

Following Gebhardt et al. (2001), we impose following data requirements. First, firms must have non-missing book value of equity. The definition of book equity is the same as the one used to construct IST factors in the preceding subsection. We remove firms with a negative book value of equity. Second, firms must have non-missing net income (IB). For firms with negative IB, we replace it with $0.06 \times AT$ if possible. Third, firms must have non-missing dividends (DVC) and long-term debt ($DLTT$). Last, we exclude firms with missing or negative earnings forecasts for the following fiscal year (I/B/E/S FPI=2).

Appendix B.3. Easton (2004) Measure

Easton (2004) uses the following equation to estimate the implied cost of capital:

$$\text{P}_t = \frac{\text{FE}_{t+2} + r_e \times \text{D}_{t+1} - \text{FE}_{t+1}}{r_e^2}$$

P_t is the stock price. r_e is the implied cost of capital. FE_{t+1} and FE_{t+2} are consensus analyst earnings forecasts for the current and next fiscal years. D_{t+1} is the expected dividend per share, and is calculated as the product of FE_{t+1} with the most recent payout ratio. The definition and criteria of the payout ratio is the same as that used in Gebhardt et al. (2001). We require firms with non-missing book value of equity, net income (IB), and dividends (DVC). Firms with a negative book value of equity are excluded. We also exclude firms with missing or negative earnings forecasts for the next fiscal year (I/B/E/S FPI=2).

Appendix B.4. Ohlson and Juettner-Nauroth (2005) Measure

Ohlson and Juettner-Nauroth (2005) construct the implied cost of capital using the following equation:

$$r_e = A + \sqrt{A^2 + \frac{\mathrm{FE}_{t+1}}{\mathrm{P}_t} \times (g - (\gamma - 1))}.$$

r_e is the implied cost of capital. $A = 0.5\Big[(\gamma-1) + \frac{\mathrm{D}_{t+1}}{\mathrm{P}_t}\Big]$. D_{t+1} is the expected dividend per share, and is calculated as the product of FE_{t+1} with the most recent payout ratio. FE_{t+1} and FE_{t+2} are consensus analyst earnings forecasts for the current and next fiscal years. P_t is the stock price. $\gamma - 1$ is set to 10-year Treasury yield minus 3%. $g = 0.5\Big[\Big(\frac{\mathrm{FE}_{t+2} - \mathrm{FE}_{t+1}}{\mathrm{FE}_{t+1}}\Big) + \mathrm{LTG}_t\Big]$. As in Gode and Mohanram (2003), we use the average of near-term and long-term growth rates to estimate g. The definition and criteria of the payout ratio is the same as that used in Gebhardt et al. (2001). We require firms with non-missing book value of equity, net income (*IB*), and dividends (*DVC*). Firms with negative book value of equity are excluded. We also exclude firms with missing or negative earnings forecasts for the next fiscal year (I/B/E/S FPI=2).

Appendix B.5. Gordon and Gordon (1997) Measure

The Gordon and Gordon (1997) measure is a special case of the finite-horizon Gordon growth model. They use the following equation to calculate the implied cost of capital:

$$\mathrm{P}_t = \frac{\mathrm{FE}_{t+1}}{r_e}.$$

r_e is the implied cost of capital. FE_{t+1} is consensus analysts earnings forecasts for the current fiscal year. Firms with missing or negative earnings forecasts for the next fiscal year (I/B/E/S FPI=2) are excluded.

Table B1 IST Factors

Variable	Definition
IK	IK is the investment-capital ratio. We measure investment as the difference between capital expenditure and PPE sales or *CAPX-SPPE*. We measure capital using lagged PPE, *PPEGT*. *SPPE* is set to zero when missing.
Tobin's Q	Tobin's Q is the market value of assets divided by their replacement costs. The market value is the difference between (MKCAP12+*DLTT*+*PSTKRV*) and (*INVT*+*TXDITC*). The replacement cost is the book value of PPE, *PPEGT*. We set *TXDITC* to zero when missing. MKCAP12 is the market capitalization, the product of the share price **PRC** with shares outstanding **SHROUT**, at the calendar year end.
PE	PE is the ratio of a firm's market value (MKCAP12+*DLTT*+*PSTKRV*-*TXDB*) to the sum of operating income, *IB*, and interest expenses, *XINT*. MKCAP12 is the market capitalization, the product of the share price **PRC** with shares outstanding **SHROUT**, at the end of the calendar year.
IMC	IMC is the return difference between the value-weighted portfolio of investment-goods producers and the value-weighted portfolio of consumption-goods producers. We use June-end market capitalization for weights.
β_{MKT}	We estimate market beta by regressing daily excess stock returns on a constant and concurrent daily excess stock market returns using a one-year rolling window. We include only stocks that have at least 200 valid daily returns in a calendar year.
β_{IMC}	We estimate IMC beta by regressing daily excess stock returns on a constant and concurrent daily IMC using a one-year rolling window. We include only stocks that have at least 200 valid daily returns in a calendar year.
IMCIV	IMCIV is the square root of the sum of squared residuals from the regression of daily excess stock returns on a constant, daily value-weighted IMC, and daily excess market returns. We include only stocks that have at least 200 valid daily returns in a calendar year.

Note: The table describes the variables that we use to construct the IST shock proxies. Unless otherwise indicated, variables in italic and bold are from Compustat and CRSP, respectively.

Table B2 Summary Statistics for monthly ICCs

ICC	PSS	GLS	Easton	OJ	Gordon
Mean	0.107	0.091	0.116	0.118	0.071
Std Dev	0.021	0.021	0.029	0.029	0.020
Kurtosis	3.310	1.715	3.914	1.424	2.227
Skew	1.654	1.301	1.914	1.407	1.346
PSS	1				
GLS	0.969	1			
Easton	0.957	0.955	1		
OJ/GM	0.894	0.921	0.963	1	
Gordon	0.968	0.987	0.928	0.871	1

Note: The table reports the summary statistics of ICC measures.